OP11: Using Lloyd's mirror to investigate double slit interference patterns and determine the wavelength of a monochromatic light source

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Abstract

This experiment investigates interference fringes under varying conditions and determines the wavelength of a monochromatic source using the Lloyd's mirror double slit setup. Key results: the equation $\Delta x = \lambda D/d$ is empirically confirmed; it's verified that there is a $\phi = \pi$ phase change upon reflection, white light is composed of a spectrum of visible light, superimposing waves form a beats pattern, and diffraction of light occurs about an edge; the wavelength of light can be relatively accurately found using the setup detailed in the report and moire fringe interference patterns (we found $\lambda = (534 \pm 12)$ nm for a LED of $\lambda_{\text{true}} = 532$ nm).

Introduction

This experiment was carried out to deduce the effects on interference fringes when changing conditions in the double slit experiment; investigate phenomenon of the wave nature of light including reflection, diffraction and superposition; determine the wavelength of light using Lloyd's mirror; and improve skills when working with optical equipment to obtain quantitative and qualitative results.

The double slit experiment was a pivotal experiment in optics developed in 1803 to provide evidence for the wave nature of light [1]. It revolutionised wave optics and has wide applications in not only physics research (e.g., building sensitive optical equipment such as telescopes/microscopes) and real life (e.g., radio transmission), but it also opened the doors to the discovery of much of modern physics, including electromagnetic waves and quantum mechanics. Lloyd's mirror (first described in 1834 [2])is a newer version of Young's two slit experiment, allowing for continuous and easier changing of the slit width by reflecting monochromatic light at a small angle from a plane mirror, effectively creating a virtual source which interferes with the direct one, creating the same interference pattern.

From here onward, this report contains the theoretical background required to understand the report, followed by experimental methods used to obtain results, a summary of said results including analysis explaining their significance, and a conclusion detailing final findings and comparison with previously obtained results and theoretical predictions.

1 Theory

1.1 Lloyd's Mirror

The fringe separation, Δx, of a two adjacent peaks in a double slit interference pattern is [3]

$$\Delta x = \frac{\lambda D}{d} \tag{1}$$

where λ is the wavelength of the incident, coherent light, D is the distance between the double slit and the plane of observation and d is the slit separation. This comes from the fact the nth bright fringe of generic two slit interference is seen at $x_n = n\lambda D/d$. For a Lloyd's mirror two slit interference pattern, it's $x_n = (n - \frac{1}{2})\frac{\lambda D}{d}$ due to a phase change upon reflection. Both have the same fringe separation, Δx.

The spacial frequency, f, (i.e., the number of fringes per unit length) is given by the relation

$$f = \frac{1}{\Delta x} = \frac{d}{\lambda D} \tag{2}$$

In the experiments original formulation by Young in 1803, the conventional double slit was used (two parallel, equal, translucent lines separated by some distance in an impenetrable material). Lloyd's mirror removes the need for having a large selection of double slits each with different separations by creating sources using reflection.

Consider S to be a real source (in our case coming from the virtual aperture) of diverging light. This source strikes a mirror at some small angle meaning some light (with the perpendicular distance between the plane of the mirror and the source being h) reflects from the mirror onto the observation plane (seemingly coming from a virtual source, S'), and some light shines directly from the source onto the plane. This is therefore equivalent to a double slit with $d = 2h$. Note Lloyd's mirror ensures that the light reaching the plane is coherent, since all light comes through the same single slit. [2]

1.2 Phase change upon reflection

Reflection at a boundary introduces a phase change of $\phi = \pi$. This is a consequence of conservation of energy and momentum. As a wave ($y_{\text{incident}}(x,t) = A\cos(\omega t - k_1 x)$) travels from a less to more dense medium ($k_1 < k_2$), the reflected wave has reflection coefficient is $r = A'/A < 0$ and equation $y_{\text{reflected}}(x,t) = A'\cos(\omega t + k_1 x) = |A'|\cos(\omega t + k_1 x + \pi)$. Taking the limit as $\rho_2 \propto k_2^2 \to \infty$ (pure reflection, $y_{\text{transmitted}} = 0$), $r = -1$ and $A' = -A$.

1.3 White light

White light is composed of roughly equal intensities of many different wavelengths of visible light. When refracted through a material whose refractive index is frequency dependant (e.g., glass), dispersion can be seen, as all wavelengths refract at different angles and are individually visible. Higher frequencies are refracted less. Dispersion is also seen in the double slit experiment since wavelength is linearly related to fringe separation. White light can be produced by passing UV rays through a phosphor coating.

