MACM 101

The Rules of Inference and Valid Argumentation

Section 1.6

Last Time

- Defined inductive and **deductive** reasoning.
- Reviewed the foundation of mathematical theories – **formal axiomatic systems**.
- Cast propositional logic as a formal axiomatic system – **The Laws of Logic**.
- Provided examples of deducing statements (theorems) from said laws (rules of deduction/inference).

Section 1.6 Summary

- Valid Arguments.
- Inference Rules for Propositional Logic.
- Using Rules of Inference to Build Arguments.
- Rules of Inference for Quantified Statements.
- Building Arguments for Quantified Statements.

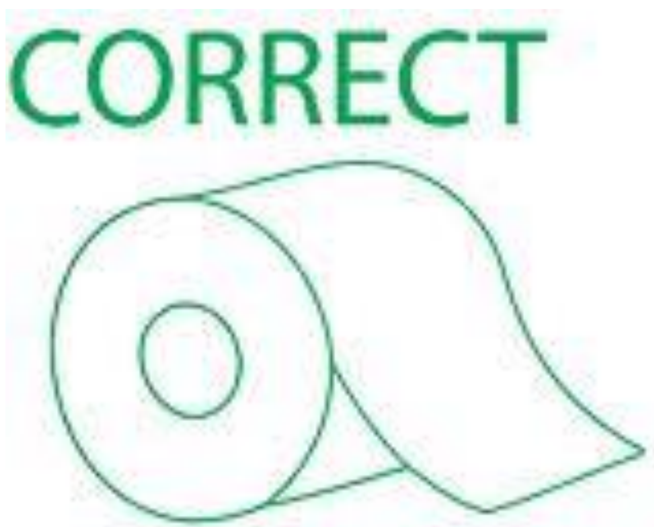
Goals

- The main purpose of Sections 1.6-1.8 is to provide a foundation for *proof theory* – a mathematical proof is purely deductive, *valid argument.*
- Review Tautologies, Contradictions, Logical Implication and the notion of a valid argument.
- Discuss the difference between *truth* and validity.
- Define the Rules of Inference and their application.

Question

How can one prove the following proposition:

"Toilet paper should be installed *over* as opposed to *under*"?



What is an Argument?

- When a scientist argues in favor of a scientific theory, it is based on supportive evidence (often observations) for the *probabilistic* truth of some general conclusion.

inductive argument

Web definitions

Inductive reasoning is reasoning in which the premises seek to supply strong evidence for the truth of the conclusion. While the conclusion of a deductive argument is supposed to be certain, the truth of an inductive argument is supposed to be probable, based upon the evidence given.
http://en.wikipedia.org/wiki/Inductive_argument

What is an Argument?

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- We are not concerned with such arguments in MACM 101, as mentioned earlier in this course, but rather those that are ***deductive*** in nature those that deal with *truth value only*.

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- When a scientist argues in favor of a scientific theory, it is based on supportive evidence (often observations) for the *probabilistic* truth of some general conclusion.
- We are not concerned with such arguments in MACM 101, but rather those that are ***deductive*** in nature those that deal with *truth value only*.
- See the following:
  https://www.lanecc.edu/sites/default/files/trio/deductive_and_inductive_arguments.pdf

Truth Versus Validity

Truth and validity are two different concepts:

- **Truth** is predicated on **propositions**, which are either true or false.
- **Validity** is predicated on **deductive arguments**, which are either valid or invalid.

Definition

Deductive Argument:

A sequence of declarative statements.

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NOTE: All such statements except the final one are called *premises* (or *assumptions*, *antecedent*, or hypotheses). The final one is called the *conclusion* or *consequents*.

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Valid Deductive Argument:

An argument such that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true.

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NOTE: This course is purely deductive, so from now on when we say, *valid argument*, we mean *valid deductive argument*.

Formal Definition of a Valid Argument

A deductive argument is **valid** *iff* the premises provide conclusive proof for its conclusion.

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In other words, either of the following must hold:

(i) If the premises of a valid argument are all true, then its conclusion must also be true.

(ii) It is impossible for the conclusion of a valid argument to be false while its premises are true.

Formal Definition of a Valid Argument

A deductive argument is **valid** *iff* the premises provide conclusive proof for its conclusion.

Put another way, a tautology is *necessary and sufficient* for a valid argument.

