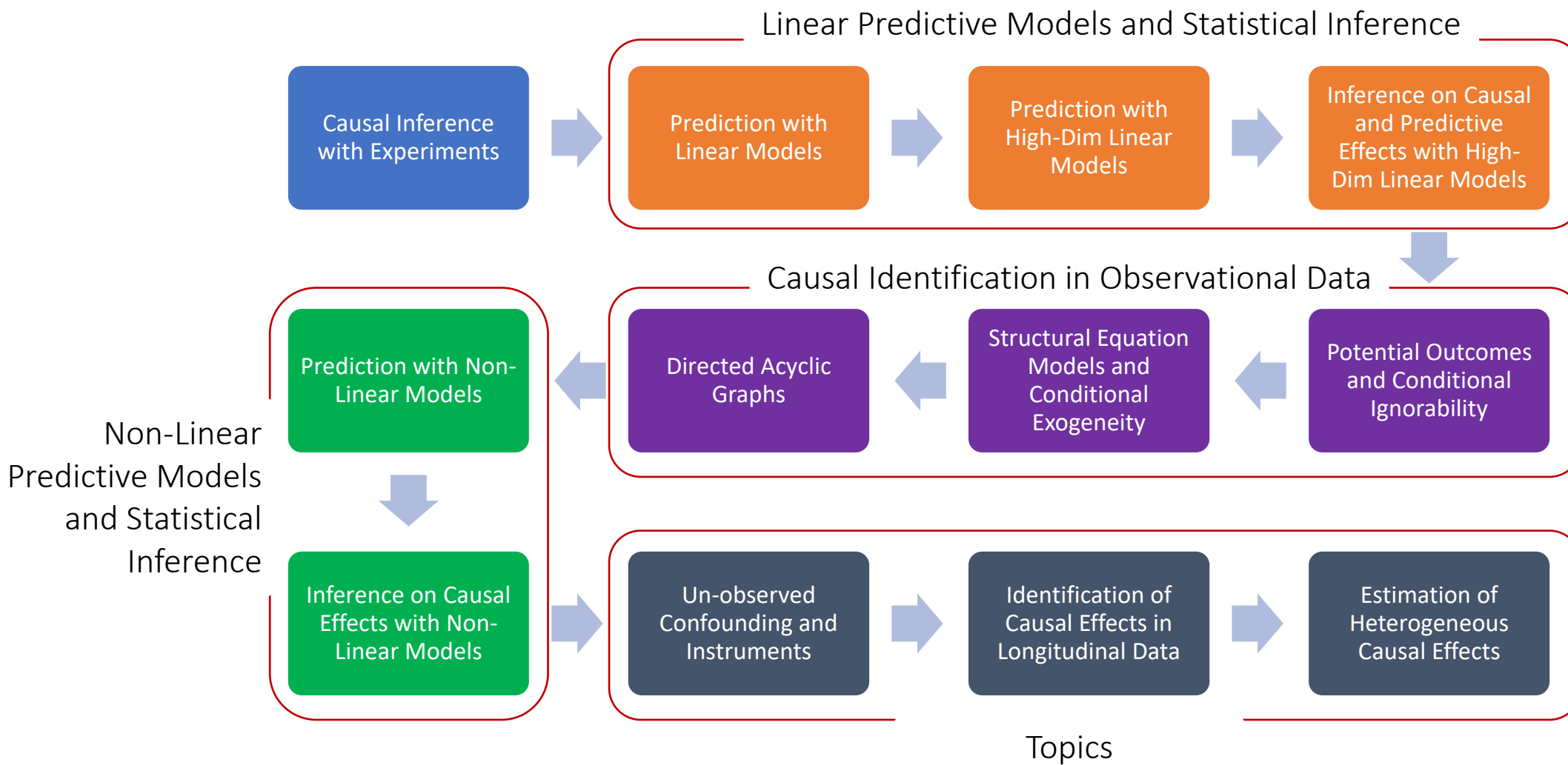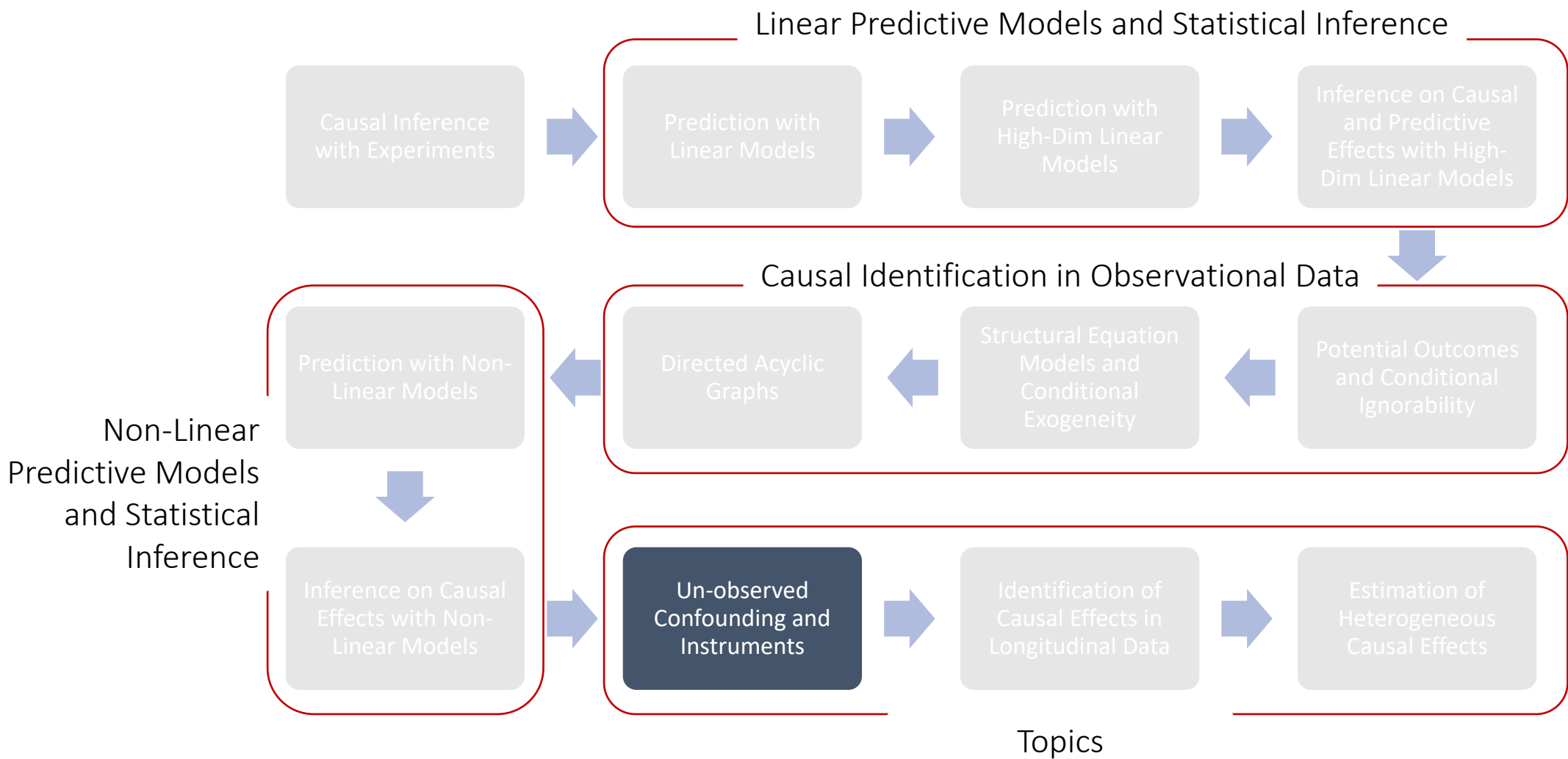# MS&E 228: Unobserved Confounding

