Question 8

Part a

Every rabbit hops.

Part b

Every animal is a rabbit and it hops.

Part c

There exists an animal such that, if it is a rabbit, then it hops.

Part d

There exists an animal that is a rabbit and it hops.

Question 20

Part a

$$P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$$

Part b

$$P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$$

Part c

$$P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$$

Part d

$$P(1) \vee P(3) \vee P(5)$$

Part e

$$(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-5) \wedge P(-3) \wedge P(-1))$$

Question 32

Part a

"All dogs have fleas"

Let the domain be dogs and $P(x)$ mean "x has fleas"

$\forall x P(x)$

Negate, then apply De Morgan's Laws for Quantifiers

$\neg(\forall x P(x)) \equiv \exists x \neg P(x)$

"There is a dog that does not have fleas"

Part b

"There is a horse that can add"

Let the domain be horses and $P(x)$ mean "x can add"

$\exists x P(x)$

Negate, then appy De Morgan's Laws.

$\neg(\exists x P(x)) \equiv \forall x \neg P(x)$

Part c

"Every koala can climb"

Let the domain be koalas and $P(x)$ mean "x can climb"

$\forall x P(x)$

Negate, then apply De Morgan's Laws

$\neg(\forall x P(x)) \equiv \exists x \neg P(x)$

"There exists a koala that cannot climb"

Part d

"No monkey can speak french"

Let the domain be monkeys and $P(x)$ mean "x can speak french"

$\forall x \neg P(x)$

Negate, then apply De Morgan's Law and the Double Negation Law

$\neg(\forall x \neg P(x)) \equiv \exists x \neg(\neg P(x)) \equiv \exists x P(x)$

Part e

"There exists a pig that can swim and catch fish"

Let the domain be pigs, $P(x)$ mean "x can swim" and $Q(x)$ mean "x can catch fish"

$\exists x(P(x) \land Q(x))$

Negate, then apply De Morgan's Law for Quantifiers and the regular De Morgan's Law $\neg(\exists x(P(x) \land Q(x))) \equiv \forall x \neg(P(x) \land Q(x)) \equiv \forall x(\neg P(x) \lor \neg Q(x))$

"All pigs can not swim or not catch fish."

Question 36

Part a

$$\forall x(-2 < x < 3)$$

$$-2 < x < 3 \text{ implies } -2 < x \text{ and } x < 3$$

$$\forall x((-2 < x) \land (x < 3))$$

Negate:

$$\neg \forall x((-2 < x) \land (x < 3))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \exists x \neg((-2 < x) \land (x < 3))$$

Using De Morgan's Laws:

$$\equiv \exists x((\neg(-2 < x)) \lor (\neg(x < 3)))$$

When $-2 < x$ is not true, we require $x \leq -2$.

When $x < 3$ is not true, we require $x \geq 3$.

$$\equiv \exists x((x \leq -2) \lor (x \geq 3))$$

Part b

$$\forall x(0 \leq x < 5)$$

$$0 \leq x < 5 \text{ implies } 0 \leq x \text{ and } x < 5$$

$$\forall x((0 \leq x) \wedge (x < 5))$$

Negate:

$$\neg \forall x((0 \leq x) \wedge (x < 5))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \exists x \neg((0 \leq x) \wedge (x < 5))$$

Using De Morgan's Laws:

$$\equiv \exists x((\neg(0 \leq x)) \vee (\neg(x < 5)))$$

When $0 \leq x$ is not true, we require $x < 0$.
When $x < 5$ is not true, we require $x \geq 5$.

$$\equiv \exists x((x < 0) \vee (x \geq 5))$$

Part c

$$\exists x(-4 \leq x \leq 1)$$

$$-4 \leq x \leq 1 \text{ implies } -4 \leq x \text{ and } x \leq 1$$

$$\exists x((-4 \leq x) \wedge (x \leq 1))$$

Negate:

$$\neg \exists x((-4 \leq x) \wedge (x \leq 1))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \forall x \neg((-4 \leq x) \wedge (x \leq 1))$$

Using De Morgan's Laws:

$$\equiv \forall x((\neg(-4 \leq x)) \vee (\neg(x \leq 1)))$$

When $-4 \leq x$ is not true, we require $x < -4$.
When $x \leq 1$ is not true, we require $x > 1$.

$$\equiv \forall x((x < -4) \lor (x > 1))$$

Part d

$$\exists x(-5 < x < -1)$$

$$-5 < x < -1 \text{ implies } -5 < x \text{ and } x < -1$$

$$\exists x((-5 < x) \land (x < -1))$$

Negate:

$$\neg\exists x((-5 < x) \land (x < -1))$$

Using De Morgan's Laws for Quantifiers:

$$\equiv \forall x \neg((-5 < x) \land (x < -1))$$

Using De Morgan's Laws:

$$\equiv \forall x((\neg(-5 < x)) \lor (\neg(x < -1)))$$

When $-5 < x$ is not true, we require $x \leq -5$.
When $x < -1$ is not true, we require $x \geq -1$.

$$\equiv \forall x((x \leq -5) \lor (x \geq -1))$$

Question 48

Part a

If $(\forall x P(x)) \lor A$ is true, then A is true or for all values y we have that $P(y)$ is true. Then $P(y) \lor A$ is true for all values of y, implying that $\forall x(P(x) \lor A)$ is true.

If $(\forall x P(x)) \lor A$ is false, then A is false and there exists a value of y such that $P(y)$ is false. Then $P(y) \lor A$ is false, implying that $\forall x(P(x) \lor A)$ is also false.

Thus, the two expressions always have the same truth value and therefore they are logically equivalent.

Part b

If $(\exists x P(x)) \vee A$ is true, then A is true or there exists a value y for which $P(y)$ is true. Then $P(y) \vee A$ is true, implying that $\exists x(P(x) \vee A)$ is true.

If $(\exists x P(x)) \vee A$ is false, then A is false, and for all values y we have that $P(y)$ is false. Then $P(y) \vee A$ is false for every value of y, implying that $\exists x(P(x) \vee A)$ is also false.

Thus, the two expressions always have the same truth value and therefore they are logically equivalent.