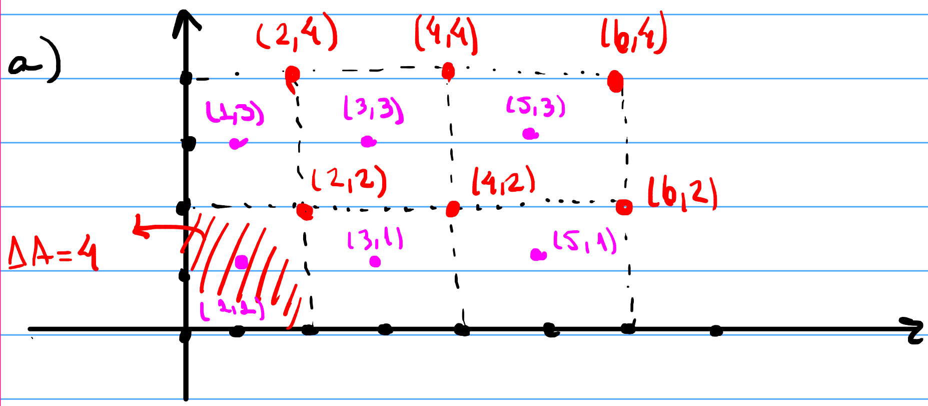
# LISTA 7

Seção 15.1: 1, 5, 6, 7, 9, 11, 15, 17, 19, 21, 23, 27, 29, 31, 33, 35, 39, 41, 43, 45, 47, 54, 55

① $z = xy$ ; $R = \{(x,y) \mid 0 \le x \le 6 \,,\; 0 \le y \le 4\}$
$m = 3$ e $n = 2$.

a)




$V \approx f(2,2)\Delta A + f(4,2)\Delta A + f(6,2)\Delta A + f(2,4)\Delta A + f(4,4)\Delta A + f(6,4)\Delta A$
$V \approx 4(4 + 8 + 12 + 8 + 16 + 24)$
$V \approx 4(28 + 20 + 24) = 4(28 + 44) = 4 \cdot 72 = \boxed{288 \text{ u.v}}$

b) $V \approx \Delta A( f(1,1) + f(3,1) + f(5,1) + f(1,3) + f(3,3) + f(5,3))$
$V \approx 4(1 + 3 + 5 + 3 + 9 + 15) = 4.(19 + 9 + 8) = 4(19 + 17) = 4.36$
$\boxed{= 144 \text{ u.v}}$

5) $f(x,y) = \sqrt{52 - x^2 - y^2}$ ; $2 \le x \le 4$; $2 \le y \le 6$

Retas $x=3$ e $x=4$.

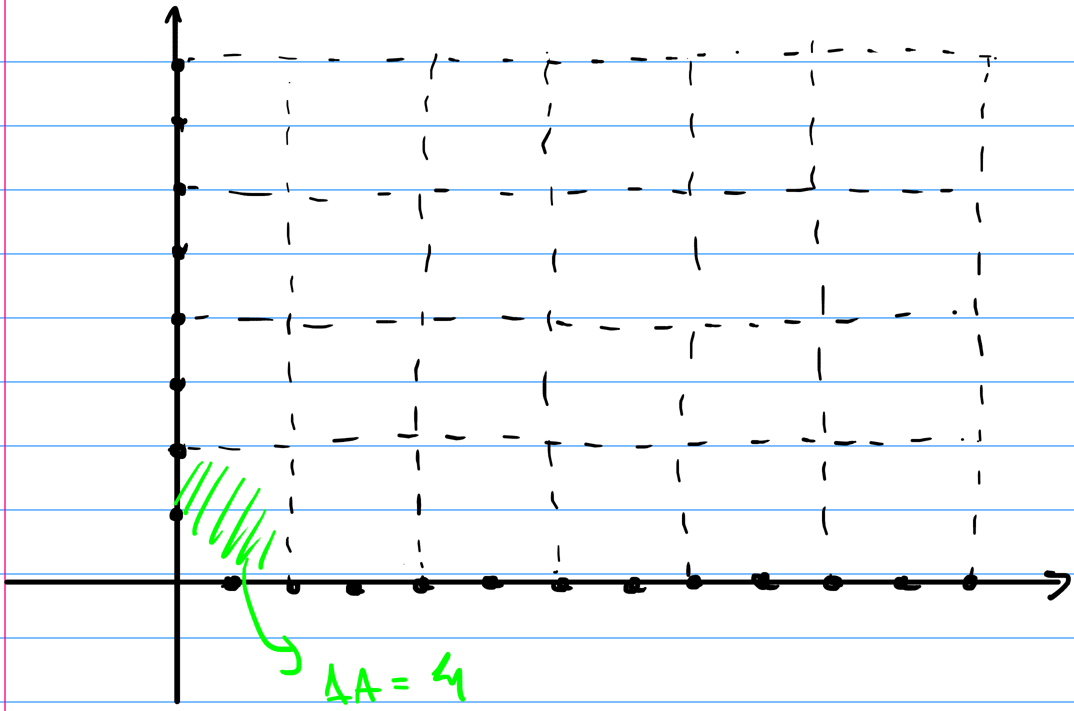
$L \Rightarrow$ inferiores esquerdos

$U \Rightarrow$ superiores direitos


$\boxed{U < V < L}$ Meio óbvio!

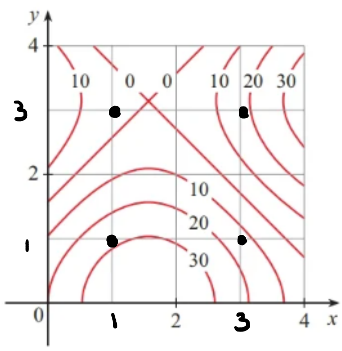
6)

| | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1,5 | 2 | 2,4 | 2,8 | 3 | 3 |
| 2 | 1 | 1,5 | 2 | 2,8 | 3 | 3,6 | 3 |
| 4 | 1 | 1,8 | 2,7 | 3 | 3,6 | 4 | 3,2 |
| 6 | 1 | 1,5 | 2 | 2,3 | 2,7 | 3 | 2,5 |
| 8 | 1 | 1 | 1 | 1 | 1,5 | 2 | 2 |

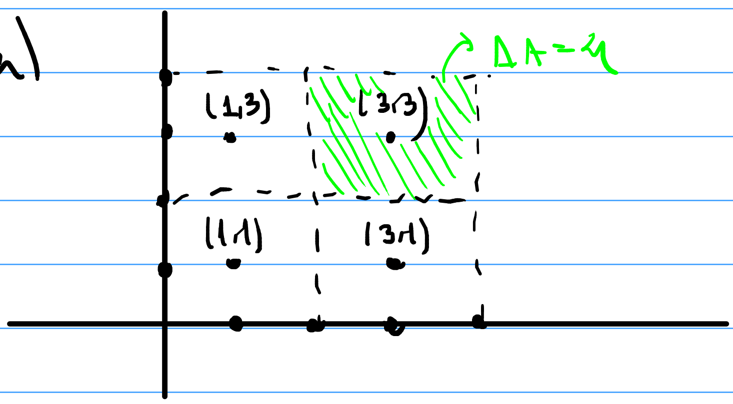



$$\boxed{V \approx 4\left(\sum f(x,y) \text{ na tabela}\right)}$$

7




a)



$$\iint_R f(x,y)\,dA \approx 4\big(f(1,1)+f(3,1)+f(1,3)+f(3,3)\big)$$

$$= 4(29+15+3+18) = \boxed{260}$$

b) $$f_{méd} = \frac{1}{A}\iint_R f(x,y)\,dA = \frac{1}{16}\cdot 260 = 16{,}25.$$