Vasilis Syrgkanis

MS&E, Stanford
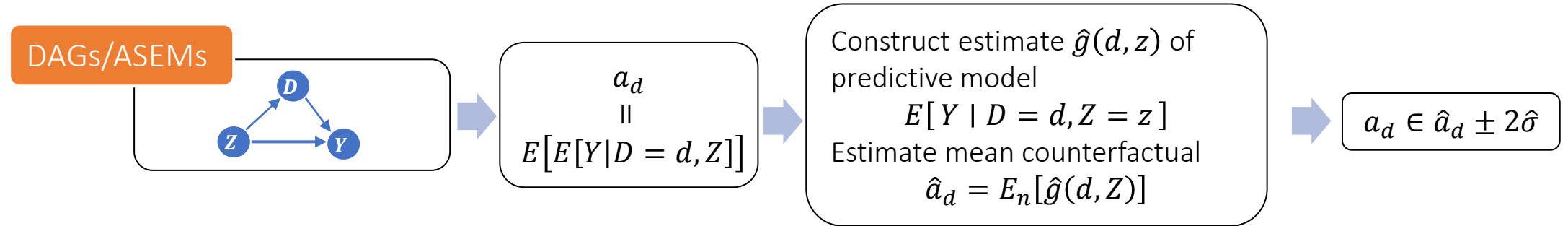
Goals for Today

- What can we do when we have un-observed confounding

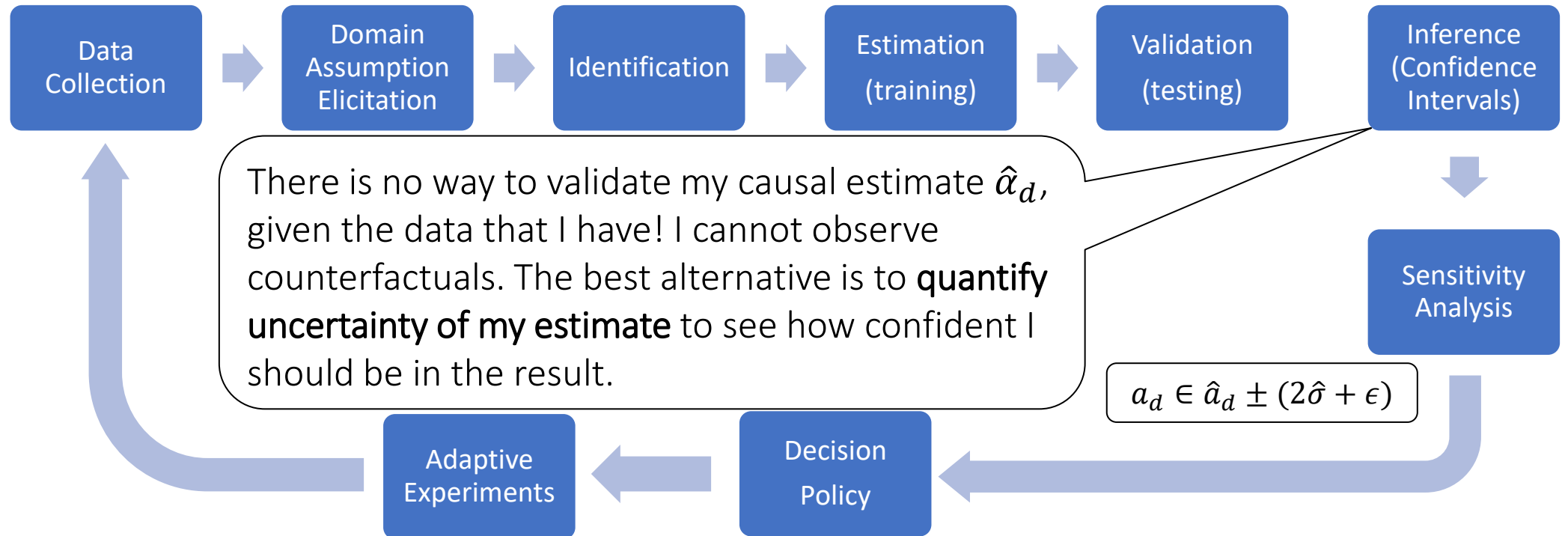- Omitted variable bias bounds

- Introduction to "Instruments"

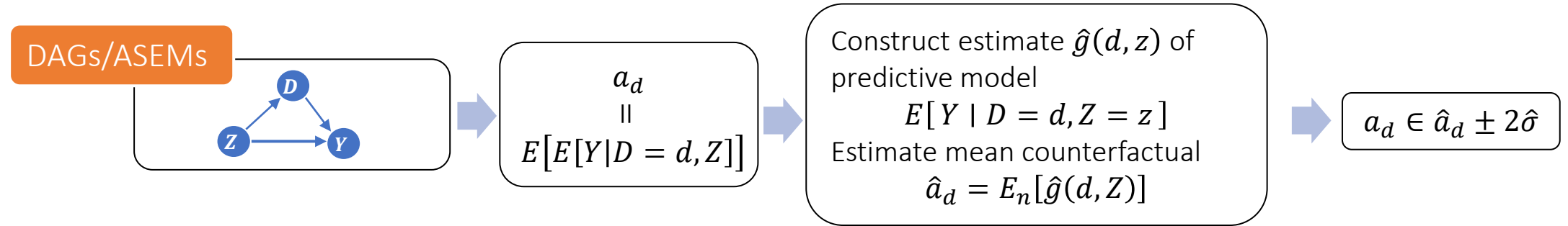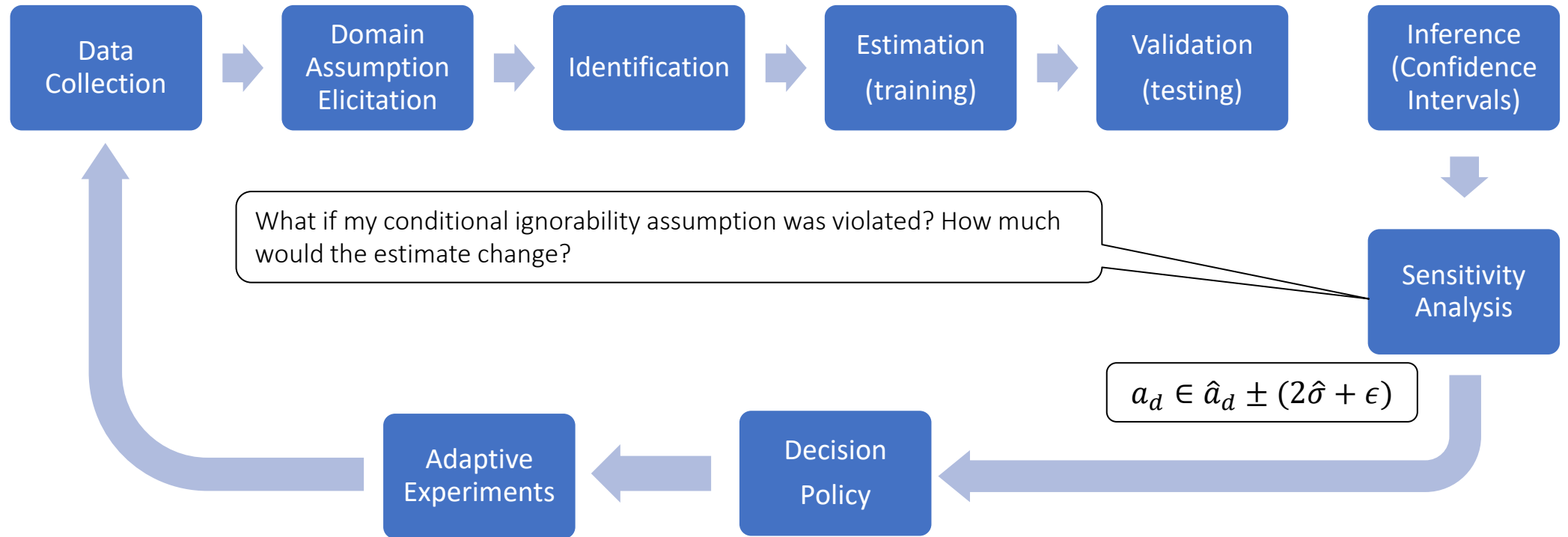# Recap of Last Lecture

Causal Inference Pipeline

Theory

DAGs/ASEMs

$$a_d$$
$$\|$$
$$E[E[Y|D=d,Z]]$$

Construct estimate $\hat{g}(d,z)$ of predictive model
$$E[Y \mid D=d, Z=z]$$
Estimate mean counterfactual
$$\hat{a}_d = E_n[\hat{g}(d,Z)]$$

$$a_d \in \hat{a}_d \pm 2\hat{\sigma}$$

Practice

Data Collection → Domain Assumption Elicitation → Identification → Estimation (training) → Validation (testing) → Inference (Confidence Intervals)

There is no way to validate my causal estimate $\hat{\alpha}_d$, given the data that I have! I cannot observe counterfactuals. The best alternative is to **quantify uncertainty of my estimate** to see how confident I should be in the result.

Inference (Confidence Intervals) → Sensitivity Analysis

$$a_d \in \hat{a}_d \pm (2\hat{\sigma} + \epsilon)$$

Adaptive Experiments ← Decision Policy ← Sensitivity Analysis

Causal Inference Pipeline

Theory

DAGs/ASEMs

$$a_d$$
$$\parallel$$
$$E\big[E[Y|D = d, Z]\big]$$

Construct estimate $\hat{g}(d, z)$ of predictive model
$$E[Y \mid D = d, Z = z]$$
Estimate mean counterfactual
$$\hat{a}_d = E_n[\hat{g}(d, Z)]$$

$$a_d \in \hat{a}_d \pm 2\hat{\sigma}$$

| Data Collection | → | Domain Assumption Elicitation | → | Identification | → | Estimation (training) | → | Validation (testing) | Inference (Confidence Intervals) |

What if my conditional ignorability assumption was violated? How much would the estimate change?

