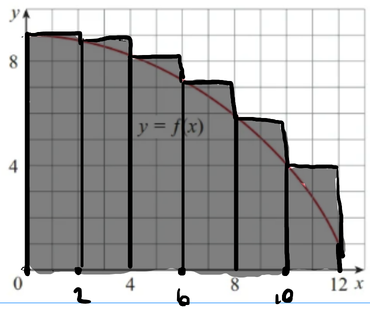
LISTA 8 - CÁLCULO

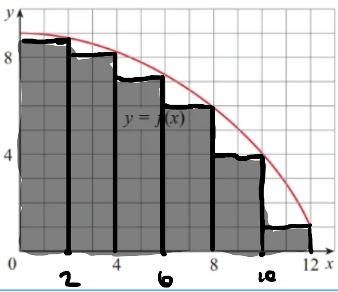
Seção 5.1: 2, 21, 24, 25, 26

(2) a) Os comprimentos dos intervalos é: $\frac{12-0}{6} = 2$

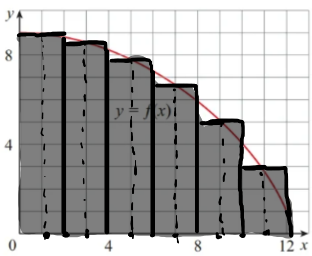
$\Rightarrow$ subintervalos: $[0,2]$; $[2,4]$; $[4,6]$; $[6,8]$; $[8,10]$; $[10,12]$

i) L_6: 

$\approx 86{,}4$

ii) R_6 

$\approx 70{,}6$

iii) M_6 

$\approx 81{,}6$

b) L_6 é superestimativa

c) R_6 é subestimativa

d) A melhor aproximante é M_6, pois os pontos amostrais são a médio dos extremos

(21) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{2}{n} \cdot \frac{1}{1+2i/n} = \lim\limits_{n\to\infty} \frac{2}{n} \sum\limits_{i=1}^{n} \frac{1}{1+i\cdot 2/n}$

Representa a área da curva

$y = \frac{1}{x+1}$ entre 0 e 2.

(24) a) Definição 2:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$$

Curva $y = x^3$ de 0 a 1

$\boxed{\Delta x = \frac{1}{n}}$ $\rightarrow$ $A = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^3 = \boxed{\lim\limits_{n\to\infty} \frac{1}{n^4} \sum\limits_{i=1}^{n} i^3 = A}$

b) $A = \lim_{n \to \infty} \frac{1}{n^4} \cdot \left(\frac{n(n+1)}{2}\right)^2 = \lim_{n \to \infty} \frac{1}{n^4} \cdot \frac{n^2(n^2+2n+1)}{2^2} =$

$\lim_{n \to \infty} \frac{1}{4} \cdot \left(1 + \frac{2}{n^3} + \frac{1}{n^4}\right) = \boxed{\frac{1}{4}}$ //.

25 a) Como f é contínua e crescente em [a,b] $f(a) < f(b)$. Logo L_n (aproximação usando os extremos esquerdos) é uma subestimativa. Portanto, R_n é uma superestimativa logo $L_n < A < R_n$.

b) $R_n - L_n = \frac{b-a}{n}(f(b) - f(a))$

$$R_n - L_n = \sum_{i=1}^{n} f(x_i)\Delta x - \sum_{i=0}^{n-1} f(x_i)\Delta x$$

$$R_n - L_n = \Delta x \left(\cancel{f(x_1)} + \cancel{f(x_2)} + \cancel{\dots} + f(x_n) - f(x_0) - \cancel{f(x_1)} - \cancel{\dots} - \cancel{f(x_{n-1})} \right)$$

