Determination of the Dynamic Viscosity of Water

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The mass flow rate of water through a capillary tube is measured for varying heights of water and the Poiseuille equation is used to calculate the dynamic viscosity of water as 1.0 ± 0.1 mPa s at $(14.7 \pm 0.2)°$C. This is found to be in reasonable agreement with literature values. The experimental method is outlined and evaluated to be suitable for viscometry; possible improvements for future experiments are also presented.

I Introduction

Dynamic viscosity is a property of all fluids which measures the shear force per unit area needed to move one horizontal plane of its molecules tangentially with respect to another plane of molecules. As the planes slide past each other, an internal friction is created by intermolecular forces acting between the planes that resists the motion.

A fluid's behaviour whilst flowing or interacting with a solid boundary is governed by its viscosity, so it is essential to both understand viscosity and have an accurate and precise value of viscosity for any fluid being studied. Water is one such fluid whose behaviour is important to understand as it has an essential role in describing weather systems, ocean currents and many other active areas of research in fluid mechanics. This provides a strong motivation to determine an accurate and precise value of the viscosity of water.

The flow of liquid through a tube is described by the Poiseuille equation [1, Pg. 1]:

$$\dot{V} = \frac{\pi a^4 \Delta p}{8\eta l} \tag{1}$$

For a liquid with viscosity η flowing at rate $\dot{V}$ through a tube of length l and radius a with a change in pressure Δp across the tube. This equation holds if the radius of the tube is many times smaller than the length and the flow is not turbulent [2, Pg. 11] i.e. the planes of molecules are parallel and slide past each other without disruption. It also assumes that the liquid is newtonian i.e. its viscosity is constant and does not depend on the shear forces acting upon it.

In practice, rate of volume flow and change in pressure are difficult to directly measure so they need to be replaced with directly measurable variables. A useful modified form of the Poiseuille equation can be found by considering how the pressure difference is related to the more easily measurable height of the water h, density of the fluid ρ and gravitational field strength g, and also considering how the flow is related to the rate of change of mass $\dot{m}$ and density ρ.

$$\Delta p = \rho g h \ , \quad \dot{m} = \frac{dm}{dt} = \rho \dot{V}$$

$$\frac{dm}{dt} = \frac{\pi \rho^2 g h a^4}{8\eta l} \tag{2}$$

By measuring the rate of change of mass for different heights of water, a graph can then be plotted such that the gradient is directly proportional to the viscosity, from which viscosity can be calculated. This assumes that the change in height is negligible over the interval while the rate of change of mass is measured. In the following sections an experimental setup for this process is outlined and the results are presented and evaluated.

II Methods

Rates of change of mass were calculated for different heights of water by dripping water through a capillary tube into a beaker resting on a mass balance. The mass of the beaker was recorded every second over a ten second period and the gradient of each dataset was the rate of change of mass at that given height. The flow of water from the capillary tube was in discrete drops so the gradients often had a large error due to the random nature of the drops. To mitigate this, repeats were taken at each height and averaged. The water was fed into the capillary tube from a large container to ensure that the change in height over 10 seconds was negligible. The temperature of the water was measured as $(14.7 \pm 0.2)°$C using a thermometer.

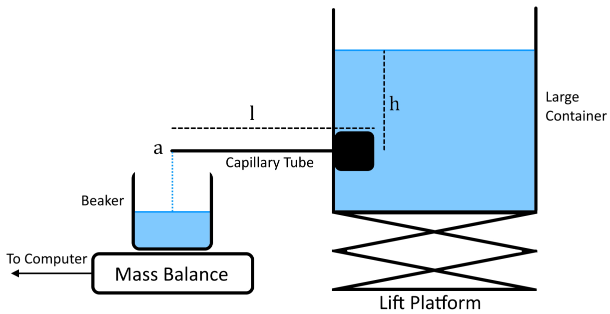

FIG. 1: Diagram of the experimental setup showing how the water in the large container flows through the capillary tube and onto the mass balance. The labels a, l and h represent the radius and length of the tube and the height of the water respectively.

