d) Write $\begin{bmatrix}1\\2\\3\end{bmatrix}$ as a linear combination of vectors in B.

$$\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}1\\3\\1\end{bmatrix} + \begin{bmatrix}0\\-1\\2\end{bmatrix}$$

$$\begin{bmatrix}1\\2\\3\end{bmatrix}_B = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

e) Do the same for col $\begin{bmatrix}2\\3\\4\end{bmatrix}$

col 5 = col 4 + col 1 + col 2

$$\begin{bmatrix}2\\3\\4\end{bmatrix} = \begin{bmatrix}1\\3\\1\end{bmatrix} + \begin{bmatrix}0\\-1\\2\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}2\\3\\4\end{bmatrix}_B = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

f) use d), e) to help find vectors, n_1, n_2 in $\mathcal{N} = \text{Null}(T)$ which are linearly independent.

$$e_1 = \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$$

Col 3 = Col 1 + col 2

$T(\underline{e_3}) = T(\underline{e_1}) + T(\underline{e_2})$

$T(\underline{e_1}) + T(\underline{e_2}) - T(\underline{e_3}) = 0$

$T(\underline{e_1} + \underline{e_2} - \underline{e_3}) = 0$ by linearity

$\underline{e_1} + \underline{e_2} - \underline{e_3} \in \mathcal{N}$.

$$\underline{e_1} + \underline{e_2} - \underline{e_3} = \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix} - \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\\0\\0\end{bmatrix} \in \mathcal{N}$$