LISTA 5

Seção 14.5: 1, 3, 5, 7, 11, 13, 15, 17, 19, 25, 27, 39, 43, 47, 49, 56, 57

(1) $z = x^2y + xy^2 \qquad x = 3t \; ; \; y = t^2$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (2xy + y^2) \cdot 3 + (x^2 + 2xy) \cdot 2t$$

$$\frac{dz}{dt} = 3(6t^3 + t^4) + 2t(9t^2 + 6t^3) \boxed{= 36t^3 + 15t^4}$$

(3) $z = xy^3 - x^2y \qquad x = t^2 + 1 \; ; \; y = t^2 - 1.$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (y^3 - 2xy) \cdot 2t + (3xy^2 - x^2) \cdot 2t$$

$$\boxed{= 2t(y^3 - x^2 + 3xy^2 - 2xy)}$$

(5) $z = \operatorname{sen} x \cos y \qquad x = \sqrt{t} \; ; \; y = 1/t$

$$\frac{dz}{dt} = (\cos x \cos y) \cdot \frac{1}{2\sqrt{t}} - \operatorname{sen} x \operatorname{sen} y \cdot (-1) \cdot \frac{1}{t^2}$$

$$\boxed{\frac{dz}{dt} = \frac{\cos x \cos y}{2\sqrt{t}} + \frac{\operatorname{sen} x \operatorname{sen} y}{t^2}}$$

(7) $w = xe^{y/z}$ $\quad x = t^2 ; y = 1 - t ; z = 1 + 2t.$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt}$$

$$\frac{dw}{dt} = e^{y/z}\cdot 2t + \frac{1}{z}\cdot xe^{y/z}\cdot(-1) + (-1)\cdot\frac{1}{z^2}yxe^{y/z}\cdot 2$$

$$\boxed{\frac{dw}{dt} = 2t\cdot e^{y/z} - \frac{xe^{y/z}}{z} - \frac{2xye^{y/z}}{z^2}}$$

(11) $z = (x-y)^5$ $\quad x = s^2 t \; ; y = st^2$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = 5(x-y)^4\cdot 2st + 5(x-y)^4\cdot t^2$$

$$\boxed{\frac{\partial z}{\partial s} = 5(x-y)^4(2st + t^2)}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = 5(x-y)^4\cdot s^2 + 5(x-y)^4\cdot 2st$$

$$\boxed{\frac{\partial z}{\partial t} = 5(x-y)^4(s^2 + 2st)}$$

(13) $z = \ln(3x+2y)$ $\quad x = s \cdot \text{sen}\,t \; ; \; y = t \cdot \cos s$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s} = \frac{3}{3x+2y} \cdot \text{sen}\,t + \frac{2}{3x+2y} \cdot (-t\,\text{sen}\,s)$$

$$\boxed{\frac{\partial z}{\partial s} = \frac{3\,\text{sen}\,t - 2t\,\text{sen}\,s}{3x+2y}}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} = \frac{3}{3x+2y} \cdot s\cos t + \frac{2}{3x+2y} \cdot \cos s$$

$$\boxed{\frac{\partial z}{\partial t} = \frac{3s\cos t + 2\cos s}{3x+2y}}$$

(15) $z = \frac{\text{sen}\,\theta}{r}$; $\quad \theta = s^2 + t^2 \quad r = st$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} + \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial s} = \frac{\cos\theta}{r} \cdot 2s - \frac{\text{sen}\,\theta}{r^2} \cdot t = \boxed{\frac{2rs\cos\theta - t\,\text{sen}\,\theta}{r^2}}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} + \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial t} = \frac{\cos\theta}{r} \cdot 2t - \frac{\text{sen}\,\theta}{r^2} \cdot s = \boxed{\frac{2rt\cos\theta - s\,\text{sen}\,\theta}{r^2}}$$

(17) $p(2) = f(4,5)$

$p'(2) = f_x(4,5) \cdot g'(2) + f_y(4,5) \cdot h'(2) = 2 \cdot (-3) + 8 \cdot 6 = \boxed{42}$

