

Time Series Analysis of GPS Observables

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BIOGRAPHY

Kai Borre is a Professor of Geodesy at the Aalborg University since 1976. His recent software developments include a large collection of MATLAB files for postprocessing of GPS observations. In 1997 he coauthored the book *Linear Algebra, Geodesy, and GPS* with Gilbert Strang, Professor of Mathematics at the Massachusetts Institute of Technology.

Christian Tiberius holds a PhD in Geodesy from the Delft University of Technology, obtained in 1998 on recursive data processing for kinematic GPS surveying. Research interests are ambiguity resolution, stochastic modelling, and quality control. At present he is a lecturer in mathematical geodesy.

ABSTRACT

Today, GPS receivers for positioning and timing are capable to output range observations at high rates, typically once per second or even higher, for instance at 5 or 10 Hz. One is easily provided with dense and extensive time series of code and phase observations to all GPS satellites in view, on both the L1 and L2 frequency.

The observations' noise is commonly assumed to be white, i.e. that consecutive observations are not correlated. Neglecting however any severe time correlation leads to significant consequences on the results of the data processing, that is, on the quality of the parameters of interest.

Therefore in this contribution the time behaviour of the GPS observations' noise is explored using standard techniques from modern time series analysis. Two different mathematical models for processing are employed and data from zero baseline experiments are used. The analyses show that the noise may not always be independent between consecutive observables. In particular at higher sampling rates, time correlation does enter the scene. Correlation between neighbouring observations turns out to be sig-

nificant for 5 Hz data; the correlation coefficient may reach a value of 0.8 to 0.9. An attempt has been made to describe the time correlation by a first order auto-regressive noise process.

INTRODUCTION

A GPS observation carries information on the geometric range between satellite and receiver (position) and on the clocks in use (time). In order to extract the desired position and/or time information by processing the collected data using a least-squares algorithm, one has to formulate a mathematical model.

Most elementarily the GPS range observable can be split into a 'signal' part and a noise part:

$$\underline{p}(t) = \tilde{\rho}(t) + \underline{e}(t)$$

where the observable \underline{p} is a stochastic variable, that is time dependent $\underline{p}(t)$, and hence a stochastic process. The deterministic signal part $\tilde{\rho}(t)$ contains the geometric range and systematic effects as for instance signal delay terms. The composition of GPS satellites, signals, propagation media and receiver constitutes a dynamic measurement system. A more involved type of formulation, incorporating also the dynamics of the system, in terms of differential equations is given in e.g. Eissfeller (1997). In practice the above equation is considered instead at discrete epochs.

For the determination of position or time, in either a static or dynamic environment, observations are usually collected during a certain period of time. A careful modeling of both the functional and stochastic aspects of the GPS observables is necessary in order to obtain—via an adequate processing of these data—meaningful results for the position coordinates and estimates of time. In order to apply least-squares, the problem with observations and unknown parameters is cast in a mathematical model. The functional model should account for the information content, or the 'signal' $\tilde{\rho}(t)$ in the random observation process. After that,

only noise remains. For a lot of applications the functional part is usually reasonably well covered.

The least-squares principle requires also the specification of a weight matrix for the observations. The second order moment properties of the observables' noise can be specified in the observables' variance covariance (vc-) matrix. This matrix must be specified such that it reflects the (true) characteristics of the noise. When the inverse of this vc-matrix is taken as the least-squares weight matrix, least-squares is identical to BLUE: Best Linear Unbiased Estimation, yielding optimal estimators for the unknown parameters. Within this setting, the vc-matrix is generally referred to as the stochastic model.

For the stochastic model, it is commonly assumed that all observables have equal noise characteristics and that they do not change over time. The underlying assumption is that the noise $\underline{e}(t)$ is a *stationary* random process; its properties do not change over time t ; only the time difference $\tau = t_2 - t_1$ between two epochs of interest may play a role. A further common simplification is stating that the noise $\underline{e}(t)$ is a *white noise* process; the noise $\underline{e}(t_1)$ is not correlated with the noise at any other epoch $\underline{e}(t_2)$, with $t_1 \neq t_2$ (in a sequence of observations, two consecutive observations are uncorrelated).

In this paper we concentrate on this last simplification, and we will question its validity. In practice one might have a desire for white noise, as then the data processing is easy to implement. The weight matrix, or its inverse, the observations' vc-matrix, for all observations together, over all epochs, is block diagonal. When the noise turns out not to be white, first the actual process should be identified in order to capture it by an adequate model. The implementation of the data processing will be more involved, see for instance Howind & Kutterer & Heck (1999); they consider temporal correlation induced by remaining differential atmospheric delays. Similarly such correlation, as a component of what is referred to as physical correlation, showed up in the residual errors in El-Rabbany (1994).

We explore in this contribution the time behaviour of the GPS observations' noise, using standard techniques from modern time series analysis. By using data from zero baseline experiments, we restrict the attention to just the receiver, out of the full measurement system. Any time correlation is then likely to be induced by the receiver's tracking loops or internal data processing. An attempt has been made to describe the time correlation by a first order autoregressive noise process.

The present research was carried out to gain more insight in the noise characteristics of series of GPS observables, once the 'signal' component has been removed. The final aim is to be able to construct an adequate stochastic model for the processing of GPS data in precise applications.

TIME SERIES ANALYSIS

According to Priestley (1981), statistical inference implies estimating unknown quantities (parameters in probability distributions) from observational data. Thereby one usually restricts to the first two order moments. With time series analysis the primary object to be estimated from experimental data is the autocorrelation (or autocovariance) function; one concentrates on the second order properties of stationary random processes.

