1.1 Exercises 4, 8, 14, 21, 28, 36.

For exercises 4, 14 and 21 first write the linear system as an augmented matrix $[A|b]$ and then do elementary row operations on $[A|b]$ to answer the question. Indicate the row operations that you do. Verify that the solutions you get are correct by substituting them into the original equations.

4. Find the point of intersection of the lines $x_1 - 5x_2 = 1$ and $3x_1 - 7x_2 = 5$.

$$\begin{bmatrix} 1 & -5 & 1 \\ 3 & -7 & 5 \end{bmatrix}$$ add $-3 \cdot$ row 1 to row 2

$$\begin{bmatrix} 1 & -5 & 1 \\ 0 & 8 & 2 \end{bmatrix}$$ divide row 2 by 2

$$\begin{bmatrix} 1 & -5 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$ add $5/4 \cdot$ row 2 to row 1

$$\begin{bmatrix} 1 & 0 & \frac{9}{4} \\ 0 & 4 & 1 \end{bmatrix}$$ Solve for unknowns

$x_1 + 0x_2 = \frac{9}{4} \sim x_1 = \frac{9}{4}$

$0x_1 + 4x_2 = 1 \sim x_2 = \frac{1}{4}$

Verify sol'ns

4. Find the point of intersection of the lines $x_1 - 5x_2 = 1$ and $3x_1 - 7x_2 = 5$.

$x_1 - 5x_2 = 1$ ✓ $\quad 3x_1 - 7x_2 = 5$ ✓

$\frac{9}{4} - 5(\frac{1}{4}) = \quad 3(\frac{9}{4}) - 7(\frac{1}{4}) =$

In Exercises 7–10, the augmented matrix of a linear system has been reduced by row operations to the form shown. In each case, continue the appropriate row operations and describe the solution set of the original system.

7. $\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$ divide row 3 by 2

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$ add $-7 \cdot$ row 3 to row 2

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$ add $-2 \cdot$ row 3 to row 1

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$ add $-1 \cdot$ row 2 to row 1

$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$ $x_1 = -5$, $x_2 = 7$, $x_3 = -1$

$(-5, 7, -1)$ is a solution set of the original system

14.

$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$ add eq 1 to eq 2

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$ swap eq 2 and 3

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{bmatrix}$$ add $2 \cdot$ eq 2 to eq 3

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{bmatrix}$$ divide eq 3 by 7

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$ ~~add row 3 to row 2~~

~~add 3·row 2 to row 1~~

~~$x_1 = 6$, $x_2 = 1$, $x_3 = 1$~~

Verify solution

14. $x_1 - 3x_2 = 5$, $-x_1 + x_2 + 5x_3 = 2$, $x_2 + x_3 = 0$

$6 - 3(1) =$?

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$ add $-1 \cdot$ row 3 to row 2

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$ add $3 \cdot$ row 2 to row 1

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$ $x_1 = 2$, $x_2 = -1$, $x_3 = 1$

Verify solution

14. $x_1 - 3x_2 = 5$, $-x_1 + x_2 + 5x_3 = 2$, $x_2 + x_3 = 0$

$2 - 3(-1) =$ ✓

$-(2) + (-1) + 5(1) =$

$(-1) + 1 =$

nice.

21. Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ have a common point of intersection? Explain.

$$\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix}$$ add $-2 \cdot$ row 1 to row 2

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ -1 & -3 & 4 \end{bmatrix}$$ add r1 to r3

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{bmatrix}$$ add $\frac{7}{4}$ r2 to r1

$$\begin{bmatrix} 1 & 0 & -35/4 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{bmatrix}$$ $\frac{7}{4} \cdot (-5) = -\frac{35}{4}$

$7x_2 = -5$

$x_2 = -\frac{5}{7}$ ✓

$-7x_2 = 5$

$x_2 = -\frac{5}{7}$ ✓

$x_1 = -\frac{35}{4}$

$x_2 = -\frac{5}{7}$

These lines all share 1 common point because the system of equations they form is consistent.

28. **(T/F)** Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

True. Otherwise, exercising elementary row operations on an augmented Matrix would not bear a valid solution to the original linear system.

36. Construct three different augmented matrices for linear systems whose solution set is $x_1 = -2$, $x_2 = 1$, $x_3 = 0$.

This is basically cheating lol

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 0 & 6 \\ 0 & 2 & 3 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 0 & 6 \\ 0 & 2 & 3 & 2 \\ 0 & 2 & 6 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 0 & 4 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$