# Question 4

Prove that there are no perfect cubes less than 1000 that are the sum of the cubes of two positive integers.

The cubes less than 1000 are 1, 8, 27, 64, 125, 216, 343, 512 and 729.

If we try to sum all possible combinations of two of these integers, we see that none of them work. By exhaustion, we conclude that no cube less than 1000 is the sum of two cubes. (See next page for the exhaustive list)

$1 + 1 = 2$
$1 + 8 = 9$
$1 + 27 = 28$
$1 + 64 = 65$
$1 + 125 = 126$
$1 + 216 = 217$
$1 + 343 = 344$
$1 + 512 = 513$
$1 + 729 = 730$
$8 + 8 = 16$
$8 + 27 = 35$
$8 + 64 = 72$
$8 + 125 = 133$
$8 + 216 = 224$
$8 + 343 = 351$
$8 + 512 = 520$
$8 + 729 = 737$
$27 + 27 = 54$
$27 + 64 = 91$
$27 + 125 = 152$
$27 + 216 = 243$
$27 + 343 = 370$
$27 + 512 = 539$
$27 + 729 = 756$
$64 + 64 = 128$
$64 + 125 = 189$
$64 + 216 = 280$
$64 + 343 = 407$
$64 + 512 = 576$
$64 + 729 = 793$
$125 + 125 = 250$
$125 + 216 = 341$
$125 + 343 = 468$
$125 + 512 = 637$
$125 + 729 = 854$
$216 + 216 = 432$
$216 + 343 = 559$
$216 + 512 = 728$
$216 + 729 = 945$
$343 + 343 = 686$
$343 + 512 = 855$
$343 + 729 = 1072$
$512 + 512 = 1024$
$512 + 729 = 1241$
$729 + 729 = 1458$