$$= \Delta x (f(x_n) - f(x_0)) = \boxed{\frac{b-a}{n}(f(b) - f(a))}$$ //

c) $L_n < A < R_n$

$L_n - R_n < A - R_n < 0$

$R_n - A < R_n - L_n$

$$\boxed{\therefore\ R_n - A < \frac{b-a}{n}\left(f(b)-f(a)\right)}$$

(26) $y = e^x$ de 1 a 3

$$R_n - A < \frac{b-a}{n}\left(f(b)-f(a)\right)$$

$$R_n - A < 0{,}0001$$

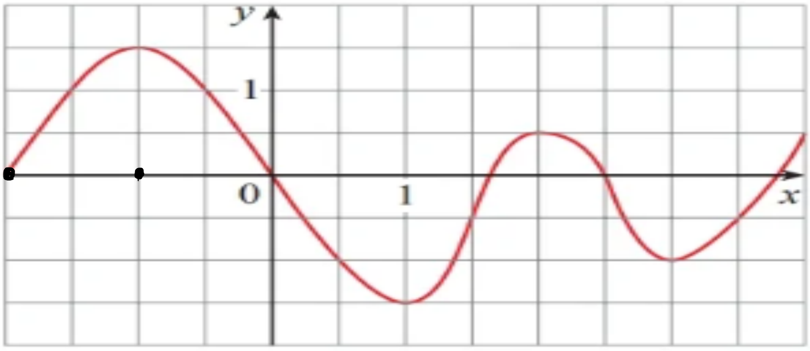
$$\Rightarrow \frac{b-a}{n}\left(f(b)-f(a)\right) = 0{,}0001$$

$$\frac{3-1}{n}\left(e^3-e\right) = \frac{1}{10000}$$

$$\boxed{n = 20\,000\,(e-1)e(e+1)}$$

Seção 5.2: 6, 7, 19, 35, 44, 45, 51, 52, 60, 67, 83

(6)



Estimar $\int_{-2}^{4} g(x)\,dx$ com 6 subintervalos.

Subintervalos: $[-2,-1]$; $[-1,0]$; $[0,1]$; $[1,2]$; $[2,3]$; $[3,4]$

a) extremidades direitas:

$$\int_{-2}^{4} g(x)\,dx \approx [g(-1)+g(0)+g(1)+g(2)+g(3)+g(4)]\cdot 1$$

$$= 1{,}5 + 0 + (-1{,}5) + 0{,}5 + (-1) + (0{,}5) = \boxed{0}$$

b) extremidades esquerdas:

$$\int_{-2}^{4} g(x)\,dx = g(-2)+g(-1)+g(0)+g(1)+g(2)+g(3)$$

$$= 0 + 1{,}5 + 0 + (-1{,}5) + 0{,}5 + (-1) = \boxed{-0{,}5}$$

c) Pontos Médios:

$$\int_{-2}^{4} g(x)\,dx = g(-1,5) + g(-0,5) + g(0,5) + g(1,5) + g(2,5) + g(3,5) = 1 + \cancel{1} + (\cancel{-1}) + (-0,5) + 0 + (-0,5) \boxed{=0}$$

(7)

x	10	14	18	22	26	30
$f(x)$	–12	–6	–2	1	3	8

Intervalos: $[10,14]$; $[14,18]$; $[18,22]$; $[22,26]$; $[26,30]$.

$$\Delta x = \frac{30-10}{5} \boxed{= 4}$$

i) extremidade esquerda:

$$4(f(10)+f(14)+f(18)+f(22)+f(26)) = 4\cdot(-12-6-2+1+3) \boxed{= -64}$$

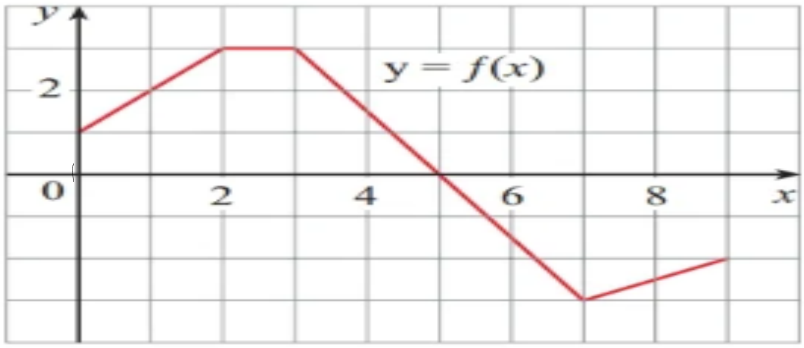
ii) extremidade direita:

