Phys 217 Measurements & Units

Santa Ana College

Chapter 1: “Units and Measurement”

- This module will cover Chapter 1 of OpenStax College Physics (2e).

- It covers the following topics and sections:

    - 1.1 Physics: An Introduction
    - 1.2 Physical Quantities and Units
    - 1.3 Accuracy, Precision, and Significant Figures
    - 1.4 Approximation

- The Chapter Summary (list of equations) is available here.

Learning Objectives

- Express quantities given in SI units using metric prefixes.
- Use conversion factors to express the value of a given quantity in different units.
- Determine the correct number of significant figures for the result of a computation.
- Describe the relationship between the concepts of accuracy, precision, uncertainty, and discrepancy.
- Calculate the percent uncertainty of a measurement, given its value and its uncertainty.

What is physics?

Physics is the systematic, formalized, study of the natural world.

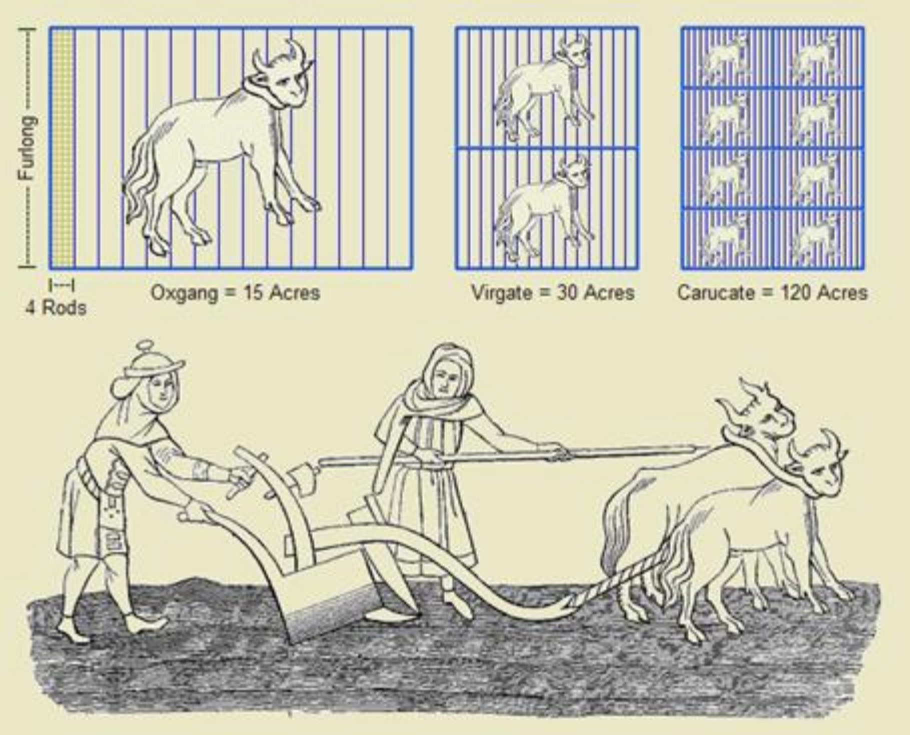
Specifically, it is formalized through the use of mathematical descriptions of the universe around us.

Measurement and comparison is *key*.

PC: wikipedia

How can we measure things?

Anthropic units:



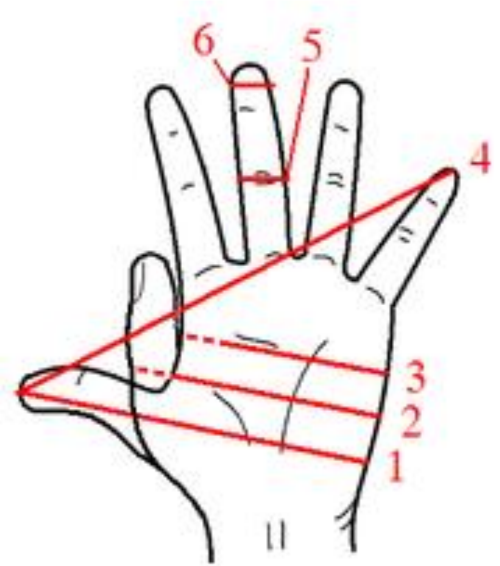
While more "prone to error," these anthropic measurements still allowed for burgeoning sciences, such as astronomy, particularly as they were 'standardized'

PC: wikimedia [1] [2] [3]

So, what did I mean by ‘prone to error’?

