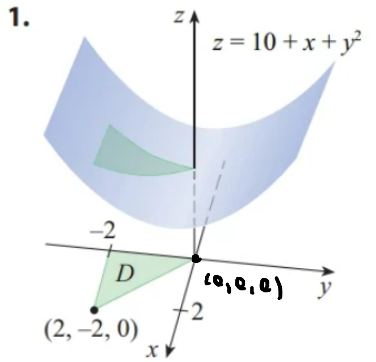
LISTA 9

Seção 15.5: 1, 3, 5, 7, 9, 11, 13, 23, 24, 25

1

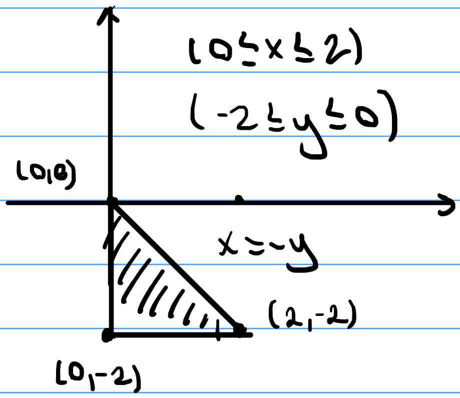


$f_x = 1$

$f_y = 2y$

$\Rightarrow \sqrt{1+1+4y^2} = \sqrt{2+4y^2}$

$$A = \int_{-2}^{0}\int_{0}^{-y} \sqrt{2+4y^2}\,dx\,dy$$



$$\Rightarrow \int_{-2}^{0} -y\sqrt{2+4y^2}\,dy$$

$$\Rightarrow -\frac{1}{8}\int_{-2}^{0} 8y\sqrt{2+4y^2}\,dy$$

$$-\frac{1}{8}\left(\frac{2}{3}(2+4y^2)^{3/2}\right)\Bigg|_{-2}^{0}$$

$$= \frac{1}{12}\left(18^{3/2} - 2^{3/2}\right) = \frac{1}{12}\left((3\sqrt{2})^3 - 2\sqrt{2}\right) = \frac{1}{\cancel{12}_{3}}\cdot\cancel{52}^{13}\sqrt{2} = \boxed{\frac{13}{3}\sqrt{2}}$$

③ $z = 5x + 3y + 6.$ on $[1,4] \times [2,6]$.

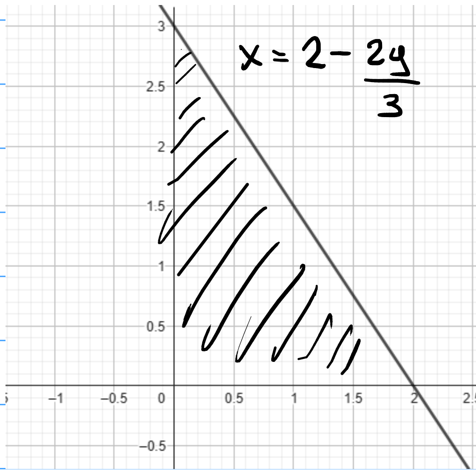
$$A = \int_4^6 \int_1^2 \sqrt{25+9+1}\, dx\, dy = \int_4^6 \int_1^2 \sqrt{35}\, dx\, dy$$

$$= \int_4^6 \sqrt{35}\, dy = \boxed{2\sqrt{35}}$$

⑤ $z = 6 - 3x - 2y$ (primeiro octante).

$3x + 2y = 6.$ $\quad 0 \le y \le 3$

$0 \le x \le 2$



$$A = \int_0^3 \int_0^{2-2y/3} \sqrt{9+4+1}\, dx\, dy$$

$$\Rightarrow \int_0^3 (2 - 2y/3)\sqrt{14}\, dy$$

$$\int_0^3 2\sqrt{14}\, dy - \int_0^3 2/3\sqrt{14}\cdot y\, dy = 6\sqrt{14} - (3\sqrt{14}) = \boxed{3\sqrt{14}}.$$

(7) $z = 1 - x^2 - y^2$ e acima de $z = -2$.

$\Rightarrow x^2 + y^2 = 3$.

$$A = \iint_D \sqrt{1 + 4x^2 + 4y^2}\, dx\, dy = \int_0^{2\pi}\int_0^{\sqrt{3}} \sqrt{1+4r^2}\, r\, dr\, d\theta.$$

$$\Rightarrow 1/8 \int_0^{\sqrt{3}} \sqrt{1+4r^2}\, 8r\, dr = \frac{1}{8} \cdot \left[\frac{2}{3} \cdot (1+4r^2)^{3/2} \right] \Bigg|_0^{\sqrt{3}}$$

$$= \frac{1}{12} \left(13^{3/2} - 1 \right) = \frac{1}{12} \left(13\sqrt{13} - 1 \right)$$

$$\Rightarrow \int_0^{2\pi} \frac{1}{12} \left(13\sqrt{13} - 1 \right) = \boxed{\pi/6 \left(13\sqrt{13} - 1 \right)}.$$

