LISTA 10

Secção 15.6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 33, 35, 37, 39

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EXEMPLO 1 Calcule a integral tripla $\iiint_B xyz^2\, dV$, onde B é a caixa retangular dada por

$$B = \{(x, y, z) \mid 0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3\}$$

Pelo Teorema de Fubini:

$$\iiint_B xyz^2\, dV = \int_0^1\int_0^3\int_{-1}^2 xyz^2\, dy\, dz\, dx$$

$$\Rightarrow \int_{-1}^2 xyz^2\, dy = xz^2y^2/2 \Big|_{-1}^2 = 2xz^2 - xz^2/2 = 3/2\, xz^2$$

$$\Rightarrow \int_0^3 3/2\, xz^2\, dz = 3/2\, x\, z^3/3 \Big|_0^3 = 27/2\, x$$

$$\Rightarrow \int_0^1 27/2\, x\, dx = \boxed{27/4}$$

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$$\int_0^2\int_0^{z^2}\int_0^{y-z} (2x-y)\, dx\, dy\, dz$$

$$\Rightarrow \int_0^{y-z} (2x-y)\, dx = (x^2 - xy)\Big|_0^{y-z} = y^2 - 2yz + z^2 - y^2 + yz = z^2 - zy.$$

$$\Rightarrow \int_0^{z^2} (z^2 - zy)\, dy = \left(z^2 y - zy^2/2\right)\Big|_0^{z^2} = z^4 - z^5/2.$$

$$\Rightarrow \int_0^2 (z^4 - z^5/2)\, dz = \left(z^5/5 - z^6/12\right)\Big|_0^2 = \frac{2^5}{5} - \frac{2^5}{6}$$

$$\frac{2^5}{6\cdot 5} = \frac{2^4}{15} = \boxed{\frac{16}{15}}.$$

(5) $$\int_1^2 \int_0^{2z} \int_0^{\ln x} x e^{-y}\, dy\, dx\, dz$$

$$\Rightarrow \int_0^{\ln x} x e^{-y}\, dy = \left(-x e^{-y}\right)\Big|_0^{\ln x} = -x e^{-\ln x} - x = -1 - x$$

$$\Rightarrow \int_0^{2z} (-1-x)\, dx = \left(-x - x^2/2\right)\Big|_0^{2z} = -2z - 2z^2$$

$$\Rightarrow \int_1^2 (-2z - 2z^2)\, dz = -\left(z^2 - 2/3\, z^3\right)\Big|_1^2 = -\left((4 - 16/3) - (1 - 2/3)\right)$$

$$= -(-4/3 - 1/3) = \boxed{5/3},$$

(7) $$\int_1^3 \int_{-1}^2 \int_{-y}^{z} \frac{z}{y}\, dx\, dz\, dy$$

$$\Rightarrow \int_{-y}^{z} z/y\, dx = z/y\, x\Big|_{-y}^{z} = z^2/y + z$$

$$\Rightarrow \int_{-1}^{2} (z^2/y + z)\, dz = \left(z^3/3y + z^2/2\right)\Big|_{-1}^{2} = \frac{8}{3y} + 2 + \frac{1}{3y} - \frac{1}{2}$$

$$= 3/y + 3/2$$

$$\Rightarrow \int_1^3 \left(3/y + 3/2\right) dy = \left(3\ln y + 3/2\, y\right)\Big|_1^3 = 3\ln 3 + 9/2 - 3/2$$

$$\boxed{= 3\ln 3 + 3}$$

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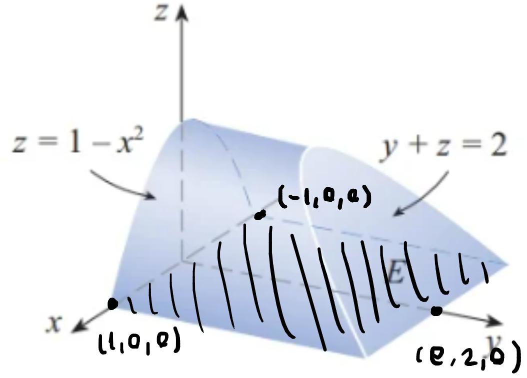

a) $\displaystyle\int_{-1}^{1}\int_0^2\int_{1-x^2}^{2-y} x \, dz\, dy\, dx$

b) $\Rightarrow \displaystyle\int_{1-x^2}^{2-y} x\,dz = xz\Big|_{1-x^2}^{2-y} = x(2-y) - x(1-x^2)$

$$= 2x - x - xy + x^3$$

$$= x^3 + x - xy.$$

$$\Rightarrow \int_0^2 (x^3 + x - xy)\, dy = (x^3 y + xy - xy^2/2)\Big|_0^2$$

$$= 2x^3 + 2x - 2x = 2x^3$$

$$\rightarrow \int_{-1}^{1} 2x^3\, dx = 2/4\, x^4 \Big|_{-1}^{1} = \boxed{0}$$

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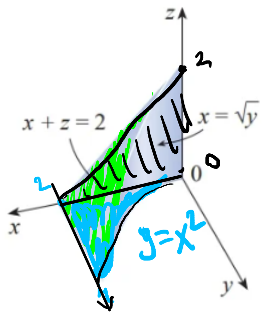
11. $f(x, y, z) = x + y$

1



a) $\displaystyle\int_0^2\int_0^{x^2}\int_0^{2-x} (x+y)\, dz\, dy\, dx$

b) $\Rightarrow \int_0^{2-x} (x+y)\,dz = (x+y)z\Big|_0^{2-x} = (x+y)(2-x)$

$= 2x - x^2 + 2y - xy$

$= -x^2 + 2x + 2y - xy$

$\Rightarrow \int_0^{x^2} (-x^2 + 2x + 2y - xy)\,dy = (-x^2y + 2xy + y^2 - xy^2/2)\Big|_0^{x^2}$

$= -x^4 + 2x^3 + x^4 - x^5/2 = 2x^3 - x^5/2$

$\Rightarrow \int_0^2 (2x^3 - x^5/2)\,dx = (x^4/2 - x^6/12)\Big|_0^2 = \frac{2^4}{2} - \frac{2^6}{12}$

$= 8 - \frac{2^4}{3} = \frac{24-16}{3} = \boxed{8/3}$

(13) $\iiint_E y\,dV = \int_0^3\int_0^x\int_{x-y}^{x+y} y \; dz\,dy\,dx$

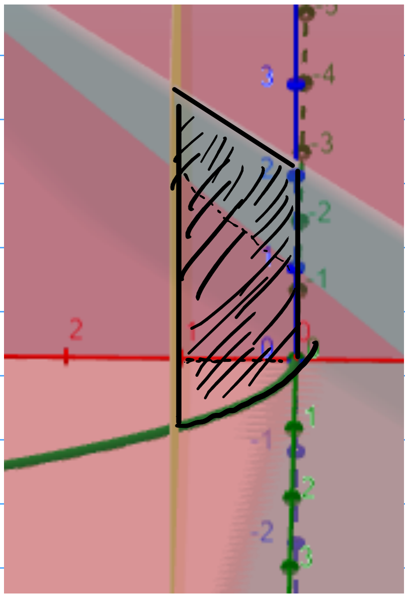
$\Rightarrow \int_{x-y}^{x+y} y\,dz = yz\Big|_{x-y}^{x+y} = y(x+y) - y(x-y) = 2y^2$

$\Rightarrow \int_0^x 2y^2\,dy = 2/3\,y^3\Big|_0^x = 2/3\,x^3$

$\Rightarrow \int_0^3 2/3\,x^3\,dx = (2/12\,x^4)\Big|_0^3 = \frac{\cancel{2}\cdot \cancel{3^4}^{\,3^3}}{\cancel{12}\;2} = \boxed{\frac{27}{2}}$

(15) Questão errada.

