Package 'probe'

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Title PaRtitiOned empirical Bayes Ecm (PROBE) algorithm

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Description This package contains functions to fit an efficient and powerful Bayesian approach for sparse high-dimensional linear regression. Unlike previous approaches, our model formulation is focused on maximum a posteriori (MAP) estimation of the regression coefficients with minimal prior assumptions on the parameters. Plug-in empirical Bayes estimates of all hyperparameters are used. Efficient estimation is completed through the use of a partitioned and extended expectation conditional maximization (ECM) algorithm which is similar to a combination of an Estimation Conditional Maximization (ECM) and a parameter expanded (PX) EM algorithm. The result is a PaRtitiOned empirical Bayes Ecm (PROBE) algorithm applied to sparse high-dimensional linear regression. We give functions to estimate credible and prediction intervals for predictions of future values.
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2 probe-package

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Description

This package contains functions to fit an efficient and powerful Bayesian approach for sparse high-dimensional linear regression. Unlike previous approaches, our model formulation is focused on maximum a posteriori (MAP) estimation of the regression coefficients with minimal prior assumptions on the parameters. Plug-in empirical Bayes estimates of all hyperparameters are used. Efficient estimation is completed through the use of a partitioned and extended expectation conditional maximization (ECM) algorithm which is similar to a combination of an Estimation Conditional Maximization (ECM) and a parameter expanded (PX) EM algorithm. The result is a PaRtitiOned empirical Bayes Ecm (PROBE) algorithm applied to sparse high-dimensional linear regression. We give functions to estimate credible and prediction intervals for predictions of future values.

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probe Fitting PaRtitiOned empirical Bayes Ecm (PROBE)

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probe-package PaRtitiOned empirical Bayes Ecm (PROBE)

algorithm

Maintainer

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Author(s)

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e_step_func	Function for fitting the empirical Bayes portion of the E-step
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Description

A wrapper function estimating posterior expectations of the γ variables using an empirical Bayesian technquee.

Usage

```
e_step_func(beta_t, beta_var, df, adj = 5, lambda = 0.1, monotone = TRUE)
```

Arguments

beta_t	Expectation of the posterior mean (assuming $\gamma = 1$)
beta_var	Current posterior variance (assuming $\gamma=1$)
df	Degrees of freedom for the t-distribution (use to calculate p-values).
adj	Bandwidth multiplier to Silverman's 'rule of thumb' for calculating the marginal density of the test-statistics (default = 5).
lambda	Value of the λ parameter for estimating the proportion of null hypothesis using Storey et al. (2004) (default = 0.1).
monotone	Logical - Should the estimated marginal density of the test-statistics be monotone non-increasing from zero (default = TRUE).

Value

```
A list including delta estimated posterior expectations of the \gamma. pi0 estimated proportion of null hypothesis
```

References

Storey, J. D., Taylor, J. E., and Siegmund, D. (2004), "Strong control, conservative point estimation and simultaneous conservative consistency of false discovery rates: A unified approach," J. R. Stat. Soc. Ser. B. Stat. Methodol., 66, 187–205. McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139.

```
#not run
#mod <- e_step_func(beta_t, beta_var, df, adj = 5, lambda = 0.1, monotone = TRUE)</pre>
```

4 m_step_regression

 ${\tt m_step_regression}$

Function for fitting the initial part of the M-step

Description

A wrapper function providing the quantities related to the M-step for α_0 and σ^2 .

Usage

```
m_{step_regression}(Y, W, W2, Z, a = -3/2, Int = TRUE)
```

Arguments

Υ	A matrix containing the outcome Y
W	Quantity $E(W_0)$ as outlined in citation, output from W_update_fun
W2	Quantity $E(W_0^2)$ as outlined in citation, output from W_update_fun
Z	A matrix or dataframe of other predictors to account for
a	(optional) parameter for changing the hyperparameter a (default, $a=-3/2$ uses $n-2$ as denominator for MAP of $\sigma^2)$
Int	(optional) Logical - should an intercept be used?

Value

A list including

coef the MAP estimates of the α_0 parameters sigma2_est the MAP estimate of σ^2 VCV posterior variance covariance matrix of α_0 , res_data dataframe containing MAP estimates, posterior variances, t-test statistics and associated p-values for α_0

References

McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139.

```
#not run
#mod <- m_step_regression(Y, W_ast, W_ast_var + W_ast^2, Z)</pre>
```

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predict_probe_func	Obtaining predictions, confidence intervals and prediction intervals from probe
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Description

A function providing predictions, along with $(1-\alpha)*100\%$ credible, and prediction intervals for new observations.

Usage

```
predict_probe_func(res, X, Z = NULL, alpha = 0.05, X_2 = NULL)
```

Arguments

res	The results from the probe function.
Χ	A matrix containing the predictors on which to apply the probe algorithm
Z	(optional) A matrix or dataframe of predictors not subjected to the sparsity assumption to account for.
alpha	significance level for $(100(1-\alpha)\%)$ credible and prediction intervals.
X_2	(optional) Square of X matrix.

Value

A dataframe with predictions, credible intervals, and prediction intervals for each new observation.

References

McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139.

```
### Example
data(Sim_data)
attach(Sim_data)
alpha <- 0.05
plot_ind <- TRUE
adj <- 10

# Run the analysis. Y_test and X_test are included for plotting purposes only
full_res <- probe( Y = Y, X = X, alpha = alpha, plot_ind = plot_ind,
Y_test = Y_test, X_test = X_test, adj = adj)

# Predicting for test data
pred_res <- predict_probe_func(full_res, X = X_test, Z = NULL, alpha = alpha)
head(pred_res)</pre>
```

6 probe

probe	Fitting PaRtitiOned empirical Bayes Ecm (PROBE) algorithm to sparse high-dimensional linear models.