1.4 Diffraction

When a system of waves pass through a narrow aperture or around an edge, the waves spread out in the phenomenon of diffraction. This is explained via Huygens Principle.[4]

1.5 Beats

When two waves ($y_i = \sin\omega_i t, \quad i \in \{1,2\}$) superimpose, the resultant is $y_{\text{resultant}} = y_1 + y_2 = 2\sin\frac{\omega_1+\omega_2}{2}t\cos\frac{\omega_1-\omega_2}{2}t$. This looks like a harmonic wave ($\sin\frac{\omega_1+\omega_2}{2}t$) modulated inside an envelope ($\cos\frac{\omega_1-\omega_2}{2}t$). [5] The number of peaks of the modulated wave in the envelope wave is $p = \langle\lambda\rangle/\Delta\lambda$ for two similar wavelengths (i.e., two close emission lines of the mercury spectrum).

2 Experimental Methods

2.1 Experimental Setup

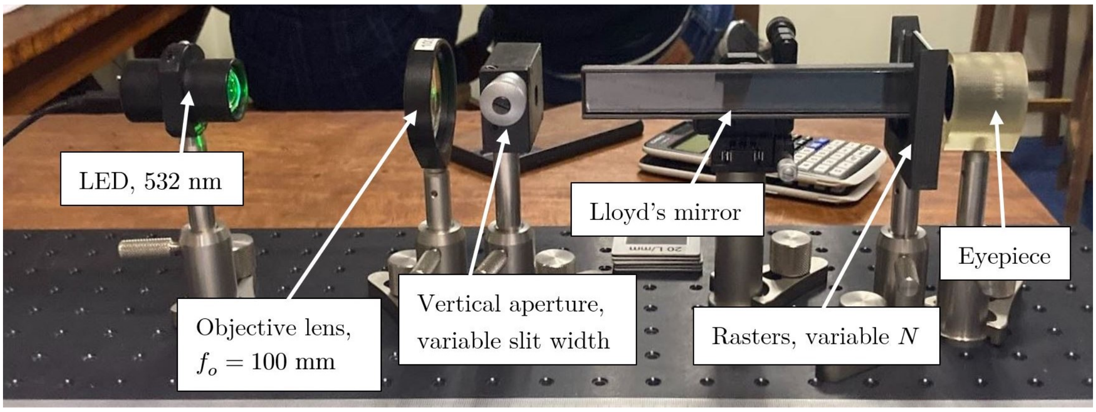


Figure 1: Labeled experimental setup diagram

The setup used for both of the subsequent sections is detailed in Fig.(1). There was a light source (an LED with a green filter, $\lambda = 532$ nm, FWHM = 3 nm) on the left, producing effectively monochromatic rays. An objective lens focused the light onto a vertical aperture (having ensured that the light was focused onto the flat side of the slit), where it was then passed into Lloyd's mirror. A raster holder was placed after the mirror, with an eyepiece behind that. When setting up the equipment, accuracy in the calibration was vital. Firstly, a line of holes in the optical breadboard was chosen and all clamps and holders were placed on it. All of the ordered optical components were roughly added as described above and then vertically aligned using an optical pin, such that the centres of the optical instruments were at the same level. It was also ensured that they were all perpendicular to the path of light. Using calibration paper, the objective lens was moved until the LED was focused on the aperture plane. Next, the aperture was opened slightly and the eyepiece was removed in order to align the mirror. It was firstly ensured it was flat with respect to the table (adjusted using the knobs on the bottom of the mirror). Secondly, the mirror was made parallel to the direction of light propagation by looking down on the setup from the top, and it was then moved left or right until two lines were visible when looking down the mirror. The second step was repeated until the positioning of the mirror was satisfactory. Once aligned, the eyepiece was clamped as close to the edge of the mirror as possible. [6]

2.2 Initial Observations

After the apparatus was operational, series of initial observations were made and in each case the interference pattern was examined: [6]

1. The slit width was varied (using the micrometer on the vertical aperture) with everything else unchanged.
2. The slit separation was varied (using the micrometer on Lloyd's mirror) with everything else unchanged.
3. The first fringe was examined at small slit separation and adequate slit width.
4. The source was swapped form the green LED to a white light source and the dispersion of light was examined.
5. The source was swapped again to a mercury lamp producing two spectral lines with close wavelengths. The relative difference in wavelengths was equal to number of resolvable fringes inside of one envelope ($\lambda/\Delta\lambda = p$).
6. The eyepiece was slowly moved from a position close to the mirrors edge to far away with everything else unchanged and the right-most fringe was examined.

