Lista 1

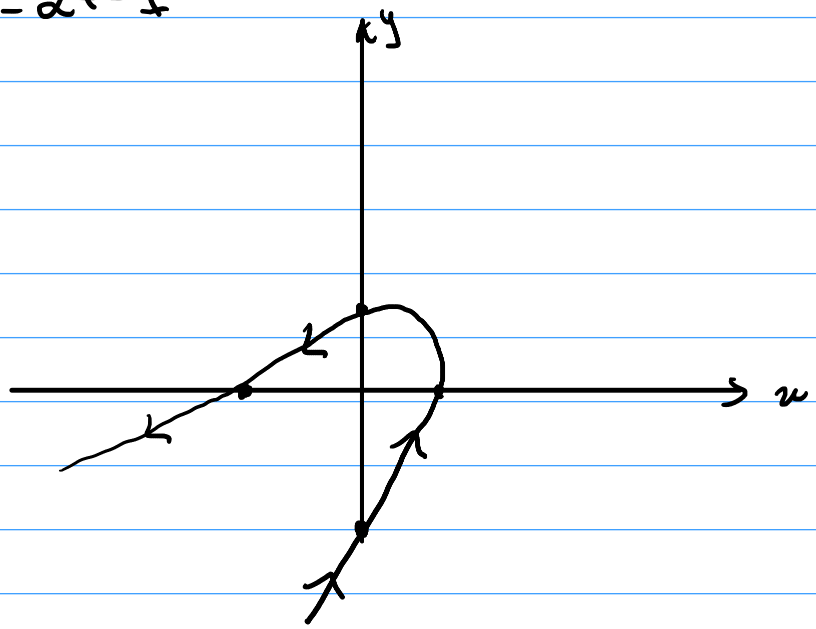
Seção 10.1: 1, 3, 5, 7, 11, 15, 23, 30, 37, 41, 53

(1) $t = -2, -1, 0, 1, 2$; $x = t^2 + t$
$y = 3^{t+t}$

$$\boxed{\text{Pares: } (2, 1/3), (0, 1), (0, 3), (2, 9), (6, 27)}$$

(3) $x = 1 - t^2$ $\quad -1 \leq t \leq 2$
$y = 2t - t^2$ $\quad (0, -3), (1, 0), (0, 1), (-3, 0)$

