MATH 240 Lecture 1.7
Matrix Operations

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1 Matrix Operations

Let A be an $m \times n$ matrix, where m is the number of rows and n is the number of columns. The notation a_{ij} refers to the entry of the matrix A in row i and column j.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

1.1 Definitions

Let A, B be $m \times n$ matrices over $\mathbb{R}$, and $r \in \mathbb{R}$.

Define

Matrix addition:

$$C = A + B \text{ is defined as } c_{ij} = a_{ij} + b_{ij}$$

Matrix subtraction:

$$D = A - B \text{ is defined as } d_{ij} = a_{ij} - b_{ij}$$

Scalar multiplication:

$$E = rA \text{ is defined as } e_{ij} = r \cdot a_{ij}$$

In each of these cases, you just apply the corresponding operation to each entry component-wise.

1.1.1 Properties

Properties of matrix addition and scalar multiplication.

These are the same as the properties of vector addition and scalar multiplication.

**Matrix addition is identical to vector addition

1.2 Matrix Multiplication $C = A \cdot B$

Definition:

Let $B = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ b_1 & b_2 & \dots & b_n \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$. $A \cdot B = \begin{bmatrix} Ab_1 & Ab_2 & \dots & Ab_n \end{bmatrix}$

Example:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 10 \\ 16 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3+5=8 \\ 6+10=16 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 4+6=10 \\ 8+12=20 \end{bmatrix}$$

1.2.1 Shortcut: The Row Column Rule for computing $A \cdot B$

For any $C = A \cdot B$, we can shortcut the computation by computing the dot product of each row of A with it's corresponding column of B.

Notice, in general, $A \cdot B \neq B \cdot A$. Matrix multiplication is not commutative.

The matrices don't have to be square.

1.2.2 Example

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

Let $B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$

In order to compute $A \cdot B$, the number of columns of A must equal the number of rows of B. A must be an $m \times n$ matrix, and B must be an $n \times p$ matrix. C becomes an $m \times p$ matrix.

Suppose we have two linear transformations $T_1(x) = Ax$ and $T_2(x) = Bx$.

Let $T_3(x) = T_1(T_2(x)) = T_1(Bx) = ABx$.

Is $T_3(x)$ also a linear transformation? Yes.

Check: (The following properties must hold for $T_3(x)$ to be a linear transformation)

1. $T_3(x+y) = T_3(x) + T_3(y)$

$A(B(x+y)) = A(Bx+By) = A(Bx)+A(By) = T_1(T_2(x))+T_1(T_2(y)) = T_3(x) + T_3(y)$

2. $T_3(cx) = cT_3(x)$

T_3's standard matrix is going to be AB.

Thm. $A(Bx) = (AB)x$