Exercises for Course: Elementary Geodesy

1. Find the length of Equator when $a = 6\,378\,137\,\mathrm{m}$.

2. Consider a levelling line observed from point A at latitude $\phi = 56^\circ$ to point B at latitude $\phi = 57^\circ$ both at elevation $0\,\mathrm{m}$. The points A and B are on the geoid; hence the potential difference is 0.

   Now increase the potential at latitude $\phi = 56^\circ$ by $C = 1\,000\,\mathrm{m}^2/\mathrm{s}^2$; and we arrive at point E. We do a similar thing at latitude $\phi = 57^\circ$ and arrive at point D. The potential difference along the closed curve $A - B - D - E$ is zero. When moving around this curve no work is performed: the field is *conservative*.

   However, if we convert the potential difference of $1\,000\,\mathrm{m}^2/\mathrm{s}^2$ to meters at the different latitudes we get different orthometric heights H!

   Compute the orthometric height of points E and D.

   Hints: $\gamma_0 = 9.780\,490\big(1 + 0.005\,288\,4 \sin^2\phi - 0.000\,005\,9 \sin^2(2\phi)\big)$. We have $f = 1/298.257\,223\,563$, and $m = \frac{(7\,292\,115 \cdot 10^{-11})^2 6\,378\,137^3}{3\,986\,004.418 \cdot 10^8}$, and $H = C/\gamma_h$ where $\gamma_h = \gamma_0\big(1 - (1 + f + m - 2f \sin^2\phi)H/a\big)$.

3. The datum ED 50 is based on the international ellipsoid of 1924 with $a = 6\,378\,388\,\mathrm{m}$. The datum WGS 84 has $a = 6\,378\,137\,\mathrm{m}$. Adding the $251\,\mathrm{m}$ to the semi-axis a, changes the position $\phi = 57^\circ$ North, and $\lambda = 10^\circ$ East. But loosely how?

4. The different position of ED 50 with respect to WGS 84 is decribed by the translational vector $t = (-87, -98, -121)$. Can you loosely describe the influence of t on the position $\phi = 57^\circ$ North, and $\lambda = 10^\circ$ East.

5. Find the length along a meridian from Equator to a point at latitude $\phi = 57^\circ$.

6. According to Figure 1 the latitude and the longitude components of the differential line element ds on the ellipsoid are $M\,d\phi$ and $N \cos\phi\, d\lambda$.

   $N \cos\phi\, d\lambda$

   $M\,d\phi$ ds

   Figure 1: Differential line element on the ellipsoid.

   Compute the metric length of a line element of $1''$ in the direction of the meridian and the parallel at points $(\phi, \lambda) = (0^\circ, 0^\circ)$ and $(57^\circ, 10^\circ)$.

7. Direct problem. Given a point with $\phi = 57^\circ$, $\lambda = 10^\circ$ and three different values of $s = 1000\,\mathrm{m}$, $10\,000\,\mathrm{m}$, and $30\,000\,\mathrm{m}$.

   Compute (ϕ_2, λ_2) and A_2 in all three cases. Hint: Use gauss_di.

8. Inverse problem. Given the results form Ex. 1: (ϕ_2, λ_2) and A_2.

   Find the distances S and azimuths between (ϕ_1, λ_1) and (ϕ_2, λ_2). Hint: Use gauss_in.

9. Given a point with $(\phi, \lambda, h) = (57°, 10°, 60\,\mathrm{m})$ in WGS 84. Find the Cartesian coordinates (X, Y, Z). Hint: Use geo2cart.

10. Given a point with the (X, Y, Z) from Ex. 3.

    Find the geographical coordinates of the point in WGS 84. Hint: Use cart2geo.

11. Given the same (X, Y, Z) as in Ex. 4, but compute the geographical coordinates now relative to the international ellipsoid 1924. Comment the result! Hint: Use cart2geo.

12. Find the changes in (ϕ, λ, h) at the point $\phi = 57°$, and $\lambda = 10°$ when we shift by $(t_x, t_y, t_z) = (-87, -98, -121)$, deminish a by 251 m $(da = -251)$ and change f by $df = 1/297 - 1/298.257223563$.

    Realize that this is the transformation from ED 50 to WGS 84. Hint: Use datumch.

13. Which changes do we experience for the point with coordinates $\phi = 56°$, and $\lambda = 10°$.

14. Convert the coordinates $X = 3429122.662$, $Y = 604646.845$, and $Z = 5325950.420$ into (N, E) in UTM, zone 32. Hint: Use cart2utm.

15. Convert $N = 6318036.28$, $E = 560828.13$ into geographical coordinates (ϕ, λ). Hint: Use utm2geo.

16. Convert $\phi = 57°$, $\lambda = 10°$ into (N, E) in UTM, zone 32. Hint: Use geo2utm.

17. Convert the UTM coordinates $N = 6317972.081$, and $E = 560749.622$ into geographical coordinates. Hint: Use utm2geo.