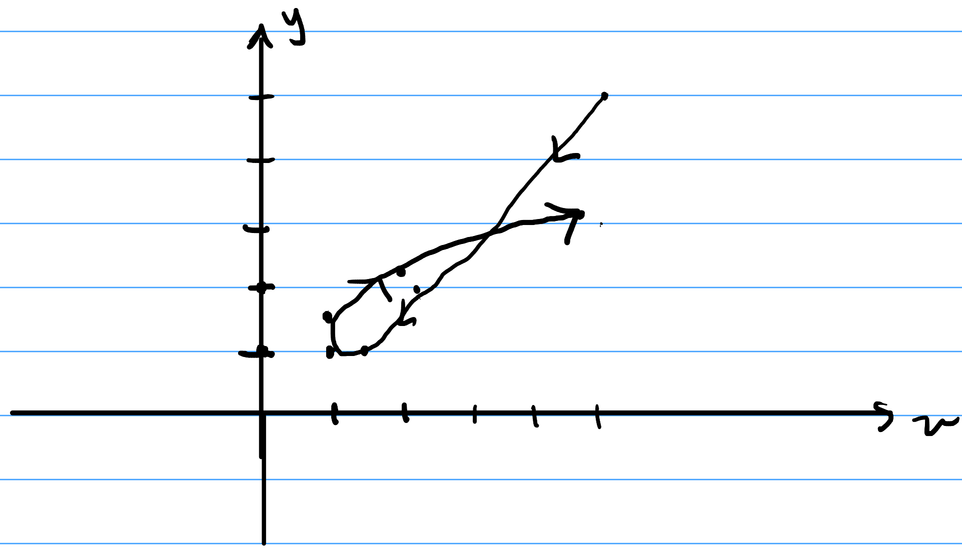
$x - y = 2t - 1$



(5) $x = 2^t - t$ $\quad -3 \leq t \leq 3$

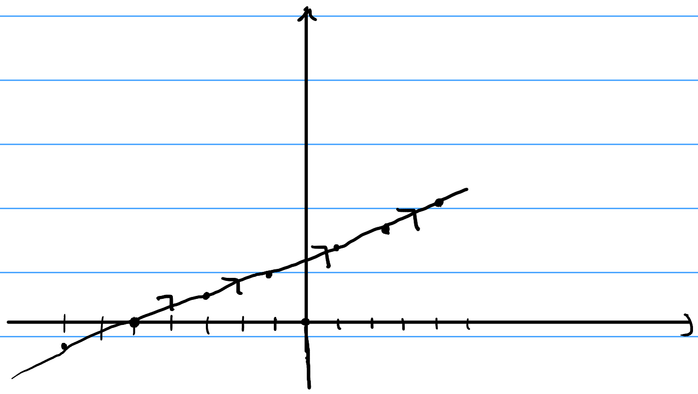
$y = 2^{-t} + t$

$(25/8, 5), (9/4, 2), (3/2, 1), (1, 1), (1, 3/2), (2, 9/4), (5, 25/8)$



(7) $x = 2t - 1; \; y = 1/2 t + 1.$

a) $(-7, -1/2), (-5, 0), (-3, 1/2), (-1, 1), (1, 3/2), (3, 2), (5, 5/2)$

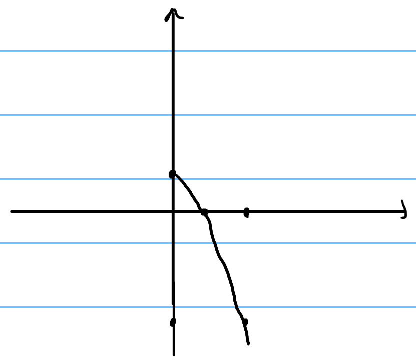


b) $x = 2t - 1; \; y = 1/2 t + 1.$

$2t = 4y - 4 \Rightarrow x = 4y - 5 \Rightarrow \boxed{x - 4y + 5 = 0}$

11) $x = \sqrt{t}$; $y = 1 - t$

a) $(0,1), (1,0), (\sqrt{2},-1), (\sqrt{3},-2), (2,-3)$



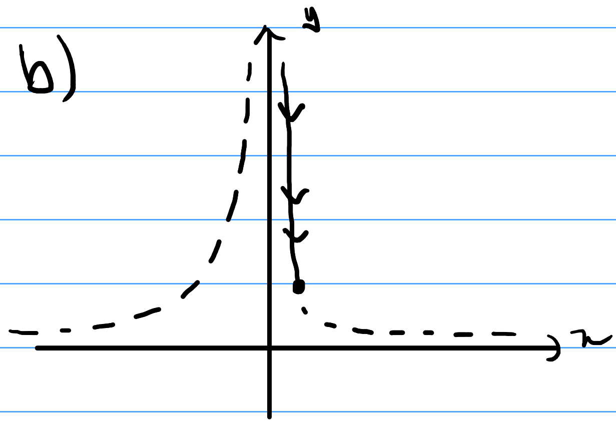
b) $t = x^2 \Rightarrow \boxed{y = 1 - x^2 \text{ (parábola, mas } x \geq 0)}$ //.

