LISTA 7:

• Seção 4.9: 1, 5, 12, 13, 14, 21, 23, 27, 35, 39, 45, 54, 59, 73

(1) a) Primitiva de $f(x) = 6$

$g'(x) = f(x) \quad \therefore \quad \boxed{g(x) = 6x + C}$

b) Primitiva de $g(x) = 3t^2$

$h'(x) = g(x) \quad \therefore \quad \boxed{h(x) = t^3 + C}$

(5) Primitiva geral de $f(x) = 4x + 7$

$\boxed{g(x) = 2x^2 + 7x + C}$

(12) $h(z) = 3z^{0,8} + z^{-2,5}$

$$g(z) = 0,6 \cdot z^{1,8} - \frac{z^{-1,5}}{1,5} + C = \boxed{\frac{3z^{1,8}}{5} - \frac{2z^{-1,5}}{3} + C}$$

(13) $f(x) = 3\sqrt{x} - 2\sqrt[3]{x} = 3x^{1/2} - 2x^{1/3}$

$$\boxed{g(x) = 2x^{3/2} - \frac{3 \cdot x^{4/3}}{2} + C}$$

(14) $g(x)=\sqrt{x}\,(2-x+6x^2) = 2x^{1/2} - x^{3/2} + 6x^{5/2}$

$$\boxed{h(x) = \frac{4x^{3/2}}{3} - \frac{2x^{5/2}}{5} + \frac{12x^{7/2}}{7} + C}$$

(21) $f(\theta) = 2\,\mathrm{sen}\,\theta - 3\sec\theta\,\mathrm{tg}\,\theta$

$$\boxed{g(\theta) = -2\cos\theta - 3\sec\theta + C}$$

(23) $f(r) = \frac{4}{1+r^2} - \sqrt[5]{r^4} = 4\cdot\frac{1}{1+r^2} - r^{4/5}$

$$\boxed{g(r) = 4\,\mathrm{arctg}\,r - \frac{5r^{9/5}}{9} + C}$$

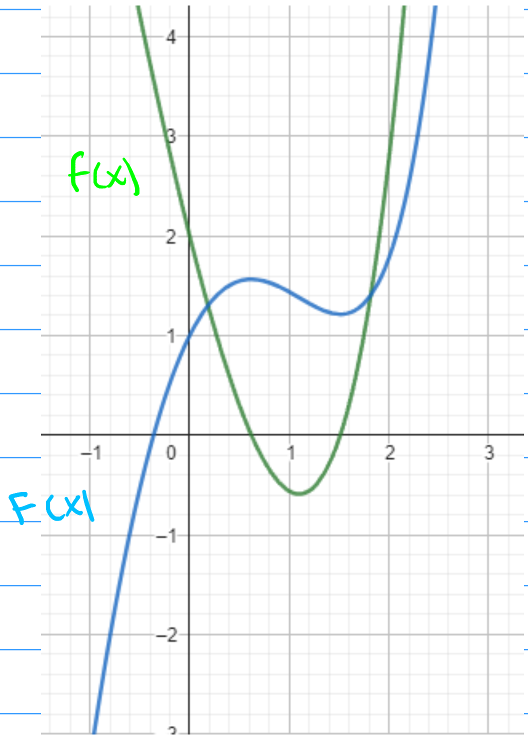
Lembre-se, primitiva de $f(x)$ é uma $g(x)$ tal que $g'(x)=f(x)$!

(27) $F(x)$ é primitiva de $f(x) = 2e^x - 6x$ e $F(0) = 1$

$$\Rightarrow \boxed{F(x) = 2e^x - 3x^2 + C}$$

$$F(0) = 1 = 2e^0 - 3 \cdot 0^2 + C \Rightarrow 1 = 2 + C \Rightarrow C = -1$$

$$\therefore \boxed{F(x) = 2e^x - 3x^2 - 1}$$



Quando $f(x) > 0$, $F(x)$ é crescente e quando $f(x) < 0$, $F(x)$ é decrescente.
Quando $f(x) = 0$, $F(x)$ possui máximo ou mínimo local.
Quando $f(x)$ possui um ponto de máximo ou mínimo (ponto crítico), $F(x)$ muda de concavidade.