Sensitivity Analysis

$$a_d \in \hat{a}_d \pm (2\hat{\sigma} + \epsilon)$$

Adaptive Experiments ← Decision Policy ←

Practice

Causal Inference Pipeline

Theory

DAGs/ASEMs

$$a_d$$
$$\parallel$$
$$E\big[E[Y|D=d,Z]\big]$$

Construct estimate $\hat{g}(d,z)$ of predictive model
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Estimate mean counterfactual
$$\hat{a}_d = E_n[\hat{g}(d,Z)]$$

$$a_d \in \hat{a}_d \pm 2\hat{\sigma}$$

Practice

Data Collection → Domain Assumption Elicitation → Identification → Estimation (training) → Validation (testing) → Inference (Confidence Intervals)

Are there other ways of identifying the treatment effect I care about (other than identification by conditioning) when I have un-observed confounding?

Sensitivity Analysis

$$a_d \in \hat{a}_d \pm (2\hat{\sigma} + \epsilon)$$

Adaptive Experiments ← Decision Policy ←

Application: Effect of 401k

Problem

- Impact of 401k eligibility on financial assets
- Working for a firm that offers access to 401k eligibility not randomly assigned
- After conditioning on important characteristics of the worker and job, assignment can be thought as exogenous
- Most important: income
- At least around 1991 (early in 401k deployment), workers would not really base their employment decision on 401k eligibility

Causal Diagrams

- F: firm characteristics
- D: eligibility for 401k
- Y: net financial assets
- X: age, income, family size, years of education, a married indicator, a two-earner status indicator, a defined benefit pension status indicator, an IRA participation indicator, and a home ownership indicator
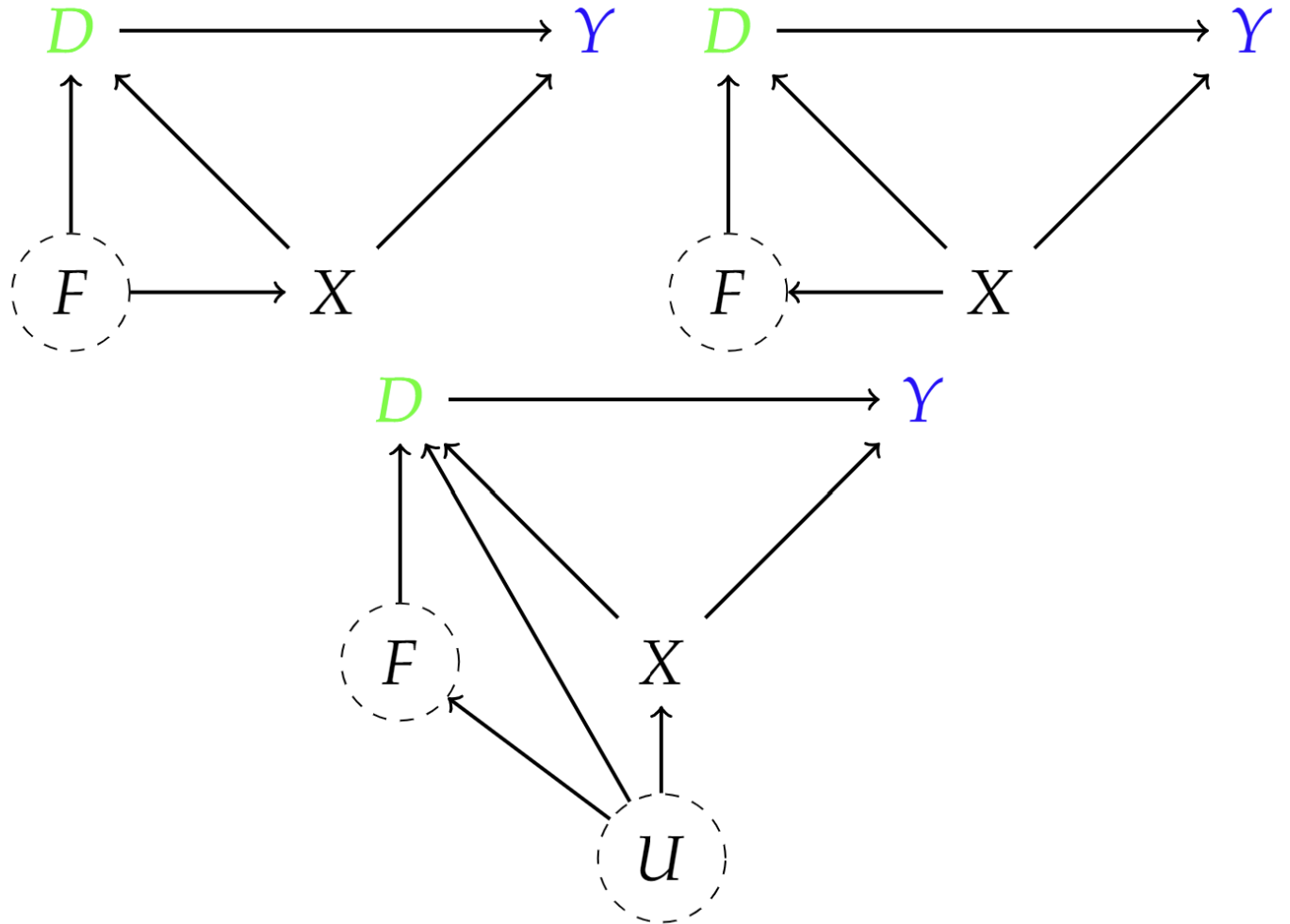
Let's try it out!

Possible Violations

- F: firm characteristics
- D: eligibility for 401k
- Y: net financial assets
- X: age, income, family size, years of education, a married indicator, a two-earner status indicator, a defined benefit pension status indicator, an IRA participation indicator, and a home ownership indicator
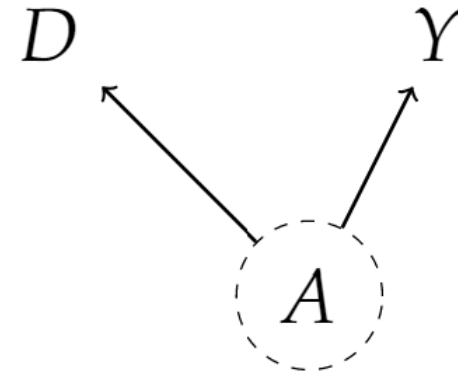- M: match amount
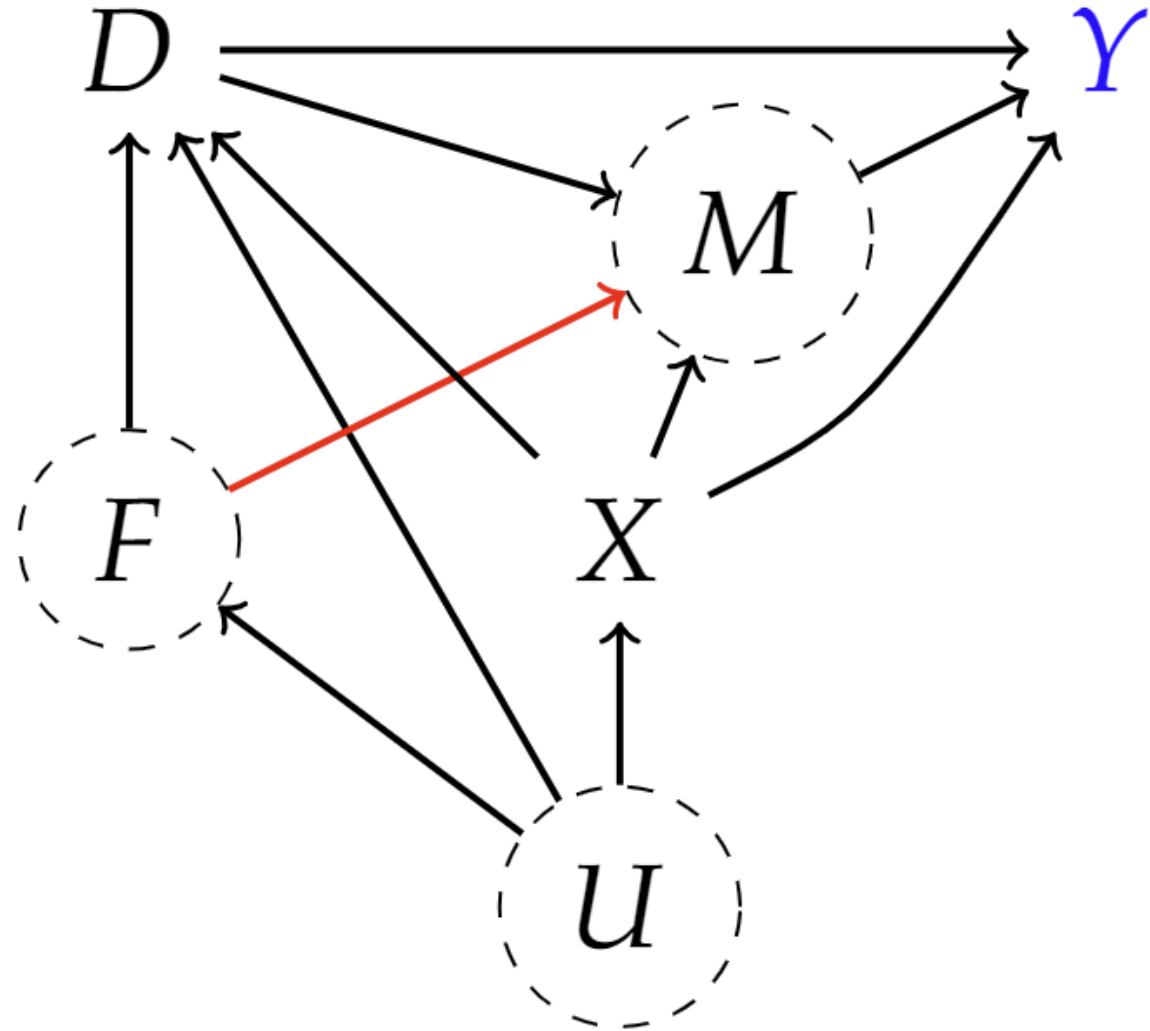
Unobserved Confounding: Overview of Today's Content

