

# MACM 101 Lecture 1.1

Alexander Ng

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## 1 Chapter Summary

- Propositional Logic
  - The Language of Propositions
  - Applications
  - Logical Equivalences and Implication
  - The Laws of Propositional Logic
- Predicate Logic
  - The Language of Quantifiers
  - Nested Quantifiers
- Proofs
  - Rules of Inference
  - Proof Methods
  - Proof Strategy

This document covers everything from Rosen 1.0 to 1.3.

## 2 Definitions

### 2.1 Deduction/Deductive Logic

Deduction is the process of deriving a conclusion from a given set of axioms or premises. In Logic, we start from the ground (axioms) and work our way up to the conclusion.

## 2.2 Truth Value

A truth value can be either true or false, but not both. This comes from the *principium tertii esclusi* of Aristotle.

### 2.2.1 True and False

We will use 0 and 1 to denote true and false, respectively.

### 2.2.2 Unknown Truth Value

The proposition  $u$  is *unknown truth value*.

## 2.3 Proposition

A proposition is a declarative sentence (or statement) that possesses truth value.

### 2.3.1 Notation

Lowercase letters denote primitive propositions, and uppercase letters denote complex propositions.

Primitive propositions are:

- Propositions that cannot be decomposed into anything simpler
- $p : 3 + 5 = 8$
- $q : \text{It is raining}$

## 2.4 Examples of things that are not propositions

- $p : \text{Sit down!} \rightarrow$  not a proposition because it is not a declarative
- $q : \text{The statement you are reading is now false.} \rightarrow$  not a proposition because it is a contradiction.
- $r : \text{The number } x \text{ is an integer.} \rightarrow$  not a proposition because it contains an unspecified variable, which means its truth value cannot be definitively determined without additional information.

## 2.5 Syntactics and Semantics

Syntactic reasoning is what can be shown.

Syntax = grammar (rules of sentence construction), the structure of propositions

Semantics reasoning is what is true

Semantics = meaning (truth value), the truth value/tables of propositions

## 2.6 Literals

A *literal* is either a primitive proposition or its negation (some textbooks use to denote a literal)

## 3 Operator Syntax

### 1. Negation - $\neg$

$q$  : it is raining,  $\neg q$  : it is not raining

Everything in this list other than  $\neg$  is known as a *logical connective*

### 2. Conjunction - $\wedge$ - Logical and

$p \wedge q$  : it is raining and it is sunny

$p \wedge \neg q$  : it is raining and it is not sunny

### 3. Disjunction - $\vee$ - Inclusive Or

$p \vee q$  : it is raining or it is sunny

$p \vee \neg q$  : it is raining or it is not sunny

### 4. Disjunction - $\oplus$ - Exclusive Or

$p \oplus q$  : it is raining xor it is sunny

$p \oplus \neg q$  : it is raining xor it is not sunny

XOR is generally what is meant in english like "the meal comes with either soup or salad"

### 5. Implication - $\rightarrow$ - "If, then"

### 6. Biconditional - $\leftrightarrow$ - "If and only if"

Nobody knows why OR and XOR are both called Disjunction

All propositions formed with logical connectives are called *compound propositions*, as opposed to *primitive propositions*

Compound propositions need not have causal relations between atomic components (they can sound nonsensical and still be valid) – material implication as opposed to causal implication, which lacks temporal ordering. (straight from the slides, p. 34)

## 4 Semantics

TRUTH TABLES. that's basically all semantics is

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