Recall Example 2.20, Grimaldi

Let us now consider the truth table in Table 2.15. The results in the last column of this table show that for any primitive statements p, r, and s, the implication

$$[p \wedge ((p \wedge r) \rightarrow s)] \rightarrow (r \rightarrow s)$$

Table 2.15

p_1				p_2	q	$(p_1 \wedge p_2) \rightarrow q$
p	r	s	$p \wedge r$	$(p \wedge r) \rightarrow s$	$r \rightarrow s$	$[(p \wedge ((p \wedge r) \rightarrow s)] \rightarrow (r \rightarrow s)$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	0	1	0	1
0	1	1	0	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

is a tautology. Consequently, for premises

$$p_1: \quad p \qquad p_2: \quad (p \wedge r) \rightarrow s$$

and conclusion q: $(r \rightarrow s)$, we know that $(p_1 \wedge p_2) \rightarrow q$ is a valid argument, and we may say that the truth of the conclusion q is *deduced* or *inferred* from the truth of the premises p_1, p_2.

The Connection Between Truth and Validity

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- This provides a direct connection between truth and validity.
- Observe that a valid argument can have one or more false premises.

Example of an Invalid Argument

1. $p \rightarrow q \vee \neg r$
2. $\underline{q \rightarrow \mathrm{p} \wedge \mathrm{r}}$
3. $\therefore p \rightarrow r$

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2. $\underline{q \rightarrow p \wedge r}$
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Verify:

p	*q*	*r*	Premise 1	Premise 2	Conclusion 3
0	0	0	**1**	**1**	**1**
0	0	1	**1**	**1**	**1**
0	1	0	**1**	**0**	**1**
0	1	1	**1**	**0**	**1**
1	0	0	**1**	**1**	**0**
1	0	1	**0**	**1**	**1**
1	1	0	**1**	**0**	**0**
1	1	1	**1**	**1**	**1**

The blackened row CANNOT happen. Hence, it is an invalid argument.

Logical Implication and Logical Equivalence

Formula $\mathcal{A}$ ***logically implies*** formula $\mathcal{B}$
if and only if
the conditional formula $\mathcal{A} \rightarrow \mathcal{B}$ is a tautology.

Formulas $\mathcal{A}$ and $\mathcal{B}$ are ***logically equivalent***
if and only if
the biconditional formula $\mathcal{A} \leftrightarrow \mathcal{B}$ is a tautology.

Example of a Simple Valid Argument

Question: What valid argument form is involved in the following:

1. If the sum of the digits of 371, 487 is divisible by 3, then 371, 487 is divisible by 3.
2. The sum of the digits of 371, 487 is divisible by 3.
3. Therefore, 371, 487 is divisible by 3.

Example of a Simple Valid Argument

Question: What valid argument form is involved in the following:

1. If the sum of the digits of 371, 487 is divisible by 3, then 371, 487 is divisible by 3.
2. The sum of the digits of 371, 487 is divisible by 3.
3. Therefore, 371, 487 is divisible by 3.

Answer: The Method of Affirming – *i.e.,* Modus Ponens or The Rule of Detachment.

$$[p \wedge (p \rightarrow q)] \rightarrow q$$

Example of a Simple Valid Argument

[redacted]

1. [redacted]
2. [redacted]
3. [redacted]

Answer: The Method of Affirming – *i.e.,* Modus Ponens or The Rule of Detachment.

$$[p \land (p \rightarrow q)] \rightarrow q$$

Alternative form:

$$\left\{\begin{array}{l} \quad p \\ \quad \underline{p \rightarrow q} \\ \therefore q \end{array}\right.$$

Example of Modus Ponens

Determine whether the argument given here is valid and determine whether its conclusion must be true because of the validity of the argument.

"If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $\left(\sqrt{2}\right)^2 = 2 > \left(\frac{3}{2}\right)^2 = \frac{9}{4}$."

Example of Modus Ponens

Determine whether the argument given here is valid and determine whether its conclusion must be true because of the validity of the argument.

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Solution: Let p be the proposition "$\sqrt{2} > \frac{3}{2}$" and q the proposition "$2 > (\frac{3}{2})^2$." The premises of the argument are $p \to q$ and p, and q is its conclusion. This argument is valid because it is constructed by using modus ponens, a valid argument form. However, one of its premises, $\sqrt{2} > \frac{3}{2}$, is false. Consequently, we cannot conclude that the conclusion is true. Furthermore, note that the conclusion of this argument is false, because $2 < \frac{9}{4}$. ◀

Rosen

Modus Ponens

p	q	$p \to q$	$p \wedge (p \to q)$	q	$p \wedge (p \to q) \to q$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	1	1	1	1	1

Tautology

Another Valid Argument Form

Question: What valid argument form is involved in the following:

1. If 870, 232 is divisible by 6, then it is divisible by 3.
2. 870, 232 is not divisible by 3.
3. Therefore, 870, 232 is not divisible by 6.

Another Valid Argument Form

Question: What valid argument form is involved in the following:

1. If 870, 232 is divisible by 6, then it is divisible by 3.
2. 870, 232 is not divisible by 3.
3. Therefore, 870, 232 is not divisible by 6.