$$4(f(14)+f(18)+f(22)+f(26)+f(30)) = 4(-6-2+1+3+8) \boxed{= 16}$$

(19) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1+x_i} \Delta x \Rightarrow [0,1]$

$$\boxed{= \int_0^1 \frac{e^x}{1+x}\,dx}$$

(35)



a) $\int_0^2 f(x)\,dx = \frac{(1+3).2}{2} = \boxed{4}$

b) $\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx = 4 + 3.1 + \frac{3.2}{2}$

$\boxed{= 10}$

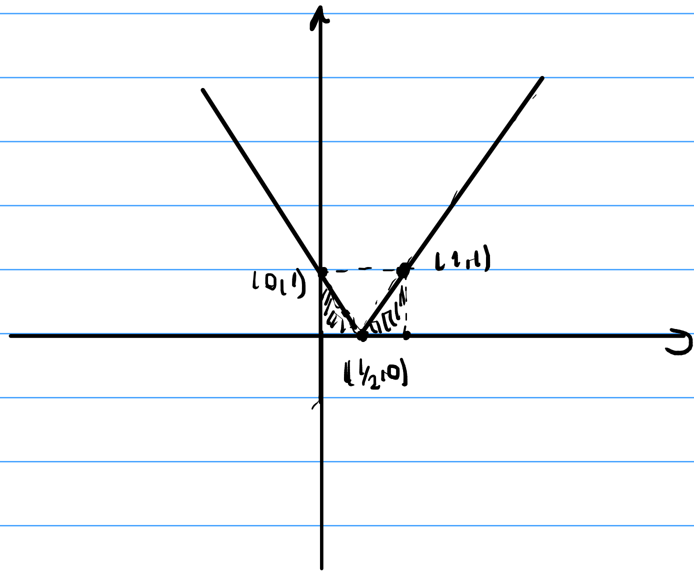
c) $\int_5^7 f(x)\,dx = -\frac{2.3}{2} = \boxed{-3}$

d) $\int_3^7 f(x)\,dx = \int_3^5 f(x)\,dx + \int_5^7 f(x)\,dx = 3-3$ $\boxed{=0}$

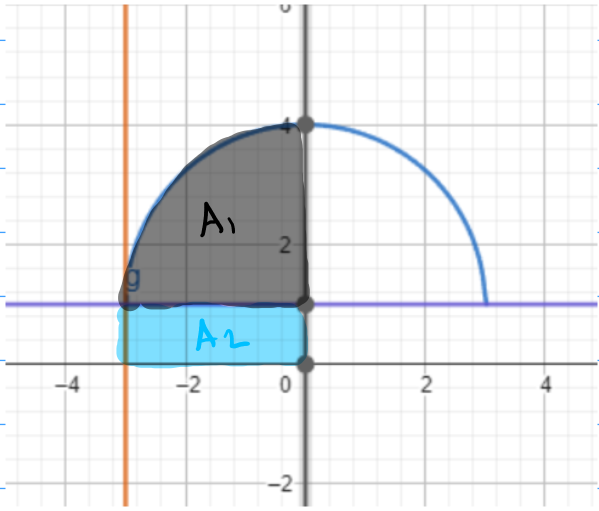
e) $\int_3^7 |f(x)|\,dx = \int_3^5 f(x)\,dx + \int_7^5 f(x)\,dx = \boxed{6}$

f) $\int_2^0 f(x)\,dx = -\int_0^2 f(x)\,dx = \boxed{-4}$

44) $\int_0^1 |2x-1|\,dx = \frac{1 \cdot 1/2}{2} + \frac{1 \cdot 1/2}{2} = \boxed{\frac{1}{2}}$



