(2) $col(3) = col(1) + col(2)$

$v_3 = v_1 + v_2$

$v_1 + v_2 - v_3 = 0$

$\Rightarrow$ Linearly Dependent.

c) Find a subset of the columns of A that form a basis

Basis: Is a set of linearly independent vectors, where any other vector in the space can be written as a linear combination of these vectors.

i.e. Pick ~~any~~ a set 3 linearly independent vectors, which will then span $\mathbb{R}3$. span $\Leftrightarrow$ basis definition.

So take columns with pivots: $= \left[\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right]$

Columns with pivots are always linearly independent.

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 3 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \qquad REF(B) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

corresponding columns from REF(A).

REF (B) has Rank 3 due to 3 linearly independent cols, (i.e. 3 pivots).

No rows of 0's $\Rightarrow Bx = b$ has a solution for any $b \in \mathbb{R}^3$ so the vectors span $\mathbb{R}3$.

so $B = \left[\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right]$ is a basis