

Testing General Relativity Using Atomic Clocks

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PHYS4281 - Applications of Atomic Clocks to Fundamental Physics

The apparent incongruence between general relativity and quantum mechanics is the central tension of modern fundamental physics research. General relativity (GR) explains gravity by describing how massive objects warp spacetime, which in turn dictates the movement of other objects. The behaviour of the universe on the largest scales is well modelled by GR since gravity is the dominant force between high mass objects on cosmic distance scales. On the other hand, quantum mechanics (QM) describes the behaviour of fundamental particles on the very smallest scales where electromagnetism and the strong and weak nuclear forces dominate. Although together these theories describe all four fundamental forces, they can only do so at certain scales. In particular, they disagree about the effect of gravity on quantum scales. Unifying QM and GR to produce a theory of *quantum gravity* is perhaps the most important endeavour in all of physics since it will, for the first time, allow us to model space-time at extreme energies and small scales which may begin to explain the behaviour of black holes and the early universe. Although many candidate theories of quantum gravity have been proposed, none have been accepted yet due to a lack of experimental evidence¹. It is therefore of paramount importance to test QM and GR, since if either were found to disagree with empirical measurements it would open the doors for theories of quantum gravity and new physics. GR is robust and well tested on cosmic scales so if it is incomplete then extremely precise measurements are needed to notice the tiny deviations expected on small scales. The continued advancements in atomic clock design have enabled extremely precise time measurements and have become cornerstones of GR testing². This essay will begin by outlining the basic principles of atomic clock operation before describing their applications in testing GR and discussing future research proposals.

Atomic Clocks

All time measurements are derived via observing oscillators. For example, the pendulum of a grandfather clock or the vibrating quartz crystal of a modern watch. An ideal oscillator has a well defined frequency that is stable over long periods and oscillates quickly so that many oscillations can be averaged across. Atomic clocks are devices that measure time by counting the oscillations of electromagnetic radiation kept at a constant frequency by driving electron transitions. If the transition has a well defined energy (i.e. a narrow linewidth), then only a small range of frequencies can drive the transition. This in turn means there is a

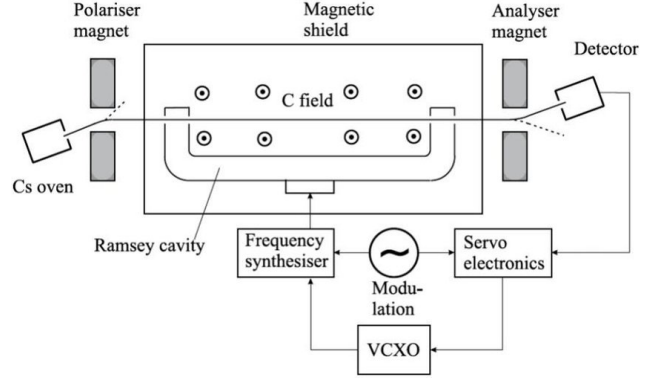


Figure 1: Schematic of a caesium beam atomic clock³.

small uncertainty in the frequency measurement which results in a precise clock. The ground state hyperfine transition of caesium-133 has a narrow linewidth and is insensitive to external magnetic fields, as a result it was commonly used in early atomic clocks⁴.

The schematic of a simple atomic clock is shown in fig. 1. A beam of caesium atoms passes through a Stern-Gerlach or polariser magnet⁵, this deflects some of the atoms, based on their hyperfine state, into the Ramsey cavity. Inside the cavity the atoms are exposed to microwave radiation that is tuned to the ground state hyperfine transition $|3, 0\rangle \rightarrow |4, 0\rangle$, meaning that the population of excited atoms increases. The second magnet deflects excited atoms towards a detector which is connected to a servo that constantly adjusts the microwave frequency such that the detection signal is maximised (i.e. the microwaves are on resonance with the transition). This feedback loop ensures that the clock frequency remains stable so that it can be counted for precise time measurements. In 1967 the second was defined based on the caesium hyperfine transition⁶. Since then, modern atomic clocks have improved by orders of magnitude by driving transitions in other elements at much higher (optical) frequencies and using lasers to trap and cool the atoms in an optical lattice⁷. The best clocks have recently achieved fractional uncertainties of $\Delta T/T \approx 10^{-21}$, an error of less than 1 s across the lifetime of the universe^{8;9}. They are the most precise measurement devices ever to exist and they enable us to perform sensitive tests of general relativity by measuring *gravitational time dilation*.