45) $\int_{-3}^0 (1+\sqrt{9-x^2})\,dx = \int_{-3}^1 dx + \int_{-3}^1 \sqrt{9-x^2}\,dx$



$$A_1 = \frac{1}{4} \cdot 3^2 \pi = \frac{9\pi}{4}$$

$$A_2 = 1.3 = 3$$

$$\boxed{\int_{-3}^{0} \sqrt{1+x^2}\, dx = 3 + \frac{9\pi}{4}}$$

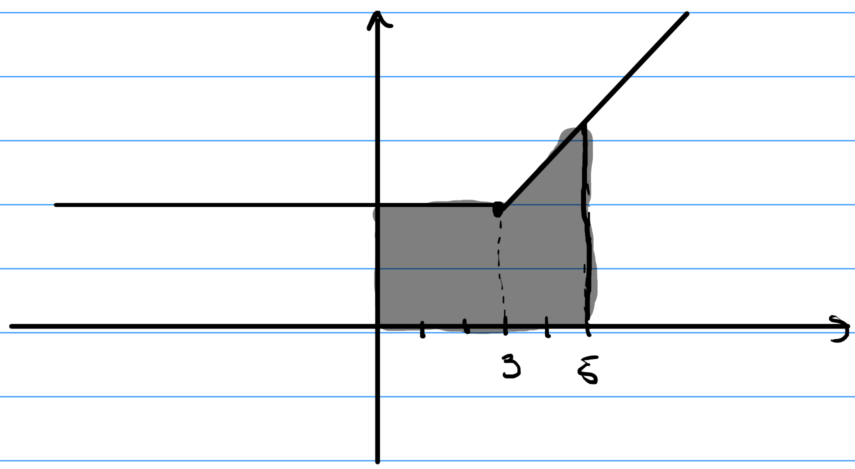
51) $\int_1^1 \sqrt{1+x^4}\,dx \; \boxed{=0}$

52) $\int_0^\pi \operatorname{sen}^4 x\,dx = \frac{3}{8}\pi$

$$\int_\pi^0 \operatorname{sen}^4 \theta\, d\theta = \boxed{-\frac{3}{8}\pi}$$

60) $\int_0^5 f(x)\,dx = 3.3 + \frac{(3+5).2}{2} = 9+8 = \boxed{17}$

$$f(x) = \begin{cases} 3, & x < 3 \\ x, & x \geq 3 \end{cases}$$



(67) $2 \leq \int_{-1}^{1} \sqrt{1+x^2}\, dx \leq 2\sqrt{2}$

Temos que no intervalo $[-1,1]$, $\sqrt{1+x^2}$ tem valor mínimo 1 e máximo $\sqrt{2}$.

$$\therefore 1(1-(-1)) \leq \int_{-1}^{1} \sqrt{1+x^2}\, dx \leq \sqrt{2}(1-(-1))$$

(propriedade 8).

$$\boxed{\therefore 2 \leq \int_{-1}^{1} \sqrt{1+x^2}\, dx \leq 2\sqrt{2}}$$

(83) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^4 \cdot \frac{1}{n}$

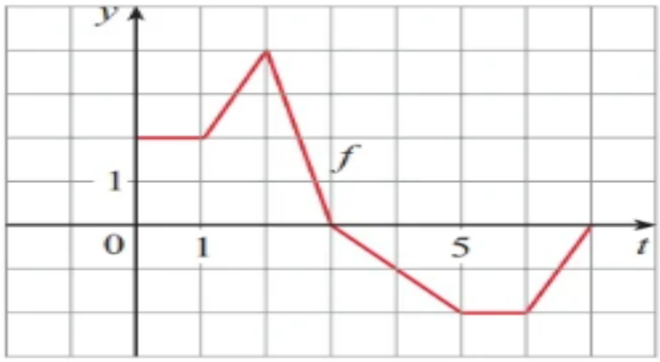
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\frac{(b-a)}{n}$$

