Question 32

Part a

"All dogs have fleas"

Let the domain be dogs and $P(x)$ mean "x has fleas"

$\forall x P(x)$

Negate, then apply De Morgan's Laws for Quantifiers

$\neg(\forall x P(x)) \equiv \exists x \neg P(x)$

"There is a dog that does not have fleas"

Part b

"There is a horse that can add"

Let the domain be horses and $P(x)$ mean "x can add"

$\exists x P(x)$

Negate, then appy De Morgan's Laws.

$\neg(\exists x P(x)) \equiv \forall x \neg P(x)$

Part c

"Every koala can climb"

Let the domain be koalas and $P(x)$ mean "x can climb"

$\forall x P(x)$

Negate, then apply De Morgan's Laws

$\neg(\forall x P(x)) \equiv \exists x \neg P(x)$

"There exists a koala that cannot climb"

Part d

"No monkey can speak french"

Let the domain be monkeys and $P(x)$ mean "x can speak french"

$\forall x \neg P(x)$

Negate, then apply De Morgan's Law and the Double Negation Law

$\neg(\forall x \neg P(x)) \equiv \exists x \neg(\neg P(x)) \equiv \exists x P(x)$

Part e

"There exists a pig that can swim and catch fish"

Let the domain be pigs, $P(x)$ mean "x can swim" and $Q(x)$ mean "x can catch fish"

$\exists x(P(x) \wedge Q(x))$

Negate, then apply De Morgan's Law for Quantifiers and the regular De Morgan's Law $\neg(\exists x(P(x) \wedge Q(x))) \equiv \forall x \neg(P(x) \wedge Q(x)) \equiv \forall x(\neg P(x) \vee \neg Q(x))$

"All pigs can not swim or not catch fish."