Subsidence Monitoring System Using Real-Time GPS Sensors

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BIOGRAPHIES

Kai Borre is a Professor of Geodesy at the Aalborg University since 1976. His recent software developments include a large collection of MATLAB files for postprocessing of GPS observations. In 1997 he coauthored the book *Linear Algebra, Geodesy, and GPS* with Gilbert Strang, Professor of Mathematics at the Massachusetts Institute of Technology. His current interests focus on filters for RTK applications. The present s/w development is one in a series of future applications.

Kees de Jong is an assistant professor at the Department of Mathematical Geodesy and Positioning of Delft University of Technology (DUT) in The Netherlands. Before joining DUT, he worked as a software engineer and consultant for a number of companies in The Netherlands and Japan. His main research interests are in GPS-based real-time positioning and quality control.

Christophe Pichot is in charge of special applications marketing and business development for Thales-Navigation in France. Since he joined this GPS manufacturer in 1989, he has been involved in the real-time technical aspects which he used for creating and developing a range of land survey products. His major task and interest are the search for particular application principles and algorithms.

ABSTRACT

Today GPS observables are so accurate that it makes sense to investigate if they can be used for subsidence monitoring at the mm-level.

This challenge, of course, asks for using sophisticated filters and data handling. Most systematic errors are eliminated to some extent by using relative positioning techniques.

The difficulties in ionospheric and tropospheric modeling set certain limits for the extension of the system: all baselines should be short and the height differences small.

The ultimate difficulty is to cope with multipath effects. We exploit the well-known repetition of the satellite constellation every 24 hours. Efficient filters model this error source down to sub-millimeter level. Hence we have a system that works at the mm-level within a 24 hours period and at the sub-mm level as time proceeds.

The software has both an initialization and a monitoring mode. The initialization period takes 24 hours, monitoring takes place in the 24 hour periods following the initialization by comparing the results from these periods with those from the initialization periods. Since the orbital repetition is not exactly 24 hours, the software determines the optimal repetition period. Also, it takes care that the same satellite configuration is used when comparing corresponding epochs.

The software runs under MS Windows (TM). The number of receivers for which data can be processed simultaneously is limited only by the capacity of the data links. The developed software builds on the Scorpio receiver from Thales-Navigation. We believe the patented method makes a break-through for a new precise application of GPS.

INTRODUCTION

For years we have had the idea to use GPS for subsidence monitoring of large constructions in real time. In recent years, receiver technology has developed tremendously. Today a phase measurement has a standard deviation of a few millimeters and other receiver related noise is even smaller.

If we by some means could handle orbit, atmospheric, and multipath errors then it should be within reach to achieve an accuracy at the millimeter level. This accuracy is interesting for a lot of static applications, such as monitoring of nuclear power plants, bridges, off-shore constructions, and geomorphology.

When using relative positioning techniques, the remaining error sources are residual ionospheric and tropospheric delays and multipath. Tropospheric delays can be modeled to within a few millimeters, or estimated, for example when station spacing or height differences become too large. Differential ionospheric delays limit the spacing between antennas. Without using a priori ionospheric information, we can support interstation distances up to a few kilometers.

Finally we introduce a methodology for handling the multipathing effect. The basic observation is that most GPS satellites make two revolutions every 24 siderial hours. For static receivers this means that the multipath error for each satellite can be mapped for a 24 hours period and used as a correction the following 24 hours.

Our system can include any number of antennas and we may define any baseline between them. If the baselines deviate from some a priori values, the system issues a visual alarm and calls up a predefined group of persons.

The data transfer from receivers to the processing center happens through modems and/or (fiber optic) cables, or local wireless telemetry links. All in all a variety of flexible solutions.

GPS MEASUREMENT MODELS

The basic observables considered here consist of single differences between receivers. These receivers are close enough to each other to assume that ionospheric effects are completely absent in the single differenced observations. Finally, it is assumed that both code (pseudo range) and carrier observations are available. These assumptions result in the following measurement models for two receivers that are tracking the same satellite s at epoch t at the GPS L1 and L2 frequencies denoted i

$$p_i^s(t_k) = \rho^s(t_k) + \delta_r(t_k) + \Delta T^s(t_k) + \eta_{p_i} + n_{p_i^s}(t_k)$$
$$\phi_i^s(t_k) = \rho^s(t_k) + \delta_r(t_k) + \Delta T^s(t_k) + \eta_{\phi_i} \qquad (1)$$
$$+ \lambda_i N_i^s + n_{\phi_i^s}(t_k)$$

