Lista 4

Seção 14.3: 1, 3, 4, 5, 9, 11, 13, 15, 17, 25, 27, 33, 37, 41, 43, 45, 47, 49, 53, 55, 57, 59, 67, 74, 100

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Tabela 1 Índice de calor I como função da temperatura e umidade

Umidade relativa (%)

Temperatura real (°C) — T \ H	40	45	50	55	60	65	70	75	80
26	28	28	29	31	31	32	33	34	35
28	31	32	33	34	35	36	37	38	39
30	34	35	36	37	38	40	41	42	43
32	37	38	39	41	42	43	45	46	47
34	41	42	43	45	47	48	49	51	52
36	43	45	47	48	50	51	53	54	56

$f(T,H)$

I) $f_T(34,75) = \lim_{h\to 0} \dfrac{f(34+h,75) - f(34,75)}{h}$

- $f_T(34,75) \approx \dfrac{f(36,75) - f(34,75)}{2} = \dfrac{54-51}{2} = \dfrac{3}{2}$
- $f_T(34,75) \approx \dfrac{f(32,75) - f(34,75)}{-2} = \dfrac{46-51}{-2} = \dfrac{5}{2}$

$\therefore f_T(34,75) \approx 2 \quad ((3/2 + 5/2)/2)$

II) $f_H(34,75) = \lim_{h\to 0} \dfrac{f(34,75+h) - f(34,75)}{h}$

- $f_H(34,75) \approx \dfrac{f(34,80) - f(34,75)}{5} = \dfrac{52-51}{5} = \dfrac{1}{5}$
- $f_H(34,75) \approx \dfrac{f(34,70) - f(34,75)}{-5} = \dfrac{49-51}{-5} = \dfrac{2}{5}$

$\therefore f_w(34,75) \simeq 0,3 \left[\left(1/5 + 2/10\right)/2\right]$

③ $T = f(x,y,t)$ (T é temperatura, x é longitude, y é latitude e t é tempo).

a) • $\frac{\partial T}{\partial x}$ = variação da temperatura em relação à longitude, mantendo a latitude e o tempo constantes

• $\frac{\partial T}{\partial y}$ = variação da temperatura em relação à latitude, mantendo a longitude e o tempo constantes

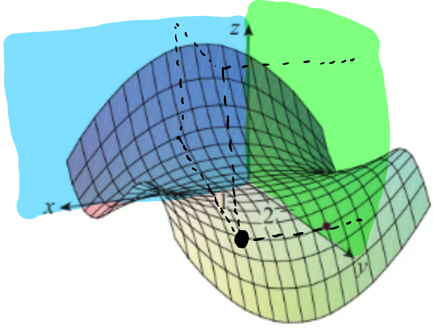
• $\frac{\partial T}{\partial z}$ = variação da temperatura em relação ao tempo, mantendo a latitude e a longitude constantes

b) • $f_x(158, 21, 9)$ ⇒ Positivo. O enunciado diz que $x = 158°$ W (oeste) e que há uma brisa quente indo para oeste

• $f_y(158, 21, 9)$ ⇒ Negativo. O enunciado diz que $y = 21°$N (norte) e que há uma brisa fria indo para o norte.

• $f_t(158, 21, 9)$ ⇒ Acho que seria positivo sempre

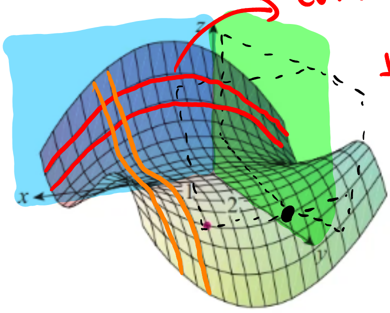
(4)



a) $f_x(1,2) > 0$

$f_y(1,2) < 0$

(5)



curvas de nivel de x ⊥ a yz

curvas de nivel de y ⊥ a xz.