Unobserved Confounding

- If there are confounding factors that are un-observed, we cannot distinguish between an effect and a spurious correlation, without further structure
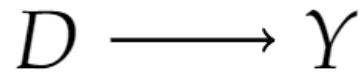
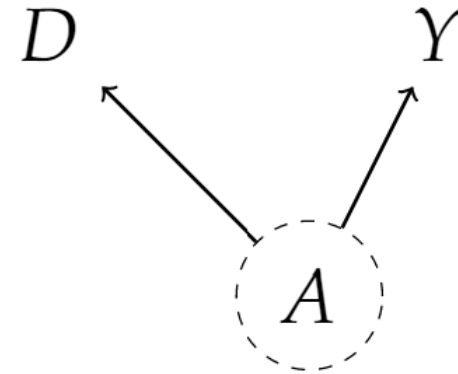$$D \longrightarrow Y$$
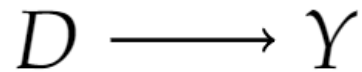
Figure 4.1: D causes Y

Figure 4.2: D and Y are caused by a latent factor A

Unobserved Confounding

- If there are confounding factors that are un-observed, we cannot distinguish between an effect and a spurious correlation, without further structure
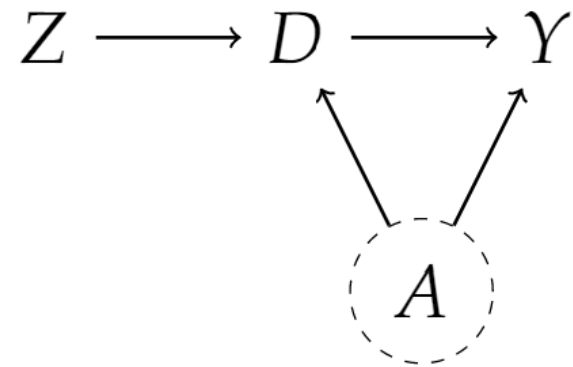
$$D \longrightarrow Y$$

Figure 4.1: D causes Y

Figure 4.2: D and Y are caused by a latent factor A

One Type of Structure: Instruments

- We already saw one example of more structure in the assignments (which one?)

- The structure we will investigate today are instruments

$$Z \longrightarrow D \longrightarrow Y$$

Figure 4.3: A DAG with Latent Confounder A and Instrument Z.

Bias Bounds

Very Frequently Unobserved Confounders

- Average treatment effect not "identifiable"
$$\theta_0 \neq E\big[E[Y|T=1,X] - E[Y \mid T=0,X]\big]$$
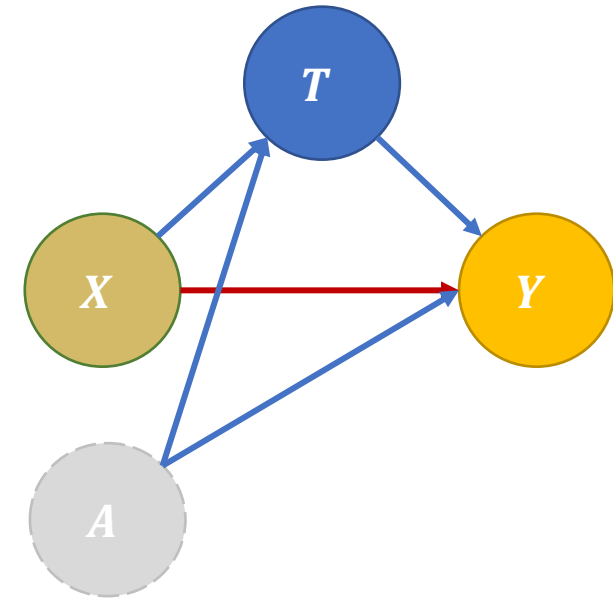
- Realistic conditional exogeneity

$$Y^{(d)} \perp T \mid X, A \qquad \text{(conditional exogeneity)}$$

- Ideal quantity: hypothetical g-formula
$$\theta_0 = E\big[E[Y|T=1,X,A] - E[Y \mid T=0,X,A]\big]$$

- Identifiable quantity:
$$\theta_s = E\big[E[Y|T=1,X] - E[Y \mid T=0,X]\big]$$

Omitted Variable Bias

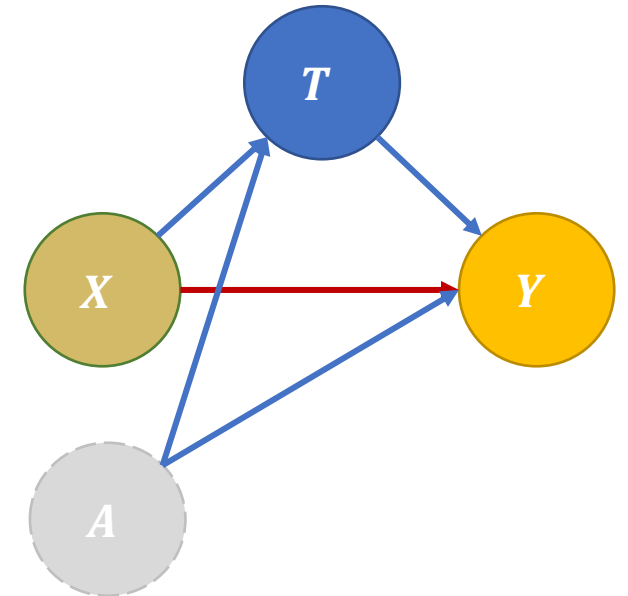- We want to estimate:

$$\theta_0 := E[g(1, X, A) - g(0, X, A)]$$
$$g(T, X, A) := E[Y \mid T, X, A]$$

- Which depends on an un-attainable "long regression"

- We can only estimate a "short regression"
$$g_s(T, X) := E[Y|T, X] = E[g(T, X, A)|T, X]$$

- And compute "short estimate"
$$\theta_s = E[g_s(1, X) - g_s(0, X)]$$

Omitted Variable Bias Bounds

- Provide expression and construct bounds on Omitted Variable Bias (OMVB)

$$\theta_s - \theta_0$$

- Under interpretable assumptions that limit the strength of unobserved confounding

- Perform statistical inference on θ_0 allowing for ML regressions

- Sensitivity analysis has a long history:
  - Rosenbaum-Rubin'83: non-parametric bounds [non-sharp]
  - A lot of follow-up work making parametric assumptions

Paul R Rosenbaum and Donald B Rubin. 'Assessing sensitivity to an unobserved binary covariate in an observational study with binary outcome'. In: *Journal of the Royal Statistical Society. Series B (Methodological)* (1983), pp. 212–218 (cited on page 9).

James M Robins. 'Association, causation, and marginal structural models'. In: *Synthese* 121.1 (1999), pp. 151–179 (cited on page 9).

Kenneth A Frank. 'Impact of a confounding variable on a regression coefficient'. In: *Sociological Methods & Research* 29.2 (2000), pp. 147–194 (cited on page 9).

Paul R Rosenbaum. 'Observational studies'. In: *Observational studies.* Springer, 2002, pp. 1–17 (cited on page 9).

Guido W Imbens. 'Sensitivity to exogeneity assumptions in program evaluation'. In: *American Economic Review* 93.2 (2003), pp. 126–132 (cited on pages 9, 16).

Babette A Brumback et al. 'Sensitivity analyses for unmeasured confounding assuming a marginal structural model for repeated measures'. In: *Statistics in medicine* 23.5 (2004), pp. 749–767 (cited on page 9).

Joseph G Altonji, Todd E Elder, and Christopher R Taber. 'Selection on observed and unobserved variables: Assessing the effectiveness of Catholic schools'. In: *Journal of political economy* 113.1 (2005), pp. 151–184 (cited on page 9).

Carrie A Hosman, Ben B Hansen, and Paul W Holland. 'The Sensitivity of Linear Regression Coefficients' Confidence Limits to the Omission of a Confounder'. In: *The Annals of Applied Statistics* (2010), pp. 849–870 (cited on page 9).

Kosuke Imai, Luke Keele, Teppei Yamamoto, et al. 'Identification, inference and sensitivity analysis for causal mediation effects'. In: *Statistical science* 25.1 (2010), pp. 51–71 (cited on page 9).