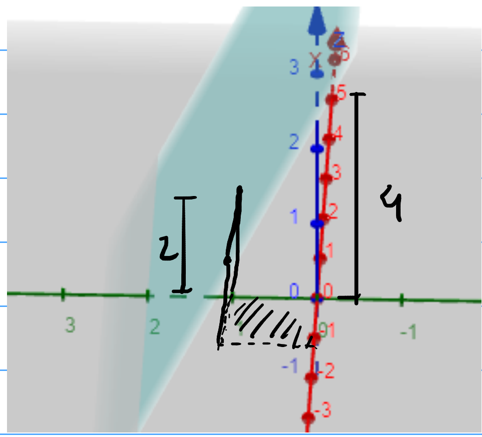
9) $$\iint_R \sqrt{2}\,dA \qquad R=\{(x,y) \mid 2\le x\le 6,\ -1\le y\le 5\}$$

Volume de um paralelepípedo reto-retângulo $4\times 6\times\sqrt{2}$ : $V = 24\sqrt{2} = \iint_R \sqrt{2}\,dA.$

11) $$\iint_R (4-2y)\,dA \qquad R=[0,1]\times[0,1]$$

$z = 4-2y$

$2y+z=4$ (plano)




Volume de um paralelepípedo reto-retângulo cortado na metade

$V = 6.1.1/2 = 3$

(15) $$\int_1^4 \int_0^2 (6x^2y - 2x)\,dy\,dx =$$

$$\int_0^2 (6x^2y - 2x)\,dy = (3x^2y^2 - 2xy)\Big|_0^2 = 3x^2.4 - 2.x.2$$
$$= 12x^2 - 4x$$

$$\int_1^4 \int_0^2 (6x^2y - 2x)\,dy\,dx = \int_1^4 (12x^2 - 4x)\,dx$$

$$= (4x^3 - 2x^2)\Big|_1^4 = (4.4^3 - 2.4^2) - (4.1 - 2)$$
$$4^4 - 2.16 - 2$$
$$\boxed{256 - 34 = 222}$$

(17) $$\int_0^1 \int_1^2 (x + e^{-y})\,dx\,dy$$

$$\int_1^2 (x + e^{-y})\,dx = (x^2/2 + x e^{-y})\Big|_1^2 = (4/2 + 2e^{-y}) - (1/2 + e^{-y})$$

$$= 2 + 2e^{-y} - 1/2 - e^{-y} = 3/2 + e^{-y}$$

$$\int_0^1 \int_1^2 (x + e^{-y})\,dx\,dy = \int_0^1 (3/2 + e^{-y})\,dy = (3/2\,y - e^{-y})\Big|_0^1$$
$$= 3/2 - e^{-1} - (0 - 1) = \boxed{5/2 - 1/e}$$

19) $\int_{-3}^{3}\int_{0}^{\pi/2}(y+y^2\cos x)\,dx\,dy$

$\int_{0}^{\pi/2}(y+y^2\cos x)\,dx = (xy+y^2 \operatorname{sen} x)\Big|_{0}^{\pi/2} = \pi/2\cdot y + y^2$

$\int_{-3}^{3}\int_{0}^{\pi/2}(y+y^2\cos x)\,dx\,dy = \int_{-3}^{3}\left(\frac{\pi}{2}y+y^2\right)dy = \Big|_{-3}^{3}\left(\pi/4\, y^2 + y^3/3\right)$

$= \pi/4\cdot 3^2 + 3^3/3 - (\pi/4\cdot 3^2 - 3^3/3) = 2\cdot 3^3/3 \boxed{=18}$

21) $\int_{1}^{4}\int_{1}^{2}(x\cdot y^{-1}+y\cdot x^{-1})\,dy\,dx$

$\int_{1}^{2}(x\cdot y^{-1}+y\cdot x^{-1})\,dy = \left(\ln|y|\cdot x + \frac{y^2}{2}\cdot x^{-1}\right)\Big|_{1}^{2}$

$= \ln 2\cdot x + \frac{2^2}{2}\cdot x^{-1} - (\ln 1\cdot x + 1/2\cdot x^{-1}) = x\cdot\ln 2 + 3/2\cdot x^{-1}$

$\int_{1}^{4}\int_{1}^{2}(x\cdot y^{-1}+y\cdot x^{-1})\,dy\,dx = \int_{1}^{4}(x\ln 2 + 3/2\cdot x^{-1})\,dx$

$= (x^2/2\cdot\ln 2 + 3/2\cdot\ln|x|)\Big|_{1}^{4} = 4^2/2\cdot\ln 2 + 3/2\ln 4 - (1/2\ln 2 + 0)$

$$= 8\ln 2 + \frac{3}{2}\cdot 2\ln 2 - \frac{1}{2}\ln 2 = 11\ln 2 - 1/2\cdot\ln 2 = \boxed{21/2\cdot\ln 2}$$

23 $$\int_0^3\int_0^{\pi/2} (t^3 \operatorname{sen}^3\phi)\, d\phi\, dt = \int_0^3 t^3\, dt \cdot \int_0^{\pi/2} \operatorname{sen}^3\phi\, d\phi$$

$$= \left(\frac{t^4}{4}\right)\Big|_0^3 \cdot \left(\frac{\cos^3\phi}{3} - \cos\phi\right)\Big|_0^{\pi/2}$$

$$= \left(\frac{3^4}{4} - 0\right)\left(0 - 0 - (1/3 - 1)\right) = \frac{3^4}{4}\cdot\frac{2}{3} = \boxed{\frac{27}{2}}$$

27 $$\iint_R x \sec^2 y\, dA \qquad R = \{(x,y) \mid 0 \le x \le 2,\ 0 \le y \le \pi/4$$

$$= \int_0^{\pi/4}\int_0^2 x\sec^2 y\, dx\, dy = \int_0^{\pi/4} \sec^2 y\, dy \cdot \int_0^2 x\, dx$$

$$= (\operatorname{tg} y)\Big|_0^{\pi/4} \cdot (x^2/2)\Big|_0^2 = 1 \cdot 4/2 = \boxed{2}$$

29 $$\iint_R \frac{xy^2}{x^2+1}\, dA \qquad R = \{(x,y) \mid 0 \le x \le 1,\ -3 \le y \le 3\}$$

$$= \int_{-3}^{3}\int_{0}^{1} \frac{xy^2}{x^2+1}\, dx\, dy = \int_{-3}^{3} y^2\, dy \cdot \int_{0}^{1} \frac{2x}{x^2+1}\, dx$$

$$= \frac{y^3}{3}\Big|_{-3}^{3} \cdot \frac{1}{2} \cdot \ln(x^2+1)\Big|_{0}^{1} = \left(\frac{3^3}{3} - \left(\frac{(-3)^3}{3}\right)\right) \cdot \frac{1}{2}\left(\ln 2 - \ln 1\right)$$

$$= 2 \cdot 3^2 \cdot \frac{1}{2} \cdot \ln 2 = \boxed{9 \ln 2}$$

31) $\iint_R x \operatorname{sen}(x+y)\, dA$ , $R = [0, \pi/6] \times [0, \pi/3]$

$$\int_{0}^{\pi/3}\int_{0}^{\pi/6} x \operatorname{sen}(x+y)\, dx\, dy$$

$$\int_{0}^{\pi/6} x \operatorname{sen}(x+y)\, dx$$

$$\boxed{\int v\, du = u \cdot v - \int u\, dv}$$

$v = x$ $\quad$ $dv = dx$

$du = \operatorname{sen}(x+y)\, dx$ $\quad$ $u = -\cos(x+y)$

$$\int x \operatorname{sen}(x+y)\, dx = -x\cos(x+y) + \int \cos(x+y)\, dx = -x\cos(x+y) + \operatorname{sen}(x+y) + C$$

