# Package 'probe'

June 19, 2025

Type	Package
Title	Sparse High-Dimensional Linear Regression with a PaRtitiOned
	Empirical Bayes Ecm (PROBE) Algorithm

**Version** 1.2 **Date** 2023-10-01

## **Description**

Implements an efficient and powerful Bayesian approach for sparse high-dimensional linear regression. It uses minimal prior assumptions on the parameters through plug-in empirical Bayes estimates of hyperparameters. An efficient Parameter-Expanded Expectation-Conditional-Maximization (PX-ECM) algorithm estimates maximum a posteriori (MAP) values of regression parameters and variable selection probabilities. The PX-ECM results in a robust computationally efficient coordinate-wise optimization, which adjusts for the impact of other predictor variables. The E-step is motivated by the popular two-group approach to multiple testing. The result is a PaRtitiOned empirical Bayes Ecm (PROBE) algorithm applied to sparse high-dimensional linear regression, implemented using one-at-a-time or all-at-once type optimization. Simulation studies found the all-at-once variant to be superior.

BugReports https://github.com/alexmclain/PROBE/issues
License GPL (>= 2)
Encoding UTF-8
RoxygenNote 7.3.2
Imports Rcpp, glmnet, RcppArmadillo
LinkingTo Rcpp, RcppArmadillo
NeedsCompilation yes
<b>Depends</b> R (>= 4.00)
Author Alexander McLain [aut, cre] ( <a href="https://orcid.org/0000-0002-5475-0670">https://orcid.org/0000-0002-5475-0670</a> ).  Anja Zgodic [aut, ctb]
Maintainer Alexander McLain <mclaina@mailbox.sc.edu></mclaina@mailbox.sc.edu>

# **Contents**

probe-package	2
e_step_func	3
hprobe	4
h_Sim_data	$\epsilon$
m step regression	6

2 probe-package

probe	1 0	robe: iOned								,		io	n	wi	ith	a	F	Pal	Rti	-
Index																				10
	Sim_data_test		 	 	•	 •		•			•	•						•		14
	Sim_data_cov		 	 																14
	Sim_data		 	 																13
	probe_one		 	 																1
	probe		 	 																9
	predict_probe_func .		 	 																8
	predict_hprobe_func		 	 																1

# **Description**

Implements an efficient and powerful Bayesian approach for sparse high-dimensional linear regression. It uses minimal prior assumptions on the parameters through plug-in empirical Bayes estimates of hyperparameters. An efficient Parameter-Expanded Expectation-Conditional-Maximization (PX-ECM) algorithm estimates maximum a posteriori (MAP) values of regression parameters and variable selection probabilities. The PX-ECM results in a robust computationally efficient coordinatewise optimization, which adjusts for the impact of other predictor variables. The E-step is motivated by the popular two-group approach to multiple testing. The result is a PaRtitiOned empirical Bayes Ecm (PROBE) algorithm applied to sparse high-dimensional linear regression, implemented using one-at-a-time or all-at-once type optimization. Simulation studies found the all-at-once variant to be superior.

#### Details

Examples for applying PROBE to sparse high-dimensional linear regression are given for one-at-a-time probe\_one or all-at-once probe type optimization.

## Author(s)

Maintainer: Alexander McLain <mclaina@mailbox.sc.edu> (ORCID)

Authors:

• Anja Zgodic [contributor]

# References

- McLain, AC, A Zgodic, H Bondell (2025). Sparse high-dimensional linear regression with a
  partitioned empirical Bayes ECM algorithm. \*\*Computational Statistics and Data Analysis\*\*
  207, 108146.
- Zgodic, A., Bai, R., Zhang, J., Wang, Y., Rorden, C., & McLain, A. (2023). Quantifying predictive uncertainty of aphasia severity in stroke patients with sparse heteroscedastic Bayesian high-dimensional regression. arXiv preprint arXiv:2309.08783.

## See Also

Useful links:

• Report bugs at https://github.com/alexmclain/PROBE/issues

e\_step\_func 3

e_step_func	Function for fitting the empirical Bayes portion of the E-step

## **Description**

A wrapper function estimating posterior expectations of the  $\gamma$  variables using an empirical Bayesian technquee.