(35) $f'''(t) = 12 + \text{sen}\,t$

$$f''(t) = 12t - \cos t + C$$
$$f'(t) = 6t^2 - \text{sen}\,t + Ct$$
$$\boxed{f(t) = 2t^3 + \cos t + \frac{Ct^2}{2}}$$

39) $f'(t) = \dfrac{4}{1+t^2}$, $f(1) = 0$

$\boxed{f(t) = 4\,\mathrm{arctg}\,t + C}$

$f(1) = 0 = 4\,\mathrm{arctg}\,1 + C \therefore \boxed{C = -\pi/4}$

$\boxed{f(t) = 4\,\mathrm{arctg}\,t - \pi/4}$

45) $f''(x) - 2 + 12x - 12x^2$ $f(0) = 4$; $f'(0) = 12$

$f'(x) = -2x + 6x^2 - 4x^3 + C$

$f'(0) = 4 \therefore C = 12$ e $f'(x) = -2x + 6x^2 - 4x^3 + 12$.

$f(x) = -x^2 + 2x^3 - x^4 + 12x + C'$

$f(0) = 4 \therefore C' = 4$ $\boxed{f(x) = -x^2 + 2x^3 - x^4 + 12x + 4}$

54) $f'''(x) = \cos x$ $f(0) = 1$; $f'(0) = 2$; $f''(0) = 3$

$f''(x) = \mathrm{sen}\,x + C \Rightarrow f''(0) = 3 \therefore C = 3$ $\boxed{f''(x) = \mathrm{sen}\,x + 3}$

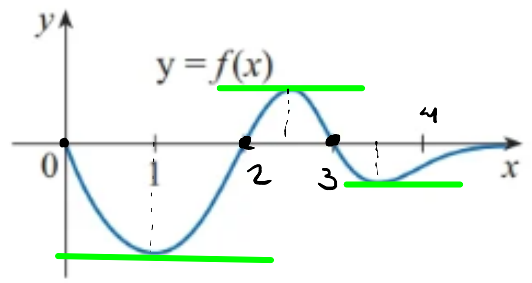
$f'(x) = -\cos x + 3x + C' \Rightarrow f'(0) = 2 \therefore \boxed{C = 3}$

$\boxed{f'(x) = -\cos x + 3x + 3}$

$f(x) = -\mathrm{sen}\,x + 3/2\,x^2 + 3x + C'' \Rightarrow f(0) = 1 \therefore \boxed{C = 1}$

$$\therefore \boxed{f(x) = \frac{3}{2}x^2 + 3x - \operatorname{sen} x + 1}$$

59



$F'(x) = f(x)$.

Informações: $F(x)$ cresc: $(2,3)$; $F(x)$ decresc: $(0,2) \cup (3,4)$

$x=0$: $F(0)=1$,

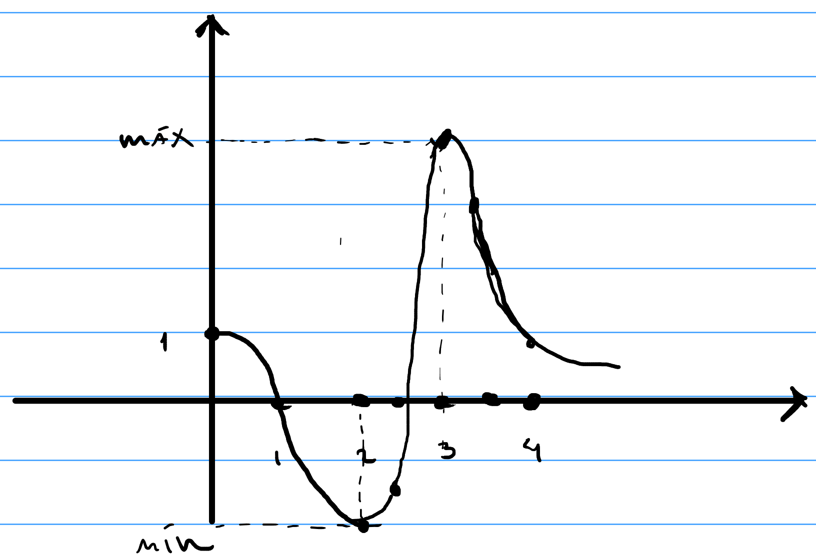
$x=2$: mínimo local; $x=3$: máximo local