$$\int_{0}^{\pi/6} x \operatorname{sen}(x+y)\, dx = \left(-x\cos(x+y) + \operatorname{sen}(x+y)\right)\Big|_{0}^{\pi/6} = \operatorname{sen}(\pi/6+y) - \frac{\pi}{6}\cos(\pi/6+y)$$

$$- \operatorname{sen} y$$

$$\int_0^{\pi/3}\int_0^{\pi/6} x\,\text{sen}(x+y)\,dx\,dy = \int_0^{\pi/3} \left(\text{sen}(\pi/6+y) - \frac{\pi}{6}\cos(\pi/6+y) - \text{sen}\,y\right) dy$$

$$= \left(-\cos(\pi/6+y) - \pi/6\,\text{sen}(\pi/6+y) + \cos y\right)\Big|_0^{\pi/3}$$

$$= \left(\cos\pi/3 - \pi/6\,\text{sen}(\pi/6+\pi/3) - \cos(\pi/6+\pi/3) - (\cos 0 - \pi/6\,\text{sen}\,\pi/6 - \cos(\pi/6))\right)$$

$$= \left(1/2 - \pi/6\cdot\text{sen}(\pi/2) - \cos\pi/2 - (1 - \pi/6\cdot 1/2 - \sqrt{3}/2)\right)$$

$$= 1/2 - \pi/6\cdot 1 - 0 - 1 + \pi/12 + \sqrt{3}/2 = \boxed{\frac{\sqrt{3}-1}{2} - \frac{\pi}{12}}$$

33 $\iint_R y e^{-xy}\,dA \qquad R = [0,2]\times[0,3]$

$$= \int_0^3\int_0^2 y e^{-xy}\,dx\,dy$$

$$\int_0^2 y e^{-xy}\,dx = y\int_0^2 -y\cdot\frac{-1}{y}\cdot e^{-xy}\,dx = -\int_0^2 e^u\,du =$$

$$= -\left(e^{-xy}\right)\Big|_0^2 = -\left(e^{-2y} - 1\right) = 1 - e^{-2y}.$$

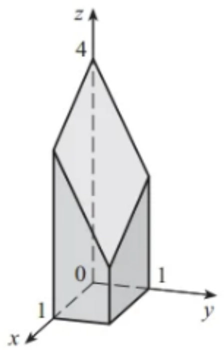
$$\int_0^3\int_0^2 y e^{-xy}\,dx\,dy = \int_0^3 (1-e^{-2y})\,dy = \left(y+\tfrac{1}{2}e^{-2y}\right)\Big|_0^3$$

$$= 3+\frac{1}{2}e^{-6} - \left(0-\frac{1}{2}\right) = 3+\frac{1}{2}+\frac{e^{-6}}{2} = \boxed{\frac{5+e^{-6}}{2}}$$

(35) $\int_0^1\int_0^1 (4-x-2y)\,dx\,dy$

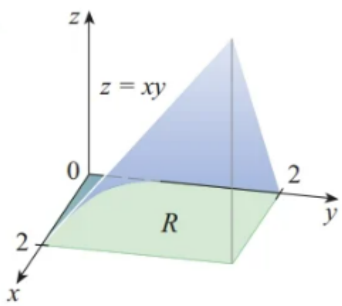
Plano $\boxed{x+2y+z=4}$ $R=[0,1]\times[0,1]$

35.




(39) a)

39.

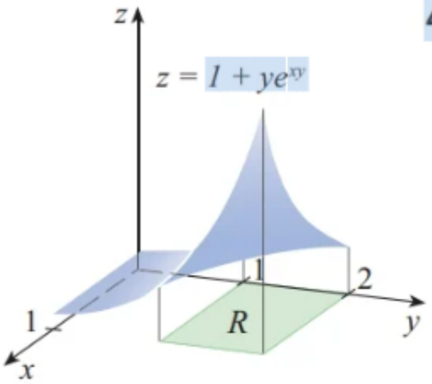



$\Rightarrow V=\int_0^2\int_0^2 xy\,dx\,dy.$

b) $V=\int_0^2 x\,dx \cdot \int_0^2 y\,dy = \frac{x^2}{2}\Big|_0^2 \cdot \frac{y^2}{2}\Big|_0^2 = \frac{4}{2}\cdot\frac{4}{2} = \boxed{4}$

41) a)

41.




$$V = \int_1^2 \int_0^1 (1 + y e^{xy}) \, dx \, dy$$

b) $$\int_0^1 (1 + y e^{xy}) \, dx = (x + e^{xy}) \Big|_0^1 = 1 + e^y - (0+1) = e^y$$

$$V = \int_1^2 e^y \, dy = e^y \Big|_1^2 = \boxed{e^2 - e}$$

43) $4x + 6y - 2z + 15 = 0$ $\quad R = \{(x,y) \mid -1 \le x \le 2, -1 \le y \le 1\}$

$$z = 2x + 3y + 15/2$$

$$V = \int_{-1}^{1} \int_{-1}^{2} (2x + 3y + 15/2) \, dx \, dy$$ $\qquad 18 + 15/2$

$$\int_{-1}^{2} (2x + 3y + 15/2) \, dx = (\cancel{2} \cdot x^2/\cancel{2} + 3xy + 15/2 \cdot x) \Big|_{-1}^{2}$$

$$= 4 + 6y + 15 - (1 - 3y - 15/2) = 4 + 6y + 15 - 1 + 3y + 15/2$$

$$= 9y + 5/2.$$

$$V = \int_{-1}^{1} (9y + 51/2)\,dy = (9y^2/2 + 51/2\,y)\Big|_{-1}^{1}$$

$$= 9/2 + 51/2 - (9/2 - 51/2)$$

$$= 60/2 + 51/2 - 9/2 = 30 + 21 = \boxed{51}$$

(45) $\frac{x^2}{4} + \frac{y^2}{9} + z = 1$ $\quad R = [-1, 1] \times [-2, 2]$

$$V = \int_{-2}^{2}\int_{-1}^{1} (1 - x^2/4 - y^2/9)\,dx\,dy$$

$$\int_{-1}^{1} (1 - x^2/4 - y^2/9)\,dx = (x - 1/4 \cdot x^3/3 - y^2/9 \cdot x)\Big|_{-1}^{1}$$

$$(1 - 1/12 - y^2/9 - (-1 + 1/12 + y^2/9)) = 1 - 1/12 - y^2/9 + 1 - 1/12 - y^2/9$$

$$= 2 - 1/6 - 2y^2/9 = 11/6 - 2/9 \cdot y^2$$

$$V = \int_{-2}^{2} (11/6 - 2/9\,y^2)\,dy = (11/6\,y - 2/9 \cdot y^3 \cdot 1/3)\Big|_{-2}^{2}$$

$$= 11/3 - 16/27 - (-11/3 + 16/27) = 22/3 - 32/27 = \boxed{\frac{166}{27}}$$

(47) $v = \int_0^1 \int_{-1}^1 (1 + x^2 y e^y)\, dx\, dy$

$\int_{-1}^1 (1 + x^2 y e^y)\, dx = (x + x^3/3\, y e^y)\Big|_{-1}^1 = 1 + 1/3\, y e^y - (-1 - 1/3\, y e^y)$

$= 2 + 2/3\, y e^y$

$v = \int_0^1 (2 + 2/3\, y e^y)\, dy = (2y + 2/3(y e^y - e^y))\Big|_0^1$

$= 2 + 2/3(e - e) - (0 + 2/3(0 - 1))$

$= 2 + 0 - 0 + 2/3 = \boxed{8/3}$

$\int y e^y\, dy \Rightarrow \int u\, dv = u.v - \int v\, du$ $\quad u = y \quad du = dy$

