# Determination of the Radius of Curvature of a Planoconvex Lens Using Newton's Rings

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This report uses the optical phenomenon Newton's rings to calculate the radius of curvature of a planoconvex lens. The radii of the rings are measured and are used to calculate the radius of curvature which turns out to be $17.3 \pm 0.3$m. The experiment is repeated to calculate the wavelength of light emitted by a sodium vapour lamp as $(5.7 \pm 0.2) \times 10^{-7}$ m which is in excellent agreement with the literature value of $\approx 5.89 \times 10^{-7}$ m [1].

## I. INTRODUCTION

Newton's rings are an optical phenomenon consisting of a series of concentric alternating bright and dark rings as a result of the interference of light passing through a planoconvex lens resting on a reflecting surface. The curved surface of the planoconvex lens touching the reflecting surface means that there is an air gap between them which increases as the distance from the centre increases. This air gap means that there is a varying path difference for light that gets reflected between the lens and the reflecting surface, this causes a varying phase difference between this light and light that does not get reflected between the surfaces. Superposition between the light rays from the two different paths creates an alternating pattern of constructive and deconstructive interference: Newton's rings. When light shines through the lens onto a distant screen (transmission) the formula for the $m^{th}$ dark ring/fringe is given by:

$$r_m^2 = mR\lambda \tag{1}$$

Where $r^2{}_m$ is the radius of the $m^{th}$ dark ring squared, $R$ is the radius of curvature of the planoconvex lens, $\lambda$ is the wavelength of the light going through the lens and $m$ is the number of the fringe i.e. the first dark fringe has $m$=1, the second has $m$=2 etc. By measuring the diameter of many dark rings, a linear graph of $r^2{}_m$ vs $\lambda m$ can be plotted such that the gradient is $R$. In following sections, the experimental set up and results are outlined as well as an evaluation of the results by using them to calculate the wavelength of light from a sodium vapour lamp and comparing it to existing literature.

## II. METHODS

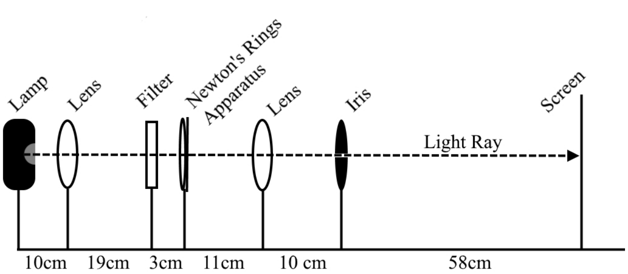


**FIG. 1:** Experimental Set Up

Figure 1 shows the arrangement of the equipment used. The Lamp used was a mercury vapour lamp which emitted white light of a mixture of known wavelengths. This light was focused by a lens through a filter which only allowed one wavelength through. The yellow long pass filter let 85% of light above 578nm through and the green band pass filter let through 80% of light between 470nm and 578nm, these filters were chosen to let wavelengths 578nm (Mercury Yellow) and 546nm (Mercury Green) through respectively. Each wavelength is diffracted by a different amount as it goes through the lens, having multiple wavelengths can therefore cause chromatic aberration and make the rings blurry and difficult to distinguish, only allowing one wavelength through the apparatus stops this effect. After going through the Newton's rings apparatus and another lens, the light goes through a fully undilated iris. This limits the amount of light that gets on the screen such that the contrast between the rings and the light is at its maximum.

The diameter of the rings could then be measured with a ruler from which the radius squared could be calculated and plotted as described in the introduction in order to calculate the radius of curvature. The diameter was measured as opposed to the radius as the centre of the rings is difficult to accurately locate and would introduce an extra uncertainty if radius was measured directly with this method. The first 5 rings were ignored as they were the least sharp and measuring them would introduce an extra uncertainty.

To check the accuracy of the values for the radius of curvature, the same experiment was repeated with a sodium vapour lamp instead of the mercury lamp. The rings' diameters were again measured so that a graph of $r^2{}_m$ vs $mR$ could be plotted such that the gradient was $\lambda$ (the wavelength of the sodium lamp light) where R is the value for radius of curvature from the weighted mean and the green and yellow filters respectively.