$x=1$; $x=2,5$; $x=3,5$ pontos de inflexão

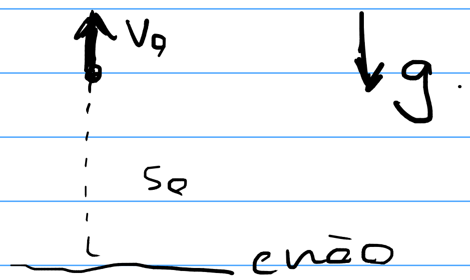
$x=1 \Rightarrow \cap \to \cup$

$x=2,5 \Rightarrow \cup \to \cap$

$x=3,5 \Rightarrow \cap \to \cup$.



(73)



Mostrar que $(v(t))^2 = v_0^2 - 19{,}6(s(t) - s_0)$

Aceleração da gravidade: $9{,}8\ m/s^2$

Como está no sentido contrário

$\therefore v'(t) = -9{,}8$

$v(t) = -9{,}8t + C \Rightarrow v(0) = v_0$

$\therefore v(t) = v_0 - 9{,}8t$

$s'(t) = v(t)$

$\therefore s(t) = v_0 t - \frac{9{,}8t^2}{2} + C \Rightarrow s(0) = s_0$

$s(t) = s_0 + v_0 t - \frac{9{,}8t^2}{2}$

Mas, $t = \frac{v_0 - v(t)}{9{,}8}$

Logo, $s(t) - s_0 = v_0 \cdot \frac{(v_0 - v(t))}{9{,}8} - \frac{\cancel{9{,}8}(v_0 - v(t))^2}{2 \cdot 9{,}8^{\cancel{2}}}$

$$19{,}6\,(s(t)-s_0) = -2v_0v(t) + \cancel{2v_0^2} - v(t)^2 + \cancel{2v_0v(t)} - v_0^2$$

$$\boxed{\therefore\ (v(t))^2 = v_0^2 - 19{,}6\,(s(t)-s_0)}$$

Seção 5.5: 3, 4, 11, 19, 21, 30, 33, 55, 59, 81, 87

(3) $\int x^2 \sqrt{x^3+1}\, dx$

$u = x^3+1$

$du = 3x^2\, dx$

$\therefore x^2 dx = du \cdot \frac{1}{3}$.

$\Rightarrow \int \frac{1}{3} \cdot u^{1/2} du = \frac{1}{3}\left(\frac{u^{3/2}}{3/2}\right) + C$

$\boxed{= \frac{2(x^3+1)^{3/2}}{9} + C}$ //.

(4) $\int \operatorname{sen}^2\theta \cos\theta \, d\theta$

$u = \operatorname{sen}\theta$.

$du = \cos\theta\, d\theta$

$\int u^2 . du = \frac{u^3}{3} + C = \boxed{\frac{\operatorname{sen}^3\theta}{3} + C}$ ///.

(11) $\int t^3 e^{-t^4} dt$

$u = -t^4$

$du = -4t^3\, dt$

$-\frac{1}{4} . du = t^3 dt$

$\Rightarrow \int e^u \cdot \left(\frac{-1}{4}\right) du = -\frac{1}{4} e^u + C = \boxed{-\frac{1}{4} e^{-t^4} + C}$ ///.

