GP01: Radioactivity and Statistics Lab Report

Alex McInerny, shil5991

26th January 2023

Abstract

This experiment investigates the random nature of radioactive decay with the use of basic statistics and a Geiger-Müller tube. Key results were measurements of background activity ((0.66 ± 0.07) s^{-1}), the activity of a KCl pellet ((3.19 ± 0.09) s^{-1}), the mass self absorption of said KCl pellet ($(0.011 \pm 2 \times 10^{-3})$ cm^{2}mg^{-1}), and it's penetration depth ($(0.064 \pm 8 \times 10^{-3})$ cm). The counter efficiency of the GM tube used was calculated to be approximately 40%. All results were consistent with the true values within one standard error where they were available.

Introduction

This experiment was carried out in order to deduce the effects of background radiation (BR); explore elementary statistics and data analysis techniques; investigate the natural radioactivity of potassium chloride (KCl) and improve ability to work with new technology including Geiger-Müller (GM) tubes and computers to record and analyse data. We also aimed to work out the mass self absorption coefficient of a KCl tablet, and it's penetration depth.

The physics discussed in this report has wide implications. The statistics in GP01 is used commonly throughout ones Physics degree, along with having far wider applications in many careers and everyday life. Lots of research requires understanding of BR to get accurate results that lack systematic errors, along with BR having important implications in areas such as treatment of illness, research into medical physics, and even smoke detectors. Understanding radioactive sources and their dangerous effects is a crucial step in minimising exposure to radiation for everyone.

From here onward, this report contains the theoretical background required to understand the report, followed by experimental methods used to obtain results, a summary of said results including analysis explaining their significance, and a conclusion detailing final findings and comparison with previously obtained results and theoretical predictions.

1 Method

1.1 Theory required for analysis

1.1.1 Background Radiation

Contributions to background radiation include; cosmic radiation (particles created/liberated by collisions envolving high energy protons from space and molcules in the atmosphere), decay of unstable elements present in rocks, other materials on earth and in our bodies, and man-made sources e.g., nuclear power and weapons testing [1]. Compared with radioactive sources (such as KCl, which itself is very weakly reactive), background radiation has very low activity, however still a noticeable effect which will need to be taken into account when doing radiological studies.

1.1.2 Radioactivity of Potassium-40

Potassium-40 is a radioactive element. It is present in very small levels (having an abundance of 0.0017% [2]) and has a half life of 1.248×10^9 years [2]. When K-40 decays, the two main possibilities and their respective probabilities are:

$$89\%:\ {}^{40}\mathrm{K} \rightarrow {}^{40}\mathrm{Ca} + \beta^- + \overline{\nu} \tag{1}$$

$$11\%:\ {}^{40}\mathrm{K} + e^- \rightarrow {}^{40}\mathrm{Ar} + \nu + \gamma \tag{2}$$

The beta particle released in Eq.(1) is of energy 1.311 MeV, and the gamma ray released in Eq.(2) is of energy 1.46 MeV. Note also that ^{40}Ca and ^{40}Ar are stable so don't decay further.

Beta particles penetrate further than alpha ones, however they're far more ionising (interact with matter more due to slower speed and larger charge) than gamma rays, therefore gamma rays have the farthest penetrating distance [3].

Due to the presence of ^{40}K, Potassium Chloride is very slightly radioactive. 1 kg of KCl emits 16350 Bq of radiation [4]. Assuming one decay emits 1.46 MeV of energy, the samples we used were approximately 1g, and we were in contact with the source for 5 hours, we received a radiation dose of around 0.917 nSv. This is practically negligible, since it's less than both; the dose received from cosmic rays in the same period (it's estimated you receive 37.6 nSv/h, meaning we received a dose of 188 nSv just during the experiment [5]); and the amount of radiation emitted by Potassium-40 inside your body (there's around 2.5 grams of potassium in the average human [6]).