where p^s and ϕ^s are the single differenced code and carrier observations, expressed in meters, ρ^s the unknown satellite-receiver single differenced range, δ_r the single differenced receiver clock error, ΔT^s the tropospheric effect, η the single differenced differential receiver delays, N^s the single differenced carrier ambiguities, λ the carrier wavelength and n_{p^s} and n_{ϕ^s} the single differenced measurement noise of code and carrier. Note that the above model is equally valid for the future GPS L5 frequency as well. The tropospheric effect may either be absent or it can be accounted for using the appropriate models.

The variance matrix $\Sigma_{p\phi}^s$ of the single differenced observations to a particular satellite s, is assumed to be given by

$$\Sigma_{p\phi}^s(t) = \text{diag}\left[\sigma_{p_1}^2(t) \quad \sigma_{p_2}^2(t) \quad \sigma_{\phi_1}^2(t) \quad \sigma_{\phi_2}^2(t)\right]. \qquad (2)$$

The argument t is included to reflect the dependence of the variances σ^2 on e.g. the satellite elevation.

The above model (1) forms the basis for our further analysis. This model, however, cannot be used directly as it is, since it is singular. In order to make it regular we have to apply a reparametrization. Another reason for a reparametrization is that (1) is expressed in terms of single differences. We would like to express the carrier ambiguities in terms of double difference ambiguities, since these can be resolved to their integer values, whereas the single difference ambiguities cannot. Therefore, assuming both receivers are tracking satellites s and r, and assuming satellite r is the reference satellite, we may write for a single differenced code and carrier observation at time t_k

$$p^s(t_k) = \rho^s(t_k) + \underbrace{\delta_r(t_k) + \eta_p}_{\delta_p(t_k)} + \Delta T^s(t_k) + n_{p^s}(t_k)$$
$$= \rho^s(t_k) + \delta_p(t_k) + \Delta T^s(t_k) + n_{p^s}(t_k)$$
$$\phi^s(t) = \rho^s(t_k) + \delta_r(t_k) + \eta_\phi + \Delta T^s(t_k)$$
$$+ \lambda(N^s - N^r + N^r) + n_{\phi^s}(t_k)$$
$$= \rho^s(t_k) + \underbrace{\delta_r(t_k) + \eta_\phi + \lambda N^r}_{\delta_\phi(t_k)} + \Delta T^s(t_k)$$
$$+ \lambda N^{rs} + n_{\phi^s}(t_k)$$
$$= \rho^s(t_k) + \delta_\phi(t_k) + \Delta T^s(t_k) + \lambda N^{rs} + n_{\phi^s}(t_k)$$
$$(3)$$

where N^{rs} is the double difference carrier ambiguity. Parameters $\delta_p(t_k)$ and $\delta_\phi(t_k)$ can be considered as receiver clock biases. They are the same for all satellites at time t_k. Substitution of (3) into (1) and applying the tropospheric correction using one of the many available models (or assuming it is zero) yields the measurement model, used in the remainder of this paper. For time t_k we get

$$p_i^s(t_k) = \rho^s(t_k) + \delta_{p_i}(t_k) + n_{p_i^s}(t_k)$$
$$\phi_i^s(t_k) = \rho^s(t_k) + \delta_{\phi_i}(t_k) + \lambda_i N_i^{rs} + n_{\phi_i^s}(t_k). \qquad (4)$$

In order to solve for the unknown station coordinates, these single differenced observation equations should be linearized with respect to the unknown parameters.

For kinematic applications we introduce a new baseline for each observation epoch. For a single satellite the linearized first order expression for the single-differenced $\rho(t_k)$ is given by

$$\Delta\rho(t_k) = \rho(t_k) - \left(\rho(t_k)\right)_0 = \left(\rho(t_k)\right)_0 + \frac{\partial\rho(t_k)}{\partial x}\bigg|_0 \Delta x(t_k)$$
$$+ \frac{\partial\rho(t_k)}{\partial y}\bigg|_0 \Delta y(t_k) + \frac{\partial\rho(t_k)}{\partial z}\bigg|_0 \Delta z(t_k) - \left(\rho(t_k)\right)_0$$
$$= \left[\frac{\partial\rho(t_k)}{\partial x} \quad \frac{\partial\rho(t_k)}{\partial x} \quad \frac{\partial\rho(t_k)}{\partial x}\right]_0 \begin{bmatrix} \Delta x(t_k) \\ \Delta y(t_k) \\ \Delta z(t_k) \end{bmatrix}$$
$$= \boldsymbol{u}(t_k)^{\text{T}} \Delta\boldsymbol{b}(t_k). \qquad (5)$$