Tyler J. Vanderweele and Onyebuchi A. Arah. 'Bias formulas for sensitivity analysis of unmeasured confounding for general outcomes, treatments, and confounders.' In: *Epidemiology (Cambridge, Mass.)* 22.1 (Jan. 2011), pp. 42–52 (cited on page 9).

Matthew Blackwell. 'A selection bias approach to sensitivity analysis for causal effects'. In: *Political Analysis* 22.2 (2013), pp. 169–182 (cited on page 9).

Kenneth A Frank et al. 'What would it take to change an inference? Using Rubin's causal model to interpret the robustness of causal inferences'. In: *Educational Evaluation and Policy Analysis* 35.4 (2013), pp. 437–460 (cited on page 9).

Nicole Bohme Carnegie, Masataka Harada, and Jennifer L Hill. 'Assessing sensitivity to unmeasured confounding using a simulated potential confounder'. In: *Journal of Research on Educational Effectiveness* 9.3 (2016), pp. 395–420 (cited on page 9).

Vincent Dorie et al. 'A flexible, interpretable framework for assessing sensitivity to unmeasured confounding'. In: *Statistics in medicine* 35.20 (2016), pp. 3453–3470 (cited on page 9).

Joel A Middleton et al. 'Bias amplification and bias unmasking'. In: *Political Analysis* 24.3 (2016), pp. 307–323 (cited on page 9).

Emily Oster. 'Unobservable selection and coefficient stability: Theory and evidence'. In: *Journal of Business & Economic Statistics* (2017), pp. 1–18 (cited on page 9).

Tyler J VanderWeele and Peng Ding. 'Sensitivity analysis in observational research: introducing the E-value'. In: *Annals of Internal Medicine* 167.4 (2017), pp. 268–274 (cited on page 9).

Nathan Kallus and Angela Zhou. 'Confounding-robust policy improvement'. In: *arXiv preprint arXiv:1805.08593* (2018) (cited on page 9).

Nathan Kallus, Xiaojie Mao, and Angela Zhou. 'Interval estimation of individual-level causal effects under unobserved confounding'. In: *The 22nd international conference on artificial intelligence and statistics*. PMLR. 2019, pp. 2281–2290 (cited on page 9).

Carlos Cinelli et al. 'Sensitivity Analysis of Linear Structural Causal Models'. In: *International Conference on Machine Learning* (2019) (cited on page 9).

AlexanderM Franks, Alexander D'Amour, and Avi Feller. 'Flexible Sensitivity Analysis for Observational Studies Without Observable Implications'. In: *Journal of the American Statistical Association* 115.532 (2020), pp. 1730–1746 (cited on page 9).

Carlos Cinelli and Chad Hazlett. 'Making sense of sensitivity: Extending omitted variable bias'. In: *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 82.1 (2020), pp. 39–67 (cited on pages 9, 16).

Carlos Cinelli and Chad Hazlett. 'An omitted variable bias framework for sensitivity analysis of instrumental variables'. In: *Work. Pap* (2020) (cited on page 9).

Matteo Bonvini and Edward H Kennedy. 'Sensitivity analysis via the proportion of unmeasured confounding'. In: *Journal of the American Statistical Association* (2021), pp. 1–11 (cited on page 9).

Daniel O Scharfstein et al. 'Semiparametric Sensitivity Analysis: Unmeasured Confounding In Observational Studies'. In: *arXiv preprint arXiv:2104.08300* (2021) (cited on page 9).

Andrew Jesson et al. 'Quantifying Ignorance in Individual-Level Causal-Effect Estimates under Hidden Confounding'. In: *arXiv preprint arXiv:2103.04850* (2021) (cited on page 9).

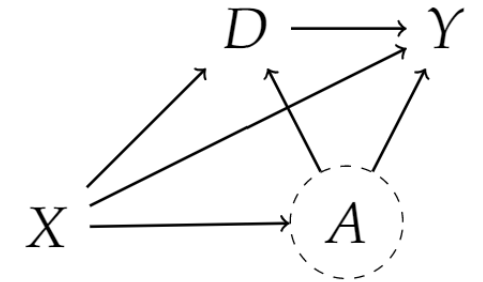# Omitted Variable Bias: Partially Linear Models

- Let's consider a simpler structural equation model

$$Y := aD + \delta A + f_Y(X) + \epsilon_Y$$
$$D := \gamma A + f_D(X) + \epsilon_D$$
$$A := f_A(X) + \epsilon_A$$
$$X := \epsilon_X$$

Figure 4.5: X are observed confounders, and A are unobserved confounders.

Omitted Variable Bias: Partially Linear Models

- Suppose we run the residual-on-residual process and first partial out X

- For convenience $E\left[\epsilon_A^2\right] = 1$

$$\tilde{Y} := a\widetilde{D} + \delta\tilde{A} + \epsilon_Y$$

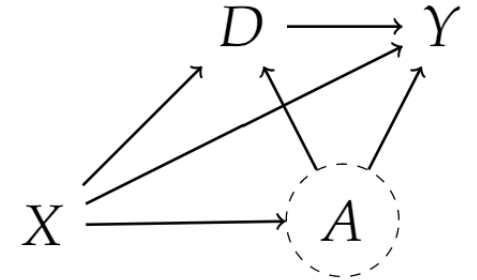$$\widetilde{D} := \gamma\tilde{A} + \epsilon_D$$

$$\tilde{A} := \epsilon_A$$

Figure 4.5: X are observed confounders, and A are unobserved confounders.

- Then the double ML method would run OLS of $\tilde{Y}$ on $\widetilde{D}$

$$\theta_s = \frac{E\left[\tilde{Y}\widetilde{D}\right]}{E\left[\widetilde{D}^2\right]} = \frac{E\left[a\,\widetilde{D}^2 + \delta\tilde{A}\widetilde{D}\right]}{E\left[\widetilde{D}^2\right]} = a + \delta\frac{E\left[\gamma\tilde{A}^2\right]}{E\left[\widetilde{D}^2\right]} = a + \frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}$$

Bias Bounds

- If the analyst can provide bounds on the strength of each relationship (e.g. $\delta, \gamma, E[\epsilon_D^2]$) then we can provide bounds on the true effect

$$\theta_0 = \theta_s \pm \left| \frac{\delta\gamma}{\gamma^2 + E[\epsilon_D^2]} \right|$$

Measurable from the data

- More interpretable bounds:

$$\frac{\delta\gamma}{\gamma^2 + E[\epsilon_D^2]} = R^2_{\tilde{Y}\sim\tilde{A}|\tilde{D}} \frac{R^2_{\tilde{D}\sim\tilde{A}}}{1 - R^2_{\tilde{D}\sim\tilde{A}}} \frac{E\left[(\tilde{Y} - \theta_s\tilde{D})^2\right]}{E[\tilde{D}^2]}$$

R^2 in linear regression of $\tilde{Y}$ on $\tilde{A}$ after linearly partialling out $\tilde{D}$

R^2 in linear regression of $\tilde{D}$ on $\tilde{A}$

Bias Bounds

- If the analyst can provide bounds on the strength of each relationship (e.g. $\delta, \gamma, E[\epsilon_D^2]$) then we can provide bounds on the true effect

$$\theta_0 = \theta_s \pm \left| \frac{\delta\gamma}{\gamma^2 + E[\epsilon_D^2]} \right|$$

Measurable from the data

- More interpretable bounds:

$$\frac{\delta\gamma}{\gamma^2 + E[\epsilon_D^2]} = R^2_{Y\sim A|D,X} \frac{R^2_{D\sim A|X}}{1 - R^2_{D\sim A|X}} \frac{\mathrm{E}\left[(\widetilde{Y} - \theta_s\widetilde{D})^2\right]}{E[\widetilde{D}^2]}$$

Reduction in unexplained variance of Y when adding A in the model that predicts Y from treatment and controls

Reduction in unexplained variance of D when adding A in the model that predicts D from controls

