Time Systems Relevant for GPS

The fundamental time scale is International Atomic Time (Temps Atomique International TAI) based on atomic clocks operated by various national agencies. At midnight on January 1, 1958 universal time and siderial times effectively ceased to function as time systems.

The theory of General Relativity implies that we have to consider the choice of an adequate inertial reference frame. For describing the equations of motion of an Earth satellite it is sufficient to use Terrestrial Dynamical Time (Temps Dynamique Terrestre TDT) which represents a uniform time scale for motion in the Earths gravity field. By definition it has the same rate as an atomic clock on Earth.

TAI is a continuous time scale related to the definition of TDT by the following definition:

$$\mathrm{TDT} = \mathrm{TAI} + 32.184\,\mathrm{sec}.$$

As mentioned the point of origin for this equation was established on January 1, 1958.

The fundamental interval unit of TAI is one SI second. The SI second was defined at the 13th general conference of the International Committee of Weights and Measures in 1967, as the »duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom«. The SI day is defined as 86 400 seconds and the Julian century as 36 525 days.

Since the apparent revolution of the sun about the Earth is non-uniform (this follows from Kepler's second law) a fictitious mean sun is defined which moves along the equator with uniform velocity. The hour angle of this fictitious sun is called universal time UT.

The time epoch denoted by the Julian date JD is expressed by a certain number of days and fraction of a day after a fundamental epoch sufficiently in the past to precede the historical record, chosen to be at 12^{h} UT on January 1, 4713 BC. The Julian day number denotes a day in this continuous count, or the length of time that has elapsed at 12^{h} UT on the day designated since this epoch. The JD of the standard epoch of UT is called J2000.0 where

$$\mathrm{J2000.0} = \mathrm{JD}\ 2\,451\,545.0 = 2000\ \text{January}\ 1.5^{\mathrm{d}}\ \mathrm{UT}.$$

The astronomic year commences at 0^{h} UT on December 31 of the previous year so that 2000 January 1.5^{d} UT = 2000 January 1 12^{h}. JD is a large number so often it is replaced by the *Modified Julian Date* MJD:

$$\boxed{\mathrm{MJD} = \mathrm{JD} - 2\,400\,000.5.}$$

Hence J2000.0 = MJD 51 544.5. Note a MJD starts at midnight.

Because TAI is a continous time scale, it does not maintain syncronization with the solar day since the Earth's rotation rate is slowing by an average of about 1 s per year. This problem is solved by defining Universal Time Coordinated UTC which runs at the same rate as TAI but is incremented by leap seconds periodically. Leap seconds are introduced by the IERS so that UTC does not vary from UT1 by more than 0.9 s. (IERS is an acronym for the International Earth Rotation Service. This service is also responsible for maintaining continuity with earlier data collected by optical instruments.) First preference is given to the end of June and December, and

Table 1: Date for introduction of leap seconds to be added to UTC to get GPS time

Total number of leap seconds	date introduced
1	30 June 1982
2	30 June 1983
3	30 June 1985
4	31 Dec. 1987
5	31 Dec. 1989
6	31 Dec. 1990
7	30 June 1992
8	30 June 1993
9	30 June 1994
10	31 Dec. 1995
11	30 June 1997
12	31 Dec. 1998
13	31 Dec. 1999

second preference to the end of March and September. DUT1 is the difference between UT1 − UTC broadcast with time signals to a precision of ± 0.1 s.

The time signals broadcast by the GPS satellites are syncronized with the atomic clock at the GPS Master Control Station in Colorado. Global Positioning System Time GPST was set to 0^{h} UTC on January 6, 1980 but is not incremented by UTC leap seconds. Therefore, there is an integer-second offset of 19 s between GPST and TAI such that

$$\text{GPST} + 19\,\text{s} = \text{TAI}.$$

In October 2001 there has been a total of 13 leap seconds since January 6, 1980 so that currently

$$\text{GPST} = \text{UTC} + 13\,\text{s}.$$

GPS week numbers and seconds of week Along with GPST, from the very begining was introduced the GPS week numbers. Since January 6, 1980 any week has been designated its own number. At the time of writing we have week number 1134. To identify a given epoch within the week, the concept of seconds of week is used. This number counts from midnight between Saturday and Sunday which is also the begining of the GPS week.

