MACM 101 Lecture 1.1

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1 Chapter Summary

- Propositional Logic

    The Language of Propositions

    Applications

    Logical Equivalences and Implication

    The Laws of Propositional Logic

- Predicate Logic

    The Language of Quantifiers

    Nested Quantifiers

- Proofs

    Rules of Inference

    Proof Methods

    Proof Strategy

This document covers everything from Rosen 1.0 to 1.3.

2 Definitions

2.1 Deduction/Deductive Logic

Deduction is the process of deriving a conclusion from a given set of axioms or premises. In Logic, we start from the ground (axioms) and work our way up to the conclusion.

2.2 Truth Value

A truth value can be either true or false, but not both. This comes from the *principium tertii eclusi* of Aristotle.

2.2.1 True and False

We will use 1 and 0 to denote true and false, respectively.

2.2.2 Unknown Truth Value

The proposition u is *unknown truth value*.

2.3 Proposition

A proposition is a declarative sentence (or statement) that possesses truth value.

2.3.1 Notation

Lowercase letters denote primitive propositions, and uppercase letters denote complex propositions.

Primitive propositions are:

- Propositions that cannot be decomposed into anything simpler
- $p : 3 + 5 = 8$
- q : It is raining

2.4 Examples of things that are not propositions

- p : Sit down! $\rightarrow$ not a proposition because it is not a declarative
- q : The statement you are reading is now false. $\rightarrow$ not a proposition because it is a contradiction.
- r : The number x is an integer. $\rightarrow$ not a proposition because it contains an unspecified variable, which means it's truth value cannot be definitively determined without additional information.

2.5 Syntactics and Semantics

Syntatic reasoning is what can be shown.

Syntax = grammar (rules of sentance construction), the structure of propositions

Semantics reasoning is what is true

Semantics = meaning (truth value), the truth value/tables of propositions

2.6 Literals

A *literal* is either a primitive proposition or its negation (some textbooks use to denote a literal)

3 Operator Syntax

1. Negation - $\neg$
   q : it is raining, $\neg q$: it is not raining
   Everything in this list other than $\neg$ is known as a *logical connective*
2. Conjunction - $\wedge$ - Logical and
   $p \wedge q$: it is raining and it is sunny
   $p \wedge \neg q$: it is raining and it is not sunny
3. Disjunction - $\vee$ - Inclusive Or
   $p \vee q$: it is raining or it is sunny
   $p \vee \neg q$: it is raining or it is not sunny

4. Disjunction - $\oplus$ - Exclusive Or
   $p \oplus q$: it is raining xor it is sunny
   $p \oplus \neg q$: it is raining xor it is not sunny
   XOR is generally what is meant in english sentences like "the meal comes with either soup or salad"

5. Implication - $\rightarrow$ - "If, then"

6. Biconditional - $\leftrightarrow$ - "If and only if"

Nobody knows why OR and XOR are both called Disjunction

All propositions formed with logical connectives are called *compound propositions*, as opposed to *primitive propositions*

Compound propositions need not have causal relations between atomic components (they can sound nonsensical and still be valid) – material implication as opposed to causal implication, which lacks temporal ordering. (straight from the slides, p. 34)

4 Logical Equivalences

TABLE 6 Logical Equivalences.	
Equivalence	***Name***
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Figure 1: Logical Equivalences

TABLE 8 **Logical Equivalences Involving Biconditional Statements.**
$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Figure 2: Logical Equivalences Involving Biconditional Statements

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Figure 3: Logical Equivalences Involving Conditional Statements

5 Semantics

Basically, all semantics is is truth tables.

5.1 Negation, $\neg$

p	$\neg p$
T	F
F	T

5.2 Logical Connectives

1. Conjunction - $\wedge$ - Logical AND
2. Disjunction - $\vee$ - Logical OR

3. Exclusive Or - $\oplus$ - Exclusive OR

4. Implication - $\rightarrow$ - Material Implication

5. Biconditional - $\leftrightarrow$ - Biconditional

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
1	1	1	1	0	1	1
1	0	0	1	1	0	0
0	1	0	1	1	1	0
0	0	0	0	0	1	1

5.3 Extra Context for Material Implication

A wide range of terminology exists to describe material implication.

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"
"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Figure 4: Ways to Express $p \rightarrow q$

5.3.1 Necessary Conditions (Examples)

For a normal car to run, it is necessary that there is fuel in the tank, its spark plugs are properly adjusted, its oil pump is working, etc.

If the car runs, then every one of the conditions must be fulfilled.
All of these propositions are $p \rightarrow q$
Let p be the statement "the car runs"
Let q be the other statement

1. If the car runs, then fuel must be in the tank. ($p \rightarrow q$)

2. If the car runs, its spark plugs must be properly adjusted. ($p \rightarrow q$)

3. If the car runs, its oil pump must be working. ($p \rightarrow q$)

From this, you can see why material implication is defined by $\neg(p \wedge \neg q) \equiv \neg p \vee q$. I think that $\neg(p \wedge \neg q)$ makes a lot more sense than $\neg p \vee q$ when you read it out loud. With this definition, you can clearly visualize the truth table. Try thinking about (not (p land not q)) and look at the truth table.