Soft Weighted Machine Unlearning

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https://arxiv.org/abs/2505.18783

Setting

Consider a prediction task (regression problem) with:

- Input space $\mathcal{X}$;
- Output space $\mathcal{Y}$;
- Training set $\mathcal{D}^n = \{z_i = (\boldsymbol{x_i}, y_i)\}_{\{i=1\}}^n \subset \mathcal{Z} = \mathcal{X} \times \mathcal{Y}$;
- Parameter $\theta \in \Theta := \mathbb{R}^d$;
- $f(\theta; x)$ estimator of $y|\boldsymbol{x}$;
- $\ell : \mathcal{Z} \times \Theta \to \mathbb{R}$ loss function (e.g., $\ell(z, \theta) \mapsto \|y - f(\theta; \boldsymbol{x})\|^2$).

We seek the empirical risk minimizer:

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} \ell(z_i, \theta).$$

Goal

For some problems (data poisoning, fairness, robustness, etc.), it is desirable to unlearn the contribution of certain training data.

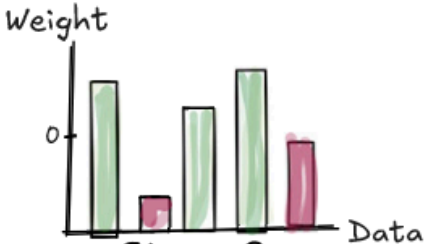
Challenges:

- Retraining the model from scratch is computationally expensive;
- The removal may decrement the performance of the model on the remaining data.

Solution: Use influence functions with soft weights.

Method

Step1:
Influence Evaluation



Step 2:
Data Reweighting

Hard

1. Compute each sample's influence on the target task

Sample z: ✗ ✗ ✓ ... ✓ ✓

Weight ϵ: -1 -1 0 0 0

Select the top-k detrimental samples on target task, and set weights to -1 (unlearn).

Soft

1. Compute each sample's influence on the target task
2. Compute each sample's influence on the utility.

Sample z: ✗ ✗ ✓ ... ✓ ✓

Weight ϵ: -1.2 -0.3 0.1 0.8 1.1

Optimize a set of weights based on the influence on the utility and the target task.

Influence Functions

Given $z_j \in \mathcal{D}^n$ and $\varepsilon_j \in \mathbb{R}$, we can define:

$$\hat{\theta}(z_j, \varepsilon_j) = \arg\min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} \ell(z_i, \theta) + \varepsilon_j \ell(z_j, \theta).$$

The *influence function* relative to z_j is:

$$\mathcal{I}(z_j) = \frac{d}{d\varepsilon_j} \hat{\theta}(z_j, \varepsilon_j) \Big|_{\varepsilon_j = -1}$$

Interpretation: It indicates how the training error would change if we removed the training data z_j.

1. If ℓ is at least C^2, we can write the influence function in a closed form:

$$\mathcal{I}(z_j, -1) = -\frac{1}{n} H_{\hat{\theta}}^{-1} \nabla_\theta \ell\left(z_j, \hat{\theta}\right)$$

where H is the Hessian matrix of the empirical risk at $\hat{\theta}$.

2. The influence function is a *first-order approximation* of the change in the estimator when we remove z_j from the training set.

In fact, if we do the *Taylor expansion* of $\hat{\theta}$ at $\varepsilon_j = -1$, we have:

$$\hat{\theta} = \hat{\theta}(z_j, -1) + \mathcal{I}(z_j) + O(\varepsilon_j^2).$$

We will consider the influence on these 3 metrics (negative influence is better):

- **Utility:** $\mathcal{I}_{\text{util}}(z_j, -1) = \sum_{z \in \mathcal{T}} \nabla_\theta \ell\left(z, \hat{\theta}\right)^\top H_{\hat{\theta}}^{-1} \nabla_\theta \ell\left(z, \hat{\theta}\right)$, where $\mathcal{T}$ is the validation set;
- **Fairness:** $\mathcal{I}_{\text{fair}}(z_j, -1) = \nabla_\theta f_{\text{fair}}\left(\mathcal{T}, \hat{\theta}\right)^\top H_{\hat{\theta}}^{-1} \nabla_\theta \ell\left(z, \hat{\theta}\right)$, with f_{fair} a fairness metric (e.g., demographic parity);
- **Robustness:** $\mathcal{I}_{\text{robust}}(z_j, -1) = \sum_{\tilde{z} \in \tilde{\mathcal{T}}} \nabla_\theta \ell\left(\tilde{z}, \hat{\theta}\right)^\top H_{\hat{\theta}}^{-1} \nabla_\theta \ell\left(z, \hat{\theta}\right)$, for a perturbed dataset $\tilde{\mathcal{T}}$.

