Quantitative

Assessment May 2018

Introduction

The goal of this practical project is to assess the quantitative skills of the candidate in relation to three main areas:

* Financial mathematics awareness
* Ability to implement a model in a programming language
* Ability to explain and discuss model’s features

To successfully complete the practical project, the candidate is not expected to undertake any significant research. The main results required to complete the project will be provided in the body of this assignment.

Rules

The candidate must return the solution to the practical project within 5 calendar days, starting from the day following the reception of this assignment.

The candidate is very welcome to choose any suitable modelling environment to complete the assignment. Examples of modelling environments readily accessible are:

* Microsoft Excel
* C# or C++
* Freemat (free download available at: <http://freemat.sourceforge.net/>)
* Scilab (free download available at: <http://www.scilab.org/>)
* Octave (free download available at: <http://www.gnu.org/software/octave/>)

The last three modelling software are popular “clones” of MATLAB.

The candidate is expected to return:

* the code
* a short note summarizing/discussing the key findings in each question

Those files must be compressed into a single archive (*.zip* or *.rar*) and sent to the following email address:

[georges.francois@willistowerswatson.com](mailto:georges.francois@willistowerswatson.com)

Candidates with a successful quantitative assessment will be shortlisted for a final round of interview that will mainly consist of a discussion of the practical project.

If you have any questions, please email [georges.francois@willistowerswatson.com](mailto:georges.francois@willistowerswatson.com)

Practical Project

In an interest rate model, one can assume that – over a chosen time interval - the change in the short term interest rate evolves according to the following stochastic process:

With:

= an arbitrarily short time interval;

= a Brownian motion;

= a parameter, often referred to as speed of mean reversion;

= a parameter, often referred to as mean short term interest rate;

= a parameter, often referred to as diffusion coefficient;

One of the key findings of the model is that the price of a zero coupon bond can be expressed as function of the unknown parameters of the model. More specifically, the price of a zero coupon bond (valued at time *t* and paying 1£ at time *T)* is given by:

Where:

= short rate = “short term (or instantaneous) spot rate”

The data in the spreadsheet represent a snapshot of the interest rate market taken at a given valuation date (data descriptions can be found in the comments added to the header of each column).

**Question 1 – Please, calibrate the model parameters based on this set of data we provided and discuss briefly (or add comments to code) any embedded assumptions you are making as well as the results of the calibration.**

*Hints:*

* *Question 1 can be regarded as an application of a nonlinear least squares estimation*
* *With we denote the value of at t = 0. This is the value which will be used to initialise your simulation (see Question 2)*
* *can be either an output of your calibration or fixed upfront*
* *If is treated as fixed parameter, please set it to a reasonable value and calibrate only the remaining model parameters*
* *Fix the sigma parameter to 0.015 for the calibration.*

**Question 2 – Using the model parameters calibrated in question 1, please simulate the stochastic process for the spot interest rate. Please, use the following configuration settings:**

**Number of simulated years = 5;**

**Number of simulated paths = 1,000;**

**Time Step Length = Annual;**

*Hints:*

* *Use a Euler scheme to discretise the process you want to simulate*
* *In case you did not manage to obtain a set of calibrated parameters from question 1, please carry on the project by using the parameterization below:* 
  + *= 0.65*
  + *= 0.015*
  + *= 0.06*
  + *= 0.01*

**Question 3 – Using the scenario set obtained at the precedent point, please value a derivative with a generic terminal payoff:**

Where:

= maturity of the option = 2 years

= maturity of the zero coupon bond underlying the option = 5 years

= value in of the zero coupon bond paying 1£ in (assume no default can occur)

= strike price of the option = 82£

*Hints:*

* *Use Monte Carlo pricing approach using the scenarios from Question 2*
* *Time-to-maturity of the underlying zero coupon bond at the time of the option expiry is in this case 3 years (= maturity of the underlying zero coupon bond – maturity of the option)*

**Question 4 – Briefly describe how the option value depends on each of the model parameters.**