1.1.3 Statistical Analysis

Poisson distributions were used throughout to model the random nature of background radiation and the decay of radioactive isotopes in KCl. For a fixed time interval where μ counts are expected, the probability of counting x counts is

$$P(x) = \frac{\mu^x}{x!}e^{-\mu} \tag{3}$$

Note for a poisson distribution, $\sigma^2 = \mu$ (4)

Further, the Poisson distribution can be modelled as the limit of a binomial distribution, $B(x) = \binom{n}{x}p^x(1-p)^{n-x}$, where $p \gg 1$ and $N \gg x$. This is shown as [7]

$$\lim_{n\to\infty} B(x) = \lim_{n\to\infty}\binom{n}{x}p^x(1-p)^{n-x} = \frac{\mu^x}{x!}e^{-\mu}, \text{ with } xp = \mu \tag{5}$$

Also, we shall compare the poisson and normal distributions when plotting histograms of counts vs. time. The normal distribution is defined as

$$N(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{6}$$

where μ is the mean, σ is the standard deviation and x is the independent variable.

Note also for large μ ($\gtrsim 200$), a possion distribution can be modelled by a discrete normal distribution with $\sigma^2 = \mu$. i.e., $P_{\text{approx}}(x) = N(x)\Delta$ (7) where $\Delta \in \mathbb{Z}$ is the width of the bin, and x is it's centre.

1.1.4 Self-absorption of beta radiation

Beta radiation is emitted from everywhere in the KCl sample, not just the surface. A large amount of this radiation will ionise a particle inside of the source itself, so it's not detected. This is called self-absorption.

For an activity per unit mass of α and density ρ, the radiation emitted by an element of volume dV is $\alpha\rho dV$ (8). Only some of this radiation is directed towards the detector, so considering this, and calling the solid angle of the detector $\Omega = A/d^2$, the radiation emitted towards the detector is $\alpha\rho\frac{\Omega}{4\pi}dV$ (9). Due to the shape of the pellets, with the detector being perpendicular to the flat surface of the pellets, the Beer-Lambert law can be used to calculate the fraction of radiation from the pellet from a specific point inside the pellet that reaches the detector. i.e., $\alpha\rho\frac{\Omega}{4\pi}e^{-z\mu}dV$ (10). Here, μ is the self-absorption constant, equal to the inverse of the penetration depth.

By integrating over the whole volume of the pellet (modelled as a cylinder of thickness L and surface are A), and then differentiating with respect to the mass, the count rate of the detector per unit mass (in Hz/kg) is

$$\frac{dR}{dM} = \epsilon\alpha\frac{\Omega}{4\pi}\frac{1}{L\mu}(1-e^{-L\mu}) \tag{11}$$

where ϵ is the efficiency of the detector and M is the sample mass.

1.2 Experimental Methods

The apparatus used within the experiment is shown in Fig.1. It consists of a GM tube and processing components connected to a power supply along with a computer to record, save and analyse the data. In the GM tube, there is a chamber containing a low pressure mixture of neon and halogen gasses along with two electrodes with a large potential difference applied across them. When ionising radiation enters the tube, some of the gas inside the tube is ionised. This means there are 'ion pairs' (one positive ion and one free electron) in the gas. The electrons are accelerated towards the positively charged wire in the tube (the anode) by the electric field produced due to the potential difference. This creates another ion pair, but also a current in the wire. The magnitude of this current is measured and the count rate of a sample is determined by a processor.

The GM tube was connected to a computer, which used the `LabVIEW Graphical Interface` software. This recorded the number of counts over a specified duration of time. `Python` was used to analyse the data. Libraries including `matplotlib.pyplot, numpy, pandas, scipy.stats and scipy.optimise` were used to analyse the data in `jupyter notebook`.

Figure 1: GM tube setup

1. We firstly measured the BR count. Having removed all sources from the vicinity which may affect the reading, we recorded data for 10 minutes at a 5 second sample rate. This data was collated and a histogram produced.
2. We placed the thickest tablet in the holder at the bottom of the GM tube, and propped it up to ensure it was as close to the end of the tube as possible. Once more using a sample rate of 5 s, we recorded data for 30 minutes.