## Usage

```
e_step_func(beta_t, beta_var, df, adj = 5, lambda = 0.1, monotone = TRUE)
```

# **Arguments**

beta_t	Expectation of the posterior mean (assuming $\gamma=1$ )
beta_var	Current posterior variance (assuming $\gamma = 1$ )
df	Degrees of freedom for the t-distribution (use to calculate p-values).
adj	Bandwidth multiplier to Silverman's 'rule of thumb' for calculating the marginal density of the test-statistics (default $= 5$ ).
lambda	Value of the $\lambda$ parameter for estimating the proportion of null hypothesis using Storey et al. (2004) (default = 0.1).
monotone	Logical - Should the estimated marginal density of the test-statistics be monotone non-increasing from zero (default = TRUE).

## Value

```
A list including delta estimated posterior expectations of the \gamma. pi0 estimated proportion of null hypothesis
```

## References

Storey, J. D., Taylor, J. E., and Siegmund, D. (2004), "Strong control, conservative point estimation and simultaneous conservative consistency of false discovery rates: A unified approach," J. R. Stat. Soc. Ser. B. Stat. Methodol., 66, 187–205. McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139.

```
#not run
#mod <- e_step_func(beta_t, beta_var, df, adj = 5, lambda = 0.1, monotone = TRUE)</pre>
```

4 hprobe

hprobe	Fitting PaRtitiOned empirical Bayes Ecm (PROBE) algorithm to
	sparse high-dimensional linear models with heterogeneous variance.

# Description

A wrapper function for the H-PROBE algorithm.

## Usage

```
\label{eq:hprobe} $$ \  \  \, $$ hprobe(Y, X, V, Z = NULL, ep = 0.1, maxit = 10000, Y_test = NULL, X_test = NULL, Z_test = NULL, verbose = FALSE, signal = NULL, eta_i = NULL, alpha = 0.05, plot_ind = FALSE, adj = 5)
```

# Arguments

X An n x M matrix of sparse predictors variables.	
V A design matrix of predictors for the variance model (including an interc	ept).
Z (optional) An n x p matrix or dataframe of other predictors not subjected sparsity assumption.	d to the
ep Value against which to compare convergence criterion (default = $0.1$ ).	
maxit Maximum number of iterations the algorithm will run for (default = 1000	00).
Y_test (optional) Test Y data used plotting purposes only (doesn't impact result	s).
X_test (optional) Test X data used plotting purposes only (doesn't impact result	s).
Z_test (optional) Test Z data used plotting purposes only (doesn't impact results	s).
V_test (optional) Test V data used plotting purposes only (doesn't impact result	s).
verbose A logical (true/false) value whether to print algorithm iteration progressummary quantities (default = FALSE).	ess and
signal (optional) A vector of indicies of the true non-null coefficients. This is calculate the true and false discovery rates by iteration for simulated data plotting purposes only (doesn't impact results).	
eta_i (optional) A vector of the true signal. This is used to calculate the MSE ation for simulated data. Used plotting purposes only (doesn't impact res	•
alpha (optional) significance level	
plot_ind A logical values (True/False) for whether to output plots on algorithm and progress (default = FALSE)	results
Bandwidth parameter for empirical Bayes E-step. The bandwidth will be to adj times Silverman's 'rule of thumb' (default = 2).	e equal

# Value

# A list including

beta\_ast\_hat MAP estimates of the regression coefficients ( $\beta^*$ ),

beta\_hat, beta\_hat\_var MAP estimates of the posterior expectation (beta\_hat) and variance (beta\_hat\_var) of the prior mean  $(\beta)$  of the regression coefficients assuming  $\gamma=1$ ,

hprobe 5

```
gamma_hat the posterior expectation of the latent \gamma variables, sigma2_est MAP estimate of the residual variance, E_step full results of the final E_step, Calb_mod results of first (\alpha_0) part of the M-step, count the total number of iterations before convergence.
```

## References

McLain, AC, A Zgodic, H Bondell (2025). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. \*\*Computational Statistics and Data Analysis\*\* 207, 108146. Zgodic, A., Bai, R., Zhang, J., Wang, Y., Rorden, C., & McLain, A. (2023). Quantifying predictive uncertainty of aphasia severity in stroke patients with sparse heteroscedastic Bayesian high-dimensional regression. arXiv preprint arXiv:2309.08783.