(19) $g(u,v) = f(e^u + \operatorname{sen} v,\ e^u + \cos v)$

	f	g	f_x	f_y
(0, 0)	3	6	4	8
(1, 2)	6	3	2	5

- $\dfrac{\partial g}{\partial u} = fx \cdot e^u + fy \cdot e^u \Rightarrow \dfrac{\partial g}{\partial u}(0,0) = 2 \cdot 1 + 5 \cdot 1 = \boxed{7}$

- $\dfrac{\partial g}{\partial v} = fx \cdot \cos v - fy \cdot \operatorname{sen} v \Rightarrow \dfrac{\partial g}{\partial v}(0,0) = 2 \cdot 1 = \boxed{2}$

(25) $z = x^4 + x^2 y$ $\qquad x = s + 2t - u\ ;\ y = stu^2$

$s = 4,\ t = 2,\ u = 1$

$x = 7\ ;\ y = 8.$

- $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial s} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial s} = 4x^3 + 2xy + x^2 \cdot tu^2 = 4 \cdot 7^3 + 2 \cdot 7 \cdot 8 + 7^2 \cdot 2 \cdot 1 = \boxed{1582}$

- $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial t} = (4x^3 + 2xy) \cdot 2 + x^2 \cdot su^2 = 2 \cdot (4 \cdot 7^3 + 2 \cdot 7 \cdot 8) + 7^2 \cdot 4 \cdot 1 = 3164$

- $\dfrac{\partial z}{\partial u} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial u} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial u} = (4x^3 + 2xy)(-1) + x^2 \cdot 2stu = -(4 \cdot 7^3 + 2 \cdot 7 \cdot 8) + 7^2 \cdot 2 \cdot 4 \cdot 2 \cdot 1 = -1484 + 784 = \boxed{-700}$

(27) $w = xy + yz + xz$

$x = r\cos\theta \,;\; y = r\sin\theta \,;\; z = r\theta$

$\boxed{r = 2 \,;\; \theta = \pi/2 \,;\; x = 0 \,;\; y = 2 \,;\; z = \pi}$

$$\cdot\ \frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r} = (y+z)\cos\theta + (x+z)\sin\theta + (x+y)\theta$$

$$\Rightarrow (2+\pi)\cdot 0 + \pi\cdot 1 + 2\cdot\pi/2 = \boxed{2\pi}$$

$$\cdot\ \frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial \theta} = (y+z)(-r\sin\theta) + (x+z)r\cos\theta + (x+y)\cdot r$$

$$\Rightarrow (2+\pi)(-2) + \pi\cdot 2\cdot 0 + 2\cdot 2 = \boxed{-2\pi}$$

(39) $T(x,y)$ em grows

$x = \sqrt{1+t}$

$y = 2 + 1/3\, t$

$$\frac{dT}{dt} = T_x(2,3)\cdot\frac{dx}{dt}(2,3) + T_y(2,3)\cdot\frac{dy}{dt}(2,3) = 4\cdot\frac{1}{2\sqrt{1+3})} + 3\cdot\frac{1}{3}$$

$$\boxed{= 2\ {}^\circ C/s}$$

(43) $l = 1m$, $w = h = 2m$.

$dl/dt = 2\,m/s$

$dw/dt = 2\,m/s$

$dh/dt = -3\,m/s$

a) $V = lwh$

$$\frac{dV}{dt} = \frac{\partial V}{\partial l}\cdot\frac{dl}{dt} + \frac{\partial V}{\partial w}\cdot\frac{dw}{dt} + \frac{\partial V}{\partial h}\cdot\frac{dh}{dt} = wh\cdot\frac{dl}{dt} + lh\frac{dw}{dt} + lw\cdot\frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\cdot2\cdot2 + 2\cdot1\cdot2 + 1\cdot2\cdot(-3) = 12-6 = \boxed{6}.$$