Time series

Let some phenomenon in the real world be of interest. One (or more) measurable characteristic(s) of the phenomenon is formalized as a random variable \underline{x} , or a random process $\underline{x}(t)$. A random process is a family of random variables indexed by the symbol t , usually time. Observations x , or x_1, \dots, x_N , are samples or realizations of this process.

In his desire to get control of the course of things, man has to gain knowledge of the process. In mathematical statistical sense, the process is fully described by the probability density function $f(x)$. The purpose of time series analysis thereby becomes to get to know, completely or partly this function $f(x)$. Inferences on the probability density function will be made on the basis of the observations, usually once a certain structure for the function $f(x)$ has been assumed a-priori.

The random variable, the observable, contains both 'signal' and noise. The first order moment of the observables is assumed to satisfy the functional model. In this signal part the parameters of interest are involved, usually intertwined with other parameters that are not of interest. The noise characteristics are to be captured by the second order moment. Noise is unavoidable, even if the system or phenomenon being observed would be deterministic, the observations are contaminated by errors of measurement Priestley (1981).

In this paper we maintain the usual notation of $\underline{x}(t)$ for the random process. In the context of the later analyses, one can think of $\underline{x}_1, \dots, \underline{x}_N$ as a sequence or series of least-squares residuals resulting from the adjustment of the observations. These residuals are estimates for the stochastic measurement errors, and have, under the working model, zero mean. They are the observables, 'corrected for' (with a best estimate for) the functional effect; the data are then detrended.

Stationarity and ergodicity

If the statistical properties of a process do not change over time, the random process is called stationary, confer Priestley (1981). Frequently used is stationary up to order 2,

which concerns the expectation and dispersion of the process. It is referred to as wide sense stationary, as opposed to (completely) stationary.

When a process is stationary, the samples in a certain (realized) record are identically distributed. For normally distributed variables, or a Gaussian process, wide sense stationarity (i.e. stationary in mean and variance) is identical to strict or full stationarity. Tiberius & Borre (1999) show that the Gaussian probability density function can be an adequate model for GPS data.

Incidentally (wide sense) stationarity of the time series has been checked in this research. The mean and standard deviation were found to be reasonably constant, except for low elevation satellites (roughly $< 30^\circ$). For satellites that are rising or setting, the standard deviation is de- or increasing, and this behaviour greatly varies for different receiver (and antenna) makes and types. Finally a series will obviously not be stationary in the mean, when a (clear) trend shows up (caused for instance by an unmodelled effect like multipath).

A stationary random process is also ergodic if every particular member of the ensemble is representative for all members, Priestley (1981). Thereby we are allowed, when estimating a certain parameter of the density function, to replace the ensemble average by the corresponding time average, obtained from the only one realization. Stationarity is a prerequisite for a process to be ergodic.

In general ergodicity of a random process can not be verified; it is (has to be) assumed. Ergodicity in relation with Gaussian random processes is discussed in section 5.3 of Bendat & Piersol (1986).

Autocovariance and autocorrelation

A key issue in time series analysis is the autocovariance function of a random process. Both the autocovariance function, and the closely related autocorrelation function are reviewed.

The (formal) autocovariance between $\underline{x}(t)$ and $\underline{x}(t + \tau)$ for a process $\underline{x}(t)$ is defined as, see Priestley (1981),

$$Q_{x_t x_{t+\tau}} = E\{(\underline{x}(t) - \mu)(\underline{x}(t + \tau) - \mu)\} \quad (1)$$

where μ is the expectation about zero, the first moment. We deal exclusively with *stationary* processes. The autocovariance $Q_{xx}(\tau)$ depends only on the time difference between instant $t + \tau$ and t . The difference τ is called the lag, alternatively the lag number when expressed as a multiple of the sampling interval T .

The autocorrelation function is defined as

$$\rho(\tau) = \frac{Q_{xx}(\tau)}{Q_{xx}(0)} \quad (2)$$

and represents the correlation coefficient between pairs of $\underline{x}(t)$ and $\underline{x}(t + \tau)$, separated by an interval of length τ . The autocovariance at lag zero

$$Q_{xx}(0) = \sigma^2$$

is the variance of \underline{x} , see section 3.3 in Priestley (1981). $\rho(\tau)$ might be referred to as the (auto) correlation coefficient function, or the normalized (auto) covariance function, see e.g. Bendat & Piersol (1986). The double subscript 'xx' will be left out.

Two standard (formal) random processes are the purely random noise process and the auto-regressive process of order 1, see e.g. also chapter 16 in Strang & Borre (1997).

A *purely random noise* process yields a sequence of uncorrelated (but not necessarily independent) random variables. The autocorrelation reads

$$\rho(\tau) = \begin{cases} 1 & \tau = 0 \\ 0 & \tau \neq 0. \end{cases}$$

A zero mean *first order auto-regressive noise* process, abbreviated to AR(1), is also called a linear Markov process (if it is Gaussian). The autocorrelation reads

$$\rho(\tau) = \beta^{|\tau|}.$$

The autocorrelation function decays to zero exponentially, when $\beta > 0$. We use $0 < \beta < 1$ (the constant β should satisfy $|\beta| < 1$ in order to let $\underline{x}(t)$ be (asymptotically) stationary up to (at least) order 2).

