Maths Tutorial #6

$T : \mathbb{R}^5 \to \mathbb{R}^3$ $A = \mathrm{Mat}_{\mathcal{E}}(T)$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 3 & -1 & 2 & 1 & 3 \\ 1 & 2 & 3 & 1 & 4 \end{bmatrix}$$

(row echelon form)

Then $A \xrightarrow{REF} \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

a) Show $Im(T)$ is all of $\mathbb{R}^3$. (~~Im~~ $Im(T)$ = Image of T)

$Im(T) = \mathbb{R}^3 \iff$ For any $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbb{R}^3$

there exists $x \in \mathbb{R}^5$ with $Ax = b$

i.e. x is a solution to $REF(A)x = b$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & 2 & \tilde{b}_1 \\ 0 & 1 & 1 & 2 & 3 & \tilde{b}_2 \\ 0 & 0 & 0 & 1 & 1 & \tilde{b}_3 \end{array}\right]$$

There are no inconsistencies (rows of 0 in $REF(A)$) so exact solutions do exist for any $b \in \mathbb{R}^3$.

b) Do the columns of A form a linearly independent set of vectors in $\mathbb{R}^3$.

Linearly independent: Not linearly dependent.

Linearly dependent: $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \alpha_4 v_4 + \alpha_5 v_5 = 0$ has non-trivial (not all 0) solutions.

① Any set of more than $> n$ in $\mathbb{R}^n$ is linearly dependent.