Question 8

Prove that if n is a perfect square, then n+2 is not a perfect square.

Let $n=m^2$. If m=0, then n+2=2, which is not a perfect square. This means $m \ge 1$. The smallest perfect square greater than n is $(m+1)^2$. We have $(m+1)^2 = m^2 + 2m + 1 = n + 2m + 1$. $n+2m+1 > n+2\cdot 1+1 > n+2$. Therefore, n+2 cannot be a perfect

square.