(17)



$$\int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 6xy \; dz\,dy\,dx$$

$$\Rightarrow \int_0^{1+x+y} 6xy \; dz = 6xy(1+x+y)$$

$$= 6xy + 6x^2y + 6xy^2$$

$$\Rightarrow \int_0^{\sqrt{x}} (6xy + 6x^2y + 6xy^2)\,dy$$

$$= (3xy^2 + 3x^2y^2 + 2xy^3)\Big|_0^{\sqrt{x}}$$

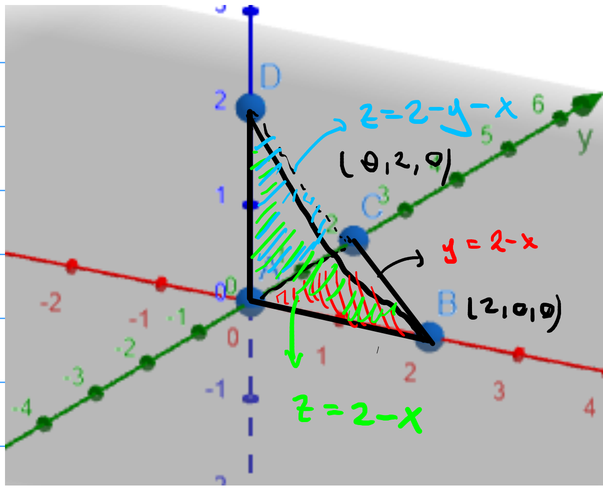
$$= 3x^2 + 3x^3 + 2x^2\sqrt{x}$$

$$= 3x^3 + 3x^2 + 2x^{5/2}$$

$$\Rightarrow \int_0^1 (3x^3 + 3x^2 + 2x^{5/2})\,dx = \left(\frac{3x^4}{4} + x^3 + \frac{2x^{7/2}}{7/2}\right)\Bigg|_0^1$$

$$= \frac{3}{4} + 1 + 2\cdot\frac{2}{7} = \frac{3}{4} + \frac{4}{7} + 1 = \frac{21+16+28}{28} = \boxed{\frac{65}{28}}$$

(19) $\iiint_T y^2 \; dV$



$$\Rightarrow \int_0^2\int_0^{2-x}\int_0^{2-x-y} y^2 \; dz\,dy\,dx \Rightarrow \int_0^{2-x-y} y^2\,dz = (2-x-y)\,y^2$$

$$= 2y^2 - xy^2 - y^3$$

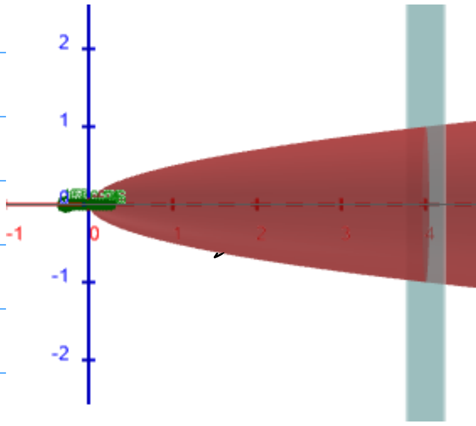
$$\Rightarrow \int_0^{2-x} (2y^2 - xy^2 - y^3)\, dy = \left(2/3\, y^3 - xy^3/3 - y^4/4\right)\Big|_0^{2-x}$$

$$= 2/3\,(2-x)^3 - x(2-x)^3/3 - (2-x)^4/4 = \frac{(2-x)^4}{3} - \frac{(2-x)^4}{4}$$

$$\frac{(2-x)^4}{12}$$

$$\Rightarrow \int_0^2 (2-x)^4/12\, dx = -\frac{1}{12} \cdot \frac{(2-x)^5}{5}\Big|_0^2 = \frac{2^5}{12 \cdot 5} = \boxed{\frac{8}{15}}$$

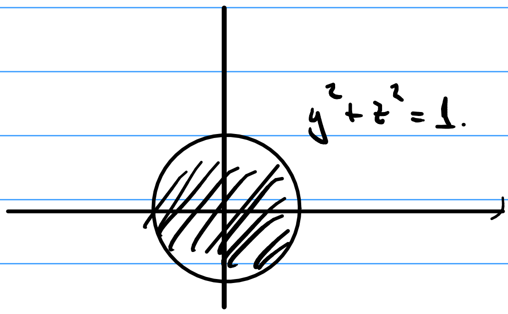
21) $$\iiint_E x\, dV$$



$x = 4y^2 + 4z^2$ $(0 \le x \le 4)$

Projeção em $zy = 4$:

$0 \le r \le 1$



$$\int_0^{2\pi}\int_0^1\int_{4r^2}^4 x\, dx\,(r\,dr\,d\theta)$$

$$\Rightarrow \int_{4r^2}^4 x\, dx = (4^2 - 4^2 r^4)/2 = 8(1 - r^4)$$

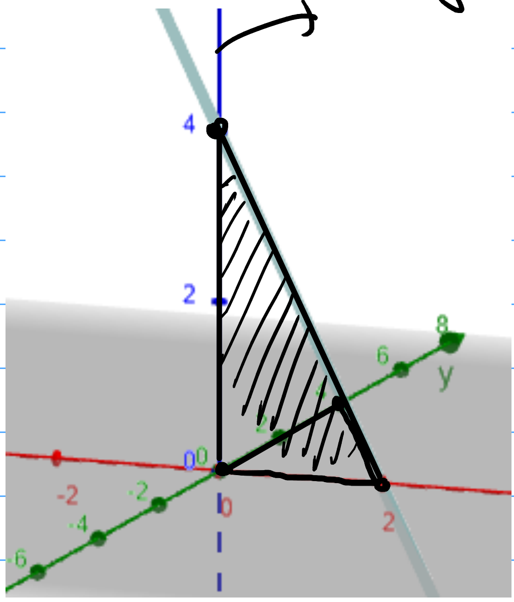
$$\Rightarrow \int_0^1 (8r - 8r^5)\,dr = 8\int_0^1 (r - r^5)\,dr = 8\left[\left(r^2/2 - r^6/6\right)\Big|_0^1\right]$$

$$= 8\left(1/2 - 1/6\right) = \frac{8 \cdot 4}{12} = 8/3$$

$$\Rightarrow \int_0^{2\pi} 8/3 \, d\theta = \boxed{16\pi/3}$$

(23)

$2x + y + z = 4$



$y = 4 - 2x$

$$V = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz\,dy\,dx$$

$$\Rightarrow \int_0^{4-2x} (4 - 2x - y)\,dy = \left(4y - 2xy - y^2/2\right)\Big|_0^{4-2x}$$

$$4(4-2x) - 2x(4-2x) - (4-2x)^2/2$$

$$16 - 8x - 8x + 4x^2 - \frac{2^2}{2}(2-x)^2$$

$$= 16 - 16x + 4x^2 - 2(4 - 2x + x^2) = 8 - 8x + 2x^2$$

$$\Rightarrow \int_0^2 (8-8x+2x^2)\,dx = \left(8x-4x^2+\tfrac{2}{3}x^3\right)\Big|_0^2$$

$$= 16-16+\tfrac{2}{3}\cdot 8 = \boxed{16/3}$$

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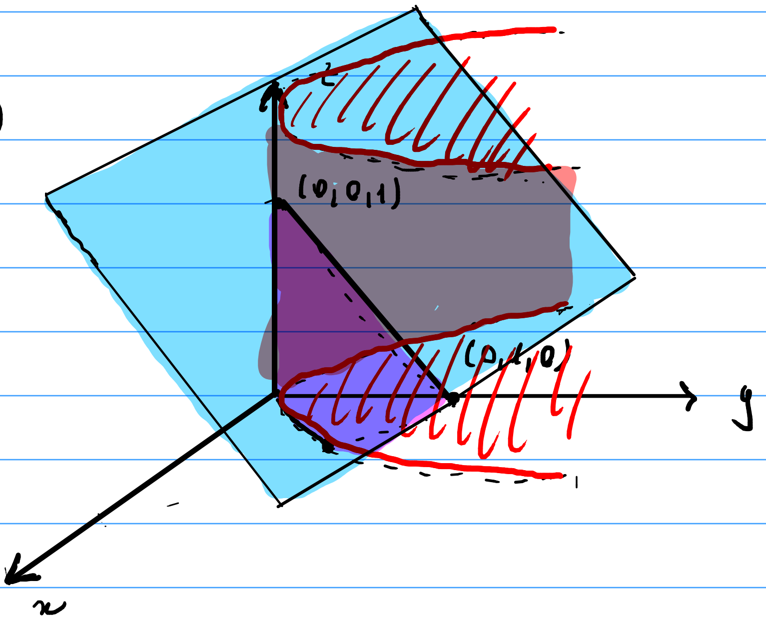

