

# Homework #3

---

Handed out Oct 31st (after exam)

Due **Nov 14**

# Last time...

---

## Types of Stellar Temperature

**Excitation** - ratio of atoms in different states of excitation, defined by Boltzmann equation

**Ionization** - ratio of atoms in different stages of ionization, defined by Saha equation

**Kinetic** - Maxwell-Boltzmann distribution

**Color** - blackbody assumption

**Effective** - at the "surface" of a star

$$L = 4\pi R^2 \sigma T^4 \leftarrow$$

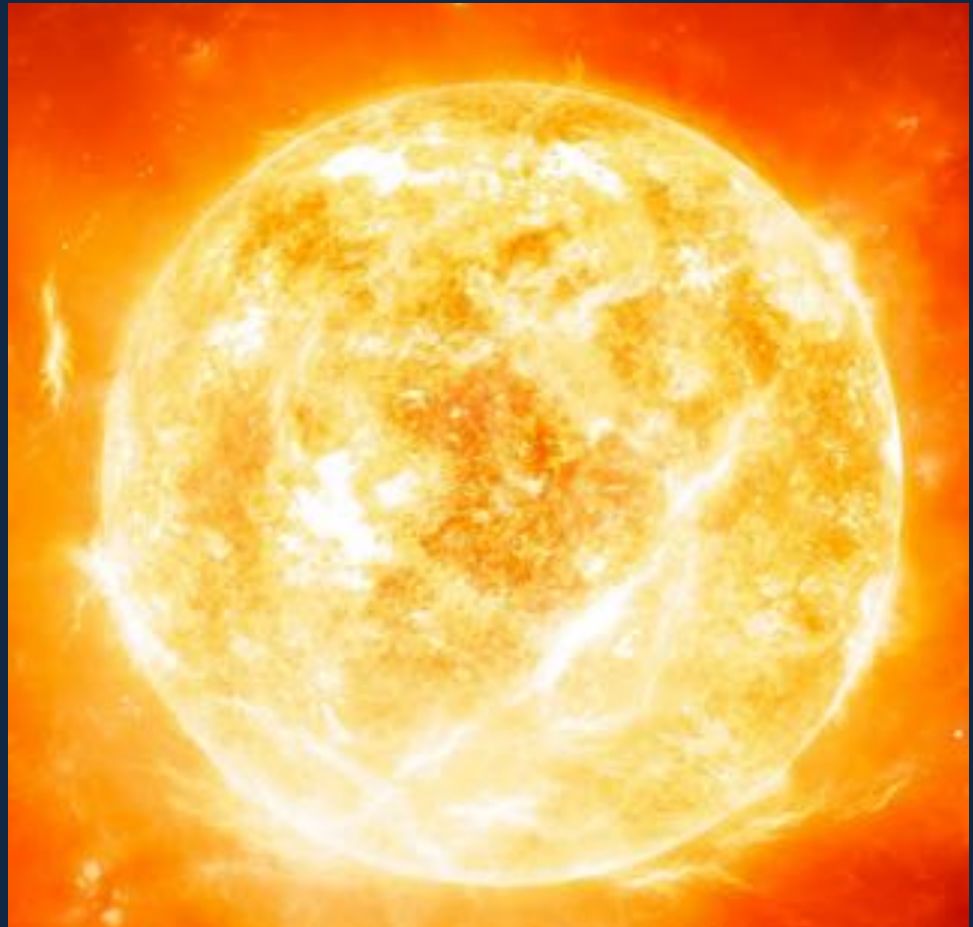
# Properties of Stars - Temperature

---

## Types of Stellar Temperature

**Why aren't these all the same?**

**Excitation**  
**Ionization**  
**Kinetic**  
**Color**  
**Effective**



# Properties of Stars - Temperature

---

## Types of Stellar Temperature

**Stars aren't in thermodynamic equilibrium...**

...but can approximate:

**L**ocal

**T**hermodynamic

**E**quilibrium

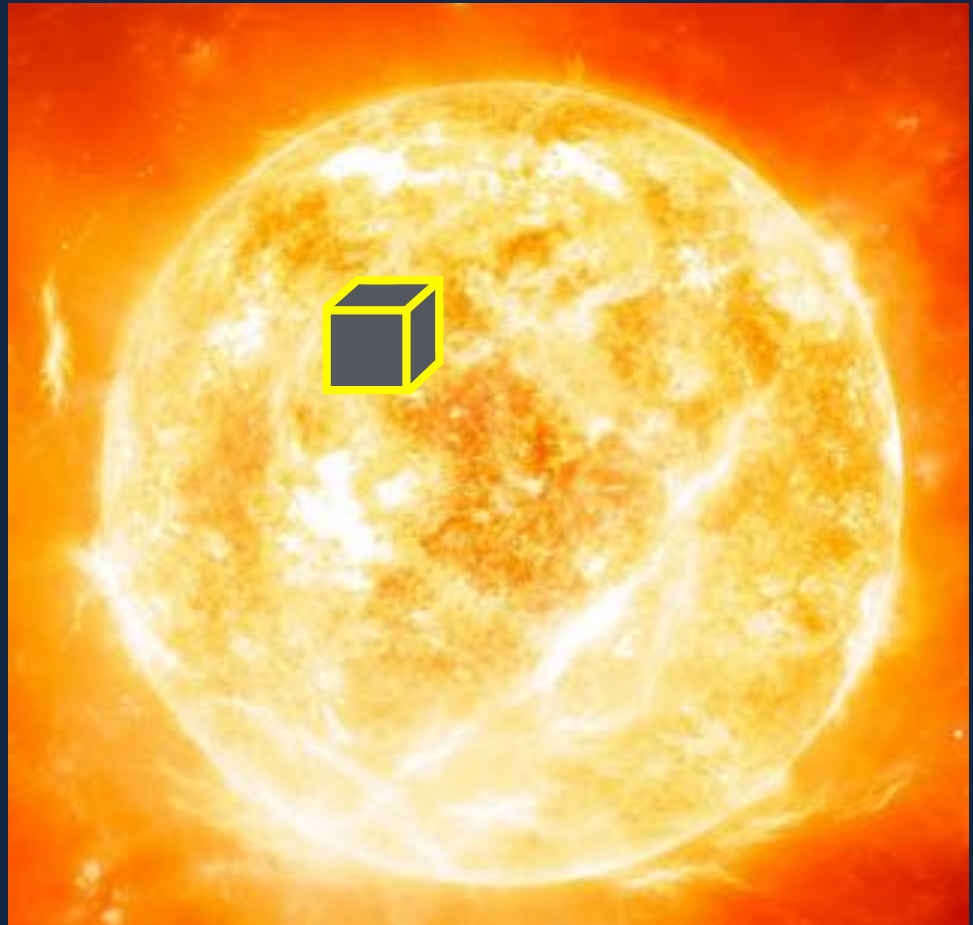
**Excitation**

**Ionization**

**Kinetic**

**Color**

**Effective**



# Properties of Stars - Mass

---

- 1) Distance - parallax, Cepheids
- 2) Velocity - proper motion, radial velocity
- 3) Brightness - magnitudes, luminosity
- 4) Temperature - effective temp (usually)
- 5) **Mass**
- 6) Radius

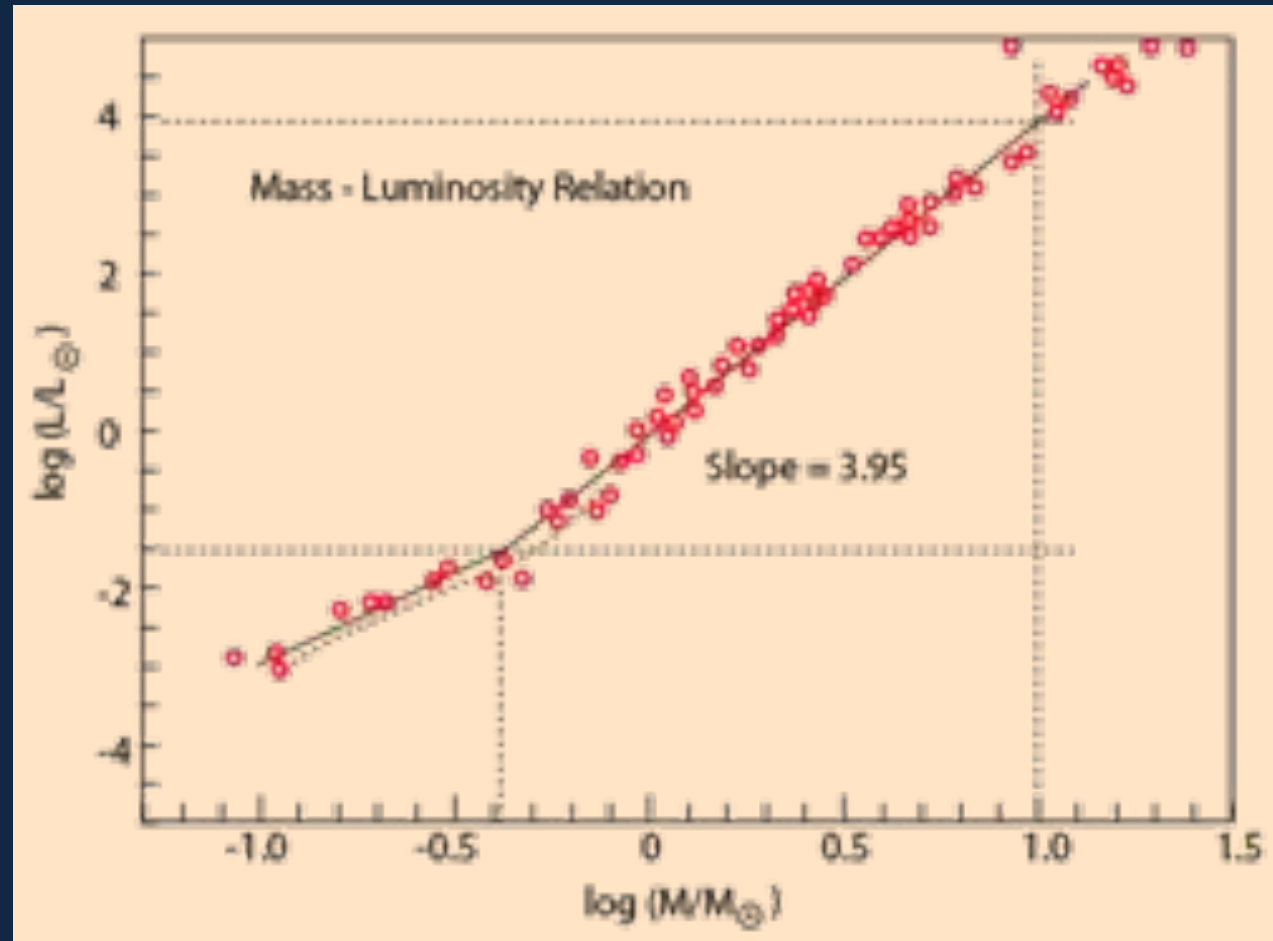
# Properties of Stars - Mass

## Determining Stellar Mass

For much of a star's lifetime it follows the **mass-luminosity relation**.

