



INTRODUCTION TO DISCRETE MATHEMATICS

MACM 101

What is Discrete Mathematics?

Discrete mathematics is concerned with mathematical structures which take on a discrete values instead of continuous ones as described by the real-number system in the case of continuous mathematics; that is, **integers are the cornerstone of discrete mathematics.**

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This branch of mathematics provides an excellent method of modeling real-world phenomena that vary between discrete states. Moreover, Discrete Mathematics provides the formal basis for computing science since **computers are finite state machines.**

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This branch of mathematics provides an excellent method of modeling real-world phenomena that vary between discrete states. Moreover, Discrete Mathematics provides the formal basis for computing science since **computers are finite state machines.**

This course will provide the formal foundation for much of what you learn in computing science. This is a difficult course, but it is one of the most valuable subjects that you will undertake at SFU.

Topics of Introductory Discrete Mathematics

- Sets, Relations and Functions
- Mathematical Logic
- Number Theory
- Counting Theory
- Algorithms and Computational Complexity
- Probability (no longer in MACM 101)
- Mathematical Induction and Recurrence Relations
- Graph Theory and Trees
- Formal Languages (only if time permits)
- Boolean Algebra (only if time permits)

MACM 101 is a course on **proof theory** within the setting of Discrete Mathematics.

Kinds of Problems Solved Using Discrete Mathematics

- How many ways can a password be chosen following specific rules?
- How many valid Internet addresses are there?
- What is the probability of winning a particular poker hand?
- Is there a link between two computers in a network?
- How can I identify spam email messages?
- How can I encrypt a message so that no unintended recipient can read it?
- How can we build a logic circuit that adds two integers?

Kinds of Problems Solved Using Discrete Mathematics

- What is the shortest path between two cities using a transportation system?
- How can we represent English sentences so that a computer can reason with them?
- How can we prove that there are infinitely many prime numbers?
- How can a list of integers be sorted so that the integers are in increasing order?
- How many steps are required to do such a sorting?
- Find the shortest tour that visits each of a group of cities only once and then ends in the starting city.

<https://www.youtube.com/watch?v=SC5CX8drAtU>

Discrete Mathematics is a Gateway Course

- Topics in discrete mathematics will be important in many courses that you will take in the future:
 - **Computer Science:** Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation, ...

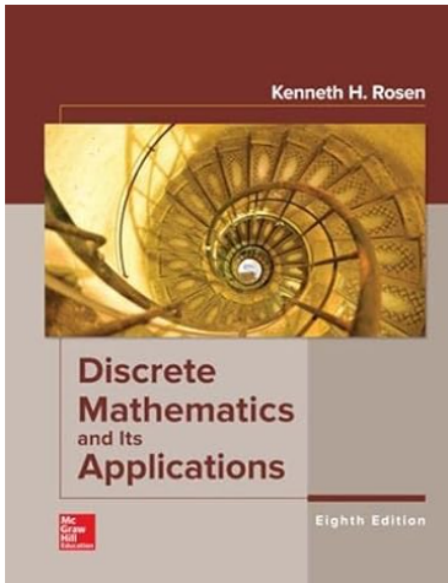
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 - **Mathematic + CoMputing = MACM.**

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 - **Mathematic + CoMputing = MACM.**
- **Other Disciplines:** You may find that concepts learned here are also useful in courses in philosophy, economics, linguistics, and other disciplines.

Textbook Topics



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Tentative Syllabus (Rosen-based)

Weeks	Topic	Chapters	Assignments
1	Introduction and background, elementary logic	1 and 2	
2	Elementary logic continued (1.1 – 1.8)	2	
3	Set theory, inclusion/exclusion (2.1 – 2.5, 8.5)	2, 8.5	Assignment 1
4	Algorithms and functions (3.1 – 3.3)	3	
5	Number theory (4.1, 4.3, 4.4)	4	Assignment 2
6	MIDTERM #1; Induction and recursion (5.1 – 5.3)	5	
7	Combinatorics	6	Assignment 3
8	Combinatorics continued (6.1 – 6.6)	6	
9	Relations (9.1 – 9.6)	9	Assignment 4
10	MIDTERM #2; Graphs and trees (10.1 – 10.2, 11.1 – 11.4)	10	
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An Example of a MACM 101 Proof

PROPOSITION: If $(0 = 1)$, then $(1 = 1)$

True or False?

An Example of a MACM 101 Proof

PROPOSITION: If $(0 = 1)$, then $(1 = 1)$

PROOF:

1. State given premise, $(0 = 1)$.

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PROPOSITION: If $(0 = 1)$, then $(1 = 1)$

PROOF:

1. State given premise, $(0 = 1)$.
2. State given conclusion, $(1 = 1)$.
3. Now, thread a *valid argument* from 1 to 2 using the rules of algebra (*axioms*) and all requisite *definitions*.

Step 3 in Detail

- ▣ $(0 = 1)$, given premise

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- ▣ $(0 = 1)$, given premise
- ▣ $(1 = 0)$, commutative law

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- ▣ $(0 = 1)$, given premise
- ▣ $(1 = 0)$, commutative law
- ▣ $(1 = 1)$, addition of previous two statements

Step 3 in Detail

- ▣ $(0 = 1)$, given premise
- ▣ $(1 = 0)$, commutative law
- ▣ $(1 = 1)$, addition of previous two statements
- ▣ This is the conclusion that we sought

QED

The Complete Proof

PROPOSITION: If $(0 = 1)$, then $(1 = 1)$

PROOF:

1. State given premise, $(0 = 1)$.
2. State given conclusion, $(1 = 1)$.
3. Now, thread a *valid argument* from 1 to 2 using the rules of algebra (*axioms*) and all requisite *definitions*.
 - a) $(0 = 1)$, given premise
 - b) $(1 = 0)$, commutative law
 - c) $(1 = 1)$, addition of previous two statements

QED

QED

Quod Erat Demonstrandum,

“What was to be demonstrated”.

Key Issues

	<i>Addition</i>	<i>Multiplication</i>
<i>Commutativity</i>	$x + y = y + x$	$x * y = y * x$
<i>Associativity</i>	$(x + y) + z = x + (y + z)$	$(x * y) * z = x * (y * z)$
<i>Identity</i>	$x + 0 = x$	$1 * x = x$
<i>Inverse</i>	$x + -x = 0$	$x * 1/x = 1$
<i>Distribution</i>	$x * (y + z) = (x * y) + (x * z)$	

Know your **definitions** and **axioms** succinctly. Here is how you learned them in grade school algebra.

Presentation Terminated