b) $A = 2(lw + lh + wh)$

$$\frac{dA}{dt} = \frac{\partial A}{\partial l}\cdot\frac{dl}{dt} + \frac{\partial A}{\partial w}\cdot\frac{dw}{dt} + \frac{\partial A}{\partial h}\cdot\frac{dh}{dt} = 2\left[(h+w)\frac{dl}{dt} + (l+h)\frac{dw}{dt} + (l+w)\frac{dh}{dt}\right]$$

$$\Rightarrow 2\left[(2+2)\cdot2 + (1+2)\cdot2 + (1+2)(-3)\right] = 2(8+6-9) = \boxed{10}.$$

c) $D = \sqrt{l^2+w^2+h^2}$

$$\frac{dD}{dt} = \frac{\partial D}{\partial l}\cdot\frac{dl}{dt} + \frac{\partial D}{\partial w}\cdot\frac{dw}{dt} + \frac{\partial D}{\partial h}\cdot\frac{dh}{dt}$$

$$\frac{dD}{dt} = \frac{\cancel{2}(l\cdot dl/dt + w\cdot dw/dt + h\cdot dh/dt)}{\cancel{2}\sqrt{l^2+w^2+h^2}} = \frac{1\cdot2 + 2\cdot2 + 2\cdot(-3)}{3}$$

$$\boxed{=0}.$$

(47) $A = \dfrac{l_1 l_2 \operatorname{sen}\theta}{2}$ $\qquad \dfrac{dl_1}{dt} = 3\,cm/s \;;\; \dfrac{dl_2}{dt} = -2\,cm/s$

$\dfrac{dA}{dt} = 0 \;;\; \dfrac{d\theta}{dt} = ?$

$l_1 = 20\,cm$

$l_2 = 30\,cm$

$\theta = \pi/6$.

$$\frac{dA}{dt} = \frac{\partial A}{\partial l_1}\cdot\frac{dl_1}{dt} + \frac{\partial A}{\partial l_2}\cdot\frac{dl_2}{dt} + \frac{\partial A}{\partial \theta}\cdot\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{-\left(\dfrac{\partial A}{\partial l_1}\cdot\dfrac{dl_1}{dt} + \dfrac{\partial A}{\partial l_2}\cdot\dfrac{dl_2}{dt}\right)}{\dfrac{\partial A}{\partial \theta}}$$

$$\frac{d\theta}{dt} = \frac{-\left(\dfrac{l_2 \operatorname{sen}\theta}{2}\dfrac{dl_1}{dt} + \dfrac{l_1 \operatorname{sen}\theta}{2}\dfrac{dl_2}{dt}\right)}{\dfrac{l_1 l_2 \cos\theta}{2}}$$

$$\frac{d\theta}{dt} = \frac{-\left(\dfrac{30 \operatorname{sen}\pi/6}{2}\cdot 3 - \dfrac{20 \operatorname{sen}\pi/6}{2}\cdot 2\right)\cdot 2}{20\cdot 30\cdot\cos\pi/6}$$

$$= \frac{-\left(\dfrac{15\cdot 3}{\cancel{2}} - \dfrac{10\cdot 2}{\cancel{2}}\right)\cdot\cancel{2}}{20\cdot 30\sqrt{3}} = \frac{20-45}{600\sqrt{3}} = \frac{-25\sqrt{3}}{3\cdot 600} = \boxed{\frac{-\sqrt{3}}{72}}$$

(49) $z = f(x,y)$ $\quad x = r\cos\theta$
$y = r\,\text{sen}\,\theta$

a) $\dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}\cdot\dfrac{\partial x}{\partial r} + \dfrac{\partial z}{\partial y}\cdot\dfrac{\partial y}{\partial r} = \dfrac{\partial z}{\partial x}\cdot\cos\theta + \dfrac{\partial z}{\partial y}\cdot\text{sen}\,\theta$

$\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x}\cdot\dfrac{\partial x}{\partial \theta} + \dfrac{\partial z}{\partial y}\cdot\dfrac{\partial y}{\partial \theta} = \dfrac{\partial z}{\partial x}\cdot(-r\,\text{sen}\,\theta) + \dfrac{\partial z}{\partial y}\cdot r\cos\theta.$

b) $\left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2 = \left(\dfrac{\partial z}{\partial x}\right)\cos^2\theta + \dfrac{\partial z}{\partial x}\cdot\dfrac{\partial z}{\partial y}\cdot\text{sen}\,2\theta + \left(\dfrac{\partial z}{\partial y}\right)^2\cdot\text{sen}^2\theta$