Answer: "The Method of Denying (the conclusion) – *i.e.,* Modus Tollens.

$$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$$

The Contrapositive form of Modus Ponens

Question: What valid argument form is involved in the following:

1. If 870, 232 is divisible by 6, then it is divisible by 3.
2. 870, 232 is not divisible by 3.
3. Therefore, 870, 232 is not divisible by 6.

Answer: "The Method of Denying (the conclusion) – i.e., Modus Tollens.

$$[(p \to q) \land \neg q] \to \neg p$$

These so-called Rules of Inference constitute *primitive* (*atomic*) *valid argument forms.*

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- Recall that **validity** is predicated on deductive arguments (either *valid* or *invalid*) whereas **truth** is predicated on propositions (either *true* or *false*).

Atomicity

- An atomic entity is one which cannot be decomposed into individual constituents.
- A primitive proposition is atomic – you cannot decompose a statement, such as “5 is an integer, into more primitive declarative statements.
- Recall that **validity** is predicated on deductive arguments (either *valid* or *invalid*) whereas **Truth** is predicated on propositions (either *true* or *false*).
- **Question**: Are there *primitive valid argument forms*?

The Rules of Inference

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore\ q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore\ \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore\ p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore\ q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore\ p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore\ p$	$(p \land q) \to p$	Simplification
p q $\therefore\ p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore\ q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

The Rules of Inference

Modus Ponens ("putting" or "pushing"): If we know $p \rightarrow q$ and p, then we can conclude q. This, of course, follows from the basic meaning of " $\rightarrow$ ".

Syllogism: If $p \rightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$. (If we also know p, then we can now conclude r. This conclusion could also be reached by two steps of modus ponens. Thus there is some freedom of choice in constructing a multistep deduction.) Reasoning by syllogism is a special case of the following:

Conditionalization: If q can be proved from the *hypothesis* (assumption) p, then $p \rightarrow q$ is a theorem. (Grimaldi does not state this principle explicitly, but it frequently appears in the examples and exercises, often as an application of "syllogism" or "disjunctive syllogism". He usually avoids it by writing, e.g.,

$$\begin{array}{r} p \\ \underline{\quad q} \\ \therefore r \end{array}$$

instead of $p \wedge q \rightarrow r$ as the conclusion of a deduction.

The Rules of Inference

Modus Tollens ("taking" or "pulling back") **and Proof by Contradiction:** If we know $p \rightarrow q$ and $\neg q$, then we can conclude $\neg p$. A special case of this pulling back on an implication is the case where q is a logical contradiction (F_0) rather than merely factually false. (Grimaldi says that modus tollens and contradiction are essentially different; I disagree.)

Proof by Cases: If $p \rightarrow r$ and $q \rightarrow r$, then $(p \vee q) \rightarrow r$. So, if we can prove that *either* p or q is true (depending upon circumstances), then we can conclude r. (Example: Let r be "$n(n-1)$ is even", p be "n is even" and q be "n is odd". The conclusion (r) is correct in both cases, but for different reasons, so two separate proofs are needed, and the principle of proof by cases combines them at the end.)

The other rules, when needed, can be reconstructed by common sense from the meaning of the logical connectives.

Example

Provide a valid argument with premises $p \rightarrow q$, $\neg p \rightarrow r$, and $r \rightarrow s$ and conclusion $\neg q \rightarrow s$.

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	Step
	1. $p \to q$
	2. $\neg q \to \neg p$
	3. $\neg p \to r$
	4. $\neg q \to r$
	5. $r \to s$
$\therefore$	6. $\neg q \to s$

Example

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	Step	**Reason**
	1. $p \rightarrow q$	Premise
	2. $\neg q \rightarrow \neg p$	Contrapositive of (1)
	3. $\neg p \rightarrow r$	Premise
	4. $\neg q \rightarrow r$	Hypothetical syllogism using (2) and (3)
	5. $r \rightarrow s$	Premise
$\therefore$	6. $\neg q \rightarrow s$	Hypothetical syllogism using (4) and (5)

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	5. $r \rightarrow s$	Premise
$\therefore$	6. $\neg q \rightarrow s$	Hypothetical syllogism using (4) and (5)

Note that one can use both the Laws of Logic and The Rules of Inference.