Application: 401k eligibility

- Y=net financial assets
- D=eligibility to enroll in 401(k) program
- X=pre-treatment worker-level covariates (observed)
- F=pre-treatment firm-level covariates (unobserved)
- M=amount of contribution matched by employer
- U=general latent factors
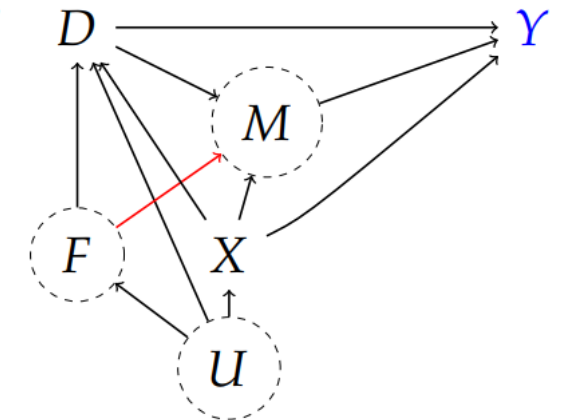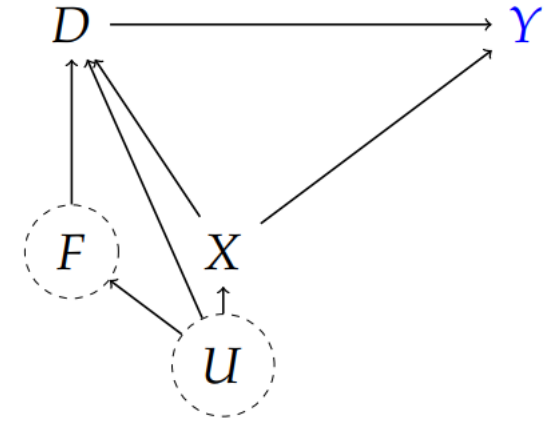
Controlling for X is sufficient in top figure but not bottom
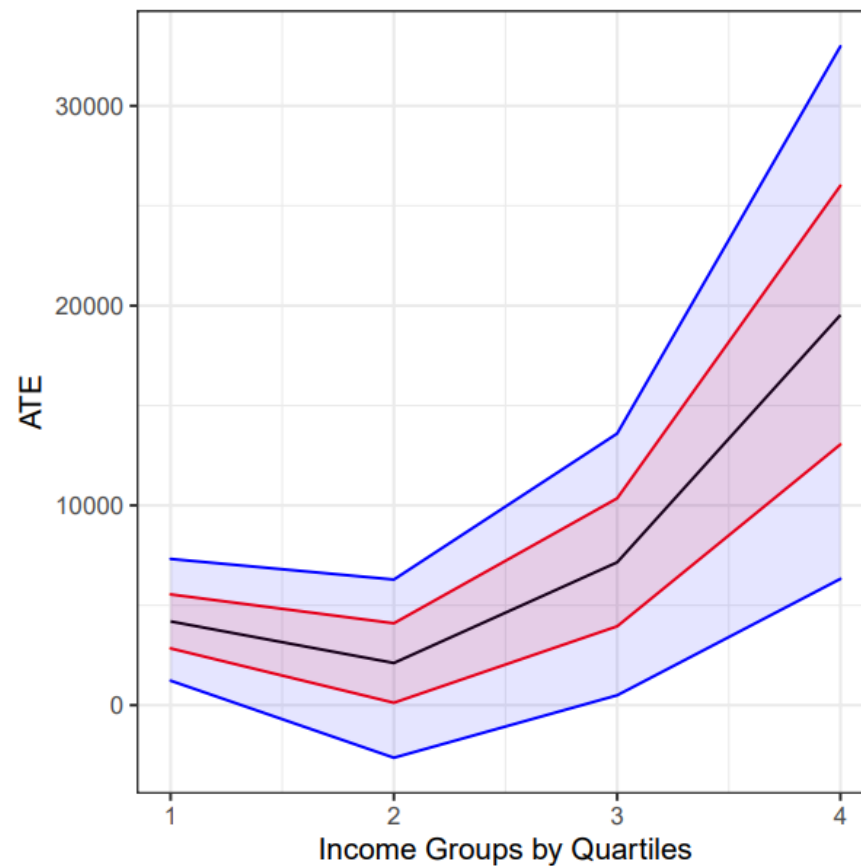
Confounding Scenario

- Posit that F explains as much variation in net financial assets as the total variation of maximal matched percentage (5%) of income over period of three years

- Posit that F explains an additional 2.5% of the variation in 401k eligibility, a 20% relative increase in the baseline R^2 of the treatment of 13%

- In PLR: translates to $C_Y^2 := R_{Y \sim A|D,X}^2 \approx 4\%, C_D^2 := R_{D \sim A|X}^2 \approx 3\%$

Robustness value (RV) = minimal equal strength of C_Y^2 and C_D^2 s.t. bound includes zero

- RV=5.5% (at 95% significance level) > 4%,3%

- Finding that 401k eligibility has positive effect is robust to this confounding scenario
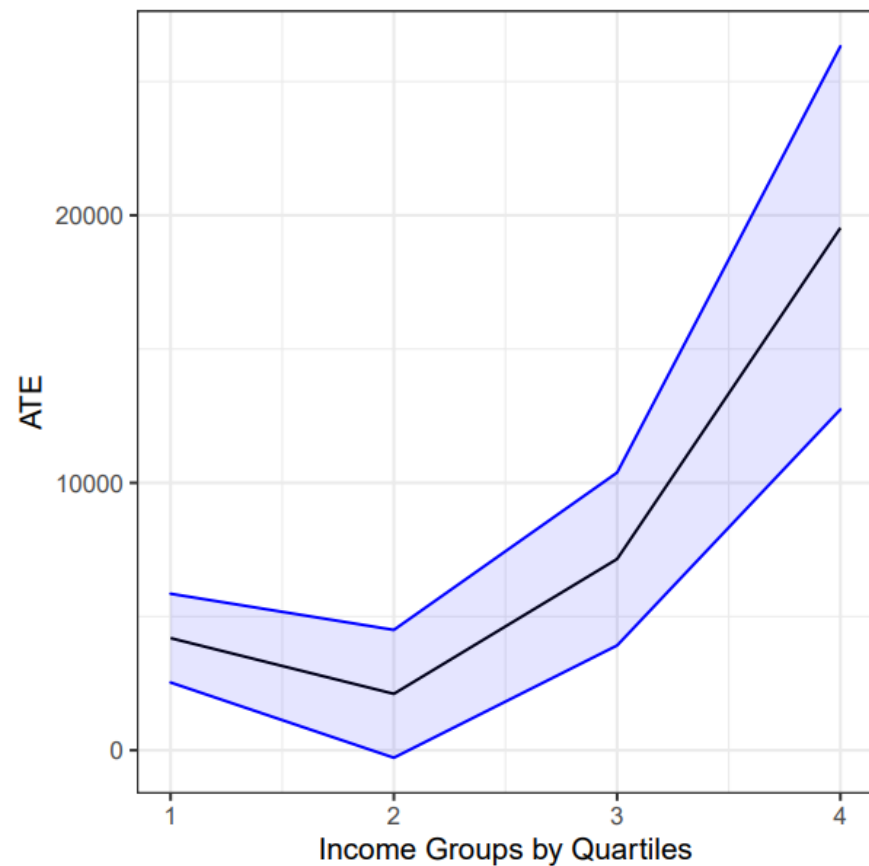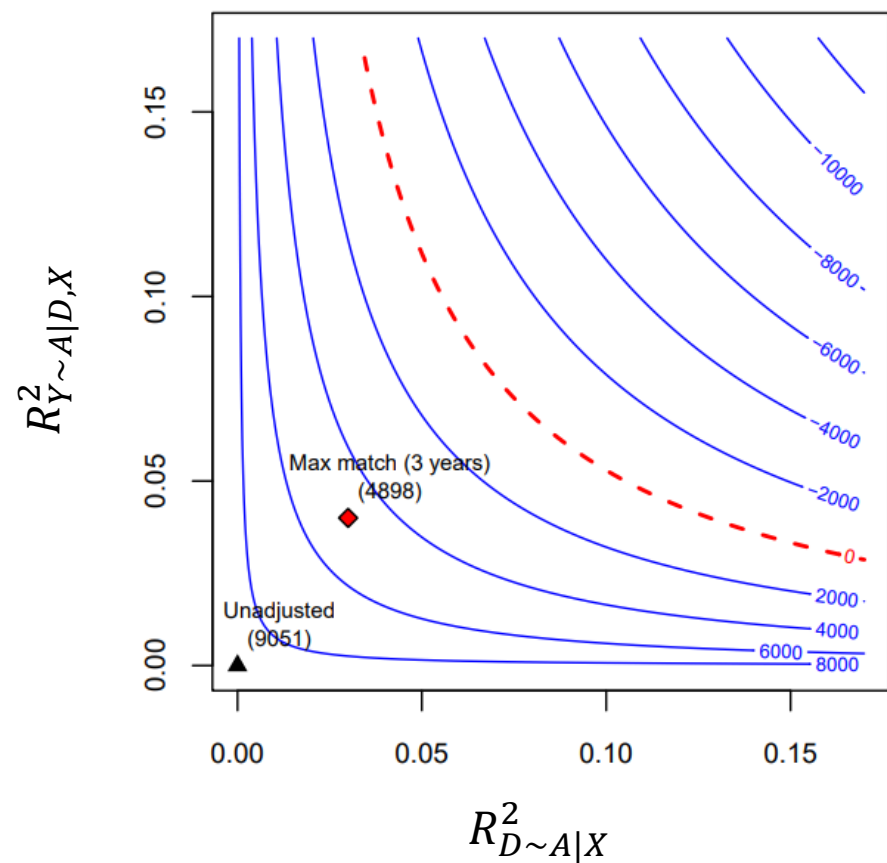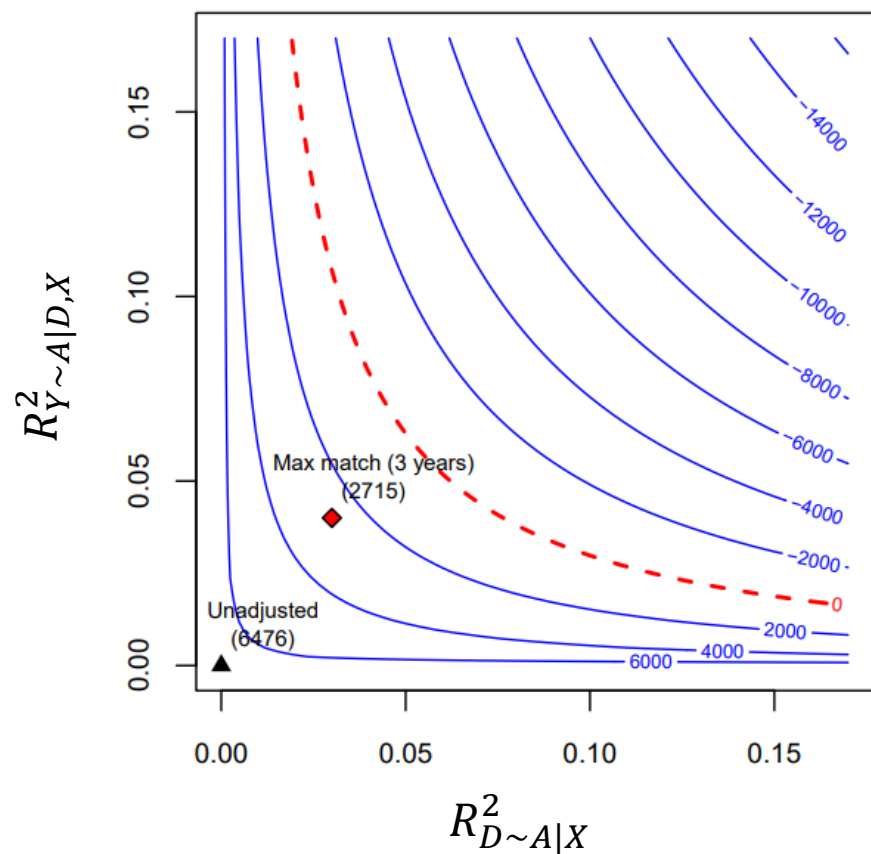
(a) Estimates under no confounding.

