Question 6

Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

Let p (r) be the proposition 'It rains.'
Let q (f) be the proposition 'It is foggy.'
Let r (s) be the proposition 'The sailing race will be held.'
Let s (l) be the proposition 'The life saving demonstration will go on.'
Let t (t) be the proposition 'The trophy will be awarded'.
We are given $(\neg p \vee \neg q) \rightarrow (r \wedge s)$, $r \rightarrow t$ and $\neg t$

1. $\neg t$ (Hypothesis)
2. $r \rightarrow t$ (Hypothesis)
3. $\neg r$ (Modus Tollens of Step 1 and 2)
4. $(\neg p \vee \neg q) \rightarrow (r \wedge s)$ (Hypothesis)
5. $(\neg(r \wedge s)) \rightarrow \neg(\neg p \vee \neg q))$ (Contrapositive of Step 4)
6. $(\neg r \vee \neg s) - > (p \wedge q)$ (De Morgan's and Double Negative)
7. $\neg r \vee \neg s$ (Following Step 3)
8. $p \wedge q$ (Modus Ponens of Step 6 and 7)
9. p (Simplification of Step 8)