3. We selected 6 different thicknesses of KCl tablet and measured the masses (using scales), m, thicknesses, L and diameters, D (both using vernier callipers). For both measurement sets, we took three readings and calculated the standard error via $\alpha = \sigma/\sqrt{N}$. We calculated the densities of the pellets $\rho = m/V = 4m/\pi D^2 L$. For each tablet, we measured the count rate for 10 minutes. We plotted count per unit mass against thickness and used a non-linear least squares fit to work out K and μ (with Eq.(11), you get $\frac{dR}{dM} = K\frac{1}{L\mu}(1-e^{-L\mu})$ (12)). We repeated the curve-fits twice more; once having deducted only background, and once the 11% gamma ray emission as well.
4. We calculated the penetration depth, equal to the inverse of the self absorption constant i.e., $\delta_p = 1/\mu$.
5. Lastly, we estimated the counter efficiency. To do this, we used the relation $K = \epsilon\alpha\frac{\Omega}{4\pi}$ (13) from Eq.(11) and Eq.(12), so measuring A and d of the GM tube (ensuring not to touch it due to the high potential difference applied to the metal tube), we estimated it's efficiency since we'd already found K.

2 Results and Analysis

2.1 Fitting Background Radiation with a Poisson distribution

As shown by Fig.2 there is a clear agreement between the superimposed poisson distribution (Eq.(3)) and the histogram of the raw data. You can see that theoretical poisson distribution is within the error bars of the histogram of counts for all counts values except on $counts = 2$, however slight deviations are expected since the probability of being within one σ is only 68%. Furthermore, small deviations on the graph can be considered to be a product of the random nature of radioactive decay, and the small sample rate and low activity of background sources.

The data for the BR (analysed using code in Appx.(A)) has $\mu = 0.66 \pm 0.07$ s^{-1} and $\sigma = 0.82$. For a poisson distribution, Eq.(4) is a required condition. We see that σ^2 is well within the errors of mu, $\sigma^2 = 0.6642 \approx \mu$ and thus conclude that background does follow a poisson distribution.

A rough estimate of BR dose (in 5 hours) is $E\mu T/5m = (4.74\times10^{-7}\pm5\times10^{-9})$ Sv/h. The script believes this should be in the range of $(1.14\times10^{-7}$ to $9.13\times10^{-7})$ Sv/h [8], so this result seems reasonable.

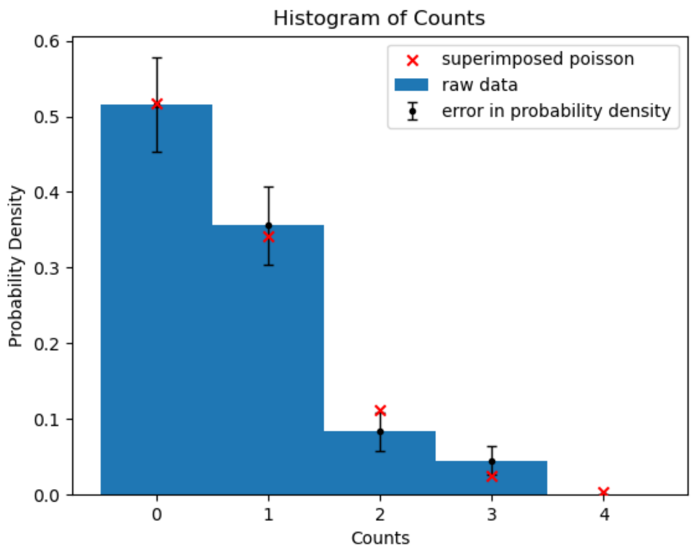


Figure 2: Histogram of counts for BR superimposed with poisson distribution

2.2 Fitting Radiation Emitted by KCl to Poisson and Normal distributions

The data collected has means and standard deviations $\mu = 3.19 \pm 0.09$ s^{-1} and $\sigma = 1.79$. Once again using Eq.(4), we see $\sigma^2 = 3.215 \approx \mu$ (they agree within the errors of μ). This indicates that a poisson distribution should be suitable.