(9) $z = y^2 - x^2$ entre $x^2 + y^2 = 1$ e $x^2 + y^2 = 4$

$$\iint_D \sqrt{1 + 4x^2 + 4x^2}\, dx\, dy = \int_0^{2\pi}\int_1^{2} \sqrt{1+4r^2}\, r\, dr\, d\theta$$

$$\Rightarrow 1/8 \int_1^{2} \sqrt{1+4r^2} \cdot (8r\, dr) = \frac{1}{8} \left[\frac{2}{3} (1+4r^2)^{3/2} \right] \Bigg|_1^{2}$$

$$\frac{1}{12}\left(17^{3/2}-5^{3/2}\right) = \frac{1}{12}\left(17\sqrt{17}-5\sqrt{5}\right)$$

$$\Rightarrow \int_0^{2\pi} \frac{1}{12}\left(17\sqrt{17}-5\sqrt{5}\right) d\theta = \boxed{\frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)}$$

(11) $z = xy \qquad x^2+y^2=1.$

$$A = \iint_D \sqrt{1+x^2+y^2}\, dx\,dy = \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\, r\,dr\,d\theta$$

$$\Rightarrow 1/2 \int_0^1 \sqrt{1+r^2}\,(2r\,dr) = \frac{1}{2}\left[\frac{2}{3}\cdot(1+r^2)^{3/2}\right]\Bigg|_0^1 = \frac{1}{3}(2\sqrt{2}-1)$$

$$\Rightarrow \int_0^{2\pi} \frac{1}{3}(2\sqrt{2}-1)\, d\theta = \boxed{\frac{2\pi}{3}(2\sqrt{2}-1)}$$

(13) $x^2+y^2+z^2=a^2$ dentro de $x^2+y^2=ax$ e acima do plano xy.

$$x^2 - 2.x.a/2 + a^2/4 + y^2 = a^2/4$$
$$y^2 + (x-a/2)^2 = (a/2)^2.$$

$z = \pm\sqrt{a^2-x^2-y^2}$. Como está acima de xy, então consideramos $z = \sqrt{a^2-x^2-y^2}$.

$$A = \iint_D \sqrt{a^2 - x^2 - y^2}\, dx\, dy =$$

$$\int_0^{\pi}\int_0^{a\cos\theta} \sqrt{1 + r^2/(a^2-r^2)}\; r\, dr\, d\theta = \int_0^{\pi}\int_0^{a\cos\theta} \sqrt{\frac{a^2}{a^2-r^2}}\; r\, dr\, d\theta$$

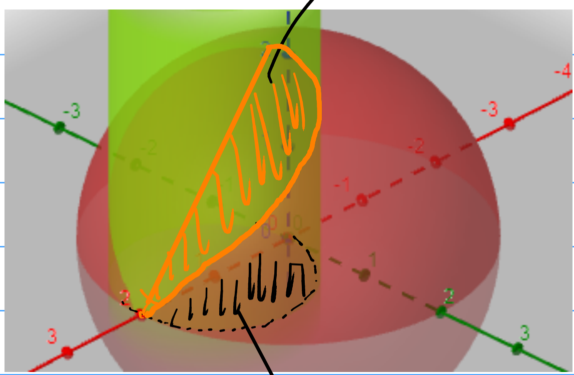
$$= a \int_0^{\pi}\int_0^{a\cos\theta} (a^2-r^2)^{-1/2}\; r\, dr\, d\theta$$

$$\Rightarrow -1/2 \int_0^{a\cos\theta} (a^2-r^2)^{-1/2} (-2r\, dr) = -\frac{1}{2} \cdot 2\, (a^2-r^2)^{1/2} \Big|_0^{a\cos\theta}$$

$$= -\left(a^2 - a^2\cos^2\theta - a^2\right) = -\left(\sqrt{a^2(1-\cos^2\theta)} - \sqrt{a^2}\right) = a\,(1 - \operatorname{sen}\theta)$$

$$\Rightarrow a^2 \int_0^{\pi} (1-\operatorname{sen}\theta)\, d\theta = a^2(\theta + \cos\theta)\Big|_0^{\pi} = \boxed{a^2(\pi - 2)}\ \text{m.}$$

Área que queremos



$x^2 + y^2 = ax.$

$r^2 = a\, r\cos\theta$

$r = a\cos\theta.$

(23) $z = ax + by + c$ projeta sobre uma região D em xy de área $A(D)$.

$$A = \iint_D \sqrt{a^2+b^2+1}\, dx\,dy = \sqrt{a^2+b^2+1} \cdot \iint_D dx\,dy$$

Temos que $\iint_D dx\,dy$ é a área da região D. Logo $\iint_D dx\,dy = A(D)$

e $\boxed{A = \sqrt{a^2+b^2+1} \cdot A(d)}$.