A “span” was a unit of measurement defined by the length of ‘4’ on this diagram, and it is now treated as a standard length of

9 inches or 0.2286 m

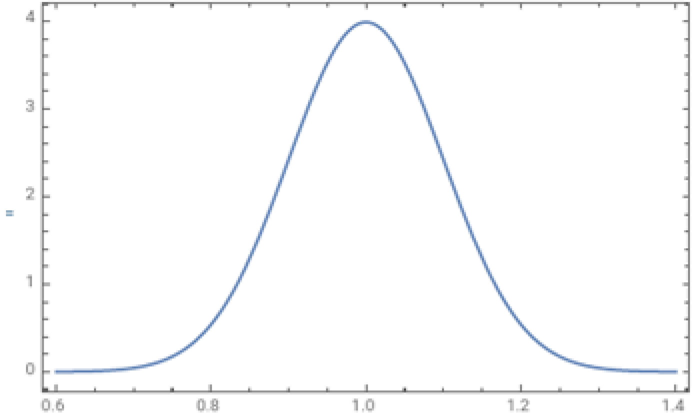


PC: wikipedia

Repeated Measurements

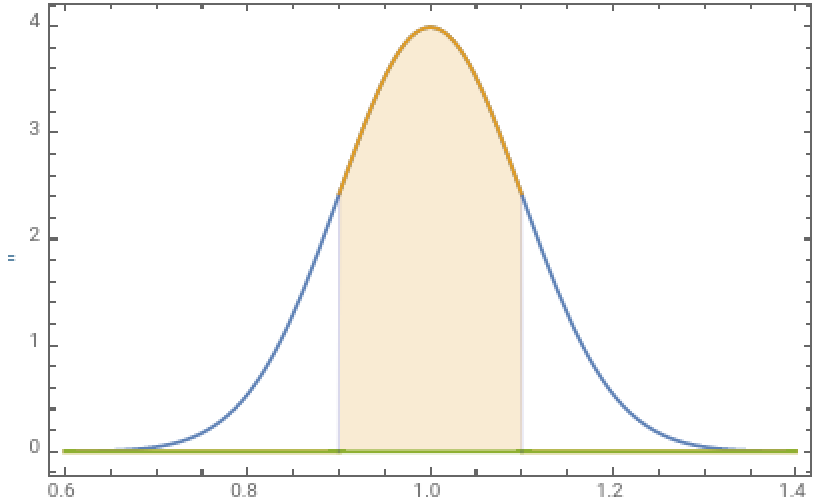
In practice we often perform the same experiment more than once and then average it. Each measurement might differ a little from the previous one (error). If we were to plot out the number of times a measurement result is found on the y axis for a given measurement, we get a *distribution* of measurements.

For a “normal” aka Gaussian distribution:



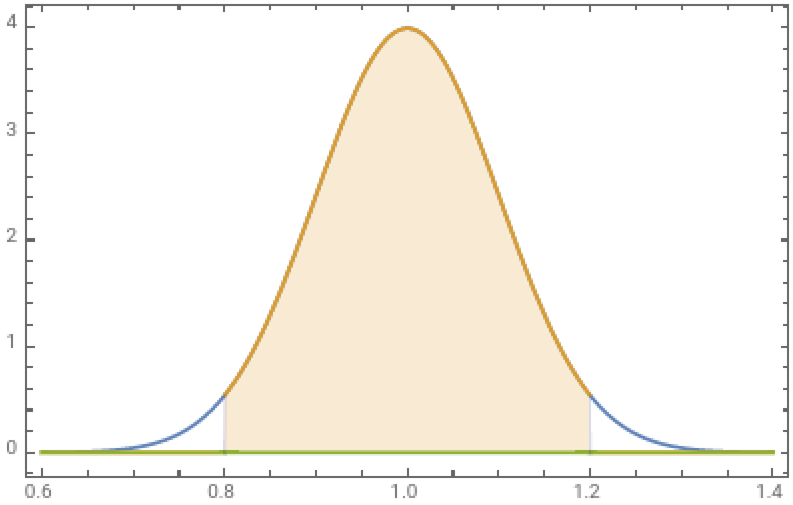
1 Standard Deviation

1 standard deviation covers 68% of the area



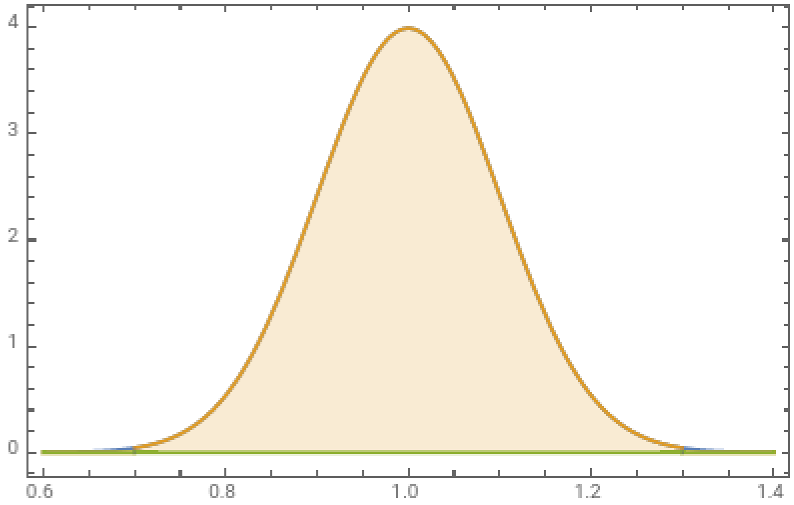
2 Standard Deviations

2 standard deviation covers 95% of the area



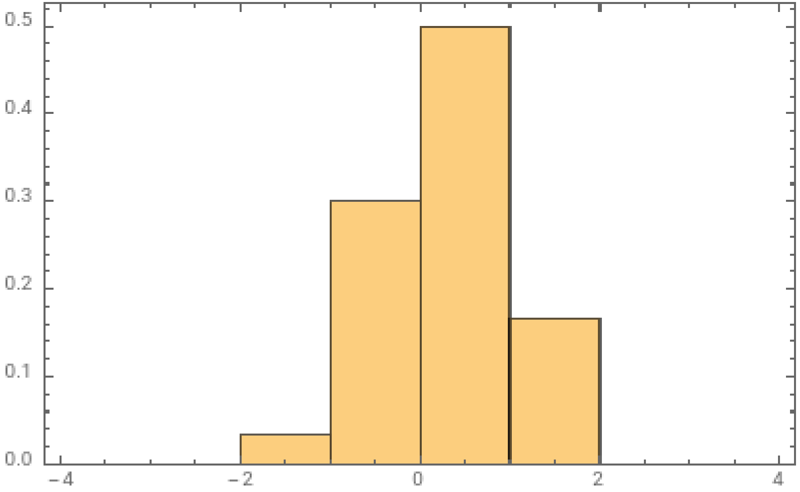
3 Standard Deviations

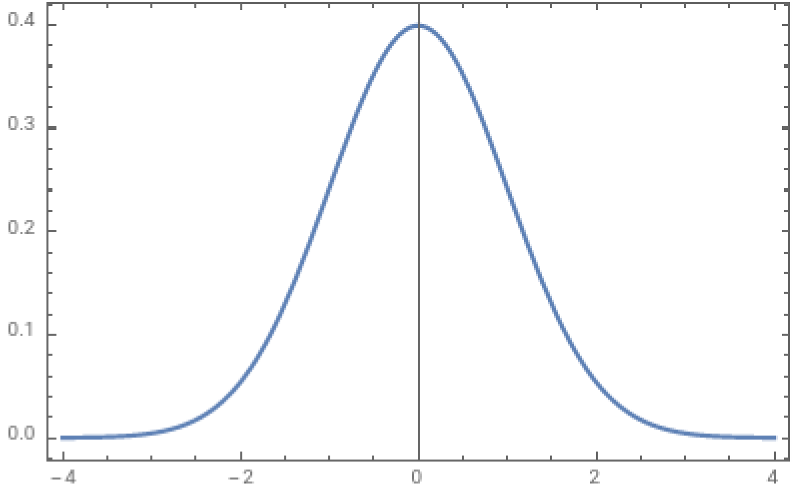
3 standard deviation covers 99.7% of the area



Ideal picture vs Reality

If we assume random error is Gaussian, then when we take N measurements our error will have additional *sampling* error illustrated below:





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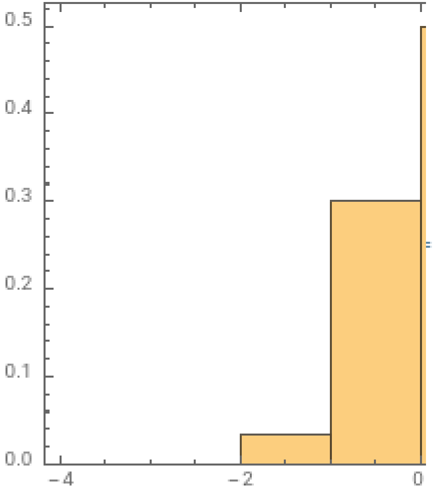
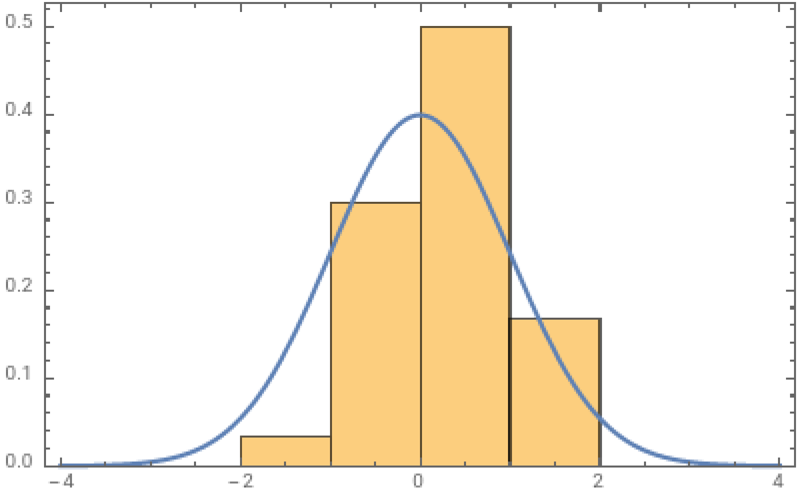
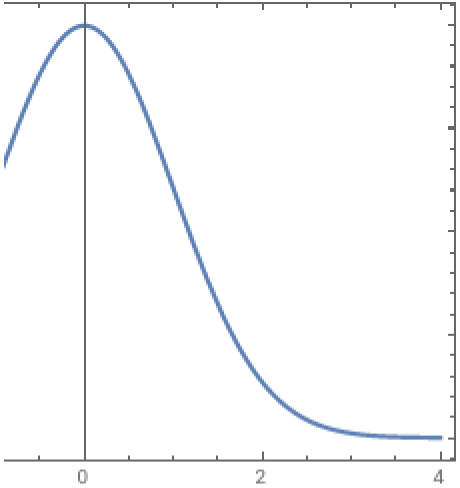