$0 \le z \le 1$

$0 \le y \le 1$

$0 \le x \le 1$

$$V = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\,dy\,dx$$

$$\Rightarrow \int_{x^2}^{1} (1-y)\,dy = \tfrac{1}{2} - x^2 + x^4/2$$

$$\Rightarrow \int_{-1}^{1}\left(\tfrac{1}{2}-x^2+x^4/2\right)\Big|_{-1}^{1} = \left(\frac{x}{2}-\frac{x^3}{3}+\frac{x^5}{10}\right)\Big|_{-1}^{1}$$

$$= \frac{1}{2}-\frac{1}{3}+\frac{1}{10}-\left(-\frac{1}{2}+\frac{1}{3}-\frac{1}{10}\right)$$

$$= 1-\frac{2}{3}+\frac{1}{5} = \frac{15-5+3}{15} = \boxed{\frac{8}{15}}$$

$$4z^2 = 4 - x^2$$

33 $\quad y = 4 - x^2 - 4z^2 \;;\; y = 0$

I) $\displaystyle\int_{-2}^{2}\int_{0}^{4-x^2}\int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x,y,z)\,dz\,dy\,dx$

II) $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}/2}^{\frac{\sqrt{4-x^2}}{2}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dy\,dz\,dx$

III) $\displaystyle\int_{0}^{4}\int_{-\frac{\sqrt{4-y}}{2}}^{\frac{\sqrt{4-y}}{2}}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dz\,dy$

IV) $\displaystyle\int_{0}^{4}\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x,y,z)\,dz\,dx\,dy$

V) $\displaystyle\int_{-1}^{1}\int_{0}^{4-4z^2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dy\,dz$

VI) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dy\,dx\,dz$

35 $\quad y = x^2 \;,\; z = 0 \qquad y + 2z = 4$

I) $\displaystyle\int_{-2}^{2}\int_{0}^{\frac{x^2-4}{2}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dz\,dx$

II) $\displaystyle\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{\frac{4-y}{2}} f(x,y,z)\,dz\,dy\,dx$

III) $\displaystyle\int_{0}^{4}\int_{0}^{\frac{4-y}{2}}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dz\,dy$

IV) $\displaystyle\int_{0}^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{\frac{4-y}{2}} f(x,y,z)\,dz\,dx\,dy$

V) $\displaystyle\int_{0}^{2}\int_{0}^{4-2y}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$

VI) $\displaystyle\int_{0}^{2}\int_{-\sqrt{4-2z}}^{\sqrt{4-2z}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dx\,dz$

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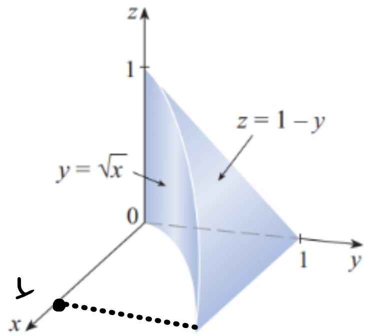

$$\int_0^1\int_{\sqrt{x}}^1\int_0^{1-y} f(x,y,z)\,dz\,dy\,dx$$

I) $\displaystyle\int_0^1\int_0^{1-\sqrt{x}}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dz\,dx$

IV) $\displaystyle\int_0^1\int_0^{1-y}\int_0^{y^2} f(x,y,z)\,dx\,dz\,dy$

II) $\displaystyle\int_0^1\int_0^{y^2}\int_0^{1-y} f(x,y,z)\,dz\,dx\,dy$

V) $\displaystyle\int_0^1\int_0^{1-z}\int_0^{y^2} f(x,y,z)\,dx\,dy\,dz$

III) $\displaystyle\int_0^1\int_0^{(1-z)^2}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dx\,dz$

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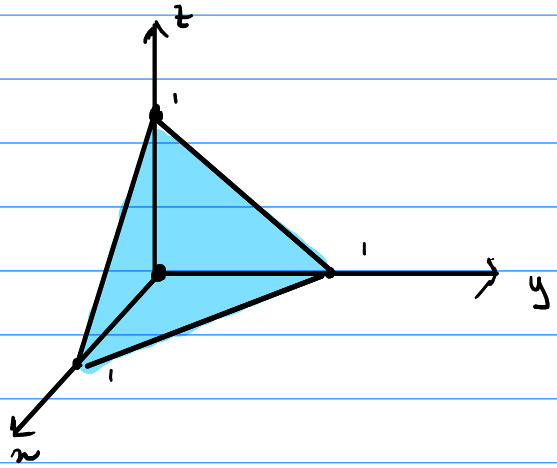
$$\int_0^1\int_y^1\int_0^y f(x,y,z)\,dz\,dx\,dy$$

$0 \le y \le 1$

$y \le x \le 1$

$0 \le z \le y$

$$\boxed{0 \le z \le y \le x \le 1}$$



I) $\displaystyle\int_0^1 \int_0^x \int_z^x f(x,y,z)\, dy\, dz\, dx$ IV) $\displaystyle\int_0^1 \int_0^y \int_y^1 f(x,y,z)\, dx\, dz\, dy$

II) $\displaystyle\int_0^1 \int_z^1 \int_y^1 f(x,y,z)\, dx\, dy\, dz$ V) $\displaystyle\int_0^1 \int_0^x \int_0^y f(x,y,z)\, dz\, dy\, dx$

III) $\displaystyle\int_0^1 \int_z^1 \int_z^x f(x,y,z)\, dy\, dx\, dz$

Seção 15.7: 1, 3, 5, 7, 9, 13, 15, 17, 19, 21, 23, 25, 31

(1) a) $(r, \theta, z) = (5, \pi/2, 2)$ $\quad \begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 5 \end{cases}$

$\Rightarrow (0, 5, 2)$.

b) $(r, \theta, z) = (6, -\pi/4, -3)$ $\quad \begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \Rightarrow \begin{cases} x = 3\sqrt{2} \\ y = -3\sqrt{2} \end{cases}$

$\Rightarrow (3\sqrt{2}, -3\sqrt{2}, -2)$

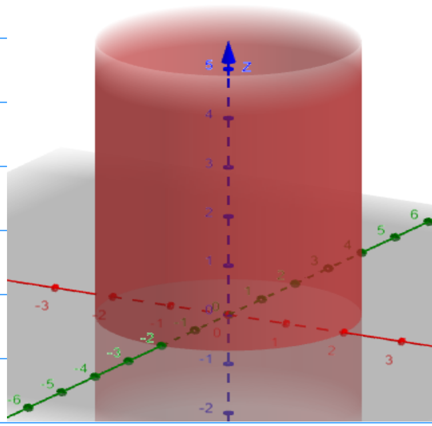
(3) a) $(4, 4, -3)$ $\Rightarrow \begin{cases} x^2+y^2 = r^2 \\ tg\,\theta = y/x \end{cases} \Rightarrow \begin{cases} r^2 = 4^2 \cdot 2 \\ tg\,\theta = 1 \end{cases}$

$\Rightarrow (4\sqrt{2}, \pi/4, -3)$

b) $(5\sqrt{3}, -5, \sqrt{3})$ $\Rightarrow \begin{cases} x^2+y^2 = r^2 \\ tg\,\theta = y/x \end{cases} \Rightarrow \begin{cases} r^2 = 4 \cdot 5^2 \\ tg\,\theta = -\sqrt{3}/3 \end{cases}$

$\Rightarrow (10, -\pi/6, \sqrt{3})$

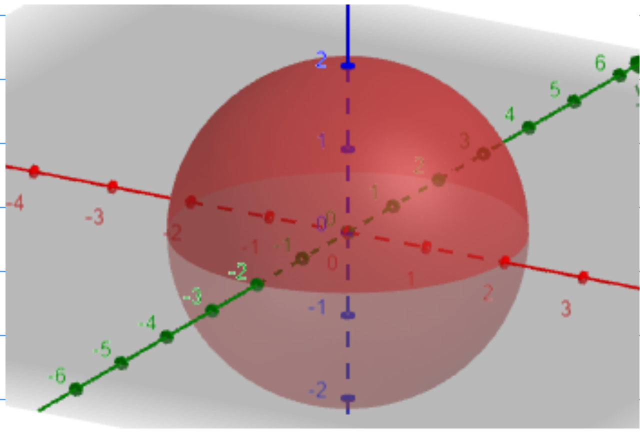
(5) $r = 2$ representa um cilindro de raio 2. e eixo de simetria z.



(7) $r^2 + z^2 = 4$

$r^2 = x^2 + y^2$

Logo, $x^2 + y^2 + z^2 = 2^2$ é uma esfera de raio 2.