$+\left(\dfrac{\partial z}{\partial x}\right)^2\cdot r^2\,\text{sen}^2\theta\cdot\dfrac{1}{r^2} - \dfrac{\partial z}{\partial x}\cdot\dfrac{\partial z}{\partial y}\cdot\text{sen}\,2\theta\cdot r^2\cdot\dfrac{1}{r^2} + \left(\dfrac{\partial z}{\partial y}\right)^2\cdot\cos^2\theta\, r^2\cdot\dfrac{1}{r^2}$

$$\boxed{\therefore \left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\cdot\left(\dfrac{\partial z}{\partial r}\right)^2 = \left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2}$$

(56) Funções Homogêneas: $f(tx, ty) = t^n f(x,y)$ e $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ contínuas.

$f(x,y) = x^2y + 2xy^2 + 5y^3$

- $f(tx,ty) = t^2x^2\cdot ty + 2tx\cdot ty^2 + 5t^3y^3 = t^3(x^2y + 2xy^2 + 5y^3)$
  $\Rightarrow f(tx,ty) = t^3 f(x,y).$

- $\frac{\partial f}{\partial x} = 2xy + 2y \Rightarrow \frac{\partial^2 f}{\partial x^2} = 2y$ (contínuo)

- $\frac{\partial f}{\partial y} = x^2 + 4xy + 15y^2 \Rightarrow \frac{\partial^2 f}{\partial y^2} = 4x + 30y$ (contínuo).

(57) f homogênea do grau n: $f(tx, ty) = t^n f(x,y)$

a) $\frac{df}{dt}(xt, yt) = \frac{\partial f}{\partial x} \cdot \frac{d(tx)}{dt} + \frac{\partial f}{\partial y} \cdot \frac{d(ty)}{dt} = n \cdot t^{n-1} \cdot f(x,y)$

$\Rightarrow \frac{\partial f}{\partial x} \cdot x + \frac{\partial f}{\partial y} \cdot y = n t^{n-1} f(x,y)$, mas $t = 1$, logo

$$\boxed{\frac{\partial f}{\partial x} x + \frac{\partial f}{\partial y} y = n f(x,y)}$$

b) $$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = n(n-1) f(x,y)$$

$\frac{d^2 f}{dt^2} = n(n-1) t^{n-1} f(x,y) \Rightarrow t = 1 \Rightarrow n(n-1) f(x,y)$

$\frac{d^2 f}{dt^2} = \frac{d}{dt}\left(\frac{\partial f}{\partial x} x\right) + \frac{d}{dt}\left(\frac{\partial f}{\partial y} y\right)$

$$x \cdot \frac{d}{dt}\left(\frac{\partial f}{\partial x}\right) + y \cdot \frac{d}{dt}\left(\frac{\partial f}{\partial y}\right)$$

$$= x \cdot \left(\frac{\partial^2 f}{\partial x^2} \cdot \frac{d(tx)}{dx} + \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{d(ty)}{dt}\right) + y\left(\frac{\partial^2 f}{\partial y^2} \frac{d(ty)}{dt} + \frac{\partial^2 f}{\partial y \partial x} \cdot \frac{d(tx)}{dt}\right)$$

$$\frac{d}{dt}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \cdot \frac{d\,tx}{dt} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y} \cdot \frac{d\,ty}{dt}\right)$$

Regra da Cadeia

$$\frac{d^2 f}{dt^2} = \frac{\partial^2 f}{\partial x^2} \cdot x^2 + 2xy \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} \cdot y^2$$

$$E \quad \frac{\partial^2 f}{\partial x^2} \cdot x^2 + 2xy \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} \cdot y^2 = n(n-1)\, f(x,y)$$

