

MACM 101 Chapter 1 Homework

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1 Section 1.1

Question 22

- a) Inclusive or, the requirement is experience with one or the other, and having both would still satisfy the requirement.
- b) Exclusive or, Lunch will come with either soup or salad, not both.
- c) Inclusive or, having both documents will not get you turned away.
- d) Exclusive or, publishing prevents perishing.

Question 24

- a) If you get promoted, then you have washed the Boss's car.
- b) If there are winds from the south, then there is a spring thaw.
- c) If you bought the computer less than a year ago, then the warranty is good.
- d) If Willy cheats, then he gets caught.
- e) If you can access the website, then you have paid a subscription fee.
- f) If you know the right people, then you get elected.
- g) If Carol is on a boat, then she gets seasick.

Question 26

- a) If you send me an e-mail message, then I will remember to send you the address.
- b) If you were born in the United States, then you are a citizen of this country.
- c) If you keep your textbook, then it will be a useful reference in your future courses.
- d) If their goalie plays well, then the Red Wings will win the Stanley Cup.
- e) If you get the job, then you had the best credentials.
- f) If there is a storm, then the beach erodes.
- g) If you log on to the server, then you have a valid password.
- h) If you do not begin your climb too late, then you will reach the summit.
- i) If you are among the first 100 customers tomorrow, then you will get a free ice cream cone.

Question 38

- a) $(p \vee q) \vee r$

p	q	r	$p \vee q$	$(p \vee q) \vee r$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

- b) $(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

c) $(p \wedge q) \vee r$

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

d) $(p \wedge q) \vee r$

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

e) $(p \vee q) \wedge \neg r$

p	q	r	$\neg r$	$p \vee q$	$(p \vee q) \wedge \neg r$
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	1	1	1
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	0

f) $(p \wedge q) \vee \neg r$

p	q	r	$\neg r$	$p \wedge q$	$(p \wedge q) \vee \neg r$
0	0	0	1	0	1
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	1	1

Question 42

Given $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$, show that the statement is true if and only if p , q and r all have the same truth value.

1. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ Original Premise
2. $((p \vee \neg q) \wedge (q \vee \neg r)) \wedge (r \vee \neg p)$ Trivial
3. $((q \wedge (p \vee \neg q)) \vee (\neg r \wedge (p \vee \neg q))) \wedge (r \vee \neg p)$ Distributive Property
4. $[r \wedge ((q \wedge (p \vee \neg q)) \vee (\neg r \wedge (p \vee \neg q)))] \vee [\neg p \wedge ((q \wedge (p \vee \neg q)) \vee (\neg r \wedge (p \vee \neg q)))]$
Distributive Property
5. Take $q \wedge (p \vee \neg q)$ and apply the distributive property,
 $(q \wedge p) \vee (q \wedge \neg q)$. Notice $q \wedge \neg q$ is a contradiction, and $p \vee \mathbf{F} \Leftrightarrow p$.
 $\therefore q \wedge (p \vee \neg q) \Leftrightarrow (q \wedge p)$
Rewritten,
 $[r \wedge ((p \wedge q) \vee (\neg r \wedge (p \vee \neg q)))] \vee [\neg p \wedge ((p \wedge q) \vee (\neg r \wedge (p \vee \neg q)))]$

6. Again, apply the distributive property.

$$[(r \wedge (p \wedge q)) \vee (r \wedge (\neg r \wedge p) \vee (\neg r \vee \neg q)))] \vee [(\neg p \wedge (p \wedge q)) \vee (\neg p \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)))]$$
7. Notice how in the above statement, we have
 $r \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q))$, which distributes to
 $r \wedge (\neg r \wedge p) \vee r \wedge (\neg r \wedge \neg q)$. This can easily be manipulated using the associative laws to show
 $((r \wedge \neg r) \wedge p) \vee ((r \wedge \neg r) \wedge \neg q) \rightarrow p \wedge \neg p \equiv \mathbf{F}$
 $(\mathbf{F} \wedge p) \vee (\mathbf{F} \wedge \neg q)$. We know that $\mathbf{F} \wedge p \equiv \mathbf{F}$.
 $\therefore r \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)) \equiv \mathbf{F}$
8. $[(r \wedge (p \wedge q)) \vee \mathbf{F}] \vee [(\neg p \wedge (p \wedge q)) \vee (\neg p \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)))]$
 Doing the same with $(\neg p \wedge ((\neg r \wedge p) \vee (\neg r \wedge \neg q)))$, we get
 $(\neg p \wedge (\neg r \wedge p)) \vee (\neg p \wedge (\neg r \wedge \neg q))$
 $\mathbf{F} \vee (\neg p \wedge (\neg r \wedge \neg q))$
9. $[(r \wedge (p \wedge q)) \vee \mathbf{F}] \vee [\mathbf{F} \vee (\neg p \wedge (\neg r \wedge \neg q))]$
 Because $p \vee \mathbf{F} \equiv p$,
10. $(r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r))$
11. $\therefore (p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \equiv (r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r))$

Proof

Show that the statement is true if and only if p , q , and r all have the same truth value.

1. Assume $p = q = r$. Then,
2. $(r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r))$
3. $(p \wedge p \wedge p) \vee (\neg p \wedge \neg p \wedge \neg p)$
 By the Idempotent laws,
4. $p \vee \neg p$. This is a Tautology.
5. \therefore when all values of p , q and r share the same truth value, the statement is true.
1. Assume $p = q$; $r = \neg p$

$$2. (r \wedge (p \wedge q)) \vee (\neg p \wedge (\neg q \wedge \neg r))$$

$$3. (p \wedge p \wedge \neg p) \vee (\neg p \wedge \neg p \wedge \neg \neg p)$$

By the Double Negation and Idempotent Laws,

$$4. (p \wedge \neg p) \vee (\neg p \wedge p)$$

Because $p \wedge \neg p$ is a Contradiction,

$$5. \mathbf{F} \vee \mathbf{F} \equiv \mathbf{F}.$$

6. \therefore when the values of p , q and r are not all the same, the statement is false.

Only after spending hours on this, I realize that I could have taken the original statement, applied the above two tests to it and proved the same thing. I am clearly, unequivocally, even, an idiot.