Fazendo $b=1$, $a=0$, $\Delta x = \frac{1}{n}$, $x_i = \frac{i}{n}$

$$\boxed{\text{Temos que } \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5} = \int_0^1 x^4\, dx}$$

Seção 5.3: 3, 9, 13, 17, 19, 27, 39, 51, 63, 67, 72, 79, 83, 86, 95

(3)



$g(x) = \int_0^x f(t)\,dt$

a) $g(0) = 0$; $g(1) = 2$; $g(2) = 5$; $g(3) = 7$; $g(6) = 3$

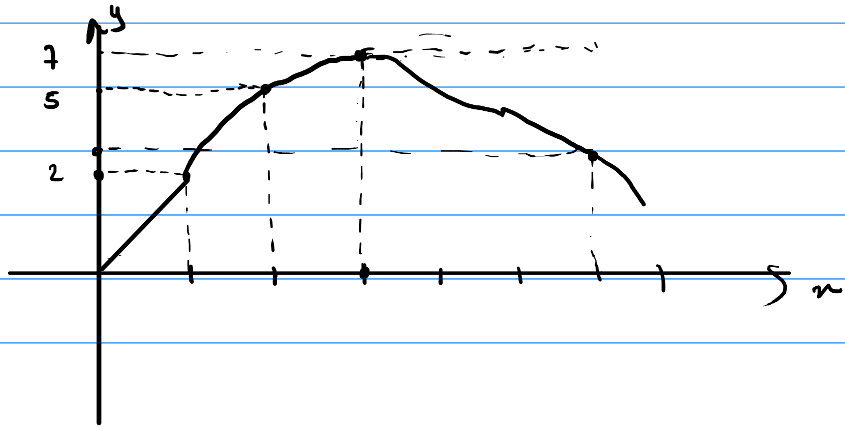
b) $g'(x) = f(x) \Rightarrow g(x)$ é crescente quando $g'(x) > 0 \Rightarrow (0, 3)$

c) Pontos críticos de $g(x)$: 3 e 7

$x = 3$: $g'(x) > 0 \rightarrow g'(x) < 0$
$x = 7$: $g'(x) < 0 \rightarrow$?

$\boxed{\text{Ponto de máximo } x = 3}$ //.

d)



(9) $g(x) = \int_0^x \sqrt{t+t^3}\, dt$

$$\boxed{g'(x) = \sqrt{x+x^3}}$$ //

(13) $F(x) = \int_x^0 \sqrt{1+\sec t}\, dt = -\int_0^x \sqrt{1+\sec t}\, dt$

$$\boxed{F'(x) = -\sqrt{1+\sec x}}$$ //.

(17) $y = \int_1^{3x+2} \frac{t}{1+t^3}\, dt \qquad y' = \frac{3x+2}{1+(3x+2)^3} \cdot 3$

$$\boxed{y' = \frac{9x+6}{27x^3+54x^2+36x+10}}$$ //.

(19) $y = \int_{\sqrt{x}}^{\pi/4} \operatorname{tg}\theta \cdot \theta\, d\theta = -\int_{\pi/4}^{\sqrt{x}} \operatorname{tg}\theta \cdot \theta\, d\theta$

$$y' = -\operatorname{tg}\sqrt{x} \cdot \sqrt{x} \cdot \frac{1}{2\sqrt{x}} = \boxed{-\frac{\operatorname{tg}(\sqrt{x})}{2}}$$ //

27 $\int_0^2 \left(\frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t\right) dt$

$= \frac{4}{5}\int_0^2 t^3\,dt - \frac{3}{4}\int_0^2 t^2\,dt + \frac{2}{5}\int_0^2 t\,dt$