By considering Fig.(3), you can quite easily see that a poisson distribution is a more suitable fit to model the radioactive decay of KCl than a normal distribution. All points except one are closer to the peak of the histogram for a poisson distribution. Furthermore, most of the poisson points are within the probability density error bars, however few of the points for the normal distribution fall within that error.

The poisson distribution is the better fit mainly because a poisson distribution tends towards a normal distribution only in the case of large μ (see Eq.(7)'s condition). Since our mean is only 3.19, the poisson is not effectively a normal, and since radioactive decay follows a poisson distribution, for samples with low activity/sample rates (and by extension low means), a normal distribution is not a good distribution model.

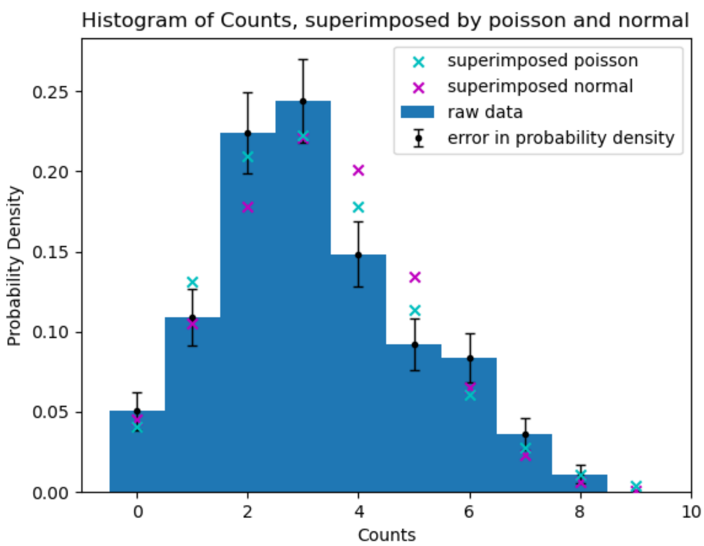
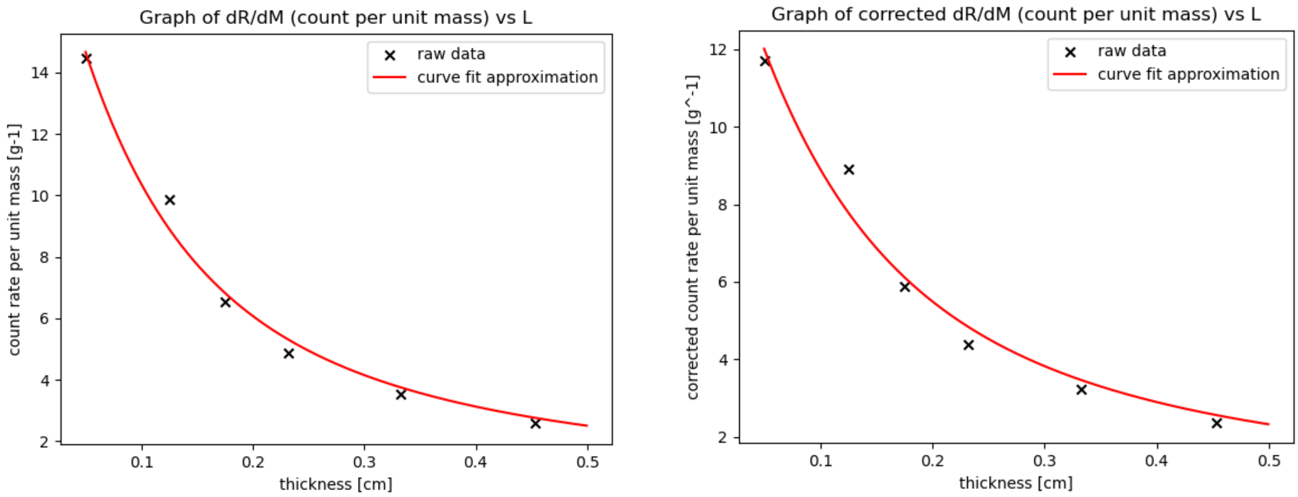


Figure 3: Histogram of counts for KCl superimposed with poisson and normal distributions

2.3 Deriving the Self-absorption Coefficient of KCl

Taking the thickest tablet (therefore reducing errors), the (averaged over three measurements) thickness is (4.53 ± 0.006) mm, diameter is (14.67 ± 0.007) mm and mass is (1.13 ± 0.003) g. This leads to a density of (1.48 ± 0.08) g cm^{-3}.



