

$$\text{col } 5 = \text{col } 1 + \text{col } 2 + \text{col } 4$$

$$T(\underline{e}_5) = T(\underline{e}_1) + T(\underline{e}_2) + T(\underline{e}_4)$$

$$T(\underline{e}_1) + T(\underline{e}_2) + T(\underline{e}_4) - T(\underline{e}_5) = 0$$

$$T(\underline{e}_1 + \underline{e}_2 + \underline{e}_4 - \underline{e}_5) = 0$$

$$\underline{e}_1 + \underline{e}_2 + \underline{e}_4 - \underline{e}_5 \in \mathcal{N}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ -1 \end{bmatrix} \in \mathcal{N}$$

clearly $\begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}$ are linearly independent because they're not scalar multiples of each other.

\Rightarrow linearly independent vectors in \mathcal{N} .

g) Show $\mathcal{N}_1, \mathcal{N}_2$ form a basis for \mathcal{N} .

$$x_1 + x_3 + x_4 + 2x_5 = 0$$

$$x_2 + x_3 + 2x_4 + 3x_5 = 0$$

$$x_4 + x_5 = 0.$$

since cols 3 and 5 don't have pivots, set them as free variables.

$$x_3 = s, x_5 = t$$

$$x = \left\{ \begin{bmatrix} -s-t \\ -s-t \\ s \\ -t \\ t \end{bmatrix} \mid t \in \mathbb{R} \right\}$$