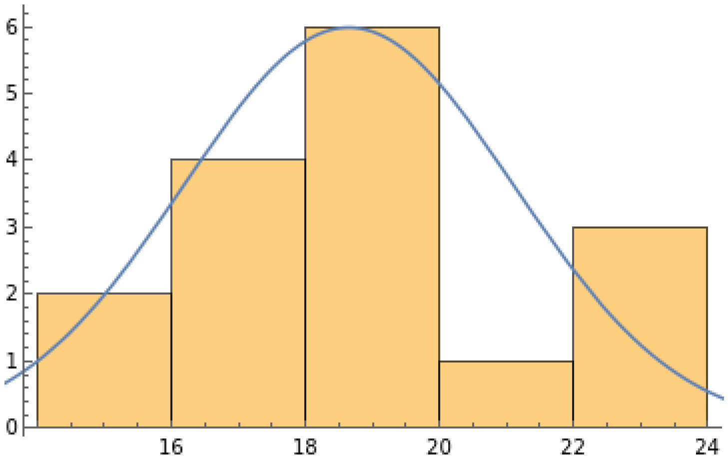


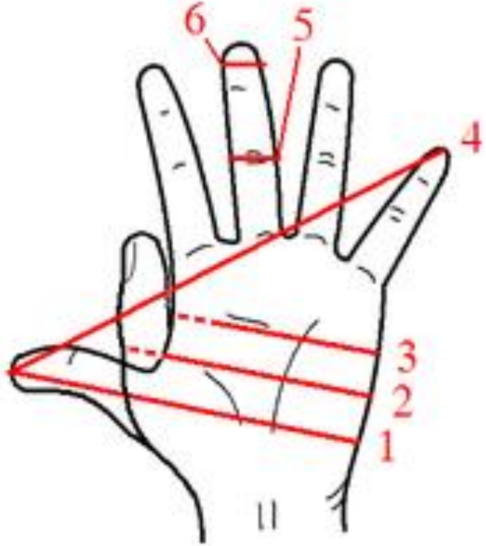
Back to hand spans…

Hand spans vary between individuals.

