Physics 212: Problem Set 2

Due on September 22, 2016

Problem 1: Consider the relic population of neutrinos. At $k_B T \gg 1$ MeV, neutrinos (and antineutrinos) interact quickly enough that they are populated at their thermal abundances. At $k_B T \approx 1$ MeV, the neutrinos stop interacting with the rest of the particles. After that time, the annihilation of the electrons and positrons heats the photons to a temperature that is $(11/4)^{1/3}$ higher than the temperature of the neutrinos.

The mass is negligible near the decoupling redshift, so one can use the relativistic limit $E = qc$. But at low redshift, we will assume the mass is large enough that the neutrinos are non-relativistic.

a) What is the velocity distribution of the massive neutrinos today? In other words, what is dn/dv? To compute this, use the fact that the neutrinos at high temperature are in a thermal distribution for a massless fermion and that the temperature at the decoupling redshift z_d is $(4/11)^{1/3}(1+z_d)2.725$ K. After decoupling, the momenta scale as $(1+z)^{-1}$. You should compute the momentum distribution at z_d and then convert is to the velocity distribution today. (Note that you do not need to compute z_d; it will cancel out).

b) Compute the mean velocity of the neutrinos today. The following integrals could be useful:

$$\int dx \frac{x^n}{e^x - 1} = n!\,\zeta(n+1) \tag{1}$$

$$\int dx \frac{x^n}{e^x + 1} = n!\,\zeta(n+1)(1 - 2^{-n}) \tag{2}$$

where $\zeta(m) = \sum_{k=1}^{\infty} k^{-m}$ is the Riemann zeta function. $\zeta(2) = \pi^2/6$, $\zeta(3) \approx 1.202$, $\zeta(4) = \pi^4/90$.

Problem 2: Big Bang Nucleosynthesis

a) Suppose an extra neutrino species is added to the Universe. Would the predicted helium abundance go up or down?

b) Suppose the weak interactions were stronger than they actually are, so that the thermal equilibrium distribution between neutrons and protons were maintained until $k_B T = 0.25$ MeV. Would the predicted helium abundance be larger or smaller than in the standard model?

c) Suppose the proton-neutron mass difference were larger than the actual value. Would the predicted helium abundance be larger or smaller than in the standard BBN calculation?

Problem 3:

Consider the following reaction that takes place in the Universe:

$$p + n \to D + \gamma$$

What fraction of neutrons are in Deuterium when the Universe is 5-minutes old? Use that $\Omega_b h^2 = 0.02$, and that the baryons consist of 14% neutrons and 86% protons.