MS&E 228: Unobserved Confounding

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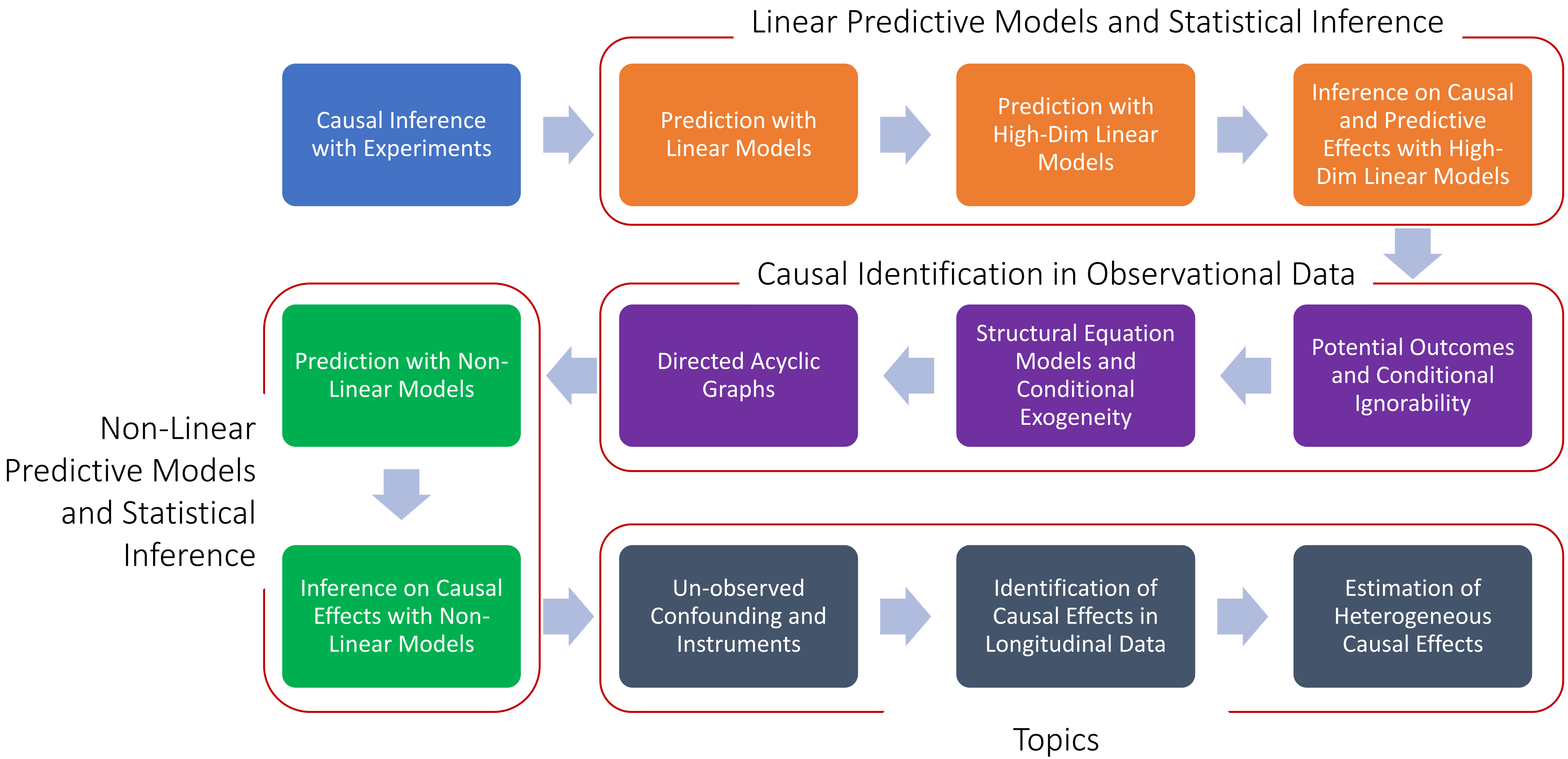
Linear Predictive Models and Statistical Inference
Causal Inference with Experiments
Prediction with Linear Models
Prediction with High-Dim Linear Models
Inference on Causal and Predictive Effects with High-Dim Linear Models
Causal Identification in Observational Data
Potential Outcomes and Conditional Ignorability
Structural Equation Models and Conditional Exogeneity
