1 Implication

$$\neg q \implies \neg p \tag{1}$$

p	q	$\neg p$	$\neg q$	$p \implies q$	$\neg q \implies \neg p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	0	0	1	1

(2)

$\therefore p \implies q$ is logically equivalent to $\neg q \implies \neg p$

So, $\neg q \implies \neg p$ is the **contrapositive** of $p \implies q$.
$p \implies q$ **material implication**
$q \implies p$ is called the **converse** of material implication.
$(\neg q \implies \neg p)$ is called the **contrapositive** of material implication, and
$(\neg p \implies \neg q)$ is called the **inverse** of material implication.

2 First Identity

$p \implies q$ means $p \therefore q$.

If the argument is valid and p is true, then q is also true.

If the argument is valid and p is false, then we can conclude nothing about q.

Thus, $p \implies q$ is logically equivalent to $\neg p \vee q$.

p	q	$\neg p$	$p \implies q$	$\neg p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

(3)

This is known as **implication reduction**.

> This is important because we *always* want to remove implication from our logical statements.

3 De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q \tag{4}$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \tag{5}$$

Many of the Axiomatic Laws come in pairs (principle of duality)

> Show the truth table (exercise)

Propositional Identities are given on the exam.

4 Principle of Duality

(exercise for the reader (???))