$$L = L_{\text{sun}} \left( \frac{M}{M_{\text{sun}}} \right)^a$$

$$3 \approx a \approx 4$$



# Properties of Stars - Mass

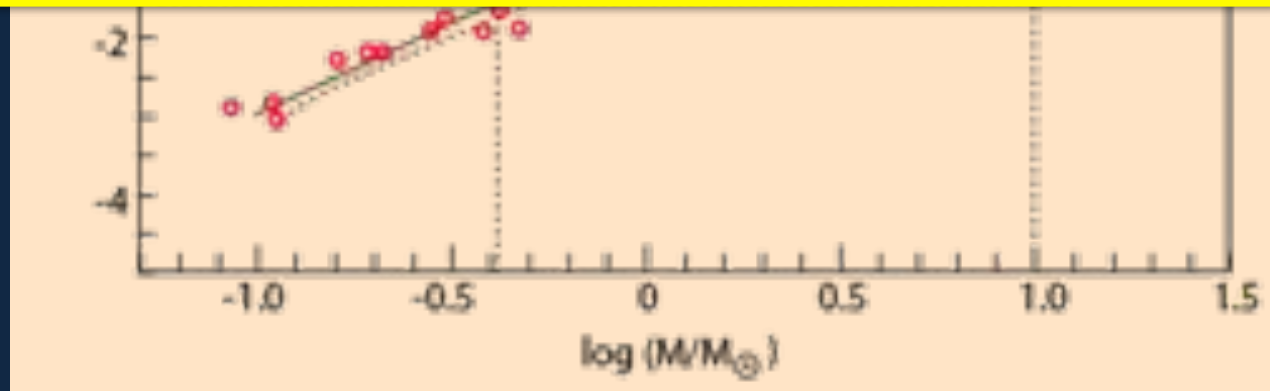
## Determining Stellar Mass

For much of a star's lifetime it follows the

### DISCUSSION QUESTION

Why - physically - might we see a mass-luminosity relation in stars?

(think, then discuss)





# Properties of Stars - Mass

## Determining Stellar Mass

For much of a star's lifetime it follows the **mass-luminosity relation**.

$$L = L_{\text{sun}} \left( \frac{M}{M_{\text{sun}}} \right)^a$$

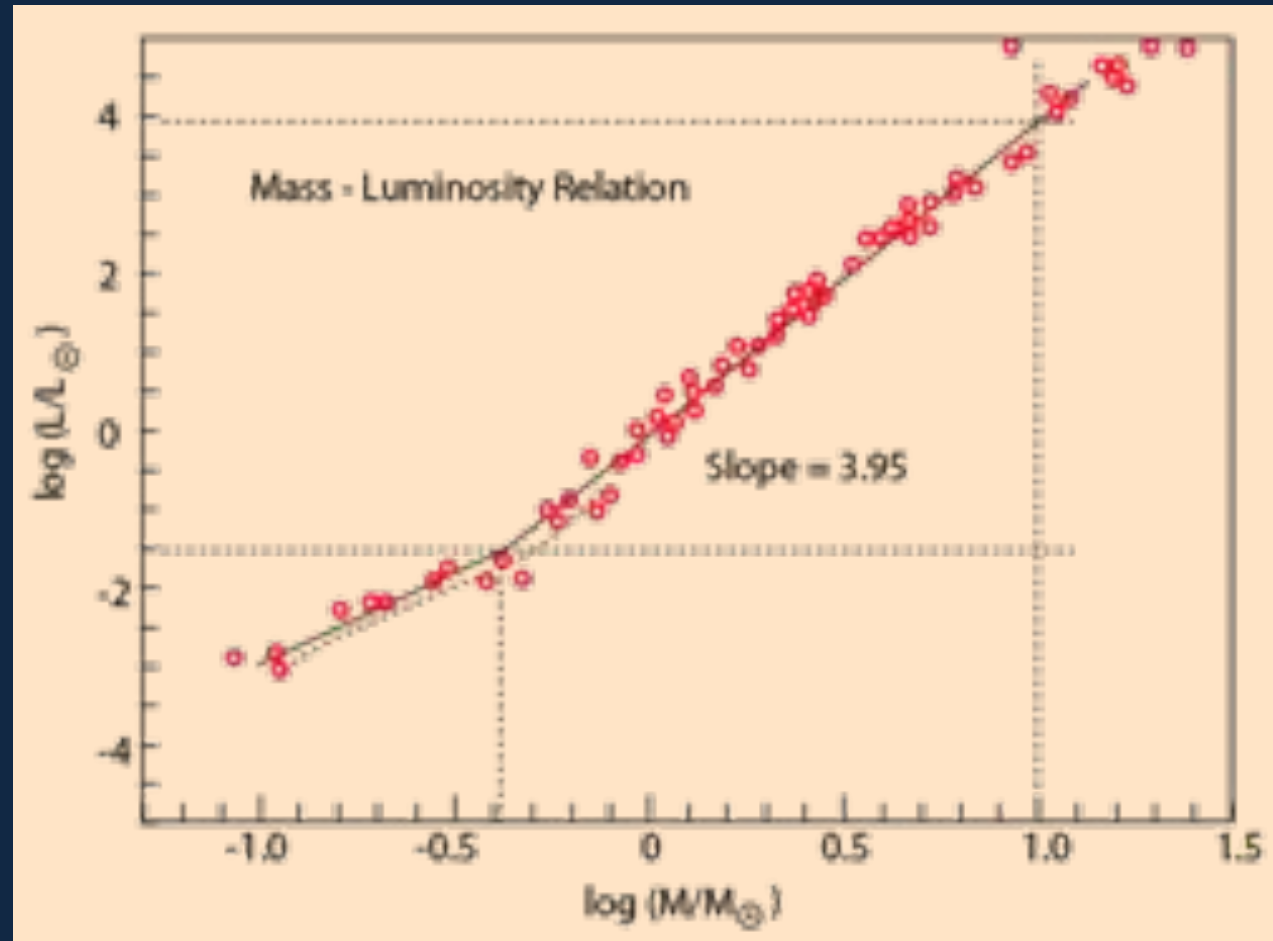
$$3 \approx a \approx 4$$

**Most massive:**

$\sim 100 M_{\text{sun}}$  (ish)

**Least massive:**

$\sim 0.01 M_{\text{sun}}$





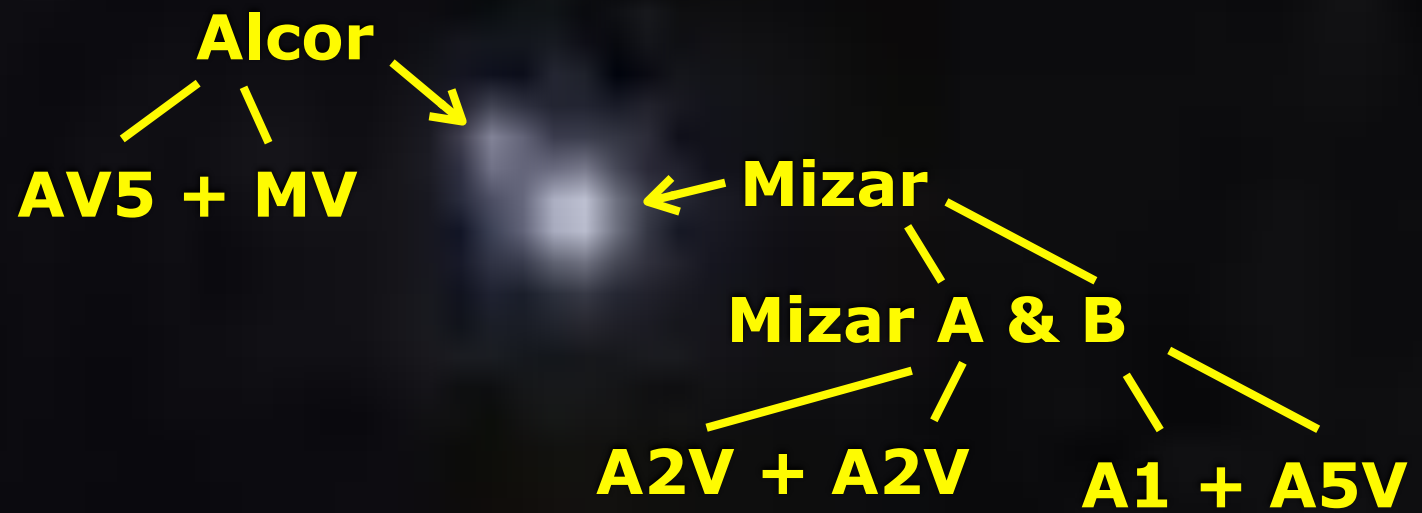
# Properties of Stars - Mass

---

## Determining Stellar Mass

Masses can be determined very precisely for stars in **spectroscopic binaries**.