Directed Acyclic Graphs
Non-Linear Predictive Models and Statistical Inference
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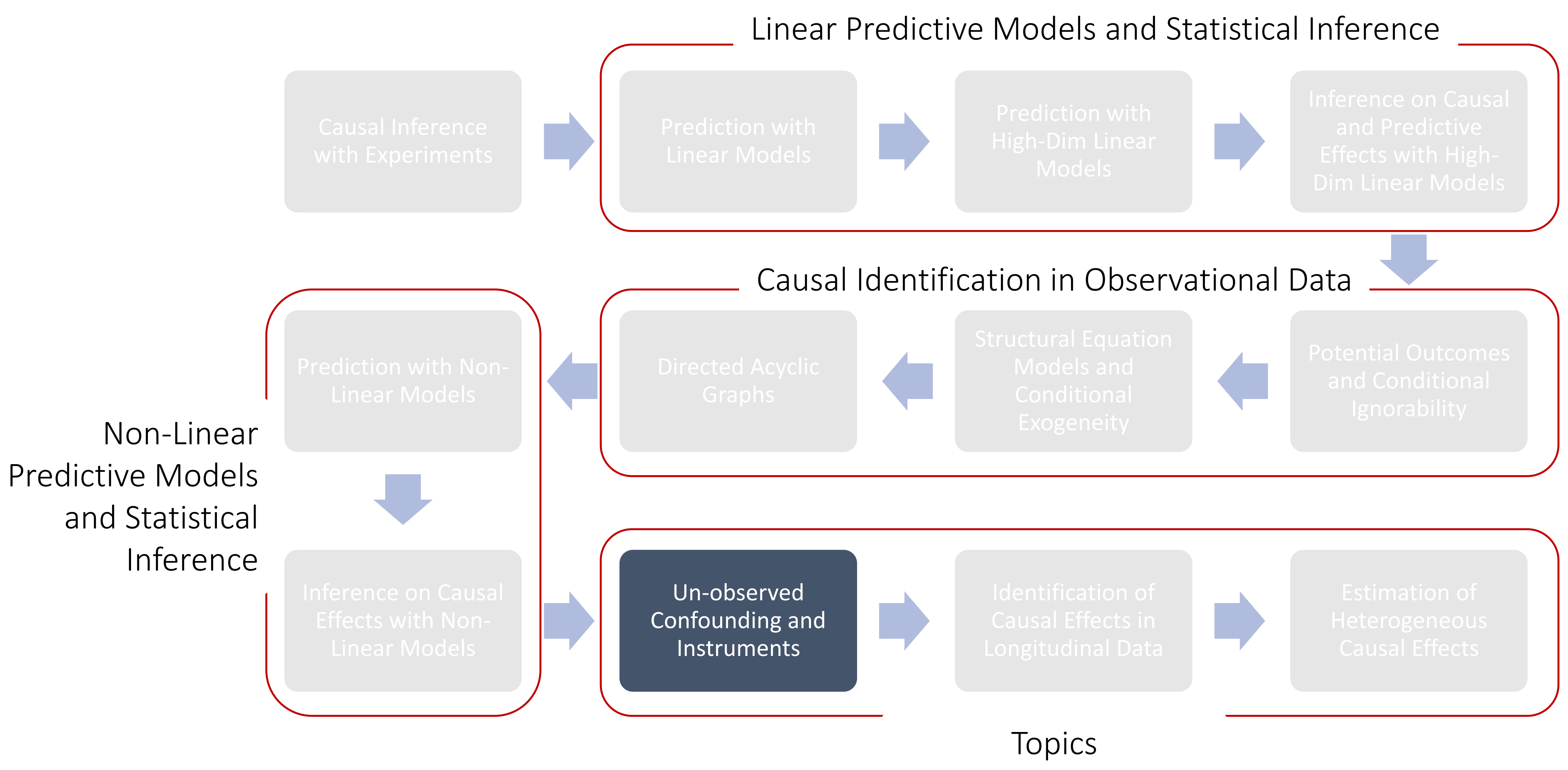
Un-observed Confounding and Instruments
Identification of Causal Effects in Longitudinal Data
Estimation of Heterogeneous Causal Effects
Topics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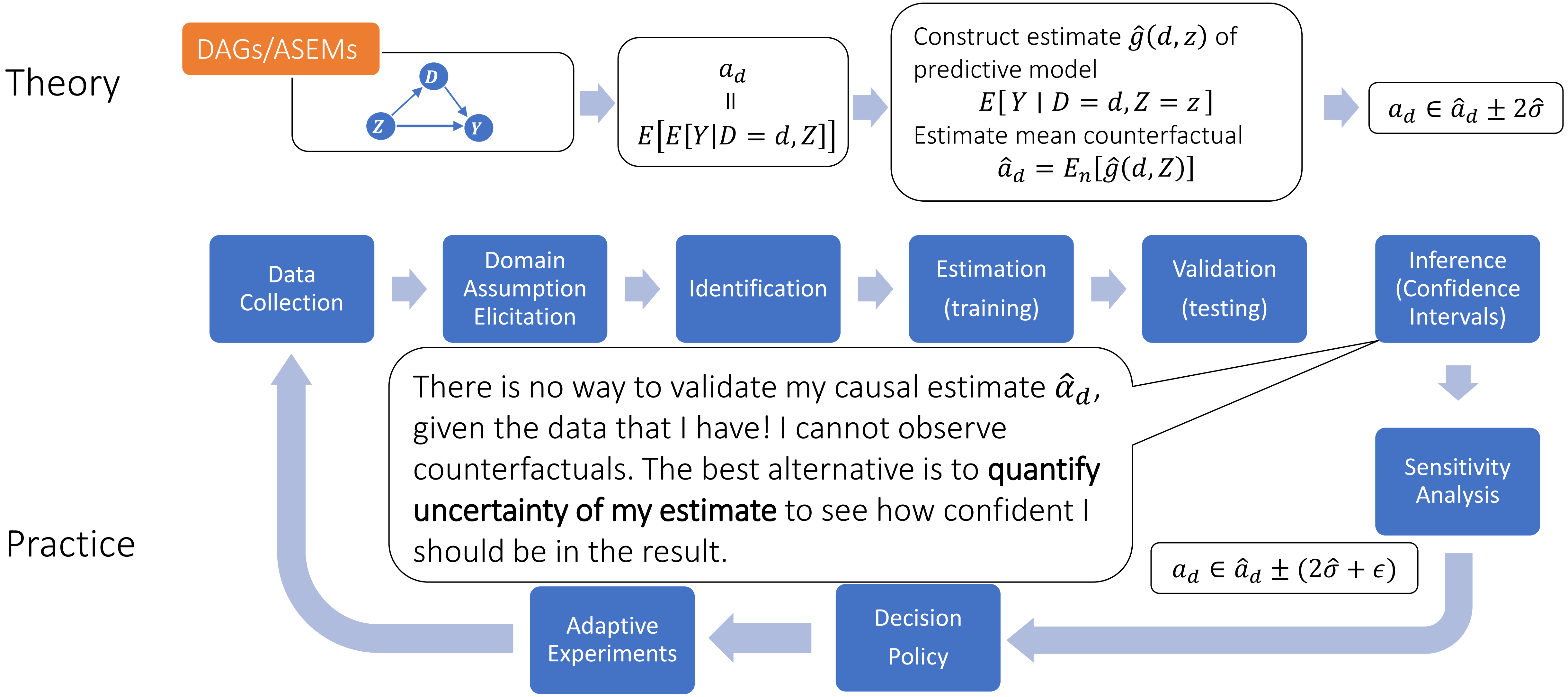
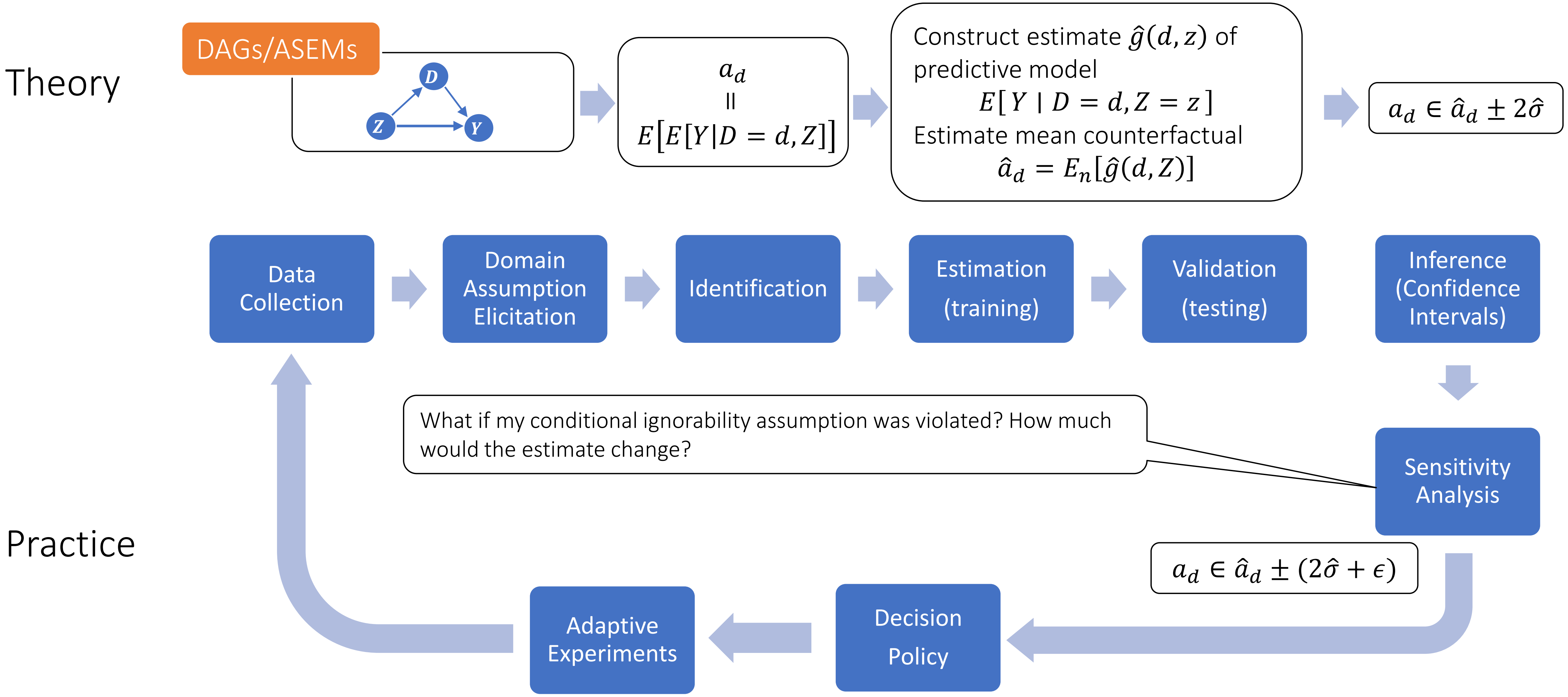
Goals for Today

