Bessel's Solution of the Direct Geodetic Problem

1 $\tan\beta_1 = (1 - f)\tan\varphi_1$

2 $\tan\sigma_1 = \dfrac{\tan\beta_1}{\cos\alpha_1}$

3 $\cos\beta_n = \cos\beta_1 \sin\alpha_1$

4 (1) $t = \frac{1}{4}e'^2 \sin^2\beta_n$

(2) $K_1 = 1 + t\Big(1 - \frac{1}{4}t(3 - t(5 - 11t))\Big)$

(3) $K_2 = t\Big(1 - t(2 - \frac{1}{8}t(37 - 94t))\Big)$

5 (1) $v = \frac{1}{4}f\sin^2\beta_n$

(2) $K_3 = v\Big(1 + f + f^2 - v(3 + 7f - 13v)\Big)$

6 (1) $\sigma = \frac{S}{K_1 b} + \Delta\sigma$

(2) $\sigma_m = 2\sigma_1 + \sigma$

7 $$\Delta\sigma = K_2 \sin\sigma\Big(\cos\sigma_m + \tfrac{1}{4}K_2(\cos\sigma\cos 2\sigma_m + \tfrac{1}{6}K_2(1+2\cos 2\sigma)\cos 3\sigma_m)\Big)$$

Steps **6** and **7** are iterated until the change in $\Delta\sigma$ becomes less than the limit value given in advance. Initially we set $\Delta\sigma = 0$.

8 (1) $$\tan\beta_2 = \frac{\sin\beta_1\cos\sigma + \cos\beta_1\sin\sigma\cos\alpha_1}{\sqrt{1-\sin^2\beta_n\sin^2(\sigma_1+\sigma)}}$$

(2) $$\tan\varphi_2 = \frac{\tan\beta_2}{1-f}$$

9 $$\Delta\omega = (1-K_3)f\cos\beta_n\Big(\sigma + K_3\sin\sigma(\cos\sigma_m + K_3\cos\sigma\cos 2\sigma_m)\Big)$$

10 (1) $$\tan\omega = \frac{\sin\sigma\sin\alpha_1}{\cos\beta_1\cos\sigma - \sin\beta_1\sin\sigma\cos\alpha_1}$$

(2) $$\lambda_2 = \lambda_1 + \omega - \Delta\omega$$

11 $$\tan\alpha_2 = \frac{\cos\beta_1\sin\alpha_1}{\cos\beta_1\cos\sigma\cos\alpha_1 - \sin\beta_1\sin\sigma}$$

Bessel's Solution of the Inverse Geodetic Problem

1 (1) $\tan\beta_1 = (1-f)\tan\varphi_1$

(2) $\tan\beta_2 = (1-f)\tan\varphi_2$

$\Delta\omega = 0$

2 $\omega = \lambda_2 - \lambda_1 + \Delta\omega$

3 $\tan\sigma =$

$$\frac{\sqrt{(\cos\beta_2\sin\omega)^2+(\cos\beta_1\sin\beta_2-\sin\beta_1\cos\beta_2\cos\omega)^2}}{\sin\beta_1\sin\beta_2+\cos\beta_1\cos\beta_2\cos\omega}$$

4 $\cos\beta_n = \dfrac{\cos\beta_1\cos\beta_2\sin\omega}{\sin\sigma}$

5 $\cos\sigma_m = \cos\sigma - \dfrac{2\sin\beta_1\sin\beta_2}{\sin^2\beta_n}$

6 Two equations are the same as step 5 in direct.

7 The same as step 9 in direct.

The procedure is iterated starting with step 2 and ending with step 7 repeatedly until the change in $\Delta\omega$ is negligible compared with the limit value given in advance. Initially we set $\Delta\omega = 0$.

8 All equations are the same as step 4 in direct.

9 The same as step 7 in direct.

10 $S = K_1 b(\sigma - \Delta\sigma)$

11 (1) $\tan\alpha_1 = \dfrac{\cos\beta_2 \sin\omega}{\cos\beta_1 \sin\beta_2 - \sin\beta_1 \cos\beta_2 \cos\omega}$

(2) $\tan\alpha_2 = \dfrac{\cos\beta_1 \sin\omega}{\cos\beta_1 \sin\beta_2 \cos\omega - \sin\beta_1 \cos\beta_2}$