MACM 101 Chapter 1 Homework

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Question 4

Part a

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
1	1	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	1	1	1	1
1	0	0	1	0	1	1
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	0	1	1	1
0	0	0	0	0	0	0

Part b

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
1	1	1	1	1	1	1
1	1	0	1	0	0	0
1	0	1	0	0	0	0
1	0	0	0	0	0	0
0	1	1	0	1	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0

Question 12

Part a

p	q	$p \vee q$	$\neg p$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
1	1	1	0	0	1
1	0	1	0	0	1
0	1	1	1	1	1
0	0	0	1	0	1

Part b

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	0	1	0	1	0	1	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

Part c

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
1	1	1	1	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	1

Part d

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r)$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r)$
1	1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0	1
0	1	1	1	1	1	1	1	1
0	1	0	1	0	0	0	0	1
0	0	1	0	1	1	0	0	1
0	0	0	0	0	1	0	0	1

Question 14

Part a

$$\neg p \wedge (p \vee q) \to q$$

$p \to q$ is false when p is true and q is false.

Assume $\neg p \wedge (p \vee q)$ is true.

When $\neg p \wedge (p \vee q)$ is true, then both $\neg p$ and $p \vee q$ are true.

When $\neg p$ is true, then p is false.

When $p \vee q$ is true with p false, then we require that q is true, and thus $\neg p \wedge (p \vee q) \to q$ is always true (as we know that the conditional statement is also true whenever $\neg p \wedge (p \vee q)$ is false). This implies that $\neg p \wedge (p \vee q) \to q \equiv \mathbb{T}$.

Part b

$$[(p \to q) \wedge (q \to r)] \to (p \to r)$$

$p \to q$ is false when p is true and q is false.

Assume $(p \to q) \wedge (q \to r)$ is true.

When $(p \to q) \wedge (q \to r)$ is true, then both $p \to q$ and $q \to r$ are true.

When p is true, because $p \to q$ is true, q must also be true, and because $q \to r$ is true, r must also be true. $p \to r$ is also true when both p and r are true.

When p is false, then $p \to r$ is true as well.

Therefore, $p \to r$ is true in each case, and thus $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is always true (as we know that the conditional statement is also true whenever $(p \to q) \wedge (q \to r)$ is false).

This implies that $[(p \to q) \wedge (q \to r)] \to (p \to r) \equiv \mathbb{T}$

Part c

$$p \wedge (p \rightarrow q) \rightarrow q$$

$p \rightarrow q$ is false when p is true and q is false.

Assume $p \wedge (p \rightarrow q)$ is true.

When $p \wedge (p \rightarrow q)$ is true, both p and $p \rightarrow q$ are true.

When $p \rightarrow q$ and p are both true, we also require that q is true. Thus, $p \wedge (p \rightarrow q) \rightarrow q$ is always true (since we know the conditional statement is also true whenever $p \wedge (p \rightarrow q)$ is false).

This implies that $p \wedge (p \rightarrow q) \rightarrow q \equiv \mathbb{T}$

Part d

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

$p \rightarrow q$ is false when p is true and q is false.

Assume $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$ is true.

When $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$ is true, then $p \vee q$, $p \rightarrow r$, and $q \rightarrow r$ are all true.

$p \vee q$ being true implies that either p or q is true.

When p is true, then r is true as well (since $p \rightarrow r$ is true).

When q is true, then r is true as well (since $q \rightarrow r$ is true).

However, r is true in each case, and thus $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is always true (as we know that the conditional statement is also true whenever $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$ is false). This then gives us that

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r \equiv \mathbb{T}$$

Question 22

Show that $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent.

1. $p \rightarrow q \equiv \neg p \vee q$ (Logical Equivalences)
2. $\equiv \neg p \vee \neg\neg q$ (Double Negation Law)
3. $\equiv \neg\neg q \vee \neg p$ (Commutative Law)
4. $\equiv \neg q \rightarrow \neg p$ (Logical Equivalences)
5. $\therefore p \rightarrow q \equiv \neg q \rightarrow \neg p$

Question 24

Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

1. $\neg(p \oplus q) \equiv \neg((p \vee q) \wedge (\neg p \vee \neg q))$ (Definition)
2. $\equiv \neg[(p \wedge (\neg p \vee \neg q)) \vee (q \wedge (\neg p \vee \neg q))]$ (Distributive Law)
3. $\equiv \neg[((p \wedge \neg p) \vee (p \wedge \neg q)) \vee ((q \wedge \neg p) \vee (q \wedge \neg q))]$ (Distributive Law)
4. $\equiv \neg[(\mathbb{F} \vee (p \wedge \neg q)) \vee ((q \wedge \neg p) \vee \mathbb{F})]$ By $p \wedge \neg p \equiv \mathbb{F}$
5. $\equiv \neg[(p \wedge \neg q) \vee (q \wedge \neg p)]$ (Identity Law)
6. $\equiv \neg(p \wedge \neg q) \wedge \neg(q \wedge \neg p)$ (De Morgan's Law)
7. $\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ (De Morgan's Law and Double Negation Law)
8. $\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (Logical Equivalence)
9. $\equiv p \leftrightarrow q$ (Logical Equivalence)

Question 40

$s^* = s$ if the compound proposition can be simplified to a variable or the negation of a variable

Question 46

Not sure how to do this one.