For an AR(1) noise process, the vc-matrix of $\underline{x}_1, \dots, \underline{x}_N$ is a full N by N matrix. It can be shown that when (zero mean) observables (or noise) are AR(1) correlated, an equivalent formulation can be given with uncorrelated variables. This is used to handle time correlation in recursive data processing (Kalman filtering, state vector augmentation), and yields the AR(1)-process as the first candidate to consider when time correlation is present in practice.

In the above discussion we did not distinguish explicitly between continuous and discrete time processes, the reader is referred to sections 3.5 through 3.7 of Priestley (1981).

Based on observations in discrete time, we will estimate the autocovariance and autocorrelation function. The samples are equidistant in time, separated by interval T . The experiment concerns a time span of $(N - 1)T$. For convenience of notation we take $T = 1$.

From the series x_1, \dots, x_N of a stationary process $\underline{x}(t)$ we can form $(N - \tau)$ pairs of observations $(x_t, x_{t+\tau})$, where the observations are separated by lag τ . As the estimate for the autocovariance function is taken

$$\hat{Q}(\tau) = \frac{1}{N} \sum_{t=1}^{N-|\tau|} (x_t - m_1)(x_{t+|\tau|} - m_1) \quad (3)$$

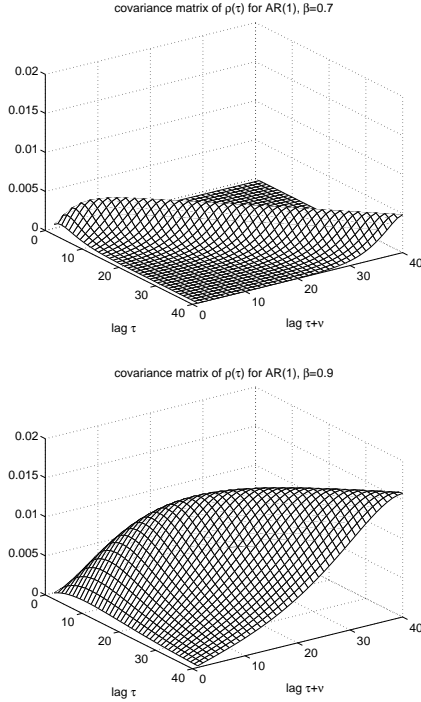


Figure 1: Covariance matrix, 40×40 , $\sigma_{\hat{\rho}(\tau)\hat{\rho}(\tau+\nu)}$ of estimators $\hat{\rho}(\tau)$ for the autocorrelation coefficients for an AR(1) noise process, with $\beta = 0.7$ (top) and $\beta = 0.9$ (bottom), $N = 600$. In the horizontal plane the diagonal runs from up left to down right. The estimators for different lags are obviously correlated.

with the divisor N rather than $N - |\tau|$, see the considerations in Priestley (1981), and m_1 the (empirical) mean.

The estimate of the autocorrelation function of a stationary process reads

$$\hat{\rho}(\tau) = \frac{\hat{Q}(\tau)}{\hat{Q}(0)} \quad \tau = 0, \pm 1, \pm 2, \dots, \pm(N-1) \quad (4)$$

and is also referred to as the sample autocorrelation function.

Expressions for (approximations to) the variance $\sigma_{\hat{\rho}(\tau)}^2$ and covariance $\sigma_{\hat{\rho}(\tau)\hat{\rho}(\tau+\nu)}$ of $\hat{\rho}(\tau)$, under the assumption that N is large and $\underline{x}(t)$ is Gaussian, are given in section 5.3 of Priestley (1981) (or section 2.1.6 of Box & Jenkins & Reinsel (1994)).

The variance and covariance as function of τ and ν are given in figure 1, for an example with $N = 600$ samples. The correlation coefficients for lag $\tau = 0$ are not stochastic by definition.

Priestley (1981) elaborates on the asymptotic distribution of the autocorrelations and proposes

$$\hat{\rho}(\tau) \sim N(\rho(\tau), \sigma_{\hat{\rho}(\tau)}^2)$$

By this, one can set a confidence interval on $\hat{\rho}(\tau)$ for a particular value of lag τ . Because of the correlation between consecutive estimators $\hat{\rho}(\tau)$ and $\hat{\rho}(\tau+\nu)$ we can in general not construct simultaneous confidence intervals for all the $\hat{\rho}(\tau)$ with $\tau = 0$ through $\tau = \pm(N-1)$.

For the purely random noise process, as $\sigma_{\hat{\rho}(\tau)\hat{\rho}(\tau+\nu)} \approx 0$, it is possible to test the sample autocorrelation function over a range of values of τ , see also chapter 4 in Chatfield (1989). This is sometimes referred to as a test for whiteness, and served as a main tool in our analyses. If whiteness was rejected, we compared the autocorrelation coefficients, as a first alternative, with those of an assumed AR(1)-process, which is still relatively easy to describe.

The estimated correlation coefficients are plotted in a graph as a function of lag τ . Such a graph is sometimes called a correlogram, see chapter 2 in Chatfield (1989). It is common practice to connect the coefficients by (straight) lines, although the actual process is discrete (sampled). In several texts, e.g. Childers (1997) and Chatfield (1989), it is recommended not to compute the autocovariance function for lags $|\tau|$ that exceed $N/10$ to $N/4$.

RESULTS

The techniques for time series analysis discussed in the previous section are now used to analyse the noise characteristics of GPS data. We try to infer whether the observation's noise can be considered to originate from a white noise process or from a first order auto-regressive process. The analysis is made on the least-squares residuals resulting from two types of adjustments.

