# Question 6

## Part a

$A \cup \emptyset = A$

$A \cup B = \{x \,| x \in A \vee x \in B\}$

By definition, $A \cup \emptyset = \{x \,| x \in A \vee x \in \emptyset\}$

The empty set does not contain any elements, so $x \notin \emptyset$

$A \cup \emptyset = \{x \,| x \in A \vee \mathbf{F}\}$

By the Identity Laws of Propositional Logic, $p \vee \mathbf{F} = p$

$A \cup \emptyset = \{x \,| x \in A\}$

$\therefore A \cup \emptyset = A$

## Part b

$A \cap U = A$

$A \cap B = \{x \,| x \in A \wedge x \in B\}$

By definition, $A \cap U = \{x \,| x \in A \wedge x \in U\}$

Because $U$ is the universal set, $x \in U \equiv \mathbf{T}$

So, $A \cap U = \{x \,| x \in A \wedge \mathbf{T}\}$

By the Identity Laws of Propositional Logic, $p \wedge \mathbf{T} = p$

$A \cap U = \{x \,| x \in A\}$

$\therefore A \cap U = A$