Example 2.25: Grimaldi

$$\begin{array}{l} p \to r \\ r \to s \\ t \vee \neg s \\ \neg t \vee u \\ \underline{\neg u \qquad} \\ \therefore \neg p \end{array}$$

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$$\begin{array}{l} p \rightarrow r \\ r \rightarrow s \\ t \vee \neg s \\ \neg t \vee u \\ \neg u \\ \hline \therefore \neg p \end{array}$$

Steps

1) $p \rightarrow r, r \rightarrow s$
2) $p \rightarrow s$
3) $t \vee \neg s$
4) $\neg s \vee t$
5) $s \rightarrow t$
6) $p \rightarrow t$
7) $\neg t \vee u$
8) $t \rightarrow u$
9) $p \rightarrow u$
10) $\neg u$
11) $\therefore \neg p$

Example 2.25: Grimaldi

$$
\begin{array}{l}
p \rightarrow r \\
r \rightarrow s \\
t \vee \neg s \\
\neg t \vee u \\
\neg u \\
\hline
\therefore \neg p
\end{array}
$$

Steps	Reasons
1) $p \rightarrow r, r \rightarrow s$	Premises
2) $p \rightarrow s$	Step (1) and the Law of the Syllogism
3) $t \vee \neg s$	Premise
4) $\neg s \vee t$	Step (3) and the Commutative Law of $\vee$
5) $s \rightarrow t$	Step (4) and the fact that $\neg s \vee t \Leftrightarrow s \rightarrow t$
6) $p \rightarrow t$	Steps (2) and (5) and the Law of the Syllogism
7) $\neg t \vee u$	Premise
8) $t \rightarrow u$	Step (7) and the fact that $\neg t \vee u \Leftrightarrow t \rightarrow u$
9) $p \rightarrow u$	Steps (6) and (8) and the Law of the Syllogism
10) $\neg u$	Premise
11) $\therefore \neg p$	Steps (9) and (10) and Modus Tollens

Example 2.29: Grimaldi

$$
\begin{array}{l}
p \rightarrow r \\
\neg p \rightarrow q \\
q \rightarrow s \\
\hline
\therefore \neg r \rightarrow s
\end{array}
$$

Example 2.29: Grimaldi

Steps	Reasons
1) $p \to r$	Premise
2) $\neg r \to \neg p$	Step (1) and $p \to r \Leftrightarrow \neg r \to \neg p$
3) $\neg p \to q$	Premise
4) $\neg r \to q$	Steps (2) and (3) and the Law of the Syllogism
5) $q \to s$	Premise
6) $\therefore \neg r \to s$	Steps (4) and (5) and the Law of the Syllogism

$$\begin{array}{l} p \to r \\ \neg p \to q \\ q \to s \\ \hline \therefore \neg r \to s \end{array}$$

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3) $\neg p \rightarrow q$	Premise
4) $\neg r \rightarrow q$	Steps (2) and (3) and the Law of the Syllogism
5) $q \rightarrow s$	Premise
6) $\therefore \neg r \rightarrow s$	Steps (4) and (5) and the Law of the Syllogism

A second way to validate the given argument proceeds as follows.

Steps	Reasons
1) $p \rightarrow r$	Premise
2) $q \rightarrow s$	Premise
3) $\neg p \rightarrow q$	Premise
4) $p \vee q$	Step (3) and $(\neg p \rightarrow q) \Leftrightarrow (\neg\neg p \vee q) \Leftrightarrow (p \vee q)$, where the second logical equivalence follows by the Law of Double Negation
5) $r \vee s$	Steps (1), (2), and (4) and the Rule of the Constructive Dilemma
6) $\therefore \neg r \rightarrow s$	Step (5) and $(r \vee s) \Leftrightarrow (\neg\neg r \vee s) \Leftrightarrow (\neg r \rightarrow s)$, where the Law of Double Negation is used in the first logical equivalence

Example 2.29: Grimaldi

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Steps	Reasons
1) $p \rightarrow r$	Premise
2) $\neg r \rightarrow \neg p$	Step (1) and $p \rightarrow r \Leftrightarrow \neg r \rightarrow \neg p$
3) $\neg p \rightarrow q$	Premise
4) $\neg r \rightarrow q$	Steps (2) and (3) and the Law of the Syllogism
5) $q \rightarrow s$	Premise
6) $\therefore \neg r \rightarrow s$	Steps (4) and (5) and the Law of the Syllogism

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5) $r \vee s$	Steps (1), (2), and (4) and the Rule of the Constructive Dilemma
6) $\therefore \neg r \rightarrow s$	Step (5) and $(r \vee s) \Leftrightarrow (\neg\neg r \vee s) \Leftrightarrow (\neg r \rightarrow s)$, where the Law of Double Negation is used in the first logical equivalence

Note: **Constructive dilemma** states that if you have a disjunction p ∨ r (which means that at least one of the two disjuncts must be true), and if each disjunct implies another statement (p implies q, and r implies s), then at least one of the two consequents, q or s, must be true also.

Valid or Invalid Argument?

- **Premise 1:** Most of the green is touching the red.
- **Premise 2:** Most of the red is touching the blue.
- **Conclusion:** Most of the green must be touching blue.
- **Argument:** Since most of the green is touching red, and most of the red is touching blue, most of the green must be touching blue.