Experiment

The data all result from a measurement campaign, carried out by the Department in Delft in spring 1999 (April 9–11). Each day the same time span of 1 hour (same satellite constellation) was used. For the analyses we restrict the time span to 10 minutes for the reason of stationarity of the time series. Changing circumstances (in particular the elevation of the satellite), may cause that the samples do not originate from a perfectly stationary random process. The standard deviation for instance, may still change slightly in the 10 minutes period, as the elevation may change by 3° or 4° .

The receivers used are geodetic dual frequency receivers; even under Anti Spoofing (AS) they provide code observations on both the L1 and L2 frequency (C1/P1 and P2) and phase observations on L1 and L2 (L1 and L2). The original observations are contained in Rinex files Gurtner (1994), and hence the random process has been already sampled and quantized; the observations are available, epochwise and numerically.

Parametrization in ranges

The model for processing the double difference data in this section, involves a parametrization in ranges from satellite to receiver, rather than in receiver baseline coordinates. This yields a simple linear model.

With dual frequency code and phase observations, C1-code, P2-code, L1-phase and L2-phase, respectively p , \bar{p} , P and \bar{P} , all in meters, we consider the following two variables, in terms of double differences

$$E\left\{\begin{pmatrix} (\frac{P}{\bar{P}} - \lambda_1 a) - \frac{p}{\bar{p}} \\ (\frac{P}{\bar{P}} - \lambda_2 \bar{a}) - \frac{p}{\bar{p}} \end{pmatrix}\right\} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$D\left\{\begin{pmatrix} (\frac{P}{\bar{P}} - \lambda_1 a) - \frac{p}{\bar{p}} \\ (\frac{P}{\bar{P}} - \lambda_2 \bar{a}) - \frac{p}{\bar{p}} \end{pmatrix}\right\} \approx \begin{pmatrix} \sigma_p^2 & \\ & \sigma_{\bar{p}}^2 \end{pmatrix}$$

as $\sigma_p \gg \sigma_P$ and $\sigma_{\bar{p}} \gg \sigma_{\bar{P}}$. λ_1 and λ_2 denote the L1 and L2 wavelengths, a and \bar{a} the (fixed integer double difference) carrier phase ambiguities, and the σ 's the standard deviations (of double difference observations). For a short baseline (or even a zero one) the differential ionospheric delay is absent. The tropospheric delay is also absent, although it could be accommodated for by the geometric range ρ , as it is non-dispersive. The above variables have expectation equal to zero. The first variable is referred to as the 'C1' or 'P1' combination, the latter as the 'P2' combination. With this model it is not directly possible to analyse the precise phase data.

We analysed data both from JPS Legacy and DSNP Scorpio 6002 receivers. The sampling rate was 5 Hz, so that the 10 minutes period yields a total of 3000 samples.

JPS receiver

Figure 2 presents the least-squares residuals for double difference combination PRN01-30. The C1- and P2-code are given; the P1-code time series is very much alike the one for the P2-code. The empirical mean m_1 and standard deviation s can be found in table 1.

Figure 3 gives, for double difference combination 04-08, the estimated autocorrelation function for the C1- and P1-code combination. Time correlation clearly is present, and slightly larger for the C1 than for the P1, the peak about zero is slightly wider. A first order auto-regressive process can provide an adequate modelling here. For the C1-code the β will be larger than 0.9 (close to 0.95), and for the P1 a very little smaller than 0.9.

Figure 4 gives the corresponding autocorrelation function for the P2-code combination. Their behaviour is very similar to that of the P1-code.

Figure 5 gives the difference of the sample autocorrelation function with an AR(1)-process, for which as a first 'try' $\beta = 0.9$ was taken. In general the difference varies around

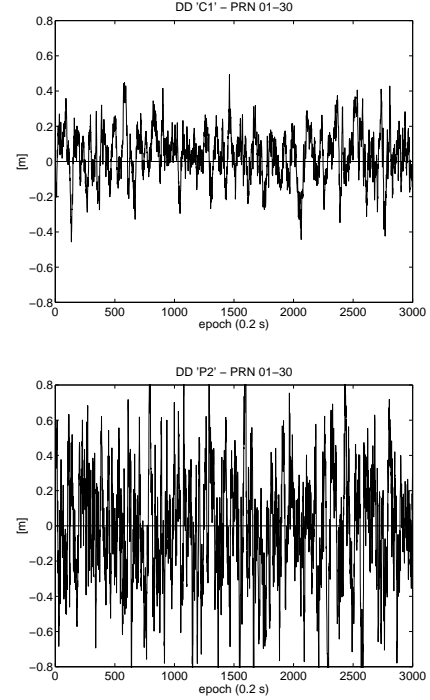


Figure 2: Time series of least-squares residual from parametrization-in-ranges model for 'C1'-code (top) and 'P2'-code (bottom) for satellites PRNs 01-30 of JPS receiver.

type	PRNs	m_1	s
C1	04-08	0.052	0.281
[m]	01-30	0.036	0.139
P1	04-08	0.073	1.043
[m]	01-30	-0.002	0.272
P2	04-08	-0.113	1.512
[m]	01-30	-0.017	0.323

Table 1: Empirical mean and standard deviation of (least-squares residual for) 'C1'-, 'P1'- and 'P2'-code combination in [m], for two selected satellite combinations; satellite PRN01 is at 35°, PRN04 at 20°, and PRNs 08 and 30 above 60° elevation; JPS receiver; $N = 3000$ samples.

zero, and the 'fit' is quite good. The dashed line gives the standard deviation $\sigma_{\hat{\rho}(\tau)}$ of the correlation coefficient estimator. It is repeated that for an AR(1) process, the correlation estimators $\hat{\rho}(\tau)$ are highly correlated for consecutive lags, see figure 1. One can therefore not judge the fit over the whole range of lags by comparing the solid line with the dashed one. This is also the reason for giving the thin dotted lines in figures 3 and 4 for only a small part on the right.