a) $f_x(-1,2) < 0$

b) $f_y(-1,2) < 0$

(9) $f(x,y) = x^4 + 5xy^3$

$$\frac{\partial f}{\partial x} = 4x^3 + 5y^3$$

$$\frac{\partial f}{\partial y} = 15xy^2$$

(11) $g(x,y) = x^3 \operatorname{sen} y$

$$\frac{\partial f}{\partial x} = 3x^2 \operatorname{sen} y$$

$$\frac{\partial f}{\partial x} = \cos y \cdot x^3$$

(13) $z = \ln(x+t^2)$

$$\frac{\partial z}{\partial x} = \frac{1}{x+t^2}$$

$$\frac{\partial z}{\partial t} = \frac{1}{x+t^2} \cdot 2t$$

(15) $f(x,y) = y e^{xy}$

$$\frac{\partial f}{\partial x} = y^2 e^{xy}.$$

$$\frac{\partial f}{\partial y} = e^{xy} + y \cdot e^{xy} \cdot x$$

(17) $g(x,y) = y(x+x^2y)^5$

$$\frac{\partial f}{\partial x} = 5(x+x^2y) \cdot y \cdot (1+2xy)$$

$$\frac{\partial f}{\partial y} = (x+x^2y)^5 + y \cdot 5(x+x^2y)^4 x^2$$

(25) $F(x,y) = \int_y^x \cos(e^t)\,dt$

$$\frac{\partial f}{\partial x} = \cos(e^x)$$

$$\frac{\partial f}{\partial y} = -\cos(e^y)$$

(27) $f(x,y,z) = x^3yz^2 + 2yz$

$$\frac{\partial f}{\partial x} = 2x^2yz^2 \; ; \quad \frac{\partial f}{\partial y} = x^3z^2 + 2z \; ; \quad \frac{\partial f}{\partial z} = 2x^3yz + 2y$$

(33) $h(x,y,z,t) = x^2y\cos(z/t)$

$$\frac{\partial f}{\partial x} = 2xy\cos(z/t) \; ; \quad \frac{\partial f}{\partial y} = x^2\cos(z/t) \; ; \quad \frac{\partial f}{\partial z} = \frac{-x^2y\,\text{sen}(z/t)}{t}$$

$$\frac{\partial f}{\partial t} = -x^2y\,\text{sen}(z/t) \cdot z\left(\frac{-1}{t^2}\right) = \frac{x^2yz\,\text{sen}(z/t)}{t^2}$$

(37) $R(s,t) = te^{s/t}$

$$R_t(0,1) \Rightarrow R_t = e^{s/t} + e^{s/t} \cdot \left(\frac{-s}{t^2}\right) = e^{s/t} - \frac{s \cdot e^{s/t}}{t^2}$$

$$R_t(0,1) = e^0 - 0 = 1$$

(41) $x^2 + 2y^2 + 3z^2 = 1$

$$\text{I)}\ \frac{\partial z}{\partial x} \Rightarrow \quad 2x + 6z \cdot \frac{\partial z}{\partial x} = 0 \quad \Rightarrow \frac{\partial z}{\partial x} = -\frac{x}{3z}$$

II) $\dfrac{\partial z}{\partial y} \Rightarrow 4y + 6z\dfrac{\partial z}{\partial y} = 0 \Rightarrow \dfrac{\partial z}{\partial y} = \dfrac{-2y}{3z}$

(43) $e^z = xyz$ $\quad xz \quad e \quad y$

I) $\dfrac{\partial z}{\partial x} \Rightarrow e^z \cdot \dfrac{\partial z}{\partial x} = yz + xy\dfrac{\partial z}{\partial x} \Rightarrow \dfrac{\partial z}{\partial x} = \dfrac{yz}{e^z - xy}$

II) $\dfrac{\partial z}{\partial x} \Rightarrow e^z \dfrac{\partial z}{\partial y} = xz + xy\dfrac{\partial z}{\partial y} \Rightarrow \dfrac{\partial z}{\partial y} = \dfrac{xz}{e^z - xy}$

(45) a) $z = f(x) + g(y)$

- $\dfrac{\partial z}{\partial x} = f'(x)$ • $\dfrac{\partial z}{\partial y} = g'(y)$

(47) $f(x,y) = x^4y - 2x^3y^2$

$\dfrac{\partial f}{\partial x} = 4x^3y - 6x^2y^2 \Rightarrow \dfrac{\partial^2 f}{\partial x^2} = 12x^2y - 12xy^2.$

$\dfrac{\partial f}{\partial y} = x^4 - 4x^3y \Rightarrow \dfrac{\partial^2 f}{\partial y^2} = -4x^3$