2.3 Determine the wavelength of light

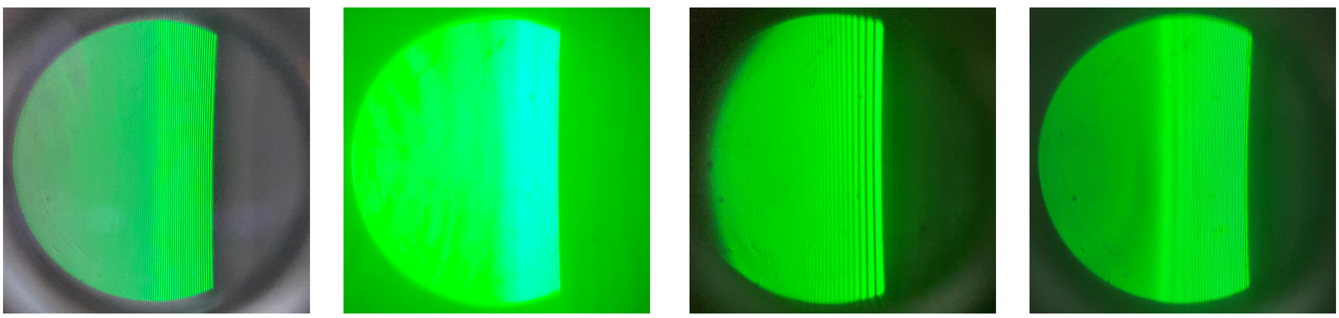
The wavelength of light was determined by measuring the fringe separation as a function of the mirror displacement. The moire fringe technique was used instead of counting the number of individual fringes. A raster (with a known number of lines per mm) was placed in just after the mirror and the eyepiece clamped as close as possible after that, but far enough such that it was focused on the lines of the raster (as to avoid diffraction, ensure the observation plane is a constant distance D away from the single slit, and to ensure the raster lines and fringes were in the same plane). The horizontal displacement of the mirror was then altered until the spacial frequencies of the raster and fringes is approximately equal, when a beat pattern/moire fringe was seen, at which point the micrometer reading was recorded. This was repeated five times for nine different rasters and the readings were averaged.

Due to the Lloyd's mirror setup, Eq.(2) was modified such that

$$\langle s \rangle = -\frac{\lambda D}{2} N + s_0 \tag{3}$$

where $\langle s \rangle$ is the average micrometer reading, N is the number of lines per mm on the raster and s_0 is an arbitrary zero error on the micrometer reading. A graph of s against n was then plotted and a curve-fit was completed. D was measured using a ruler, and the wavelength was then calculated. `Python` libraries including `matplotlib.pyplot`, `numpy`, `pandas` and `scipy.optimise` were used to analyse the data in `jupyter notebook`. [6]

3 Results and Analysis



(a) Small aperture slit width (b) Large aperture slit width (c) Small slit separation (d) Large slit separation

Figure 2: 2a and 2b are images of the interference pattern for different aperture slit widths at constant slit separation, and 2c and 2d are images of the interference pattern for different slit separations at constant aperture slit width.

Fig.(2a) and Fig.(2b) (relating to Sec.(2.2) Itm.(1)) shows as the aperture slit width increases, more light passes through the system, so the observed intensity increases. Note there were no noticeable changes to the fringe separation.

Fig.(2c) and Fig.(2d) (relating to Sec.(2.2) Itm.(2)) shows as the slit separation, d, increases, the fringe separation, Δx, decreases as expected in Eq.(1). This qualitatively agrees with Eq.(1). Note there was no change to the intensity of the interference pattern.

