MACM 316 Lecture 5

Alexander Ng

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1 Review

Why did we expand around $h_0 = 0$?

Because we want to know what happens with h_0, choosing 0 as our expansion point makes sense. The Taylor series approximation is more accurate the closer you are to your expansion point.

2 Another Example

Find the rate of convergence of the following $h \to 0$.

$$\lim_{h\to 0} \cos h + \frac{1}{2}h^2 = 1$$

$\alpha = 1$
$\alpha_h = cosh + \frac{1}{2}h^2$

$$\begin{aligned}
\alpha_h - \alpha &= cosh + \frac{1}{2}h^2 - 1 \\
&= 1 - \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^6) + \frac{1}{2}h^2 - 1 \\
&= \frac{h^4}{4!} + O(h^6)
\end{aligned}$$
$$= O(h^4)$$

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3 Chapter 6 - Direct methods for solving linear systems

Preface

In MATH 240, we learned methods for solving linear systems of equations where our $n \times m$ matrix has few rows and few columns. In Numerical Analysis, we will learn methods for solving linear systems where our matrix has thousands or millions of rows and columns.

We will first study methods that give an answer ina fixed number of steps, subject only to roundoff errors. This method only has error from the accumulation of numerical representation errors.

3.1 Linear Systems of Equations

Linear Systems of Equations

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{bmatrix}$$

3.2 Elementary Row Operations

1. Multiply row i by a constant $\lambda \neq 0$, denoted by $\lambda E_i \rightarrow E_i$
2. Add a multiple of row i to row j, denoted by $E_i + \lambda E_j \rightarrow E_i$
3. Interchange rows i and j, denoted by $E_i \leftrightarrow E_j$

3.3 Notes on Gaussian Elimination

The idea behind <u>Gaussian Elimination</u> is to transform the matrix into a *triangular equivalent matrix problem of the upper triangular or lower triangular form.

3.4 Example 1

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -8 \\ 2 & -2 & 3 & -3 & -20 \\ 1 & 1 & 1 & 0 & -2 \\ 1 & -1 & 4 & 3 & 12 \end{bmatrix}$$

4 Complexity of Gaussian Elimination

How does the number of operations change with the size of the matrices?

We can count up the number of multiplications and divisions to go to upper triangular form.

$$\begin{bmatrix} * & \dots & * & x_1 \\ \vdots & \ddots & \vdots & \vdots \\ * & \dots & * & x_n \end{bmatrix} \rightarrow \begin{bmatrix} * & * & \dots & * & x_1 \\ 0 & * & & \vdots & \vdots \\ \vdots & \vdots & & * & x_n \\ 0 & * & \dots & * & x_{n+1} \end{bmatrix}$$

This proceeds as follows:

Provided $a_{11} \neq 0$, the operation corresponding to

$$E_j - \frac{a_{ji}}{a_{11}} E_i \rightarrow E_j$$