$t = 1$

Seção 14.6: 3, 5, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31, 34, 35, 39, 41, 45, 46, 49, 51, 55, 61

(3) $w = f(t, v)$. $D_u f(-20, 30)$ em $u = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

Velocidade do vento (km/h)

Temperatura real (°C): T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

$$D_u f(-20,30) = \frac{\partial f}{\partial x}(-20,30)\cdot\frac{\sqrt{2}}{2} + \frac{\partial f}{\partial y}(-20,30)\cdot\frac{\sqrt{2}}{2}$$

$$D_u f(-20,30) \approx \left(\frac{-39-(-33)}{2\cdot(-5)} + \frac{-26-(-33)}{2\cdot 5}\right)\frac{\sqrt{2}}{2} + \left(\frac{-34-(-33)}{2\cdot 10} + \frac{-30-(-33)}{2(-10)}\right)\frac{\sqrt{2}}{2}$$

$$\approx \frac{13\sqrt{2}}{20} - \frac{2\sqrt{2}}{20} = \frac{11\sqrt{2}}{20} \boxed{\approx 0,778}$$

(5) $D_u f(x_0, y_0) = \|\nabla f(x_0, y_0)\| \cdot \|u\| \cdot \cos\theta$

$$\frac{\partial f}{\partial x} = -\operatorname{sen}(xy)\, y^2 ; \quad = \boxed{0}$$

$$\frac{\partial f}{\partial y} = \cos(xy) - xy\,\text{sen}(xy) \Rightarrow 1 - 0 \boxed{=1}$$

$$D_u f(0,1) = \|(0,1)\| \cdot 1 \cdot \cos \pi/4 \boxed{= \sqrt{2}/2}$$

(9) $f(x,y) = x/y$; $P(2,1)$; $u = 3/5\,\hat{i} - 4/5\,\hat{j} = (3/5, -4/5)$.

a) $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \left(\frac{1}{y}, -\frac{x}{y^2}\right) \boxed{= \frac{1}{y}\hat{i} - \frac{x}{y^2}\hat{j}}$

b) $\nabla f(2,1) = \boxed{\hat{i} - 2\hat{j}}$

c) $D_u f(2,1)$ en u: $(1,-2) \cdot (3/5, -4/5) \boxed{= 11/5}$

(11) $f(x,y,z) = x^2yz - xyz^3$; $P = (2,-1,1)$; $u = (0, 4/5, -3/5)$.

a) $\nabla f = \left(2xyz - yz^3,\ x^2z - xz^3,\ x^2y - 3xyz^2\right)$

b) $\nabla f(2,-1,1) = \left(2\cdot2(-1) - (-1)\cdot1^3,\ 2^2\cdot1 - 2\cdot1,\ 2^2(-1) - 3\cdot2\cdot(-1)\cdot1^2\right)$
$\boxed{= (-3, 2, 2)}$

c) $D_u f(2,-1,1) = (-3,2,2)(0, 4/5, -3/5) = 0 + 8/5 - 6/5 \boxed{= 2/5}$

(13) $f(x,y) = e^x \operatorname{sen} y$ $\quad P = (0, \pi/3) \quad v = (-6, 8)$

$\frac{\partial f}{\partial x} = e^x \operatorname{sen} y \Rightarrow \boxed{\sqrt{3}/2}$; $\frac{\partial f}{\partial y} = e^x \cos y \Rightarrow \boxed{1/2}$

$u = (-6/10, 8/10)$

$D_u f(0, \pi/3) = (\sqrt{3}/2, 1/2) \cdot (-6/10, 8/10) = -3\sqrt{3}/10 + 4/10$
$\boxed{= \frac{4 - 3\sqrt{3}}{10}}$

(16) $g(s,t) = s\sqrt{t}$; $P = (2,4)$; $v = 2\hat{i} - \hat{j}$

$\frac{\partial g}{\partial s} = \sqrt{t} \Rightarrow \boxed{2}$; $\frac{\partial g}{\partial t} = \frac{s}{2\sqrt{t}} \Rightarrow \boxed{1/2}$; $u = (2/\sqrt{5}, -1/\sqrt{5})$.