Measuring Gravitational Time Dilation

General relativity (GR) predicts that time will pass more slowly for clocks at low gravitational potential

(i.e. close to massive objects) when compared to a clock at high potential. This is known as gravitational time dilation (GTD). In the relatively weak gravitational field of Earth these time differences are tiny (on the order $\sim 10^{-8}$ s); only atomic clocks have the precision required to measure them. Gravitational redshift (GRS) is the phenomenon that electromagnetic radiation gets redshifted as it increases its gravitational potential. Since redshift corresponds to a decrease in frequency, GRS can be thought of as GTD slowing down the photons' oscillation as it leaves the potential well¹⁰.

GTD was measured using clocks for the first time in 1972 by the Hafele-Keating experiment¹¹. Four caesium beam atomic clocks were placed on a commercial aircraft which then performed two equatorial circumnavigations, first eastwards then westwards. Compared to a clock at rest on Earth's surface, the flying clocks should experience time dilation from two sources. Firstly, the altitude of the aircraft will bring the flying clocks into a higher gravitational potential and cause them to run faster; this is the gravitational time dilation. Secondly, the flying clocks are travelling at high speed which causes them to run slow when travelling in the direction of Earth's rotation (eastwards) and vice versa; this is kinematic time dilation as described by special relativity. GR predicted that the clocks would experience a net time difference of -40 ± 23 ns eastwards and $+275 \pm 21$ ns westwards respectively. The clocks were in excellent agreement and measured -59 ± 10 ns and $+273 \pm 7$ ns respectively, this paved the way for similar future experiments.

Advances in atomic clock precision have enabled tests of GR to extreme sensitivity. While the Hafele-Keating experiment measured the effects of GTD at the length scale of aircraft altitudes (~ 10 km), in 2010 it was shown that they can be measured over distances of less than one metre¹². Further still, in 2021 it was shown that they can be measured across single millimetres⁹. Rather than having to fly the clocks themselves in aircraft, GTD can instead be measured across the sample of ultra-cool strontium atoms trapped in the optical lattice. Using a camera to image the lattice directly means that the frequency can be measured separately across the atomic sample, rather than as a global parameter as with standard clocks. This *in situ* survey allows the frequency changes due to GRS to be measured between atoms at different gravitational potentials in the lattice. The resulting intra-lattice frequency map is used to measure a frequency gradient of $(-9.8 \pm 2.3) \times 10^{-20} \text{ mm}^{-1}$, in excellent agreement with the theoretical GR prediction of $-1.09 \times 10^{-20} \text{ mm}^{-1}$. The clock used for this result demonstrated intra-lattice measurements for the first time and has the lowest clock uncertainty to date of 7.6×10^{-21} , an improvement of 2 orders of magnitude from the previous state-of-the-art only a few years prior^{13;14}. It is likely that this novel technique will

evolve to produce GR tests of increasing sensitivity in the coming decades.

Future Research

In 2008 the best clocks had an uncertainty of $\sim 10^{-16}$, only 13 years later this has improved by 5 orders of magnitude to $\sim 10^{-21}$. The rapid progress in atomic clock technology means that there are a wide array of research proposals outlining theoretical tests for GR. Although many satellites already contain basic atomic clocks, it is expected that placing laser-cooled, optical atomic clocks in an orbit around Earth will enable GR tests of unprecedented sensitivity since they would be able to send signals of ultra-stable frequency down to Earth and allow long term monitoring of the effects of gravitational redshift. The ACES (Atomic Clock Ensemble in Space) in an ESA mission due to begin in the coming years which will place a laser-cooled caesium clock (uncertainty $\sim 10^{-13}$) on the ISS¹⁵, theoretically enabling a direct measurement of GRS frequency shift with an accuracy of 2×10^{-6} . There are plans to place an optical atomic clock (uncertainty $\sim 10^{-18}$) on the recently launched chinese space station CSS¹⁶. This is expected to achieve an accuracy of $(0.27 \pm 2.15) \times 10^{-7}$.

Conclusion

Atomic clocks are the linchpins of gravitational time dilation measurements which have been crucial in testing general relativity so far. Despite the unprecedented increases in clock precision in recent years, the theory of general relativity is yet to noticeably deviate from any experimental results. However, the advent of novel intra-lattice frequency measurements and the placement of atomic clocks in orbit are set to continue this progress in coming decades and perhaps notice the minute perturbations expected from quantum gravity.

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