(9) a) $x^2 - x + y^2 + z^2 = 1$

$$\begin{cases} x = r\cos\theta \\ y = r\,sen\,\theta \end{cases}$$

$r^2 - r\cos\theta + z^2 = 1$

$z^2 = 1 - r^2 + r\cos\theta$.

b) $z^2 = x^2 - y^2 \Rightarrow z^2 = r^2\cos^2\theta - r^2 sen^2\theta$

$z^2 = r^2(\cos^2\theta - sen^2\theta)$

$z^2 = r^2 \cos 2\theta$

(13) Traduzindo em coordenadas cilíndricas.

$$\begin{cases} 6 \le r \le 7 \text{ (raio interno e externo).} \\ 0 \le \theta < 2\pi \text{ (completa a volta do círculo).} \\ 0 \le z \le 20 \text{ (20 de comprimento).} \end{cases}$$

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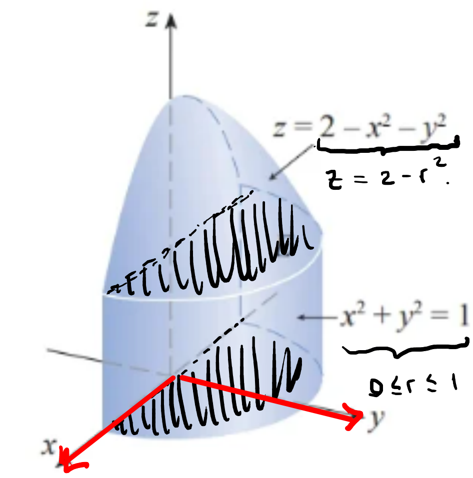
15. $f(x, y, z) = x^2 + y^2$



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a) $$\int_0^{\pi}\int_0^{1}\int_0^{2-r^2} r^2 \cdot r \, dz\, dr\, d\theta$$

$$= \int_0^{\pi}\int_0^{1}\int_0^{2-r^2} r^3 \, dz\, dr\, d\theta$$

b) $$\int_0^{2-r^2} r^3 \, dz = r^3 z \Big|_0^{2-r^2} = (2-r^2)\, r^3 = 2r^3 - r^5.$$

$$\Rightarrow \int_0^1 (2r^3 - r^5)\, dr = \left(r^4/2 - r^6/6\right)\Big|_0^1 = 1/2 - 1/6 = \boxed{1/3}$$

$$\Rightarrow \int_0^{\pi} 1/3 \, d\theta = \boxed{\pi/3}$$

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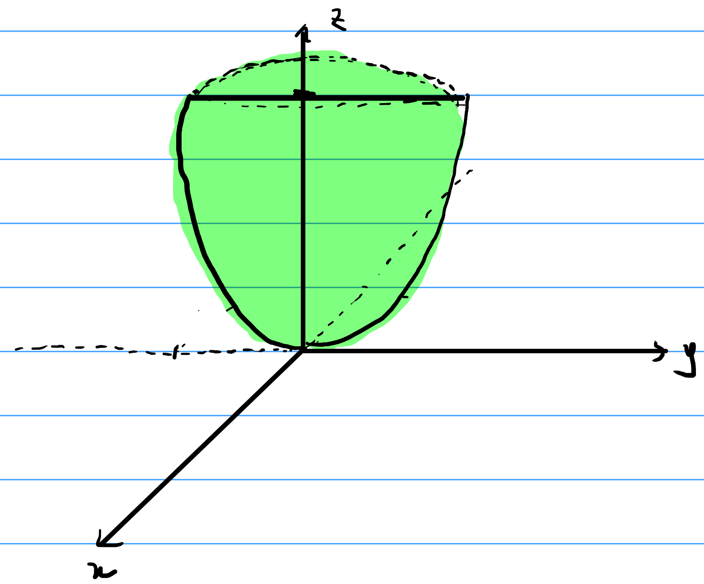
$$\int_{\pi/2}^{3\pi/2}\int_0^{3}\int_{r^2}^{9} r \, dz\, dr\, d\theta$$

$z = x^2 + y^2$



(19) $$\int_0^{2\pi}\int_0^{4}\int_{-5}^{4} r\cdot r \, dz\, dr\, d\theta$$

$$\Rightarrow \int_0^4 9r^2 \, dr \Rightarrow 3r^3\Big|_0^4 \Rightarrow 3.4^3$$

$$\Rightarrow \int_0^{2\pi} 3.4^3 \, d\theta = 6.4^3\pi = 6.64\pi = \boxed{384\pi}$$

(21) $$\int_0^{\pi/2}\int_0^{2}\int_0^{4-r^2} (r\cos\theta + r\operatorname{sen}\theta + z)\, r \, dz\, dr\, d\theta$$

$$\Rightarrow \int_0^{4-r^2} \left(r^2(\operatorname{sen}\theta + \cos\theta) + rz\right) dz = \left(zr^2(\operatorname{sen}\theta+\cos\theta) + rz^2/2\right)\Big|_0^{4-r^2}$$

$$(4-r^2)\, r^2 (\operatorname{sen}\theta + \cos\theta) + r(4-r^2)^2/2$$

$$\Rightarrow (4r^2 - r^4)(\operatorname{sen}\theta + \cos\theta) + r/2(16 - 8r^2 + r^4)$$

$$\Rightarrow (4r^2 - r^4)(\operatorname{sen}\theta + \cos\theta) + 8r - 4r^3 + r^5/2$$

$$\Rightarrow \int_0^2 \left[(4r^2 - r^4)(\operatorname{sen}\theta+\cos\theta) + 8r - 4r^3 + r^5/2\right] dr$$

$$\Rightarrow \left[\left(\frac{4}{3}r^3 - \frac{r^5}{5}\right)\left(\operatorname{sen}\theta + \cos\theta\right) + 4r^2 - r^4 + \frac{r^6}{12}\right]\Bigg|_0^2$$

$$\left(\frac{4}{3}\cdot 8 - \frac{32}{5}\right)(\operatorname{sen}\theta + \cos\theta) + 4.4 - 16 + \frac{\cancel{64}^{16}}{\cancel{12}_3}$$

$$2.32/15\,(\operatorname{sen}\theta + \cos\theta) + 16/3 = 16/3\left(4/5(\operatorname{sen}\theta+\cos\theta) + 1\right)$$

$$\Rightarrow \int_0^{\pi/2} 16/3 \left(4/5 \left(\text{sen}\theta + \cos\theta \right) + 1 \right) d\theta$$

$$16/3 \left(-4/5 \cos\theta + 4/5 \,\text{sen}\theta + \theta \right) \Big|_0^{\pi/2}$$

$$16/3 \left[4/5 + \pi/2 + 4/5 \right] = \boxed{8/3\,\pi + 128/15}$$

(23) $\int_0^{2\pi} \int_0^{1} \int_0^{2r} r^3 \cos^2\theta \, dz\, dr\, d\theta$

$$\Rightarrow \int_0^1 2r^4 \cos^2\theta \, dr = 2/5 \cos^2\theta$$

$$\Rightarrow \int_0^{2\pi} 1/5 (1 + \cos 2\theta) \, d\theta = 1/5 \left(\theta + 1/2 \,\text{sen}\, 2\theta \right) \Big|_0^{2\pi}$$

$$\boxed{= 2\pi/5}$$

(25) $V = \int_0^{2\pi} \int_0^{1} \int_r^{\sqrt{2-r^2}} r \, dz\, dr\, d\theta$

$$= \int_0^1 \left(\sqrt{2-r^2}\, r - r^2 \right) dr = -1/2 \int_0^1 \sqrt{2-r^2}\,(-2r)\, dr - \int_0^1 r^2 \, dr$$

$$= -\frac{1}{2} \left((2-r^2)^{3/2} \cdot \frac{2}{3} \right) \Big|_0^1 - \frac{r^3}{3} \Big|_0^1$$

$$= -\frac{1}{3} \left(1 - 2\sqrt{2} \right) - \frac{1}{3} = \frac{2\sqrt{2}}{3} - \frac{2}{3} = \frac{2}{3} \left(\sqrt{2} - 1 \right)$$

$$\int_0^{2\pi} 2/3(\sqrt{2}-1)\,d\theta = \boxed{4/3\pi(\sqrt{2}-1)}$$

(31) $$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy.$$

$$\Rightarrow \int_0^{2\pi}\int_0^{2}\int_r^{2} r\cos\theta\, z\,dz\,dx\,dy$$

$$\Rightarrow \int_r^2 r^2\cos\theta\, z\,dz = z^2 \cdot \frac{r^2\cos\theta}{2}\Big|_r^2 = 2r^2\cos\theta - \frac{r^2}{2}\cos\theta$$

$$= \frac{3}{2}\cos\theta\, r^2$$

$$\Rightarrow \int_0^2 3/2\cos\theta\, r^2\,dr = \frac{r^3}{\cancel{3}}\cdot\frac{\cancel{3}}{2}\cos\theta\Big|_0^2 = 4\cos\theta$$

$$\Rightarrow \int_0^{2\pi} 4\cos\theta\,d\theta = 4\sin\theta\Big|_0^{2\pi} \boxed{=0}$$

Seção 15.8: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25, 27, 29.