- What can we do when we have un-observed confounding
- Omitted variable bias bounds
- Introduction to “Instruments”

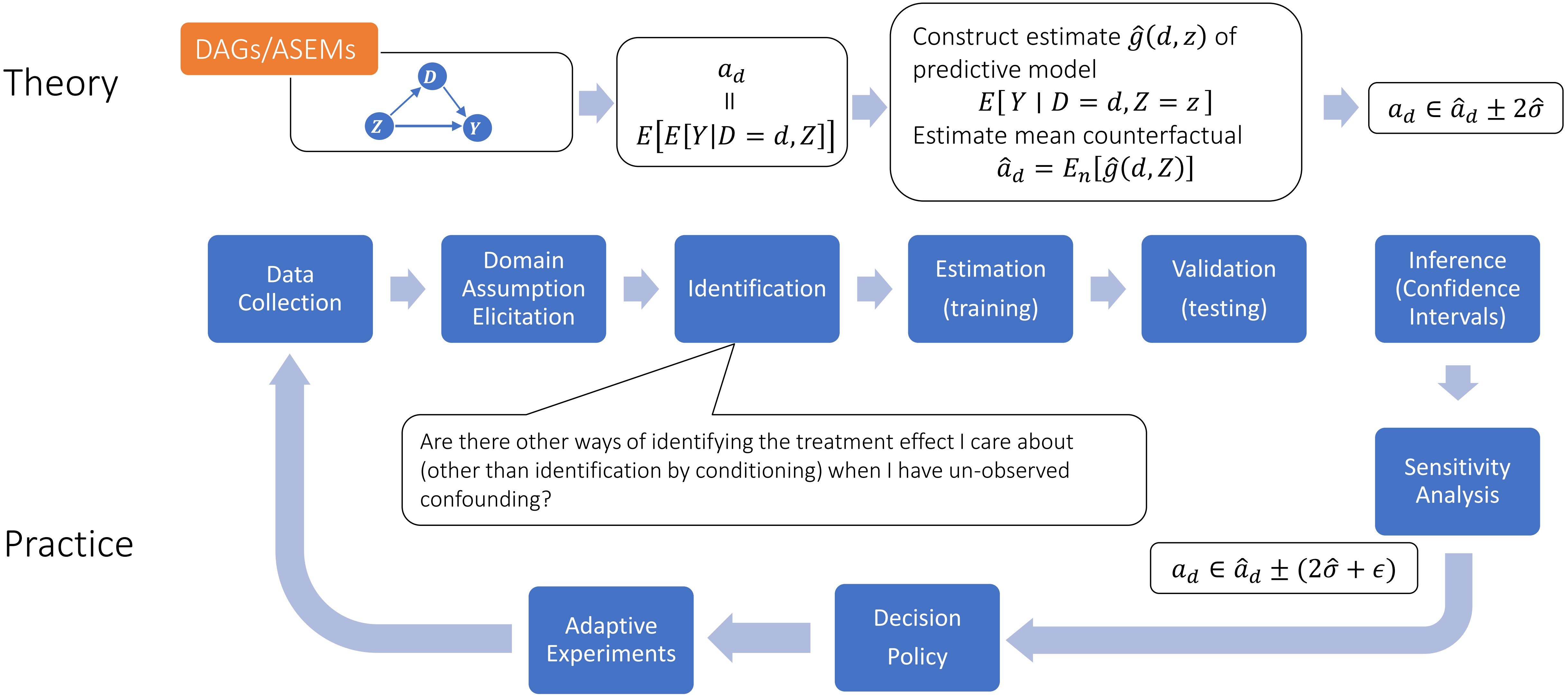
Causal Inference Pipeline



Causal Inference Pipeline

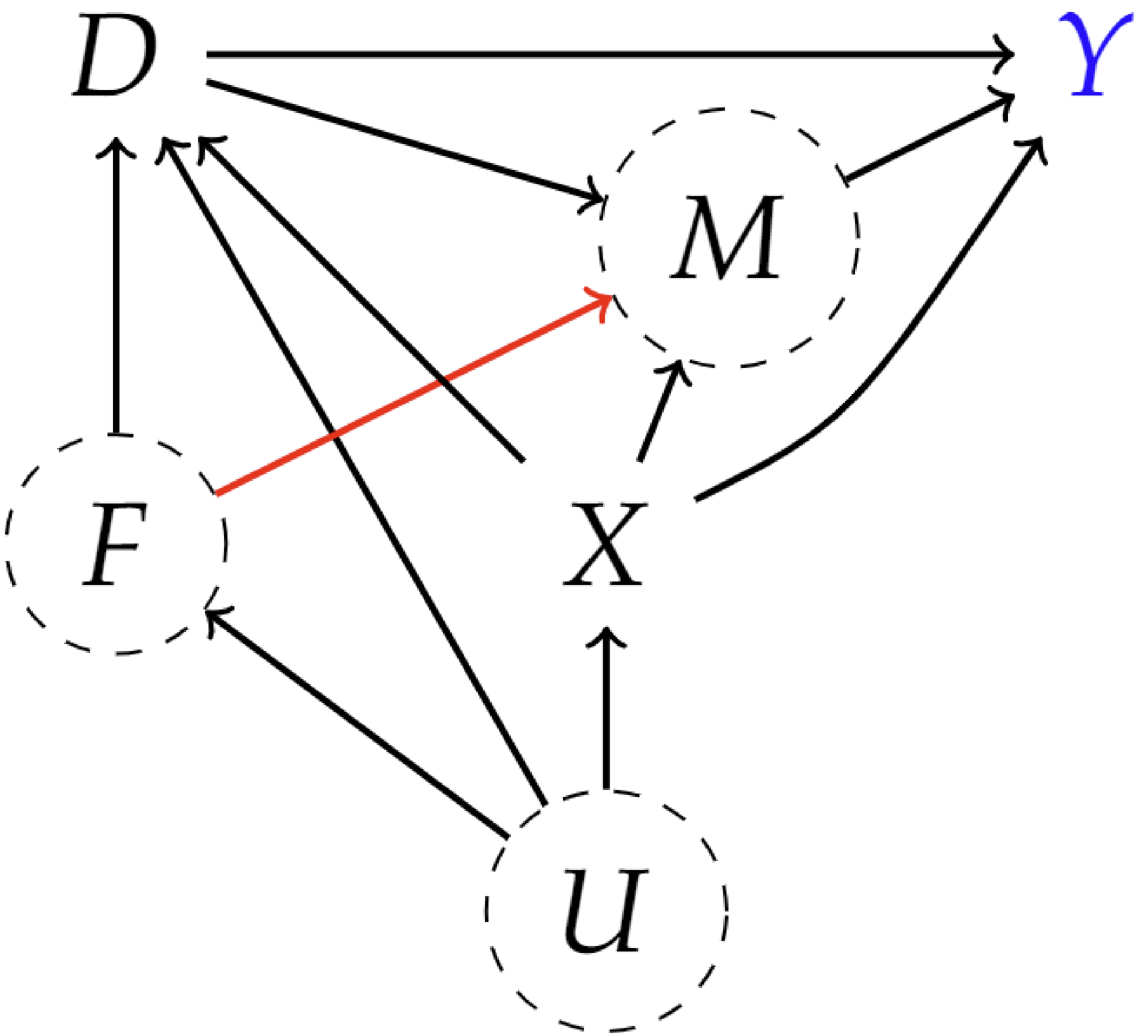


Causal Inference Pipeline



Possible Violations

- F: firm characteristics
- D: eligibility for 401k
- Y: net financial assets
- X: age, income, family size, years of education, a married indicator, a two-earner status indicator, a defined benefit pension status indicator, an IRA participation indicator, and a home ownership indicator
- M: match amount



Unobserved Confounding: Overview of Today’s Content

Unobserved Confounding

- If there are confounding factors that are un-observed, we cannot distinguish between an effect and a spurious correlation, without further structure

$$D \longrightarrow Y$$

Figure 4.1: D causes Y

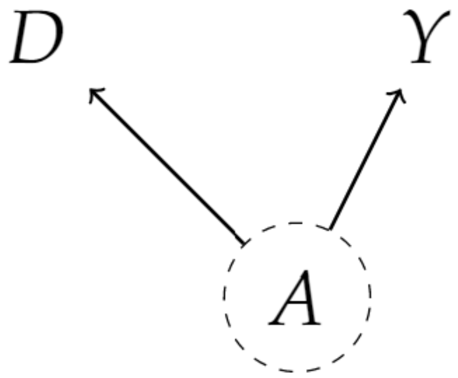


Figure 4.2: D and Y are caused by a latent factor A

Unobserved Confounding

- If there are confounding factors that are un-observed, we cannot distinguish between an effect and a spurious correlation, without further structure

$$D \longrightarrow Y$$

Figure 4.1: D causes Y

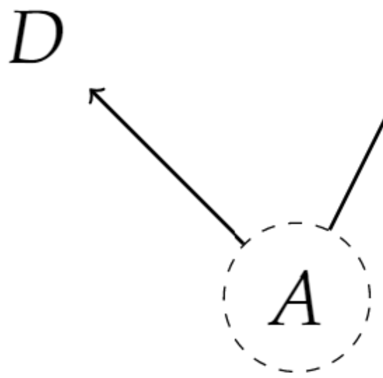


Figure 4.2: D and Y are caused by a latent factor A

One Type of Structure: Instruments

- We already saw one example of more structure in the assignments (which one?)

