MACM 316 Theorems

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Thm. Let A be an $n \times m$ matrix, B be an $m \times k$ matrix, C be an $k \times p$ matrix, D be an $m \times k$ matrix and λ a real number.

(a) $A(BC) = (AB)C$ (associativity)

(b) $A(B + D) = AB + AD$ (distributivity)

(c) $IB = B$ and $BI = B$ (idempotency)

(d) $\lambda(AB) = (\lambda A)B = A(\lambda B)$ (scalar associativity)

Def. An $n \times n$ matrix A is said to be nonsingular (or invertible) if an $n \times n$ matrix A^{-1} exists with

$$AA^{-1} = A^{-1}A = I.$$

The matrix A^{-1} is called the inverse of A. A singular matrix does not have an inverse.

Thm. For any nonsingular $n \times n$ matrix A

(a) A^{-1} is unique

(b) A^{-1} is nonsingular and $(A^{-1})^{-1} = A$

(c) If B is also a nonsingular $n \times n$ matrix, then $(AB)^{-1} = B^{-1}A^{-1}$

You can find A^{-1} by row reducing the augmented matrix $[A|I]$, which looks like this:

$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right]$$