Question 8

Part a

$A \cup A = A$

By definition, $A \cup A = \{x \,|x \in A \vee x \in A\}$

The left and right side of the disjunction are the same set, so by the Idempotent Law of Propositional Logic, $p \vee p \equiv p$,

$x \in A \vee x \in A \equiv x \in A$

$A \cup A = \{x \,|x \in A\}$

$\therefore A \cup A = A$

Part b

$A \cap A = A$

By definition, $A \cap A = \{x \,|x \in A \wedge x \in A\}$

The left and right side of the conjunction are the same set, so by the Idempotent Law of Propositional Logic, $p \wedge p \equiv p$,

$x \in A \wedge x \in A \equiv x \in A$

$A \cap A = \{x \,|x \in A\}$

$\therefore A \cap A = A$