(b) Bounds under posited confounding.

Note: Estimate (black), bounds (red), and confidence bounds (blue) for the ATE. Confounding scenario: $\rho^2 = 1$; $C_Y^2 \approx 0.04$; $C_D^2 \approx 0.031$. Significance level of 5%.

**(a)** Contours for $\theta_- = \theta_s - |B|$.

(b) Contours lower limit confidence bound.

Can we recover the true effect? Instrumental Variables

Instrumental Variables and 2SLS

Instrumental Variable: any random variable Z that affects the treatment (log-price) D but does not affect the outcome (log-demand) Y other than through the treatment [Wright'28, Bowden-Turkington'90, Angrist-Krueger'91, Imbens-Angrist'94]
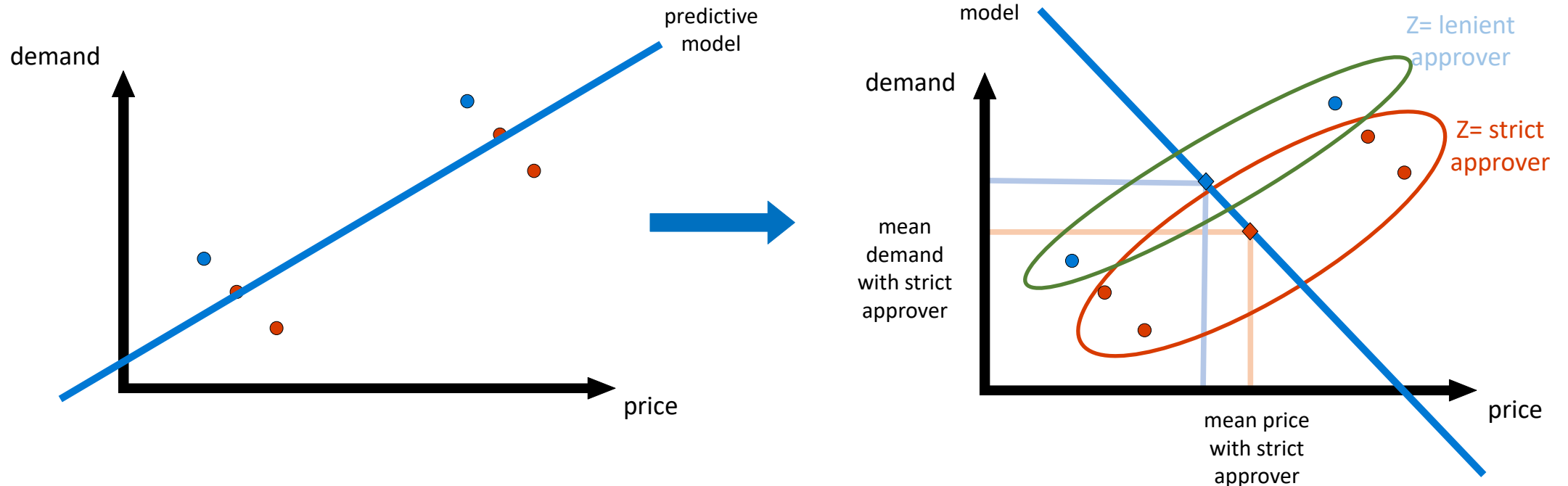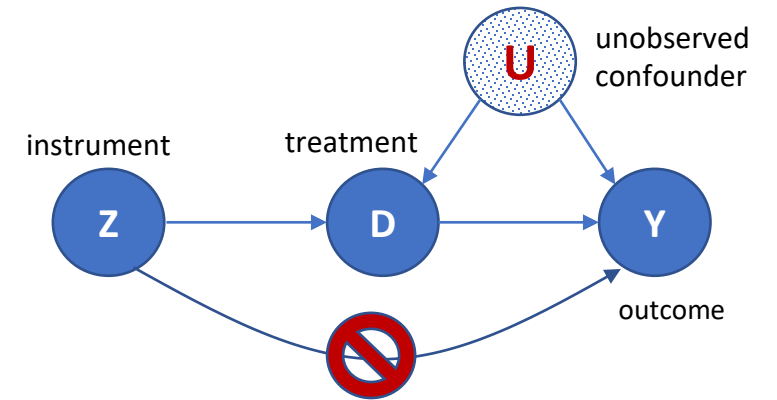
Identification of Causal Effects via Instruments

Phillip Wright's idea (1928): the first causal path diagram analysis
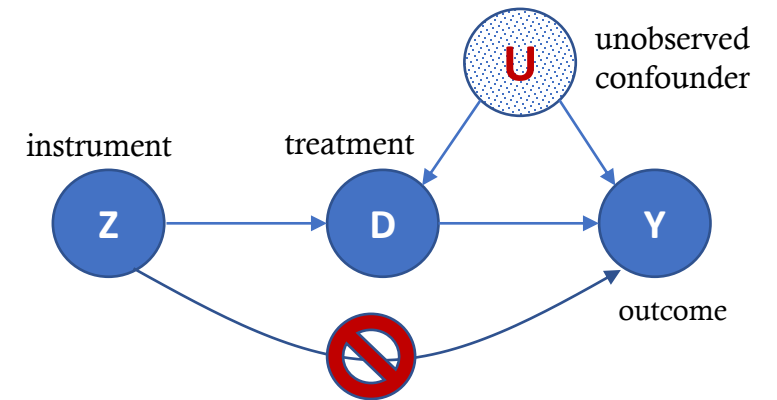
◈ We can estimate effect of Z on y via a regression

$$\gamma = \frac{\mathbb{E}[\tilde{Z}\tilde{y}]}{\mathbb{E}[\tilde{Z}^2]}$$

◈ We can estimate the effect of Z on D via a regression

$$\delta = \frac{\mathbb{E}[\tilde{Z}\tilde{D}]}{\mathbb{E}[\tilde{Z}^2]}$$

◈ The effect of Z on Y (γ) is the product of the effect of Z on T (δ) multiplied by the effect of T on y (θ)

$$\theta = \frac{\gamma}{\delta} = \frac{\mathbb{E}[\tilde{Z}\tilde{y}]}{\mathbb{E}[\tilde{Z}\tilde{D}]}$$

unobserved confounder

instrument   treatment

outcome

Partially Linear Instrumental Variable Model

- Typically for continuous treatment/instrument a partially linear structural equation assumed

$$Y := \theta_0 D + f_Y(X) + A + \epsilon_Y$$
$$D := \beta Z + f_D(X) + \beta A + \epsilon_D$$
$$Z := f_Z(X) + \epsilon_Z$$
$$A := f_A(X) + \epsilon_A$$

All errors are exogenous and un-correlated

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} := \theta_0 \tilde{D} + \tilde{A} + \epsilon_Y$$
$$\tilde{D} := \beta \tilde{Z} + \beta \tilde{A} + \epsilon_D$$
$$\tilde{Z} := \epsilon_Z$$
$$\tilde{A} := \epsilon_A$$

- We see immediately that:

$$\tilde{Y} := \theta_0 \tilde{D} + U, \qquad U \perp \tilde{Z}$$

- Since $\epsilon_A, \epsilon_Y, \epsilon_Z$ are un-correlated: $E\big[(\tilde{A} + \epsilon_Y)\tilde{Z}\big] = 0$
- Thus we have the moment restriction: $E\big[(\tilde{Y} - \theta_0 \tilde{D})\tilde{Z}\big] = 0$