#### See Also

predict\_probe\_func to obtain predictions, credible intervals and prediction intervals from PROBE.

```
### Example
data(h_Sim_data)
attach(h_sim_data)
# Run Analysis
res <- hprobe(Y = Y, X = X, V = V)
# Predicting for test data
pred_res <- predict_hprobe_func(res, X_test, V = V_test)</pre>
sqrt(mean((Y_test - pred_res$Pred)^2))
head(cbind(Y_test, pred_res))
plot(Y_test, pred_res$Pred, ylab = "Prediction", xlab = "Test Outcome")
abline(coef = c(0,1))
# Proportion of explained variance
1 - var(Y_test - pred_res$Pred)/var(Y_test)
## Omega coefficients (versus true values)
cbind(omega_tr, res$omega)
## True versus estimated beta coefficcients
plot(beta_tr,
     res$beta_ast_hat,
     xlab = "True Beta",
     ylab = "Estimated Beta")
abline(coef = c(0,1))
\#\# Confusion matrix of true versus estimated signals using 0.5 cutoff.
table(beta_tr==0, res$gamma_hat<0.5)</pre>
```

6 m\_step\_regression

h_Sim_data	Simulated high-dimensional data set for sparse linear regression with heterogeneous errorss

## **Description**

This dataset was simulated using a  $20 \times 20$  2-dimensional setting described in the reference. The data contains 200 subjects with one outcome, 400 predictor variables, and 6 variables that are related to the variance of the error term. The data contains training and test data, along with the true values of the parameters used to generate the data..

# Usage

```
data("h_Sim_data")
```

## **Format**

A data frame with 200 observations and the following objects:

Y Outcome variable of length 200.

X A  $200 \times 400$  matrix of binary predictor variables.

V A  $200 \times 7$  design matrix to model variance of the residuals (includes an intercept).

beta\_tr The true values of all 400 regression coefficients.

omega\_tr The true values of all 7 variance regression coefficients.

## Source

Simulated data.

## **Examples**

```
data(h_Sim_data)
attach(h_sim_data)
length(Y)
dim(X)
```

m\_step\_regression

Function for fitting the initial part of the M-step

## **Description**

A wrapper function providing the quantities related to the M-step for  $\alpha_0$  and  $\sigma^2$ .

# Usage

```
m_{step\_regression}(Y, W, W2, Z = NULL, a = -3/2, Int = TRUE)
```

predict\_hprobe\_func 7

# **Arguments**

Υ	A matrix containing the outcome Y
W	Quantity $E(W_0)$ as outlined in citation, output from W_update_fun
W2	Quantity ${\cal E}(W_0^2)$ as outlined in citation, output from W_update_fun
Z	A matrix or dataframe of other predictors to account for
a	(optional) parameter for changing the hyperparameter $a$ (default, $a=-3/2$ uses $n-2$ as denominator for MAP of $\sigma^2)$
Int	(optional) Logical - should an intercept be used?

## Value

## A list including

coef the MAP estimates of the  $\alpha_0$  parameters sigma2\_est the MAP estimate of  $\sigma^2$  VCV posterior variance covariance matrix of  $\alpha_0$ , res\_data dataframe containing MAP estimates, posterior variances, t-test statistics and associated p-values for  $\alpha_0$ 

## References

McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139.

## **Examples**

```
#not run
#mod <- m_step_regression(Y, W_ast, W_ast_var + W_ast^2, Z)</pre>
```

## **Description**

A function providing predictions, along with  $(1-\alpha)*100\%$  credible, and prediction intervals for new observations.

## Usage

```
predict_hprobe_func(res, X, V, Z = NULL, alpha = 0.05, X_2 = NULL)
```

# Arguments

res	The results from the probe function.
Χ	A matrix containing the predictors on which to apply the probe algorithm
V	A design matrix of predictors for the variance model.
Z	(optional) A matrix or dataframe of predictors not subjected to the sparsity assumption to account for.
alpha	significance level for $(100(1-\alpha)\%)$ credible and prediction intervals.
X 2	(optional) Square of X matrix.