Here's some real data: $mean = 18.6\ cm$

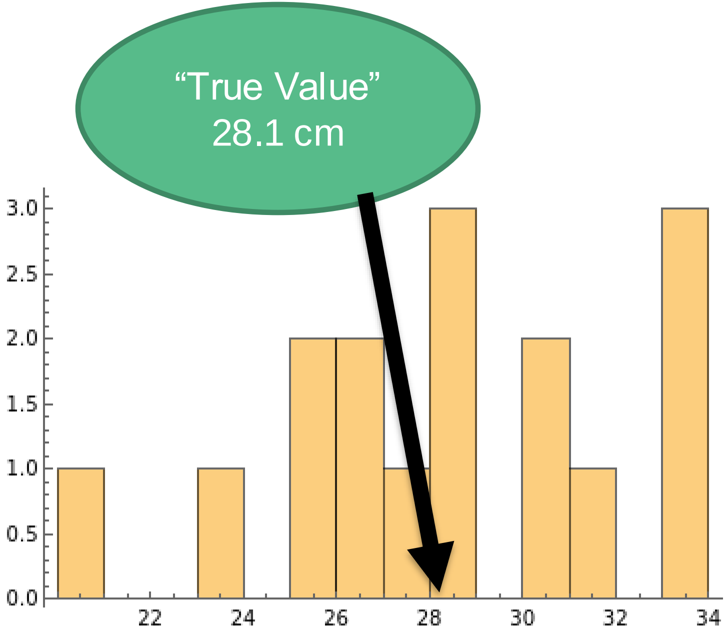
$standard\ deviation = 2.5\ cm$

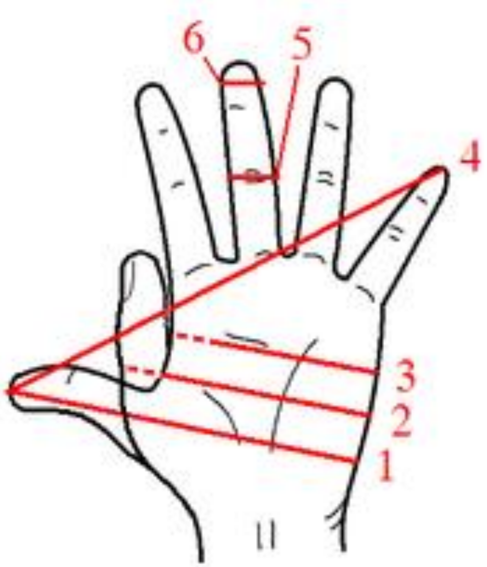




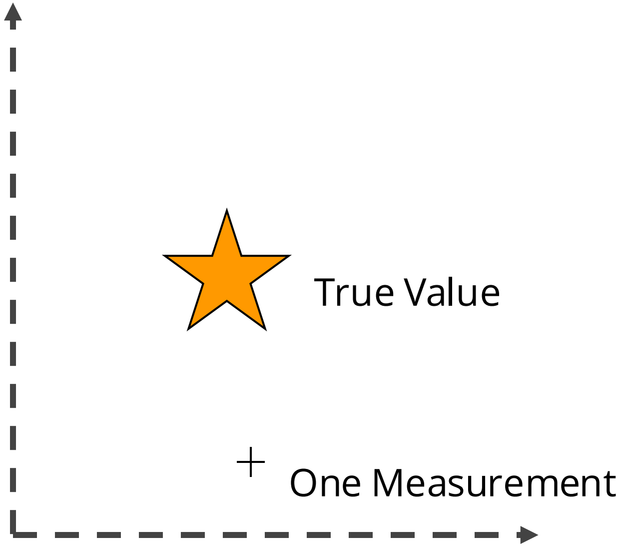
So, what did I mean by 'prone to error'?

Using each lab group's average hand span, the length of an object in cm was found, here is all of the data:





Accuracy vs Precision

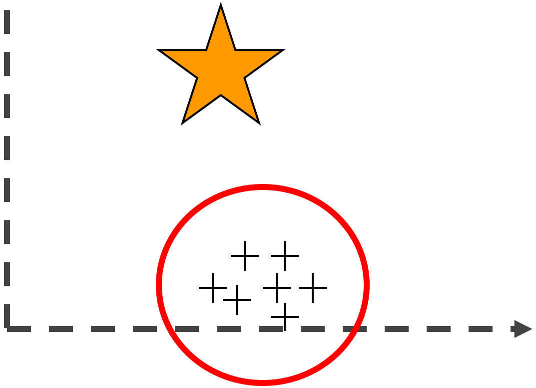


Suppose you’re trying to use a Map app and GPS to find where you are. There’s where the GPS says you are, and there’s where you are in fact. From experience, we know you aren’t always where the app says you are. This is shown above in cartoon form.

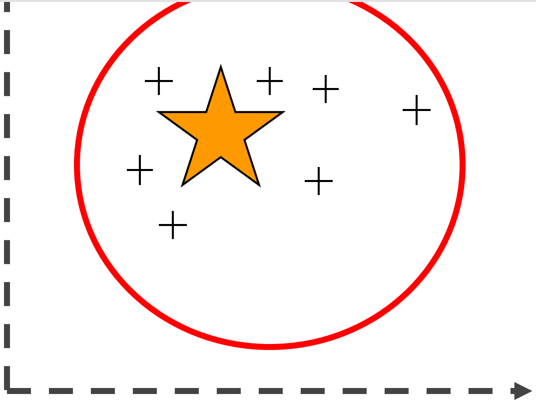
Accuracy vs Precision

If we make many measurements, say using different GPS apps or using your friend’s phone, and display them all at once we can make *distributions* of measurements (ideally we’d use the same tool and repeat it many times with the same setup):

- **Precision implies the measurements have a small neighborhood**
- **Accuracy implies the neighborhood includes the “true value”**

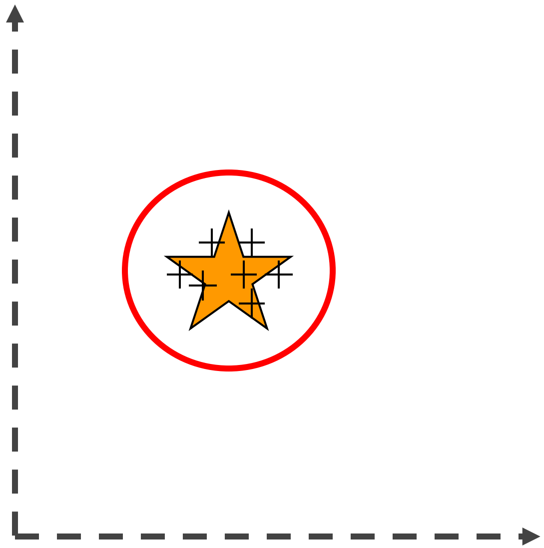


Precise (does not include ‘true value’)



Accuracy vs Precision

Ideal is to have *both*.