It turns out that the noise in the code observations of the JPS receiver can be modelled adequately by a first order auto-

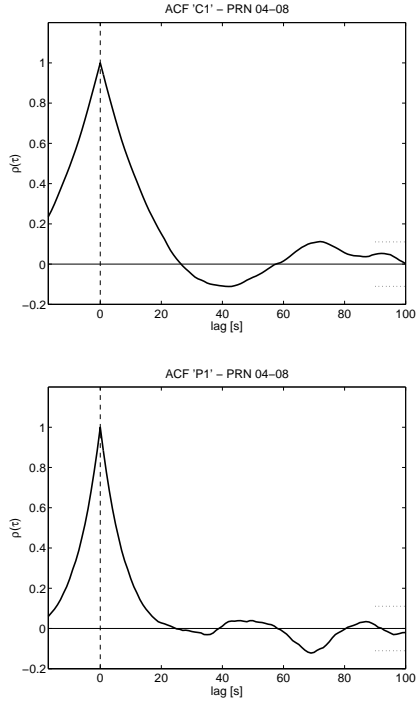


Figure 3: Sample autocorrelation coefficients for ‘C1’-code (top) and ‘P1’-code (bottom) for satellites PRN04-08 of JPS receiver. The thin dotted lines indicate the 95% confidence interval for an AR(1) process, computed with $\beta = 0.9$.

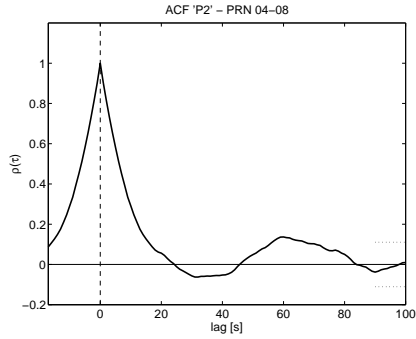


Figure 4: Sample autocorrelation coefficients for ‘P2’-code for satellites PRN04-08 of JPS receiver. The dotted lines indicate the 95% confidence interval for an AR(1) process, computed with $\beta = 0.9$.

regressive process. For these data the correlation between consecutive observations at 5 Hz sampling rate is severe. In Braasch & van Dierendonck (1999) it is discussed that, at high sampling rates, 5–10 Hz, a default (code) tracking loop bandwidth may not be sufficiently large to provide independent samples; this is a design compromise as enlarging the bandwidth in return yields noisier observations.

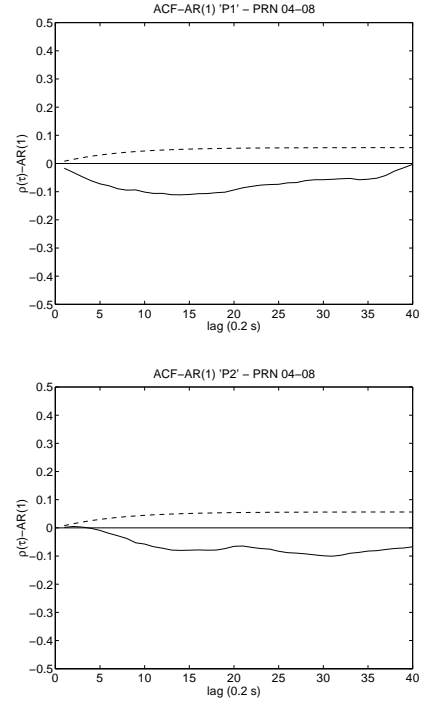


Figure 5: Difference of sample autocorrelation coefficient with theoretical first order auto-regressive process with $\beta = 0.9$ for ‘P1’-code (top) and ‘P2’-code (bottom) for satellites PRN04-08; JPS receiver. The dashed line indicates the standard deviation $\sigma_{\hat{\rho}}$, for $\beta = 0.9$, $N = 3000$ samples.

DSNP receiver

Figure 6 shows the least-squares residual for the C1-code combination and for the P2-code combination, for satellites 01 and 29 with the DSNP receivers. Note that the vertical axis ranges here from -2 to 2 meter. The empirical mean is -0.002 , 0.026 and -0.059 , and the standard deviation 0.794 , 0.578 and 0.814 (all in meter), for respectively the C1-, P1- (not shown) and P2-code combination.

Figure 7 gives the estimated autocorrelation coefficients and the difference with an AR(1)-process with parameter $\beta = 0.80$ for the C1-code combination and $\beta = 0.97$ for the P2-code combination. The coefficients for the ‘C1’-code well agree with the assumed AR(1)-process. For the ‘P2’-code it can be observed that the actual correlation is smaller than the assumed first order auto-regressive process in the middle range of lags, the difference is ‘temporarily’ below zero. This phenomenon shows up also for the ‘P1’-code, and is much more pronounced. The autocorrelation function starts with $\rho(\tau = 1) = 0.99$ (which implies really severe time correlation), but then, after a couple of epochs, starts to drop quicker than the corresponding first order auto-regressive process. The fit is generally poor for the ‘P1’- and ‘P2’-code.