15) $x = \cos\theta$ $\quad 0 \leq \theta < \pi/2$

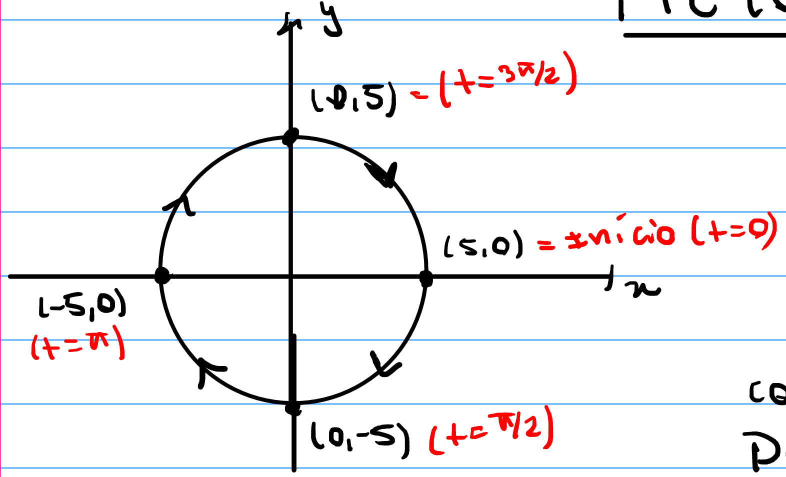
$y = \sec^2\theta$

a) $\Rightarrow \boxed{y = \frac{1}{x^2}, \quad 0 < x \leq 1.}$ //. $\Rightarrow \sec\theta = \frac{1}{\cos\theta}$.

b)

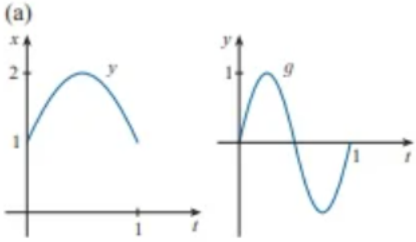


23 $x = 5\cos t$
$y = -5\,\mathrm{sen}\,t$ $\Rightarrow$ $\boxed{x^2+y^2=25}$
$\boxed{t \in [0,\infty)}$.



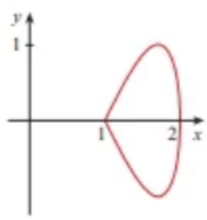
Movimento horário começando em (5,0). Demora 2π s para dar uma volta

30 (a)

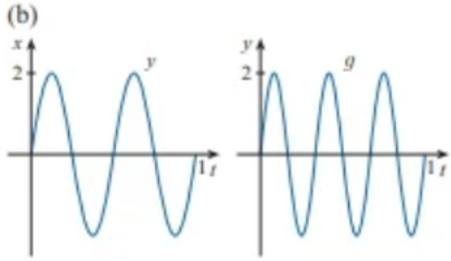


$0 \leq t \leq 1$.
$1 \leq x \leq 2$
$-1 \leq y \leq 1$. $\Rightarrow$

III

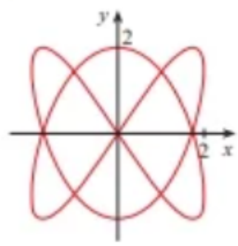


(b)

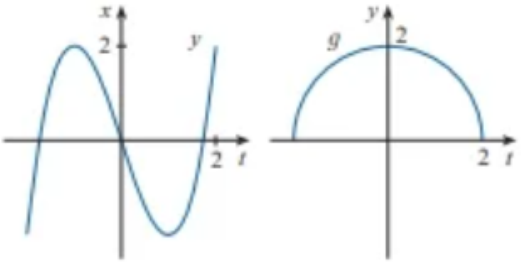


$0 \leq t \leq 1$
$-2 \leq x \leq 2$ $\Rightarrow$
$-2 \leq y \leq 2$
formatos senoidais
(curva de lissajous)

I



(c)

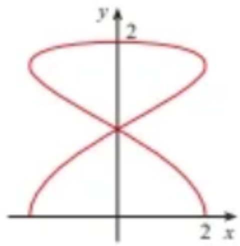


$-2 \leq t \leq 2$

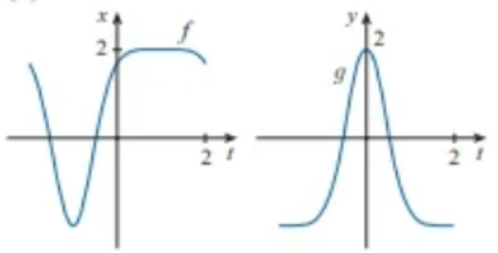
$-2 \leq x \leq 2 \Rightarrow$

$0 \leq y \leq 2$

IV



(d)