## III. RESULTS AND DISCUSSION

Graphs of $r^2{}_m$ vs $\lambda m$ were plotted for the green and yellow data sets respectively and a linear least squares regression line was performed on them to produce a value of the gradient (radius of curvature) (see Table II) [2, pg. 69-70]. These results have similar errors and are in reasonable agreement with each other (as they are between 1 and 2 standard errors away from each other [2, pg. 28]) so it is reasonable to combine them with a weighted mean [2, pg. 49-51].

**TABLE I:** Radius of Curvature (R)

| | Value (m) | Uncertainty (m) |
|---|---|---|
| Green | 17.8 | $\pm 0.4$ |
| Yellow | 17.3 | $\pm 0.4$ |
| Weighted Mean | 17.6 | $\pm 0.3$ |

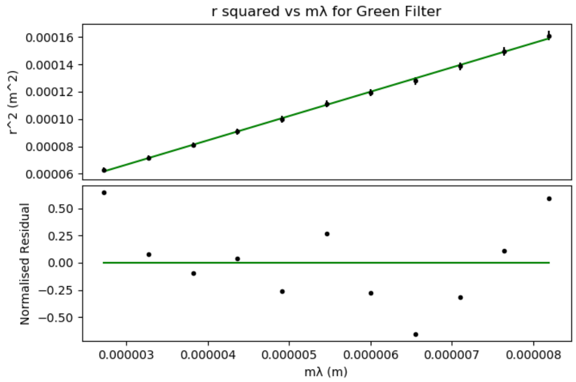


**FIG. 2:** Green Filter Graph

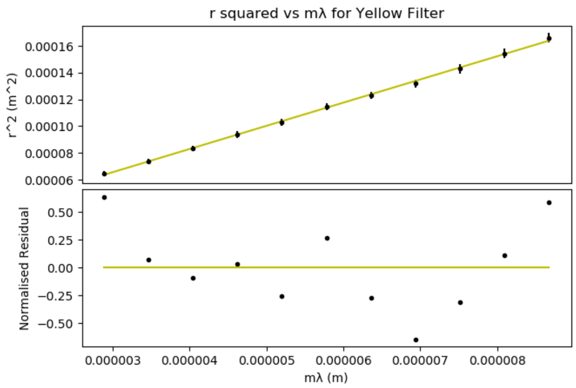


**FIG. 3:** Yellow Filter Graph

Graphs of $r^2_m$ vs $mR$ were plotted for each value of R respectively and a value for λ was found with a linear regression line as before [2, pg. 69-70].

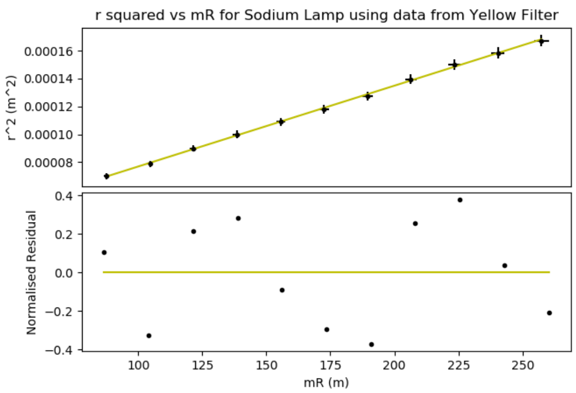


**FIG. 4:** Yellow Sodium Lamp Graph

The literature value for the wavelength of light from a sodium vapour lamp is ≈ 5.89 x $10^{-7}$ m [1] (in reality there are 2 wavelengths that are so close together they are indistinguishable from one single wavelength for the purposes of this experiment[1]). The wavelength calculated from the yellow dataset is in excellent agreement with the literature value (as it is less than one standard error away [2, pg. 28]) and so the yellow dataset's value for R and λ are the most accurate. The values from the green and weighted mean datasets are only in reasonable agreement (as they are between one and two standard errors apart [2, pg. 28]).