Algorithm 1: Soft-Weighted Unlearning Framework

Input: Model $\hat{\theta}$, Training Dataset $\mathcal{D}$, Validationa and Testing Dataset $\mathcal{T}$, Adversarial Samples $\tilde{z} \in \tilde{\mathcal{T}}$

 # Step 1: Influence Evaluation.

for *each sample* $z_i \in \mathcal{D}$ **do**

  Evaluate influence of z_i on validation set;

  Utility: $\mathcal{I}_{\text{util}}(z_i; -1) \leftarrow$ Equation (3).

  Fairness: $\mathcal{I}_{\text{fair}}(z_i; -1) \leftarrow$ Equation (4).

  Robustness: $\mathcal{I}_{\text{robust}}(z_i; -1) \leftarrow$ Equation (5).

end

 # Step 2: Weights Optimization.

Weights $\{\epsilon_i^*\}_{i=1}^n \leftarrow$ Equation (7)

 # Step 3: Model Correction.

if $f \leftarrow f_{fair}(\mathcal{T}; \theta)$ *or* $\sum_{\tilde{z} \in \tilde{\mathcal{T}}} \nabla_\theta \ell(\tilde{z}; \theta) \leq \delta$ **then**

  $\theta \leftarrow$ Equation (8) or Other Unlearning Algorithms

end

Output: θ

Soft weights

Instead of a binary decision (keep or remove), we want to assign a customized weight to each training data.
Namely, we want to find $\varepsilon^\star = (\varepsilon_1, ..., \varepsilon_n)$ and use it for the one-shot updating rule:

$$\hat{\theta}(\mathcal{D}, \varepsilon^\star) \approx \hat{\theta} + \varepsilon^\star H_\theta^{-1} \left(\nabla_\theta \ell\left(z_1, \hat{\theta}\right), ... \nabla_\theta \ell\left(z_n, \hat{\theta}\right) \right)^\top.$$

I.e., we update each training data with a re-scaled loss function

$$\varepsilon_i \mathcal{J}(z_i, -1) = \mathcal{J}(z_i, \varepsilon_i).$$

We need weights to be easy to retrieve, since they must be computed for each training data.

They are defined as the solution of the following optimization problem:

$$\min_{\boldsymbol{\varepsilon} \in \mathbb{R}^n} \sum_{i=1}^{n} \mathcal{J}_{\text{fair}}(z_i, \varepsilon_i) + \lambda \, \|\boldsymbol{\varepsilon}\|_2^2$$

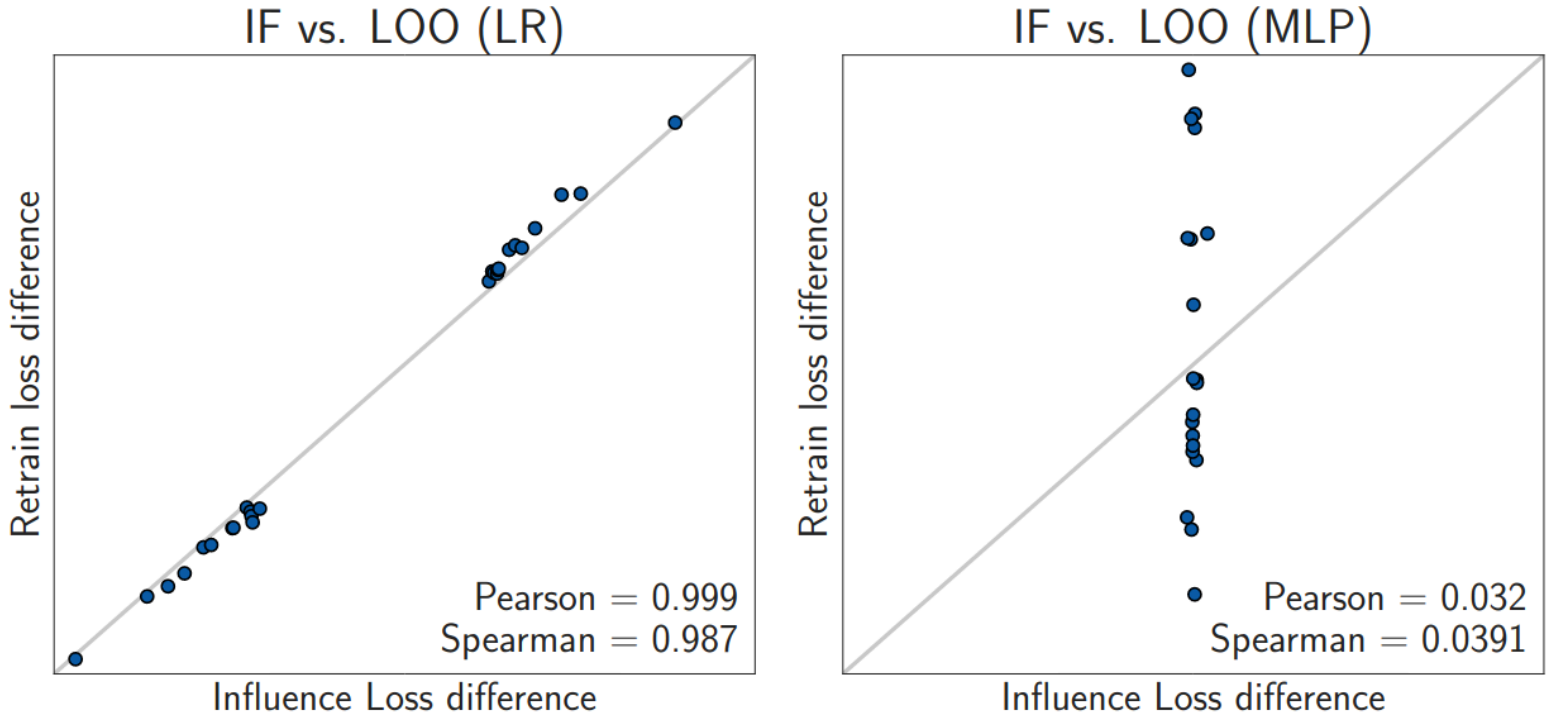
(quadratic problem) subject to:

$$\sum_{i=1}^{n} \mathcal{J}_{\text{fair}}(z_i, \varepsilon_i) \geq -f_{\text{fair}}\left(\mathcal{T}; \hat{\theta}\right), \quad \sum_{i=1}^{n} \mathcal{J}_{\text{util}}(z_i, \varepsilon_i) \leq 0$$

Main issues

- We assume to know $\hat{\theta}$ so that $H_{\hat{\theta}}$ is PD, thus invertible. In practice we only have an approximation;
- Computing the inverse of the hessian is expensive. We must use approximations;
- Such approximations don't work well for neural networks.

Bad approximation



["If Influence Functions are the Answer, Then What is the Question?", J. Bae, N. Ng, A. Lo, M. Ghassemi, R. Grosse, 2022]

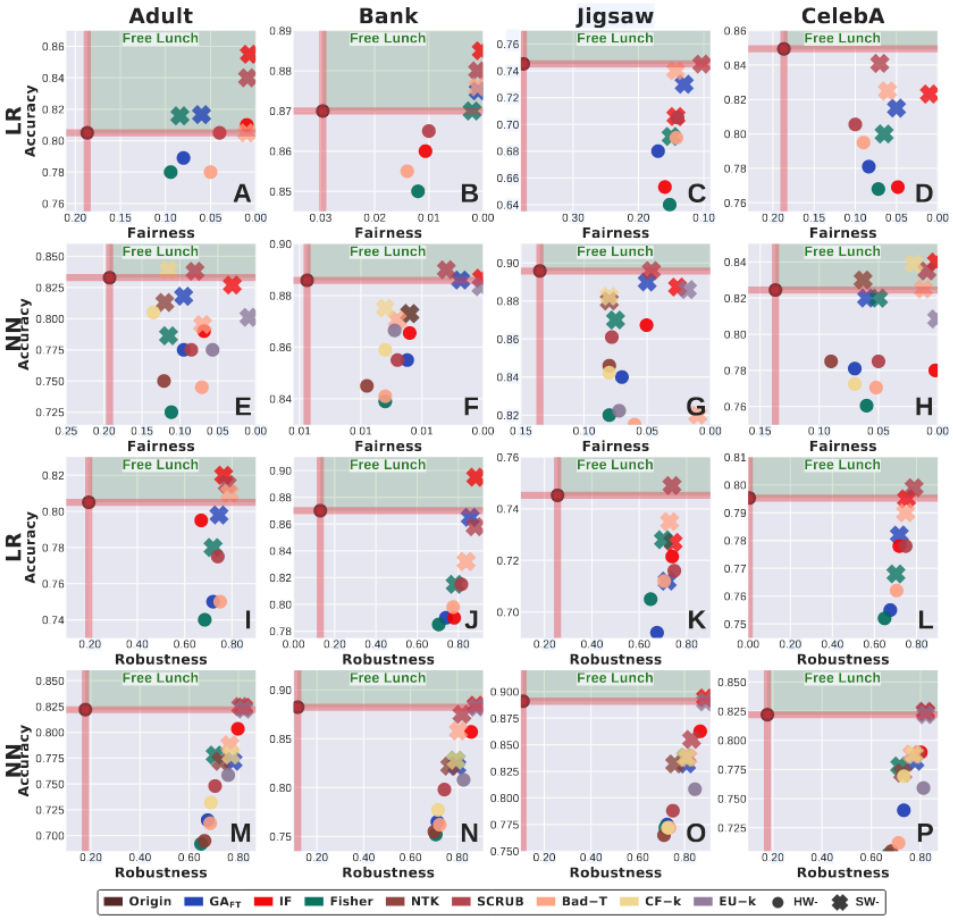
A solution

1. Add a regularization term to the loss function. This way, $H_{\hat{\theta}}$ is guaranteed to be PD.
2. Approximate the hessian with σI. This way, we have:

$$\hat{\theta}\left(z_j, \varepsilon_j^\star\right) - \hat{\theta} \approx \varepsilon_j^\star \frac{\sigma}{n} \nabla_\theta \ell\left(z_j, \hat{\theta}\right).$$

If we see $\frac{\sigma}{n}$ as a learning rate, we can index it by time and use also others unlearning algorithms instead of IFs to do the update in an iterative fashion(?).

Numerical evidence



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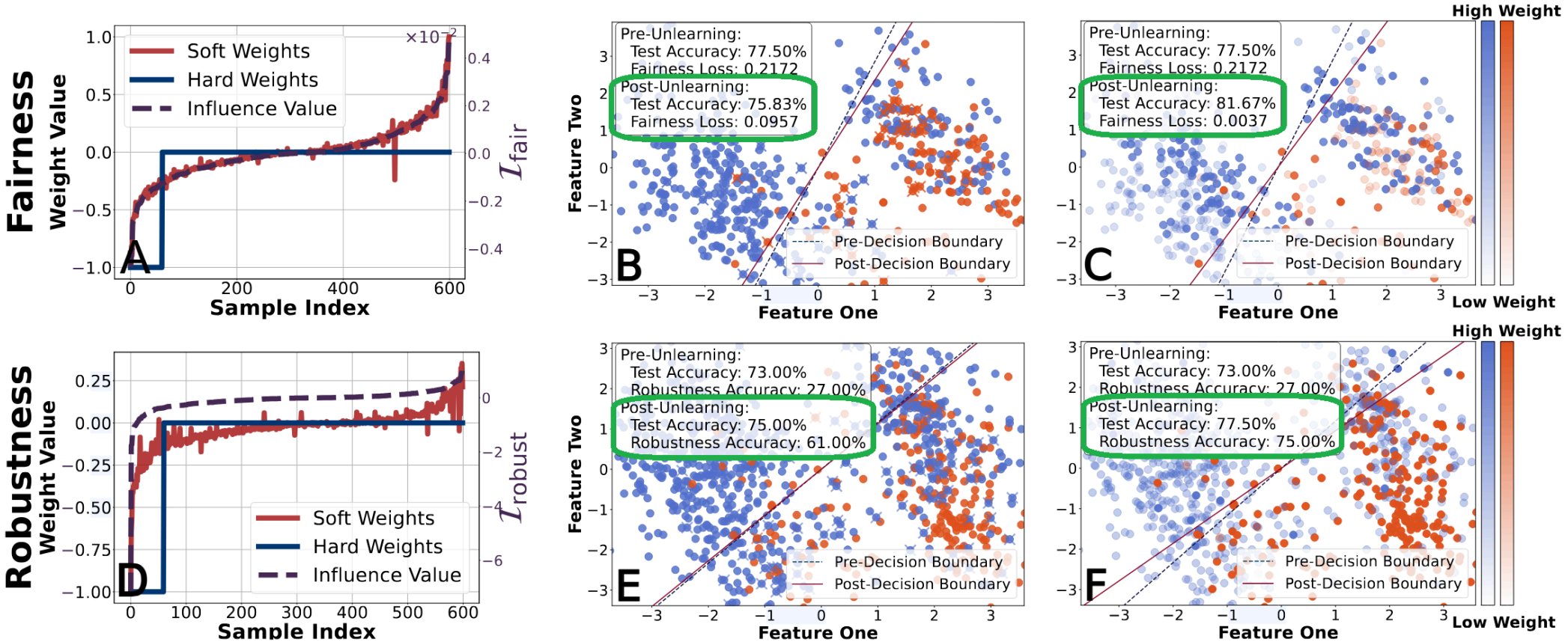

Take-away

Using some smart approximations, we can adapt all the existing unlearning algorithms to the soft weighted setting.
This family of methods performs better than the HW version for non-privacy applications and can also improve the performance of the model on the remaining data.

["Soft Weighted Machine Unlearning", X. Qiao, N. Ding, Y. Cheng, M. Zhang, 2025]