$\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x} = 4x^3 - 12x^2y$

49. $z = \dfrac{y}{2x+3y}$

$$\frac{\partial z}{\partial x} = \frac{-2y}{(2x+3y)^2} \Rightarrow \frac{\partial^2 z}{\partial x^2} = \frac{-2y \cdot (-2)}{(2x+3y)^3} \cdot 2 = \frac{8y}{(2x+3y)^3}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2x+3y} + \frac{-y}{(2x+3y)^2} \cdot 3 = \frac{2x}{(2x+3y)^2} \Rightarrow \frac{\partial^2 z}{\partial y^2} = \frac{-12x}{(2x+3y)^3}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \frac{-2}{(2x+3y)^2} + \frac{(-2y)\cdot(-2)}{(2x+3y)^3} 3 = \frac{-2(2x+3y)+12y}{(2x+3y)^3}$$

$$\boxed{\frac{-4x+6y}{(2x+3y)^3}}$$

53. $u = x^4 y^3 - y^4$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(3x^4y^2 - 4y^3\right) = 12x^3y^2 \checkmark$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(4x^3y^3) = 12x^3y^2. \checkmark$$

55 $u = \cos(x^2y)$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left[-\operatorname{sen}(x^2y).x^2\right] = -\cos(x^2y).2x^3 - 2x\operatorname{sen}(x^2y) \checkmark$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left[-\operatorname{sen}(x^2y).\,2xy\right] = -\cos(x^2y).2x^3 - 2x\operatorname{sen}(x^2y) \checkmark$$

57 $f(x,y) = x^4y^2 - x^3y$

I) $f_{xxx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\right) = \frac{\partial}{\partial x}\left(\frac{\partial(4x^3y^2-3x^2y)}{\partial x}\right) = \frac{\partial}{\partial x}(12x^2y^2-6xy)$

$= 24xy^2 - 6y$

II) $f_{xyx} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\right) = \frac{\partial}{\partial y}(12x^2y^2-6xy) = 24x^2y - 6x$

59 $f(x,y,z) = e^{xyz^2}$

$f_{xyz} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right)\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(e^{xyz^2}.2xyz\right)\right).$

$= \frac{\partial}{\partial x}\left(e^{xyz^2}.xz^2.2xyz + 2xz.e^{xyz^2}\right) = \frac{\partial}{\partial x}\left(2e^{xyz^2}(x^2yz^3+xz)\right)$

$= 2e^{xyz^2}.yz^2(x^2yz^3+xz) + 2e^{xyz^2}.(2xyz^3+z)$

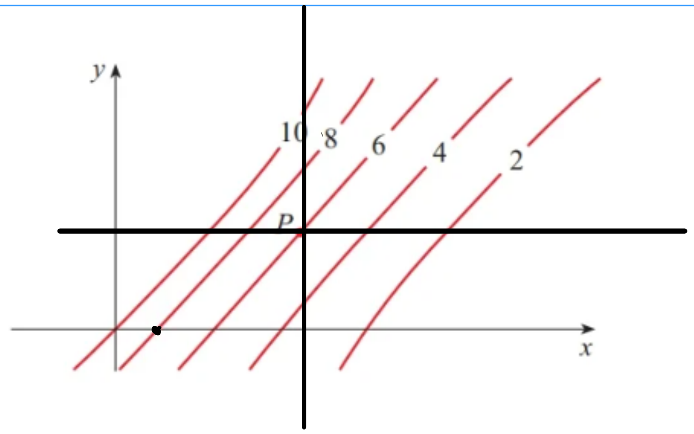
$= 2e^{xyz^2} \cdot x^2y^2z^5 + 2e^{xyz^2} \cdot xyz^3 + 4e^{xyz^2} \cdot xyz^3 + 2e^{xyz^2} \cdot z$

$= 2e^{xyz^2} \cdot x^2y^2z^5 + 6e^{xyz^2} \cdot xyz^3 + 2e^{xyz^2} \cdot z$

(67) $f(x,y,z) = xy^2z^3 + \arcsin(x\sqrt{z})$

$f_{xyz} = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\right) = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x}(2xyz^3)\right) = \frac{\partial}{\partial z}(2yz^3) = 6yz^2$

(74)



a) $f_x(P) \Rightarrow$ Os valores no eixo x vão decrescendo, mantido y. Logo $\partial f/\partial x(P) < 0$

b) $f_y(P) \Rightarrow$ Os valores no eixo y vão crescendo, mantido x. Logo $\partial f/\partial y(P) > 0$

c) $f_{xx}(P) \Rightarrow$

(100) $f(x,y) = \sqrt[3]{x^3+y^3}$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = \frac{\sqrt[3]{(0+h)^3 + 0^3} - \sqrt{0^3+0^3}}{h}$$

$$= \frac{\sqrt[3]{(h^3)+0} - 0}{h} = \frac{h}{h} = \boxed{1}$$

Seção 14.4: 1, 3, 5, 9, 15, 19, 23, 25, 27, 29, 31, 33, 35, 39, 41, 43, 51

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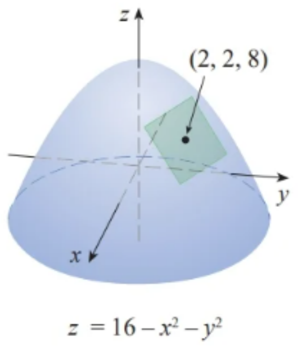