**TABLE II:** Wavelength of Sodium Vapour Lamp Light (λ)

| | Value (x $10^{-7}$ m) | Uncertainty (x $10^{-7}$ m) |
|---|---|---|
| Green | 5.5 | ±0.2 |
| Yellow | 5.7 | ±0.2 |
| Weighted Mean | 5.5 | ±0.1 |

This implies that there is a systematic error on the green value which has skewed the data and the weighted mean and caused the yellow data to be the most accurate. One possible cause of this could be the green filter, as it has a profile that ends very close to the value of the mercury yellow wavelength (578nm) and it is possible that some of the yellow light went through the green filter and affected the results. The filter has a half maximum transmission of 465nm - 555nm, assuming that the filter's profile is a normal distribution, it can be calculated that the filter lets through light with a mean wavelength of 510nm and a standard deviation of 66.7nm. If this is the case, then there is a 15.4% chance that light of wavelength greater than or equal to 578nm passes through the filter and this may be enough to cause some chromatic aberration which made the rings less sharp and hence induced a systematic error in the data. As the green filter is a band pass filter, the distribution resembles a normal distribution [3] so this approximation is somewhat valid although not conclusive, in comparison the yellow filter is a long pass filter and so does not resemble a normal distribution [4] and is unlikely to allow the green wavelength through.

## IV. CONCLUSIONS

The yellow dataset is therefore the most accurate, hence the radius of curvature has been determined to be 17.3 ± 0.4 m. The wavelength of the light from the sodium lamp determined using this value, (5.7 ± 0.2) x $10^{-7}$ m, is in excellent agreement with the literature value of ≈ 5.89 x $10^{-7}$ m [1]. This adds credence to this method of calculating the radius of curvature using Newton's rings.

## V. ACKNOWLEDGEMENTS

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### References

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[2] Hughes, I. and Hase, T. (2011). Measurements and their uncertainties. Oxford: Oxford University Press.

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[4] Edmund Optics (n.d.). [image] Available at: https://www.edmundoptics.com/contentassets/1cbf040e481a4510bd5dd78c3 1-of.gif [Accessed 19 Feb. 2019].

## VI. ERROR APPENDIX

Unless stated otherwise all following equations[2, pg. 44] and propagations [2, Ch. 4] come from *Measurements and their Uncertainties*.

The wavelength of the mercury vapour lamp's light ($\lambda$) was given and taken as an exact value and the fringe number ($m$) is an integer and as such $\lambda m$ has no error. The diameter of the rings was measured with a ruler with markings every mm and so the error for each measurement was $\pm$0.5mm, but since the ruler's measurement depends on the position of the 0 mark, as well as the measured mark, the total uncertainty is doubled to be $\pm$1mm. Diameter was turned into radius by halving it and propagating the errors through equation 2. Errors were propagated for turning radius into radius squared through equation 3.

$$\alpha_Z = k\alpha_A \; \forall \; Z = kA \tag{2}$$

$$|\frac{\alpha_Z}{Z}| = |n\frac{\alpha_A}{A}| \; \forall \; Z = A^n \tag{3}$$

The measured values for diameter were the size of the visible rings on the screen but the size of the rings on the lens were smaller by a scale factor that was measured with a ruler by measuring the size of the scale projected on the screen. The measured values were multiplied by this scale factor to give the true values; these errors were propagated through equation 4.

$$\frac{\alpha_Z}{Z} = \sqrt{(\frac{\alpha_A}{A})^2 + (\frac{\alpha_B}{B})^2} \; \forall \; Z = A \times B \tag{4}$$

The errors of the gradients of the graphs were calculated using equation 5 [2, pg.70].

$$\alpha_m = \sqrt{\frac{\Sigma_i w_i}{\Delta'}} \; \forall \; \Delta' = \Sigma_i w_i \Sigma_i w_i x_i^2 - (\Sigma_i w_i x_i)^2 \tag{5}$$