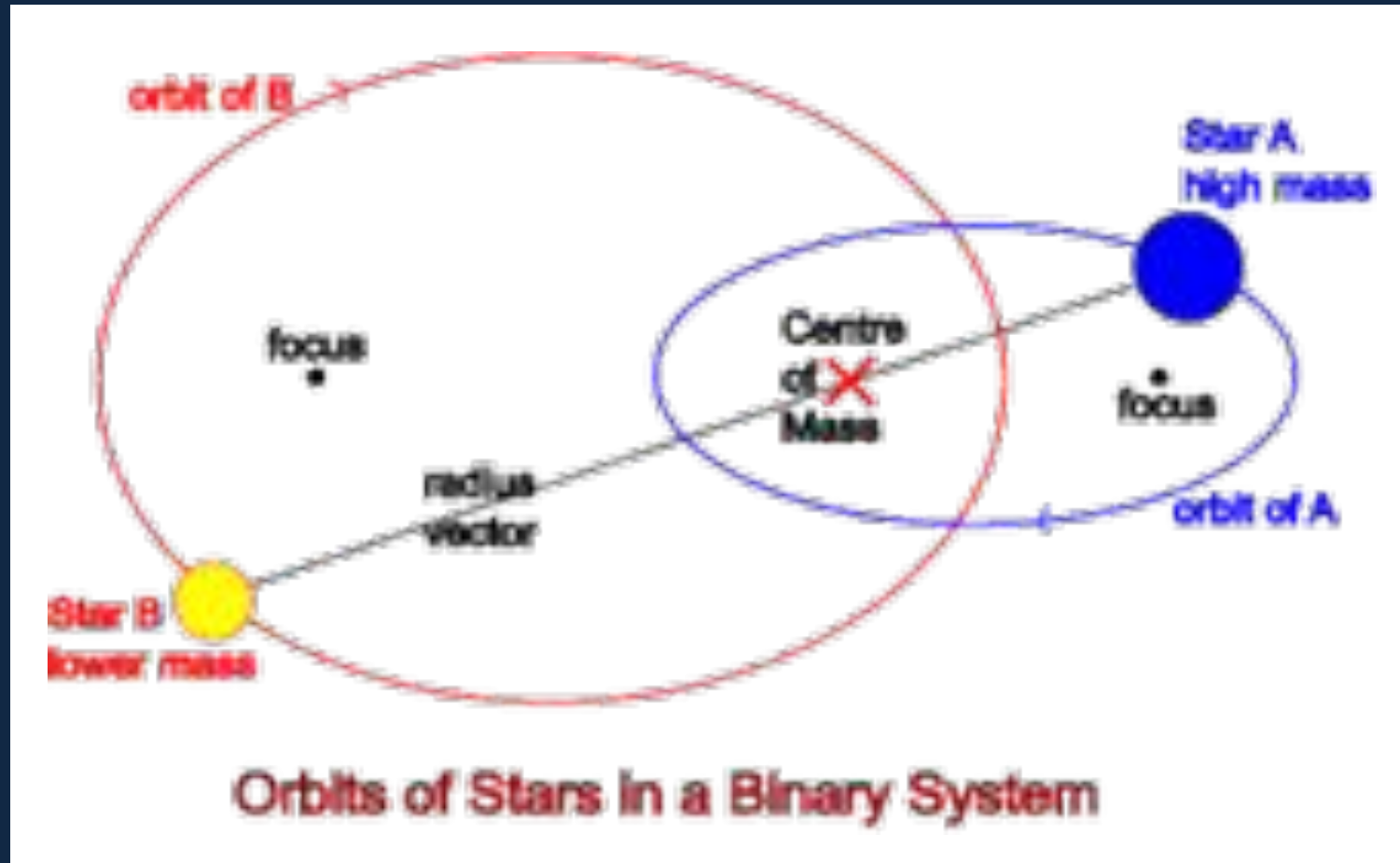




# Properties of Stars - Mass

## Determining Stellar Mass

A spectroscopic binary is a classic example of center-of-mass physics:



# Properties of Stars - Mass

---

## Determining Stellar Mass

A spectroscopic binary is a classic example of center-of-mass physics:



$$m_1 d_1 = m_2 d_2$$

$$v = 2\pi d/P \text{ so } d = vP/2\pi$$

$$m_1 v_1 P/2\pi = m_2 v_2 P/2\pi$$

# Properties of Stars - Mass

## Determining Stellar Mass

A spectroscopic binary is a classic example of center-of-mass physics:



$$m_1 d_1 = m_2 d_2$$

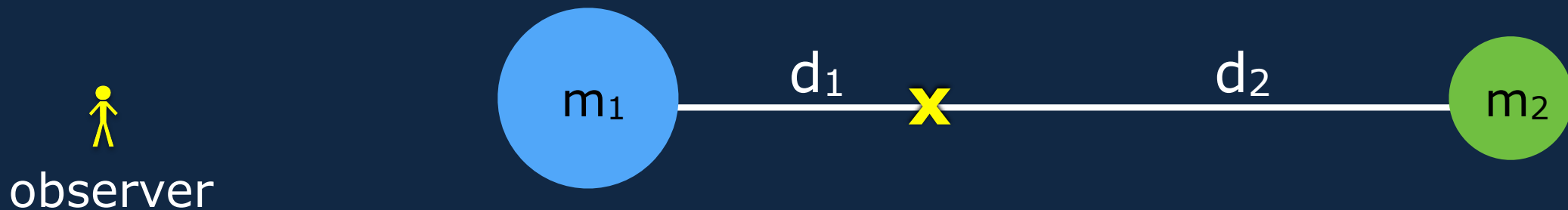
$$v = 2\pi d/P \text{ so } d = vP/2\pi$$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

# Properties of Stars - Mass

## Determining Stellar Mass

A spectroscopic binary is a classic example of center-of-mass physics:



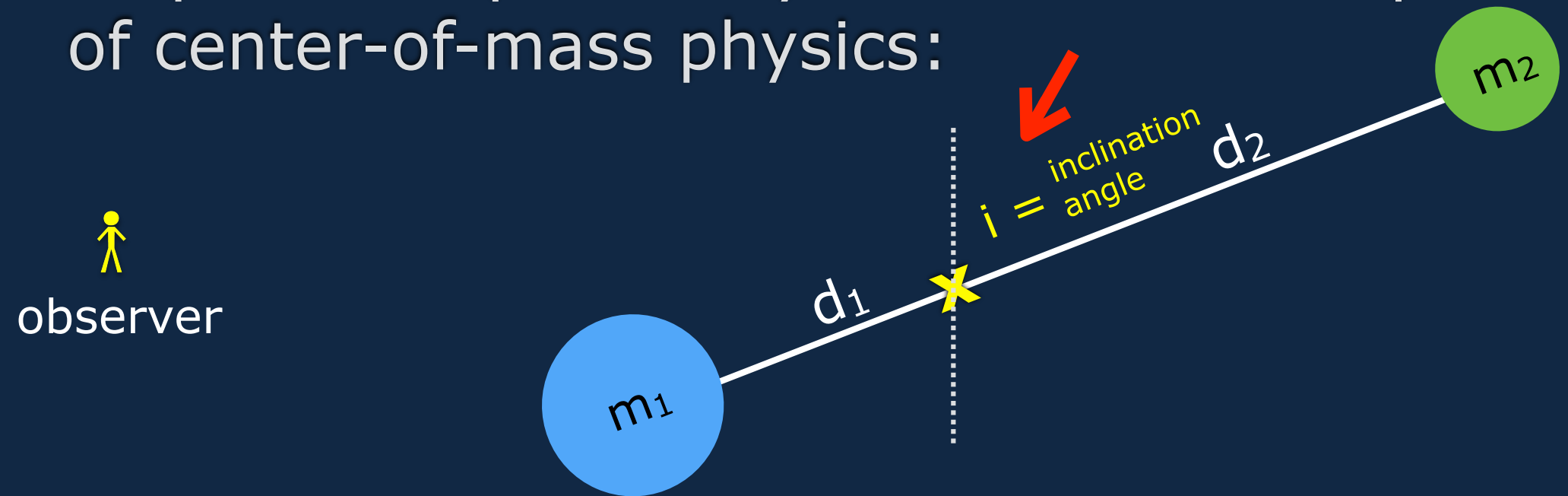
$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$



# Properties of Stars - Mass

## Determining Stellar Mass

A spectroscopic binary is a classic example of center-of-mass physics:



$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

# Properties of Stars - Mass

## Determining Stellar Mass

A spectroscopic binary is a classic example of center-of-mass physics



observer

$i = 0$ : face on

$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

inclination  
angle

$i =$

$d_1$

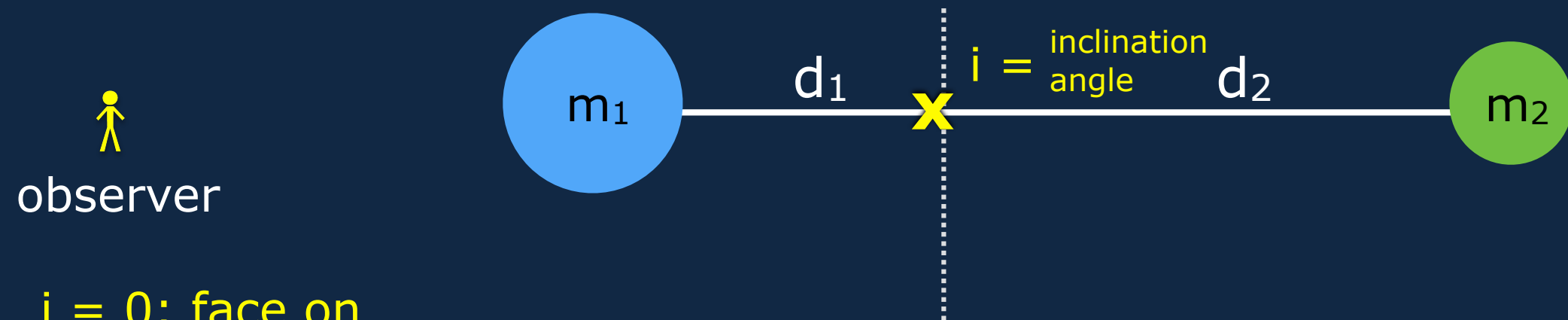
$m_2$

$m_1$

# Properties of Stars - Mass

## Determining Stellar Mass

A spectroscopic binary is a classic example of center-of-mass physics:



$i = 0$ : face on

$i = 90$ : along orbital  
plane

$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

# Properties of Stars - Mass

## Determining Stellar Mass

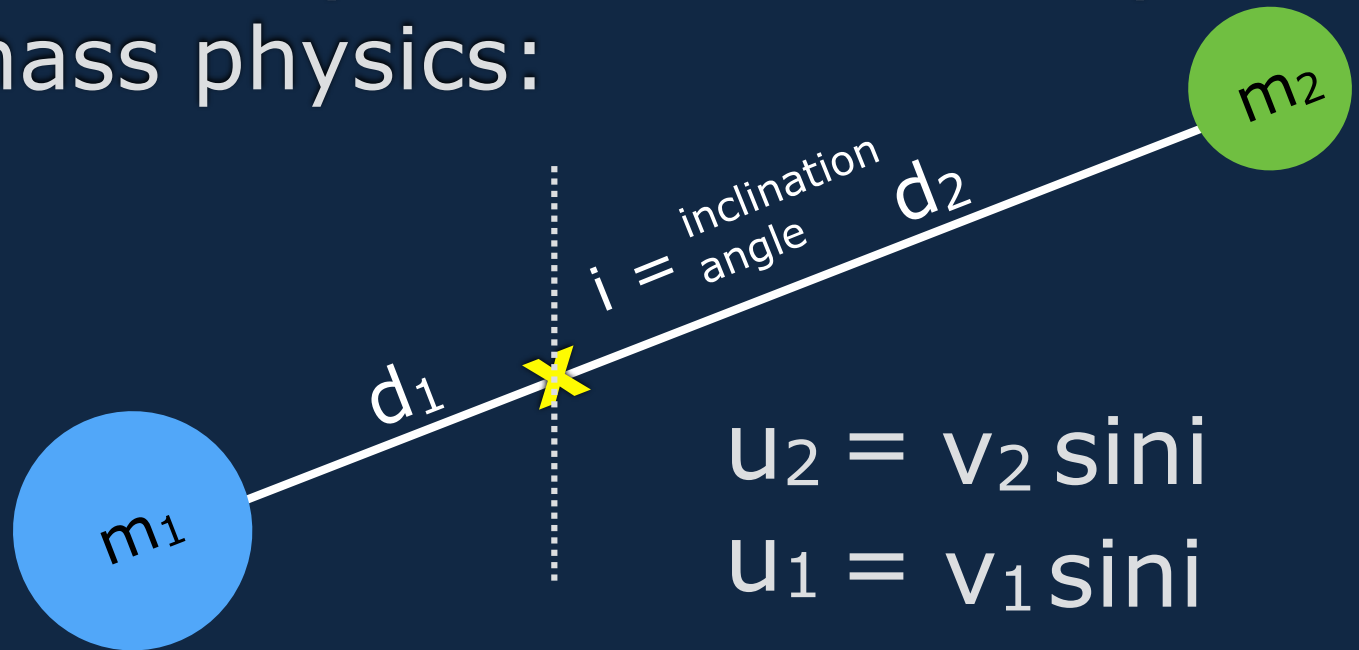
A spectroscopic binary is a classic example of center-of-mass physics:



observer

$i = 0$ : face on

$i = 90$ : along orbital plane



$$u_2 = v_2 \sin i$$

$$u_1 = v_1 \sin i$$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

# Properties of Stars - Mass

## Determining Stellar Mass

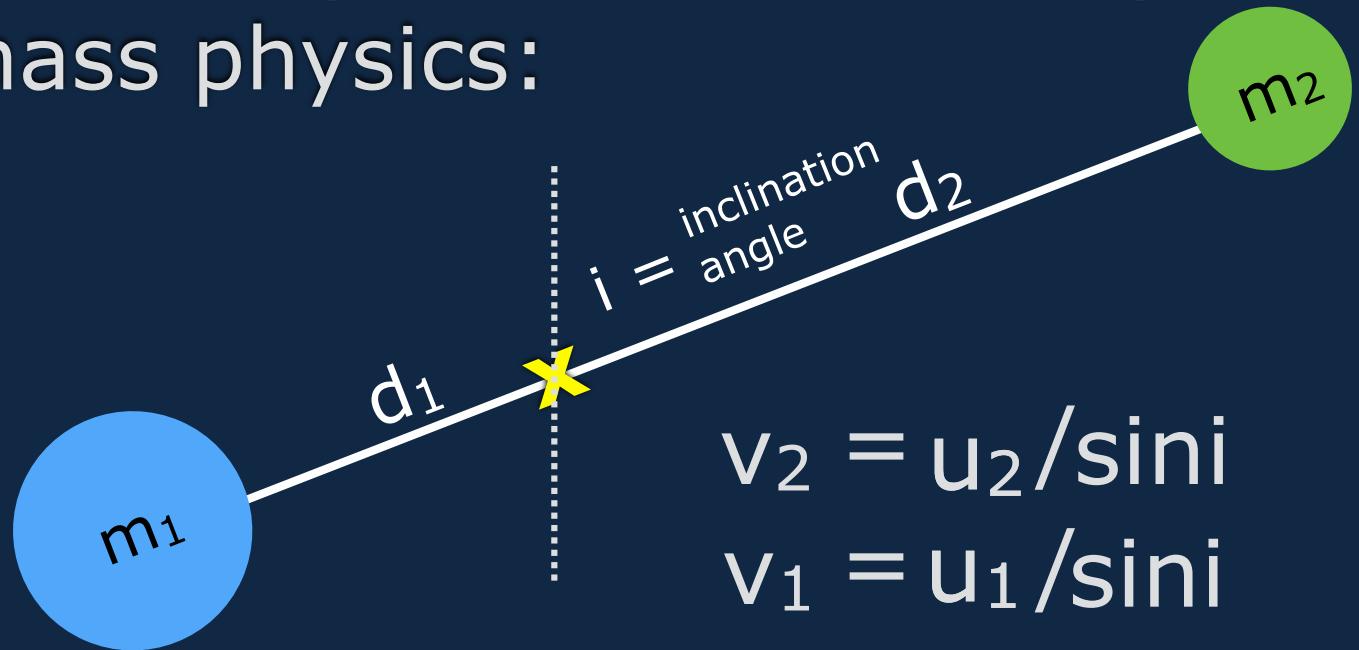
A spectroscopic binary is a classic example of center-of-mass physics:



observer

$i = 0$ : face on

$i = 90$ : along orbital plane



$$v_2 = u_2 / \sin i$$

$$v_1 = u_1 / \sin i$$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

# Properties of Stars - Mass

## Determining Stellar Mass

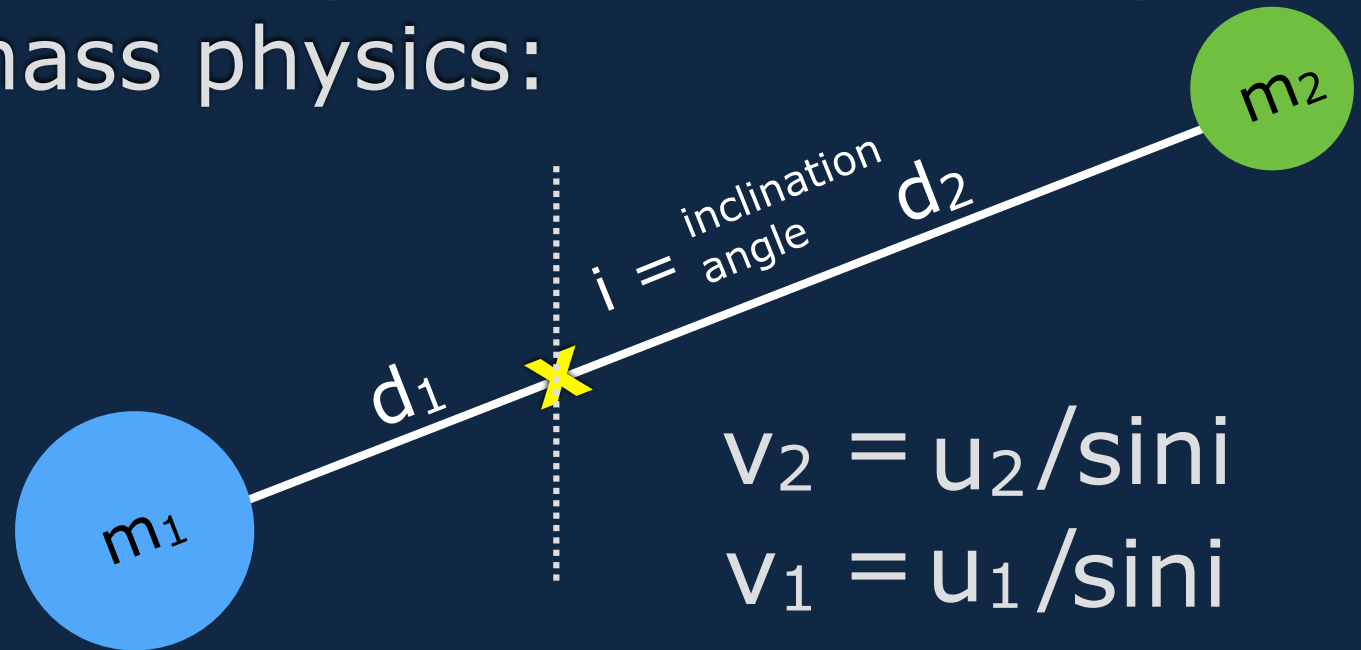
A spectroscopic binary is a classic example of center-of-mass physics:



observer

$i = 0$ : face on

$i = 90$ : along orbital plane



$$v_2 = u_2 / \sin i$$

$$v_1 = u_1 / \sin i$$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{u_2}{u_1}$$



# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{u_2}{u_1} , \quad v_1 = u_1 / \sin i$$

# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{u_2}{u_1} , \quad v_1 = u_1 / \sin i$$

We also know Kepler's 3rd Law:

$$m_1 + m_2 = \frac{4\pi^2 (a_1 + a_2)^3}{GP^2} , \quad v_1 = \frac{2\pi a_1}{P}$$

where  $a_1$  and  $a_2$  are the semimajor axes of the two masses' elliptical orbits.

# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{u_2}{u_1} ,$$

$$m_1 + m_2 = \frac{4\pi^2 (a_1 + a_2)^3}{GP^2} ,$$

$$\frac{2\pi a_1}{P} = u_1 / \sin i$$

# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{u_2}{u_1} ,$$

$$m_1 + m_2 = \frac{4\pi^2 (a_1 + a_2)^3}{GP^2} ,$$

$$a_1 = \frac{u_1 P}{2\pi \sin i}$$

$$a_2 = \frac{u_2 P}{2\pi \sin i}$$

# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{u_2}{u_1} ,$$

$$m_1 + m_2 = \frac{4\pi^2 (a_1 + a_2)^3}{GP^2} ,$$

$$a_1 + a_2 = \frac{(u_1 + u_2) P}{2\pi \sin i}$$

# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{u_2}{u_1} ,$$

$$m_1 + m_2 = \frac{4\pi^2 \left( \frac{(u_1 + u_2) P}{2\pi \sin i} \right)^3}{GP^2}$$



# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{u_2}{u_1}$$

$$m_1 + m_2 = \frac{P (u_1 + u_2)^3}{2G\pi \sin^3 i}$$

# Properties of Stars - Mass

---

## Determining Stellar Mass

$$\frac{m_1}{m_2} = \frac{u_2}{u_1}$$

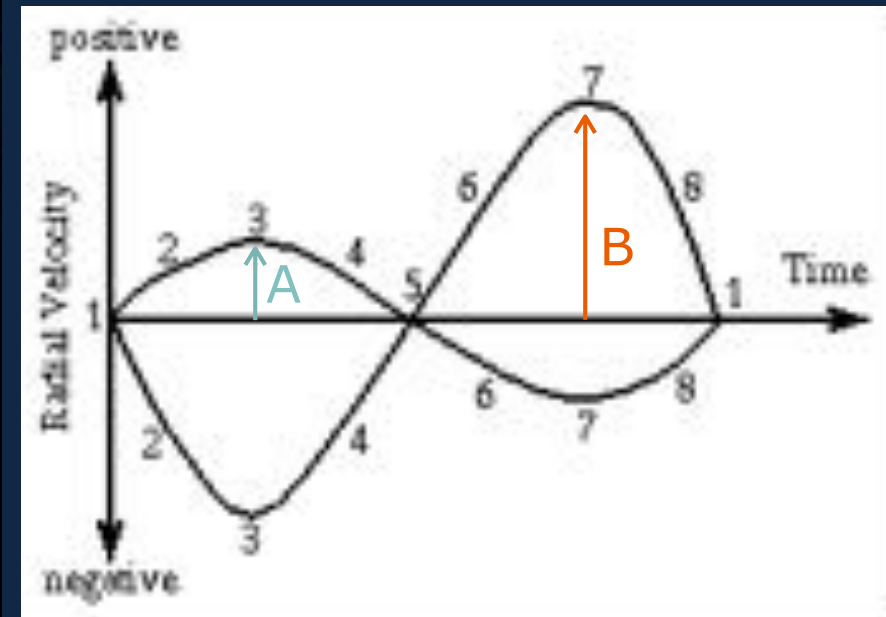
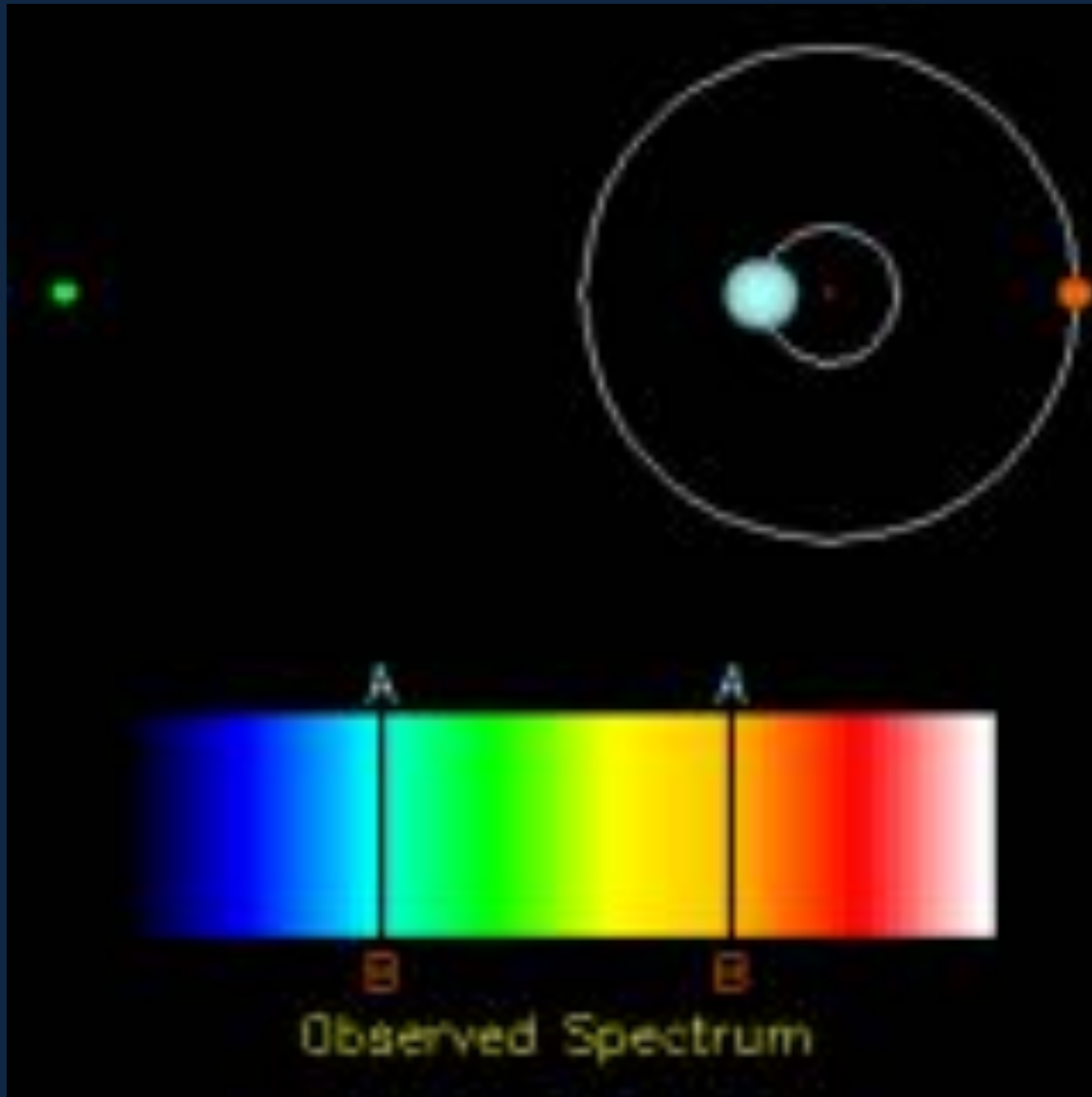
and

$$m_1 + m_2 = \frac{P (u_1 + u_2)^3}{2G\pi \sin^3 i}$$

give us masses from a spectroscopic binary

# Properties of Stars - Mass

## Determining Stellar Mass



# Properties of Stars - Radius

---

- 1) Distance - parallax, Cepheids
- 2) Velocity - proper motion, radial velocity
- 3) Brightness - magnitudes, luminosity
- 4) Temperature - effective temp (usually)
- 5) Mass - luminosity, binaries
- 6) Radius

# Properties of Stars - Radius

---

## Determining Stellar Radius

Lunar occultation

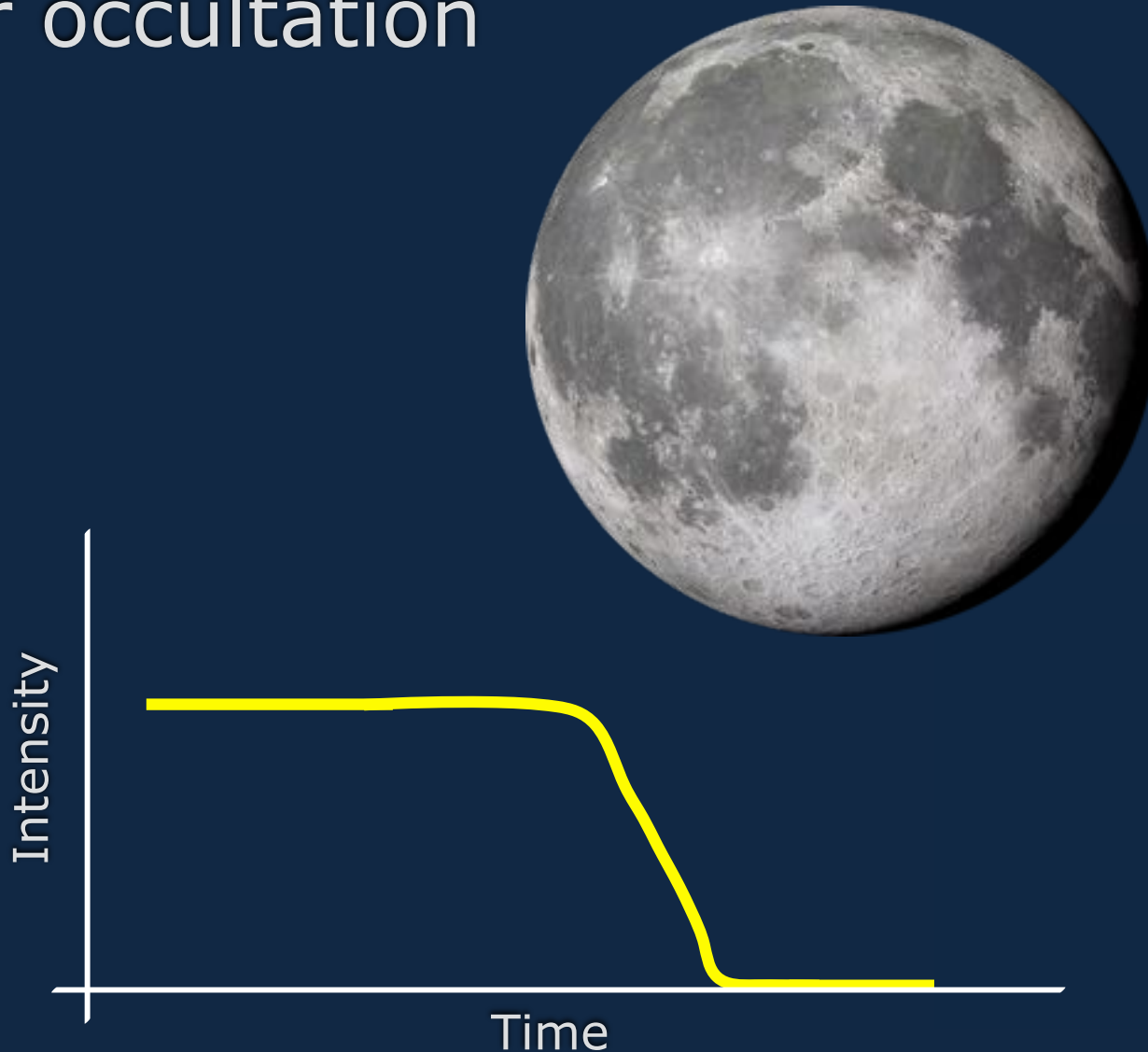


# Properties of Stars - Radius

---

## Determining Stellar Radius

Lunar occultation

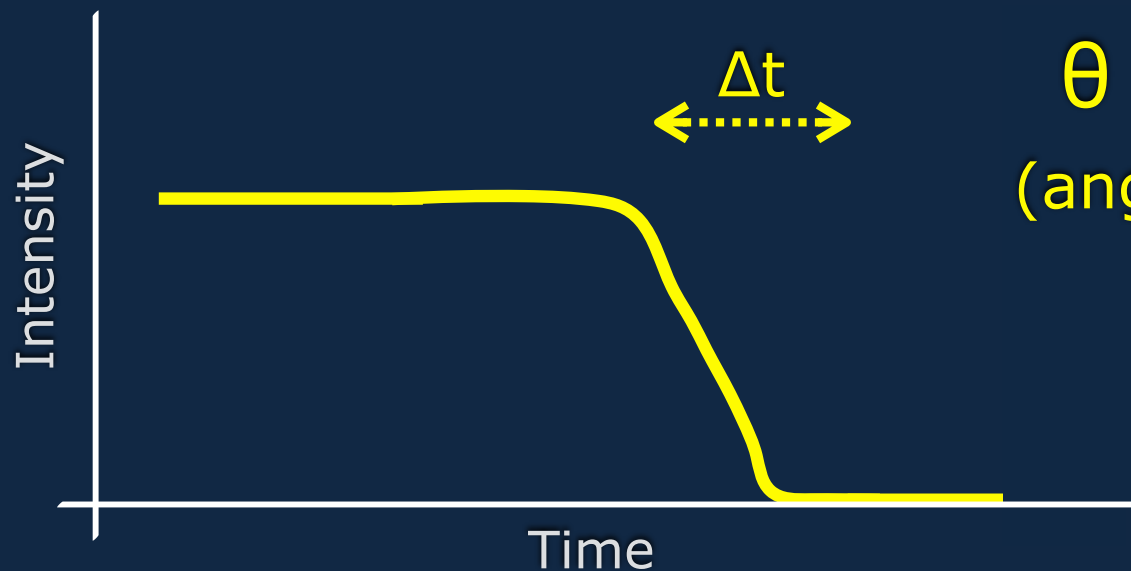




# Properties of Stars - Radius

## Determining Stellar Radius

Lunar occultation



$$\theta = v_{\text{Moon}} \Delta t$$

(angular distance)

# Properties of Stars - Radius

## DISCUSSION QUESTION

Which of the following factors limit the use of the lunar occultation technique?