$= \frac{\cancel{4}}{5}\left(\frac{2^4}{\cancel{4}}\right) - \frac{\cancel{3}}{4}\cdot\left(\frac{2^3}{\cancel{3}}\right) + \frac{\cancel{2}}{5}\cdot\frac{2^2}{\cancel{2}}$

$= \frac{16}{5} - 2 + \frac{4}{5} = \frac{16+4-10}{5} = \boxed{2}$

30 $\int_1^8 x^{-2/3}\,dx \Rightarrow \int x^{-2/3}\,dx = 3\cdot x^{1/3} + C$

$\Rightarrow \int_1^8 x^{-2/3}\,dx = 3\left(8^{1/3} - 1^{1/3}\right) = 3(2-1) = \boxed{3}$

51 $\int_0^4 2^s\,ds = \int 2^s\,ds = \frac{2^s}{\ln 2} + C$

$\Rightarrow (2^4 - 2^0)\cdot\frac{1}{\ln 2} = \boxed{\frac{15}{\ln 2}}$

(63) $\int_{-2}^{1} x^{-4}\,dx \Rightarrow \int_{-2}^{1} \frac{1}{x^4}\,dx$

Perceba que para $x=0$, $1/x^4$ não está definida. Como $0 \in [-2,1]$, então a integral $\nexists \int_{-2}^{1} \frac{1}{x^4}\,dx$

(67) $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du = -\int_0^{2x} f(u)\,du + \int_0^{3x} f(u)\,du$

$$g'(x) = \frac{(3x)^2-1}{(3x)^2+1}\cdot 3 - \frac{(2x)^2-1}{(2x)^2+1}\cdot 2 = \boxed{\frac{3(9x^2-1)}{9x^2+1} - \frac{2(4x^2-1)}{4x^2+1}}$$

(72) $f(x) = \int_0^x (1-t^2)\,e^{t^2}\,dt$

$f'(x) = (1-x^2)\,e^{x^2}$

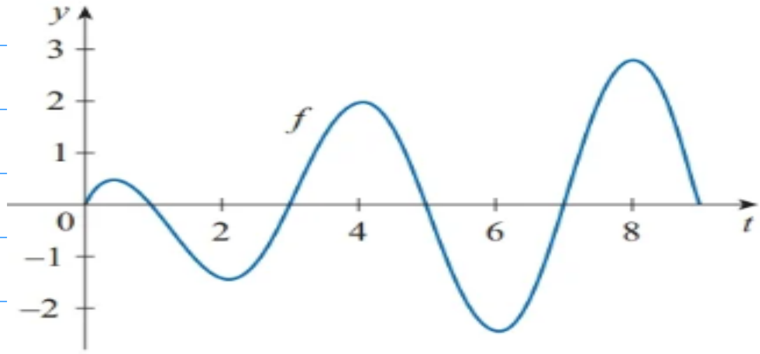
$f(x)$ é cresente quando $f'(x)>0$
Isso ocorre em $(-1,1)$.

(79) $f(1)=12$, f' é contínua $\int_1^4 f'(x)\,dx = 17$

$f(4)=?$ $\Rightarrow \int_1^4 f'(x)\,dx = F(4)-F(1) = 17$
$F'(x) = f'(x)$ (F é primitiva de f'). Logo,
$F(x) = f(x) + C$

$17 = f(9) + \cancel{C} - f(1) - \cancel{C} \Rightarrow f(9) = 17 + f(1) \boxed{= 29}$.