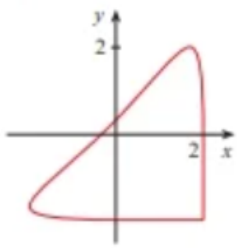


$-2 \leq t \leq 2$

$-2 \leq x \leq 2 \Rightarrow$

$-2 \leq x \leq 2$

II



37 a) $x = x_1 + (x_2 - x_1)t \quad \Rightarrow \quad 0 \leq t \leq 1$

$y = y_1 + (y_2 - y_1)t$

Reta que passa por $(x_1, y_1), (x_2, y_2)$:

$$\boxed{y = \frac{y_2 - y_1}{x_2 - x_1} \cdot x + \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1}, \quad x \in [x_1, x_2]}$$

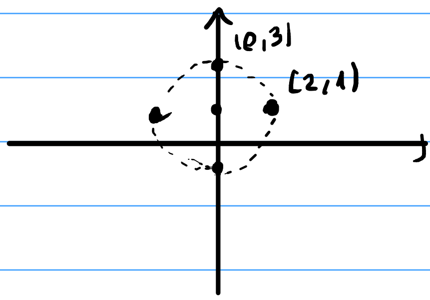
• Isolando t: $\boxed{t = \frac{x - x_1}{x_2 - x_1}}$

$$\Rightarrow y = y_1 + \frac{(y_2 - y_1)}{x_2 - x_1} \cdot (x - x_1) = \boxed{\frac{y_2 - y_1}{x_2 - x_1} \cdot x + \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1}}$$

b) $(-2, 7)$
$(3, -1)$

$$\boxed{\begin{array}{l} x = -2 + 5t \quad \Rightarrow \quad 0 \leq t \leq 1 \\ y = 7 - 8t \end{array}}$$

41 $\quad x^2 + (y-1)^2 = 4$



a) Uma volta - Horário - Início em $(2,1)$

$$\begin{cases} x = 2\cos t \\ y = -2\,\text{sen}\,t + 1 \end{cases} \qquad 0 \leq t \leq 2\pi$$

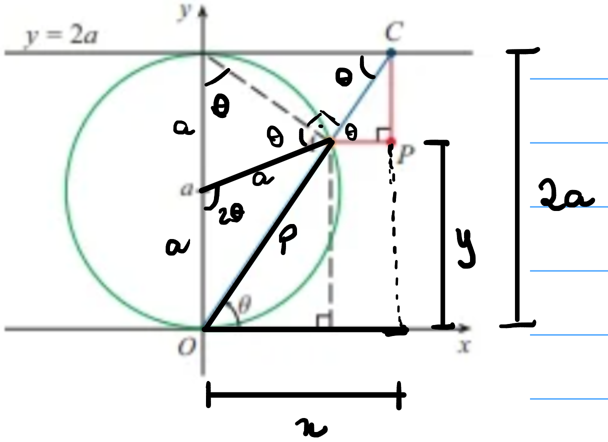
b) Três voltas - Anti-horário - Início em $(2,1)$

$$\begin{cases} x = 2\cos t \\ y = 2\,\text{sen}\,t + 1 \end{cases} \qquad 0 \leq t \leq 6\pi.$$

c) Meia volta - Anti-horário - Início em $(0,3)$

$$\begin{cases} x = 2\cos t \\ y = 2\,\text{sen}\,t + 1 \end{cases} \qquad \pi/2 \leq t \leq 3\pi/2$$

53



$$\operatorname{tg}\theta = \frac{2a}{x} \Rightarrow x = \frac{2a}{\operatorname{tg}\theta} = \boxed{2a\operatorname{cotg}\theta}$$

$$\rho^2 = a^2 + a^2 - 2\cdot a\cdot a\cos 2\theta = 2a^2(1-\cos 2\theta) = 4a^2\operatorname{sen}^2\theta$$