- The structure we will investigate today are instruments

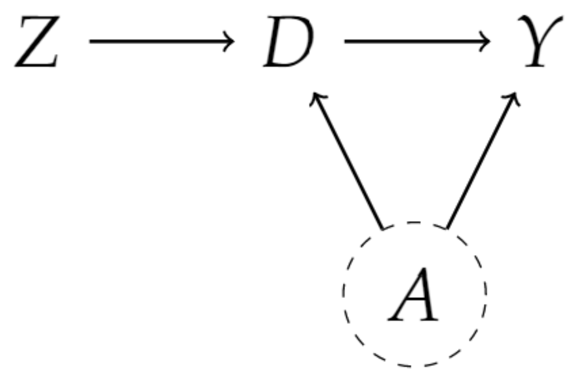


Figure 4.3: A DAG with Latent Confounder A and Instrument Z.

Bias Bounds

Very Frequently Unobserved Confounders

- Average treatment effect not “identifiable”

$$\theta_0 \neq E\big[E[Y|T=1,X] - E[\,Y \mid T=0,X\,]\big]$$

- Realistic conditional exogeneity

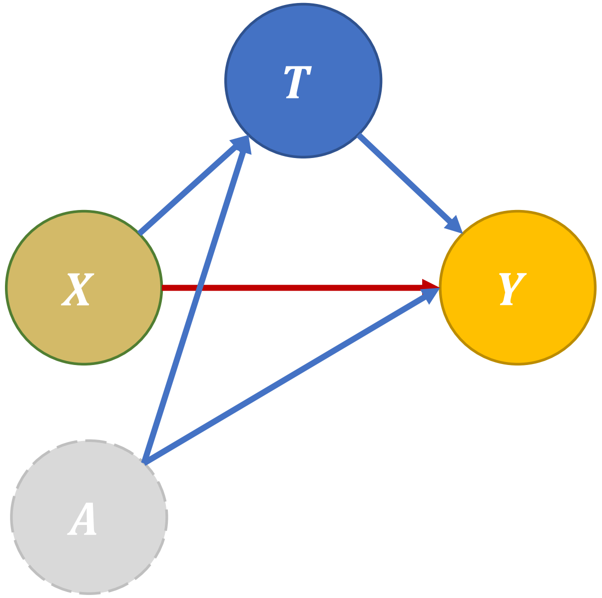
$$Y^{(d)} \perp T \mid X, A \qquad \text{(conditional exogeneity)}$$

- Ideal quantity: hypothetical g-formula

$$\theta_0 = E\big[E[Y|T=1,X,A] - E[\,Y \mid T=0,X,A\,]\big]$$

- Identifiable quantity:

$$\theta_s = E\big[E[Y|T=1,X] - E[\,Y \mid T=0,X\,]\big]$$



Omitted Variable Bias

- We want to estimate:

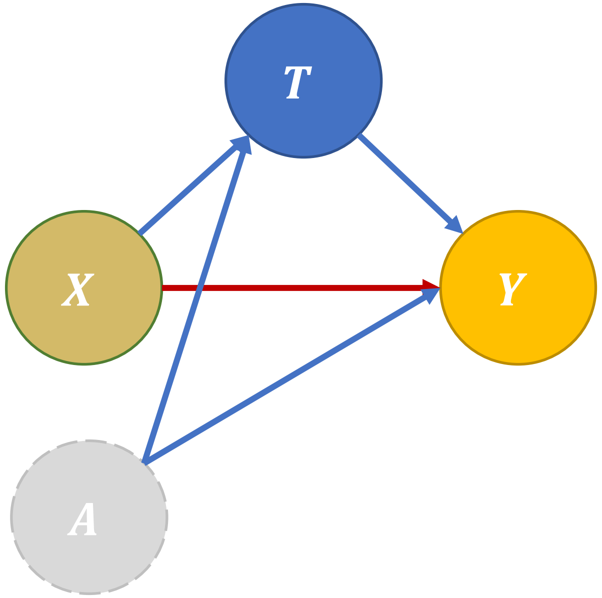
$$\theta_0 \coloneqq E[g(1,X,A) - g(0,X,A)]$$
$$g(T,X,A) \coloneqq E[\,Y \mid T,X,A\,]$$

- Which depends on an un-attainable “long regression”
- We can only estimate a “short regression”

$$g_s(T,X) \coloneqq E[Y|T,X] = E[g(T,X,A)|T,X]$$

- And compute “short estimate”

$$\theta_s = E[g_s(1,X) - g_s(0,X)]$$



Omitted Variable Bias Bounds

- Provide expression and construct bounds on Omitted Variable Bias (OMVB)
$$\theta_s - \theta_0$$
- Under interpretable assumptions that limit the strength of unobserved confounding
- Perform statistical inference on θ_0 allowing for ML regressions

- Sensitivity analysis has a long history:
  - Rosenbaum-Rubin'83: non-parametric bounds [non-sharp]
  - A lot of follow-up work making parametric assumptions

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Omitted Variable Bias: Partially Linear Models

- Let's consider a simpler structural equation model

$$
\begin{aligned}
Y &:= \alpha D + \delta A + f_Y(X) + \epsilon_Y \\
D &:= \gamma A + f_D(X) + \epsilon_D \\
A &:= f_A(X) + \epsilon_A \\
X &:= \epsilon_X
\end{aligned}
$$

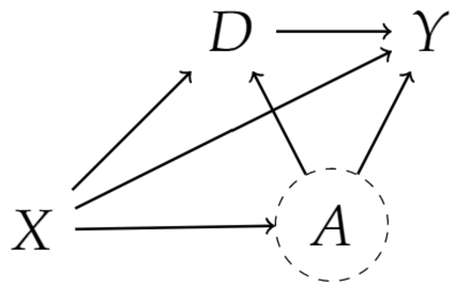


Figure 4.5: X are observed confounders, and A are unobserved confounders.

Omitted Variable Bias: Partially Linear Models

- Suppose we run the residual-on-residual process and first partial out X
- For convenience $E\left[\epsilon_A^2\right] = 1$

$$\tilde{Y} := a\tilde{D} + \delta\tilde{A} + \epsilon_Y$$
$$\tilde{D} := \gamma\tilde{A} + \epsilon_D$$
$$\tilde{A} := \epsilon_A$$

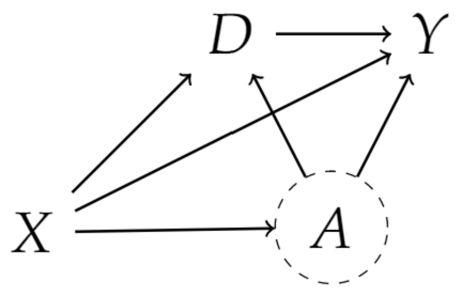


Figure 4.5: X are observed confounders, and A are unobserved confounders.