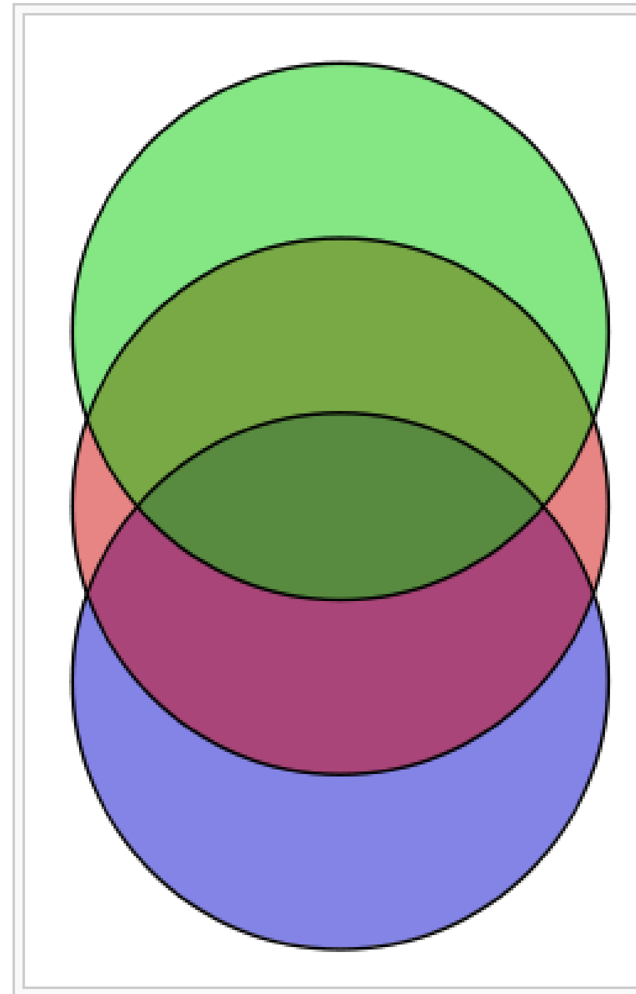
Valid or Invalid Argument?

$$\boldsymbol{p \wedge q \rightarrow r}$$

or, equivalently,

$$\underline{\boldsymbol{p \wedge q}}$$

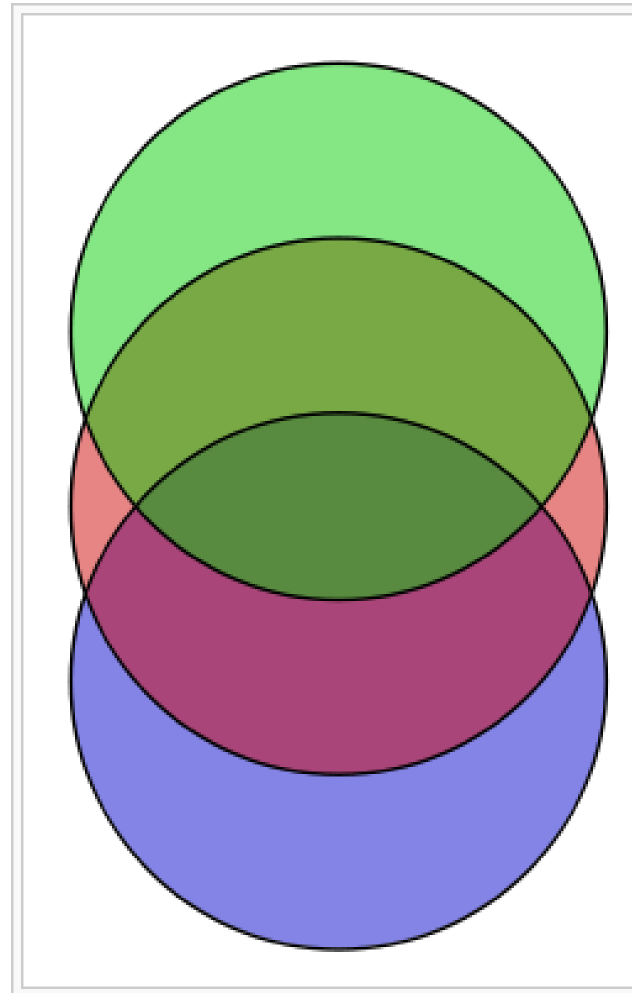
$$\therefore r$$



Example of possibilities

Answer: Invalid Argument

- **Premise 1:** Most of the green is touching the red.
- **Premise 2:** Most of the red is touching the blue.
- **Conclusion:** Most of the green must be touching blue.
- **Logical fallacy:** Since most of the green is touching red, and most of the red is touching blue, most of the green must be touching blue – a *false conclusion*.



