

1.1 Exercises 4, 8, 14, 21, 28, 36.

For exercises 4, 14 and 21 first write the linear system as an augmented matrix $[A|b]$ and then do elementary row operations on $[A|b]$ to answer the question. Indicate the row operations that you do. Verify that the solutions you get are correct by substituting them into the original equations.

4. Find the point of intersection of the lines $x_1 - 5x_2 = 1$ and $3x_1 - 7x_2 = 5$.

$$\begin{bmatrix} 1 & -5 & 1 \\ 3 & -7 & 5 \end{bmatrix} \text{ add } -3 \cdot \text{row 1} \\ \text{to row 2}$$

$$\begin{bmatrix} 1 & -5 & 1 \\ 0 & 8 & 2 \end{bmatrix} \text{ divide row 2} \\ \text{by 2}$$

$$\begin{bmatrix} 1 & -5 & 1 \\ 0 & 4 & 1 \end{bmatrix} \text{ add } \frac{5}{4} \cdot \text{row 2} \\ \text{to row 1}$$

$$\begin{bmatrix} 1 & 0 & \frac{9}{4} \\ 0 & 4 & 1 \end{bmatrix} \text{ solve for} \\ \text{unknowns}$$

$$x_1 + 0x_2 = \frac{9}{4} \sim x_1 = \frac{9}{4}$$

$$0x_1 + 4x_2 = 1 \sim x_2 = \frac{1}{4}$$

Verify solns

4. Find the point of intersection of the lines $x_1 - 5x_2 = 1$ and $3x_1 - 7x_2 = 5$.

$$x_1 - 5x_2 = 1$$

$$\frac{9}{4} - 5\left(\frac{1}{4}\right) =$$

$$3x_1 - 7x_2 = 5$$

$$3\left(\frac{9}{4}\right) - 7\left(\frac{1}{4}\right) =$$

In Exercises 7
been reduced to
continue the ap
set of the origi

$$7. \begin{bmatrix} 1 & 7 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$(-5, 7)$
of the

augmented matrix $[A|b]$ and
question. Indicate the row
are correct by substituting

$x_2 = 1$ and

In Exercises 7–10, the augmented matrix of a linear system has
been reduced by row operations to the form shown. In each case,
continue the appropriate row operations and describe the solution
set of the original system.

7.
$$\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

8.
$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$
 divide row 3 by 2

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$
 add $-7 \cdot$ row 3 to row 2

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$
 add $-2 \cdot$ row 3 to row 1

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$
 add $-1 \cdot$ row 2 to row 1

$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad \begin{array}{l} x_1 = -5 \\ x_2 = 7 \\ x_3 = -1 \end{array}$$

$(-5, 7, -1)$ is a solution set
of the original system

$x_2 = 1$ and

$x_1 = 5$ ✓
 $x_3 = \frac{1}{4}$

14.
$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ add eq 1 to eq 2}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ swap eq 2 and 3}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{bmatrix} \text{ add 2 \cdot eq 2 to eq 3}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{bmatrix} \text{ divide eq 3 by 7}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ add row 3 to row 2}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ add 3 \cdot row 2 to row 1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{aligned} x_1 &= 6 \\ x_2 &= 1 \\ x_3 &= 1 \end{aligned}$$

verify solution

14.
$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

$$6 - 3(1) = ?$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ add -1 \cdot row 3 to row 2}$$

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ add 3 \cdot row 2 to row 1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{aligned} x_1 &= 2 \\ x_2 &= -1 \\ x_3 &= 1 \end{aligned}$$

verify solution

14.
$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

$$\begin{aligned} 2 - 3(-1) &= \\ -(2) + (-1) + 5(1) &= \\ (-1) + 1 &= \end{aligned}$$

nice.

21. Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ have a common point of intersection? Explain.

$$\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix} \text{ add } -2 \cdot \text{row 1 to row 2}$$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ -1 & -3 & 4 \end{bmatrix} \text{ add } r_1 \text{ to } r_3$$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{bmatrix} \text{ add } \frac{7}{4} r_2 \text{ to } r_1$$

$$\begin{bmatrix} 1 & 0 & -35/4 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{bmatrix} \quad \frac{7}{4} \cdot (-5) = -\frac{35}{4}$$

$$x_1 = -\frac{35}{4}$$

$$x_2 = -\frac{5}{7}$$

$$7x_2 = -5$$

$$x_2 = -\frac{5}{7}$$

$$-7x_2 = 5$$

$$x_2 = -\frac{5}{7}$$

These lines all share
1 common point because
the system of equations
they form is consistent.

28. (T/F) Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

True. Otherwise, exercising elementary row operations on an augmented Matrix would not bear a valid solution to the original linear system.

36. Construct three different augmented matrices for linear systems whose solution set is $x_1 = -2$, $x_2 = 1$, $x_3 = 0$.

This is basically cheating lol

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} -2 & 2 & 0 & 6 \\ 0 & 2 & 3 & 2 \\ 0 & 0 & 3 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} -2 & 2 & 0 & 6 \\ 0 & 2 & 3 & 2 \end{array} \right]$$

36. Construct three different augmented matrices for linear systems whose solution set is $x_1 = -2$, $x_2 = 1$, $x_3 = 0$.

This is basically cheating lol

$$\left[\begin{array}{cccc} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc} -2 & 0 & 0 & 4 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc} -2 & 2 & 0 & 6 \\ 0 & 2 & 3 & 2 \\ 0 & 0 & 3 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc} -2 & 2 & 0 & 6 \\ 0 & 2 & 3 & 2 \\ 0 & 2 & 6 & 2 \end{array} \right]$$