Augmented State Vector x_k

Consider the usual model

$$\begin{aligned} x_k &= F_{k-1}x_{k-1} + G_k\epsilon_k \\ b_k &= A_k x_k + e_k. \end{aligned}$$

In practice the process noise ϵ_k often is *correlated over time*. However, this can be handled correctly by an *augmentation of the state vector* x_k. Suppose ϵ_k can be split into correlated quantities $\epsilon_{1,k}$ and uncorrelated quantitites $\epsilon_{2,k}$:

$$\epsilon_k = \epsilon_{1,k} + \epsilon_{2,k}.$$

We assume that $\epsilon_{1,k}$ can be modeled as a difference equation

$$\epsilon_{1,k} = G_\epsilon \epsilon_{1,k-1} + \epsilon_{3,k-1}.$$

The augmented state vector x'_k is

$$x'_k = \begin{bmatrix} x_k \\ \epsilon_{1,k} \end{bmatrix}$$

and the augmented state equation, driven only by uncorrelated disturbances, is

$$x'_k = \begin{bmatrix} x_k \\ \epsilon_{1,k} \end{bmatrix} = \begin{bmatrix} F & G \\ 0 & G_\epsilon \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \epsilon_{1,k-1} \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \epsilon_{2,k-1} \\ \epsilon_{3,k-1} \end{bmatrix}.$$