(a) Uncorrected

(b) Corrected for background and gamma emission

Figure 4: Plots of count rate per unit mass against thickness

Using a very simple model (Fig.(4a)) (assuming no background radiation nor gamma ray emission), using the curve-fit (code included in Appx.(B)) we concluded values of the self-absorption constant and pre-factor were $\mu = (18 \pm 2)$ cm^{-1} and $K = (0.07 \pm 0.01)$ g^{-1}.

Slightly modifying the model, when background was removed however the effects of gamma weren't taken into account, $\mu' = (14 \pm 2)$ cm^{-1} and $K' = (0.07 \pm 0.02)$ g^{-1}. As expected the graph for this has a peak value of corrected dR/dM of around 10 g^{-1} and a very similar looking shape.

With the fully modified model (Fig.(4b)), talking into account both background radiation and the emission of 11% gamma rays, the same quantities were $\mu'' = (15 \pm 2)$ cm^{-1} and K'' still $= (0.07 \pm 0.02)$ g^{-1}. Note how although the shape of the graphs is very similar, there is a higher count rate per unit mass in the uncorrected graph than the corrected one. This is because for a low activity sample like KCl, background has a large effect and the 11% gamma is less profound, so including the gamma radiation is outweighed by the removal of data for background.

We could then work out the mass self absorption constant, μ/ρ, in each case. First, consider the uncorrected data. This provides a mass absorption constant of $\mu/\rho = (0.012 \pm 1 \times 10^{-3})$ cm^{2}mg^{-1}. The slightly corrected data gives $(\mu/\rho)' = (9.5 \times 10^{-3} \pm 7 \times 10^{-4})$ cm^{2}mg^{-1}. The fully corrected data gives $(\mu/\rho)'' = (0.011 \pm 2 \times 10^{-3})$ cm^{2}mg^{-1}.

The literature value for the mass self absorption is $(\mu/\rho)_{\text{true}} = 0.0115$ cm^{2}mg^{-1} [8]. Interestingly, the simple model gave a value to high, and the fully corrected model was slightly too low, though both results agree to the literature value within their errors. We believe the corrected value, μ'', was a bit too low because we assumed that the GM tube detected no gamma radiation at all. I believe this is not the case, and it can detect some of the high energy photons emitted, so removing all 11% of the gamma was too much. If we were to take it in the middle (say around 5% of radiation were gamma rays which weren't detected), results would be closer to the true value. More substantially, if the true efficiency of the GM tube was known, we could also take that into account in all readings of the number of counts per second, far improving the results.

2.4 Calculating the penetration depth in the KCl tablet

The penetration depth of the tablet was then calculated via $\delta_p = 1/\mu''$ to be $(0.064 \pm 8 \times 10^{-3})$ cm.

2.5 Calculating the counter efficiency

Taking rough measurements of the GM tube, we find $A \approx 1.76 \times 10^{-4}$ m^2 and $d \approx 3.62 \times 10^{-4}$ m which means $\Omega = 0.134$. Calculating the efficiency, $\epsilon = 4\pi K/\alpha\Omega \approx 40\%$. This is far from perfect and may have caused substantial discrepancies through the previous two sections.