Definition

Fallacy:

An error in reasoning that leads to an invalid argument.

i.e., using ambiguous premises, or assuming that which is to be proved (*begging the argument*), or jumping to unjustified conclusions, *etc.*

Formal Fallacies

- **Affirming a disjunct** – concluding that one disjunct of a logical disjunction must be false because the other disjunct is true; *A or B; A; therefore not B*.

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p	$\neg p$	q	$\neg q$	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$
0	1	0	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	0	1	0	1	1	1	1

Formal Fallacies

- **Affirming a disjunct** – concluding that one disjunct of a logical disjunction must be false because the other disjunct is true; *A or B; A; therefore not B*.
- **Affirming the consequent** – the hypothesis of a material conditional is claimed to be true because the conclusion is true; *if A, then B; B, therefore A*.
- **Denying the antecedent** – the conclusion in a material conditional is claimed to be false because the hypothesis is false; *if A, then B; not A, therefore not B*.

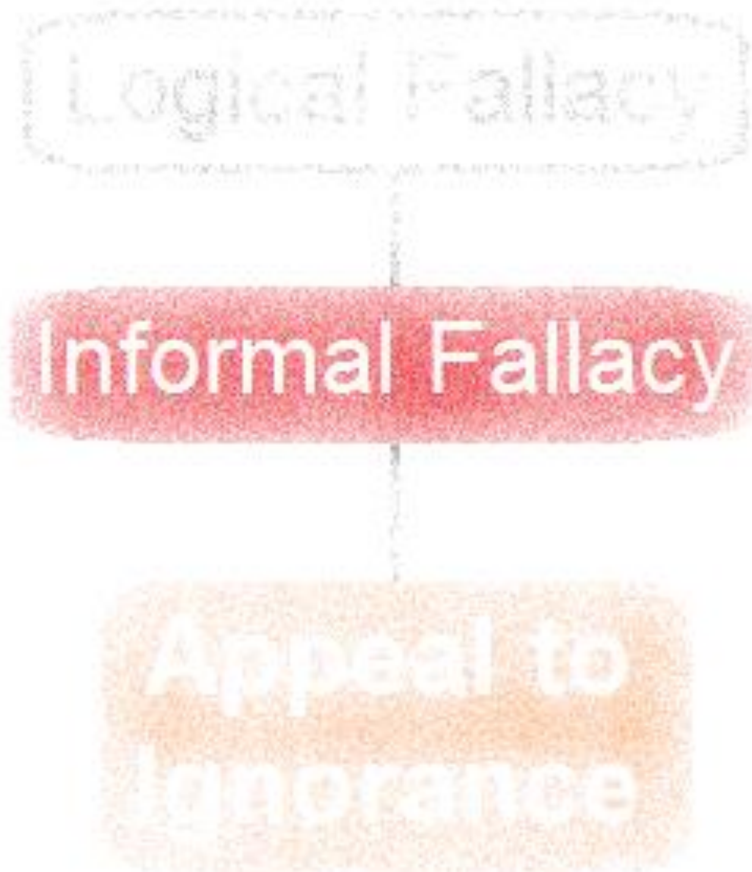
Formal Fallacies:
Recall from Elementary Logic PPT

p	$\neg p$	q	$\neg q$	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$
0	1	0	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	0	1	0	1	1	1	1

Clearly, $\boldsymbol{(p \to q) \Leftrightarrow (\neg q \to \neg p)}$ and $\boldsymbol{(q \to p) \Leftrightarrow (\neg p \to \neg q)}$

Terminology:
$(p \to q)$ is called material implication,
$(q \to p)$ is called the **CONVERSE** of material implication,
$(\neg q \to \neg p)$ is called the **CONTRAPOSITIVE** of material implication, and
$(\neg p \to \neg q)$ is called the **INVERSE** of material implication.

Informal Fallacies



See http://en.wikipedia.org/wiki/List_of_fallacies

Circular Reasoning

- The components of a circular argument are often logically valid because if the premises are true, the conclusion must be true.

Circular Reasoning

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e.g., Vancouver is in Canada. Therefore, Vancouver is in Canada.

Circular Reasoning

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- Circular logic cannot prove a conclusion because, if the conclusion is doubted, the premise which leads to it will also be doubted.
- Begging the question is a form of circular reasoning.

Begging the Question

- **Theorem:** n is an even integer whenever n^2 is an even integer.

Proof:

Suppose that n^2 is even. Then $n^2 = 2k$ for some integer k. Let $n = 2l$ for some integer, l. This shows that n is even.