$D_u g(2,4) = (2, 1/2) \cdot (2/\sqrt{5}, -1/\sqrt{5}) = \frac{4}{\sqrt{5}} - \frac{1}{2\sqrt{5}} = \boxed{\frac{7}{2\sqrt{5}}}$

(17) $f(x,y,z) = x^2 y + y^2 z \quad P = (1,2,3) \quad v = (2,-1,2)$

$\frac{\partial f}{\partial x} = 2xy \Rightarrow \boxed{4}$; $\frac{\partial f}{\partial y} = x^2 + 2yz \Rightarrow \boxed{13}$; $\frac{\partial f}{\partial z} = y^2 \Rightarrow \boxed{4}$

$u = (2/3, -1/3, 2/3)$

$D_u f(1,2,3) = (4, 13, 4)(2/3, -1/3, 2/3) = 8/3 - 13/3 + 8/3 = \boxed{1}$

21 $f(x,y) = x^2y^2 - y^3$ $\quad P(1,2)$ e $Q(-3,5)$

$\frac{\partial f}{\partial x} = 2xy^2 \Rightarrow \boxed{8}$; $\frac{\partial f}{\partial y} = 2x^2y - 3y^2 \Rightarrow \boxed{-8}$

$\overrightarrow{PQ} = (-4,3) \Rightarrow u = (-4/5, 3/5)$

$D_u f = (8,-8)(-4/5, 3/5) = -32/5 - 24/5 = \boxed{-56/5}$

25 $f(x,y,z) = xy - xy^2z^3$ $\quad P = (2,-1,1).$ $\quad Q = (5,1,7).$

$\frac{\partial f}{\partial x} = y - y^2z^3 \Rightarrow \boxed{-2}$; $\frac{\partial f}{\partial y} = x - 2xyz^3 \Rightarrow \boxed{6}$;

$\frac{\partial f}{\partial z} = -3xy^2z^2 \Rightarrow \boxed{-6}$

$\overrightarrow{PQ} = (3,2,6) \Rightarrow u = (3/7, 2/7, 6/7)$

$D_u f = (-2,6,-6) \cdot (3/7, 2/7, 6/7) = -\frac{6}{7} + \frac{12}{7} - \frac{36}{7} = \boxed{-\frac{30}{7}}$

(27) $f(x,y) = 5xy^2$ $(3,-2)$

$$\nabla f(3,-2) = \left(\frac{\partial f}{\partial x}(3,-2), \frac{\partial f}{\partial y}(3,-2)\right) = (20,-60).$$

$$\text{Var máx} = \|\nabla f\| = \sqrt{400+3600} = \sqrt{4000} = \boxed{20\sqrt{10}}$$

(29) $f(x,y) = \operatorname{sen}(xy)$ $(1,0)$

$$\nabla f(1,0) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (0,1)$$

$$\|\nabla f(1,0)\| = \boxed{1}$$

(31) $f(x,y,z) = \dfrac{x}{y+z}$ $(8,1,3)$

$$\frac{\partial f}{\partial x} = \frac{1}{y+z} \Rightarrow \boxed{\frac{1}{4}} ; \quad \frac{\partial f}{\partial y} = -\frac{x}{(y+z)^2} \Rightarrow \boxed{-\frac{1}{2}}$$

$$\frac{\partial f}{\partial z} = -\frac{x}{(y+z)^2} \Rightarrow \boxed{-\frac{1}{2}}$$

$$\|\nabla f(8,1,3)\| = \|(1/4, -1/2, -1/2)\| = \boxed{\sqrt{9/16}} = 3/4$$

34) $f(x,y) = x^2 + xy^3$ em $(2,1)$ e $D_u f = 2$

$D_u f(2,1) = \langle \nabla f(2,1), u \rangle = 2$

$2 = \langle (2.2+1, 3.2), (a,b) \rangle$, $\|u\| = 1$ e $a^2+b^2=1$.