Furthermore for convenience the individual days of the week are numbered: Sunday 1, Monday 2, Tuesday 3, Wednesday 4, Thursday 5, Friday 6, and Saturday 7.

Professional GPS softwares use the day of week for numerical reasons. The seconds of week may be as large as $7 \times 24 \times 60 \times 60 = 604\,800$ s. In order to keep track of the mm in a point position we have to know time at the level of 0.01 nanosecond. Using seconds of week with twelve decimals is beyond the limits of most computers. So either you may split the real number holding the seconds of week into an integer part and a decimal part or you may calculate time in terms of GPS week number, day of week and seconds of day.

Reference Systems Relevant for GPS

The terrestrial reference system used by the U.S. Department of Defence for the GPS positioning is the World Geodetic System 1984 WGS 84. The GPS navigation message includes Earth-fixed satellite ephemerides expressed in this system.

The IERS terrestrial reference frame ITRF is defined by the adopted geocentric Cartesian coordinates and velocities of global tracking stations derived from the analysis of VLBI, SLR, and GPS data. The ITRF coordinates implicitly define the frame origin, reference direction and scale. The unit of length is the SI meter. The latest in a series of annual ITRF frames is ITRF96 with coordinates given at epoch 1996.0. Also included are station velocities computed by the IERS from a combination of the adopted NNR-NUVEL1 model and long-term space geodetic measurements. Annual refinements of the ITRF are to be expected at up to the 1 cm level in position and several mm/year in velocity, with a gradual increase in the number of defining stations (mainly GPS).

WGS 84 is a global geocentric coordinate system defined originally by DoD based on Doppler observations of the TRANSIT satellite system. WGS 84 was first determined by aligning as closely as possible, using a similarity transformation, the DoD reference frame NSWC-9Z2 and the BIH Conventional Terrestrial System—this is now

Table 2: WGS 84 Station Set G873: Cartesian Coordinates, 1997.0 Epoch. Coordinates are at the antenna electrical center. ΔE, ΔN, Δh are deltas between G730 and G873 coordinate sets, where E, N, and h represent the East, North and Ellipsoidal height components

Station Location	X m	Y m	Z m	ΔE mm	ΔN mm	Δh mm
Air Force Stations						
Colorado	−1 248 597.221	−4 819 433.246	3 976 500.193	1	13	33
Ascension	6 118 524.214	−1 572 350.829	−876 464.089	20	40	−11
Diego G., < 2 Mar 97	1 917 032.190	6 029 782.349	−801 376.113	−33	−85	52
Diego G., > 2 Mar 97	1 916 197.323	6 029 998.996	−801 737.517			
Kwajalein	−6 160 884.561	1 339 851.686	960 842.977	47	3	41
Hawaii	−5 511 982.282	−2 200 248.096	2 329 481.654	6	26	27
National Imagery and Mapping Agency Stations						
Australia	−3 939 181.976	3 467 075.383	−3 613 221.035	−62	−27	75
Argentina	2 745 499.094	−4 483 636.553	−3 599 054.668	−10	41	67
England	3 981 776.718	−89 239.153	4 965 284.609	88	71	11
Bahrain	3 633 910.911	4 425 277.706	2 799 862.677	−43	−48	−81
Ecuador	1 272 867.278	−6 252 772.267	−23 801.890	−20	25	107
Washington D.C.	1 112 168.441	−4 842 861.714	3 985 487.203	391	78	−37
China	−2 148 743 914	4 426 641.465	4 044 656.101	310	−81	−15

replaced by ITRF—at the epoch 1984.0. It was realized by the adobted coordinates of a globally distributed set of tracking stations with an accuracy of 1–2 m (compare to the 1–2 cm of ITRF).