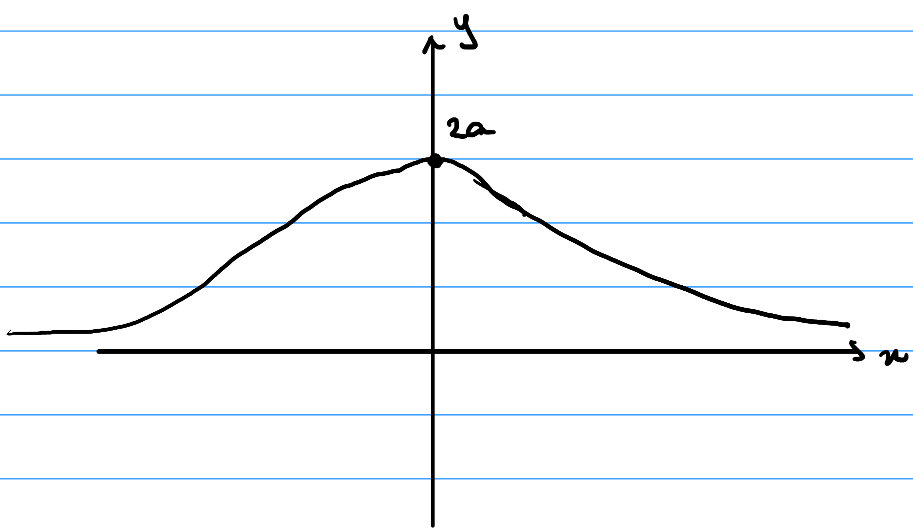
$\therefore \boxed{\rho = 2a\operatorname{sen}\theta.}$

Mas, $\operatorname{sen}\theta = \dfrac{y}{\rho}$ $\therefore \boxed{y = 2a\operatorname{sen}^2\theta}$.

$$\therefore \begin{cases} x = 2a\operatorname{cotg}\theta \\ y = 2a\operatorname{sen}^2\theta \end{cases}$$

$\theta \to 0 \Rightarrow x \to \pm\infty$; $y \to 0$

$y_{máx} = 2a$ $(\theta = \pi/2 + k\pi)$.



Secão 10.2: 1, 3, 5, 7, 11, 15, 17, 19, 21, 23, 33, 35, 37, 39, 40

(1) $x = 2t^3 + 3t$ $\Rightarrow$ $dx/dt = 6t^2 + 3$ $\Rightarrow$ $\boxed{\dfrac{dy}{dx} = \dfrac{4 - 10t}{6t^2 + 3}}$

$y = 4t - 5t^2$ $\quad$ $dy/dt = 4 - 10t$

(3) $x = te^t$ $\Rightarrow$ $dx/dt = e^t(1+t)$ $\Rightarrow$ $\boxed{\dfrac{dy}{dx} = \dfrac{1 + \cos t}{e^t(1+t)}}$

$y = t + \operatorname{sen} t$ $\quad$ $dy/dt = 1 + \cos t$

(5) $\dfrac{dy}{dx} = \dfrac{2^t \ln 2 - 2}{2t + 2}$ $\quad$ $\begin{cases} x = t^2 + 2t \\ y = 2^t - 2t \end{cases}$ $\Rightarrow$ ponto $(15, 2)$ $\hookrightarrow \boxed{t = 3}$

$\therefore \dfrac{dy}{dx}(t=3) = \dfrac{2^3 \ln 2 - 8}{2 \cdot 4} = \boxed{\dfrac{4\ln 2 - 1}{4}}$

(7) $\begin{cases} x = t^3 + 1 \\ y = t^4 + t \end{cases}$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{4t^3 + 1}{3t^2}$ $\Rightarrow t = -1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-3}{3} = \boxed{-1}$ $\Rightarrow$ $\boxed{x = 0 \; ; \; y = 0}$

Reta: $(y - 0) = -1(x - 0) \Rightarrow \boxed{y = -x}$

(11) $\begin{cases} x = \operatorname{sen} t \\ y = \cos^2 t \end{cases}$ no ponto $\left(\frac{1}{2}, \frac{3}{4}\right)$. $\Rightarrow$ $\boxed{\begin{aligned} t &= \pi/6 \\ x &= 1/2 \\ y &= 3/4 \end{aligned}}$ //.