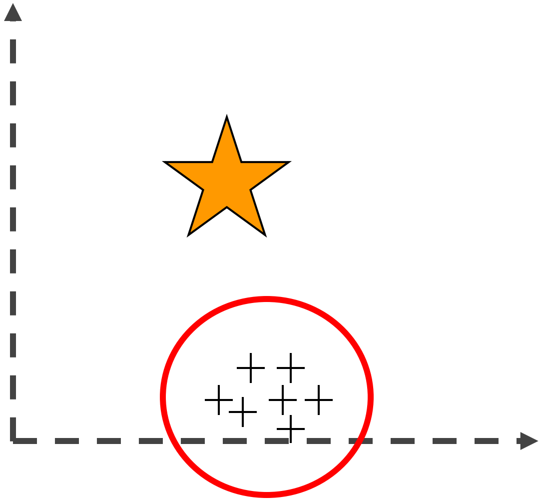


Precise **and** accurate

Types of error & Measurement

If I use the same experimental apparatus, and get a *spread* of values, this is

Random error (related to the precision)



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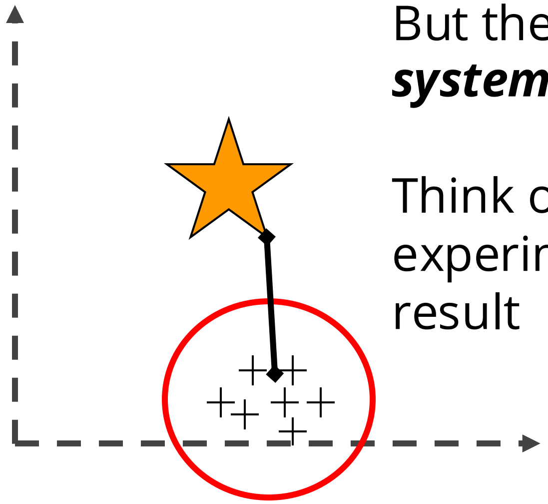
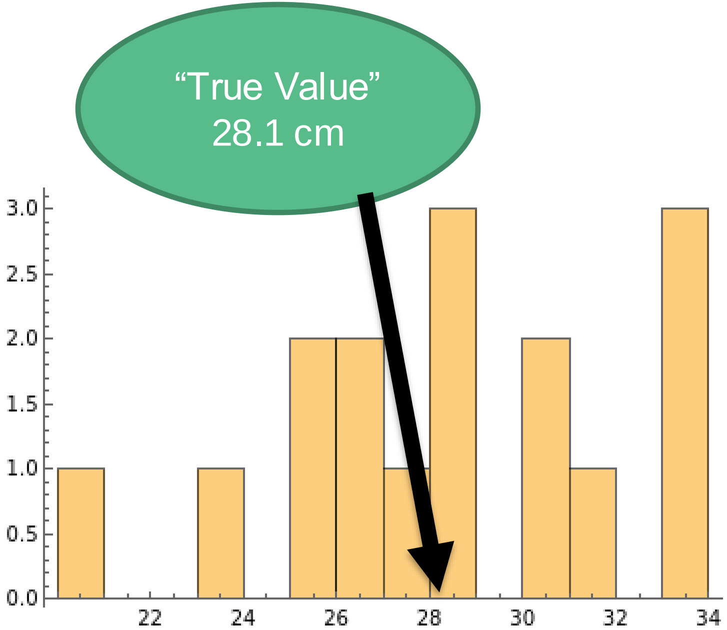


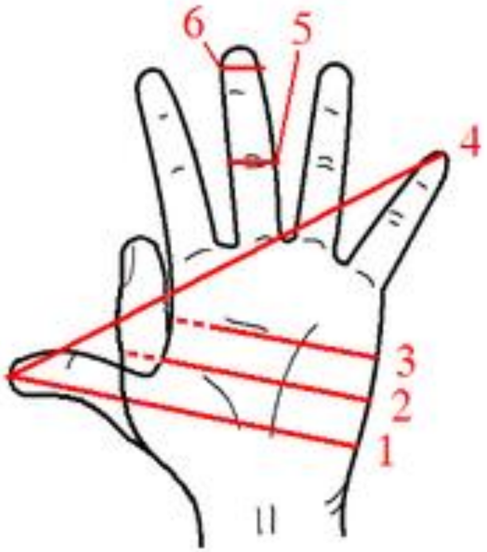
But the distance to the “true value” is ***systematic* error**

Think of it like a mistake in the experiment *systematically* skewing the result

So, what did I mean by ‘prone to error’?

Anthropomorphic can be accurate, making them useful, but being imprecise is a limitation.