$dv = e^y dy \quad v = e^y$

$= y e^y - \int e^y dy = y e^y - e^y + C.$

55 $$\int_0^1\int_{-1}^1 \frac{xy}{1+x^4}\,dx\,dy = \int_0^1 y\,dy \cdot \frac{1}{2}\int_{-1}^1 \frac{2x}{1+x^4}\,dx$$

$$= \left(y^2/2\right)\Big|_0^1 \cdot 1/2 \cdot \left(\operatorname{Arctg} x^2\right)\Big|_{-1}^1 = 1/2 \cdot 1/2\left(\pi/4 - (\pi/4)\right)$$

$$= 1/4 \cdot 0 = \boxed{0}$$

$$\boxed{\frac{1}{2}\int_{-1}^1 \frac{2x}{1+x^4}\,dx = \frac{1}{2}\int_{-1}^1 \frac{1}{1+u^2}\,du = \frac{1}{2}\operatorname{Artg} x^2\Big|_{-1}^1}$$

$u = x^2$

Seção 15.2: 1, 3, 5, 7, 9, 11, 13, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 39, 55, 57, 59, 63, 65, 68, 71

(1) $\displaystyle\int_1^5\int_0^x (8x-2y)\,dy\,dx$

$\displaystyle\int_0^x (8x-2y)\,dy = \left(8xy-\cancel{2}\cdot y^2/\cancel{2}\right)\Big|_0^x = 8x^2 - x^2 = 7x^2$

$\displaystyle\int_1^5 7x^2\,dx = \left(7x^3/3\right)\Big|_1^5 = 7/3\,(5^3-1^3) = 7\cdot 124/3 = \boxed{868/3}$ //

(3) $\displaystyle\int_0^1\int_0^y x e^{y^3}\,dx\,dy$

$\displaystyle\int_0^y x e^{y^3}\,dx = \left(\frac{x^2 e^{y^3}}{2}\right)_0^y = \frac{y^2 e^{y^3}}{2} = \frac{3y^2}{6}\cdot e^{y^3}$

$\displaystyle\frac{1}{6}\int_0^1 3y^2 e^{y^3}\,dy = \frac{1}{6}\int_0^1 e^u\,du = \frac{1}{6}\cdot e^{y^3}\Big|_0^1 = \boxed{\frac{1}{6}(e-1)}$ //

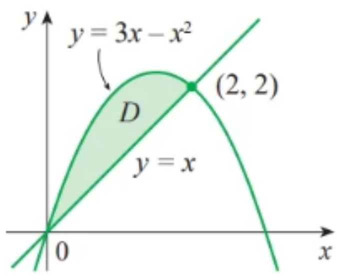
(5) $\displaystyle\int_0^1\int_0^{s^2} (\cos s^3)\,dt\,ds \Rightarrow \int_0^{s^2} \cos s^3\,dt = \frac{3s^2}{3}\cdot \cos s^3$

$$\Rightarrow \frac{1}{3}\int_0^1 3s^2 \cos s^3 \, ds = \frac{1}{3}\int_0^1 \cos u \, du = \frac{1}{3} \operatorname{sen} s^3 \Big|_0^1$$

$$\boxed{= \frac{1}{3} \operatorname{sen} 1}$$

7

7. $f(x, y) = 2y$




a) $$\int_0^2 \int_x^{3x-x^2} 2y \, dy \, dx$$

b) $$\int_x^{3x-x^2} 2y \, dy = y^2 \Big|_x^{3x-x^2}$$

$$= 9x^2 - 6x^3 + x^4 - x^2 = 8x^2 - 6x^3 + x^4$$

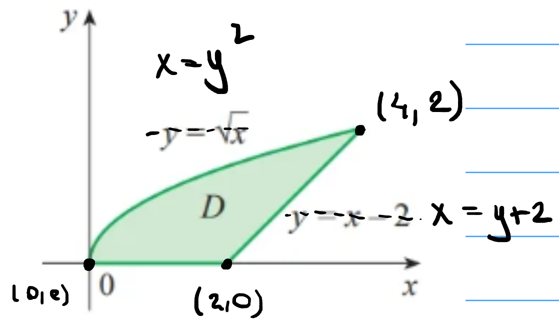
$$\Rightarrow \int_0^2 (8x^2 - 6x^3 + x^4) \, dx = \left( \frac{8x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5} \right) \Bigg|_0^2$$

$$= \frac{8 \cdot 8}{3} - \frac{6 \cdot \cancel{16}^{4}}{\cancel{4}} + \frac{32}{5} = \frac{64 \cdot 5}{15} - \frac{6 \cdot 4 \cdot 15}{15} + \frac{32 \cdot 3}{15}$$

$$= \boxed{56/15}$$

(9)

9. $f(x, y) = xy$




$x - 2 = \sqrt{x}$

$x^2 - 4x + 4 = x$

$x^2 - 5x + 4 = 0$

$x = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5\pm 3}{2}$ 4, $\cancel{1}$

a) $\int_0^2 \int_{y^2}^{y+2} xy \, dx \, dy$

b) $\int_0^2 \int_{y^2}^{y+2} xy \, dx \, dy \Rightarrow \int_{y^2}^{y+2} xy \, dx = \frac{x^2 y}{2} \Big|_{y^2}^{y+2}$

$= \frac{1}{2}(y^3 + 4y^2 + 4y - y^5) \Rightarrow \frac{1}{2}\int_0^2 (y^3 + 4y^2 + 4y - y^5)\, dy$

$= \frac{1}{2} \cdot \left(\frac{2^4}{4} + \frac{4 \cdot 2^3}{3} + \frac{4 \cdot 2^2}{2} - \frac{2^6}{6}\right)$

$= \frac{1}{2} \cdot \frac{1}{6}\left(2^3 \cdot 3 + 4 \cdot 2^4 + 3 \cdot 4 \cdot 2^2 - 2^6\right) = \frac{1}{12} \cdot 72 = \boxed{6}$

(11) $\iint_D \frac{y}{x^2+1}\, dA$, $D = \{(x,y) \mid 0 \le x \le 4,\ 0 \le y \le \sqrt{x}\}$

$$\int_0^4 \int_0^{\sqrt{x}} \frac{y}{x^2+1}\, dy\, dx \Rightarrow \int_0^{\sqrt{x}} \frac{y}{x^2+1}\, dy = \frac{1}{2} \cdot \frac{y^2}{x^2+1} \Big|_0^{\sqrt{x}}$$

$$= \frac{1}{4} \cdot \frac{2x}{x^2+1} \quad \Rightarrow \frac{1}{4}\int_0^4 \frac{2x\, dx}{x^2+1} = \frac{1}{4}\int_0 \frac{1}{u}\, du = \frac{1}{4} \ln(x^2+1)\Big|_0^4$$

