MACM 101 Chapter 1 Homework

Alexander Ng

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1 Section 1.1

Question 22

a) Inclusive or, the requirement is experience with one or the other, and having both would still satisfy the requirement.

b) Exclusive or, Lunch will come with either soup or salad, not both.

c) Inclusive or, having both documents will not get you turned away.

d) Exclusive or, publishing prevents perishing.

Question 24

a) If you get promoted, then you have washed the Boss's car.

b) If there are winds from the south, then there is a spring thaw.

c) If you bought the computer less than a year ago, then the warranty is good.

d) If Willy cheats, then he gets caught.

e) If you can access the website, then you have paid a subscription fee.

f) If you know the right people, then you get elected.

g) If Carol is on a boat, then she gets seasick.

Question 26

a) If you send me an e-mail message, then I will remember to send you the address.

b) If you were born in the United States, then you are a citizen of this country.

c) If you keep your textbook, then it will be a useful reference in your future courses.

d) If their goalie plays well, then the Red Wings will win the Stanley Cup.

e) If you get the job, then you had the best credentials.

f) If there is a storm, then the beach erodes.

g) If you log on to the server, then you have a valid password.

h) If you do not begin your climb too late, then you will reach the summit.

i) If you are among the first 100 customers tomorrow, then you will get a free ice cream cone.

Question 38

a) $(p \vee q) \vee r$

p	q	r	$p \vee q$	$(p \vee q) \vee r$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

b) $(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

c) $(p \wedge q) \vee r$

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

d) $(p \wedge q) \vee r$

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

e) $(p \vee q) \wedge \neg r$

p	q	r	$\neg r$	$p \vee q$	$(p \vee q) \wedge \neg r$
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	1	1	1
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	0

f) $(p \wedge q) \vee \neg r$

p	q	r	$\neg r$	$p \wedge q$	$(p \wedge q) \vee \neg r$
0	0	0	1	0	1
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	1	1

Question 42

Given $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$, show that the statement is true if and only if p, q and r all have the same truth value.

1. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ Original Premise
2. $((p \vee \neg q) \wedge (q \vee \neg r)) \wedge (r \vee \neg p)$ Trivial
3. $((q \wedge (p \vee \neg q)) \vee (\neg r \wedge (p \vee \neg q))) \wedge (r \vee \neg p)$ Distributive Property
4. $[r \wedge ((q \wedge (p \vee \neg q)) \vee (\neg r \wedge (p \vee \neg q)))] \vee [\neg p \wedge ((q \wedge (p \vee \neg q)) \vee (\neg r \wedge (p \vee \neg q)))]$
   Distributive Property
5. Take $q \wedge (p \vee \neg q)$ and apply the distributive property,
   $(q \wedge p) \vee (q \wedge \neg q)$. Notice $q \wedge \neg q$ is a contradiction, and $p \vee \mathbf{F} \Leftrightarrow p$.
   $\therefore q \wedge (p \vee \neg q) \Leftrightarrow (q \wedge p)$
   Rewritten,
   $[r \wedge ((p \wedge q) \vee (\neg r \wedge (p \vee \neg q)))] \vee [\neg p \wedge ((p \wedge q) \vee (\neg r \wedge (p \vee \neg q)))]$

6. Again, apply the distributive property.
   $[(r \wedge (p \wedge q)) \vee (r \wedge (\neg r \wedge p) \vee (\neg r \vee \neg q)))] \vee [(\neg p \wedge (p \wedge q)) \vee (\neg p \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)))]$

7. Notice how in the above statement, we have
   $r \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q))$, which distributes to
   $r \wedge (\neg r \wedge p) \vee r \wedge (\neg r \wedge \neg q)$. This can easily be manipulated using the associative laws to show
   $((r \wedge \neg r) \wedge p) \vee ((r \wedge \neg r) \wedge \neg q)$. $\rightarrow p \wedge \neg p \equiv \mathbf{F}$
   $(\mathbf{F} \wedge p) \vee (\mathbf{F} \wedge \neg q)$. We know that $\mathbf{F} \wedge p \equiv \mathbf{F}$.
   $\therefore r \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)) \equiv \mathbf{F}$

8. $[(r \wedge (p \wedge q)) \vee \mathbf{F})] \vee [(\neg p \wedge (p \wedge q)) \vee (\neg p \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)))]$
   Doing the same with $(\neg p \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)))$, we get
   $(\neg p \wedge (\neg r \wedge p)) \vee (\neg p \wedge (\neg r \wedge \neg q))$
   $\mathbf{F} \vee (\neg p \wedge (\neg r \wedge \neg q))$

9. $[(r \wedge (p \wedge q)) \vee \mathbf{F})] \vee [\mathbf{F} \vee (\neg p \wedge (\neg r \wedge \neg q))]$
   Because $p \vee \mathbf{F} \equiv p$,

10. $(r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r))$

11. $\therefore (p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \equiv (r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r))$

Proof

Show that the statement is true if and only if p, q, and r all have the same truth value.

1. Assume $p = q = r$. Then,

2. $(r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r)$

3. $(p \wedge p \wedge p) \vee (\neg p \wedge \neg p \wedge \neg p)$
   By the Idempotent laws,

4. $p \vee \neg p$. This is a Tautology.

5. $\therefore$ when all values of p, q and r share the same truth value, the statement is true.

1. Assume $p = q$; $r = \neg p$

2. $(r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r)$

3. $(p \wedge p \wedge \neg p) \vee (\neg p \wedge \neg p \wedge \neg\neg p)$
   By the Double Negation and Idempotent Laws,

4. $(p \wedge \neg p) \vee (\neg p \wedge p)$
   Because $p \wedge \neg p$ is a Contradiction,

5. $\mathbf{F} \vee \mathbf{F} \equiv \mathbf{F}$.

6. $\therefore$ when the values of p, q and r are not all the same, the statement is false.

Only after spending hours on this, I realize that I could have taken the original statement, applied the above two tests to it and proved the same thing. I am clearly, unequivocally, even, an idiot.