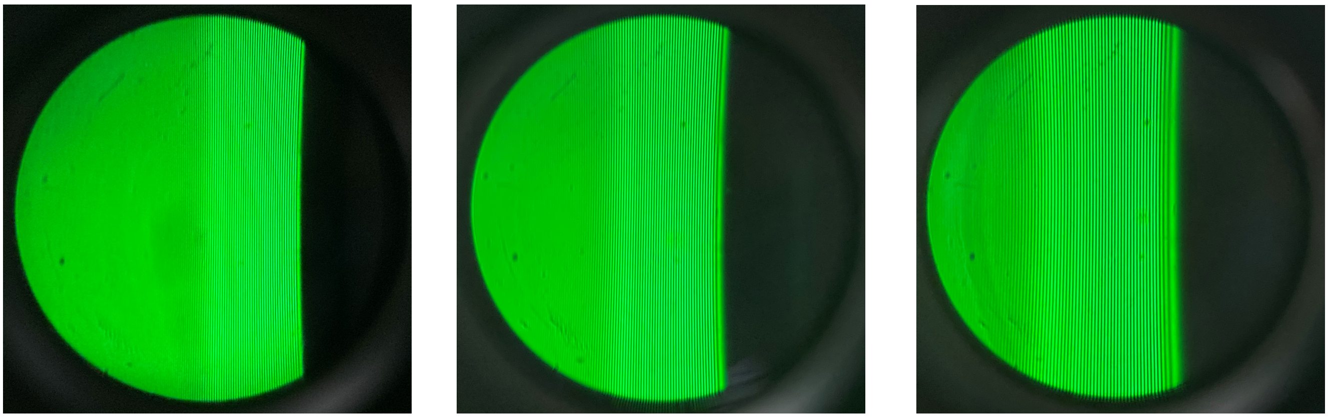
(a) Zoomed in on phase relation (b) Zoomed in on dispersion effect (c) Zoomed in on envelope effect

Figure 3: 3a is a zoomed image of the phase relation of the 0th order maximum, 3b is an image of the interference pattern of white light exhibiting dispersion, and 3c is an image of interference pattern for superimposed individual mercury spectral lines

Fig.(3a) (relating to Sec.(2.2) Itm.(3)) shows the phase relationship of the Lloyd's mirror interference pattern. Note that in each image only half of the interference pattern is shown since the other half is blocked by the mirror. With a phase of $\phi = 0$, it would be expected that the 0th order maximum (in the centre) would be half thickness, since half of it was cut off by the mirror. It's only at a phase shift of $\phi = \pi$ that the central maximum (shifted by $\frac{1}{2}\Delta x$ would be fully visible, and the exact same width as the 1st maximum. Since the only difference between Lloyd's and Young's double slit experiment is light reflecting off a mirror in Lloyd's, we can conclude that reflection at a boundary does cause a phase shift of $\phi = \pi$.

Fig.(3b) (relating to Sec.(2.2) Itm.(4)) shows the dispersion of white light. Since red light has larger $\lambda \approx 700$ nm, by Eq.(1), it has larger Δx and appears further left. The converse is true for blue light, so it appears on the right. In the middle, notice the light is still white. This is because the wavelengths are too close to, so they don't spread out enough and still appear merged into white.

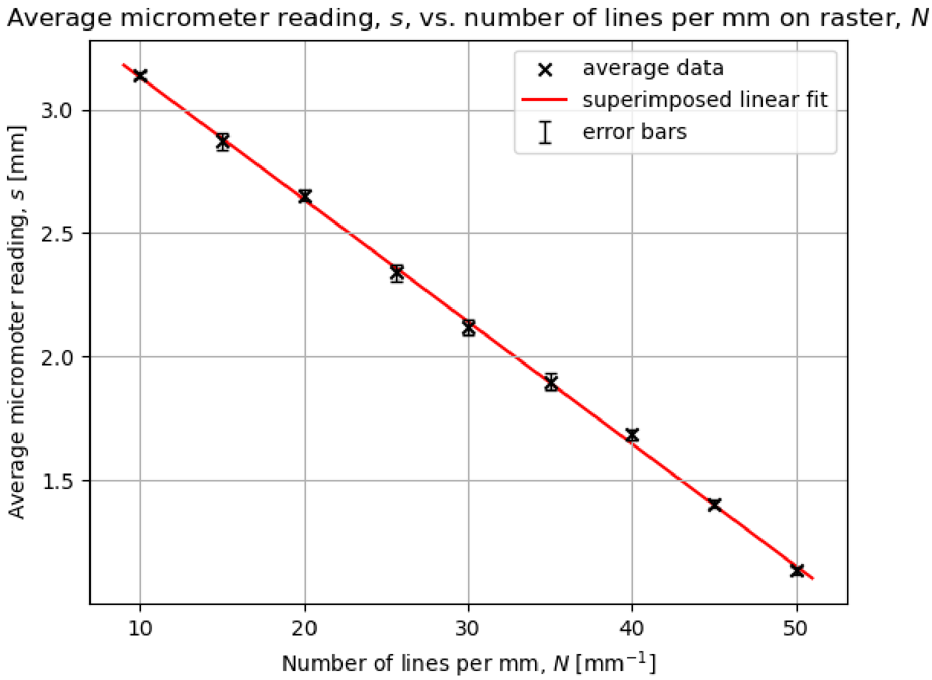
Fig.(3c) (relating to Sec.(2.2) Itm.(5) shows superposition of two spectral lines with close wavelengths forming an envelope and beast pattern (i.e., fringes within fringes). There were $\sim$16 sharp fringes between two fuzzy peaks. Considering the mercury spectrum, this corresponds to the emission wavelengths of 546.07 and 576.97 nm, as expected since both wavelengths correspond to green light, and the mercury lamp had a green filter.