Two different capillary tubes were used in order to get multiple values for viscosity that could be averaged together: a narrow blue tube and a wider white tube. For the blue tube, 3 repeats were taken at every centimetre between 11cm and 19cm. For the white tube, 2 repeats were taken every centimetre between 11cm and 14cm and one repeat was taken for heights between 15cm and 17cm.

Colour	Radius (mm)	Length (mm)
Blue	0.38 ± 0.02	141 ± 1
White	0.57 ± 0.02	142 ± 1

TABLE I: Table showing the dimensions of each capillary tube.

In equation 2 the errors are dominated by the radius of the capillary tube as it is to the fourth power. Therefore, the accurate and precise measurement of the capillary tube radii was essential as otherwise the error in the final value would be very large. The radii were measured using a travelling microscope on a vernier scale to maximise precision. The crosshair of the microscope was lined up with one edge of the capillary tube hole and the vernier scale reading was recorded, then the crosshair was realigned to the other edge of the hole and the vernier scale was recorded: the difference of the vernier scale readings was then the diameter of the capillary tube. As the capillary tubes were not exact circles, they were each measured 3 times at different orientations and averaged.

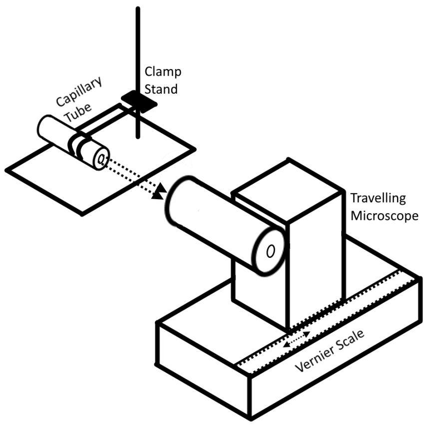

FIG. 2: Diagram of the setup used to measure the radii of the capillary tubes. Note that the size of the capillary tube has been enlarged for clarity.

III Results and Discussion

A regression line was used to calculate the rate of change of mass from each dataset at a particular height. Graphs of rate of change of mass against height were then plotted for each capillary tube respectively.

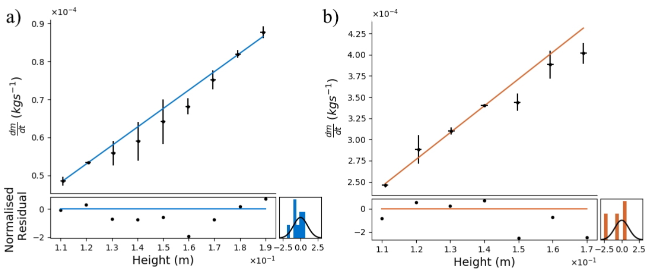

FIG. 3: Rate of mass change vs height graphs for the blue tube (Fig a) and white tube (Fig b) respectively.

Equation 2 implies a linear relationship between rate of change of mass and height so a two parameter χ^2 analysis was conducted. For the blue tube; $\chi^2_\nu = 0.907 \approx 1$ and $P(\chi^2_{min};\nu) = 0.4996 \approx 0.5$ which both imply that a linear fit is suitable. For the white tube; $\chi^2_\nu = 2.939$ and $P(\chi^2_{min};\nu) = 0.0117$ which suggest that the linear fit should not be rejected. Over 65% of the normalised residuals are within one standard error of the trend line and they appear to be randomly distributed so this implies that the line is appropriate. A linear fit is therefore appropriate for the data and it is reasonable to calculate viscosity from their gradients.

Viscosity values of 1.2 ± 0.3 mPa s and 0.9 ± 0.1 mPa s were calculated for the blue and white tubes respectively. These values are within one standard error of eachother and have errors of similar magnitude so they can be combined with a weighted mean to give a final value of 1.0 ± 0.1 mPa s. This in reasonable agreement with the literature value of 1.1382 ± 0.0001 mPa at $15°C$[3]. To calculate viscosity, g was taken as 9.81 and density of water was taken as (999.10168 ± 0.00064) kgm^{-3} [4]; the other variables were measured directly.