- Then the double ML method would run OLS of $\tilde{Y}$ on $\tilde{D}$

$$\theta_s = \frac{E\left[\tilde{Y}\tilde{D}\right]}{E\left[\tilde{D}^2\right]} = \frac{E\left[a\,\tilde{D}^2 + \delta\tilde{A}\tilde{D}\right]}{E\left[\tilde{D}^2\right]} = a + \delta\frac{E\left[\gamma\tilde{A}^2\right]}{E\left[\tilde{D}^2\right]} = a + \frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}$$

Bias Bounds

- If the analyst can provide bounds on the strength of each relationship (e.g. $\delta, \gamma, E\left[\epsilon_D^2\right]$) then we can provide bounds on the true effect

$$\theta_0 = \theta_s \pm \left|\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right|$$

- More interpretable bounds:

$$\left(\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right)^2 = \boxed{R^2_{\tilde{Y}\sim\tilde{A}|\tilde{D}}}\frac{R^2_{\tilde{D}\sim\tilde{A}}}{1-\boxed{R^2_{\tilde{D}\sim\tilde{A}}}}\boxed{\frac{\mathrm{E}\left[\left(\tilde{Y}-\theta_s\tilde{D}\right)^2\right]}{E\left[\tilde{D}^2\right]}}$$

Measurable from the data

R^2 in linear regression of $\tilde{Y}$ on $\tilde{A}$ after linearly partialling out $\tilde{D}$

R^2 in linear regression of $\tilde{D}$ on $\tilde{A}$

Bias Bounds

- If the analyst can provide bounds on the strength of each relationship (e.g. $\delta, \gamma, E\left[\epsilon_D^2\right]$) then we can provide bounds on the true effect

$$\theta_0 = \theta_s \pm \left|\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right|$$

- More interpretable bounds:

$$\left(\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right)^2 = \boxed{R^2_{Y\sim A|D,X}} \frac{R^2_{D\sim A|X}}{1 - \boxed{R^2_{D\sim A|X}}} \boxed{\frac{\mathrm{E}\left[\left(\tilde{\mathrm{Y}} - \theta_s\tilde{D}\right)^2\right]}{E\left[\tilde{D}^2\right]}}$$

Measurable from the data

Reduction in unexplained variance of Y when adding A in the model that predicts Y from treatment and controls

Reduction in unexplained variance of D when adding A in the model that predicts D from controls

For more details:
Making Sense of Sensitivity: Extending Omitted Variable Bias

For more general analysis see:
Long Story Short: Omitted Variable Bias in Causal Machine Learning

Bias Bounds

- The analyst provides bounds on the partial R^2

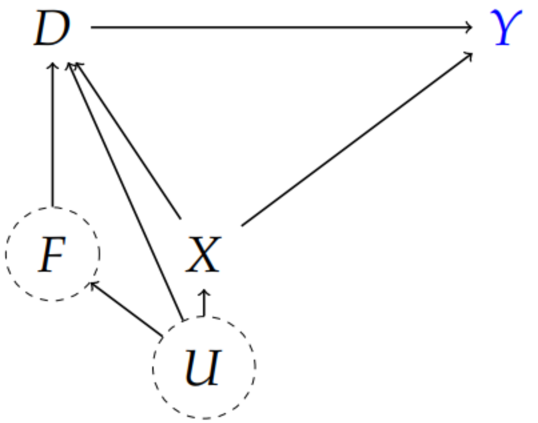
$$R^2_{Y\sim A|D,X} \leq C_Y^2, \qquad R^2_{D\sim A|X} \leq C_D^2$$

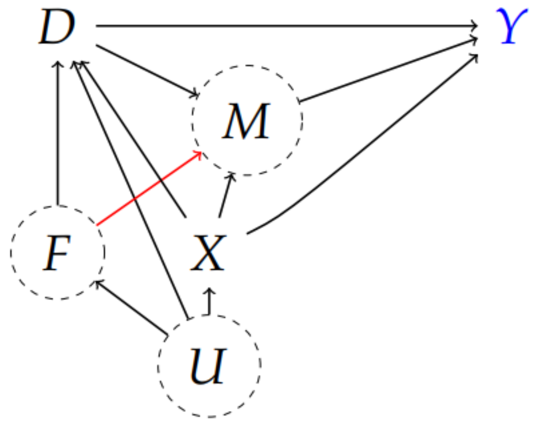
- Based on these bounds we can conclude that

$$\theta_0 \in \theta_s \pm \sqrt{C_Y^2 \frac{C_D^2}{1-C_D} \frac{E\left[\left(\tilde{Y} - \theta_s \widetilde{D}\right)^2\right]}{E\left[\widetilde{D}^2\right]}}$$

Application: 401k eligibility

- Y=net financial assets
- D=eligibility to enroll in 401(k) program
- X=pre-treatment worker-level covariates (observed)
- F=pre-treatment firm-level covariates (unobserved)
- M=amount of contribution matched by employer
- U=general latent factors





Controlling for X is sufficient in top figure but not bottor

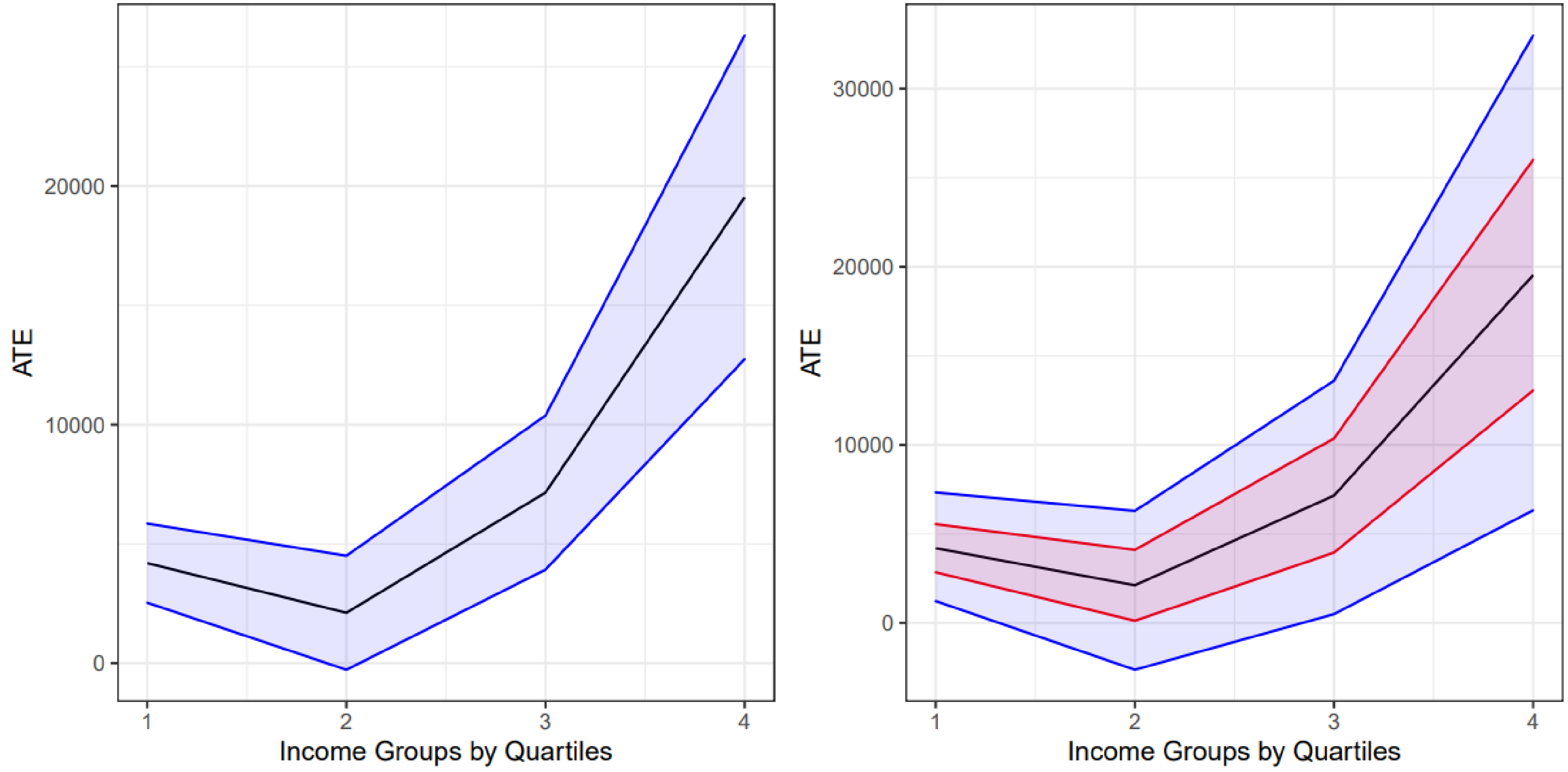
Confounding Scenario

- Posit that F explains as much variation in net financial assets as the total variation of maximal matched percentage (5%) of income over period of three years
- Posit that F explains an additional 2.5% of the variation in 401k eligibility, a 20% relative increase in the baseline R^2 of the treatment of 13%

