

# MACM 316 Final Review

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Monday, April 7, 2025

## 1 Example 1

The following sequence converge to 0 of order  $\alpha$ . What is  $\alpha$ ? Identify any examples with sublinear convergence. Justify your answers using the definitions

(a)  $p_n = n^{-e}$

(b)  $p_n = e^{-n}$

(c)  $p_n = e^{-(e^n)}$

### 1.1 a)

Conjecture  $\alpha = 1$

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{|p_{n+1} - 0|}{|p_n - 0|} &= \lim_{n \rightarrow \infty} \frac{(n+1)^{-e}}{n^{-e}} \\ &= \lim_{n \rightarrow \infty} \left[1 + \frac{1}{n}\right]^{-e} \\ &= 1\end{aligned}$$

$\therefore \alpha = 1$ . This is less than linear (sublinear) because  $\lambda = 1$

**1.2 c)**

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{|p_{n+1} - 0|}{|p_n - 0|} &= \lim_{n \rightarrow \infty} \frac{e^{-(e^{n+1})}}{(e^{-(e^n)})^\alpha} \\ &= \lim_{n \rightarrow \infty} \frac{e^{-e^n \cdot e}}{e^{-e^n \cdot \alpha}} \\ &= 1 \text{ if } \alpha = e\end{aligned}$$