19) $\int \cos^3\theta \cdot \operatorname{sen}\theta \, d\theta$

$u = \cos\theta$
$du = -\operatorname{sen}\theta \, d\theta$

$$\Rightarrow \int -u^3 \cdot du = -\frac{u^4}{4} + C = \boxed{-\frac{\cos^4\theta}{4} + C}$$

21) $\int \frac{e^u}{(1-e^u)^2} du$

$s = 1 - e^u$
$-ds = +e^u du$

$$\Rightarrow \int \frac{-ds}{s^2} = -\int s^{-2} ds = -\frac{s^{-1}}{-1} + C$$

$$= s^{-1} + C = \boxed{\frac{1}{1-e^u} + C}$$

30) $\int \frac{dx}{ax+b}$ $(a \neq 0)$

$u = ax + b$
$du = a\,dx \Rightarrow dx = \frac{1}{a} \cdot du$

$$\Rightarrow \int \frac{1}{a} \cdot \frac{du}{u} \Rightarrow \frac{1}{a} \ln|u| + C = \boxed{\frac{\ln|ax+b|}{a} + C}$$

(33) $\int \frac{\sec^2\theta}{tg\theta}\,d\theta$

$u = tg\,\theta$
$du = \sec^2\theta\,d\theta.$

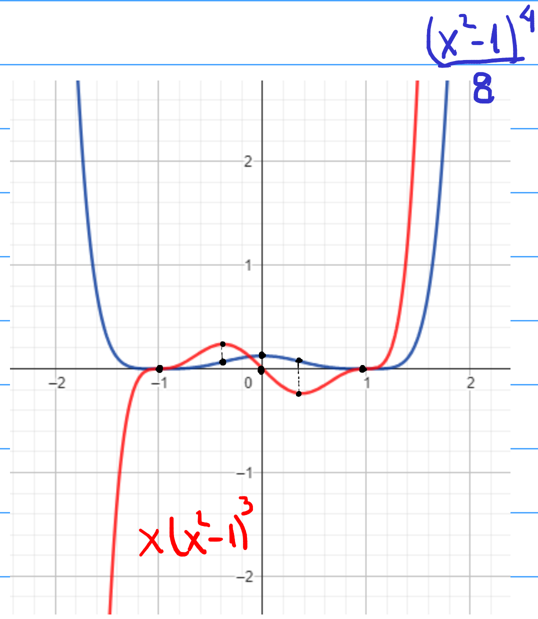
$\Rightarrow \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|tg\,\theta| + C}$

(55) $\int x(x^2-1)^3\,dx$

$u = x^2-1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$\Rightarrow \int \frac{1}{2}u^3\,du = \frac{1}{2}\cdot\frac{u^4}{4} + C = \boxed{\frac{(x^2-1)^4}{8} + C}$

Fazendo $C=0$ e fazendo os gráficos de $\frac{(x^2-1)^4}{8}$ e $x(x^2-1)^3$:



- Quando $x(x^2-1)^3 < 0$, $\frac{(x^2-1)^4}{8}$ é decrescente e quando $x(x^2-1)^3 > 0$, $\frac{(x^2-1)^4}{8}$ é crescente
- Os pontos críticos de $x(x^2-1)^3$ são pontos de máximo ou mínimo locais.

• Quando $x(x^2-1)^3$ possui máximo ou mínimo locais, $\frac{(x^2-1)^4}{8}$ possui ponto de inflexão.

• Pelos gráficos e pela verificação reversa, as funções são plausíveis.

59 $\int_0^1 \cos(\pi t/2)\, dt$

$u = \pi t/2$
$du = \pi/2\, dt$
$\frac{2}{\pi} du = dt$

$= \int_0^1 \frac{2}{\pi} \cos u\, du \Rightarrow \frac{2}{\pi} \cdot (+\text{sen}\, \pi t/2) + C$

$\Rightarrow \frac{2}{\pi}\left(+\text{sen}\, \pi/2 - \text{sen}\, 0\right) = \boxed{+\frac{2}{\pi}}$