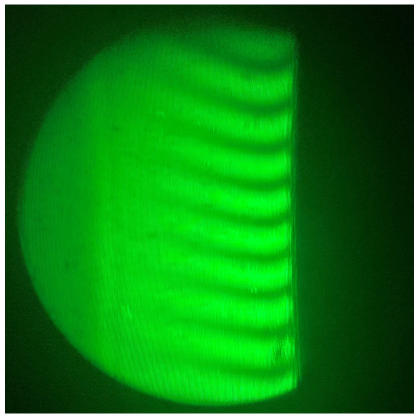
(a) Eyepiece close to mirror's edge (b) Eyepiece in-between positions (c) Eyepiece far from mirror's edge

Figure 4: Images of interference pattern for different positions of the eyepiece at constant aperture slit width and slit separation

Fig.(4) (relating to Sec.(2.2) Itm.(6)) shows how diffraction occurs at the edge of the mirror. For small eyepiece distances, $s = D - D_0$ (where D_0 is the eyepiece distance from the slit when it's in contact with the edge of the mirror), (Fig.(4a)), there is no blurring i.e., very little diffraction on the rightmost fringe. For large eyepiece distances (Fig.4c)) the right-most fringe is very blurred due do diffraction of the wave about the edge of the mirror (and the 1st maximum also has some discernible blurring). When close, the wavefront hasn't spread out much so observed diffraction is minimal. When further away, the wavefront has spread out a lot, so you see the effect of diffraction more.



(a) Average data superimposed with linear regression line of best fit



(b) Moire fringes on turning point

Figure 5: 5a is a graph of average micrometer reading against number of lines per mm, and 5b is the moire fringes on their turning point

The procedure as detailed in Sec.(2.3) was carried out. Fig.(5b) shows the interference pattern containing moire fringes at which the micrometer reading was recorded, when the moire fringes changed direction. The data collected tabulated in Appx.(A) and the distance between the slit and observation plane was naively measured to be $D = (185 \pm 2)$ mm. Fig.(5a) shows a graph of average micrometer reading , $\langle s \rangle$, against the number of lines per mm of the raster, N superimposed with the a line of best fit ($R^2 = 0.9992$) (code included in Appx.(B)). From Eq.(3), wavelength of light is calculated from the graph using $\lambda = -2m/D$ where $m = (0.0494 \pm 0.0005)$ mm^2 is the gradient. This provides $\lambda = (534 \pm 12)$ nm. This easily agrees with the true value of $\lambda_{\text{true}} = 532$ nm within it's errors. Said error was calculated by adding the percentages error of D and m (due to the rasters not being exactly perpendicular to the rays, the effective lines per mm was increased. It was assumed an angular deviation of 10 degs leading to $\delta N/N = \delta m/m = \theta^2/2 \approx 0.02$) in quadrature (the error in the curvefit was $\sim$20 times smaller than that from the angular deviation, therefore was effectively negligible).

4 Conclusion

In this experiment using Lloyd's mirror qualitative findings were initially recorded, where we empirically validated Eq.(1) for two slit interference patterns, confirmed that a phase changed of $\phi = \pi$ occurs upon reflection, verified that white light is made from the visible spectrum (while further confirming Eq.(1)), authenticated that superposition of similar wavelengths leads to a beats interference pattern, and shows how diffraction occurs at an edge, and is more prominent when viewed from afar. All findings agreed with theoretical predictions where applicable.