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} := \theta_0 \widetilde{D} + \tilde{A} + \epsilon_Y$$
$$\widetilde{D} := \beta \tilde{Z} + \beta \tilde{A} + \epsilon_D$$
$$\tilde{Z} := \epsilon_Z$$
$$\tilde{A} := \epsilon_A$$

- We see immediately that:
$$\tilde{Y} := \theta_0 \widetilde{D} + U, \qquad U := \tilde{A} + \epsilon_Y \perp \tilde{Z}$$

- Since $\epsilon_A, \epsilon_Y, \epsilon_Z$ are un-correlated: $E\big[(\tilde{A} + \epsilon_Y)\tilde{Z}\big] = 0$

- Thus we have the moment restriction: $E\big[(\tilde{Y} - \theta_0 \widetilde{D})\tilde{Z}\big] = 0$

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} := \theta_0 \tilde{D} + \tilde{A} + \epsilon_Y$$
$$\tilde{D} := \beta \tilde{Z} + \beta \tilde{A} + \epsilon_D$$
$$\tilde{Z} := \epsilon_Z$$
$$\tilde{A} := \epsilon_A$$

- Thus we have the moment restriction: $E\big[(\tilde{Y} - \theta_0 \tilde{D})\tilde{Z}\big] = \mathbf{0}$

- We re-derive a generalization of Wright's formula

$$\theta_0 = \frac{E\big[\tilde{Y}\tilde{Z}\big]}{E\big[\tilde{D}\tilde{Z}\big]}$$

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} := \theta_0 \widetilde{D} + \tilde{A} + \epsilon_Y$$
$$\widetilde{D} := \beta \tilde{Z} + \beta \tilde{A} + \epsilon_D$$
$$\tilde{Z} := \epsilon_Z$$
$$\tilde{A} := \epsilon_A$$

- Setting falls into the general moment estimation framework

$$M(\theta, h, p, m) = E\left[\left(Y - h(X) - \theta\left(D - p(X)\right)\right)\left(Z - m(X)\right)\right] = 0$$

- Where $h(X) = E[Y|X], p(X) = E[D|X], m(Z) = E[Z|X]$

Orthogonal Method: Double ML for IV

Double ML. Split samples in half

- Regress $Y \sim X$ with ML on first half, to get estimate $\hat{h}(S)$ of $E[Y|X]$

- Regress $D \sim X$ with ML on first half, to get estimate $\hat{p}(S)$ of $E[D|X]$

- Regress $Z \sim X$ with ML on first half, to get estimate $\hat{m}(S)$ of $E[Z|X]$

- Construct residuals on other half, $\hat{Z} = Z - \hat{m}(X), \widehat{D} := D - \hat{p}(X)$ and $\hat{Y} := Y - \hat{h}(X)$

- Solve moment condition:

$$E_n\big[\big(\hat{Y} - \theta\widehat{D}\big)\widehat{D}\big] = 0$$

```python
from econml.iv.dml import OrthoIV
orthoiv = OrthoIV()
orthoiv.fit(y, D, Z, W=X).effect_inference()
```

The Binary Case
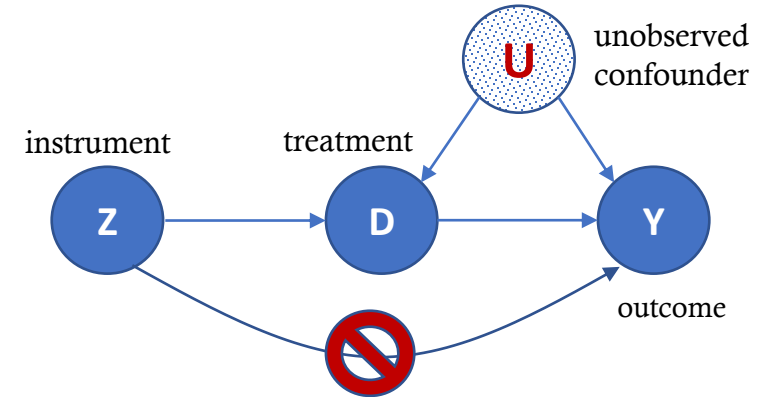
Imbens-Angrist (1994): core contribution of Nobel 2022 award

◈ Instrument/Treatment are binary (instrument=recommended treatment)

◈ Assume monotonicity: $D^{(1)} \geq D^{(0)}$

◈ Recommended treatment cannot reverse taken treatment

◈ Object of interest: Local Average Treatment Effect (ATE among compliers)

$$\theta_0 = E\left[Y^{(1)} - Y^{(0)}\middle|D^{(1)} > D^{(0)}\right]$$

◈ Proof [Angrist-Imbens'94]:

$$\theta_0 = \frac{E\left[\left(Y^{(1)} - Y^{(0)}\right)1\{D^{(1)} > D^{(0)}\}\right]}{E\left[1\{D^{(1)} > D^{(0)}\}\right]} = \frac{E\left[Y^{(D(1))} - Y^{(D(0))}\right]}{E\left[D^{(1)} - D^{(0)}\right]} = \frac{ATE(Z \to Y)}{ATE(Z \to D)}$$

$$\gamma = \frac{\mathbb{E}\left[\tilde{Z}\tilde{y}\right]}{\mathbb{E}\left[\tilde{Z}^2\right]}$$

$$\delta = \frac{\mathbb{E}\left[\tilde{Z}\tilde{D}\right]}{\mathbb{E}\left[\tilde{Z}^2\right]}$$
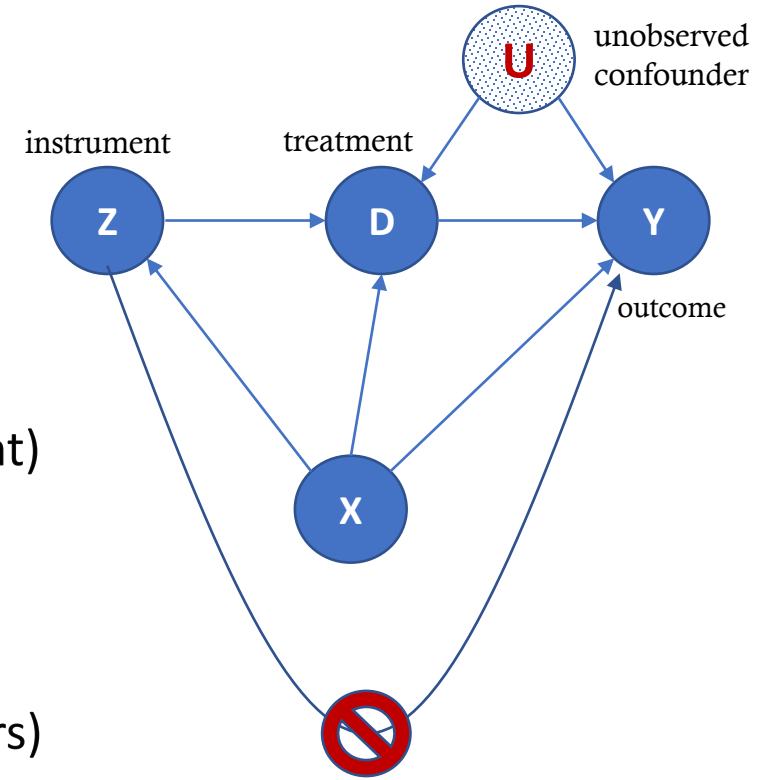
The Binary Case

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$$\theta_0 = E[\mathrm{Y}^{(1)} - \mathrm{Y}^{(0)} | \mathrm{D}^{(1)} > \mathrm{D}^{(0)}]$$

❖ Proof [Angrist-Imbens'94]:

$$\theta_0 = \frac{E[(Y^{(1)} - Y^{(0)})1\{D^{(1)} > D^{(0)}\}]}{E[1\{D^{(1)} > D^{(0)}\}]} = \frac{E[Y^{(D(1))} - Y^{(D(0))}]}{E[D^{(1)} - D^{(0)}]} = \frac{ATE(Z \to Y)}{ATE(Z \to D)}$$

$$E[E[Y|Z=1,X] - E[Y|Z=0,X]]$$

$$E[E[D|Z=1,X] - E[D|Z=0,X]]$$

LATE in the Binary Case

- Under monotonicity

$$\theta_0 = \frac{E\big[E[Y \mid Z = 1, X] - E[Y|Z = 0, X]\big]}{E\big[E[D|Z = 1, X] - E[D|Z = 0, X]\big]}$$

- Moment formulation

$$E[E[Y|Z = 1, X] - E[Y|Z = 0, X] - \theta_0(E[D|Z = 1, X] - E[D|Z = 0, X])] = 0$$

$$+$$
$$a_{Z \to Y}(Z, X)(Y - E[Y|Z, X])$$

$$+$$
$$a_{Z \to D}(Z, X)(D - E[D|Z, X])$$

- Orthogonal moment formulation: apply ATE debiasing twice