By $\Delta \boldsymbol{b}(t_k)$ we denote the corrections to the initial baseline vector $\boldsymbol{b}(t_k)_0$ at epoch t_k, and $\boldsymbol{u}(t_k)$ is a 3-vector containing the partial derivatives of ρ with respect to the unknown coordinates and the subscript $_0$ indicates the single differenced range was computed at some initial value. Defining the *computed* single differences $p_i^s(t_k)_0$ and $\phi_i^s(t_k)_0$ as

$$\begin{aligned}
\left(p_i^s(t_k)\right)_0 &= \left(\rho^s(t_k) + \delta_{p_i}(t_k)\right)_0 \\
\left(\phi_i^s(t_k)\right)_0 &= \left(\rho^s(t_k) + \delta_{\phi_i}(t_k) + \lambda_i N_i^{rs}\right)_0
\end{aligned} \tag{6}$$

and substituting (6) into (4) yields

$$\begin{aligned}
p_i^s(t_k) - \left(p_i^s(t_k)\right)_0 &= \boldsymbol{u}^s(t_k)^{\mathrm{T}} \Delta \boldsymbol{b}(t_k) + \Delta \delta_{p_i}(t_k) + n_{p_i^s}(t_k) \\
\phi_i^s(t_k) - \left(\phi_i^s(t_k)\right)_0 &= \boldsymbol{u}^s(t_k)^{\mathrm{T}} \Delta \boldsymbol{b}(t_k) + \Delta \delta_{\phi_i}(t_k) \\
&\quad + \lambda_i \Delta N_i^{rs} + n_{\phi_i^s}(t_k).
\end{aligned} \tag{7}$$

In the sequel we assume that the two receivers are tracking m satellites and that, for notational convenience, the last satellite is the reference satellite. The complete linearized single difference measurement model for a single epoch is given in matrix-vector notation as

$$\begin{aligned}
\begin{bmatrix} \boldsymbol{p}_1 - \boldsymbol{p}_{1,0} \\ \boldsymbol{p}_2 - \boldsymbol{p}_{2,0} \\ \boldsymbol{\phi}_1 - \boldsymbol{\phi}_{1,0} \\ \boldsymbol{\phi}_2 - \boldsymbol{\phi}_{2,0} \end{bmatrix}_k &= \begin{bmatrix} U \\ U \\ U \\ U \end{bmatrix}_k \Delta \boldsymbol{b}_k + \begin{bmatrix} \boldsymbol{e}_m & & & \\ & \boldsymbol{e}_m & & \\ & & \boldsymbol{e}_m & \\ & & & \boldsymbol{e}_m \end{bmatrix} \\
&\times \begin{bmatrix} \Delta \delta_{p_1} \\ \Delta \delta_{p_2} \\ \Delta \delta_{\phi_1} \\ \Delta \delta_{\phi_2} \end{bmatrix} + \begin{bmatrix} \Theta & \Theta \\ \Theta & \Theta \\ \lambda_1 E & \Theta \\ \Theta & \lambda_2 E \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_{p_1} \\ \boldsymbol{n}_{p_2} \\ \boldsymbol{n}_{\phi_1} \\ \boldsymbol{n}_{\phi_2} \end{bmatrix}_k
\end{aligned} \tag{8}$$

where $\boldsymbol{e}_m$ is an m-vector, having all ones as its components, E an m by $m-1$ matrix, defined as $E = [\, I_{m-1} \quad \boldsymbol{0}^{\mathrm{T}} \,]^{\mathrm{T}}$ and $\boldsymbol{p}$, $\boldsymbol{\phi}$, N, $\boldsymbol{n}_p$, and $\boldsymbol{n}_\phi$ m-vectors. The m by 3 design matrix U is defined as $U = [\, \boldsymbol{u}^1 \ldots \boldsymbol{u}^m \,]^{\mathrm{T}}$. Finally Θ is an m by $m-1$ zero matrix.