81 $y = \sqrt{2x+1}$ $\quad 0 \leq x \leq 1.$

como $y \geq 0$, então a área sob a curva é $\int_0^1 \sqrt{2x+1}\, dx$

$u = 2x+1.$ $\quad du = 2dx \quad dx = \frac{1}{2} du$

$\Rightarrow \frac{1}{2}\int_0^1 u^{1/2} du \Rightarrow \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C \Rightarrow \frac{1}{2} \cdot \frac{2}{3} (2x+1)^{3/2} + C = \boxed{\frac{(2x+1)^{3/2}}{3} + C}$

$$\int_0^1 \sqrt{2x+1}\,dx = \frac{(2.1+1)^{3/2}}{3} - \frac{(2.0+1)^{3/2}}{3} = \frac{3\sqrt{3}-1}{3}$$

$$\boxed{= \sqrt{3} - 1/3}$$

(87) $r(t) = 100\, e^{-0{,}01t}$ $\quad r(t)$ en L/min

$r(t) = f'(t)$

$\int r(t)\,dt \Rightarrow f(t).$

$$\int_0^{60} 100\, e^{-0{,}01t}\,dt \Rightarrow \qquad u = -0{,}01t.$$

$$du = -0{,}01\,dt$$

$$-100\,du = dt$$

$$\int_0^{60} -100^2 e^u\,du = -100^2 \int_0^{60} e^u\,du = -100^2\left(e^{-0{,}01.60} - e^0\right)$$

$$= 100^2 - 100^2 \cdot e^{-0{,}6} = 100^2\left(1 - e^{-0{,}6}\right) = 100^2(1 - 0{,}55)$$

$$= 100^2 \cdot 0{,}45 = \boxed{4590L}$$

Problemas Quentes: 1, 5, 6, 8, 10, 13, 18, 19

1) $x\,\text{sen}(\pi x) = \int_0^{x^2} f(t)\,dt$, $f(4) = ?$

$$\frac{d(x\,\text{sen}\,\pi x)}{dx} = \frac{d}{dx}\int_0^{x^2} f(t)\,dt$$

$$x\cos\pi x \cdot \pi + \text{sen}\,\pi x = f(x^2)\cdot 2x$$

$$x = 2 \Rightarrow 2\pi\cos 2\pi + \text{sen}\,2\pi = f(4)\cdot 4$$

$$f(4) = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$$

5) $$f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}}\,dt \qquad f'(\pi/2).$$

$$g(x) = \int_0^{\cos x} (1+\text{sen}\,t^2)\,dt$$

$$g(\pi/2) = \int_0^{0} (1+\text{sen}\,t^2)\,dt = 0$$

$$g'(\pi/2) = (1+\text{sen}(\cos\pi/2)^2)(-\text{sen}\,\pi/2) = -1$$

$$f'(\pi/2) = \frac{1}{\sqrt{1+(g(\pi/2))^3}} \cdot g'(\pi/2)$$

$$f'(\pi/2) = \frac{1}{\sqrt{1+0}} \cdot (-1) = \boxed{-1}$$

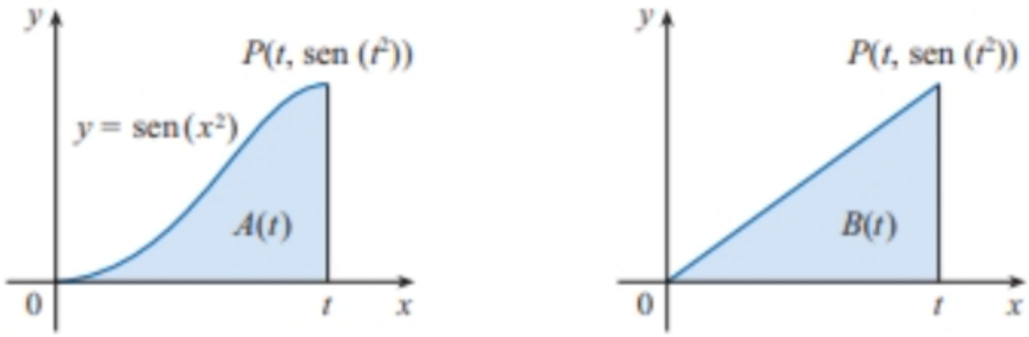
6 $f(x) = \int_0^x x^2 \operatorname{sen}(t^2)\,dt \qquad f'(x) = ?$

$$f(x) = x^2 \int_0^x \operatorname{sen}(t^2)\,dt.$$

$$\boxed{f'(x) = 2x \int_0^x \operatorname{sen}(t^2)\,dt + x^2 \operatorname{sen}(x^2).}$$

