MACM 101 Chapter 1.4

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1 Quantifiers

1.1 Universal vs. Existential Quantifiers

- The Universal Quantifier, $\forall$, reads:
  $\forall x P(x)$
  for all x, $P(x)$ is true
- The Existential Quantifier $\exists$
- u is the universal discourse

1.2 The Problem

Free Variables are bad

Bound variables are good

A predicate is a function that maps variables to truth values, allowing one to go beyond atomic propositions.

Quantifiers are things that allow us to bind variables to a domain.

Predicates are not propositions, unless we replace the free variable with a logical constant, or bind it's variables with a quantifier.

Going from a generalization to an instance is called instantiation, which is very important for acting on quantified predicates using the rules of inference and the laws of logic.

1.2.1 P and Q

Let $P(x)$ and $Q(x)$ be open statements defined for **u**.

These two statements are logically equivalent, written

$$\forall x[P(x) \Leftrightarrow Q(x)]$$

when $P(a) \leftrightarrow Q(a) \forall a \in \mathbf{u}$.

*State the contrapositive, converse and inverse. (exercise)

$\forall x P(x) | x \in \mathbb{R} \implies$ *something* how to specify the domain of a quantifier

Quantifiers have higher precedence than all logical connectives

Don't mix quantifiers unless they are the same kind

1.2.2 Arguing with Quantified Statements

1. $\exists m \in \mathbb{Z}^+ \forall n \in \mathbb{Z}^+ (m \leq n)$

2. $\forall x \in \mathbb{R}^+ \exists y \in R^+ (x \leq y)$