a) Sem eliminar o parâmetro:

$$dy/dx = \frac{2\cancel{\cos t}\cdot(-\operatorname{sen} t)}{\cancel{\cos t}} = \boxed{-2\operatorname{sen} t}$$

$dy/dx\,(t = \pi/6) = -1$; Reta: $(y - 3/4) = (-1)(x - 1/2)$

$$\boxed{\Rightarrow y = -x + \frac{5}{4}} \; //.$$

b) Eliminando o parâmetro:

$y = 1 - \operatorname{sen}^2 t \Rightarrow \boxed{y = 1 - x^2}$ // $\Rightarrow \frac{dy}{dx} = -2x$ $\boxed{\Rightarrow \frac{dy}{dx}(x = \tfrac{1}{2}) = -1}$

Reta: $(y - 3/4) = (-1)(x - 1/2)$ $\boxed{\Rightarrow y = -x + \frac{5}{4}}$ //.

(15) $\begin{aligned} x &= t^2 + 1 \\ y &= t^2 + t \end{aligned}$ $\Rightarrow$ $\frac{dy}{dx} = \boxed{\frac{2t+1}{2t}}$ $\Rightarrow$ $\frac{d^2y}{dx^2} = \dfrac{\frac{d(dy/dx)}{dt}}{\frac{dx}{dt}}$

$$\frac{d^2y}{dx^2} = \dfrac{\frac{4 \cdot t - (2t+1) \cdot 2}{4t^2}}{2t} = \boxed{-\frac{1}{4t^3}} \; //$$

A função terá concavidade para cima, quando $\frac{d^2y}{dx^2} > 0$. Isso ocorre em $x \in (-\infty, 0)$.

(17) $x = e^t$

$y = te^{-t}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^{-t} - t.e^{-t}}{e^t} = \boxed{e^{-2t}(1-t)}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d(dy/dx)}{dt}}{\frac{dx}{dt}} = \frac{e^{-2t}.(-2)(1-t) + e^{-2t}.(-1)}{e^t}$$

$$\boxed{= e^{-3t}(2t-3)}$$

A função é côncava para cima em $x \in (3/2, \infty)$

(19) $x = t - \ln t$

$y = t + \ln t$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + 1/t}{1 - 1/t} = \boxed{\frac{t+1}{t-1}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d(dy/dx)}{dt}}{\frac{dx}{dt}} = \frac{\frac{1.(t-1) - 1(t+1)}{(t-1)^2}}{\frac{t-1}{t}} = -\frac{2t}{(t-1)^3}$$

A função é côncava para cima em $x \in (0, 1)$

21 $x = t^3 - 3t$ $\qquad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \boxed{\dfrac{2t}{3t^2-3}}$

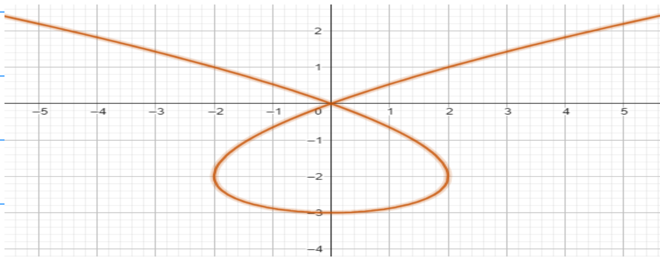
$y = t^2 - 3$

• Tangentes Horizontais: $2t = 0$; $3t^2 - 3 \neq 0$ $\Rightarrow \boxed{t = 0}$

Ponto: $(0, -3)$

• Tangentes verticais: $2t \neq 0$; $3t^2 - 3 = 0$ $\Rightarrow \boxed{t = \pm 1}$

Pontos: $(-2, -2)$, $(2, -2)$



23 $x = \cos\theta$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{-\operatorname{sen} 3\theta \cdot 3}{-\operatorname{sen}\theta} = \boxed{\dfrac{3\operatorname{sen} 3\theta}{\operatorname{sen}\theta}}$