I: star brightness

II: star size

III: star position

IV: star temperature

A) II and III

C) III only

**B) I, II, and III**

D) all of the above

# Properties of Stars - Radius

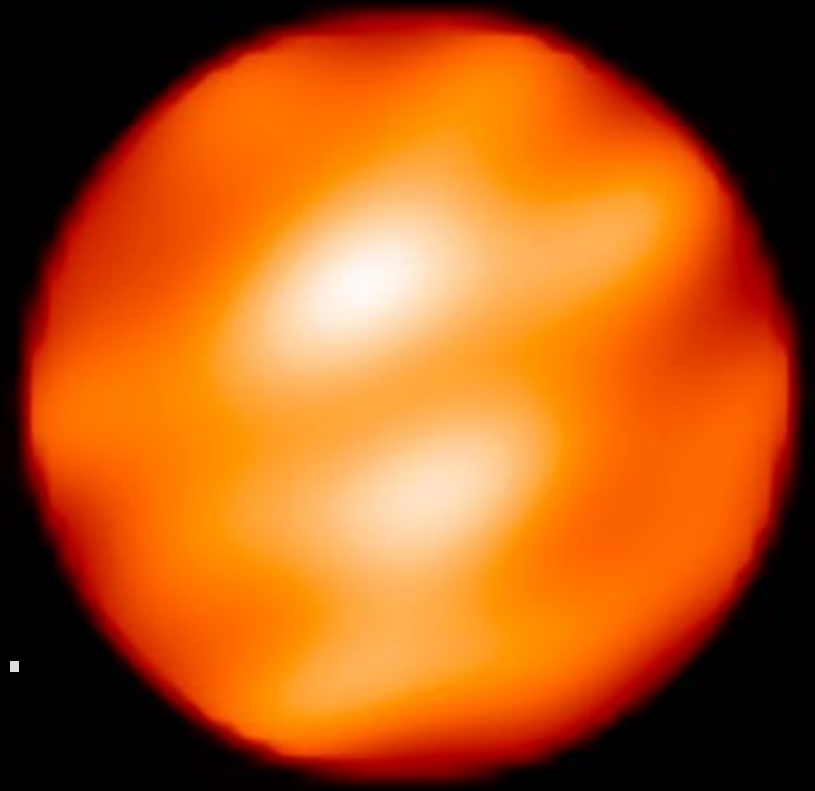
---

## Determining Stellar Radius

Lunar occultation  
Interferometry

We can use interferometry to push down towards diffraction-limited astro and image very large stars.

Easier at longer wavelengths.



Betelgeuse in the IR

10 mas

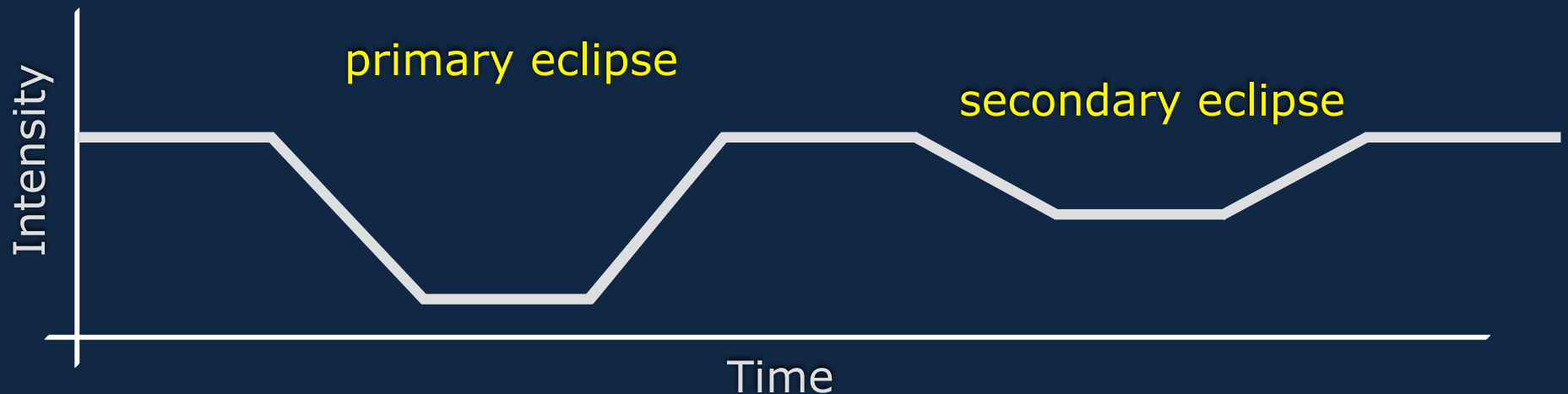
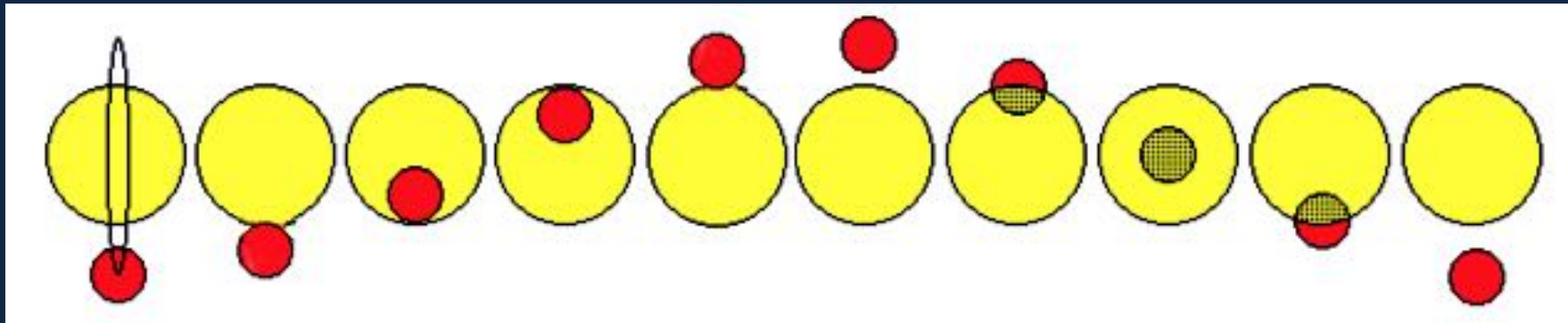
# Properties of Stars - Radius

## Determining Stellar Radius

Lunar occultation

Interferometry

Eclipsing binaries



# Properties of Stars - Radius

---

## Determining Stellar Radius

Lunar occultation

Interferometry

Eclipsing binaries

from T and L ( $L = 4\pi R^2 \sigma T^4$ )