QED

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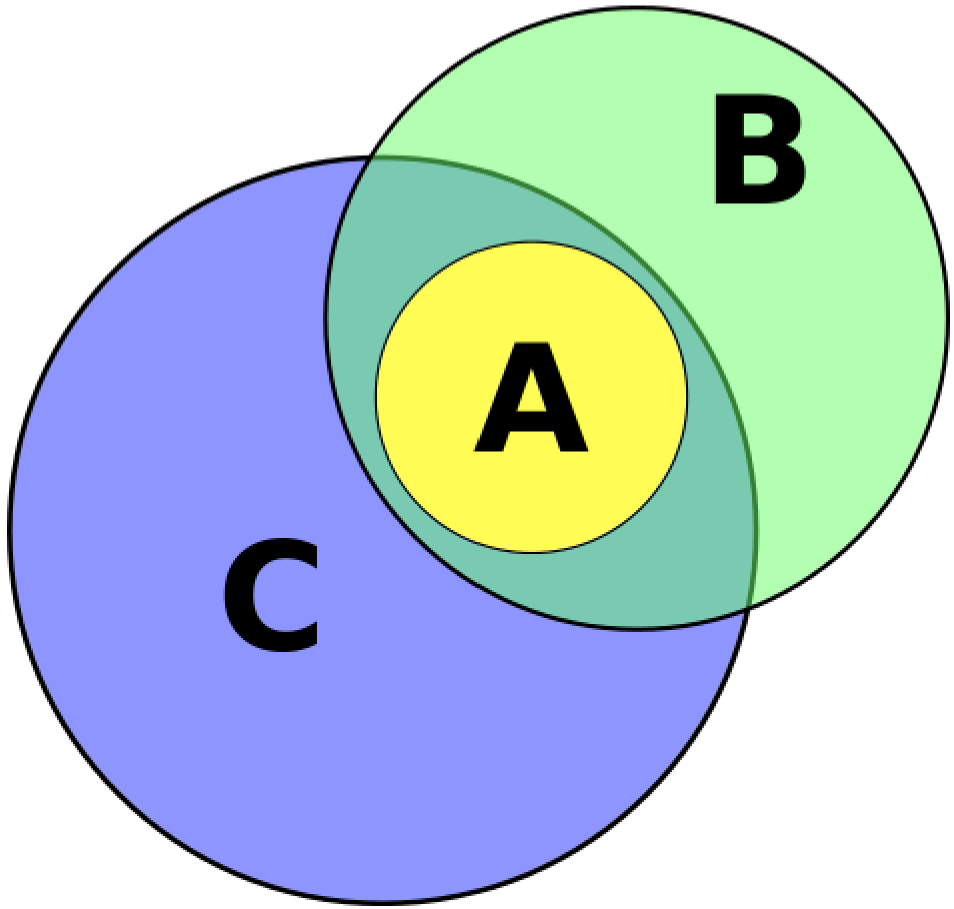
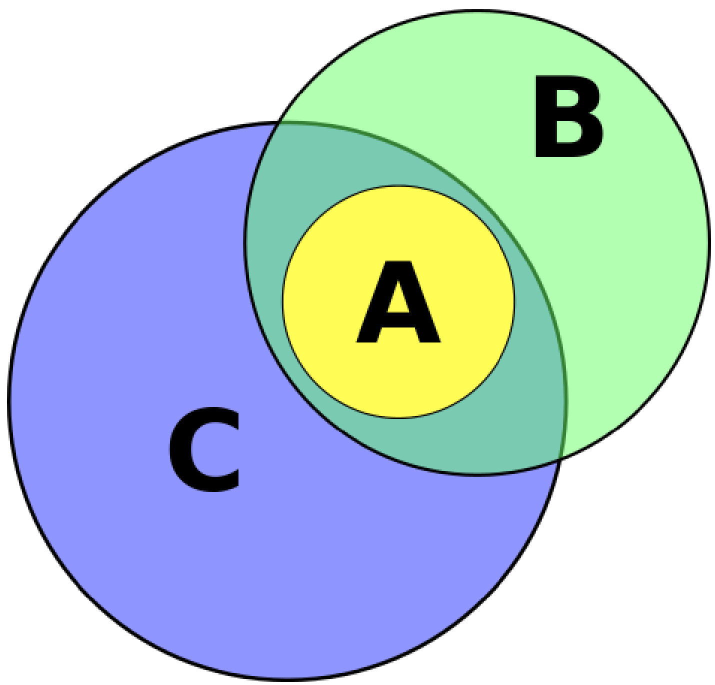
Solution: This argument is incorrect. The statement "let $n = 2l$ for some integer l" occurs in the proof. No argument has been given to show that n can be written as $2l$ for some integer l. This is circular reasoning because this statement is equivalent to the statement being proved, namely, "n is even." Of course, the result itself is correct; only the method of proof is wrong. ◀

