Inflation Expectations, Hypothesis Testing

# Hypothesis Testing: An Application to Inflation Rates

Wilcoxon Signed Ranked Test: Do Consumers and Investors Agree on Inflation Expectations?

Consumer expectations of future inflation rates have decreased in recent months according to data from the University of Michigan Survey of Consumers.

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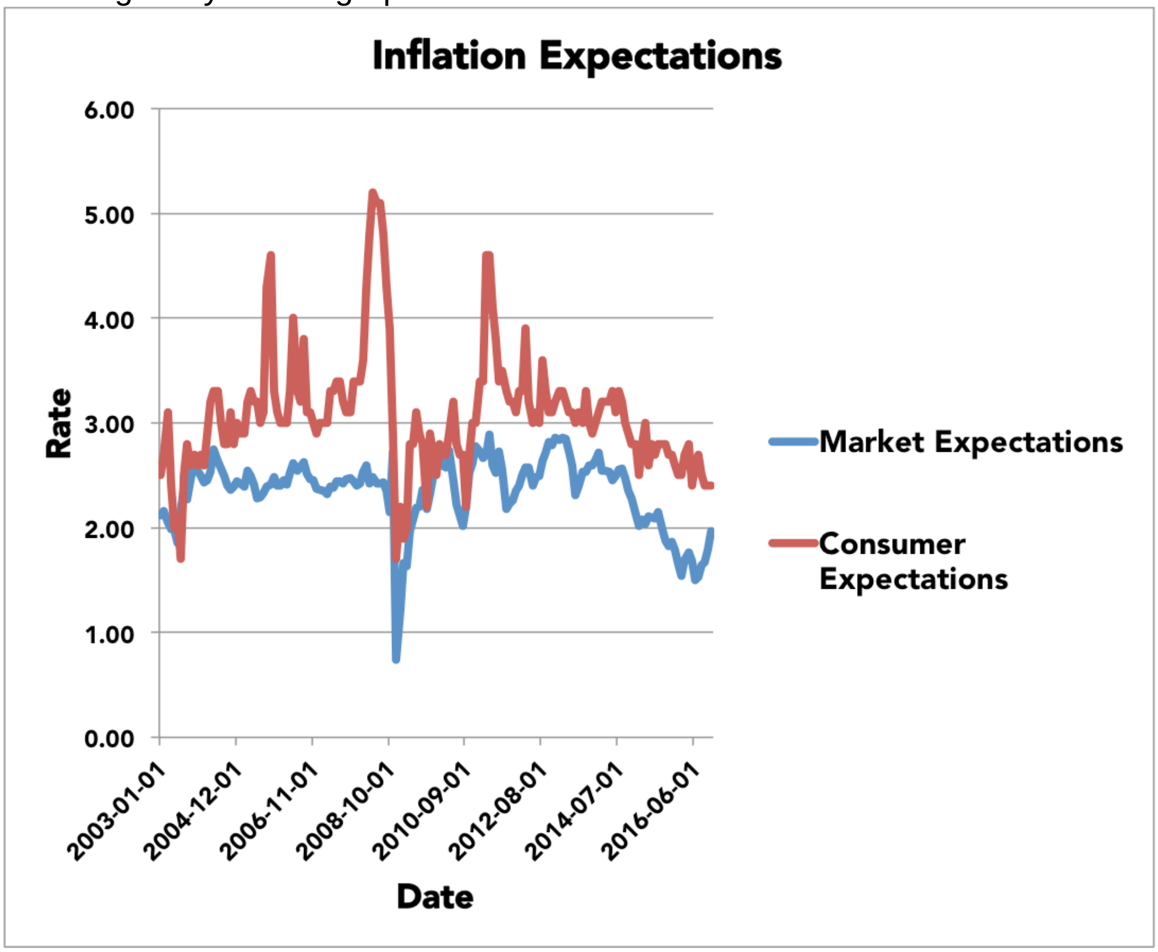
Inflation expectations are important to investors, consumers, lenders, and borrowers. For example, bond investors care about inflation expectations because higher inflation rates chip away at their returns. Consumers, on the other hand, care about inflation expectations because higher inflation rates mean the purchasing power of their savings will decrease. As seen in Venezuela and other countries with hyperinflation, millions of people can have their entire savings wiped away by periods of extremely fast price increases.

Moderate changes in the inflation rate are usually not an issue if investors and consumers can forecast these rate changes. For example, when inflation expectations build, bond yields typically increase, as investors predict that future inflation will chip away at their investment at a faster pace, so they simply demand a higher return. Similarly, when consumers expect faster inflation, they may choose to save less money if the interest rate on their savings does not increase. Thus, banks often have to increase interest rates when high inflation is expected in upcoming years. In this snapshot report, we analyze and compare both investor inflation expectations and consumer inflation expectations with the Wilcoxon Signed Rank Test.