Quick Check 1.1

Accuracy is the measure of how close two different measurements with the same instrument are:

A. True
B. False

Quantifying error, first steps

- If we know an accepted value we can calculate ***percent error***
- If we have no known value, we can calculate ***relative percent difference***

$$\boldsymbol{percent\ error = \frac{|Expected\ - Measured|}{Expected}\, x\ 100\%}$$

$$\boldsymbol{rel.\%\ difference = \frac{|Experiment\ 1 - Experiment\ 2|}{\frac{Experiment\ 1 + Experiment\ 2}{2}}\, x\ 100\%}$$

Quantifying error, first steps

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I do the experiment twice, finding **10.0** m/s^2 and **9.59** m/s^2

Quantifying error, first steps

- If we know an accepted value we can calculate ***percent error***
- If we have no known value, we can calculate ***relative percent difference***

I expect to find g = **9.81** m/s^2 instead find it to be **10.0** m/s^2

$$\boldsymbol{percent\ error = \frac{\left|9.81\,\frac{m}{s^2} - 10.0\,\frac{m}{s^2}\right|}{9.81\ m/s^2}\ x\ 100\% = 1.94\ \%}$$

$$\boldsymbol{rel.\%\ difference = \frac{\left|9.59\,\frac{m}{s^2} - 10.0\ m/s^2\right|}{\frac{9.59\,\frac{m}{s^2} + 10.0\ m/s^2}{2}}\ x\ 100\% = 4.18\ \%}$$

I do the experiment twice, finding **10.0** m/s^2 and **9.59** m/s^2

Back to Measurements

- Where did something occur? *Length*
- When did it occur? *Time*
- How much stuff was involved? *Mass*
- How much energy was involved? *Temperature**

Four "base" units: Length, mass, time, temp.

SI Unit System

- The **International System** (SI for **Système International**) is the most widely used system of units. SI base units include:
    - Meters (length)
    - Seconds (time)
    - Kilograms (mass)
- SI is one type of metric system, but not the only one (defined by base units).
- SI has many derived units, which are written in terms of base units
    - Joules (work-energy): $1\ J = 1\ kg\ m^2/s^2$
    - Watts (power): $1\ W = 1\ J/s = 1\ kg\ m^2/s^3$

Need for Accuracy & Precision

- Needs for accuracy in science have driven changes in the standards for units
- In the past, 1 meter of length has been defined by:
    1. One ten-millionth of the distance from the North pole to the equator
    2. A platinum-iridium **standard meter bar** kept in France
    3. 1 650 763.73 wavelengths of an emission line of Kr-86
- In each transition, the new distance was chosen so that the approximate length of 1 meter was preserved, but accuracy, precision, and reproducibility changed as systems of measurement were changed.

From Standards To Universality

- Second: caesium frequency $\Delta\nu$ of caesium 133 atom is measured, think of this as the atom wiggling for 9,192,631,770 times and this time frame is 1 s.

- Meter: the distance light travels in vacuum in 1/299792458 of a second, where the second is defined in terms of the caesium frequency $\Delta\nu$

- Kilogram: is defined by taking the fixed numerical value of the Planck constant, h, to be $6.62607015 \times 10^{-34}$ kg m^2 s^{-1}which can be measured through any number of fundamental physics processes (some necessary to modern technology)
- We will discuss Kelvin (K) and how it is defined later in the semester.

Unit Consistency and Conversions

- An equation must be **dimensionally consistent**. Terms to be added or equated must **always** have the same units. (Be sure you're adding "apples to apples.")
- Always carry units through calculations.
- Convert to standard units as necessary, by forming a ratio of the same physical quantity in two different units and using it as a multiplier.
- For example, to find the number of seconds in 3 min, we write:

$$3\ \text{min} = (3\ \cancel{\text{min}})\left(\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right) = 180\ \text{s}$$

Unit Conversions

Suppose we want to figure out how many weeks are in the month of February in a non-leap year, when February has exactly 28 days. Starting from our measurement (28 days), we can multiply our measurement by a conversion factor to find the same duration measured in our new units.

$$28\ days \times \frac{1\ week}{7\ days} = \frac{28 \times 1}{7} \frac{days \times week}{days} = \frac{28}{7}\ week = \mathbf{4\ weeks}$$

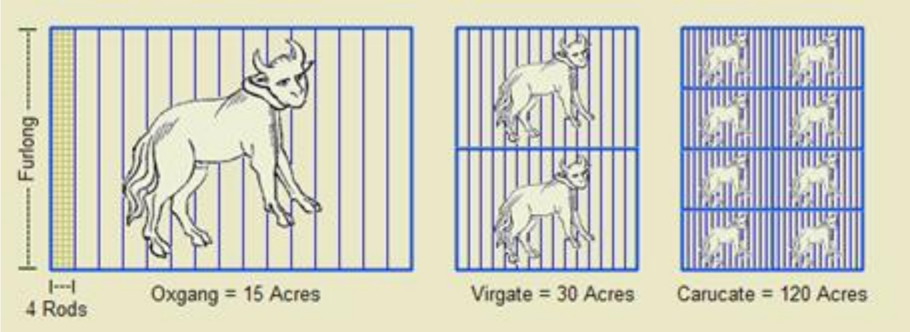
Quick Check 1.2

The basic SI units are

A. Second, meter, gram.
B. Second, meter, kilogram.
C. Second, centimeter, gram.
D. Meter, meter/second, meter/second^2.
E. Yard, span, cubit.

