Question 34

Suppose $x = \frac{a}{b}$ where a and b are integers with $b \neq 0$. Then, $\frac{x}{2} = \frac{a}{2b}$, which is rational since a and $2b$ are both integers with $2b \neq 0$.

Suppose $\frac{x}{2} = \frac{a}{b}$ where a and b are integers with $b \neq 0$. Then $x = \frac{2a}{b}$, and $3x - 1 = \frac{6a}{b} - 1 = \frac{6a-b}{b}$. This is rational since $6a - b$ and b are integers with $b \neq 0$.

Suppose that $3x - 1 = \frac{a}{b}$ where a and b are integers with $b \neq 0$. Then $x = \frac{\frac{a}{b}+1}{3} = \frac{a+b}{3b}$ and this is rational, since $a + b$ and $3b$ are both integers with $3b \neq 0$.

This shows that (i) implies (ii), (ii) implies (iii) and (iii) implies (i), which is sufficient to show that these three statements are equivalent.