MACM 101 Chapter 2 Homework - Halting Problem

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Assume H is a Turing machine that solves the halting problem.

This function takes an input P, which is a program, and x, which is an input to the program, and returns **T** if P halts on input x, and **F** otherwise.

$$H(P, x) = \begin{cases} P \text{ halts on } x & \mathbf{T} \\ P \text{ does not halt on } x & \mathbf{F} \end{cases}$$

Let D be another Turing machine that takes an input P, which is a program, and returns $H(P, P)$. That is, D takes a program P as input and returns **T** if P halts on P, and **F** otherwise.

$$D(P) = \begin{cases} H(P, P) & \text{D never halts} \\ & \text{D halts immediately} \end{cases}$$

Consider $D(D)$:

$$D(D) = \begin{cases} \text{If } H(D, D) = \mathbf{T}, \text{ then } D(D) \text{ goes into an infinite loop.} \\ \text{If } H(D, D) = \mathbf{F}, \text{ then } D(D) \text{ halts immediately.} \end{cases}$$

Now:

- If $H(D, D) = \mathbf{T}$, this implies $D(D)$ should halt, but by the definition of D, if $H(D, D) = \mathbf{T}$, $D(D)$ enters an infinite loop, which is a contradiction.

- If $H(D, D) = \mathbf{F}$, this implies $D(D)$ does not halt, but by the definition of D, if $H(D, D) = \mathbf{F}$, $D(D)$ halts immediately, which is also a contradiction.

Since both cases lead to contradictions, the assumption that H exists is false.

Therefore, the halting problem is unsolvable.

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