# Properties of Stars - Radius

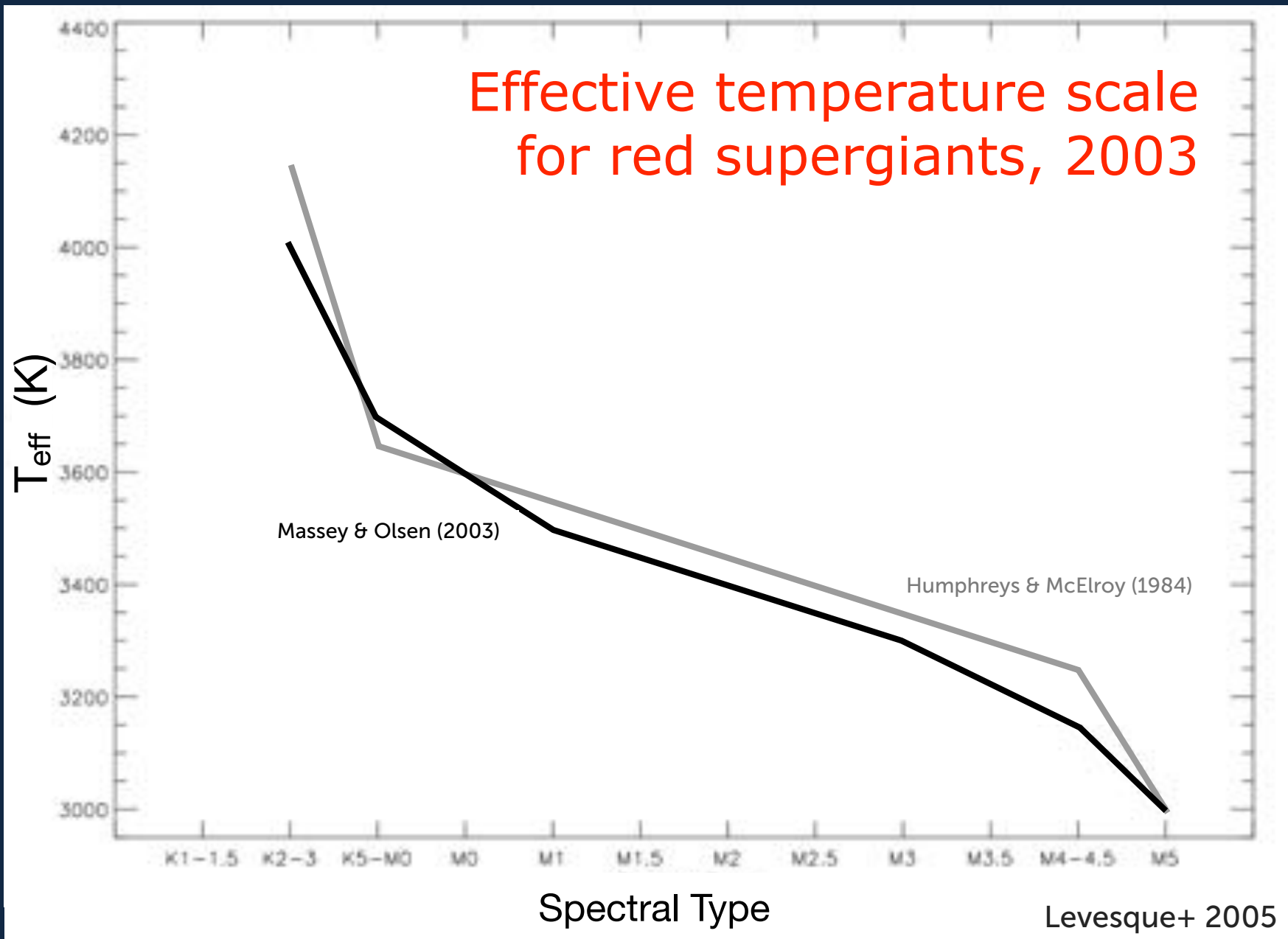
## Determining Stellar Radius

Smallest stars: neutron stars; ~city of Seattle

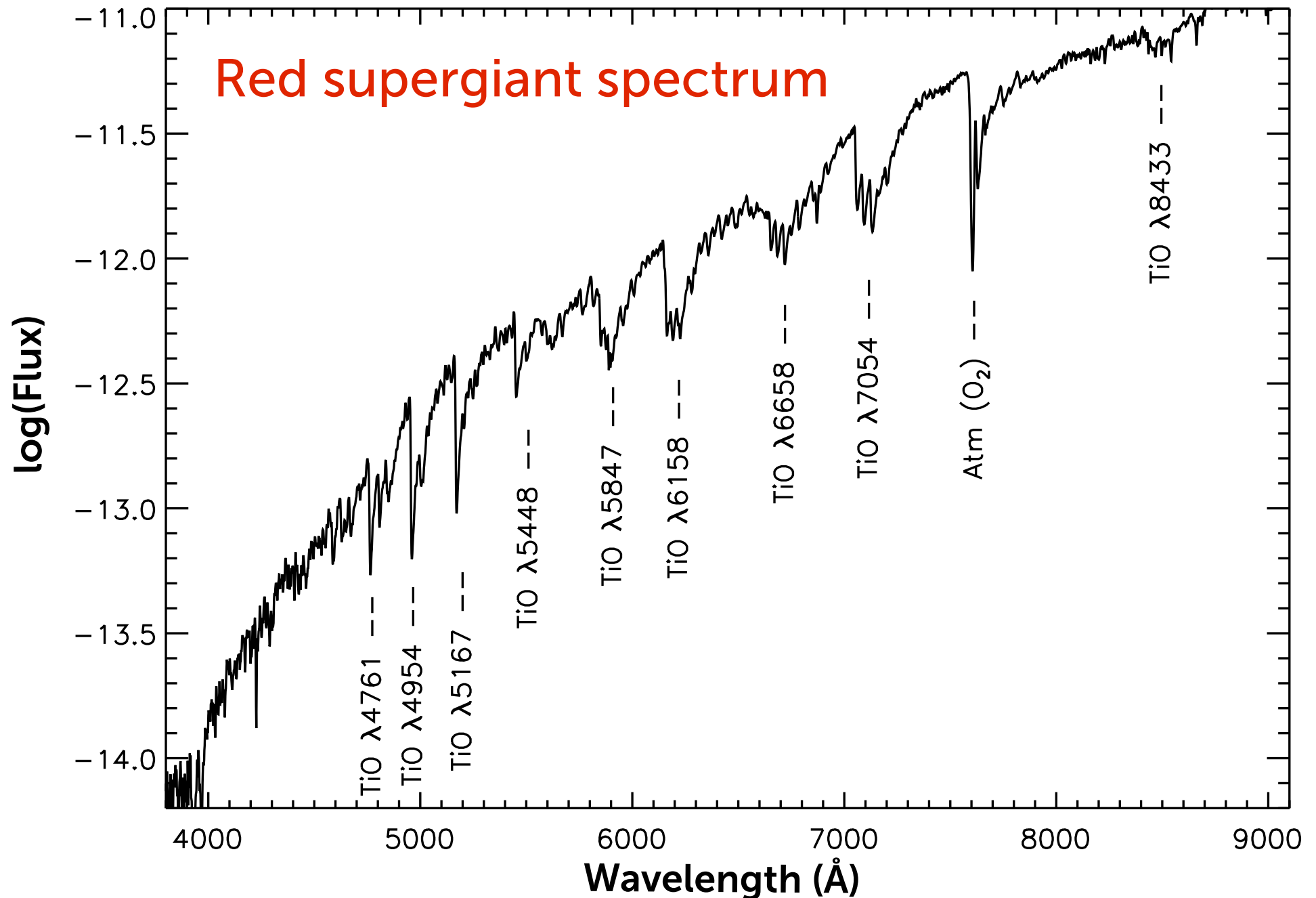
Largest stars: red supergiants...