Funçã de fres nel!

8



$$\lim_{t\to 0^+} \frac{A(t)}{B(t)}$$

$$\begin{cases} A(t) = \int_0^t \operatorname{sen}(t^2) \\ B(t) = \frac{t \operatorname{sen}(t^2)}{2} \end{cases}$$

$$\lim_{t\to 0^+} \frac{\int_0^t \operatorname{sen}(t^2)\,dt}{\frac{t\,\operatorname{sen}(t^2)}{2}} = \frac{0}{0} \text{ (Indeterminado)}$$

Aplicando L'Hospital:

$$\lim_{t\to 0^+} \frac{2\,\operatorname{sen} t^2}{\operatorname{sen} t^2 + t\cos t^2 \cdot 2t} = \lim_{t\to 0^+} \frac{2\operatorname{sen} t^2}{\operatorname{sen} t^2 + 2t^2\cos t^2} = \frac{0}{0}$$

Aplicando novamente:

$$\lim_{t\to 0^+} \frac{2\cos t^2 \cdot 2t}{2t\cos t^2 + 4t\cos t^2 - 2t^2\operatorname{sen} t^2 \cdot 2t}$$

$$= \lim_{t\to 0^+} \frac{{}^{2}\cancel{4t}\cos t^2}{\cancel{2t}(3\cos t^2 - 2t^2\operatorname{sen} t^2)} = \lim_{t\to 0^+} \frac{2\cos t^2}{3\cos t^2 - 2t^2\operatorname{sen} t^2}$$

$$= \frac{2 \cdot 1}{3 \cdot 1 - 0} = \boxed{\frac{2}{3}}$$

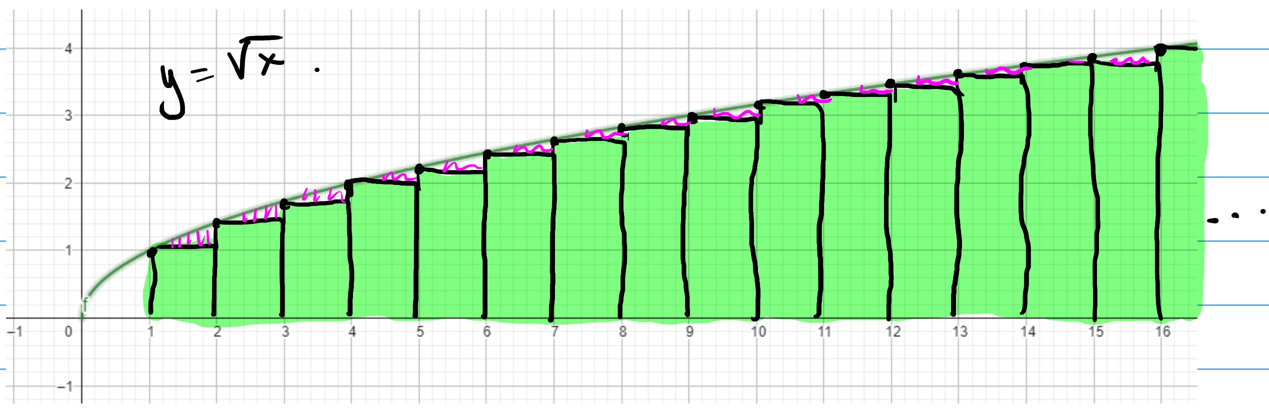
$$\boxed{\therefore \lim_{t\to 0^+} \frac{\int_0^t \operatorname{sen}(t^2)\,dt}{\frac{t\,\operatorname{sen}(t^2)}{2}} = \frac{2}{3}}$$