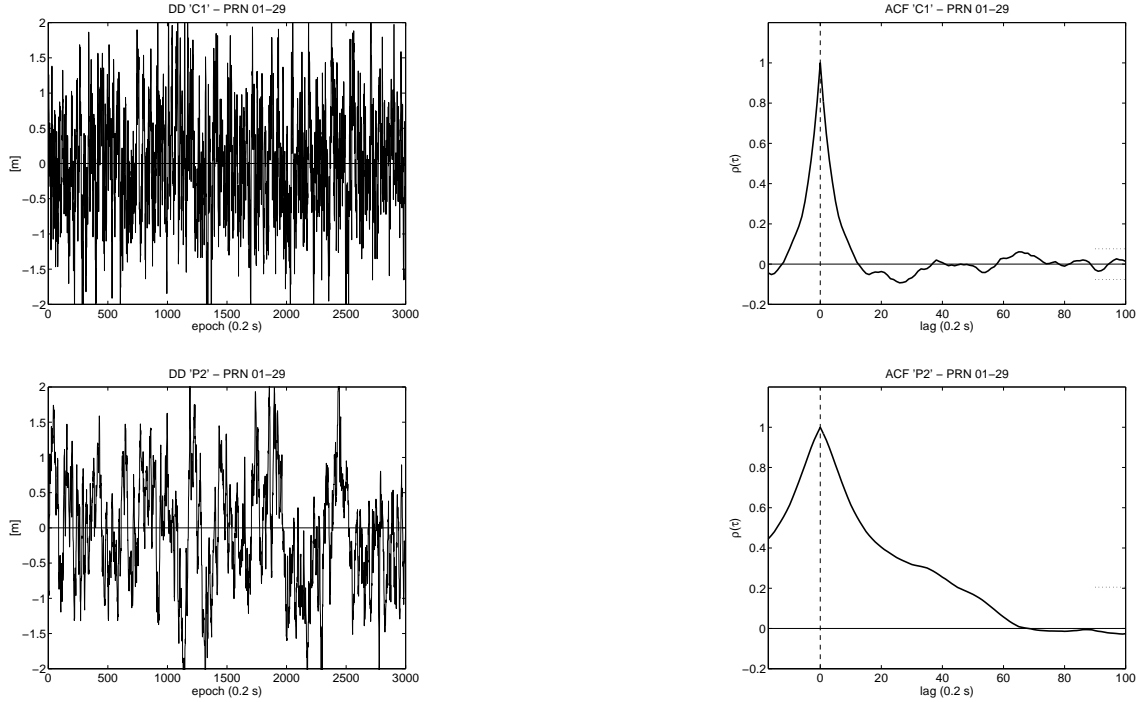


Figure 6: Time series of least-squares residual, for ‘C1’-code (top) and ‘P2’-code (bottom) for satellites PRN01-29, both at an about 35° elevation angle; DSNP receiver.

Parametrization in coordinates

In this section we analyse time series that result from a more or less ordinary adjustment for precise relative positioning. Two receivers are involved that simultaneously track m satellites. The second, or rover receiver is to be positioned with respect to the first, or reference receiver.

The geometric range is parametrized explicitly in (baseline) coordinates. The unknown parameters, in a model with single difference observations over short distances, are baseline coordinates, ambiguities in case of phase observations, and the (differential) receiver clock error. In this analysis, the baseline coordinates are precisely known (in this case of a zero baseline, even exactly) and the integer carrier phase ambiguities are known at a very high degree of certainty. These parameters are kept fixed in the adjustment: a so-called coordinates and ambiguities constrained solution.

Per epoch, per observation type, there remains only one unknown parameter, namely the receiver clock. All single difference observations (‘corrected’ for the known parameters) are related to this unknown parameter by the same coefficient. This model is then repeated for every epoch, with each time a new unknown clock error. Carrying out the constrained adjustment for every epoch, yields a series of least-squares residuals per satellite/channel.

Here we consider all four observation types, C1-code, P2-code, L1-phase and L2-phase, of data gathered with Trim-

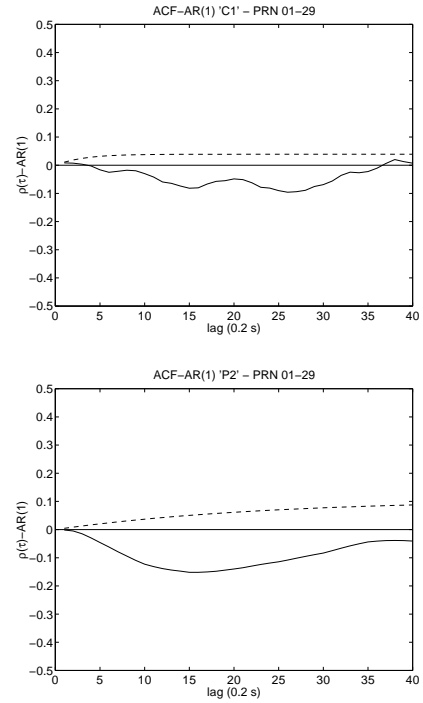


Figure 7: Sample autocorrelation function (top) and difference with theoretical first order auto-regressive process (bottom) for satellites PRN01-29; DSNP receiver; $\beta = 0.80$ for ‘C1’-code and $\beta = 0.97$ for ‘P2’-code. The dashed line indicates the standard deviation $\sigma_{\hat{\rho}}$, $N = 3000$ samples.

ble 4000 SSi receivers. With 10 minutes of data at a 1 second interval, there are $N = 600$ epochs, and thus N single

