MACM 101 Chapter 1.5 Homework

Alexander Ng

Tuesday, October 1, 2024

Question 2

Part a

$\exists x \forall y(xy = y)$

There exists a number x such that forall y, $x \times y = y$

Part b

$\forall x \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$

For every real number x and every real number y, if x is greater than or equal to 0 and y is less than 0, $x - y$ is greater than 0.

Part c

$\forall x \forall y \exists z(x = y + z)$

For every real number x and every real number y, there exists a real number z such that $x = y + z$.

Question 10

(a) $\forall x F(x, \text{Fred})$

(b) $\forall y F(\text{Evelyn}, y)$

(c) $\forall x \exists y F(x, y)$

(d) $\forall x \exists y \neg F(x, y)$

(e) $\forall x \exists y F(y, x)$

(f) $\neg\forall x(F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

(g) $\exists x\exists y(F(\text{Nancy}, x) \wedge F(\text{Nancy}, y) \wedge y \neq z \wedge \forall w(F(\text{Nancy}, w) \rightarrow (w = y) \vee (w = z)))$

(h) $\exists y(\forall x F(x, y) \wedge \forall z(\forall w F(w, z)) \rightarrow z = y)$

(i) $\neg\exists x F(x, x)$

(j) $\exists x\exists y(F(x, y) \wedge \forall z(F(x, z) \rightarrow ((z = y) \vee (z = x))))$

Question 24

(a) There exists a real number x such that for all real numbers y, the sum $x + y$ is equal to y

(b) For all real numbers x and all real numbers y, if x is a nonzero positive and y is negative, $x - y$ is positive.

(c) There exists a real number x and there exists a real number y such that x is negative, y is negative and $x - y$ is greater than 0.

(d) For all real numbers x and all real numbers y, if and only if $x \neq 0$ and $y \neq 0$, then $x \times y \neq 0$.

Question 26

Let $Q(x, y)$ be the statement $x + y = x - y$

(a) False, $1 + 1 \neq 1 - 1$

(b) True, $2 + 0 = 2 - 0$

(c) False, since there are many values of y such that $1 + y \neq 1 - y$

(d) False, $x + 2 = x - 2$ has no solution

(e) True, when $x = y = 0$, $x + y = x - y$

(f) True, when $y = 0$, $x + y = x - y$

(g) True, when $y = 0$, $x + y = x - y$

(h) False, since there is no x such that $x + y = x - y$ for all y.

(i) False