(10) $$\sum_{i=0}^{10000} \sqrt{i} \approx \int_0^{10000} \sqrt{i}\, di$$

$$\Rightarrow \int_0^{10000} \sqrt{i}\, di = \left.\frac{i^{3/2}}{3/2}\right]_0^{10000} = \frac{(10\,000)^{3/2}}{3/2}$$

$$= \frac{2.\left(\sqrt{100^2}\right)^3}{3} = \frac{2\,000\,000}{3} \approx 666666{,}7$$

Representação gráfica:



(13) $P(x) = a + bx + cx^2 + dx^3$

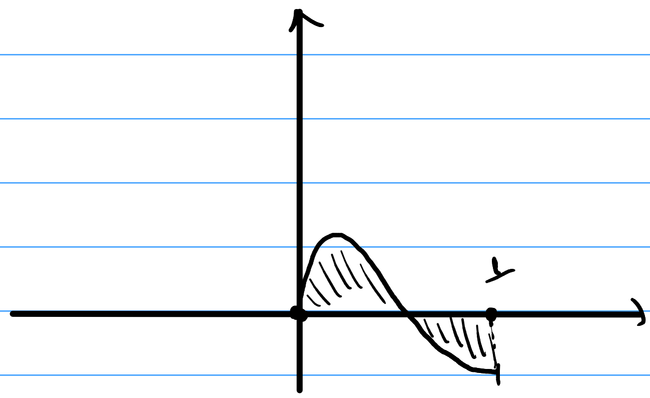
$$a + \frac{b}{2} + \frac{c}{3} + \frac{d}{4} = 0$$

Prove que $\exists x \in (0,1) \mid P(x) = 0$
Generalize para o grau n.

$\rightarrow \int P(x)\,dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4} + C. \quad (C = 0).$

$$\int_0^1 P(x)\,dx = a + \frac{b}{2} + \frac{c}{3} + \frac{d}{4} - 0 = 0.$$

A função teria esse cara:



Como $\int_0^1 P(x)\,dx = 0$, então temos certeza

que para algum $c \in (0,1)$ temos $\int_0^c P(x)\,dx = \int_c^1 P(x)\,dx$
Pelo teorema do valor int e de Rolle, temos certeza
que $P(x)$ tem solução no intervalo $(0,1)$.

Generalização:

$$P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

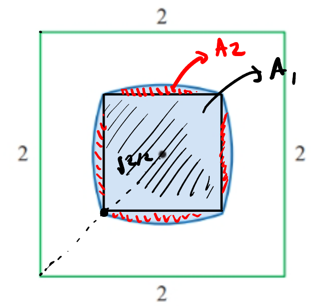
$$a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \dots + \frac{a_n}{n+1} = 0$$

Fazendo $\int P(x) = a_0x + \frac{a_1x^2}{2} + \dots + \frac{a_n \cdot x^{n+1}}{n+1} + C \quad (C=0)$

$$\int_0^1 P(x) = a_0 + \frac{a_1}{2} + \dots + \frac{a_n}{n+1} - 0 = 0.$$

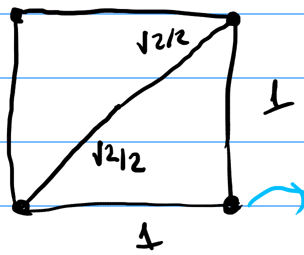
Pelo mesmo argumento do raciocínio anterior, existe um $c \in (0,1) \mid P(c) = 0$.

18



$$\text{Área} = A_1 + 4A_2$$

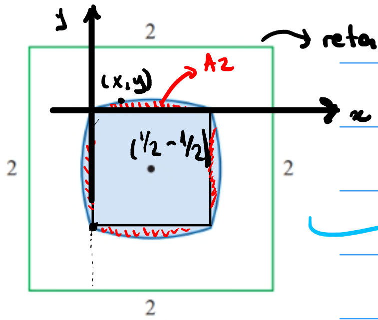
I) $A_1 =$



$$\boxed{A_1 = 1^2 = 1}$$

→ (A borda são pontos em que a distância é igual).

