MACM 101 Chapter 2.1 Homework

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Question 12

(a) Given $\emptyset \in \{\emptyset\}$.
$\{\emptyset\}$ is a set containing only the $\emptyset$, therefore, the given statement is **True**.

(b) Given $\emptyset \in \{\emptyset, \{\emptyset\}\}$.
$\emptyset$ is an element of $\{\emptyset, \{\emptyset\}\}$, therefore, the given statement is **True**.

(c) Given $\{\emptyset\} \in \{\emptyset\}$.
$\{\emptyset\}$ is not an element of $\{\emptyset\}$, therefore, the given statement is **False**.

(d) Given $\{\emptyset\} \in \{\{\emptyset\}\}$.
$\{\emptyset\}$ is an element of $\{\{\emptyset\}\}$, therefore, the given statement is **True**.

(e) Given $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$.
Every element in $\{\emptyset\}$ is also an element of $\{\emptyset, \{\emptyset\}\}$, therefore the given statement is **True**.

(f) Given $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$.
Every element in $\{\{\emptyset\}\}$ is also an element of $\{\emptyset, \{\emptyset\}\}$, therefore the given statement is **True**.

(g) Given $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$.
Every element in $\{\{\emptyset\}\}$ is also an element of $\{\{\emptyset\}, \{\emptyset\}\}$. However, the sets are equal, therefore the given statement is **False**.

Question 34

Let $A = \{a, b, c\}$, $B = \{x, y\}$ and $C = \{0, 1\}$.

(a) $A \times B \times C$
$= (A \times B) \times C$
$= \{(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)\} \times C$
$= \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1),$
$(b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$

(b) $C \times B \times A$
$= (C \times B) \times A$
$= \{(0, x), (0, y), (1, x), (1, y)\} \times A$
$= \{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c),$
$(1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$

(c) $C \times A \times B$
$= (C \times A) \times B$
$= \{(0, a), (0, b), (0, c), (1, a), (1, b), (1, c)\} \times B$
$= \{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y)$
$(1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)\}$

(d) $B \times B \times B$
$= (B \times B) \times B$
$= \{(x, x), (x, y), (y, x), (y, y)\} \times B$
$= \{(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y)\}$

Question 44

Prove or disprove that if A, B, and C are nonempty sets and $A \times B = A \times C$, then $B = C$.

Assume $B \neq C$.

This means $\exists x(x \in B \land x \notin C)$

However, because $A \times B$ is defined as $\{(a, b)|a \in A \land b \in B\}$, and $A \times C$ is defined as $\{(a, c)|a \in A \land c \in B\}$, if $\exists x(x \in B \land x \notin C)$, this implies $\exists x((a, x) \in A \times B \land (a, x) \notin A \times C)$

$\therefore A \times B \neq A \times C$

This is a contradiction, thus $B = C$.

$\square$

Question 50

This exercise presents Russell's paradox. Let S be the set that contains a set x if the set x does not belong to itself, so that $S = \{x \mid x \notin x\}$.

Part a

Show the assumption that S is a member of S leads to Links a contradiction.

Assume $S \in S$.

By definition of S, this means $S \notin S$.

$S \in S$ implies $S \notin S$ is a contradiction.

Part b

Show the assumption that S is not a member of S leads to a contradiction.

Assume $S \notin S$.

By definition of S, this means $S \in S$.

$S \notin S$ implies $S \in S$ is a contradiction.