1 a) $(\rho, \theta, \phi) = (2, 3\pi/4, \pi/2)$

$$\begin{cases} x = \rho \operatorname{sen}\phi \cos\theta \\ y = \rho \operatorname{sen}\phi \operatorname{sen}\theta \\ z = \rho \cos\phi \end{cases}$$

$$\boxed{\Rightarrow (-\sqrt{2}, \sqrt{2}, 0)}$$ //.

b) $(\rho, \theta, \phi) = (4, -\pi/3, \pi/4)$

$$\begin{cases} x = \rho \operatorname{sen}\phi \cos\theta \\ y = \rho \operatorname{sen}\phi \operatorname{sen}\theta \\ z = \rho \cos\phi \end{cases}$$

$$\boxed{\Rightarrow (\sqrt{2}, -\sqrt{6}, 2\sqrt{2})}$$ //.

3 a) $(3, 3, 0)$

$$\begin{cases} \rho = \sqrt{x^2 + y^2 + z^2} \\ \theta = \operatorname{arctg}(y/x) \\ \phi = \arccos\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{cases}$$

$$\boxed{\Rightarrow (3\sqrt{2}, \pi/4, \pi/2)}$$ /

b) $(1, -\sqrt{3}, 2\sqrt{3})$

$$\begin{cases} \rho = \sqrt{x^2 + y^2 + z^2} \\ \theta = \operatorname{arctg}(y/x) \\ \phi = \arccos\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{cases}$$

$$\boxed{\Rightarrow (4, -\pi/3, \pi/6)}$$ //.

5 $\phi = 3\pi/4$ é um cone com as coordenadas z negativas

$$\frac{z}{\sqrt{x^2+y^2+z^2}} = \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad z^2 = \frac{1}{2}(x^2+y^2+z^2) \Rightarrow \boxed{z = -\sqrt{x^2+y^2}}$$

(7) $\rho\cos\phi = 1 \quad \Rightarrow \quad \boxed{z=1}$ (plano horizontal)

(9) a) $x^2+y^2+z^2 = 9 \Rightarrow \rho^2 = 9 \Rightarrow \boxed{\rho = 3}$

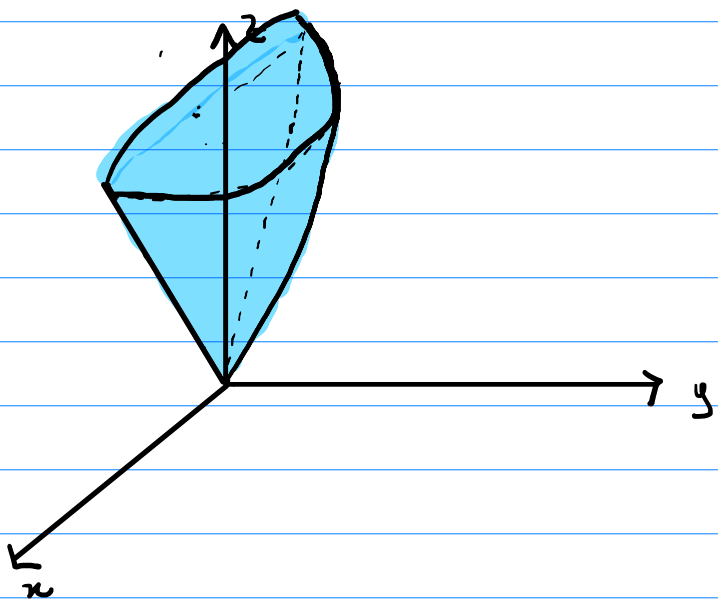
b) $x^2 - y^2 - z^2 = 1$

$\Rightarrow \rho^2 \operatorname{sen}^2\phi \cos^2\theta - \rho^2 \operatorname{sen}^2\phi \operatorname{sen}^2\theta - \rho^2\cos^2\phi = 1$

$\rho^2 \operatorname{sen}^2\phi \cdot \cos 2\theta - \rho^2\cos^2\phi = 1$

$$\boxed{\rho^2(\operatorname{sen}^2\phi \cos 2\theta - \cos^2\phi) = 1}$$

(11) $\rho \leq 1$, $0 \leq \phi \leq \pi/6$, $0 \leq \theta \leq \pi$.

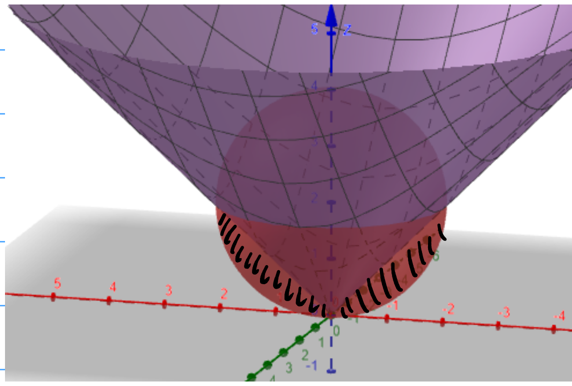


(15) Interno à esfera $x^2+y^2+z^2=4z$.
Externo ao cone $z=\sqrt{x^2+y^2}$.

$\rho^2 = 4\rho\cos\phi \qquad \rho = 4\cos\phi$

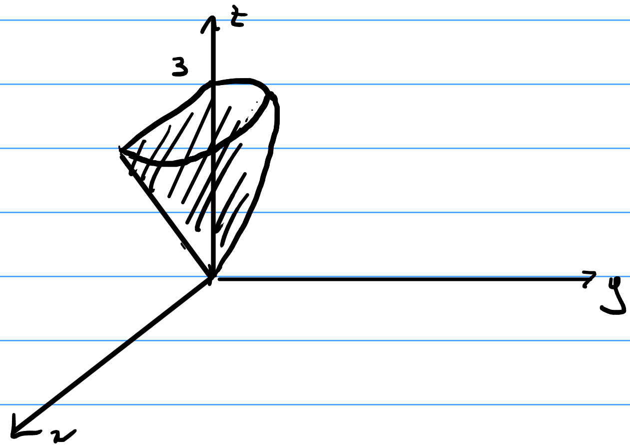
$\rho\cos\phi = \rho|\operatorname{sen}\phi| \Rightarrow \rho(\operatorname{sen}\phi - \cos\phi) = 0 \Rightarrow \rho = 0$ ou $\phi = \pi/4$.

$$\boxed{\begin{array}{c} 0 \le \rho \le 4\cos\phi \\ \pi/4 \le \phi \le \pi/2 \end{array}}$$



(17) $$\int_0^{\pi/6}\int_0^{\pi/2}\int_0^{3} \rho^2 \operatorname{sen}\phi \, d\rho \, d\theta \, d\phi$$

$0 \le \rho \le 3$
$0 \le \theta \le \pi/2$
$0 \le \phi \le \pi/6$



$$\Rightarrow \int_0^3 \rho^2 \operatorname{sen}\phi \, d\rho = \rho^3/3 \operatorname{sen}\phi \Big|_0^3 = 9\operatorname{sen}\phi$$

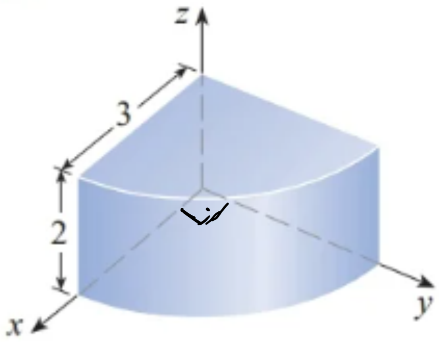
$$\Rightarrow \int_0^{\pi/2} 9 \operatorname{sen}\phi \, d\theta = 9\pi/2 \operatorname{sen}\phi.$$

$$\Rightarrow \int_0^{\pi/6} 9\pi/2 \operatorname{sen}\phi \, d\phi = -9\pi/2 \cos\phi \Big|_0^{\pi/6} = 9\pi/2\left(1 - \sqrt{3}/2\right)$$

$$\boxed{= 9\pi/4\left(2 - \sqrt{3}\right)}$$

19

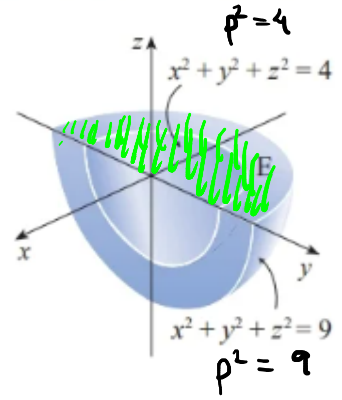
19.



