Question 24

Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

1. $\neg(p \oplus q) \equiv \neg((p \vee q) \wedge (\neg p \vee \neg q))$ (Definition)
2. $\equiv \neg[(p \wedge (\neg p \vee \neg q)) \vee (q \wedge (\neg p \vee \neg q))]$ (Distributive Law)
3. $\equiv \neg[((p \wedge \neg p) \vee (p \wedge \neg q)) \vee ((q \wedge \neg p) \vee (q \wedge \neg q))]$ (Distributive Law)
4. $\equiv \neg[(\mathbb{F} \vee (p \wedge \neg q)) \vee ((q \wedge \neg p) \vee \mathbb{F})]$ By $p \wedge \neg p \equiv \mathbb{F}$
5. $\equiv \neg[(p \wedge \neg q) \vee (q \wedge \neg p)]$ (Identity Law)
6. $\equiv \neg(p \wedge \neg q) \wedge \neg(q \wedge \neg p)$ (De Morgan's Law)
7. $\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ (De Morgan's Law and Double Negation Law)
8. $\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (Logical Equivalence)
9. $\equiv p \leftrightarrow q$ (Logical Equivalence)