The Three-Observation Combinations

In the following we deal with one-way observations between one receiver and one satellite. However, most results derived are valid for DD as well.

We assume that P_1, Φ_1, and Φ_2 observations are available. With $\alpha = (f_1/f_2)^2$ this can be collected in the matrix equation

$$\begin{bmatrix} P_1 \\ \Phi_1 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & \lambda_1 & 0 \\ 1 & -\alpha & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \rho^* \\ I \\ N_1 \\ N_2 \end{bmatrix}. \tag{1}$$

This system of equations has three observations and four unknowns. So there is a rank defect of one. This is handled by zeroing the ambiguities.

Subtracting row 2 from row 3 yields

$$\begin{aligned} \Phi_2 - \Phi_1 &= \rho^* - \alpha I + \lambda_2 N_2 - \rho^* + I - \lambda_1 N_1 \\ &= (1-\alpha)I + \lambda_2 N_2 - \lambda_1 N_1. \end{aligned}$$

The ambiguities N_1 and N_2 are constants and we set $N_1 = N_2 = 0$ and get an expression for the ***ionospheric delay*** I

$$\Phi_2 - \Phi_1 = (1-\alpha)I$$

or

$$I = \tfrac{1}{1-\alpha}(\Phi_2 - \Phi_1). \tag{2}$$

For a moment we investigate the ordinary four-observation combination

$$\begin{bmatrix} P_1 \\ \Phi_1 \\ P_2 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & \lambda_1 & 0 \\ 1 & \alpha & 0 & 0 \\ 1 & -\alpha & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \rho^* \\ I \\ N_1 \\ N_2 \end{bmatrix} = A\boldsymbol{x}. \tag{3}$$

We find that $\det(A) = (1-\alpha)\lambda_1\lambda_2$ and the inverse coefficient matrix is

$$A^{-1} = \tfrac{1}{1-\alpha} \begin{bmatrix} -\alpha & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ \frac{1+\alpha}{\lambda_1} & \frac{1-\alpha}{\lambda_1} & -\frac{2}{\lambda_1} & 0 \\ \frac{2\alpha}{\lambda_2} & 0 & \frac{1-\alpha}{\lambda_2} & \frac{1-\alpha}{\lambda_2} \end{bmatrix} \tag{4}$$

and (3) can be written as

$$\begin{bmatrix} \rho^* \\ I \\ N_1 \\ N_2 \end{bmatrix} = \frac{1}{1-\alpha} \begin{bmatrix} -\alpha & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ \frac{1+\alpha}{\lambda_1} & \frac{1-\alpha}{\lambda_1} & -\frac{2}{\lambda_1} & 0 \\ \frac{2\alpha}{\lambda_2} & 0 & \frac{1-\alpha}{\lambda_2} & \frac{1-\alpha}{\lambda_2} \end{bmatrix} \begin{bmatrix} P_1 \\ \Phi_1 \\ P_2 \\ \Phi_2 \end{bmatrix}. \tag{5}$$

The second row reads

$$I = \tfrac{1}{1-\alpha}(P_1 - P_2). \tag{6}$$

Hence (6) is an alternative expression for the ionospheric delay.

Multipath

Multipath M on pseudorange observations can be several meters while multipath on phase observations at most can be $\lambda_i/4$. So it makes sense to investigate the following model

$$P_1 = \rho^* + I + M_1 \tag{7}$$

$$\Phi_1 = \rho^* - I + \lambda_1 N_1 \tag{8}$$

$$\Phi_2 = \rho^* - \alpha I + \lambda_2 N_2. \tag{9}$$

We subtract (9) from (8):

$$\Phi_1 - \Phi_2 = -I + \alpha I + \lambda_1 N_1 - \lambda_2 N_2$$

and isolate I:

$$I = \tfrac{1}{1-\alpha}(\Phi_1 - \Phi_2) + \tfrac{1}{1-\alpha}\big(\lambda_1 N_1 - \lambda_2 N_2\big). \tag{10}$$

Subtracting (8) from (7) yields

$$I = (P_1 - \Phi_1 - M_1 + \lambda_1 N_1)/2. \tag{11}$$

Equating (10) and (11) gives

$$P_1 - \Phi_1 - M_1 + \lambda_1 N_1 = \tfrac{2}{\alpha-1}(\varphi_1 - \Phi_2) - \tfrac{2}{\alpha-1}\big(\lambda_1 N_1 - \lambda_2 N_2\big)$$

or

$$M_1 - \big(\tfrac{\alpha+1}{\alpha-1}\lambda_1 N_1 + \tfrac{2}{\alpha-1}\lambda_2 N_2\big) = P_1 - \tfrac{\alpha+1}{\alpha-1}\Phi_1 + \tfrac{2}{\alpha-1}\Phi_2.$$

The value of the paranthesis is constant over time; it is equalled to zero; the result is

$$M_1 = P_1 - \tfrac{\alpha+1}{\alpha-1}\Phi_1 + \tfrac{2}{\alpha-1}\Phi_2. \tag{12}$$