One source of error could possibly be due to the temperature of the room being greater than the temperature of the water. The temperature of the water was (14.7 ± 0.2)°C as it came from a tap whose pipes went outside whereas the room temperature was (21.0 ± 0.2)°C. This meant that over the course of multiple experiments the water's temperature rose which, if it reached 20°C, could have reduced its viscosity to (1.0020 ± 0.0001) mPa s [3] and density to (998.20569 ± 0.00069) kgm^{-3} [4]. These temperature fluctuations may be the reason that some of the heights in Figure 3 have much larger errors than others e.g. repeat readings taken when the water was at a different temperature may have been very different to consecutive repeats. This may explain why the determined value of viscosity is slightly lower than the value at 15°C as the temperature was likely higher than this.

Another possible reason for these heteroscedastic errors is the high sensitivity of the mass balance. It had a high precision (as was necessary for measuring individual water drops) but this meant it was affected by vibrations in the room and often produced erroneous data. Obvious cases of this were deleted e.g. data points that had a mass of zero or when the mass decreased, but tiny errors caused by small vibrations were undetectable.

IV Conclusions

The dynamic viscosity of water has been determined and found to be in reasonable agreement with the literature values [3]. This gives credibility to this experimental method for determining viscosity and could be used to determine more precise values in future if the systematic and measurement errors are reduced. The dominant source of error was the capillary tube measurement; so in future experiments additional repeat measurements are needed to get a more precise value. The dominant systematic errors were temperature changes in the water and the fluctuations of the mass balance which can be fixed by using water and equipment at thermal equilibrium with the surroundings and running the experiment in an isolated environment.

Capillary Tube	Viscosity ($mPas$)
Blue	1.2 ± 0.3
White	0.9 ± 0.1
Combined	1.0 ± 0.1

TABLE II: The determined values for dynamic viscosity of water at $(14.7 \pm 0.2)°C$ for different capillary tube radii and the combined final value.

References

[1] PFITZNER, J. (1976). Poiseuille and his law. Anaesthesia, 31(2), pp.273-275.

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[4] Patterson, J. and Morris, E. (1994). Measurement of Absolute Water Density, 1 °C to 40 °C. Metrologia, 31(4), p.288.

[5] Hughes, I. and Hase, T. (2011). Measurements and their uncertainties. Oxford: Oxford University Press.

V Error Appendix

The computer recorded time with error $\pm 1 \times 10^{-6}$ s and the mass balance recorded mass with error ± 0.0001 g so the mass-time datasets were homoscedastic which meant it was appropriate to use a least-squares regression line to calculate the gradient m and gradient error $\alpha_m$of each dataset using the following equations [5, Pg. 58], where x is time, y is mass and N is the number of data points:

$$m = \frac{N\Sigma_i x_i y_i - \Sigma_i x_i \Sigma_i y_i}{\Delta} \tag{3}$$

$$\Delta = N\Sigma_i x_i^2 - (\Sigma_i x_i)^2 \tag{4}$$

$$\alpha_m = \alpha_{CU}\sqrt{\frac{N}{\Delta}} \tag{5}$$

$$\alpha_{CU} = \sqrt{\frac{1}{N-2}\Sigma_i (y_i - mx_i - c)^2} \tag{6}$$

The lengths of the capillary tubes and the height of the water was measured with a ruler with an error of ± 1 mm, the radii of the capillary tubes were measured with a travelling vernier microscope with a small absolute error of ± 0.01 mm, the final error of ± 0.02 mm came from averaging repeats and taking the standard error. The errors were propagated using the following equation [5, Pg. 44]. Note that π and g were taken as exact. In this case, b is the gradient of Figure 3:

$$\frac{\alpha_\eta}{\eta} = \sqrt{(4\frac{\alpha_a}{a})^2 + (2\frac{\alpha_\rho}{\rho})^2 + (\frac{\alpha_l}{l})^2 + (\frac{\alpha_b}{b})^2} \tag{7}$$

The gradient of Figure 3 was calculated using chi squared minimisation by fitting a two parameter linear model to the data as described in [5, Chapter 6]