Furthermore, we used Lloyd's mirror to measure the wavelength of a source of light (found to be $\lambda = (534 \pm 12)$ nm, which agreed with the true value within errors). The method used to calculate the wavelength should be highly accurate and is mainly let down by the measurement of the distance between the slit plane and observation plane, D (since it was tricky to know exactly where the slit plane was in the aperture, and measuring with a ruler may have introduced parallax errors). (If D was measured accurately, an expected difference on λ would be $\sim$0.1%, whereas our measurement had a percentage difference of 0.4%) (note the angular deviation of the rasters was a major overestimate, and wouldn't contribute much to the overall error). To improve on this, all other apparatus excluding the aperture and rasters should have been removed, and the distance measured using calipers poked through the aperture onto the slit plane.

References

[1] Double-slit experiment, Feb 2023. URL https://en.wikipedia.org/wiki/Double-slit_experiment.

[2] Lloyd's mirror, Oct 2022. URL https://en.wikipedia.org/wiki/Lloyd%27s_mirror.

[3] Leila Hodgkins. Young's double slit experiment. URL https://www.schoolphysics.co.uk/age16-19/Wave%20properties/Interference/text/Young's_double_slits/index.html.

[4] Diffraction. URL https://isaacphysics.org/concepts/cp_diffraction?stage=all.

[5] Physics tutorial: Interference and beats. URL https://www.physicsclassroom.com/class/sound/Lesson-3/Interference-and-Beats.

[6] Op11 lloyd's mirror and double-slit interference fringes, Jan 2019. URL https://www-teaching.physics.ox.ac.uk/practical_course/scripts/srv/local/rscripts/trunk/Optics/OP11/OP11.pdf.

A Results table

s_1	s_2	s_3	s_4	s_5	$\langle s \rangle$	$\pm\delta s$	N **mm**$^{-1}$
3.100	3.145	3.155	3.180	3.115	3.139	0.014265	10.00
2.760	2.835	2.900	2.925	2.940	2.872	0.033264	15.00
2.590	2.605	2.690	2.680	2.700	2.653	0.023000	20.00
2.220	2.325	2.365	2.400	2.395	2.341	0.033068	25.59
2.020	2.100	2.180	2.175	2.125	2.120	0.029198	30.00
1.765	1.950	1.900	1.950	1.930	1.899	0.034728	35.00
1.615	1.655	1.705	1.730	1.720	1.685	0.021737	40.00
1.350	1.410	1.400	1.440	1.420	1.404	0.015033	45.00
1.090	1.135	1.175	1.110	1.165	1.135	0.016047	50.00

B Curve fit and plotting code

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

s = [2.341,1.899,2.872,2.12,1.685,3.139,1.135,2.653,1.404]
N = [25.59,35,15,30,40,10,50,20,45]

def linear(t, m, c):
    return m*t+c

parameters, covariant_matrix = curve_fit(linear, N, s, p0=(0,0))
gradient = parameters[0]
intercept = parameters[1]
gradientError = np.sqrt(covariant_matrix.diagonal())[0]
interceptError = np.sqrt(covariant_matrix.diagonal())[1]

print(gradient)
print(gradientError)
print(intercept)
print(interceptError)

s_errorbar = [0.033068112,0.034727511,0.033264095,0.029197603,0.021737065,0.014265343, ...
        0.016046807,0.023,0.015033296]

x_arr = np.arange(9,52)

plt.grid()
plt.scatter(N,s,label = ("average data"), zorder = 2, color = 'k', marker = "x")
plt.plot(x_arr, linear(x_arr, gradient, intercept), label = ("superimposed linear fit"), ...
        zorder = 1, color = 'r')
plt.errorbar(x = N, y = s, yerr = s_errorbar, linewidth = 1, fmt=" ", capsize = 3, ...
        color = "k", zorder = 2, label = "error bars")
plt.ylabel('Average micromoter reading, $s$ [mm]')
plt.xlabel('Number of lines per mm, $N$ [mm$^{-1}$]')
plt.title('Average micrometer reading, $s$, vs. number of lines per mm on raster, $N$')
plt.legend();
plt.savefig("slit_separation_vs_lines_per_mm_graph.png")
plt.show()
```