- In PLR: translates to $\mathrm{C}_{\mathrm{Y}}^2 \coloneqq R_{Y\sim A|D,X}^2 \approx 4\%, C_D^2 \coloneqq R_{D\sim A|X}^2 \approx 3\%$

Robustness value (RV) = minimal equal strength of C_Y^2 and C_D^2 s.t. bound includes zero

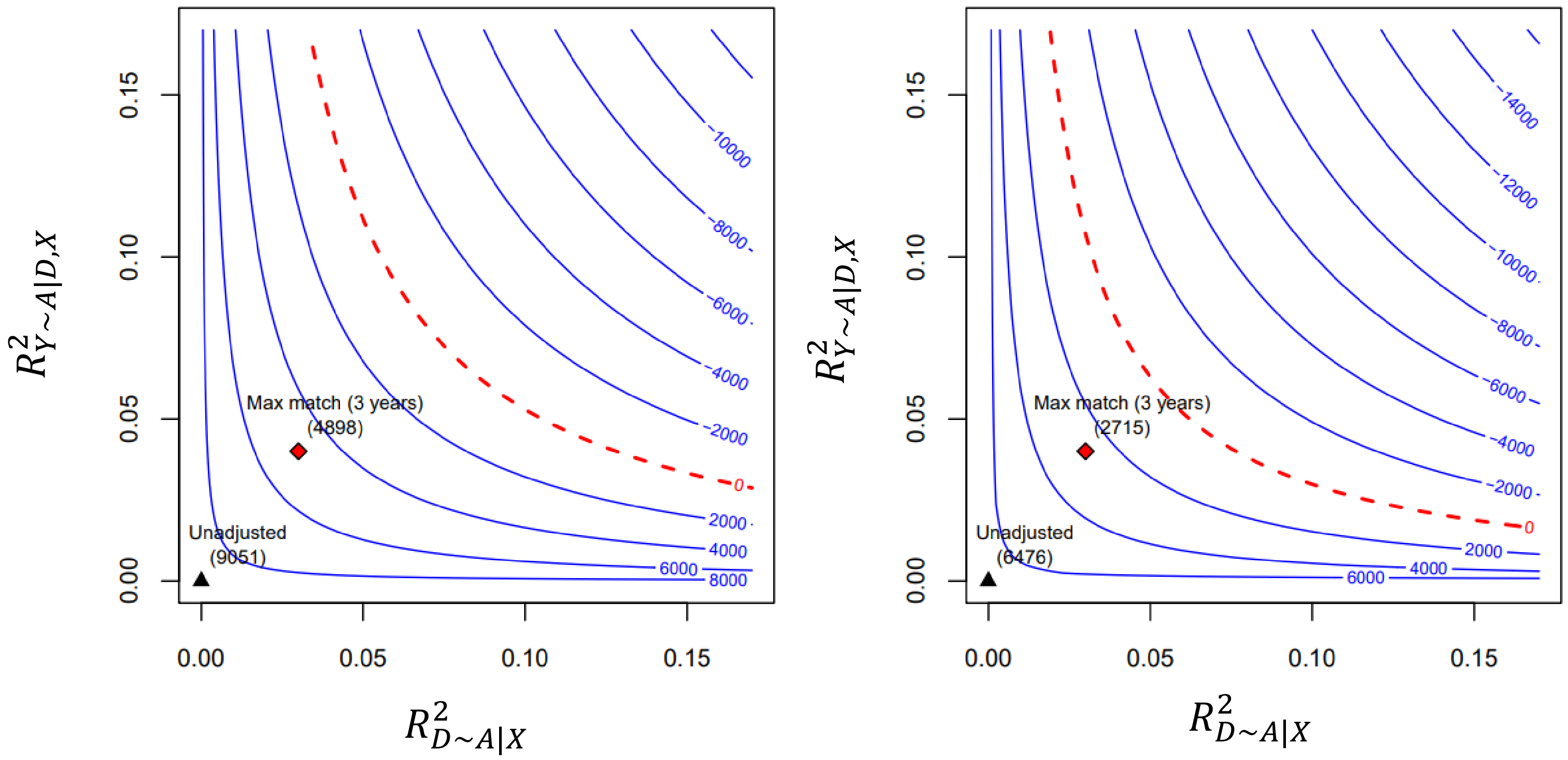
- RV=5.5% (at 95% significance level) > 4%,3%
- Finding that 401k eligibility has positive effect is robust to this confounding scenario



(a) Estimates under no confounding.

(b) Bounds under posited confounding.

Note: Estimate (black), bounds (red), and confidence bounds (blue) for the ATE. Confounding scenario: $\rho^2 = 1$; $C_Y^2 \approx 0.04$; $C_D^2 \approx 0.031$. Significance level of 5%.



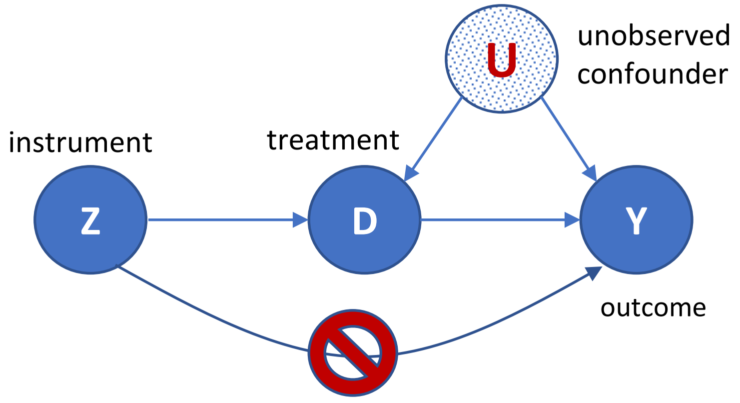
(a) Contours for $\theta_{-} = \theta_s - |B|$.

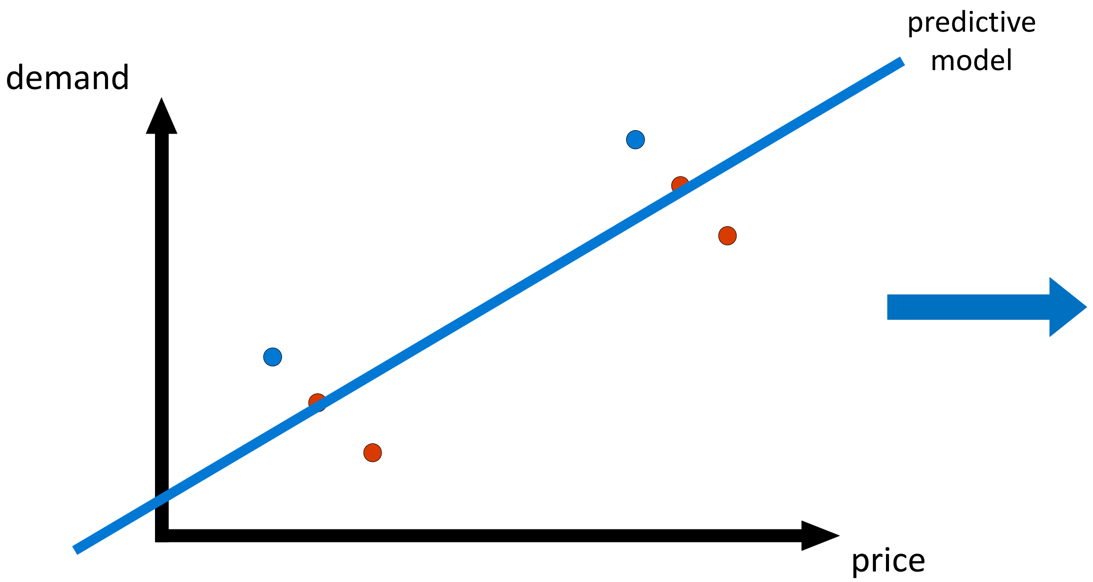
(b) Contours lower limit confidence bound.