$2 = 5a + 6b$

$$\begin{cases} 5a+6b=2 \\ a^2+b^2=1 \end{cases} \Rightarrow b = \pm\sqrt{1-a^2}$$

$$5a \pm 6\sqrt{1-a^2} = 2$$

$$36(1-a^2) = 4 - 20a + 25a^2$$

$$36 - 36a^2 = 4 - 20a + 25a^2$$

$$61a^2 - 20a - 32 = 0$$

$$a = \frac{20 \pm \sqrt{400 + 4.32.61}}{2.61} = \frac{20 \pm 4\sqrt{513}}{2.61}$$

$$\boxed{a = \frac{10 \pm 2\sqrt{513}}{61}}$$

$$a^2 = \frac{100 \pm 40\sqrt{513} + 8208}{3721} = \boxed{\frac{8308 \pm 40\sqrt{513}}{3721}}$$

$\boxed{a^2 > 1}$ $\therefore$ $\boxed{\text{Não há}}$

35) $f(x,y) = x^2 + y^2 - 2x - 4y$

Todos os pontos em que a direção de maior variação é (1,1).

$$\nabla f = (2x-2,\ 2y-4) = \boxed{2(x-1,\ y-2)}.$$

Tem maior variação na direção do vetor gradiente

$$D_u f\ máx = 2\sqrt{(x-1)^2 + (y-2)^2}$$

39) $V(x,y,z) = 5x^2 - 3xy + xyz$; $P = (3,4,5)$

a) $v = (1,1,-1) \Rightarrow u = (1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$.

$\nabla V(3,4,5) = (10.3 - 3.4 + 4.5, -3.3 + 3.5, 3.4) = (38, 6, 12)$

$D_u V(3,4,5) = (38, 6, 12)(1/\sqrt{3}, 1/\sqrt{3} + -1/\sqrt{3}) = \boxed{\dfrac{32}{\sqrt{3}}}$

b) Direção do gradiente $(38, 6, 12)$

c) $D_u V_{máx} = \|(38,6,12)\| = \sqrt{1624} = \boxed{4\sqrt{406}}$.

41) $f(x,y)$ $\quad \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ contínuas.

$A(1,3)$ $\quad D_u f(A)$ em $\vec{AB}$ é 3
$B(3,3)$ $\quad D_u f(A)$ em $\vec{AC}$ é 26
$C(1,7)$ $\quad D_u f(A)$ em $\vec{AD}$ é ?
$D(6,15)$

$\vec{AB} = (2,0) \Rightarrow u = (1,0)$
$\vec{AC} = (0,4) \Rightarrow u = (0,1)$
$\vec{AD} = (5,12) \Rightarrow u = (5/13, 12/13)$.

$$\begin{cases} \partial f/\partial x(A) = 3 \\ \partial f/\partial y(A) = 26 \end{cases} \Rightarrow D_u f(A) \text{ em } AD \text{ é } \frac{15}{13} + \frac{312}{13} = \boxed{\frac{327}{13}}$$

(45) $\partial_u^2 f(x,y) = D_u\left(D_u f(x,y)\right)$

$f(x,y) = x^3 + 5x^2y + y^3$; $u = (3/5, 4/5)$; $P = (2,1)$

$$D_u^2 f = f_{xx} \cdot a^2 + 2f_{xy} \cdot ab + f_{yy} \cdot b$$

$f_{xx} = 6x + 10y \Rightarrow 12 + 10 = 22$
$f_{yy} = 6y. \Rightarrow 6$
$f_{xy} = 10x \Rightarrow 20.$

$$D_u^2 f(2,1) = 22 \cdot \frac{9}{25} + 2 \cdot 20 \cdot \frac{12}{25} + 6 \cdot \frac{16}{25} = \boxed{\frac{774}{25}}$$

(46) a) $D_u^2 f = f_{xx}\, a^2 + 2f_{xy}\, ab + f_{yy}\, b^2.$

$$D_u\left[f_x \cdot a + f_y \cdot b\right] = D_u(h(x,y))$$

$$= \lim_{t \to 0} \frac{h(x+a.t,\, y+b.t) - h(x,y)}{t}$$

$$= h_x \cdot a + h_y \cdot b$$

$$= \frac{\partial}{\partial x}\left(f_x \cdot a + f_y \cdot b\right) \cdot a + \frac{\partial}{\partial y}\left(f_x \cdot a + f_y \cdot b\right) \cdot b$$