To complete this analysis, we first gather consumer inflation expectation data from the Survey of Consumers of the University of Michigan

(UMichigan), which is collected each month. Market expectations of inflation, on the other hand, were gathered from the 5-year, 5-year forward break-even inflation rate. This rate tracks the difference in yield between inflation-protected Treasurys (TIPS) and nominal Treasurys. Thus, this rate shows, in five years, what the market predicts inflation will be over the course of the following five years. A graph of these rates can be seen below.




To analyze these rate expectations, we will use the Wilcoxon Signed Rank Test, a common non-parametric hypothesis test. For i = 1…n, we let $X_i$ represent the difference in inflation expectations for investors and consumers in the $i^{th}$ time period. As an example of this notation, for the tenth entry in this dataset (October, 2003), we define $X_{10}$ = (consumer expectations of inflation from October 2003) - (market expectations of inflation from 2003), where, again, consumer expectations are gathered from the UMichigan survey report, and market/investor expectations of inflation are retrieved from the 5-year, 5-year forward break-even rate.

Let $X_1….X_n$ be independent and identically distributed (typical IID assumption) from an arbitrary distribution *f*, where *f* is symmetric about some constant c. Similar to my previous report using the Mann-Whitney U-test, we will again drop the normality assumption in this analysis. As stated previously, although normal distributions and lognormal distributions are mathematically convenient, it is rare that financial data truly comes from a normal distribution. Thus, in this analysis, we want to make as few unnecessary assumptions as possible, and we only assume the distribution *f* is symmetric.

In this analysis, we use a significance level of $\alpha = 0.05$, and the null and alternative hypotheses are defined as follows: $H_0$: *f* is symmetric about $\mu = 0$ and $H_1$: *f* is symmetric about $\mu = c$ for some constant c, where $c \neq 0$. Intuitively, since $X_i$ is the difference in inflation expectations between consumers and investors, the null hypothesis $H_0$ is essentially saying that consumer expectations of inflation are not statistically significantly different than market expectations of inflation. Thus, if we reject the null hypothesis ($p < \alpha = 0.05$), the data is supporting that inflation expectations from consumers and investors are statistically different at the given significance level. On the other hand, if $p > \alpha = 0.05$, the data does not support that inflation expectations are statistically different for investors and consumers.

Similar to the Mann-Whitney U-test described in the previous report, the Wilcoxon signed rank test exploits symmetry under $H_0$. To complete this test, the absolute values of $X_1 \ldots X_n$ are sorted in increasing order, where, again, 'n' is the total number of entries in the dataset. After sorting these values, the smallest absolute value of $X_i$ is assigned a rank of 1, the next smallest absolute value of $X_i$ is assigned a rank of 2, etc. The largest absolute value of $X_i$ is assigned a rank of 'n.' Finally, our test statistic, which we will call $W_+$ in this analysis, is the sum of the ranks corresponding to *only* the positive values of $X_1 \ldots X_n$.

As an example of the Wilcoxon signed rank test described above, imagine a scenario where we have only four data points ($X_1 = -4$, $X_2 = 15$, $X_3 = -6$, and $X_4 = 3$). Following the procedures stated above, after sorting the $X_i$ in terms of increasing absolute value ($X_4$, $X_1$, $X_3$, $X_2$), we see $W_+ = 1 + 4 = 5$ (rank $X_4$ + rank $X_2$) in this scenario. Of course, our dataset used in the analysis of inflation rates was much larger, so the Wilcoxon Signed Rank Test was completed in Python (with import pandas).

Returning to the analysis of inflation expectations data, we must recall that our alternative hypothesis was formulated above as a two-sided test ($H_1$: *f* is symmetric about $\mu = c$ for some constant c, where $c \neq 0$). Because this test is two-sided, either abnormally large or small values of $W_+$ indicate we should reject $H_0$. According to statistical theory, the distribution of $W_+$ under $H_0$ is approximately normal with $\mu = n*(n+1) \div 4$, where $n = 167$ is the number of entries in our dataset.

In this analysis, the Wilcoxon Signed Rank Test from Python returns $p = 8.94 * 10^{-29}$. Thus, we reject the null hypothesis $H_0$, and this dataset supports that inflation expectations of consumers and investors come from different distributions. Please note: although this result may seem trivial, it is not. When looking at the dataset of inflation expectations for both consumers and investors, it is clear that consumers almost always predict higher inflation rates than the investors predict. Consequently, investors and consumers are clearly not on the same page when it comes to forecasting where inflation is headed in upcoming years.

Inflation expectations are especially important now, as the difference in inflation expectations between consumers and investors has begun to narrow after Trump was elected. In recent months, investors have increased inflation expectations for upcoming years, with President-elect Trump expected to cut taxes, spend on infrastructure, increase import tariffs, and force companies to pay higher wages, all potential drivers of higher inflation rates. For these reasons, money has flocked into funds holding TIPS in recent months. On the other hand, consumer expectations of inflation from the UMichigan Consumer Survey decreased the past two months. We should continue to watch these inflation expectations in the upcoming months when Trump takes office.


Click here to download the data used in this snapshot financial report.