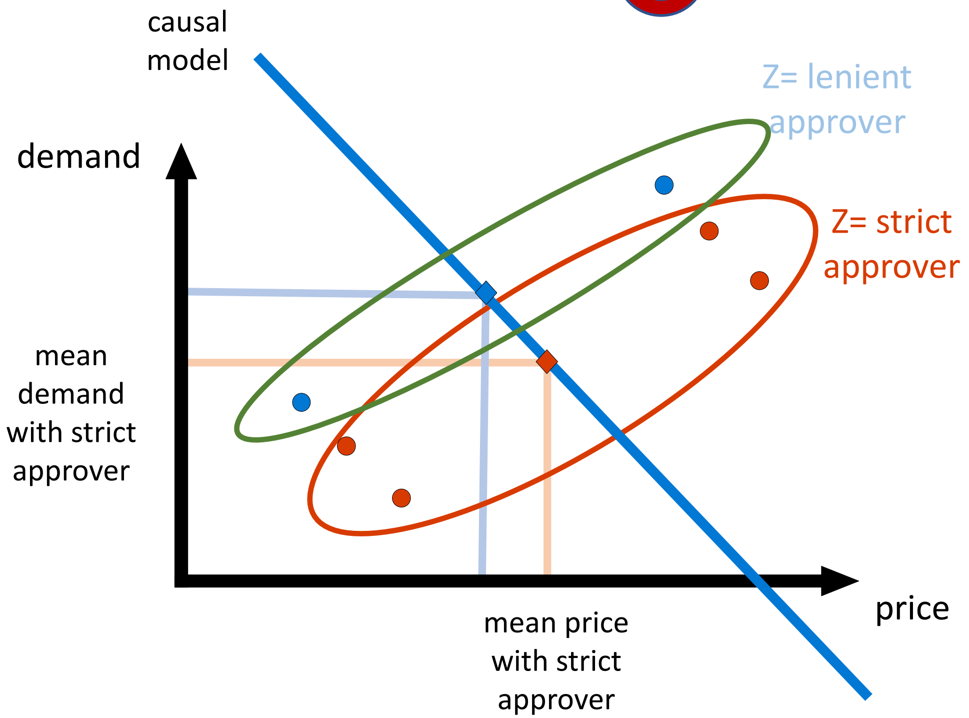
Can we recover the true effect?
Instrumental Variables

Instrumental Variables and 2SLS

Instrumental Variable: any random variable Z that affects the treatment (log-price) D but does not affect the outcome (log-demand) Y other than through the treatment [Wright'28, Bowden-Turkington'90, Angrist-Krueger'91, Imbens-Angrist'94]



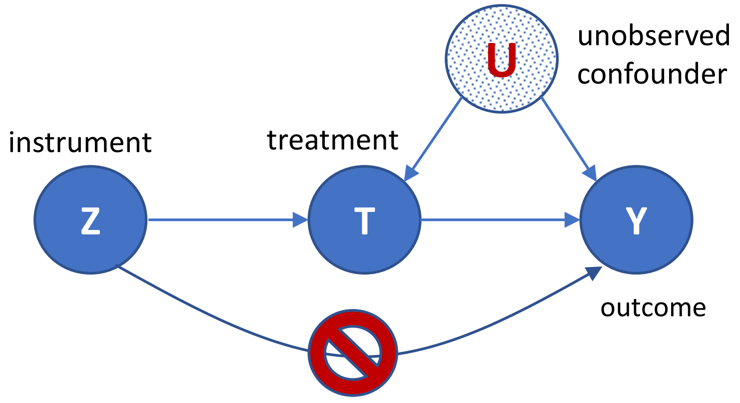




Instrumental Variables and 2SLS

Instruments are widely used

- In the discount example (see also [Kling AER06] for effects of incarceration)
  - Discounts are sent to an approver desk
  - Approver assignment is random and different approvers are more or less "lenient"
  - Approver leniency is an instrument

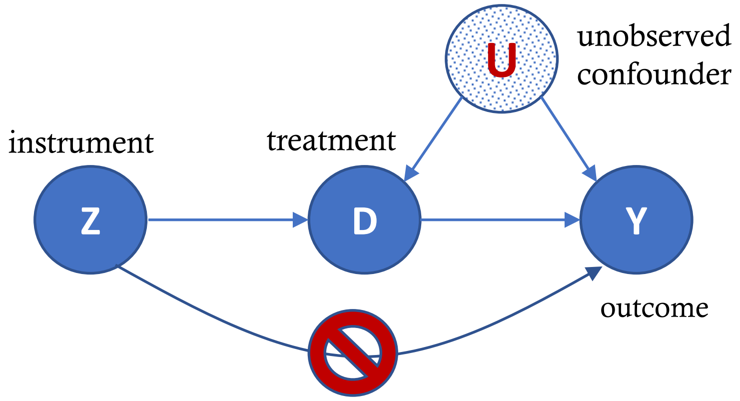


- In healthcare [Doyle et al., JPE15]
  - Random assignment to ambulance companies of nearby patients is an instrument for measuring hospital quality

- In Tech [S., NeurIPS19]
  - Recommendation A/B tests as instruments for the effects of downstream actions

Identification of Causal Effects via Instruments

Phillip Wright's idea (1928): the first causal path diagram analysis



- We can estimate effect of Z on y via a regression

$$\gamma = \frac{\mathbb{E}[\tilde{Z}\tilde{y}]}{\mathbb{E}[\tilde{Z}^2]}$$

- We can estimate the effect of Z on D via a regression

$$\delta = \frac{\mathbb{E}[\tilde{\mathrm{Z}}\tilde{\mathrm{D}}]}{\mathbb{E}[\tilde{\mathrm{Z}}^2]}$$

- The effect of Z on Y (γ) is the product of the effect of Z on T (δ) multiplied by the effect of T on y (θ)

$$\theta = \frac{\gamma}{\delta} = \frac{\mathbb{E}[\tilde{Z}\tilde{y}]}{\mathbb{E}[\tilde{Z}\tilde{D}]}$$

Partially Linear Instrumental Variable Model

- Typically for continuous treatment/instrument a partially linear structural equation assumed

$$
\begin{gathered}
Y \coloneqq \theta_0 D + f_Y(X) + \delta A + \epsilon_Y \\
D \coloneqq \beta Z + f_D(X) + \gamma A + \epsilon_D \\
Z \coloneqq f_Z(X) + \epsilon_Z \\
A \coloneqq f_A(X) + \epsilon_A
\end{gathered}
$$

All errors are exogenous and un-correlated

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$
\begin{aligned}
\tilde{Y} &:= \theta_0 \widetilde{D} + \delta \tilde{A} + \epsilon_Y \\
\widetilde{D} &:= \beta \tilde{Z} + \gamma \tilde{A} + \epsilon_D \\
\tilde{Z} &:= \epsilon_Z \\
\tilde{A} &:= \epsilon_A
\end{aligned}
$$

- We see immediately that:

$$\tilde{Y} := \theta_0 \widetilde{D} + U, \qquad U := \delta \tilde{A} + \epsilon_Y \perp \tilde{Z}$$

- Since $\epsilon_A, \epsilon_Y, \epsilon_Z$ are un-correlated: $E\left[\left(\delta \tilde{A} + \epsilon_Y\right)\tilde{Z}\right] = 0$
- Thus we have the moment restriction: $E\left[\left(\tilde{Y} - \theta_0 \widetilde{D}\right)\tilde{Z}\right] = 0$