Model (8) results in the so-called 'float' solution. Once the ambiguities have been resolved, they will no longer appear in the model (8). Due to the high precision of the carrier observations with respect to the code observations, the final ('fixed') solution will be governed mainly by the carrier observations. Thus, the measurement model for the 'fixed' solution reads

$$\begin{aligned}
\begin{bmatrix} \boldsymbol{\phi}_1 - \boldsymbol{\phi}_{1,0} - \lambda_1 E N_1 \\ \boldsymbol{\phi}_2 - \boldsymbol{\phi}_{2,0} - \lambda_2 E N_2 \end{bmatrix}_k &= \begin{bmatrix} U \\ U \end{bmatrix} \Delta \boldsymbol{b}_k \\
&\quad + \begin{bmatrix} \boldsymbol{e}_m & \\ & \boldsymbol{e}_m \end{bmatrix} \begin{bmatrix} \Delta \delta_{\phi_1} \\ \Delta \delta_{\phi_2} \end{bmatrix}_k + \begin{bmatrix} \boldsymbol{n}_{\phi_1} \\ \boldsymbol{n}_{\phi_2} \end{bmatrix}_k.
\end{aligned} \tag{9}$$

Δ-FILES

In order to compare GPS positions of corresponding epochs, i.e., epochs that are almost 24 hours (or multiples thereof) apart, in real-time, data from all observation epochs have to be stored. Since the satellites used to compute a solution at corresponding epochs may not be the same, effects due to e.g. multipath may not be the same in the corresponding positions. It is therefore not sufficient to store just the estimated positions. Instead, the following data has to be stored for each epoch:

- Time of observation

- Estimated position

- Satellite identifiers

- L1 and L2 carrier residuals for each satellite

- Partial derivatives with respect to the unknown coordinates for each satellite.

There is no need to store the coefficients of the clock parameters since they are constant. In addition, the stochastic model for the carrier observations should be stored. This has to be done only once. Finally, the satellite navigation messages have to be stored, in order to be able to determine the repetition period of the satellite orbits.

The above data is stored in binary Δ-files. Since the residuals are close to zero, they do not have to be stored as double-precision (8-byte) numbers. Also, for single differences, the partial derivates have the property that, for each satellite, the sum of their squares is equal to one. Therefore only the sign of the third partial derivative has to be stored. Again, it is not necessary to store the remaining derivatives with the highest possible precision. As a result, the size of the Δ-files can be reduced considerably.

COMPARING POSITIONS

As already mentioned, it only makes sense to compare positions when they are computed using the same satellite configuration. If this is not the case for the current epoch and the corresponding epoch from the Δ-file, at least one position should be recomputed. To accomplish this, only the carrier residuals and partial derivatives are required. Since the correction will be small, no iterations, and therefore no recomputation of the partial derivatives, are necessary.

For example, assume at the current epoch satellites 1, 2, 3, 4, 5, 6, 7, and 8 are used to compute a position, and at the corresponding epoch almost 24 hours ago, satellites 3, 4, 5, 6, 7, 8, and 9 were used. Then for both epochs a new position will be estimated using satellites 3, 4, 5, 6, 7, and 8, since these are the satellites both epochs have in common. Next, these positions, e.g., each coordinate or functions thereof, such as baseline length, are compared to find if a deformation has occurred.

EXPERIMENTS AND RESULTS

Thales-Navigation made initial test measurements in 1998. They demonstrated the potential of the presented method. August 1999 another test series was made at Aalborg University. This test series was processed in various ways to obtain insight into the nature of double differenced observations freed from multipath errors.

In August 2001 another test series was made at Aalborg University. The rover receiver was placed in a closed yard with possibly large multipath errors.

The multipath M for the C/A code is estimated as follows. The starting set of simplified undifferenced observation equations is

$$p_1 = \rho + I_1 + M$$
$$\phi_1 = \rho - I_1 + \lambda_1 N_1$$
$$\phi_2 = \rho - I_2 + \lambda_2 N_2.$$

These equations neglect possible multipath on phase observations. In fact we also assume that all noise is due to multipath! We introduce $\alpha = (f_1/f_2)^2 = I_2/I_1$ and arrive at

$$p_1 - \frac{\alpha+1}{\alpha-1}\phi_1 + \frac{2}{\alpha-1}\phi_2 = M - \frac{\alpha+1}{\alpha-1}\lambda_1 N_1 + \frac{2}{\alpha-1}\lambda_2 N_2. \quad (10)$$

The ambiguities N_1 and N_2 are unknown, but constant parameters. The multipath M is plotted in Figure 1. The data shown are for the time span 7 h 04 m to 7 h 36 m local time. Multipath for the following day, shifted -246 s, is plotted as well. The shift is estimated by a clever procedure. It is obvious that the variation in M is repeated from day to day for the given fixed site.