Description

A wrapper function for the main PROBE algorithm function. The R package is a work in progress.

Usage

```
probe(Y, X, Z = NULL, ep = 0.1, maxit = 10000, Y_test = NULL, X_test = NULL,
Z_test = NULL, verbose = FALSE, signal = NULL, eta_i = NULL, alpha = 0.05,
plot_ind = FALSE, adj = 5)
```

Arguments

Υ	The outcome variable.
Χ	An n x M matrix of sparse predictors variables.
Z	(optional) An $n \times p$ matrix or dataframe of other predictors not subjected to the sparsity assumption.
ер	Value against which to compare convergence criterion (default = 0.1).
maxit	Maximum number of iterations the algorithm will run for (default = 10000).
Y_test	(optional) Test Y data used plotting purposes only (doesn't impact results).
X_test	(optional) Test X data used plotting purposes only (doesn't impact results).
Z_test	(optional) Test Z data used plotting purposes only (doesn't impact results).
verbose	A logical (true/false) value whether to print algorithm iteration progress and summary quantities (default = FALSE).
signal	(optional) A vector of indicies of the true non-null coefficients. This is used to calculate the true and false discovery rates by iteration for simulated data. Used plotting purposes only (doesn't impact results).
eta_i	(optional) A vector of the true signal. This is used to calculate the MSE by iteration for simulated data. Used plotting purposes only (doesn't impact results).
alpha	(optional) significance level
plot_ind	A logical values (True/False) for whether to output plots on algorithm results and progress (default = FALSE)
adj	Bandwidth parameter for empirical Bayes E-step. The bandwidth will be equal to adj times Silverman's 'rule of thumb' (default = 2).

Value

A list including

```
beta_ast_hat MAP estimates of the regression coefficients (\beta^*),
```

beta_hat,beta_hat_var MAP estimates of the posterior expectation (beta_hat) and variance (beta_hat_var) of the prior mean (β) of the regression coefficients assuming $\gamma=1$,

 $\ensuremath{\mathsf{gamma_hat}}$ the posterior expectation of the latent γ variables,

sigma2_est MAP estimate of the residual variance,

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```
E_step full results of the final E_step, {\tt Calb\_mod\ results\ of\ first\ }(\alpha_0)\ part\ of\ the\ M-step, count the total number of iterations before convergence.
```

References

McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139..

See Also

predict_probe_func to obtain predictions, credible intervals and prediction intervals from PROBE.

```
### Example
data(Sim_data)
attach(Sim_data)
alpha <- 0.05
plot_ind <- TRUE</pre>
adj <- 10
# Run the analysis. Y_test and X_test are included for plotting purposes only
full_res <- probe( Y = Y, X = X, Y_test = Y_test,</pre>
X_test = X_test, alpha = alpha, plot_ind = plot_ind, adj = adj)
# Predicting for test data
pred_res <- predict_probe_func(full_res, X = X_test, alpha = alpha)</pre>
head(pred_res)
# Estimate of the residual variance
full_res$sigma2_est
### Example with additional covariate data Z (not subjected to the sparsity assumption)
data(Sim_data_cov)
attach(Sim_data_cov)
# Calculating the true signal (the impact of X only)
eta_i <- apply(t(X)*beta_tr,2,sum)</pre>
# Run the analysis. eta_i (true signal) and signal are included for plotting purposes only.
full_res <- probe( Y = Y, X = X, Z = Z, signal = signal,</pre>
eta_i = eta_i, alpha = alpha, plot_ind = plot_ind, adj = adj)
# Final estimates of the impact of X versus the true values:
data.frame(true_values = beta_X_tr, full_res$Calb_mod$res_data[-2,])
#Compare to a standard linear model of X on Y:
summary(lm(Y~Z\$Cont\_cov + Z\$Binary\_cov))\$coefficients
```

8 Sim_data_cov

Sim_data

Simulated high-dimensional data set for sparse linear regression.

Description

This dataset was simulated using a 100×100 2-dimensional setting described in the reference. The data contains 400 subjects with one outcome and 10,000 predictor variables. There is also test outcomes and predictor variables.

Usage

```
data("Sim_data")
```

Format

A data frame with 400 observations and the following objects:

Y Outcome variable of length 400.

X A 400×10000 matrix of binary predictor variables.

beta_tr The true values of all 10000 regression coefficients.

signal The locations of the non-zero regression coefficients.

Y_test Outcome variable of length 400 for test set.

Z_test A 400×10000 matrix of binary predictor variables for test set.

Examples

```
data(Sim_data)
attach(Sim_data)
length(Y)
dim(X)
```

Sim_data_cov

Simulated high-dimensional data set for sparse linear regression with non-sparse covariates.

Description

This dataset was simulated using a 100×100 2-dimensional setting described in the reference only two covariates are added. The data contains 400 subjects with one outcome, 10000 predictor variables which are to be subjected to the sparsity assumption, and 2 covariates which are not to be subjected to the sparsity assumption.

Usage

```
data("Sim_data_cov")
```

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Format

A data frame with 400 observations and the following objects:

Y Outcome variable of length 400.

Z A dataframe of a continuous (Cont_cov) and binary (Binary_cov) covariate.

X A 400×10000 matrix of binary predictor variables.

 $beta_{tr}$ The true values of all 10000 regression coefficients.

 $beta_Z_tr \ \ The \ true \ values \ of \ the \ intercept, \ Cont_cov, \ and \ Binary_cov.$

signal The locations of the non-zero regression coefficients.

Examples

data(Sim_data_cov)
attach(Sim_data_cov)
length(Y)
summary(Z)
dim(X)

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