Question 36

Part a

$$\forall x(-2 < x < 3)$$

$$-2 < x < 3 \text{ implies } -2 < x \text{ and } x < 3$$

$$\forall x((-2 < x) \land (x < 3))$$

Negate:

$$\neg\forall x((-2 < x) \land (x < 3))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \exists x \neg((-2 < x) \land (x < 3))$$

Using De Morgan's Laws:

$$\equiv \exists x((\neg(-2 < x)) \lor (\neg(x < 3)))$$

When $-2 < x$ is not true, we require $x \leq -2$.
When $x < 3$ is not true, we require $x \geq 3$.

$$\equiv \exists x((x \leq -2) \lor (x \geq 3))$$

Part b

$$\forall x(0 \leq x < 5)$$

$$0 \leq x < 5 \text{ implies } 0 \leq x \text{ and } x < 5$$

$$\forall x((0 \leq x) \land (x < 5))$$

Negate:

$$\neg\forall x((0 \leq x) \land (x < 5))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \exists x \neg((0 \leq x) \wedge (x < 5))$$

Using De Morgan's Laws:

$$\equiv \exists x((\neg(0 \leq x)) \vee (\neg(x < 5)))$$

When $0 \leq x$ is not true, we require $x < 0$.
When $x < 5$ is not true, we require $x \geq 5$.

$$\equiv \exists x((x < 0) \vee (x \geq 5))$$

Part c

$$\exists x(-4 \leq x \leq 1)$$

$$-4 \leq x \leq 1 \text{ implies } -4 \leq x \text{ and } x \leq 1$$

$$\exists x((-4 \leq x) \wedge (x \leq 1))$$

Negate:

$$\neg \exists x((-4 \leq x) \wedge (x \leq 1))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \forall x \neg((-4 \leq x) \wedge (x \leq 1))$$

Using De Morgan's Laws:

$$\equiv \forall x((\neg(-4 \leq x)) \vee (\neg(x \leq 1)))$$

When $-4 \leq x$ is not true, we require $x < -4$.
When $x \leq 1$ is not true, we require $x > 1$.

$$\equiv \forall x((x < -4) \vee (x > 1))$$

Part d

$$\exists x(-5 < x < -1)$$

$$-5 < x < -1 \text{ implies } -5 < x \text{ and } x < -1$$

$$\exists x((-5 < x) \wedge (x < -1))$$

Negate:

$$\neg\exists x((-5 < x) \wedge (x < -1))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \forall x \neg((-5 < x) \wedge (x < -1))$$

Using De Morgan's Laws:

$$\equiv \forall x((\neg(-5 < x)) \vee (\neg(x < -1)))$$

When $-5 < x$ is not true, we require $x \leq -5$.
When $x < -1$ is not true, we require $x \geq -1$.

$$\equiv \forall x((x \leq -5) \vee (x \geq -1))$$