8 predict\_probe\_func

## Value

A dataframe with predictions, credible intervals, and prediction intervals for each new observation.

## References

McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139.

## **Examples**

```
### Example
#not run
# pred_res <- predict_probe_func(full_res, X = X_test, Z = NULL, alpha = alpha)
# head(pred_res)</pre>
```

predict\_probe\_func Obtaining predictions, confidence intervals and prediction intervals from probe

## **Description**

A function providing predictions, along with  $(1 - \alpha) * 100\%$  credible, and prediction intervals for new observations.

# Usage

```
predict_probe_func(res, X, Z = NULL, alpha = 0.05, X_2 = NULL)
```

# **Arguments**

res	The results from the probe function.
X	A matrix containing the predictors on which to apply the probe algorithm
Z	(optional) A matrix or dataframe of predictors not subjected to the sparsity assumption to account for.
alpha	significance level for $(100(1-\alpha)\%)$ credible and prediction intervals.
X_2	(optional) Square of X matrix.

## Value

A dataframe with predictions, credible intervals, and prediction intervals for each new observation.

# References

McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139.

probe 9

## **Examples**

```
### Example
#not run
# pred_res <- predict_probe_func(full_res, X = X_test, Z = NULL, alpha = alpha)
# head(pred_res)</pre>
```

probe

Fitting PaRtitiOned empirical Bayes Ecm (PROBE) algorithm to sparse high-dimensional linear models.

# Description

A wrapper function for the all-at-once variant of the PROBE algorithm.

## Usage

```
probe(Y, X, Z = NULL, ep = 0.1, maxit = 10000, Y_test = NULL, X_test = NULL,
Z_test = NULL, verbose = FALSE, signal = NULL, eta_i = NULL, alpha = 0.05,
plot_ind = FALSE, adj = 5)
```

# **Arguments**

Υ	The outcome variable.
Χ	An n x M matrix of sparse predictors variables.
Z	(optional) An $n \times p$ matrix or dataframe of other predictors not subjected to the sparsity assumption.
ер	Value against which to compare convergence criterion (default = $0.1$ ).
maxit	Maximum number of iterations the algorithm will run for (default = 10000).
Y_test	(optional) Test Y data used plotting purposes only (doesn't impact results).
X_test	(optional) Test X data used plotting purposes only (doesn't impact results).
Z_test	(optional) Test Z data used plotting purposes only (doesn't impact results).
verbose	A logical (true/false) value whether to print algorithm iteration progress and summary quantities (default = FALSE).
signal	(optional) A vector of indicies of the true non-null coefficients. This is used to calculate the true and false discovery rates by iteration for simulated data. Used plotting purposes only (doesn't impact results).
eta_i	(optional) A vector of the true signal. This is used to calculate the MSE by iteration for simulated data. Used plotting purposes only (doesn't impact results).
alpha	(optional) significance level
plot_ind	A logical values (True/False) for whether to output plots on algorithm results and progress (default = FALSE)
adj	Bandwidth parameter for empirical Bayes E-step. The bandwidth will be equal to adj times Silverman's 'rule of thumb' (default = 2).

10 probe

#### Value

```
A list including beta_ast_hat MAP estimates of the regression coefficients (\beta^*), beta_hat, beta_hat_var MAP estimates of the posterior expectation (beta_hat) and variance (beta_hat_var) of the prior mean (\beta) of the regression coefficients assuming \gamma=1, gamma_hat the posterior expectation of the latent \gamma variables, sigma2_est MAP estimate of the residual variance, E_step full results of the final E_step, Calb_mod results of first (\alpha_0) part of the M-step, count the total number of iterations before convergence.
```

#### References

McLain, AC, A Zgodic, H Bondell (2025). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. \*\*Computational Statistics and Data Analysis\*\* 207, 108146.