II)



→ reta $y = \frac{1}{2}$, $\Rightarrow 0.x + 1.y - \frac{1}{2} = 0$

→ Novo sistema ordenado

$$\sqrt{(x-\tfrac{1}{2})^2 + (y+\tfrac{1}{2})^2} = \frac{|0.x + 1.y - \tfrac{1}{2}|}{1}$$

$$\sqrt{(x-\tfrac{1}{2})^2 + (y+\tfrac{1}{2})^2} = |y - \tfrac{1}{2}|$$

$$x^2 - x + \tfrac{1}{4} + \cancel{y^2} + y + \cancel{\tfrac{1}{4}} = \cancel{y^2} - y + \cancel{\tfrac{1}{4}}$$

$$2y = x - x^2 - \tfrac{1}{4}.$$

$$\boxed{y = \frac{x - x^2 - 1/4}{2}}$$

Fazendo $\int_0^2 y\,dx = \int_0^2 \frac{x - x^2 - 1/4}{2}\,dx$

$\Rightarrow \int y = \frac{1}{2}\left[\int x\,dx - \int x^2\,dx - \int \frac{1}{4}\,dx\right]$

$$\boxed{= \frac{1}{2}\left[\frac{x^2}{2} - \frac{x^3}{3} - \frac{1}{4}x\right] + C}$$

//

$\int_0^2 y = \frac{1}{2}\left[\frac{4}{2} - \frac{8}{3} - \frac{1}{2}\right] - 0 = \frac{1}{2}\left[2 - \frac{1}{2} - \frac{8}{3}\right)$

$= \frac{1}{2}\left(\frac{3}{2} - \frac{8}{3}\right) = \frac{1}{2}\left(\frac{9-16}{6}\right) = -\frac{5}{12}$

Como $y \leq 0$ $\forall x \in [0,2]$, então $A_2 = \frac{5}{12}$

$4A_2 = \frac{5}{3}$.

$\Rightarrow A_1 + 4A_2 = 1 + \frac{5}{3} = \boxed{\frac{8}{3}}$ //

(19) $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \dots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right)$

Fazendo pela definição de integral:

$$\lim_{n\to\infty} \sum_{i=0}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx \;;\; \Delta x = \frac{b-a}{n}.$$

O limite do exercício pode ser escrito como:

$\lim_{n\to 0} \sum_{i=0}^{n} \frac{1}{\sqrt{n}\sqrt{n+i}}$. Pela definição de integral,

$$\lim_{n\to 0} \sum_{i=0}^{n} \frac{1}{\sqrt{n}\sqrt{n+i}} = \int_0^n \frac{1}{\sqrt{n}\sqrt{n+i}}\,di = \frac{1}{\sqrt{n}} \int_0^n \frac{1}{\sqrt{n+i}}\,di$$

$$\Rightarrow \int \frac{1}{\sqrt{n+i}}\,di \quad \Rightarrow \quad u = n+i \quad du = di$$

$$\Rightarrow \int \frac{1}{\sqrt{n+1}}\,di = \int \frac{du}{\sqrt{u}} = \frac{u^{1/2}}{1/2} + C = 2\sqrt{u} + C = 2\sqrt{n+i} + C$$

$$\Rightarrow \frac{1}{\sqrt{n}} \int_0^n \frac{1}{\sqrt{n+i}}\,di = \frac{1}{\sqrt{n}} \left(2\sqrt{n+n} - 2\sqrt{n+0} \right) = \frac{2}{\cancel{\sqrt{n}}} \left(\sqrt{2}\cdot\cancel{\sqrt{n}} - \cancel{\sqrt{n}} \right)$$

$$\boxed{= 2(\sqrt{2}-1)}.$$