$$\boxed{= \frac{1}{4} \cdot \ln 17}$$

(13) $\iint_D e^{-y^2}\, dA$ $D = \{(x,y) \mid 0 \le y \le 3,\ 0 \le x \le y\}$

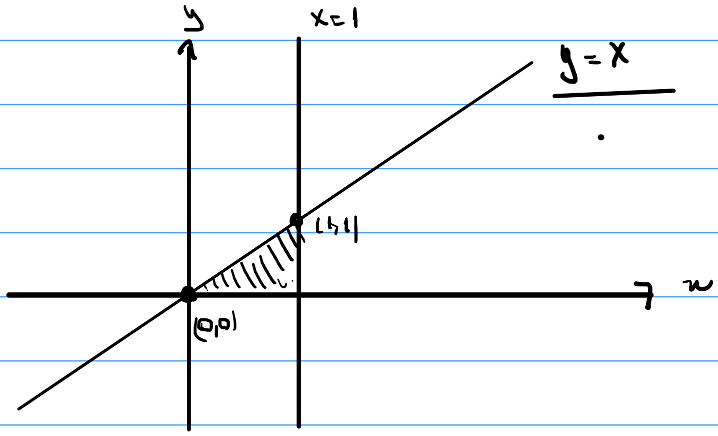
$$\int_0^3 \int_0^y e^{-y^2}\, dx\, dy \Rightarrow \int_0^y e^{-y^2} dx = x e^{-y^2}\Big|_0^y = \frac{-2y}{-2} \cdot e^{-y^2}$$

$$\Rightarrow -\frac{1}{2}\int_0^3 -2y\, e^{-y^2} dy = -\frac{1}{2}\int_0^3 e^{u}\, du = -\frac{1}{2} e^{-y^2}\Big|_0^3$$

$$\boxed{= \frac{1}{2}\left(1 - \frac{1}{e^9}\right)}$$

(17) $\iint_D x \, dA \qquad y=x,\ y=0,\ x=1.$

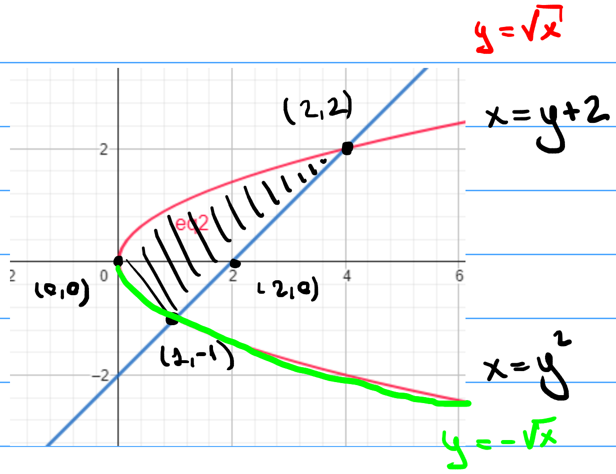
D:




I) $\int_0^1 \int_0^x x \, dy \, dx \qquad$ II) $\int_0^1 \int_y^1 y \, dx \, dy$

- $\int_0^1 \int_0^x x \, dy \, dx = \int_0^1 x^2 dx = \frac{1}{3}.$

(19) $\iint_D y \, dA \qquad$ D: $y = x-2 \quad x = y^2$



$$\int_0^1\int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx + \int_1^4\int_{x-2}^{\sqrt{x}} y\,dy\,dx = \int_1^2\int_{y^2}^{y+2} y\,dx\,dy$$

$$\int_1^2\int_{y^2}^{y+2} y\,dx\,dy \Rightarrow \int_{y^2}^{y+2} y\,dx = xy\Big|_{y^2}^{y+2} = y^2+2y-y^3$$

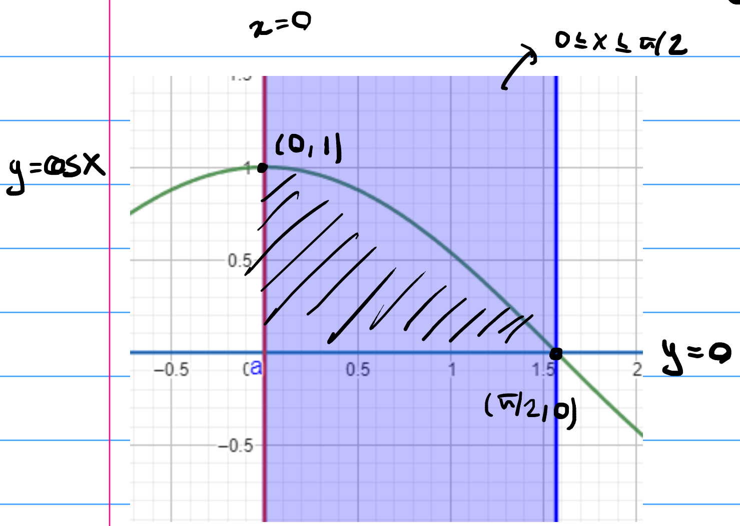
$$= \int_1^2 (y^2+2y-y^3)\,dy = (y^3/3 + y^2 - y^4/4)\Big|_1^2 = 19/12$$

(21) $\iint_D \mathrm{sen}^2 x\, dA$

$y = \cos x$

$0 \leq x \leq \pi/2$

$y=0 \ , \ x=0$



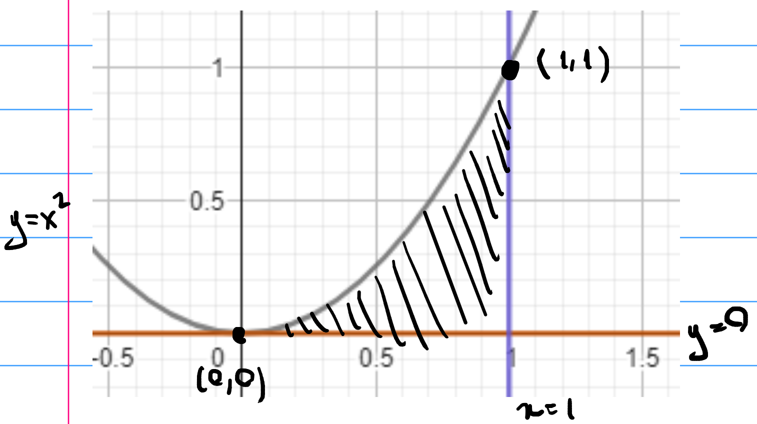

$$\int_0^1\int_0^{\arccos y} \mathrm{sen}^2 x\,dx\,dy = \int_0^{\pi/2}\int_0^{\cos x} \mathrm{sen}^2 x\,dy\,dx$$

$$\int_0^{\cos x} \operatorname{sen}^2 x \, dy \Rightarrow y \operatorname{sen}^2 x \Big|_0^{\cos x} = \operatorname{sen}^2 x \cos x.$$

$$\int_0^{\pi/2} \operatorname{sen}^2 x \cos x \, dx = \int_0^{\pi/2} u^2 \, du \Rightarrow \left(\frac{1}{3} \operatorname{sen}^3 x\right)\Big|_0^{\pi/2} = \frac{1}{3}$$

23 $\displaystyle\iint_D x \cos y \, dA$

$y=0$
$y=x^2$
$x=1$




$$\int_0^1 \int_0^{x^2} x \cos y \, dy \, dx \Rightarrow \int_0^{x^2} x \cos y \, dy = x \operatorname{sen} y \Big|_0^{x^2}$$