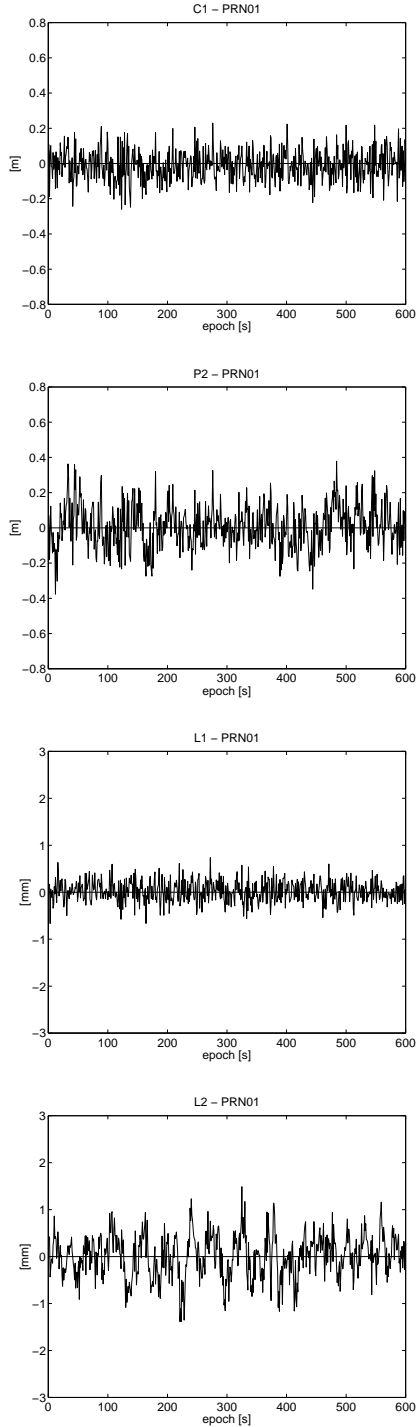


Figure 8: Time series of least-squares residual from baseline model for C1- and P2-code in [m] and L1- and L2-phase in [mm] for satellite PRN01 (at 35° elevation); Trimble receiver.

difference residuals in each time series per channel/satellite.

Figure 8 gives time series of the least-squares residual for satellite PRN01, for all four observation types; the vertical

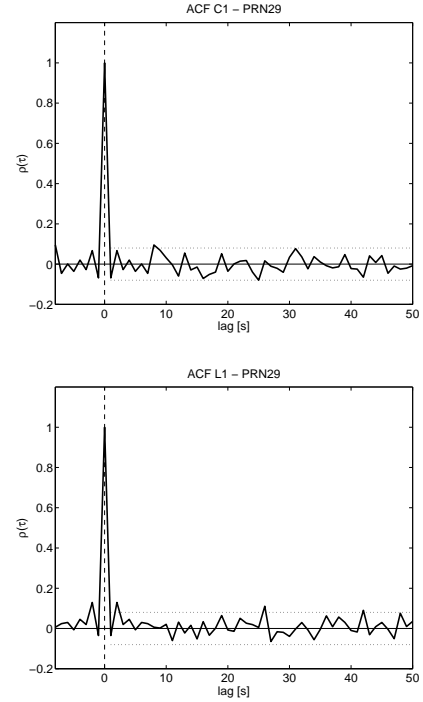


Figure 9: Sample autocorrelation functions for least-squares residuals of geometry-based model for L1-frequency observations: C1-code (top) and L1-phase (bottom) for satellite PRN29 of Trimble receiver; $N = 600$ samples.

type	PRN	m_1	s
C1	01	-0.011	0.087
[m]	29	-0.005	0.098
P2	01	0.008	0.123
[m]	29	-0.012	0.132
L1	01	0.022	0.224
[mm]	29	0.028	0.207
L2	01	0.017	0.456
[mm]	29	-0.059	0.401

Table 2: Empirical mean and standard deviation of least-squares residual of baseline model, for C1- and P2-code in [m], and for L1- and L2-phase in [mm], of Trimble receiver, for two selected satellites, PRN01, and 29, at medium elevation, $N = 600$ samples.

axis is in [m] for code and in [mm] for phase. Mean m_1 and standard deviation s are given in table 2. Concerning time correlation, a difference can be expected between the C1-code and L1-phase on one hand, and the P2-code and L2-phase on the other. For the estimated autocorrelation functions in figures 9 and 10, satellite PRN29 was selected at a medium elevation angle. The 95% confidence intervals are indicated in these graphs by dotted lines, based on a

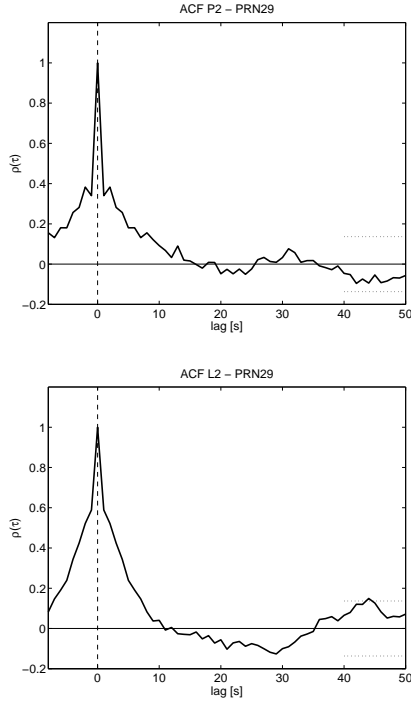


Figure 10: Sample autocorrelation functions for least-squares residuals of geometry-based model for L2-frequency observations: P2-code (top) and L2-phase (bottom) for satellite PRN29 of Trimble receiver; $N = 600$ samples.

white noise process for C1-code and L1-phase, figure 9, and based on an AR(1)-process with $\beta = 0.7$ for P2-code and L2-phase, figure 10.