## See Also

predict\_probe\_func to obtain predictions, credible intervals and prediction intervals from PROBE.

```
### Example
data(Sim_data)
data(Sim_data_test)
attach(Sim_data)
attach(Sim_data_test)
alpha <- 0.05
plot_ind <- TRUE
adj <- 10
# Run the analysis. Y_test and X_test are included for plotting purposes only
full_res \leftarrow probe(Y = Y, X = X, Y_test = Y_test,
X_test = X_test, alpha = alpha, plot_ind = plot_ind, adj = adj)
# Predicting for test data
pred_res <- predict_probe_func(full_res, X = X_test)</pre>
sqrt(mean((Y_test - pred_res$Pred)^2))
# Estimate of the residual variance and true value
full_res$sigma2_est
sigma2\_tr
# RMSE of estimated beta coefficients
beta_ast_est <- full_res$beta_ast_hat</pre>
sqrt(mean((beta_ast_est - beta_tr)^2))
# Posterior expectation of gamma by true
gamma_est <- full_res$E_step$gamma</pre>
sum(gamma_est)
sum(gamma_est[beta_tr>0])
```

probe\_one 11

probe\_one Fitting PaRtitiOned empirical Bayes Ecm (PROBE) algorithm to sparse high-dimensional linear models.

# Description

A wrapper function for the one-at-a-time variant of the PROBE algorithm.

The outcome veriable

## Usage

```
probe_one(Y, X, ep = 0.001, maxit = 10000, Y_test = NULL, X_test = NULL,
verbose = FALSE, signal = NULL, eta_i = NULL, alpha = 0.05, plot_ind = FALSE,
order.method = "lasso", adj = 10, delta = 0.4, update_order= NULL, beta_start= NULL)
```

## **Arguments**

Υ	The outcome variable.
Χ	An n x M matrix of sparse predictors variables.
ер	Value against which to compare convergence criterion (default = $0.001$ ).
maxit	Maximum number of iterations the algorithm will run for (default = 10000).
Y_test	(optional) Test Y data used plotting purposes only (doesn't impact results).
X_test	(optional) Test X data used plotting purposes only (doesn't impact results).
verbose	A logical (true/false) value whether to print algorithm iteration progress and summary quantities (default = FALSE).
signal	(optional) A vector of indicies of the true non-null coefficients. This is used to calculate the true and false discovery rates by iteration for simulated data. Used plotting purposes only (doesn't impact results).
eta_i	(optional) A vector of the true signal. This is used to calculate the MSE by iteration for simulated data. Used plotting purposes only (doesn't impact results).
alpha	(optional) significance level
plot_ind	A logical values (True/False) for whether to output plots on algorithm results and progress (default = FALSE) $$

probe\_one

order.method Updating order and initial values of the algorithm. For lasso (default) or ridge, a lasso or a ridge regression model (fit with 10-fold CV) will be fitted and used. The update\_order is defined by the absolute values of the coefficient and beta\_start is the coefficient values. When using none, update\_order and beta\_start must be given. random will randomly select the updating order

and use very small values for beta\_start.

adj Bandwidth parameter for empirical Bayes E-step. The bandwidth will be equal

to adj times Silverman's 'rule of thumb' (default = 10).

delta Learning rate for iteration t is  $(1 + t)^{-1}$  + delta) (default delta = 0.4).

update\_order Manual value for the updating order for when order.method = "none" is used.
beta\_start Manual value for the starting beta coefficients for when order.method = "none"

is used.

seed Seed value to ensure reproducibility when order.method = "lasso", order.method

= "ridge", or order.method = "random".

#### Value

A list including

beta\_ast\_hat MAP estimates of the regression coefficients ( $\beta^*$ ),

beta\_hat, beta\_hat\_var MAP estimates of the posterior expectation (beta\_hat) and variance (beta\_hat\_var) of the prior mean  $(\beta)$  of the regression coefficients assuming  $\gamma=1$ ,

gamma\_hat the posterior expectation of the latent  $\gamma$  variables,

sigma2\_est MAP estimate of the residual variance,

E\_step full results of the final E\_step,

count the total number of iterations before convergence.

## References

McLain, A. C., Zgodic, A., & Bondell, H. (2022). Sparse high-dimensional linear regression with a partitioned empirical Bayes ECM algorithm. arXiv preprint arXiv:2209.08139..