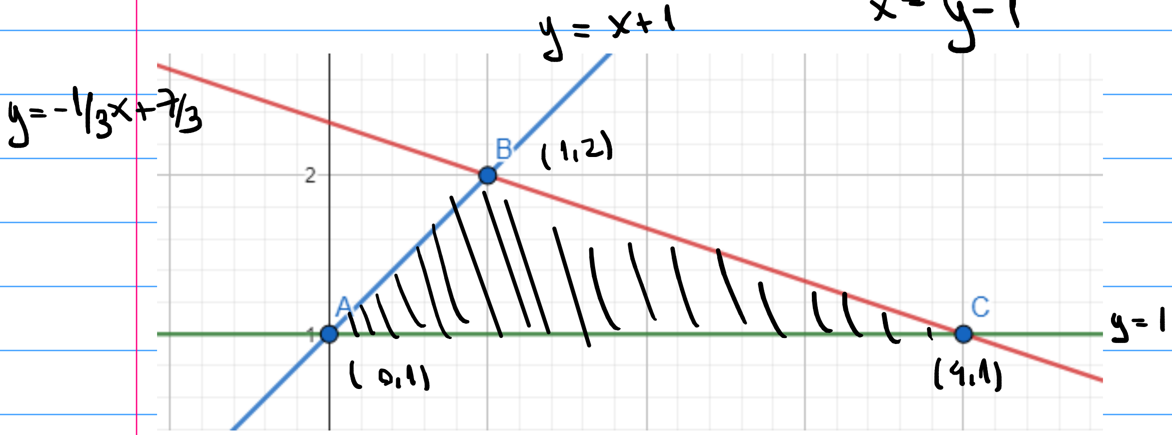
$$= \frac{2x}{2} \operatorname{sen} x^2 \Rightarrow \frac{1}{2} \int_0^1 2x \operatorname{sen} x^2 \, dx = \frac{1}{2} \int_0^1 \operatorname{sen} u \, du$$

$$= -\frac{1}{2} (\cos x^2)\Big|_0^1 = -\frac{1}{2} (\cos 1 - 1) = \boxed{\frac{1}{2}(1 - \cos 1)}$$

(25) $\iint_D y^2 \, dA$ $\quad (0,1),\ (1,2),\ (4,1).$

$x = 7-3y$

$x = y-1$




$$\int_1^2 \int_{y-1}^{7-3y} y^2 \, dx \, dy \Rightarrow xy^2 \Big|_{y-1}^{7-3y} \Rightarrow 7y^2-3y^3-y^3+y^2$$

$$= 8y^2-4y^3$$

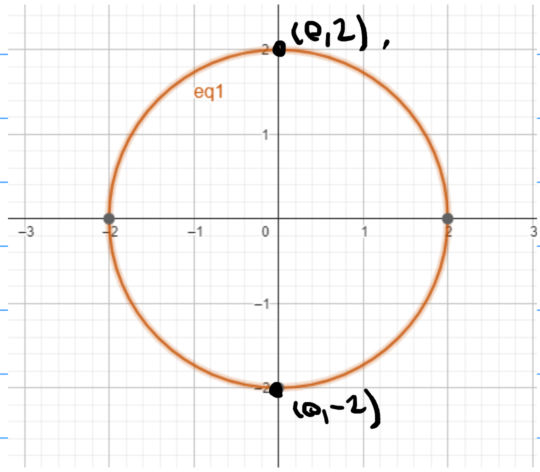
$$\int_1^2 8y^2-4y^3 \Rightarrow \left(\frac{8y^3}{3}-y^4\right)\Big|_1^2 = \frac{64}{3}-16-\left(\frac{8}{3}-1\right)$$

$$= \frac{64}{3}-\frac{8}{3}-15 = \boxed{\frac{11}{3}}$$

27 $\iint (2x-y)\,dA$

$x^2+y^2=4.$

$x = \pm\sqrt{4-y^2}.$




$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (2x-y)\,dx\,dy \Rightarrow (x^2-xy)\Big|_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}$$

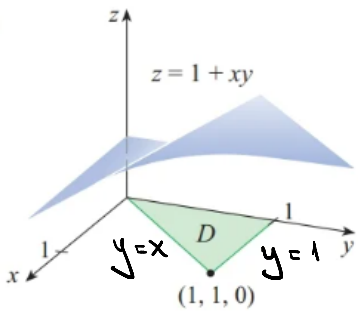
$$= 4-y^2 - y\sqrt{4-y^2} - (4-y^2 + y\sqrt{4-y^2})$$

$$= -2y\sqrt{4-y^2}$$

$$\int_{-2}^{2} \sqrt{u}\,du \Rightarrow \frac{u^{3/2}}{3/2} \Rightarrow \frac{(4-y^2)^{3/2}}{3/2}\Big|_{-2}^{2} = 0$$

29

29.




a) $\int_0^1\int_0^y (1+xy)\,dx\,dy$

b) $\int_0^y (1+xy)\,dx = \left(x + x^2 y/2\right)\Big|_0^y = y + y^3/2$

$\int_0^1 \left(y + y^3/2\right) dy = \left(y^2/2 + y^4/8\right)\Big|_0^1 = 1/2 + 1/8 \boxed{= 5/8}$

(31) $3x + 2y - z = 0$ $\qquad$ $y = x^2 \quad x = y^2$.

$\boxed{z = 3x + 2y}$ $\qquad$ $x = x^4 \Rightarrow \boxed{x = 0,\ x = 1}$.

$\int_0^1 \int_{x^2}^{\sqrt{x}} (3x+2y)\,dy\,dx \Rightarrow \int_{x^2}^{\sqrt{x}} (3x+2y)\,dy = \left(3xy + y^2\right)\Big|_{x^2}^{\sqrt{x}}$

$= 3x\sqrt{x} + x - 3x^3 - x^4 = 3x^{3/2} + x - 3x^3 - x^4$

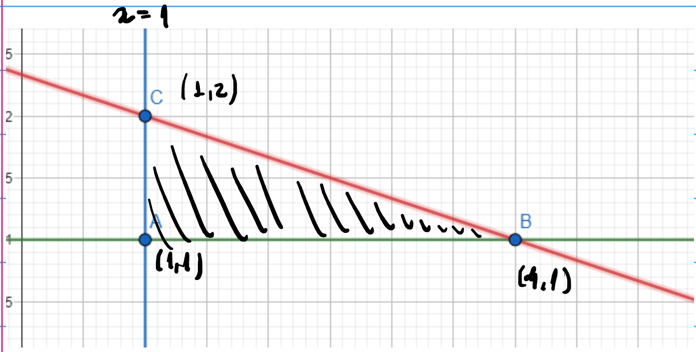
$\int_0^1 \left(3x^{3/2} + x - 3x^3 - x^4\right) = \left(\frac{3x^{5/2}}{5/2} + \frac{x^2}{2} - \frac{3x^4}{4} - \frac{x^5}{5}\right)\Big|_0^1$

$\frac{3 \cdot 2}{5} + \frac{1}{2} - \frac{3}{4} - \frac{1}{5} = 1 - \frac{1}{4} \boxed{= \frac{3}{4}}$.

(33) $z = xy$ $\quad (1,1)\ ,(4,1)\ ,(1,2)$

$x = 7-3y$

$x=1$




$y=1$

$$\int_1^2\int_1^{7-3y} xy\,dx\,dy \Rightarrow \int_1^{7-3y} xy\,dx = \left(x^2/2\,y\right)\Big|_1^{7-3y}$$

$$= \frac{1}{2}y\left(49-42y+9y^2-1\right) = y/2\left(48-42y+9y^2\right)$$

$$= 24y - 21y^2 + 9/2\,y^3$$

$$\Rightarrow \int_1^2 \left(24y-21y^2+9/2\,y^3\right)dy \Rightarrow \left(12y^2-7y^3+9/8\,x^4\right)\Big|_1^2$$

$$= 48-7\cdot\cancel{8}+\frac{9}{\cancel{8}}\cdot\overset{2}{\cancel{16}} - (12-7+9/8)$$