$f(2,2) = 8$

$\frac{\partial f}{\partial x} = -2x \Rightarrow -4$

$\frac{\partial f}{\partial y} = -2y \Rightarrow -4$

$z = 8 + (-4)(x-2) + (-4)(y-2)$

$z = 8 - 4x + 8 - 4y + 8$

$\boxed{z = 24 - 4x - 4y}$

3 $z = 2x^2 + y^2 - 5y$ em $(1, 2, -4)$

$f(1,2) = -4$; $\frac{\partial z}{\partial x} = 4x \Rightarrow 4$; $\frac{\partial z}{\partial y} = 2y - 5 \Rightarrow -1$.

$z = -4 + 4(x-1) + (-1)(y-2)$

$z = -4 + 4x - 4 - y + 2$

$\boxed{z = -6 + 4x - y}$

5 $z = e^{x-y}$ em $(2, 2, 1)$

$f(2,2) = 1$; $\frac{\partial z}{\partial x} = e^{x-y} \Rightarrow 1$; $\frac{\partial z}{\partial y} = -e^{x-y} \Rightarrow -1$

$z = 1 + 1.(x-2) + (-1)(y-2)$

$z = 1 + x - 2 - y + 2$

$\boxed{z = 1 + x - y}$

(9) $z = x\,\text{sen}(x+y)$ $(-1, 1, 0)$

$f(-1,1) = 0$; $\dfrac{\partial z}{\partial x} = \text{sen}(x+y) + x.\cos(x+y) \Rightarrow -1$; $\dfrac{\partial z}{\partial y} = x\cos(x+y) \Rightarrow -1$

$z = 0 + (-1)(x+1) + (-1)(y-1)$

$\boxed{z = -x - y}$

(15) $f(x,y) = x^3y^2$ em $(-2,1)$

Para $f(x,y)$ ser diferenciável em $(-2,1)$ suas derivadas parciais devem existir perto de $(-2,1)$ e serem contínuas em $(-2,1)$.

Além disso Δz pode ser escrito:

$$\Delta z = f_x(a,b)\Delta x + f_y(a,b)\Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

com $\varepsilon_1, \varepsilon_2 \to 0$ quando $(\Delta x, \Delta y) \to (0,0)$.

$\dfrac{\partial z}{\partial x} = 3x^2y^2 \Rightarrow 12$; $\dfrac{\partial z}{\partial y} = 2yx^3 \Rightarrow -16$; $f(-2,1) = -8$

$L(x,y) = -8 \quad 12(x+2) + (-16)(y-1)$

$\boxed{L(x,y) = 12x - 16y + 32}$

(19) $f(x,y) = x^2 e^y$ — $(1,0)$

$\frac{\partial f}{\partial x} = 2xe^y \Rightarrow 2$; $\frac{\partial f}{\partial y} = e^y \cdot x^2 \Rightarrow 1$; $f(1,0) = 1$

$L(x,y) = 1 + 2(x-1) + 1(y-0)$

$\boxed{L(x,y) = 2x + y - 1}$ //

(23) $e^x \cos(xy)$ — $(0,0)$

$f(0,0) = 1$; $\frac{\partial f}{\partial x} = e^x \cos xy - e^x \operatorname{sen} xy \cdot y \Rightarrow 1$

$\frac{\partial f}{\partial y} = -e^x \operatorname{sen} xy \cdot y \Rightarrow 0$

$L(x,y) = 1 + 1(x-0) + 0.(y-0) = x+1$

$\boxed{\text{Verdade}}$ //.

