Dynamic Models

Constant speed, straight line

$$
\begin{aligned}
x_k &= x_{k-1} + v_{x,k-1}\Delta t \\
y_k &= y_{k-1} + v_{y,k-1}\Delta t \\
v_{x,k} &= v_{x,k-1} \\
v_{y,k} &= v_{y,k-1}
\end{aligned}
$$

Constant speed, straight line again

$$
\begin{aligned}
x_k &= x_{k-1} + v_{k-1}\sin\alpha_{k-1}\Delta t \\
y_k &= y_{k-1} + v_{k-1}\cos\alpha_{k-1}\Delta t \\
\alpha_k &= \alpha_{k-1} \\
v_k &= v_{k-1}
\end{aligned}
$$

Constant speed, circle

$$
\begin{aligned}
x_k &= x_{k-1} + v_{k-1}\sin\alpha_{k-1}\Delta t + \tfrac{1}{2}a_{cr,k-1}\cos\alpha_{k-1}\Delta t^2 \\
y_k &= y_{k-1} + v_{k-1}\cos\alpha_{k-1}\Delta t - \tfrac{1}{2}a_{cr,k-1}\sin\alpha_{k-1}\Delta t^2 \\
\alpha_k &= \alpha_{k-1} + a_{cr,k-1}/v_{k-1}\Delta t \\
v_k &= v_{k-1} \\
a_{cr,k} &= a_{cr,k-1}
\end{aligned}
$$

Tracking a GPS Satellite, 2D-Version

Observation equations

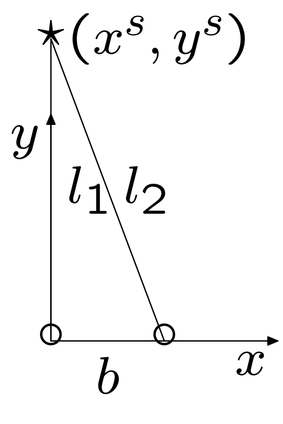
$$
\begin{aligned}
l_1 &= (y^s)^0 + \Delta y^s \\
l_2 &= \sqrt{b^2 + l^2} = \sqrt{b^2 + (l^2)^0} \\
&\qquad + \frac{b}{\sqrt{b^2 + (l^2)^0}} \Delta x^s + \frac{l}{\sqrt{b^2 + (l^2)^0}} \Delta y^s
\end{aligned}
$$

or

$$
\begin{aligned}
\Delta l_1 &= l_1 - (y^s)^0 = \Delta y^s \\
\Delta l_2 &= l_2 - \sqrt{b^2 + (l^2)^0} \\
&= \frac{b}{\sqrt{b^2 + (l^2)^0}} \Delta x^s + \frac{l}{\sqrt{b^2 + (l^2)^0}} \Delta y^s
\end{aligned}
$$

or in matrix form

$$
\begin{bmatrix} 0 & 1 \\ \frac{b}{\sqrt{b^2+(l^2)^0}} & \frac{l^0}{\sqrt{b^2+(l^2)^0}} \end{bmatrix} \begin{bmatrix} \Delta x^s \\ \Delta y^s \end{bmatrix} = \begin{bmatrix} \Delta l_1 \\ \Delta l_2 \end{bmatrix}.
$$



Tracking a GPS Satellite on L1 and L2

Observation equations

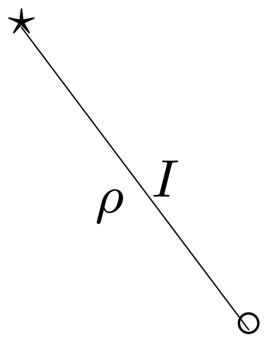
$$
\begin{aligned}
P_1 &= \rho + I \\
P_2 &= \rho + (f_1/f_2)^2 I
\end{aligned}
$$

or in matrix form

$$
\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & \alpha & 0 \end{bmatrix} \begin{bmatrix} \rho \\ \dot\rho \\ \ddot\rho \\ I \\ \dot I \end{bmatrix}.
$$

Dynamical model

$$
\begin{bmatrix} \rho_k \\ \dot\rho_k \\ \ddot\rho_k \\ I_k \\ \dot I_k \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \frac{1}{2}(\Delta t)^2 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho_{k-1} \\ \dot\rho_{k-1} \\ \ddot\rho_{k-1} \\ I_{k-1} \\ \dot I_{k-1} \end{bmatrix}.
$$



Quality Control for Recursive Solution

Innovation

$$e_k = b_k - A_k \widehat{x}_{k|k-1} \text{ with covariance matrix } \Sigma_{e,k}.$$

The local overall model test* for epoch k:

$$T = e_k^\top \Sigma_{e,k}^{-1} e_k \sim \chi^2.$$

Test for cycle slip: Let ∇ be the size of a (gross) error, and c_d and c_b denote (specified) vectors

$$\begin{aligned} d_{k,k-1} + c_d \nabla &= x_k - F_{k,k-1} x_{k-1} + \epsilon_k \\ b_k + c_b \nabla &= A_k x_k + e_k. \end{aligned}$$