Fallacy of Guilt By Association



Fallacy of Guilt By Association

- **Premise 1: A is a B**
- **Premise 2: A is also a C**
- **Conclusion: Therefore, all Bs are Cs**



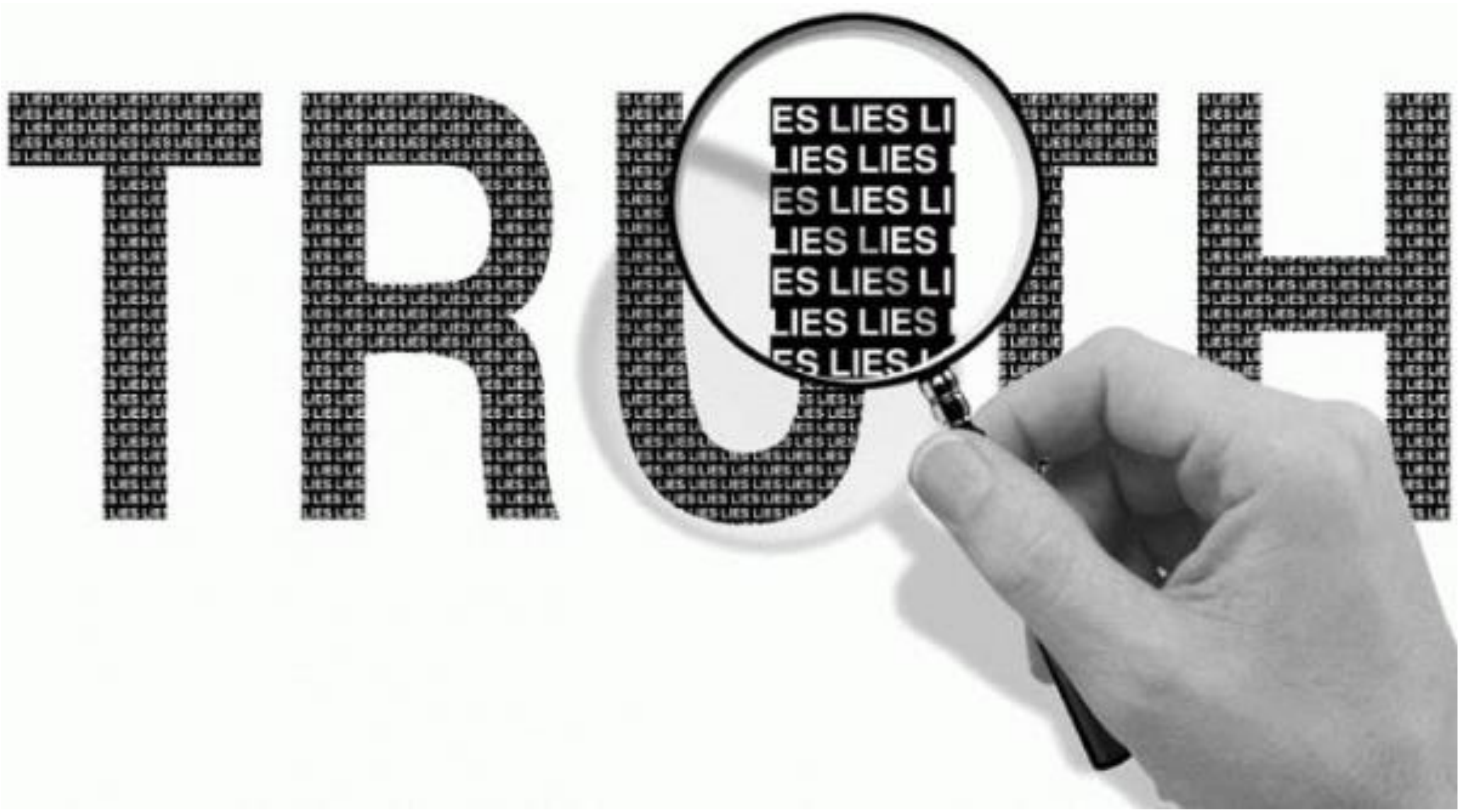
Fallacy of Guilt By Association

- **Premise 1:** A is a B
- **Premise 2:** A is also a C
- **Conclusion:** Therefore, all Bs are Cs

- **Example:**
  - **Bill, Karl, Jared, and Brett are all friends of *Josh*, and they are all petty criminals.**
  - **Jill is a friend of *Josh*;**
  - **Therefore, Jill is a petty criminal.**

Summary of Terms

Argument
Premise
Conclusion
Valid Argument (validity)
Conditional statements
Modus ponens
Modus tollens
Antecedent
Consequent
Denying the antecedent (fallacy of)
Affirming the consequent (fallacy of)



Presentation Terminated