REGRA DA CADEIA

CASO 1: $\left.\begin{array}{l} z = f(x,y) \\ x = g(t) \\ y = h(t) \end{array}\right] \Rightarrow z = (g(t), h(t))$

$$\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}}$$

Temos que $\Delta z = f_x(x_0,y_0)\,\Delta x + f_y(x_0,y_0)\,\Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$. onde $\varepsilon_1, \varepsilon_2 \to 0$ quando $(\Delta x, \Delta y) \to (0,0)$.

Quando varia para $t+\Delta t$, x varia para $x+\Delta x$ e y varia para $y+\Delta y$, onde $\Delta x = g(t+\Delta t) - g(t)$ e $\Delta y = h(t+\Delta t) - h(t)$.

Quando $\Delta t \to 0$, como g, h são contínuas (são diferenciáveis), então $\Delta x \to 0$ e $\Delta y \to 0$.

Além disso, lembre-se que $\frac{dz}{dt} = \lim\limits_{\Delta t \to 0} \left(\frac{\Delta z}{\Delta t}\right)$.

Fazendo $\frac{\Delta z}{\Delta t} = f_x(x_0,y_0)\,\frac{\Delta x}{\Delta t} + f_y(x_0,y_0)\,\frac{\Delta y}{\Delta t}$

Assim $\frac{dz}{dt} = \lim\limits_{\Delta t \to 0} \frac{\Delta z}{\Delta t} = f_x(x_0,y_0) \cdot \lim\limits_{\Delta t \to 0} \frac{\Delta x}{\Delta t} + f_y(x_0,y_0) \lim\limits_{\Delta t \to 0} \frac{\Delta y}{\Delta t} +$

$$\lim_{\Delta t\to 0} \cancel{\varepsilon_1}^{\,0} \cdot \lim_{\Delta t\to 0} \frac{\Delta x}{\Delta t} + \lim_{\Delta t\to 0} \cancel{\varepsilon_2}^{\,0} \cdot \lim_{\Delta t\to 0} \frac{\Delta y}{\Delta x}$$

$$\boxed{\Delta t \to 0 \;,\; \varepsilon_1, \varepsilon_2 \to 0.}$$

$$\boxed{\therefore \frac{dz}{dt} = f_x(x_0,y_0)\cdot\frac{dx}{dt} + f_y(x_0,y_0)\frac{dy}{dt}}$$

CASO 2:

$$\left.\begin{array}{l} z = f(x,y) \\ x = g(s,t) \\ y = h(s,t) \end{array}\right] \Rightarrow z = f\big(g(s,t), h(s,t)\big)$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}.$$

CASO 3 (generalizado):

$$z = f(x_1,\dots,x_n)$$
$$x_i = g_i(t_1,\dots,t_m)$$

$$\boxed{\frac{\partial z}{\partial t_j} = \frac{\partial z}{\partial x_1}\cdot\frac{\partial x_1}{\partial t_j} + \frac{\partial z}{\partial x_2}\cdot\frac{\partial x_2}{\partial t_j} + \dots + \frac{\partial z}{\partial x_n}\cdot\frac{\partial x_n}{\partial t_j}}$$

Ex: $z = xye^y$

$x = t^2$

$y = 5t$

I) $\dfrac{dz}{dt} = ye^y \cdot 2t + (xe^y + xye^y) \cdot 5 = e^y(2yt + 5x + 5xy)$

$= e^{5t}(10t^2 + 5t^2 + 25t^3) = 5e^{5t}(3t^2 + 5t^3) =$

$\boxed{= 5t^2 e^{5t}(5t+3)}$

II) $z = 5t^3 \cdot e^{5t} \Rightarrow \dfrac{dz}{dt} = 15t^2 \cdot e^{5t} + 5t^3 e^{5t} \cdot 5$

$\boxed{= 5t^2 e^{5t}(5t+3)}$.

Ex: $z = \sqrt{x}\, e^{xy}$

$x = 1 + st$

$y = s^2 - t^2$

I) $\dfrac{\partial z}{\partial s} = \left(\dfrac{1}{2\sqrt{x}} e^{xy} + \sqrt{x} \cdot e^{xy} \cdot y\right) \cdot t + \sqrt{x} \cdot e^{xy} \cdot x \cdot 2s$

$\dfrac{\partial z}{\partial t} \left(\dfrac{1}{2\sqrt{x}} e^{xy} + \sqrt{x} e^{xy} \cdot y\right) \cdot s + \sqrt{x} e^{xy} \cdot x \cdot (-2t)$.

Ex: $W(T,R)$

$\frac{dT}{dt} = 0{,}15°C/ano$, $\frac{dR}{dt} = -0{,}1cm/ano$, $\frac{\partial W}{\partial T} = -2$, $\frac{\partial W}{\partial R} = 8$

a) Taxa de variação da produção de trigo em relação à temperatura média, mantendo o volume anual das chuvas constante.

Taxa de variação da produção de trigo em relação ao volume anual das chuvas, mantendo a temperatura média constante.

b) $\frac{dW}{dt} = \frac{\partial W}{\partial R} \cdot \frac{dR}{dt} + \frac{\partial W}{\partial T} \cdot \frac{dT}{dt} = 8 \cdot (-0{,}1) - 2(0{,}15)$

$= -0{,}8 - 0{,}3 = \boxed{-1{,}1}$.

Ex: $P(L,K) = 1{,}47 \cdot L^{0{,}65} \cdot K^{0{,}35}$

$\frac{\partial P}{\partial L} = 1{,}47 \cdot K^{0{,}35} \cdot 0{,}65 \cdot L^{-0{,}35} \Rightarrow 1{,}47 \cdot 8^{0{,}35} \cdot 0{,}65 \cdot 30^{-0{,}35}$

$\simeq 0{,}6016$

$\frac{\partial P}{\partial K} = 1{,}47 \cdot L^{0{,}65} \cdot 0{,}35 \cdot K^{-0{,}65} \Rightarrow 1{,}47 \cdot 0{,}35 \cdot 30^{0{,}65} \cdot 8^{-0{,}65}$

$\approx 0{,}4$

$\frac{dP}{dt} = 0{,}6016 \cdot (-2) + 0{,}4 \cdot (0{,}5) = \boxed{-1{,}0032}$.