Perform the tests

$$t_d = \frac{c_{d_k}^\top \Sigma_{d,k}^{-1} d_k}{\sqrt{c_{d_k}^\top \Sigma_{d,k}^{-1} c_{d_k}}} \sim N(0,1)$$

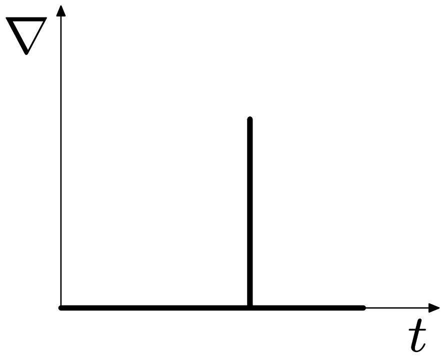
$$t_b = \frac{c_{b_k}^\top \Sigma_{b,k}^{-1} b_k}{\sqrt{c_{b_k}^\top \Sigma_{b,k}^{-1} c_{b_k}}} \sim N(0,1).$$

If $\widehat{\nabla}$ with variance σ_∇^2 and $\widehat{x}_{k|k}$ have been identified the filter is corrected as follows

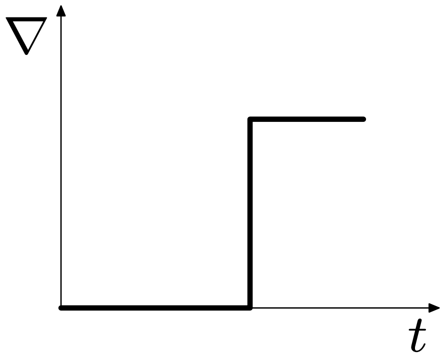
$$\begin{aligned} \widehat{x}_{k|k} &= \widehat{x}_{k|k} - K_k c_b \widehat{\nabla} \\ P_{k|k} &= P_{k|k} + K_k c_b \sigma_\nabla^2 c_b^\top K_k^\top. \end{aligned}$$

*local means for epoch k

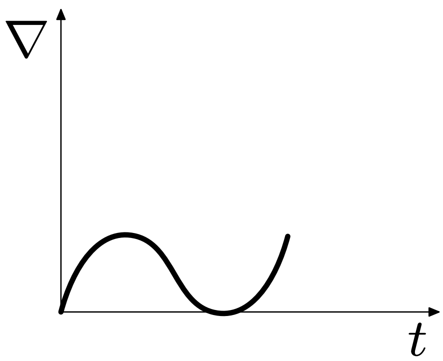
Error Types



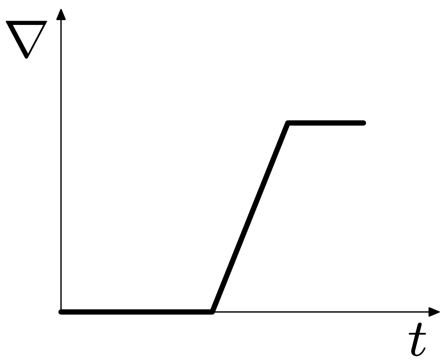
Outlier



Jump



Periodic



Ramp

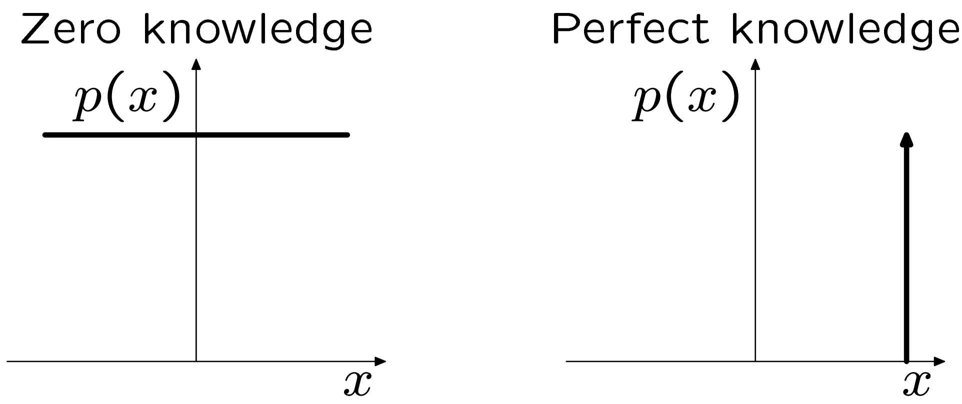
Filter Implementations

Covariance filters: P

- (Standard) Kalman Filter
- Square Root Filter
- U-D Decomposition

Information filters: P^{-1}

- Information Filter
- Square Root Information Filter (SRIF)



$P = \infty$ versus $P^{-1} = 0$ $\quad$ $P = 0$ versus $P^{-1} = \infty$