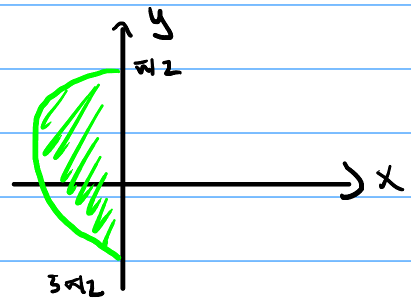
A melhor escolha seria coordenadas cilíndricas, já que o eixo z é eixo de simetria e a projeção em xy é conveniente para coordenadas polares:

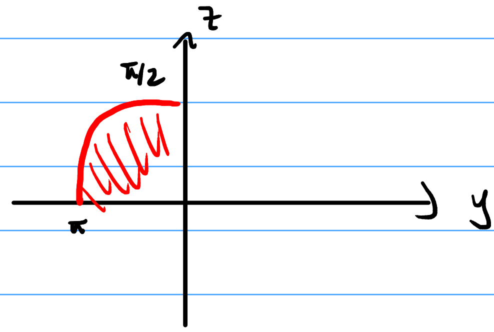
$$\int_0^{\pi/2}\int_0^{3}\int_0^{2} f(r\cos\theta, r\operatorname{sen}\theta, z)\, r \, dz \, dr \, d\theta$$

(21) **21.** $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$



a) $\displaystyle\int \int_{\pi/2}^{3\pi/2} \int_{2}^{3} \rho \cdot \rho^2 \operatorname{sen}\phi \, d\rho \, d\theta \, d\phi$





b) $\displaystyle\int_{\pi/2}^{\pi} \int_{\pi/2}^{3\pi/2} \int_{2}^{3} \rho^3 \operatorname{sen}\phi \, d\rho \, d\theta \, d\phi = \boxed{\frac{65}{4}\pi}$

(23) $\displaystyle\iiint_B (x^2+y^2+z^2)^2 \, dV$

B é a "bola" de centro na origem e raio 5

$\Rightarrow \displaystyle\int_0^{\pi} \int_0^{2\pi} \int_0^{5} \rho^4 \rho^2 \operatorname{sen}\phi \, d\rho \, d\theta \, d\phi$

$$\Rightarrow \int_0^5 \rho^6 \operatorname{sen}\phi \, d\rho = 5^7/7 \operatorname{sen}\phi$$

$$\Rightarrow \int_0^{2\pi} 5^7/7 \operatorname{sen}\phi \, d\theta = 2\pi/7 \cdot 5^7 \operatorname{sen}\phi$$

$$\Rightarrow \int_0^{\pi} (2\pi/7 \cdot 5^7 \operatorname{sen}\phi) \, d\phi = \frac{2\pi}{7} \cdot 5^7 (-\cos\phi)\Big|_0^{\pi}$$

$$\boxed{= \frac{4\pi}{7} \cdot 5^7}$$

25 $\Rightarrow \iiint_E (x^2+y^2) \, dV$ entre as esferas $x^2+y^2+z^2=4$ e $x^2+y^2+z^2=9$

$$\Rightarrow \int_0^{\pi} \int_0^{2\pi} \int_2^3 \rho^2 \operatorname{sen}^2\phi \cdot \rho^2 \operatorname{sen}\phi \, d\rho \, d\theta \, d\phi$$

$$\Rightarrow \int_0^{\pi} \int_0^{2\pi} \int_2^3 \rho^4 \operatorname{sen}^3\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_2^3 \rho^4 d\rho \cdot \int_0^{2\pi} d\theta \cdot \int_0^{\pi} \operatorname{sen}^3\phi \, d\phi$$

$$= \frac{3^5 - 2^5}{5} \cdot 2\pi \cdot \frac{4}{3} = \boxed{\frac{1688\pi}{15}}$$

27 $\iiint x e^{x^2+y^2+z^2} \, dV$

Porção da esfera $x^2+y^2+z^2 \leq 1$ no primeiro octante.

$$\Rightarrow \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} \rho \, sen\phi \cos\theta \, e^{\rho^2} \cdot \rho^2 sen\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} \rho^3 e^{\rho^2} sen^2\phi \cos\theta \, d\rho \, d\theta \, d\phi .$$

$$= \int_0^1 \rho^3 e^{\rho^2} d\rho \cdot \int_0^{\pi/2} \cos\theta \, d\theta \cdot \int_0^{\pi/2} sen^2\phi \, d\phi = 1/2 \cdot 1 \cdot \pi/4 = \boxed{\pi/8}$$

$$\Rightarrow 1/2 \int_0^1 u e^u du = 1/2 \left(u e^u - e^u\right)\Big|_0^1 = +1/2$$

$u = \rho^2$

$$\Rightarrow \int_0^{\pi/2} sen^2\phi \, d\phi = 1/2 \int_0^{\pi/2} (1-\cos 2\phi) d\phi = 1/2 \left(\phi - 1/2 \, sen 2\phi\right)\Big|_0^{\pi/2}$$

$$= 1/2 \left(\pi/2 - 0 - 0\right)$$

29 Volume da parte da "bola" $\rho \le a$ entre os cones $\phi = \pi/6$ e $\phi = \pi/3$.

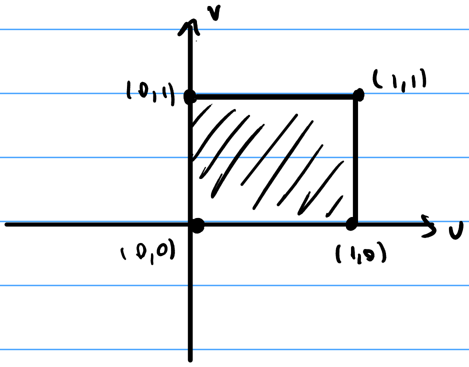
$$\int_{\pi/6}^{\pi/3}\int_0^{2\pi}\int_0^{a} \rho^2 \operatorname{sen}\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^a \rho^2 d\rho \cdot \int_0^{2\pi} d\theta \cdot \int_{\pi/6}^{\pi/3} \operatorname{sen}\phi \, d\phi = a^3/3 \cdot 2\pi \cdot (-1/2 + \sqrt{3}/2)$$

$$= \boxed{\frac{\pi}{3} a^3 (\sqrt{3}-1)}$$ //.

Seção 15.9: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 25, 27

(1) $S = \{(u,v) \mid 0 \le u \le 1,\ 0 \le v \le 1\}$



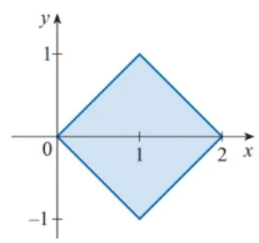
a) $x = u+v$ $\Rightarrow$ $0 \le x \le 2$ $\Rightarrow$ $u=0$: $y=-x$

$y = u-v$ $\quad$ $0 \le y \le 1$ $\quad$ $v=0$: $y=x$

$u=1$: $y=2-x$

$v=1$: $y=x-2$

VI



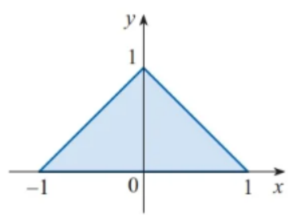
b) $x = u-v$ $\Rightarrow$ $-1 \le x \le 1$ $\Rightarrow$ $u=0$: $x=-v,\ y=0$

$y = uv$ $\quad$ $0 \le y \le 1$ $\quad$ $v=0$: $x=u,\ y=0$

$u=1$: $y=1-x$

$v=1$: $y=1+x$

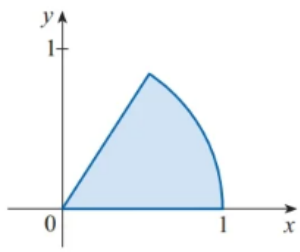
I



c) $x = u\cos v \implies 0 \leq x \leq 1$ (por polar, vai ser um

$y = u\,\text{sen}\, v \qquad 0 \leq y \leq 1$ setor circular)