(24) Parte superior de $x^2+y^2+z^2=a^2$:
$z = \sqrt{a^2-x^2-y^2}$.

$$\frac{\partial z}{\partial x} = \frac{1}{2} \cdot \frac{1}{\sqrt{a^2-x^2-y^2}} \cdot (-2x) = \frac{-x}{\sqrt{a^2-x^2-y^2}}$$

$$\frac{\partial y}{\partial y} = \frac{-y}{\sqrt{a^2-x^2-y^2}}$$

$$\therefore \sqrt{1+(\partial z/\partial x)^2+(\partial z/\partial y)^2} = \sqrt{\frac{x^2+y^2}{a^2-(x^2+y^2)}}$$

$\sqrt{\frac{x^2+y^2}{a^2-(x^2+y^2)}}$ possui descontinuidade infinita em $x^2+y^2=a$ que é justamente a projeção da esfera em xy.

contudo, poderes fazer a integração em $x^2+y^2 \leq t^2$, com $t \to a^-$ (não pode ser $t \to a^+$ pois pegaríamos pontos fora do $x^2+y^2=a^2$).

Logo, a área da parte superior da esfera pode ser dada por:

$$A = \lim_{t \to a^-} \int_0^{2\pi}\int_0^{t} \sqrt{\frac{a}{a^2-r^2}}\; r\,dr\,d\theta$$

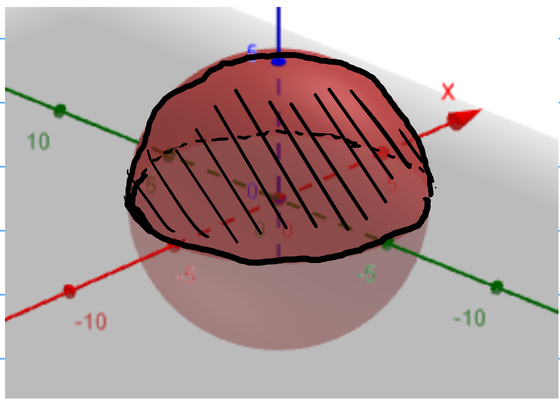
$$= \lim_{t \to a^-} \left[t \cdot \int_0^{2\pi}\int_0^{t} (a^2-r^2)^{-1/2}\; r\,dr\,d\theta \right]$$

$$\Rightarrow -1/2 \int_0^{t} (a^2-r^2)^{-1/2}(-2r\,dr) = -\frac{1}{2} \cdot 2\,(a^2-r^2)^{1/2}\Big|_0^{t} = (a - \sqrt{a^2-t^2})$$

$$\int_0^{2\pi} (a-\sqrt{a^2-t^2}) = 2\pi\,(a-\sqrt{a^2-t^2})$$

$$\lim_{t\to a^-} 2\pi t\,(a-\sqrt{a^2-t^2}) = 2\pi . a . a = 2\pi a^2$$

Como a esfera é simétrica, então a área total de sua superfície é $4\pi a^2$.



(25) $y = x^2+z^2$ limitado por $y=25$.

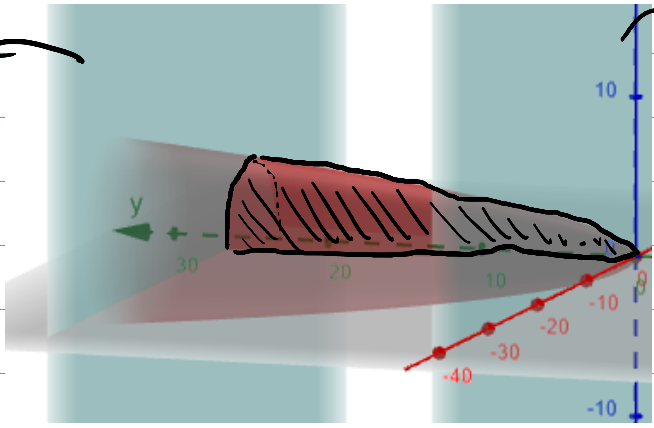
Projetando $y=x^2+z^2$ com $y=25$ em xz, ficamos com o círculo $x^2+z^2=25$.

Assim, $A = \int_0^{2\pi}\int_0^5 \sqrt{1+4r^2}\, r\,dr\,d\theta =$

$$\frac{1}{8} \int_0^5 \sqrt{1+4r^2}\,(8r\,dr) = \frac{1}{8}\cdot\frac{2}{3} (1+4r^2)^{3/2}\Big|_0^5 = \frac{1}{12}(101\sqrt{101}-1)$$

e $\int_0^{2\pi} \frac{1}{12}(101\sqrt{101}-1)\,d\theta = \boxed{\frac{\pi}{6}(101\sqrt{101}-1)}$.

$y=25$.



$y=0$ (plano xz)