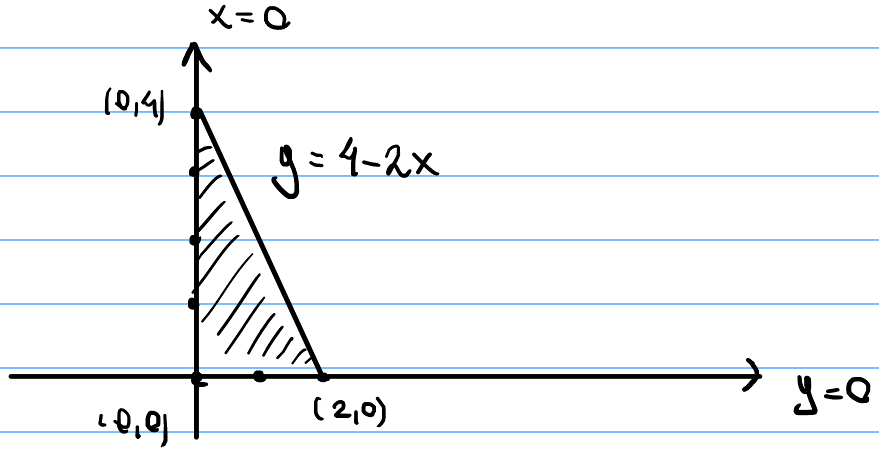
$$= 48-56+18-12+7-9/8 = 5-9/8 = \boxed{\frac{31}{8}}$$

(35) $2x + y + z = 4$ $\quad x=0$

$y=0$

$z=0$

D :




$$\int_0^2\int_0^{4-2x} (4-2x-y)\,dy\,dx \Rightarrow \int_0^{4-2x} (4-2x-y)\,dy = \left(4y-2xy-y^2/2\right)\Big|_0^{4-2x}$$

$$= 4(4-2x) - 2x(4-2x) - (4-2x)^2/2 =$$

$$16-8x-8x+4x^2-(16-16x+4x^2)/2 = 16-16x+4x^2-8+8x-2x^2$$

$$= 8-8x+2x^2$$

$$\int_0^2 (8-8x+2x^2)\,dx = \left(8x-4x^2+2x^3/3\right)\Big|_0^2$$

$$= \frac{32}{2} - \frac{64}{2} + \frac{2.64}{2\cdot3} = \frac{128}{2\cdot3} - \frac{32}{2\cdot3}.3 = \boxed{\frac{16}{3}}$$

39) $x^2+y^2=1$. $y=z$ primeiro octante $(x,y,z\geq 0)$

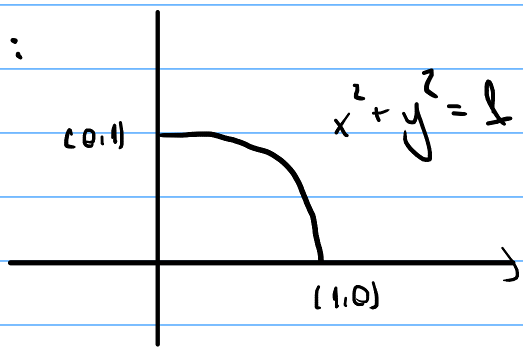
$x=0$

$z=0$

$\boxed{0\leq y\leq 1}$

$\boxed{0\leq x\leq 1.}$

D:

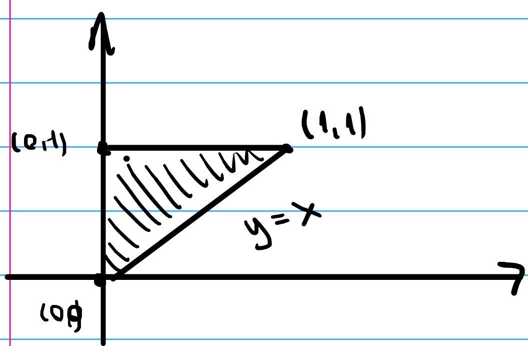



$$\int_0^1\int_0^{\sqrt{1-y^2}} y\,dx\,dy$$

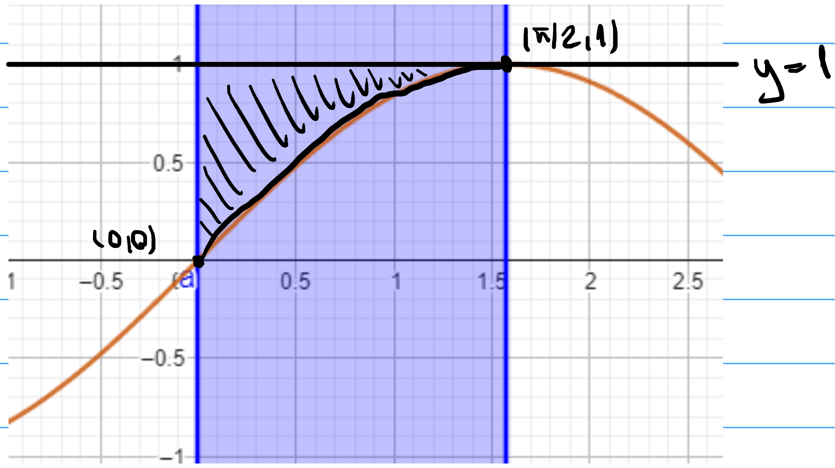
$$-\frac{1}{2}\int_0^1 -2\cdot y\sqrt{1-y^2}\,dy = -\frac{1}{2}\int_0^1 \sqrt{u}\,du = -\frac{1}{2}\,\frac{(1-y^2)^{3/2}}{3/2}\Bigg|_0^1$$

$$= -\frac{1}{2}\cdot\frac{2}{3}(1-y^2)^{3/2}\Big|_0^1 = \boxed{1/3}$$

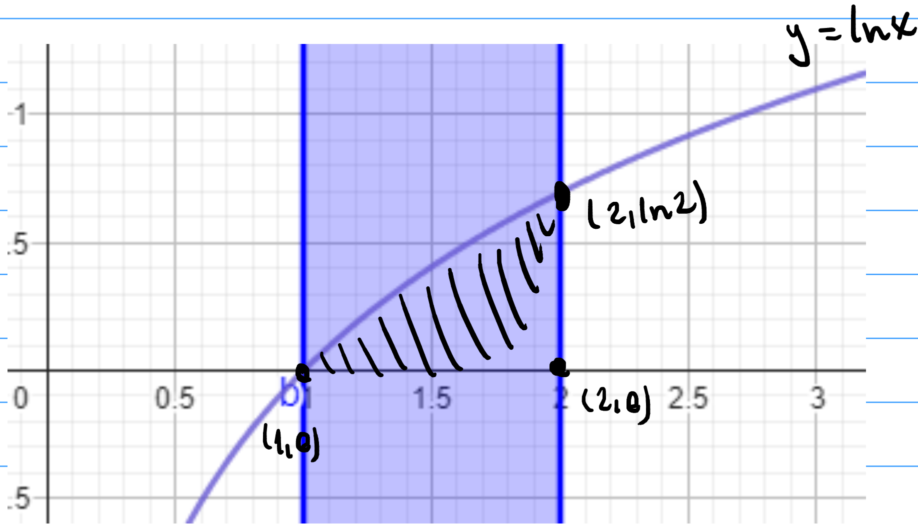
55. $$\int_0^1\int_0^y f(x,y)\,dx\,dy = \int_0^1\int_x^1 f(x,y)\,dy\,dx$$



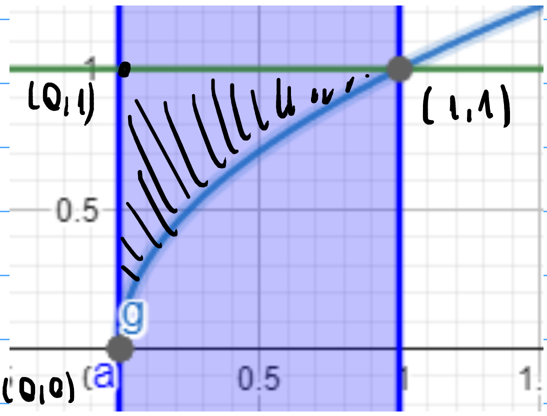
57 $\int_0^{\pi/2}\int_{\operatorname{sen}x}^{1} f(x,y)\,dy\,dx = \int_0^1\int_0^{\arcsin y} f(x,y)\,dx\,dy$




