Question 44

Prove or disprove that if A, B, and C are nonempty sets and $A \times B = A \times C$, then $B = C$.

Assume $B \neq C$.

This means $\exists x (x \in B \wedge x \notin C)$

However, because $A \times B$ is defined as $\{(a, b) | a \in A \wedge b \in B\}$, and $A \times C$ is defined as $\{(a, c) | a \in A \wedge c \in B\}$, if $\exists x (x \in B \wedge x \notin C)$, this implies $\exists x ((a, x) \in A \times B \wedge (a, x) \notin A \times C)$

$\therefore A \times B \neq A \times C$

This is a contradiction, thus $B = C$.

$\square$