IV



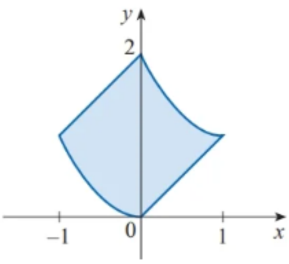
d) $x = u - v \implies -1 \leq x \leq 1 \implies u=0: \; y = x^2, \; -1 \leq x \leq 0$

$y = u + v^2 \qquad 0 \leq y \leq 2 \qquad v=0: \; y = x$

$u=1: \; y = x^2 - 2x + 2, \; 0 \leq x \leq 1$

$v=1: \; y = x + 2, \; -1 \leq x \leq 0$

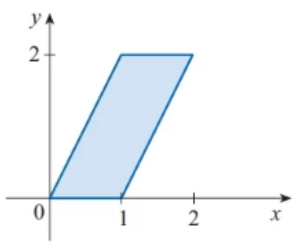
V



e) $x = u + v \implies 0 \leq x \leq 2$

$y = 2v \qquad 0 \leq y \leq 2$

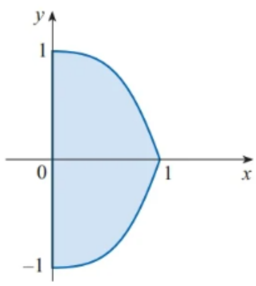
III



f) $x = uv \implies 0 \leq x \leq 1$

$y = u^3 - v^3 \qquad -1 \leq y \leq 1$

II

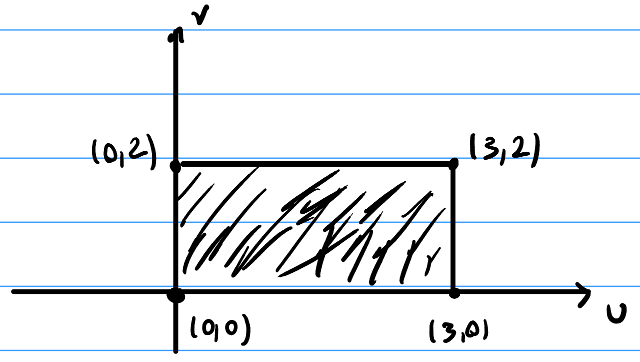


3) $S = \{(u,v) \mid 0 \le u \le 3,\ 0 \le v \le 2\}$

$x = 2u + 3v$

$y = u - v$

S:



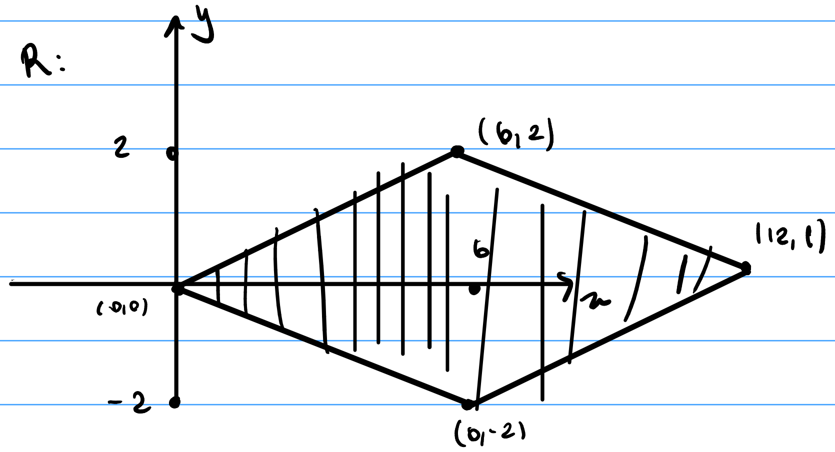
$u = 0: \quad y = -\tfrac{1}{3}x,$

$u = 3: \quad y = 5 - x/3$

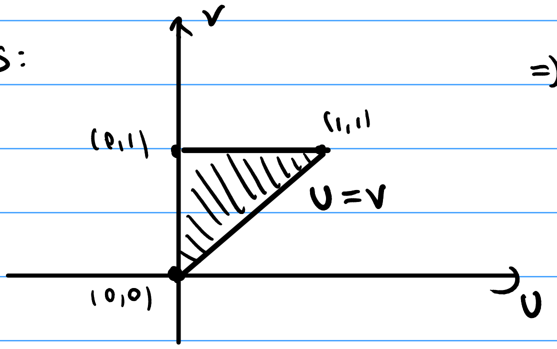
$v = 0: \quad y = \tfrac{1}{2}x$

$v = 2: \quad y = x/2 - 5$

R:



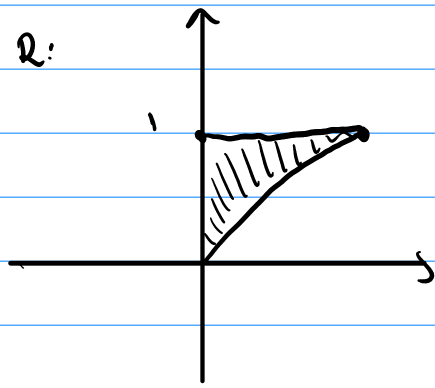
5) S:



$\Rightarrow S = \{(u,v) \mid 0 \le u \le v,\ 0 \le v \le 1\}$

$x = u^2, \quad y = v$

R:



$u = 0 \quad : \quad x = 0, \quad 0 \le y \le 1$

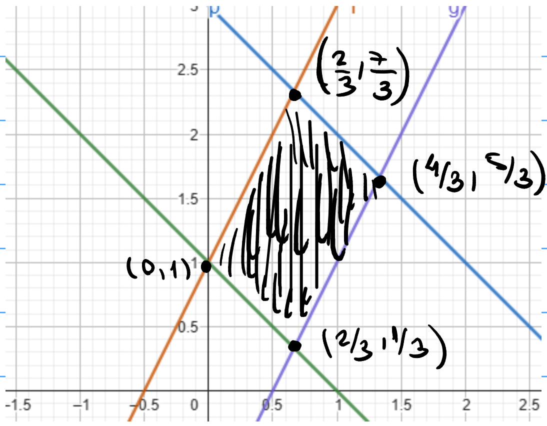
$u = v \quad : \quad x = y^2, \quad y = \sqrt{x}, \quad 0 \le x \le 1$

$v = 0 \quad : \quad x = u^2, \quad y = 0 \quad \Rightarrow x = 0$

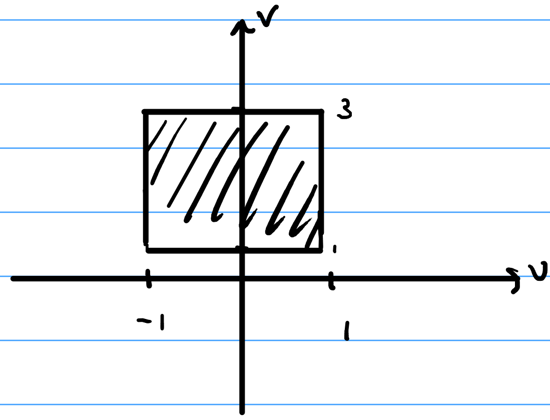
$v = 1 \quad : \quad x = u^2, \ y = 1 \quad \Rightarrow \quad 0 \le x \le 1.$

(7) S é um retângulo em uv.
R é limitado por $y = 2x-1$ $\quad y = 1-x$
$y = 2x+1$ $\quad y = 3-x$

R:



S:

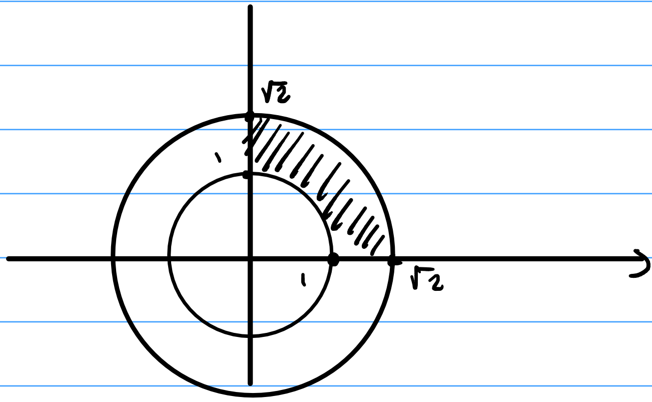


$0 \leq x \leq 4/3 \quad , \quad 1/3 \leq y \leq 7/3$

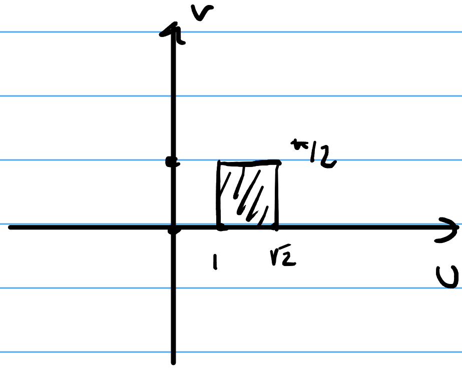
$y = 2x-1 \qquad y = 1-x$
$y = 2x+1 \qquad y = 3-x$

$$\begin{cases} 2x-y = 1 \; , \quad x+y = 1 \\ 2x-y = -1 \; , \quad x+y = 3 \end{cases} \Rightarrow \boxed{\begin{array}{ll} 2x-y = u & -1 \leq u \leq 1 \\ x+y = v & 1 \leq v \leq 3 \end{array}}$$

(9) R:



$x^2+y^2=1$
$x^2+y^2=2$



$$\boxed{\begin{array}{ll} x = u\cos v & 1 \le u \le \sqrt{2} \\ y = u \,\text{sen}\, v & 0 \le v \le \pi/2 \end{array}}$$

Polar.