In January 1987 the U.S. Defence Mapping Agency began using WGS 84 in their computation of precise ephemerides for the TRANSIT satellites. These ephemerides were used to point position—using Doppler tracking—the coordinates of ten DoD GPS monitoring stations. GPS tracking from these stations were used until recently to generate the GPS broadcast orbits, fixing the Doppler derived coordinates (tectonic plate motions were ignored).

Within the last several years, the coordinates for these DoD GPS stations have been refined twice, once in 1994 and again in 1997. The two sets of self-consistent GPS-realized coordinates (Terrestrial Reference Frames) derived to date have been designated: WGS 84 G730 and WGS 84 G873 where the »G« indicates these coordinates were obtained through GPS techniques and the number following the »G« indicates the GPS week number when these coordinates were implemented in the NIMA precise GPS ephemeris estimation process. Week 730 starts on January 2, 1994 and week 873 starts on September 30, 1996. The dates when these refined station coordinate sets were implemented in the GPS Operational Control Segment were: 29 June 1994 and 29 January 1997.

The most recent set of coordinates for these permanent DoD stations is provided in Table 2. The changes between the G730 and G873 coordinate sets are provided as well. Note that the most recent additions to the NIMA station network, the station located at the U. S. Naval Observatory in Washington D. C. and the station located near Beijing China, exhibit the largest change between the two coordinate sets. This result is due to the fact that these two stations were not part of the G730 general geodetic solution conducted in 1994.

Further improvements and future realizations of the WGS 84 reference frame are anticipated.

In addition, the original WGS 84 GM value was replaced by the IERS 1992 standard value of $3\,986\,004.418 \times 10^8 \mathrm{m}^3/\mathrm{s}^2$ in order to remove a 1.3 m bias in the DoD orbit fit. It is now estimated that the level of coincidence between ITRF94 and WGS 84 G873 is of the order of 10 cm.

Given that both the orbit-based and station-based WGS 84 to ITRF comparisons indicate in general, a lack of statistical significance in the estimated similarity parameters, readers are advised to avoid application of any transformation between these two

Table 3: WGS 84 G873 to ITRF94 Similarity Transformation Based on 17 Station Positions. Based on NIMA solitary point positioning of 17 IGS stations using the NIMA GASP algorithm

	ΔX	ΔY mm	ΔZ	ϵ	ψ milliarcsecond	ω	ΔL ppb
value	96	60	44	−2.2	−0.1	1.1	−14.3
σ	55	55	54	2.1	2.1	2.2	8.4

global reference frames. The standard deviation σ of the parameters shown in Table 3 indicate that *application of such a transformation is not warranted*. Application of a transformation to either of these station categories is not advised since the uncertainty at an individual station will not be reduced by the application of a similarity transformation.

Earth centered and Earth fixed coordinate system The theory of general relativity influences the GPS user in the way he chooses his coordinate system. In geodesy we are acoustomed to work in Earth fixed coordinate systems. In case of GPS we want to maintain that useful tradition. As the GPS is global it also is reasonable to perform our calculations in an Earth centered system—in short ECEF. One candidate is WGS 84.

According the ephemerides all satellite positions are calculated in WGS 84 which is an ECEF system. This implies that we must rotate the satellite position vector about the 3-axis an amount equal to the angular rotation of the Earth in the time it takes the signal to travel from the satellite to the receiver. The height of a GPS satellite is about 20 000 km. Thus the signal transit time is about 66 ms. The Earth rotates 15 arcsec/s so the angular displacement of the Earth about its rotation axis during signal travel is roughly 1 arcsec. So if ECEF coordinates are used and the correction is *not* applied, the recovered station coordinates will be biased by about one arcsec in longitude.

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Information on the International GPS Service can be found at

- `igscb.jpl.nasa.gov`