83



$g(x) = \int_0^x f(t)\,dt$

a) Máximos e mínimos de g: $g' = 0$ ou $\nexists\, g'$

$\Rightarrow g'(x) = f(x). \Rightarrow f(x) = 0 \quad x = \{0, 1, 3, 5, 7, 9\}$

g crescente: $(0,1) \cup (3,5) \cup (7,9)$
g decrescente: $(1,3) \cup (5,7)$

Pontos de máximo: $x = 1$; $x = 5$
Pontos de mínimo: $x = 3$; $x = 7$.

b) Candidatos para máximo absoluto:
$x = 0$, $x = 1$, $x = 5$, $x = 9$

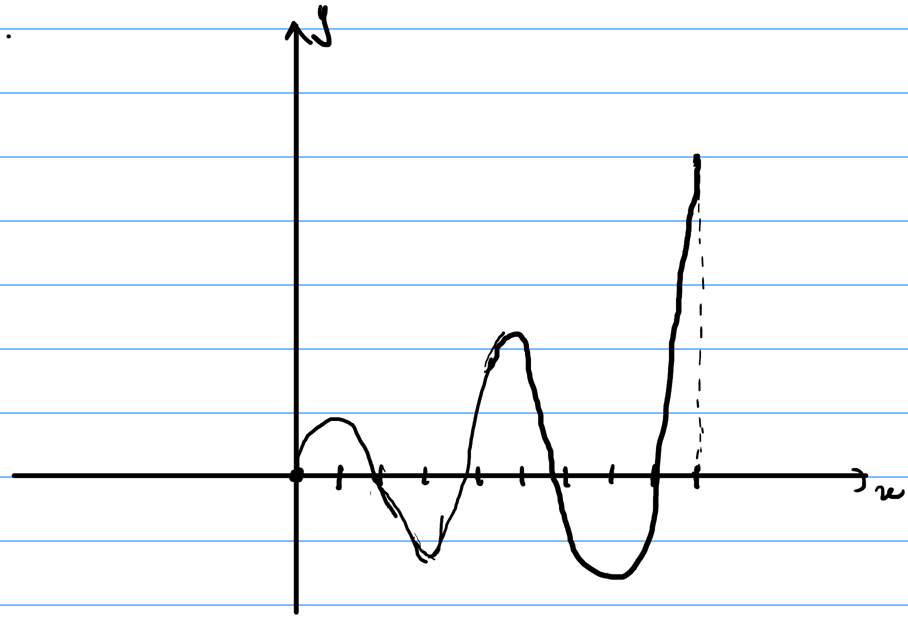
$g(0) = 0 \qquad g(1) > g(0)$

$g(5) > g(1)$ e $g(9) > g(5) \;\therefore\; \boxed{x = 9}$.

c) g é côncava para baixo quando $g''<0$ ou g' é decrescente.

Observando o gráfico de $f(g')$, f é decrescente em: $(1/2, 2) \cup [4, 6] \cup (8, 9)$

d)



86) $\lim\limits_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \dots + \sqrt{\frac{n}{n}}\right)$

$= \lim\limits_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\sqrt{\frac{i}{n}} = \int_0^1 \sqrt{x}\, dx = \left(\frac{1^{3/2}}{3/2} - 0\right) = \boxed{\frac{2}{3}}$

(95) a) Se deprecia a uma taxa $f(t)$

$$\int_0^t f(s)\,ds = h(t)$$

$$h'(t) = f(t)$$

Temos que a derivada de $h(t)$ representa a taxa que a máquina se deprecia em relação ao seu último recondicionamento.

Logo, $h(t) = \int_0^t f(s)ds$ representa a perda de valor da máquina desde o último recondicionamento.

b) $$C = C(t) = \frac{1}{t}\left[A + \int_0^t f(s)\,ds\right]$$

$C(t)$ representa o gasto médio em $t \in [0,t]$ (lembre-se: A = custo fixo; $\int_0^t f(s)ds$ = perda de valor).