(25) $L(x,y) = 6 + 1(x-2) + (-1)(y-5)$

$L(2,2;4,9) = 6 + (2,2-2) - (4,9-5)$
$= 6 + 0,2 - (-0,1) = 6 + 0,3 \boxed{= 6,3}$ //

(27) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ em $(3,2,6)$

$f(3,2,6) = 7$; $\frac{\partial f}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2+z^2}} \Rightarrow 3/7$

$\frac{\partial f}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2+z^2}} \Rightarrow 2/7$

$\frac{\partial f}{\partial z} = \frac{2z}{2\sqrt{x^2+y^2+z^2}} \Rightarrow 6/7$

$L(x,y,z) = 7 + \frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6)$

$$\boxed{L(x,y,z) = \frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z}$$

$f(3{,}02;\ 1{,}97;\ 5{,}99) = \frac{3.3{,}02 + 2.1{,}97 + 6.5{,}99}{7} \boxed{\approx 6{,}9915}$

(29)

Umidade relativa (%)

Temperatura real (°C)

T \ H	40	45	50	55	60	65	70	75	80
26	28	28	29	31	31	32	33	34	35
28	31	32	33	34	35	36	37	38	39
30	34	35	36	37	38	40	41	42	43
32	37	38	39	41	42	43	45	46	47
34	41	42	43	45	47	48	49	51	52
36	43	45	47	48	50	51	53	54	56

$I = f(T, H)$

Calcular a aprox. linear quando $(T,H) \to (32,65)$ e $(T,H) \to (33,63)$.

$$f_T(32,65) \approx \frac{\frac{48-43}{2} + \frac{40-43}{-2}}{2} = \frac{5+3}{4} = 2$$

$$f_H(32,65) \approx \frac{\frac{45-43}{5} + \frac{42-43}{-5}}{2} = \frac{2+1}{10} = \frac{3}{10}$$

$$f(32,65) = 43$$

$$L(x,y) = 43 + 2(T-32) + 3/10\,(H-65)$$
$$= 43 + 2T - 64 + 3/10\,H - 19,5$$
$$\boxed{= 2T + 3/10\,H - 40,5}$$

$$f(33,63) \approx L(33,63) = \boxed{44,4\,°C}$$

(31) $m = p^5 q^3$

$$dm = \frac{\partial m}{\partial p}dp + \frac{\partial m}{\partial q}dq = \boxed{5p^4q^3\,dp + 3q^2p^5\,dq}$$

(33) $z = e^{-2x} \cos 2\pi t$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial t} dt = \boxed{-2e^{-2x} \cos 2\pi t \, dx - e^{-2x} \operatorname{sen} 2\pi t . 2\pi \, dt}$$

(35) $H = x^2 y^4 + y^3 z^5$

$$\boxed{dH = 2xy^4 \, dx + (4y^3 x^2 + 3y^2 z^5) \, dy + 5z^4 y^3 \, dy}$$

(39) $z = 5x^2 + y^2$ $\quad (1,2) \to (1{,}05; 2{,}1)$

$$\Delta z = 5.(1{,}05)^2 + (2{,}1)^2 - (5.1^2 + 2^2)$$

$$\Delta z = 9{,}9225 - (9) = 0{,}9225$$

$$dz = 10x\,dx + 2y\,dy = 10.1{,}05.0{,}05 + 2.2{,}1.0{,}1$$

$$= 0{,}945$$

(41) $A = xy$ $\quad x \to$ comp. $\quad y \to$ larg. $\qquad dx = dy = \varepsilon$

$$dA = y\,dx + x\,dy$$

$$dA = 24.\varepsilon + 30.\varepsilon = 54.\varepsilon \Rightarrow 54.0{,}1 \boxed{= 5{,}4\,cm^2}$$

$$A = 30.24 = 720\,cm^2$$

$$\text{Erro} = \boxed{0{,}75\%}$$

(43) $V = \pi r^2 h$

$r = 4cm$

$h = 12cm$

$dV = 2r\pi h\,dr + 2\pi r h\,dr$ $\quad \varepsilon = 0{,}04cm = dr = dh$

$dV = 4\pi r h\,dr = 4 \cdot 3{,}14 \cdot 4 \cdot 12 \cdot 0{,}04 \approx \boxed{16cm^3}$ //.

(51) $B(m,h) = m/h^2$

a) Aprox. linear em $(23;1{,}1)$

$B(23;1{,}1) \approx 19$

$\frac{\partial B}{\partial m} = \frac{1}{h^2} \Rightarrow 0{,}83$

$\frac{\partial B}{\partial h} = \frac{-2m}{h^3} \Rightarrow -34{,}5$

$L(m,h) = 19 + 0{,}83(x-23) - 34{,}5(y-1{,}1)$

$\boxed{L(m,h) \approx 0{,}83m - 34{,}5y + 38{,}02}$ //.

b) $L(24;1{,}13) = 0{,}83 \cdot 24 - 34{,}5 \cdot 1{,}13 + 38{,}02 = 18{,}955$

$B(24;1{,}13) = 24/(1{,}13)^2 = 18{,}7958$