MACM 101 Chapter 2.2 Homework

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Question 6

Part a

$A \cup \emptyset = A$

$A \cup B = \{x \,| x \in A \vee x \in B\}$
By definition, $A \cup \emptyset = \{x \,| x \in A \vee x \in \emptyset\}$
The empty set does not contain any elements, so $x \notin \emptyset$
$A \cup \emptyset = \{x \,| x \in A \vee \mathbf{F}\}$
By the Identity Laws of Propositional Logic, $p \vee \mathbf{F} = p$
$A \cup \emptyset = \{x \,| x \in A\}$
$\therefore A \cup \emptyset = A$

Part b

$A \cap U = A$

$A \cap B = \{x \,| x \in A \wedge x \in B\}$
By definition, $A \cap U = \{x \,| x \in A \wedge x \in U\}$
Because U is the universal set, $x \in U \equiv \mathbf{T}$
So, $A \cap U = \{x \,| x \in A \wedge \mathbf{T}\}$
By the Identity Laws of Propositional Logic, $p \wedge \mathbf{T} = p$
$A \cap U = \{x \,| x \in A\}$
$\therefore A \cap U = A$

Question 8

Part a

$A \cup A = A$

By definition, $A \cup A = \{x\,|x \in A \vee x \in A\}$

The left and right side of the disjunction are the same set, so by the Idempotent Law of Propositional Logic, $p \vee p \equiv p$,

$x \in A \vee x \in A \equiv x \in A$

$A \cup A = \{x\,|x \in A\}$

$\therefore A \cup A = A$

Part b

$A \cap A = A$

By definition, $A \cap A = \{x\,|x \in A \wedge x \in A\}$

The left and right side of the conjunction are the same set, so by the Idempotent Law of Propositional Logic, $p \wedge p \equiv p$,

$x \in A \wedge x \in A \equiv x \in A$

$A \cap A = \{x\,|x \in A\}$

$\therefore A \cap A = A$

Question 10

Part a

$A - \emptyset = A$

$= A \cap \overline{\emptyset}$

The complement of the empty set is the universal set, so $\overline{\emptyset} = U$

By Question 6, Part b, $A \cap U = A$

$= A \cap U = A$

$\therefore A - \emptyset = A$

Part b

$\emptyset - A = \emptyset$

By definition,

$= \emptyset \cap \overline{A}$

By definition,

$= \{x\,\big|x \in \emptyset \wedge x \in \overline{A}\}$

Because $x \in \emptyset \equiv \mathbf{F}$

$= \{x\,\big|\mathbf{F} \wedge x \in \overline{A}\}$

By the Domination Laws of Propositional Logic, $p \wedge \mathbf{F} = \mathbf{F}$

$= \{x\,|\mathbf{F}\}$

$\therefore \emptyset - A = \emptyset$