3 Conclusion

In this experiment, we measured the count rate for background radiation (found to be around $\mu = 0.66 \pm 0.07$ s^{-1}, which is within the accepted range), the activity of a sample of KCl pellet (measured as $\mu = 3.19 \pm 0.09$ s^{-1}), and the mass absorption constant and penetration depth of KCl tablets (which were measured as $(\mu/\rho) = (0.011 \pm 2 \times 10^{-3})$ cm^{2}mg^{-1} and $\delta_p = (0.064 \pm 8 \times 10^{-3})$ cm respectively). Both of these seem reasonable, and agree to the true values within their uncertainties where the true values are available, however the errors on our measurement for the mass absorption constant were quite large ($\pm 18\%$). The counter efficiency for the GM tube we used was approximated as 40%.

Due to the random nature of radioactive decay, it's impossible to get perfectly accurate results (hence reasonably large errors). All errors in the experiment would have been reduced by using a GM tube able which a higher counter efficiency that was able to measure both counts of beta and gamma radiation to a high accuracy, by using a more radioactive sample which a higher activity (although this was not feasible in our working environment due to the danger of working with radioactive materials), and by taking readings for a considerably longer duration of time with a longer sample rate.

References

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Appendix A: Example of code used in section 2.1 and section 2.2

```
import pandas as pd
from scipy.stats import poisson
import numpy as np
import matplotlib.pyplot as plt

df = pd.read_csv("backgroundRadiationData1.csv")
#print(df)
counts = df["counts"].values
time = df["time/s"].values

N = len(counts)
mu = np.mean(counts)
print(f"mean = {mu}")
median = np.median(counts)
print(f"median = {median}")
sigma = np.std(counts)
print(f"standard deviation = {sigma}")
alpha = sigma/np.sqrt(N)
print(f"standard error = {alpha}")

x = np.arange(0,5,1)
theory = poisson.pmf(x,mu)
bins = [-0.5,0.5,1.5,2.5,3.5,4.5]

height, binEdges, patches = plt.hist(counts, bins = bins, density = True, zorder = 1)
heightErrors = np.sqrt(height/N)
plt.errorbar(x = x, y = height, yerr = heightErrors, linewidth = 1, fmt=".", capsize = 3,
    color = "k", zorder = 2)
plt.title('Histogram of Counts')
plt.scatter(x,theory,color = "r", marker = "x", zorder = 3)
plt.xlabel('Counts')
plt.ylabel('Probability Density')

plt.legend(["superimposed poisson","raw data","error in probability density"])
plt.savefig('BR_Counts_vs_time')
```

Appendix B: Example of code used in section 2.3

```
import pandas as pd
from scipy.stats import poisson
from scipy.stats import norm
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
pd.options.display.max_rows = 999

mu = [muF,muG,muH,muI,muJ,muK]
mass = [1.13,.85,.58,.43,.29,.14]
RdivideM = [0,0,0,0,0,0]
for i in range(0,len(mu)):
    RdivideM[i] = mu[i]/mass[i]
thickness = [.453,.333,.232,.175,.125,0.05]

df = pd.read_csv("BackgroundRadiationWithCapsuleNoTablet.csv")
counts = df["counts"].values
background = np.mean(counts)
print(background)
mu_corrected1 = [0,0,0,0,0,0]
```

```
R_correcteddivideM1 = [0,0,0,0,0,0]

for i in range(0,len(mu)):
    mu_corrected1[i] = mu[i]/0.89 - background
    R_correcteddivideM1[i] = mu_corrected1[i]/mass[i]

parameters, covariant_matrix = curve_fit(equation, thickness, R_correcteddivideM1, p0=(0.7,1))
k_corrected_fitted1, u_corrected_fitted1 = parameters
k_corrected_error1, u_corrected_error1 = covariant_matrix.diagonal()**(1/2)

plt.scatter(thickness,R_correcteddivideM1, marker = "x", color = "k")
plt.plot(x_arr, equation(x_arr, k_corrected_fitted1, u_corrected_fitted1), 'r')
plt.title('Graph of corrected dR/dM (count per unit mass) vs L')
plt.xlabel('thickness [cm]')
plt.ylabel('corrected count rate per unit mass [g^-1]')
plt.legend(["raw data","curve fit approximation"])
plt.savefig('corrected_dmdr')
```