Extended Kalman Filter
and
Correlated Observation and Process Noise

Kai Borre

AALBORG UNIVERSITY

Extended Kalman filter

Estimation of GPS receiver position and phase ambiguities on L1 and L2

```
% error covariance for state
Q = diag([0.0001*ones(1,3) zeros(1,2*noprn)]);
P = 500*eye(3+2*noprn);          % covariance for state vector
R = 0.003^2*kron(eye(2),D*D');   % error covariance for observations
tt-loop
    b(tt,:) = Phi1-lambda1*X(3+tt,1); %+X(3+noprn+tt,1);
    b(noprn+tt,:) = Phi2-lambda2*X(3+noprn+tt,1);
    bk(tt,:) =  rhok_i-rhok_j-rhol_i+rhol_j;
    bk(noprn+tt,:) =  rhok_i-rhok_j-rhol_i+rhol_j;
 % Extended filter
        P = P+Q;
        K = P*A'*inv(A*P*A'+R);
        X = X+K*(b-bk);
end; %tt-loop
```

kalman_ext

The Kalman Filter With Correlated Process and Observation Noise

System equation Observation equation	$\boldsymbol{x}_k = F_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{\epsilon}_{k-1}; \qquad \boldsymbol{\epsilon}_k \sim N(\boldsymbol{0}, \Sigma_{\epsilon,k})$ $\boldsymbol{b}_k = A_k\boldsymbol{x}_k + \boldsymbol{e}_k; \qquad \boldsymbol{e}_k \sim N(\boldsymbol{0}, \Sigma_{e,k})$
Initial conditions Other conditions	$E\{\boldsymbol{x}_0\} = \boldsymbol{x}_0$ $E\{(\boldsymbol{x}_0 - \boldsymbol{x}_{0\|0})(\boldsymbol{x}_0 - \boldsymbol{x}_{0\|0})^{\mathrm{T}}\} = P_{0\|0}$ $E\{\boldsymbol{\epsilon}_k \boldsymbol{e}_j^{\mathrm{T}}\} = C_k, \quad$ for all k, j
Prediction of state vector Prediction of covariance matrix	$\boldsymbol{x}_{k\|k-1} = F_{k-1}\boldsymbol{x}_{k-1\|k-1}$ $P_{k\|k-1} = F_{k-1}P_{k-1\|k-1}F_{k-1}^{\mathrm{T}} + \Sigma_{\epsilon,k}$
Kalman gain matrix Updating of state vector Covariance matrix for updating	$K_k = (P_{k\|k-1}A_k^{\mathrm{T}} + C_k)(A_k P_{k\|k-1} A_k^{\mathrm{T}}$ $\qquad + \Sigma_{e,k} + A_k C_k + C_k^{\mathrm{T}} A_k^{\mathrm{T}})^{-1}$ $\boldsymbol{x}_{k\|k} = \boldsymbol{x}_{k\|k-1} + K_k(\boldsymbol{b}_k - A_k\boldsymbol{x}_{k\|k-1})$ $P_{k\|k} = (I - K_k A_k)P_{k\|k-1} - K_k C_k^{\mathrm{T}}$

AALBORG UNIVERSITY

Augmented State Vector x_k

Consider the usual model

$$x_k = F_{k-1}x_{k-1} + G_k\epsilon_k$$
$$b_k = A_k x_k + e_k.$$

In practice the process noise ϵ_k often is *correlated over time*. However, this can be handled correctly by an *augmentation of the state vector* x_k. Suppose ϵ_k can be split into correlated quantities $\epsilon_{1,k}$ and uncorrelated quantitites $\epsilon_{2,k}$:

$$\epsilon_k = \epsilon_{1,k} + \epsilon_{2,k}.$$

We assume that $\epsilon_{1,k}$ can be modeled as a difference equation

$$\epsilon_{1,k} = G_\epsilon \epsilon_{1,k-1} + \epsilon_{3,k-1}.$$

The augmented state vector x'_k is

$$x'_k = \begin{bmatrix} x_k \\ \epsilon_{1,k} \end{bmatrix}$$

and the augmented state equation, driven only by uncorrelated disturbances, is

$$x'_k = \begin{bmatrix} x_k \\ \epsilon_{1,k} \end{bmatrix} = \begin{bmatrix} F & G \\ 0 & G_\epsilon \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \epsilon_{1,k-1} \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \epsilon_{2,k-1} \\ \epsilon_{3,k-1} \end{bmatrix}.$$