59 $\int_1^2\int_0^{\ln x} f(x,y)\,dy\,dx = \int_0^{\ln 2}\int_{e^y}^{2} f(x,y)\,dx\,dy$



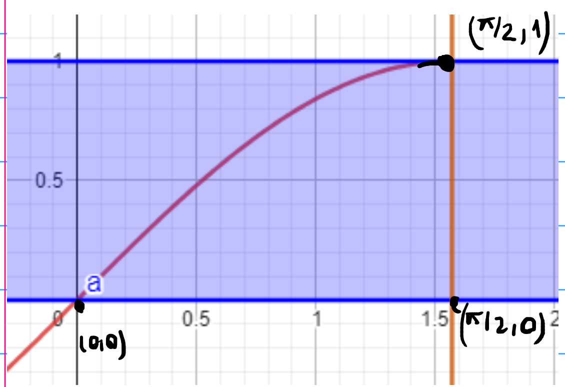
(63) $\int_0^1\int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx = \int_0^1\int_0^{y^2} \sqrt{y^3+1}\,dx\,dy$




$\int_0^1\int_0^{y^2} \sqrt{y^3+1}\,dx\,dy \Rightarrow \int_0^{y^2} \sqrt{y^3+1}\,dx = y^2\sqrt{y^3+1}$

$\frac{1}{3}\int_0^1 3y^2\sqrt{y^3+1} = \frac{1}{3}\,\frac{(y^3+1)^{3/2}}{3/2}\Big|_0^1 = \frac{2}{9}\cdot(2^{3/2}-1) = \boxed{\frac{2(2\sqrt{2}-1)}{9}}$

(65) $\int_0^1\int_{\arcsin y}^{\pi/2} \cos x\sqrt{1+\cos^2 x}\,dx\,dy$



$$= \int_0^{\pi/2}\int_0^{\operatorname{sen}x} \cos x\sqrt{1+\cos^2 x}\,dy\,dx$$

$$= \int_0^{\pi/2} \frac{\operatorname{sen}2x}{2}\cdot\sqrt{1+\frac{1}{2}+\frac{1}{2}\cos 2x}\,dx$$

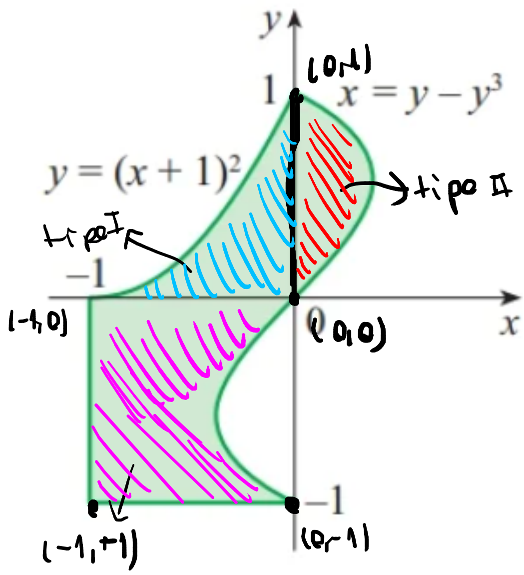
$$= -\frac{1}{2}\int_0^{\pi/2}\left(-\operatorname{sen}2x\sqrt{\frac{3}{2}+\frac{\cos 2x}{2}}\right)dx$$

$$-\frac{1}{2}\int_0^{\pi/2}\sqrt{u}\,du = -\frac{1}{2}\left(\frac{3}{2}+\frac{\cos 2x}{2}\right)^{3/2}\cdot\frac{2}{3}\Bigg|_0^{\pi/2}$$

$$-\frac{2}{6}\left(\left(\frac{3}{2}-\frac{1}{2}\right)^{3/2}-\left(\frac{3}{2}+\frac{1}{2}\right)^{3/2}\right) = \frac{2}{6}\left(2^{3/2}-(1)^{3/2}\right)$$

$$\boxed{=\frac{1}{3}\cdot\left(2\sqrt{2}-1\right)=}$$

68




$$\iint_D y\, dA.$$

$$\int_{-1}^{0}\int_{(x+1)^2}^{0} y\,dy\,dx + 2\cdot\int_{0}^{1}\int_{0}^{y-y^3} y\,dx\,dy + \int_{-1}^{0}\int_{-1}^{0} y\,dx\,dy$$

$$\Rightarrow \int_{0}^{(x+1)^2} y\,dy = \frac{y^2}{2}\Big|_{(x+1)^2}^{0} = -\frac{1}{2}\cdot(x+1)^4$$

$$\Rightarrow \int_{-1}^{0} y\,dx = xy\Big|_{-1}^{0} = y$$

$$\Rightarrow \int_{0}^{y-y^3} y\,dx = xy\Big|_{0}^{y-y^3} = y^2-y^4$$

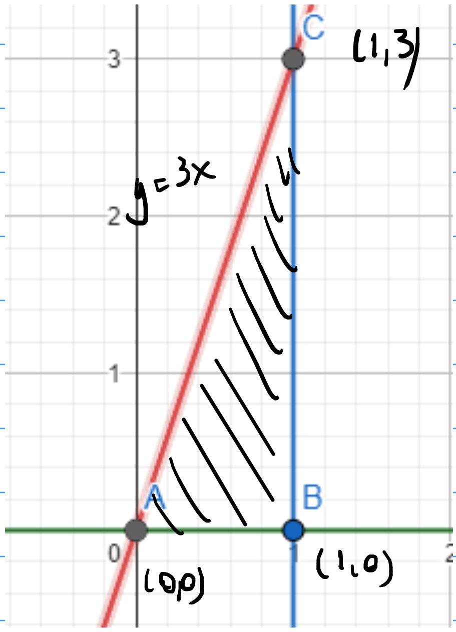
$$\Rightarrow -\frac{1}{2}\int_{-1}^{0}(x+1)^4\,dx = -\frac{1}{2}\cdot\frac{(x+1)^5}{5}\Big|_{-1}^{0} = \frac{1}{10}$$

$$\Rightarrow \int_{-1}^{0} y\,dy = \frac{y^2}{2}\Big|_{-1}^{0} = -\frac{1}{2}$$

$$\Rightarrow \int_{0}^{1}(y^2-y^4)\,dy = \left(\frac{y^3}{3}-\frac{y^5}{5}\right)\Big|_{0}^{1} = \frac{1}{3}-\frac{1}{5}$$

$$2.\ \left(\frac{1}{3}-\frac{1}{5}\right)-\frac{1}{2}+\frac{1}{10} = -\frac{2}{15}$$

71




$$f_{\text{réd}} = \frac{2}{3} \cdot \int_0^1 \int_{3x}^{3} xy \, dy \, dx$$

$$\int_{3x}^{3} xy \, dy = \left( xy^2/2 \right)\Big|_{3x}^{3}$$

$$= 9/2\, x - 9/2\, x^3$$

$$\frac{9}{2} \cdot \int_0^1 (x - x^3)\, dx = \frac{9}{2} \left( x^2/2 - x^4/4 \right)\Big|_0^1$$

$$= \frac{9}{2} \cdot \left( \frac{1}{2} - \frac{1}{4} \right) = \frac{9}{8}$$

$$f_{\text{réd}} = \frac{2}{3} \cdot \frac{9}{8} \; \boxed{= \frac{3}{4}}$$