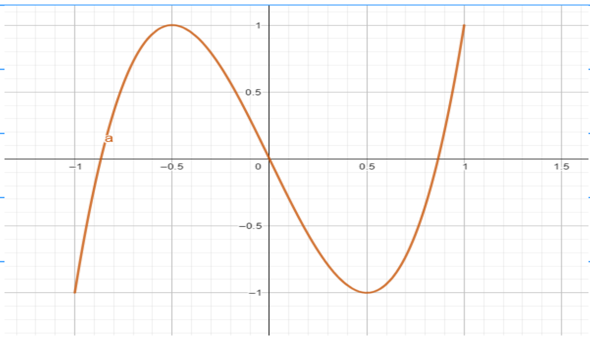
$y = \cos 3\theta$

• Tangentes Horizontais: $3\operatorname{sen} 3\theta = 0$; $\operatorname{sen}\theta \neq 0$ $\boxed{\theta = k\pi/3}$

Pontos: $(1/2, -1)$, $(-1/2, 1)$.

• Tangentes verticais: $3\operatorname{sen}3\theta \neq 0$; $\operatorname{sen}\theta = 0$ $\boxed{\theta = k\pi}$

Pontos: Não há (quando $\operatorname{sen}\theta = 0$, $3\operatorname{sen}3\theta = 0$).

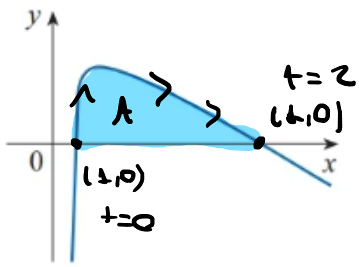


(33) $x = 3t^2 + 1$
$y = t^3 - t$

$\frac{dy}{dx} = \frac{3t^2}{6t} = \boxed{\frac{t}{2}}$

$\frac{dy}{dx} = \frac{1}{2} \Rightarrow \boxed{t = 1} \Rightarrow \boxed{\text{Ponto} = (4,0)}$.

(35)



Calcular a área sabendo que
$\begin{cases} x = t^3 + 1 \\ y = 2t - t^2 \end{cases}$

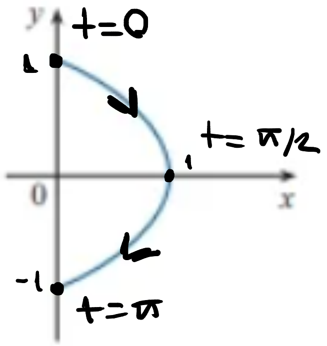
Observando o gráfico: $y = 0 \Rightarrow \boxed{t = 0 \text{ ou } t = 2}$.

$$A = \int_{x=a}^{x=b} y\, dx = \int_0^2 (2t - t^2) \cdot (3t^2)\, dt = \int_0^2 (6t^3 - 3t^4)\, dt$$

$$= \left(\frac{6}{4} \cdot t^4 - \frac{3}{5} \cdot t^5 \right) \Bigg|_0^2 = 2^4 \cdot \frac{6}{4} - \frac{3 \cdot 2^5}{5} = 4 \cdot 6 - \frac{96}{5}$$

$\boxed{= 4{,}8 \text{ u.a}}$

37



$$\begin{cases} x = \operatorname{sen}^2 t \\ y = \cos t \end{cases}$$

$$x = 0 \Rightarrow \boxed{t = k\pi,\ k \in \mathbb{Z}}$$

$$A = \int_1^{-1} x\,dy = \int_0^{\pi} \operatorname{sen}^2 t \cdot (-\operatorname{sen} t)\,dt = \int_0^{\pi} (1-\cos^2 t)(-\operatorname{sen} t)\,dt$$