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} \coloneqq \theta_0 \widetilde{D} + \delta \tilde{A} + \epsilon_Y$$
$$\widetilde{D} \coloneqq \beta \tilde{Z} + \gamma \tilde{A} + \epsilon_D$$
$$\tilde{Z} \coloneqq \epsilon_Z$$
$$\tilde{A} \coloneqq \epsilon_A$$

- Thus we have the moment restriction: $E\left[\left(\tilde{Y} - \theta_0 \widetilde{D}\right)\tilde{Z}\right] = 0$
- We re-derive a generalization of Wright's formula

$$\theta_0 = \frac{E\left[\tilde{Y}\tilde{Z}\right]}{E\left[\widetilde{D}\tilde{Z}\right]}$$

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} := \theta_0 \widetilde{D} + \tilde{A} + \epsilon_Y$$
$$\widetilde{D} := \beta \tilde{Z} + \beta \tilde{A} + \epsilon_D$$
$$\tilde{Z} := \epsilon_Z$$
$$\tilde{A} := \epsilon_A$$

- Setting falls into the general moment estimation framework

$$M(\theta, h, p, m) = E\left[\left(Y - h(X) - \theta\left(D - p(X)\right)\right)\ \left(Z - m(X)\right)\right] = 0$$

- Where $h(X) = E[Y|X]$, $p(X) = E[D|X]$, $m(Z) = E[Z|X]$

Orthogonal Method: Double ML for IV

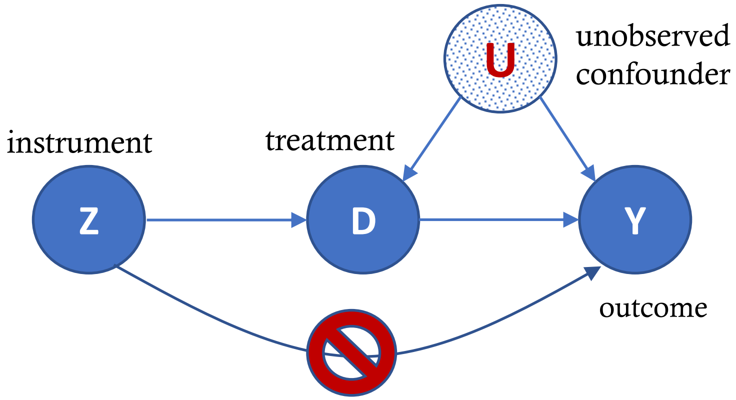
Double ML. Split samples in half

- Regress $Y \sim X$ with ML on first half, to get estimate $\hat{h}(S)$ of $E[Y|X]$
- Regress $D \sim X$ with ML on first half, to get estimate $\hat{p}(S)$ of $E[D|X]$
- Regress Z $\sim X$ with ML on first half, to get estimate $\widehat{m}(S)$ of $E[Z|X]$
- Construct residuals on other half, $\hat{Z} = Z - \widehat{m}(X), \widehat{D} \coloneqq D - \hat{p}(X)$ and $\hat{Y} \coloneqq Y - \hat{h}(X)$
- Solve moment condition:

$$E_n\left[\left(\hat{Y} - \theta\widehat{D}\right)\widehat{D}\right] = 0$$

```
from econml.iv.dml import OrthoIV
orthoiv = OrthoIV()
orthoiv.fit(y, D, Z, W=X).effect_inference()
```

The Binary Case



Imbens-Angrist (1994): core contribution of Nobel 2022 award

- Instrument/Treatment are binary (instrument=recommended treatment)
- Assume monotonicity: $D^{(1)} \geq D^{(0)}$
- Recommended treatment cannot reverse taken treatment
- Object of interest: Local Average Treatment Effect (ATE among compliers)

$$\theta_0 = E\left[\mathrm{Y}^{(1)} - \mathrm{Y}^{(0)} \middle| \mathrm{D}^{(1)} > \mathrm{D}^{(0)}\right]$$

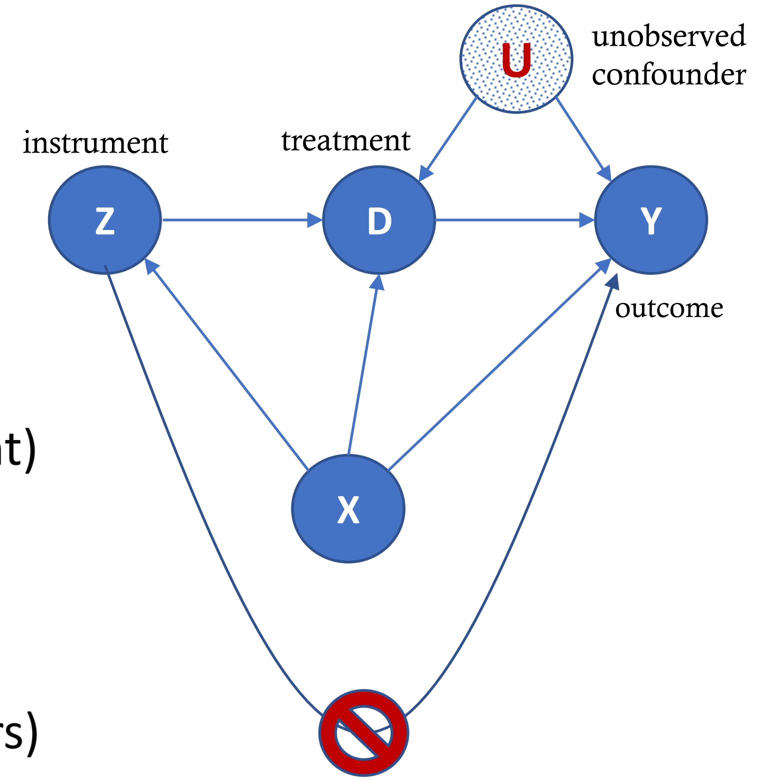
- Proof [Angrist-Imbens'94]:

$$\theta_0 = \frac{E\left[\left(Y^{(1)} - Y^{(0)}\right)1\left\{D^{(1)} > D^{(0)}\right\}\right]}{E\left[1\left\{D^{(1)} > D^{(0)}\right\}\right]} = \frac{E\left[Y^{(D(1))} - Y^{(D(0))}\right]}{E\left[D^{(1)} - D^{(0)}\right]} = \frac{\boxed{ATE(Z \to Y)}}{\boxed{ATE(Z \to D)}}$$

$$\boxed{\gamma = \frac{\mathbb{E}\left[\tilde{Z}\tilde{y}\right]}{\mathbb{E}\left[\tilde{Z}^2\right]}}$$

$$\boxed{\delta = \frac{\mathbb{E}\left[\tilde{\mathrm{Z}}\tilde{\mathrm{D}}\right]}{\mathbb{E}\left[\tilde{\mathrm{Z}}^2\right]}}$$

The Binary Case



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$$E\left[E[Y|Z=1,X] - E[Y|Z=0,X]\right]$$

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$$E\left[E[D|Z=1,X] - E[D|Z=0,X]\right]$$

LATE in the Binary Case

- Under monotonicity

$$\theta_0 = \frac{E\big[E[\,Y \mid Z = 1, X\,] - E[Y|Z = 0, X]\big]}{E\big[E[D|Z = 1, X] - E[D|Z = 0, X]\big]}$$

- Moment formulation

$$E[E[Y|Z = 1, X] - E[Y|Z = 0, X] - \theta_0(E[D|Z = 1, X] - E[D|Z = 0, X])] = 0$$

$$+ \qquad\qquad\qquad\qquad +$$

$$\boldsymbol{a_{Z\to Y}(Z, X)(Y - E[Y|Z, X])} \qquad\qquad \boldsymbol{a_{Z\to D}(Z, X)(D - E[D|Z, X])}$$

- Orthogonal moment formulation: apply ATE debiasing twice