When subtracting multipath as computed from (10) between two consecutive days we generate a time independent correction series that can be used for future data. This series is shown in Figure 2.

In Figures 3 and 4 we demonstrate the noise level of the processed data for another baseline of 15 meters. Figure 3 includes the processed data for the first 20 hours after the initialization period, and Figure 4 includes the data from the first 2.5 days after the intialization period. From both figures it is evident that the environment in which the data were collected is quite good.

We computed the averages of

- instantaneous height differences based on the original satellite constellation: $m_o = -0.70$ mm

- instantaneous height differences based on the same satellite constellation: $m_s = 0.73$ mm

- averages of n consecutive, instantaneous height differences based on the original satellite constellation:
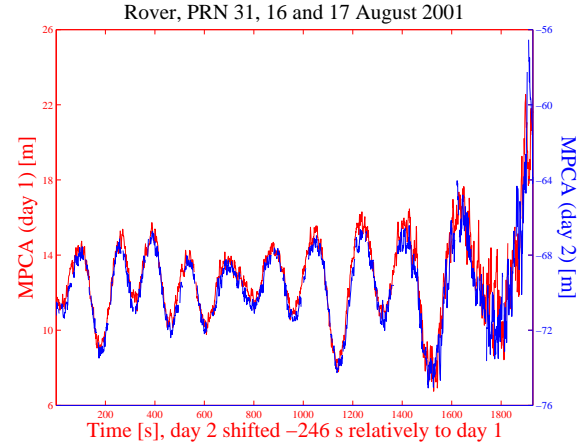

Figure 1: Multipath as derived from C/A code at the rover site. The plot shows multipath on two consecutive days, day 2 is shifted -246 s compared to day 1. There is a pronounced repeatability from one day to the next.
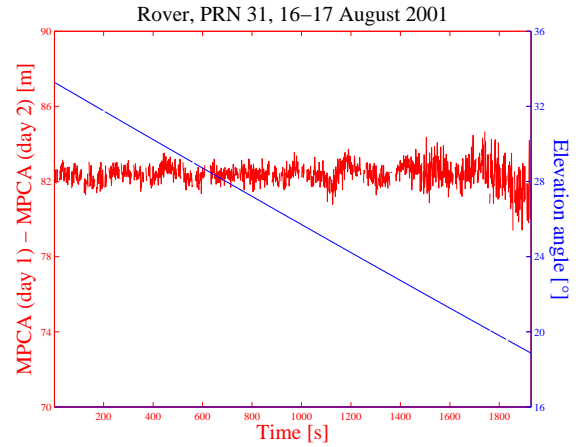

Figure 2: Difference of multipath between two consecutive days. The data for day 2 are shifted -246 s compared to day 1. The difference is reduced considerably in magnitude compared to the original values. The solid line shows the elevation angle of the satellite.

$m_a = 0.72$ mm. Normally $n = 30$, see Figures 3 and 4.

The corresponding standard deviations are

$$\sigma_o = 4.1 \text{ mm}, \qquad \sigma_s = 4.4 \text{ mm}, \qquad \sigma_a = 2.5 \text{ mm}.$$

Our conclusion is that there are hardly any biases in the data. This is very fortunate, but one may think: Why do we have to do all these difficult operations? The answer is given in the last section.