(11) $x = 2u+v$
$y = 4u - v.$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\det J = -2-4 = \boxed{-6.}$$

(13) $x = s\cos t$
$y = s\,\text{sen}\, t$

$J = s$ (polar)

(15) $x = uv$

$y = vw$

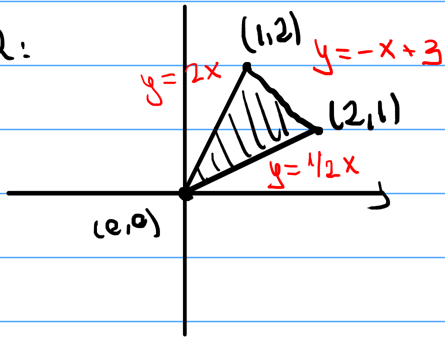
$z = wu$

$$J = \begin{bmatrix} \partial x/\partial u & \partial x/\partial v & \partial x/\partial w \\ \partial y/\partial u & \partial y/\partial v & \partial y/\partial w \\ \partial z/\partial u & \partial z/\partial v & \partial z/\partial w \end{bmatrix} = \begin{bmatrix} v & u & 0 \\ 0 & w & v \\ w & 0 & u \end{bmatrix}$$

$$\det J = v \begin{vmatrix} w & v \\ 0 & u \end{vmatrix} - u \begin{vmatrix} 0 & v \\ w & 0 \end{vmatrix} + 0 = v.wu + u.v.w = \boxed{2uvw.}$$

(17) $$\iint_R (x-3y)\,dA$$

R:



$x = 2u+v$

$y = u+2v$

Para $(0,0)$: $u=v=0$

Para $(1,2)$: $u=0$, $v=1$

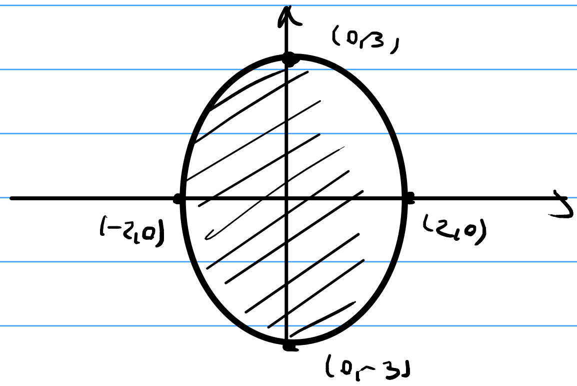
Para $(2,1)$: $v=0$, $u=1$

Assim, a nova região é o triângulo em uv do vértices $(0,0)$, $(0,1)$, $(1,0)$. Logo, uma forma de escrever a nova região seria: $S=\{(u,v)|$

$$\Rightarrow \int_0^1 \int_0^{1-v} (-u-5v) \left\| \begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix} \right\| du\,dv = -3 \int_0^1 \int_0^{1-v} (u+5v)\,du\,dv.$$

$$= \boxed{-3}$$

(19) $\iint_R x^2\,dA$ R:



$9x^2+4y^2=36$

$x=2u$

$y=3v$

$\Rightarrow \boxed{u^2+v^2=1}$

$$\Rightarrow \iint_S 4u^2 \left\| \begin{matrix}2 & 0\\ 0 & 3\end{matrix} \right\| du\,dv = \iint_S 24u^2\,du\,dv$$

$$= \int_0^{2\pi}\int_0^1 24r^2\cos^2\theta \cdot r\,dr\,d\theta$$

$$\Rightarrow \int_0^1 24r^3\,dr \int_0^{2\pi} \cos^2\theta\,d\theta$$

$$= 6\cdot\left(\frac{1}{2}\theta+\frac{1}{4}\,\mathrm{sen}\,2\theta\right)\Big|_0^{2\pi} = 6\cdot\pi = \boxed{6\pi}$$

(25) $\iint_R \frac{x-2y}{3x-y}\,dA$

R: paralelogramo $x-2y=0$, $x-2y=4$, $3x-y=1$, $3x-y=8$

$u=x-2y \rightarrow x=(2v-u)/5$

$v=3x-y \quad y=(v-3u)/5$

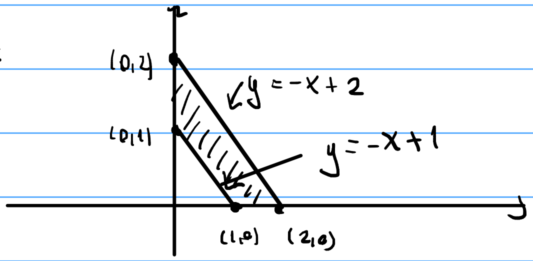
$$\Rightarrow \int_1^8 \int_0^4 \frac{u}{v} \left\| \begin{vmatrix} -1/5 & 2/5 \\ -3/5 & 1/5 \end{vmatrix} \right\| du\,dv = \int_1^8 \int_0^4 \frac{1}{5} \cdot \frac{u}{v}\, du\,dv$$

$$1/5 \cdot \int_1^8 1/v\, dv \cdot \int_0^4 u\, du = \frac{u^2}{2}\Big|_0^4 \cdot \ln v\Big|_1^8 \cdot 1/5$$

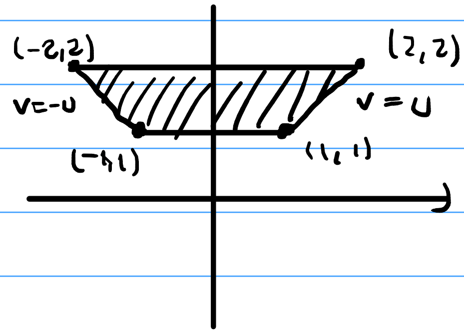
$$= \frac{16}{2} \cdot \frac{\ln 8}{5} = \boxed{\frac{8}{5} \ln 8}$$

(27) $$\iint_R \cos\left(\frac{y-x}{y+x}\right) dA \qquad u = y - x \quad v = y + x \qquad x = (v-u)/2 \quad y = (u+v)/2$$

R:



S:



Para (0,1) : $u = 1,\ v = 1$
Para (0,2): $u = 2,\ v = 2$
Para (1,0) : $u = -1,\ v = 1$
Para (2,0) : $u = -2,\ v = 2$

$$\int_1^2 \int_{-v}^{v} \cos(u/v) \left\| \begin{vmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{vmatrix} \right\| du\,dv$$

$$\int_1^2 \int_{-v}^{v} 1/2 \cos(u/v)\, du\,dv = 1/2 \int_1^2 \int_{-v}^{v} \cos(u/v)\, du\,dv$$

$$= \int_{-v}^{v} \cos(u/v)\, du = v \cdot \int_{-v}^{v} \cos k\, dk = v \cdot \operatorname{sen}\left(\frac{u}{v}\right)\Bigg|_{-v}^{v}$$

$k = u/v$

$$\Rightarrow v \cdot (\operatorname{sen} 1 - \operatorname{sen}(-1)) = 2v \operatorname{sen} 1$$

$dk = 1/v\, d$

$$= \tfrac{1}{2}\int_{1}^{2} 2v \sin 1\, dv = \frac{1}{2}\, v^2 \Big|_1^2 \sin 1 = \boxed{\frac{3}{2} \sin 1}$$