A metric system: base 10 units

- SI is a “metric” system
- ALL units can be shifted by powers of 10
- This is different from other systems:



Prefix	Power of 10	Abbreviation
giga-	10^{9}	G
mega-	10^{6}	M
kilo-	10^{3}	k
centi-	10^{-2}	c
milli-	10^{-3}	m
micro-	10^{-6}	µ
nano-	10^{-9}	n
pico-	10^{-12}	p

PC: wikimedia [1]

Unit Prefixes and Scientific Notation

- **Scientific notation** employs powers of 10 to write large or small numbers by using exponents to express powers of 10 in terms of exponents.

- Prefixes can be used to create larger and smaller units for the fundamental quantities. Some examples are:

$1\ \mu\text{m} = 10^{-6}\ \text{m}$ (size of some bacteria and living cells)

$1\ \text{km} = 10^{3}\ \text{m}$ (distance of a 10-minute walk)

$1\ \text{mg} = 10^{-6}\ \text{kg}$ (mass of a grain of salt)

$1\ \text{g} = 10^{-3}\ \text{kg}$ (mass of a paper clip)

$1\ \text{ns} = 10^{-9}\ \text{s}$ (time for light to travel 0.3 m)

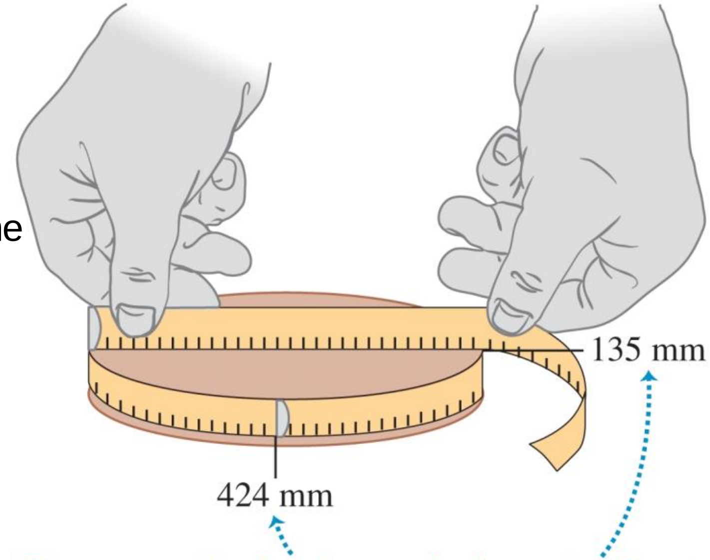
Sig Figs

- **Significant figures** are meaningful digits (rough quantity of uncertainty)
- Generally, round to the least number of significant figures of the given data
  - 25 x 18 → 2 significant figures; 25 x 18975 → still 2
  - Round up for 5+ (13.5 → 14, but 13.4 → 13)
- Significant figures are not decimal places
  - 0.00356 has 5 decimal places, 3 significant figures
- In general, trailing zeros are not significant

In other words, 3000 *may* have 4 significant figures

but usually 3000 will have only 1 significant figure!

When in doubt, use scientific notation 3.000×10^3 or 3×10^3



The measured values have only three significant figures, so their calculated ratio (π) also has only three significant figures.

Operations with Significant Figures

Video Tutor Solution: Example 1.3

- For multiplication and division, the answer can have no more significant figures than the **smallest** number of significant figures in the factors.

- For addition and subtraction, the number of significant figures is determined by the term having the fewest digits to the right of the decimal point.

Example of multiplication with Constants

The area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if r = 1.2 m (two significant figures):

Example of addition

Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer?

Quick Check 1.3

The density of a material is equal to its mass divided by its volume. What is the density in kg/m^3 of a rock of mass 1.80 kg and volume 6.0×10^{-4} m^3?

A. $3 \quad \times 10^3$ kg/m^3

B. 3.0×10^3 kg/m^3

C. 3.00×10^3 kg/m^3

D. 3.000×10^3 kg/m^3

E. Any of these — all of these answers are mathematically equivalent.

Unit Conversions Examples [Group Exercise]

A light-year is a unit of measurement of *distance* that is equal to the amount of distance an object traveling at 3×10^8 m/s travels in 1 year worth of time. To find this distance, you can multiply 1 year in seconds by the speed in m/s. If 1 mile = 1609 m, how many miles is a distance of 40 light years?

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