In order to test the whole concept under field conditions we arranged measurement campaigns in May 2001 at tectonic
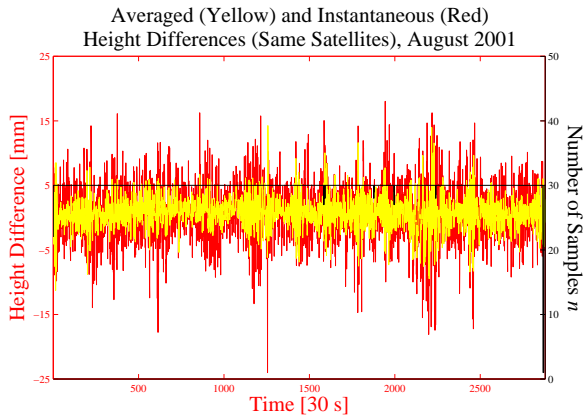
Figure 3: Differences in height difference of baseline as processed from the first 20 hours after the initialization period. Unit along the left vertical axis is meters, time runs along the horizontal axis, and the solid line shows the number of consecutive epochs used for computing the mean value. The number n is shown at the right vertical axis
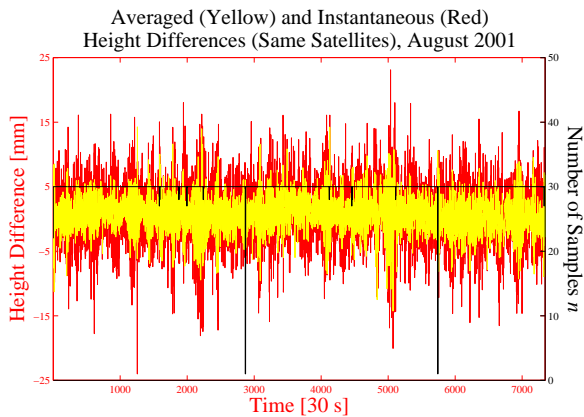

Figure 4: Differences in height differences of a baseline. The data show the period of 2.5 days after the initialization period. Unit along the left vertical axis is meters, time runs along the horizontal axis, and the solid line shows the number of consecutive epochs used for computing the mean value. The number n is shown at the right vertical axis

active sites in Europe. In cooperation with Dr. Giuseppe Cello, University of Camerino, Italy and his group we selected two sites near the city of Norcia. Two groups each of three Scorpio receivers collected data every second during May 2–7. The distance between the two point groups was 8 km and height difference ca. 150 meters.

During May 11–15 similar measurements were made close to the Triada fault in Patras, Greece. This site was planned in cooperation with Dr. George Drakatos and Dr. Nikolaos

Melis, National Observatory of Athens. Here the distance between point groups was some 700 meters and height differences within 10 meters.

The postprocessing of the Italian and Greek data is not yet finished.

DATA TRANSFER

Since the baseline processing is done at a central location, data (observations and navigation messages) have to be transferred from the receivers to this location. This can be done in a number of ways. It should be realized, however, that the basic communication channel between receiver and outside world is a serial (RS232 or RS422) port.

If the environment permits a wireless radio link may be used. In general, this means that each receiver has to use a dedicated frequency for transmitting its data. With the limited number of frequencies available nowadays, this may not be feasible, especially when the number of receivers becomes relatively large. An alternative would be to use multiplexing techniques, which allows different receivers to use the same frequency.

Another option would be to use modems and (mobile) telephone links for transferring data between receivers and processing facility. However, although prices are dropping, such links are still expensive when used 24 hours a day. A direct line between modems, connected to receiver and central computer, is also possible.

For short distances between receivers and processing center it is also possible to use an RS422 cable connection. The RS422 protocol allows data to be transferred over distances over one kilometer. However, it is important to realize that although the receiver may be within this radius from the processing center, the required cable length may be much longer. Fortunately, serial hubs are available nowadays, which combine the RS232/RS422 serial interface protocol with ethernet connectivity. In this way, receivers and computing center can be located in a local area network and there is no need to bother about the distances between them.

A number of data formats are supported by the software, ranging from the native formats of all major receiver brands to the RTCM SC-104 format.

FEATURES AND REQUIREMENTS

The features of the system are:

- Real-time system that is 'GPS L5 ready'

- Unlimited number of receivers and baselines (only limited by the capacity of the computer)

- All major GPS brands supported

- RTCM format supported

- Expert/layman graphical user interface (GUI)

- mm-level accuracy in real time

- Advanced atmospheric modelling for longer baselines

- Rigorous statistical testing procedures included

and the requirements are:

- At least dual-frequency code and carrier data

- Asyncronous data transmission using RS232/RS422 protocol

- 700 MHz processor and 10 GB hard disk.

DEVELOPMENT AND OUTLOOK

The presented system can be used as a standard monitoring system or as the advanced system described in this paper.

We expect the real-time software to be operational early 2002. It will include extended models for atmospheric delays to extend baseline lengths and height differences. At the moment we monitor the X, Y, and Z coordinates of the baselines; however, we want to include other functions of the coordinates, e.g. 2-D displacements, as well.