The sample autocorrelation functions for the C1-code and L1-phase yield a strong evidence for the noise of these observations being white. The autocorrelation coefficients in figure 9 immediately drop off to about zero. This is confirmed by the numerical values. At lag $\tau = 1$, the correlation coefficient is generally smaller than 0.1; the departures from zero are not statistically significant.

The residuals of the observations on the second frequency show a clearly non-white noise process. Time correlation is present over some 10 seconds. From the graphs it can be seen, and this holds most severely for the P2-code, that the correlation first drops quickly from $\tau = 0$ to $\tau = 1$ (too quickly compared with an AR(1) process), and next decreases (too) slowly. The P2-code autocorrelation function typically starts as an AR(1)-process with about $\beta = 0.4$, but ends with $\beta = 0.6$ to 0.7 . Analysing the numerical values, and taking into account the variance of the estimator $\hat{\rho}(\tau)$, see figure 1, it turns out that an AR(1)-process does *not* fit well to these data. The problem seems less severe for the L2-phase data, starting with $\beta = 0.6$ and changing to over $\beta = 0.7$, but they show an ‘overshoot’ at later lags (from lag $\tau = 10$ to about 30, the correlation is negative),

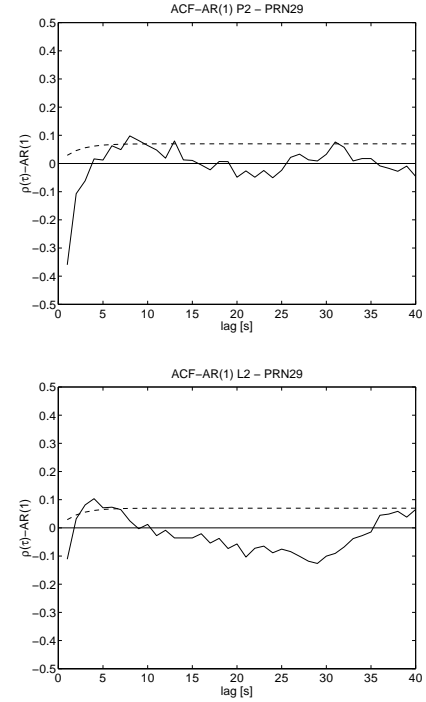


Figure 11: Difference of sample autocorrelation coefficient with theoretical first order auto-regressive process with $\beta = 0.7$ for P2-code (top) and L2-phase (bottom) for satellite PRN29; Trimble receiver. The dashed line indicates the standard deviation $\sigma_{\hat{\rho}}$ (for $\beta = 0.7$, $N = 600$ samples).

and this can not be accommodated by an AR(1)-process. The overshoot is however not clearly significant.

Figure 11 summarizes the conclusions by explicitly showing the differences of the estimated correlation coefficients with a first order auto-regressive process, for the lags $\tau = 1$ through 40, for the satellite in figure 10.

SUMMARY

Data from three different receivers have been analysed, using two different models for processing. The mean of the least-squares residual was usually found to be close to zero; departures from zero were in general not significant. Using the parametrization in ranges, the data, observed at a high sampling rate (5 Hz), showed a severe time correlation, on all three codes C1, P1 and P2, for both receiver-pairs considered. A first order auto-regressive process turned out to provide an adequate modelling for the JPS receiver. The agreement was actually very good. This holds also for the C1-code of the DSNP receiver, but not for the P1- and P2-codes.

In the last section, the common baseline model was used to process the data at a 1 second sampling interval. The coordinates and ambiguities were constrained. It can be

concluded that the receiver analysed (Trimble) gave white noise observations on the L1-frequency. For both the C1-code and L1-phase, the autocorrelation coefficient drops off very quickly. After 1 second they are generally below 0.1. A (discrete) white noise process turns out to really exist in practice. The L2-frequency observations were time correlated, over some 10 seconds, but the AR(1) process does not seem to provide the ultimate model. In particular the L2-phase observations show some overshoot, the correlation is negative over a certain range of lags, and this can not be accommodated by an AR(1) process.

CONCLUDING REMARKS

In this contribution time series analysis was applied to GPS data, in order to gain insight into the noise characteristics and to build and improve on mathematical models for the processing of observed GPS data.

We concentrated on analysis of the data in the time domain, rather than the frequency domain. Attention was focussed on the second order moment of the probability density function, and in particular on the time aspect of this moment by the autocorrelation function. The full distribution of the GPS observables—whether it is Gaussian or not—was not considered here.

From a practical point of view it is often desired that the noise in GPS code and phase observables originates from a white noise process, i.e. samples are purely random from one (discrete) epoch to the next. It was shown that a GPS receiver is capable indeed of providing observations at a once per second rate without any time correlation. In many cases however, and in particular at higher sampling rates, time correlation does enter the scene. Correlation between consecutive observations turns out to be significant for the 5 Hz data analysed; the correlation coefficient lies in the order of 0.8–0.9. As the data analysed resulted from zero baseline experiments, the correlation found, most likely is due to the receiver; the signal processing by the receiver invokes a dependence in a series of observations.

Time correlation implies that concerning the dispersion of the observables, covariances between epochs need to be introduced. A first attempt was made to capture the time correlation by a mathematical model, namely by a first order auto-regressive process. The agreement between the data and this model was found to be poor for one receiver make, but good for another.

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