Question 48

Part a

If $(\forall x P(x)) \vee A$ is true, then A is true or for all values y we have that $P(y)$ is true. Then $P(y) \vee A$ is true for all values of y, implying that $\forall x(P(x) \vee A)$ is true.

If $(\forall x P(x)) \vee A$ is false, then A is false and there exists a value of y such that $P(y)$ is false. Then $P(y) \vee A$ is false, implying that $\forall x(P(x) \vee A)$ is also false.

Thus, the two expressions always have the same truth value and therefore they are logically equivalent.

Part b

If $(\exists x P(x)) \vee A$ is true, then A is true or there exists a value y for which $P(y)$ is true. Then $P(y) \vee A$ is true, implying that $\exists x(P(x) \vee A)$ is true.

If $(\exists x P(x)) \vee A$ is false, then A is false, and for all values y we have that $P(y)$ is false. Then $P(y) \vee A$ is false for every value of y, implying that $\exists x(P(x) \vee A)$ is also false.

Thus, the two expressions always have the same truth value and therefore they are logically equivalent.