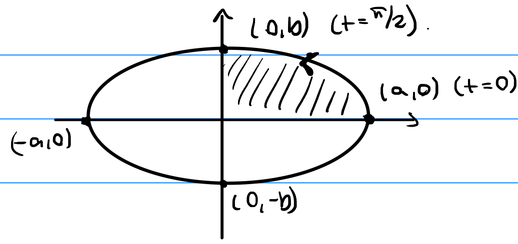
$$\Rightarrow \begin{array}{l} u = \cos t \\ du = -\operatorname{sen} t\,dt \end{array} \quad \Rightarrow \int_0^{\pi} (1-u^2)\,du = \left(u - \frac{u^3}{3}\right)\Big|_0^{\pi}$$

$$= \left(\cos t - \frac{\cos^3 t}{3}\right)\Big|_0^{\pi} = \left(\left(-1 + \frac{1}{3}\right) - \left(1 - \frac{1}{3}\right)\right) = \frac{2}{3} - 2$$

$$= -4/3 \quad \therefore \text{Área} = 4/3$$

39

$$\begin{array}{l} x = a\cos\theta \\ y = b\operatorname{sen}\theta \end{array} \qquad 0 \le \theta \le 2\pi.$$

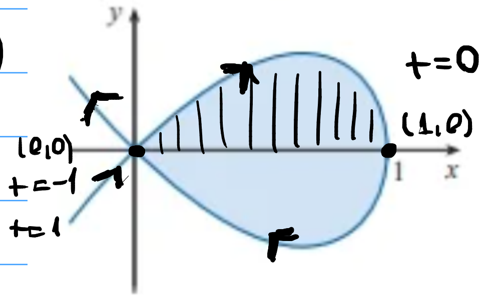


$$A = 4 \cdot \int_0^{\pi/2} b\operatorname{sen}\theta \cdot a(-\operatorname{sen}\theta)\,d\theta$$

$$= -\frac{4ab}{2} \cdot \int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta = \frac{4ab}{2} \cdot \left(\theta + \frac{\operatorname{sen} 2\theta}{2}\right)\Big|_0^{\pi/2},$$

$$= -2ab \cdot \pi/2 = -\pi ab \quad \boxed{\therefore \text{Área é } \pi ab}$$

40



$$\begin{cases} x = 1 - t^2 \\ y = t - t^3 \end{cases}$$

$y = 0 \Rightarrow t = 0 \,;\, t = 1 \,;\, t = -1$

$$\Rightarrow 2 \cdot \int_{-1}^{0} (t - t^3) \cdot (-2t)\, dt = 4 \int_{-1}^{0} (t^4 - t^2)\, dt$$

$$= 4 \left(\frac{t^5}{5} - \frac{t^3}{3} \right) \Big|_{-1}^{0} = 4 \left(0 - \left(-\frac{1}{5} + \frac{1}{3} \right) \right)$$

$$= 4 \left(\frac{1}{5} - \frac{1}{3} \right) = \frac{-8}{15}$$

$\boxed{\therefore \text{ Área é } 8/15}$ //.

$$\bullet \begin{cases} x = f(t) \\ y = g(t) \end{cases} \qquad \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \quad \text{(Regra da cadeia)}$$

$$\boxed{\therefore \frac{dy}{dt} = \frac{dy/dt}{dx/dt}}$$

$$\bullet \begin{cases} x = f(t) \\ y = g(t) \end{cases} \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{df}{dx}$$

Mas, $\dfrac{dy}{dt} = \dfrac{dy/dt}{dx/dt}$, logo $\dfrac{d^2y}{dx^2} = \dfrac{df/dt}{dx/dt}$ $\boxed{= \dfrac{\frac{d}{dt}(dy/dx)}{\frac{dx}{dt}}}$

$$\bullet \begin{cases} x = f(t) \\ y = g(t) \end{cases} \Rightarrow \int_a^b y\,dx$$

$$\boxed{\begin{array}{l} f(\alpha) = a. \\ f(\beta) = b. \\ y = g(t) \\ dx = f'(t).dt \end{array}}$$

$$\boxed{\therefore \int_a^b y\,dx = \int_\alpha^\beta g(t).f'(t)\,dt \quad \text{ou} \quad \int_\beta^\alpha g(t).f'(t)\,dt}$$

Se a curva for descrita no sentido anti-horário $\Rightarrow A > 0$

Se a curva for descrita no sentido horário $\Rightarrow A < 0$