## See Also

predict\_probe\_func to obtain predictions.

```
### Example
data(Sim_data)
data(Sim_data_test)
attach(Sim_data)
attach(Sim_data_test)
plot_ind <- TRUE
adj <- 10

# Run the analysis. Y_test and X_test are included for plotting purposes only
full_res <- probe_one( Y = Y, X = X, Y_test = Y_test, order.method = "lasso",
X_test = X_test, plot_ind = plot_ind, adj = adj)

# Predicting for test data
pred_res <- predict_probe_func(full_res, X = X_test)</pre>
```

Sim\_data 13

```
sqrt(mean((Y_test - pred_res$Pred)^2))

# Estimate of the residual variance and true value
full_res$sigma2_est
sigma2_tr

# RMSE of estimated beta coefficients
beta_ast_est <- c(full_res$beta_ast_hat)
sqrt(mean((beta_ast_est - beta_tr)^2))

# Posterior expectation of gamma by true
gamma_est <- full_res$E_step$gamma
table(gamma_est > 0.5, beta_tr > 0)
sum(gamma_est)
sum(gamma_est[beta_tr>0])
```

Sim\_data

Simulated high-dimensional data set for sparse linear regression

## **Description**

This dataset was simulated using a  $100 \times 100$  2-dimensional setting described in the reference. The data contains 400 subjects with one outcome and 10,000 predictor variables. The test outcomes and predictor variables are contained in Sim\_data\_test.

# Usage

```
data("Sim_data")
```

## **Format**

A data frame with 400 observations and the following objects:

Y Outcome variable of length 400.

X A  $400 \times 10000$  matrix of binary predictor variables.

signal The locations of the non-zero regression coefficients.

beta\_tr The true values of all 10000 regression coefficients.

sigma2\_tr The true value of the residual variance.

# Source

Simulated data.

```
data(Sim_data)
attach(Sim_data)
length(Y)
dim(X)
```

Sim\_data\_test

Sim_data_cov	Simulated high-dimensional data set for sparse linear regression with non-sparse covariates.
--------------	--

## **Description**

This dataset was simulated using a  $100 \times 100$  2-dimensional setting described in the reference only two covariates are added. The data contains 400 subjects with one outcome, 10000 predictor variables which are to be subjected to the sparsity assumption, and 2 covariates which are not to be subjected to the sparsity assumption.

# Usage

```
data("Sim_data_cov")
```

#### **Format**

A data frame with 400 observations and the following objects:

Y Outcome variable of length 400.

Z A dataframe of a continuous (Cont\_cov) and binary (Binary\_cov) covariate.

X A  $400 \times 10000$  matrix of binary predictor variables.

beta\_tr The true values of all 10000 regression coefficients.

beta\_Z\_tr The true values of the intercept, Cont\_cov, and Binary\_cov.

signal The locations of the non-zero regression coefficients.

## **Examples**

```
data(Sim_data_cov)
str(Sim_data_cov)
```

Sim\_data\_test

Simulated high-dimensional test data set for sparse linear regression

## **Description**

A test set of outcomes and predictor variables to be used with Sim\_data.

## Usage

```
data("Sim_data_test")
```

## **Format**

A data frame with 400 observations and the following objects:

Y\_test Outcome variable of length 400 for test set.

Z\_test A  $400 \times 10000$  matrix of binary predictor variables for test set.

Sim\_data\_test 15

# Source

Simulated data.

# Examples

data(Sim\_data\_test)
attach(Sim\_data\_test)
length(Y\_test)
dim(X\_test)

# **Index**

```
* datasets
    h_Sim_data, 6
    Sim_data, 13
     Sim_data_cov, 14
    {\tt Sim\_data\_test}, \\ \frac{14}{}
h_Sim_data, 6
hprobe, 4
m\_step\_regression, 6
predict_hprobe_func, 7
predict_probe_func, 8
probe, 2, 9
{\tt probe-package}, \color{red} 2
probe_one, 2, 11
Sim_data, 13
Sim_data_cov, 14
Sim_data_test, 14
```