$= (f_{xx} \cdot a + f_{xy} \cdot b) a + (f_{yx} \cdot a + f_{yy} \cdot b) \cdot b$

$$\boxed{D_u^2 f = f_{xx} \cdot a^2 + 2 f_{xy} \cdot ab + f_{yy} \cdot b^2}$$

$$\boxed{f_{xy} = f_{yx}}$$

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b) $f(x,y) = x e^{2y}$ em $v = (4,6) \therefore u = (4/\sqrt{52}, 6/\sqrt{52})$.

$f_{xx} = 0$; $f_{yy} = 4x e^{2y}$; $f_{xy} = 2 \cdot e^{2y}$

$$D_u^2 f = \cancel{0 \cdot \frac{16}{52}} + 2 \cdot \frac{24}{52} \cdot 2e^{2y} + 4x \cdot e^{2y} \cdot \frac{36}{52}$$

$$= \frac{e^{2y}}{52}(96 + 144x) = \boxed{\frac{e^{2y}}{13}(24 + 36x)}$$

(49) $f(x,y,z) = xy^2z^3 - 8$ em $(2,2,1)$

$\nabla f = (y^2z^3, 2xyz^3, 3xy^2z^2) = (4, 8, 24)$

a) Plano tangente: $4 \cdot (x-2) + 8(y-2) + 24(z-1) = 0$

$\boxed{e \;\; 4x + 8y + 24z = 48}$ $\Rightarrow x + 2y + 6z = 12$

b) Plano normal:

$$\frac{x-2}{4} = \frac{y-2}{8} = \frac{z-1}{24} \Rightarrow \boxed{x-2 = \frac{y-2}{2} = \frac{z-1}{6}}$$

(51) $f(x,y,z) = x+y+z-e^{xyz}$ em $(0,0,1)$

$\nabla f = (1-yze^{xyz}, 1-xze^{xyz}, 1-xye^{xyz})$.

$\nabla f(0,0,1) = (1,1,1)$

a) $x+y+z=1$

b) $x=y=z-1$.

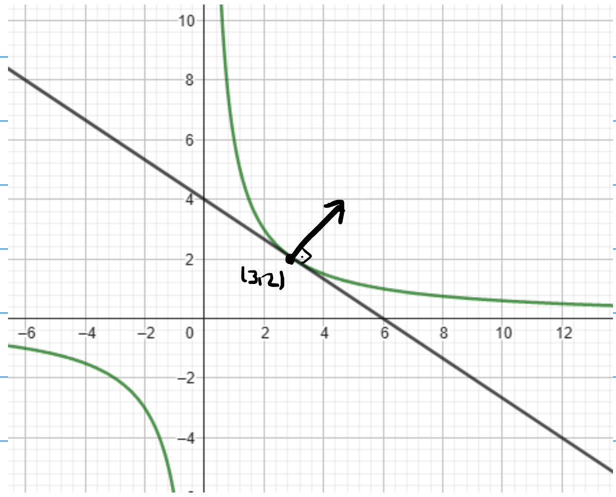
(55) $f(x,y) = xy$

$\nabla f(3,2) = (2,3)$.

Reta tangente: $2(x-3)+3(y-2)=0$
$2x+3y=12$.

vetor. grad.: $(2,3)$.

Curva nível $f(x,y)=6$:



(61) $f(x,y,z) = x^2 - y^2 - z^2 - 1 = 0$

$\nabla f = (2x, -2y, -2z) \Rightarrow \boxed{x = 1/2,\ y = -1/2,\ z = 1/2}$ Impossível.

Plano : $x + y - z = 0$.

$\hookrightarrow (1,1,-1)$.

$(x-a) + (y-b) - (z-c) = 0$

$x + y - z - (a+b+c) = 0$

Não é possível.