# Properties of Stars - Radius



# Properties of Stars - Radius

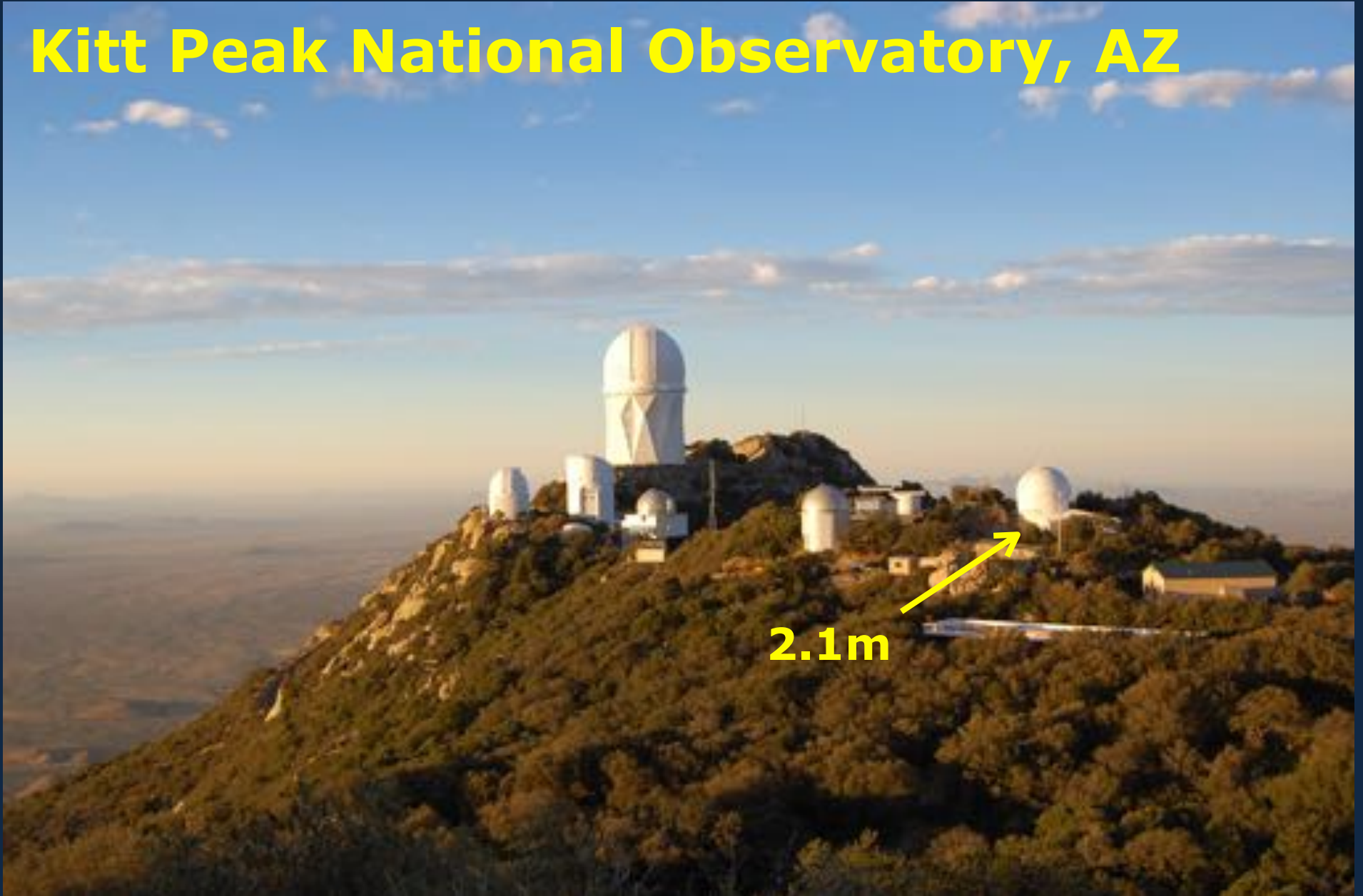




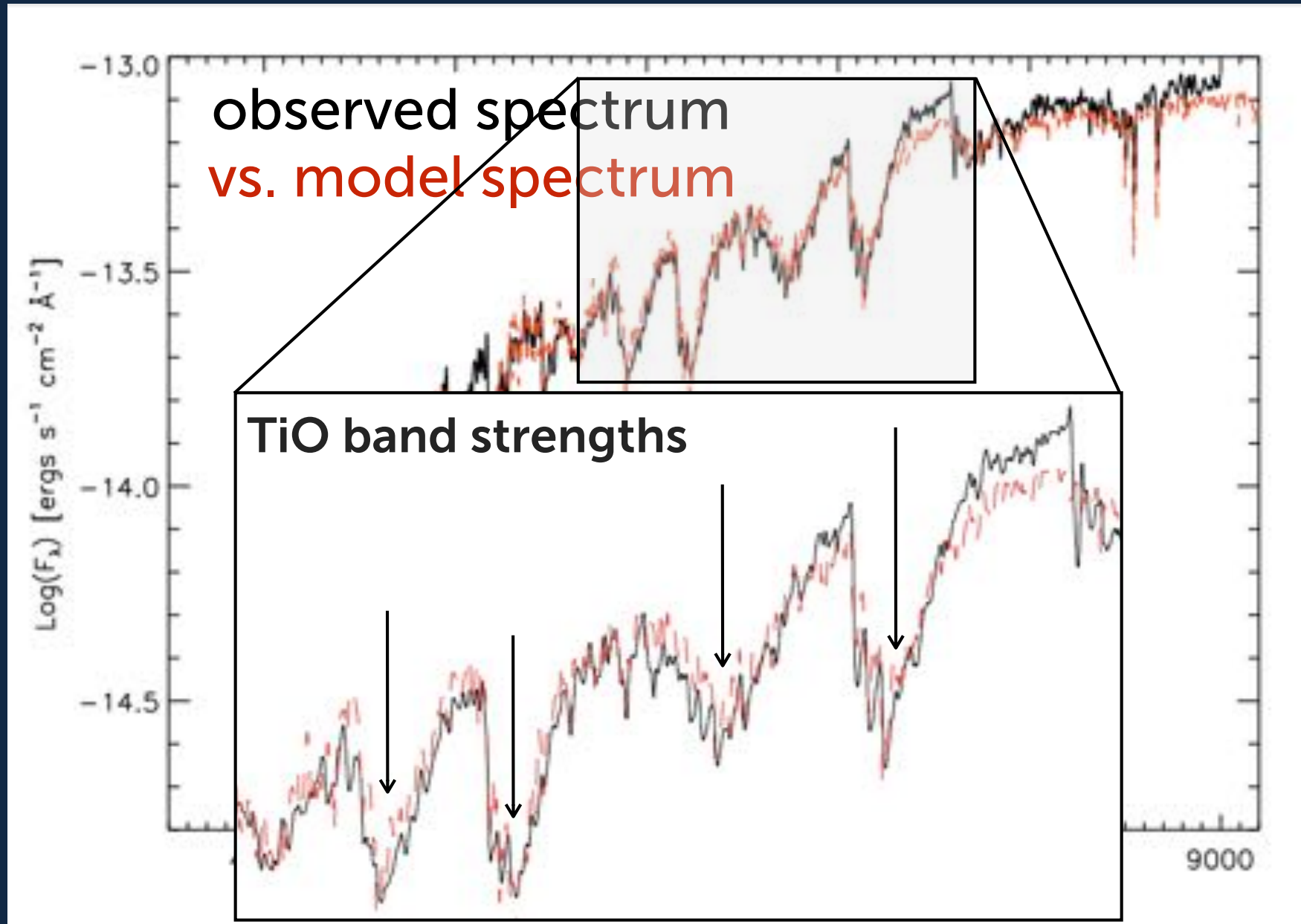
# Properties of Stars - Radius

---

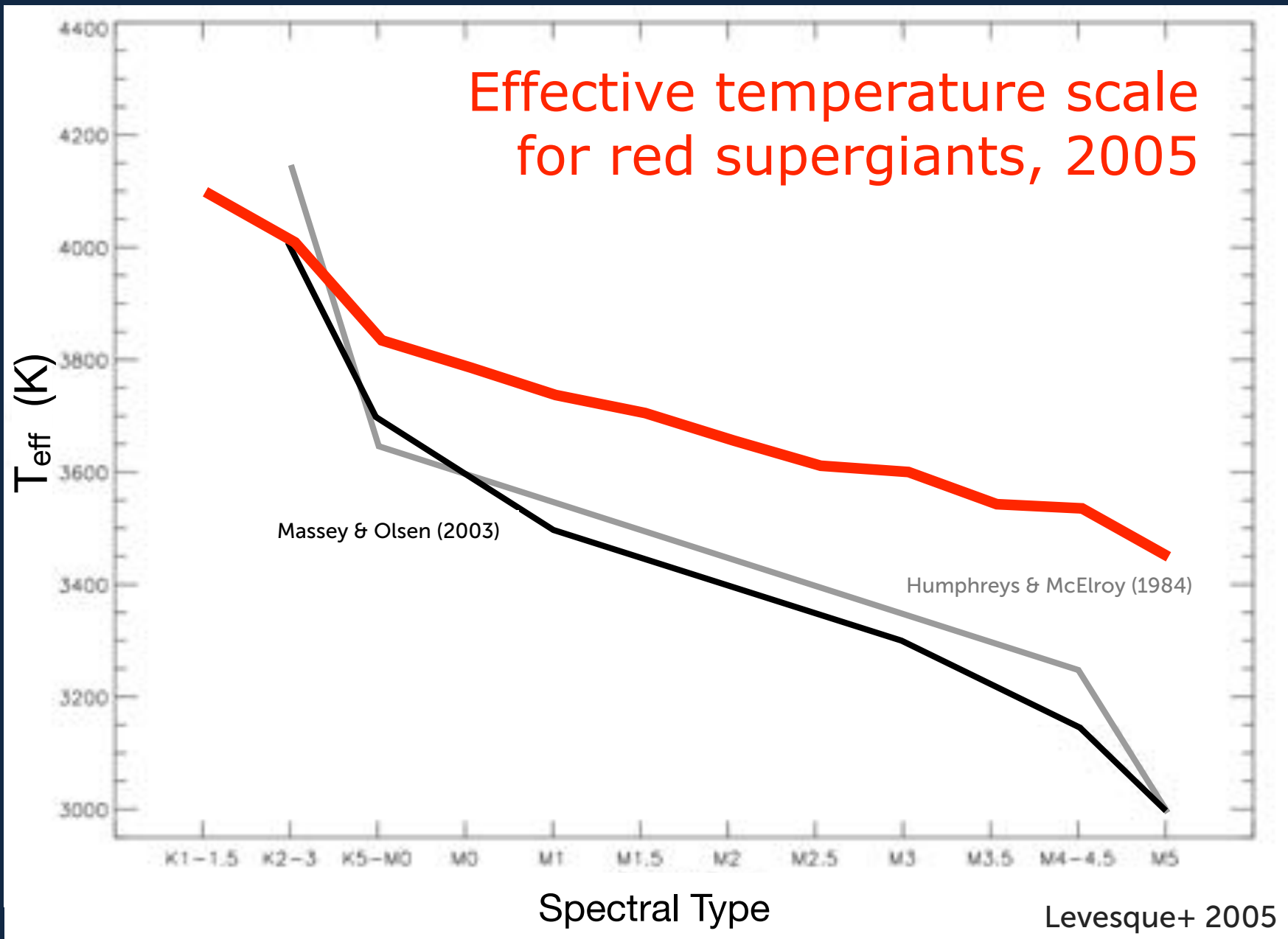
## Kitt Peak National Observatory, AZ



# Properties of Stars - Radius



# Properties of Stars - Radius



# Properties of Stars - Radius

$$L = 4\pi R^2 \sigma T^4$$

Star	Sp Type	V	B-V	m-M	T <sub>eff</sub> (K)	M <sub>bol</sub>	R/R <sub>sun</sub>
HD 90382	M3-M4 I	7.45	2.21	11.7	3550	-8.27	1060
HD 91093	M2 I	8.31	2.21	11.9	3625	-8.26	640
HD 303250	M2 I	8.92	2.51	11.9	3625	-7.60	750
HD 93420	M4 I	7.55	1.87	12.0	3525	-7.60	790
HD 94096	M2 I	7.38	2.24	12.0	3650	-8.08	920
HD 95687	M3 I	7.35	2.12	11.6	3625	-7.63	760
HD 95950	M2 I	6.75	2.04	11.6	3700	-7.54	700
V396 Cen	M3-M4 I	7.85	2.15	11.6	3550	-8.29	1070
HD 160371	K2.5 I	6.14	1.82	8.4	3900	-3.63	100
KY Cygni	M3-M4 I	10.57	3.64	11.0	3500	-10.36	2850
HD 339034	K3 I	9.36	3.05	11.8	4000	-8.63	980

Levesque et al. (2005)

# Properties of Stars - Radius

---

## Determining Stellar Radius

Smallest stars: neutron stars; ~city of Seattle

Largest stars: red supergiants; ~orbit of Jupiter





# Properties of Stars - Radius

## NSF REU Programs in Astronomy

- ~2-month summer research internships at research sites around the world
- **paid** research with an REU adviser or group
- apply directly to specific sites; typical application includes essay, transcripts, and reference letters
- most deadlines are ~Jan-Feb
- *excellent* opportunity to present, publish, and strengthen graduate school applications

# Properties of Stars - Radius

## NSF REU Programs in Astronomy

### REU Sites: Astronomical Sciences

Please report errors in the list below by writing to [reu.ast@nsf.gov](mailto:reu.ast@nsf.gov).

[Search Again](#)

Export results: [CSV](#) | [Excel](#) | [XML](#)

24 items found, displaying 1 to 20.

[\[First/Prev\]](#) [1 2](#) [\[Next/Last\]](#)

Site Information	Site Location	Contact Information	Additional Information
<b>American Museum of Natural History</b> <a href="#">REU Program in Earth Sciences and Astrophysics at the American Museum of Natural History</a> Division of Physical Sciences	New York, New York	<b>Primary:</b> Dr. Charles Liu (212) 496-3579 <a href="mailto:cliu@amnh.org">cliu@amnh.org</a> <b>Secondary:</b> James D. Webster (212) 769-5401 <a href="mailto:jdw@amnh.org">jdw@amnh.org</a>	<b>Research Topics/Keywords:</b> earth science, planetary science, astrophysics <a href="#">Abstract of Award</a> <b>Cofunded:</b> Earth Sciences
<b>Arecibo Observatory</b> <a href="#">REU at the Arecibo Observatory</a>	Arecibo, Puerto Rico	<b>Primary:</b> Dr. Hilda Colón (787) 878-2812 <a href="mailto:reu-program@naic.edu">reu-program@naic.edu</a>	<b>Research Topics/Keywords:</b> astronomy, astrophysics, planetary science, instrumentation, atmospheric physics <a href="#">Abstract of Award</a> <b>Cofunded:</b> Atmospheric and Geospace Sciences
<b>Boston University</b> <a href="#">Magnetic Fields on Planetary to</a>	Boston, Massachusetts	<b>Primary:</b> Prof. Andrew West (617) 358-5879 <a href="mailto:west@bu.edu">west@bu.edu</a>	<b>Research Topics/Keywords:</b> Astrophysics, space physics, planetary science, magnetic fields. <a href="#">Abstract of Award</a>

**Link on class website!**

Saturn

# Properties of Stars

---

- 1) Distance - parallax, Cepheids
- 2) Velocity - proper motion, radial velocity
- 3) Brightness - magnitudes, luminosity
- 4) Temperature - effective temp (usually)
- 5) Mass - luminosity, binaries
- 6) Radius - lunar, interferometry,  
binaries, L & T



# Exam #1 - Policies

PAA A10

- format: short answers (problems, paragraphs)
- exam goes from 1:30pm - 2:50pm
- scientific calculators are the **only** devices allowed
- one 8.5x11 page of notes (both sides) is allowed

You will be given this:

USEFUL CONSTANTS	
length of a solar day:	86,400 s
length of a calendar year:	$3.156 \times 10^7$ s
radius of sun ( $R_{\odot}$ ):	$6.96 \times 10^8$ m
luminosity of sun ( $L_{\odot}$ ):	$4 \times 10^{33}$ ergs s <sup>-1</sup>
$M_{\text{bol}}$ of sun:	4.74 mag
mass of sun ( $M_{\odot}$ ):	$1.99 \times 10^{30}$ kg
$T_{\text{eff}}$ of sun:	5777 K
1 parsec:	$3.086 \times 10^{16}$ m
1 AU:	$1.5 \times 10^{11}$ m
Rydberg constant ( $R$ ):	$1.097 \times 10^7$ m <sup>-1</sup>
Planck's constant ( $h$ ):	$6.63 \times 10^{-27}$ ergs s
Boltzmann constant ( $k$ ):	$1.381 \times 10^{-16}$ erg s <sup>-1</sup> K <sup>-1</sup>
Stefan-Boltzmann constant ( $\sigma$ ):	$0.567$ erg cm <sup>-2</sup> s <sup>-1</sup> K <sup>-4</sup>
Gravitational constant ( $G$ ):	$6.67 \times 10^{-11}$ m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>

You will **not** be given:

- equations
- relations
- basic units (microns, angstroms, Hz)

# Coming up next...

---

**Thursday, Oct 26: exam review  
slidecast; link on class website!**

**exam prep: extra office hours  
Mon Oct 30: 10-11am, 1-4pm**

**Tuesday, Oct 31: Exam #1**