É importante para diminuir o gasto médio a cada recondicionamento da máquina

c) $$C(t) = A \cdot t^{-1} + t^{-1}\int_0^t f(s)\,ds$$

$$C'(t) = -A \cdot t^{-2} + t^{-1} f(t) - t^{-2}\int_0^t f(s)\,ds$$

$$c'(t) = \frac{f(t) \cdot t - A - \int_0^t f(s)ds}{t^2}$$

$$C'(T) = 0 \Rightarrow f(T) \cdot t = A + \int_0^t f(s)ds \Rightarrow \boxed{f(T) = \frac{A + \int_0^t f(s)ds}{t}}$$

Seção 5.4: 5, 15, 39, 51, 65, 70, 71

5) $\int (3x^2+4x+1)\,dx = 3.\frac{x^3}{3} + \frac{4x^2}{2} + x + C = \boxed{x^3+2x^2+x+C}$ //

15) $\int \frac{1+\sqrt{x}+x}{x}\,dx = \int \frac{1}{x}\,dx + \int x^{-1/2}\,dx + \int 1\,dx$

$= \boxed{\ln|x| + 2\sqrt{x} + x + C}$ //.

39) $\int_0^1 x\left(\sqrt[3]{x} + \sqrt[4]{x}\right)dx = \int_0^1 \left(x^{4/3} + x^{5/4}\right) dx$

$= \int_0^1 x^{4/3}\,dx + \int_0^1 x^{5/4}\,dx = \frac{3}{7} + \frac{4}{9} \boxed{= \frac{55}{63}}$ //.

51) $\int_0^{1/\sqrt{3}} \frac{t^2-1}{t^4-1}\,dt = \int_0^{1/\sqrt{3}} \frac{1}{t^2+1}\,dt = \operatorname{arctg}\frac{\sqrt{3}}{3} - \operatorname{arctg} 0$

$= \boxed{\pi/6}$ //

65) $h(t)$ = frequência cardíaca (bpm)

O que é $\int_0^{30} h(t)\,dt = H(30) - H(0)$

$\rightarrow H'(t) = h(t)$ $H(t)$ é a quantidade de

de batimentos no instante t. Logo, $\int_0^{30} h(t)\,dt$
é o total de batimentos entre 0 e 30 minutos
de exercícios

70) $v(t) = t^2 - 2t - 3$, $2 \le t \le 4$

a) Deslocamento: $\int_2^4 v(t)\,dt = \left(\frac{t^3}{3} - 2\cdot\frac{t^2}{2} - 3x\right)\Big|_2^4$

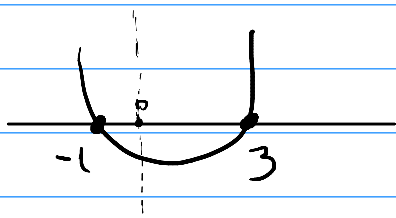
$\frac{64}{3} - 16 - \overset{-3}{\cancel{12}} - \frac{8}{3} + \cancel{4} + \cancel{6}$

$= \frac{56}{3} - 18 = \boxed{\frac{2}{3}\,m}$

b) Distância percorrida:

$t^2 - 2t - 3 = v(t)$

$t = \frac{2 \pm \sqrt{4+12}}{2}$ $\quad \frac{2\pm 4}{2}$ $\quad 1 \pm 2$



$v(t) \le 0 \Rightarrow [0,3]$
$v(t) > 0 \Rightarrow [3,\infty]$

$\Rightarrow -\int_2^3 v(t)\,dt + \int_3^4 v(t)\,dt = \frac{64}{3} - 16 - 12 - \frac{27}{3} + 9 + 9 - \frac{27}{3} + 9 + 9$
$+ \frac{8}{3} - 4 - 6 =$

$= \cancel{6} - \cancel{16} - \cancel{12}^{-1} + \cancel{9}^{\cancel{x}} + \cancel{9} + \cancel{9} + 9 - 4 - \cancel{6} = \boxed{4m}$

71) $a(t) = t + 4$, $v(0) = 5$, $0 \leq t \leq 10$

$$\int a(t)\,dt = \frac{t^2}{2} + 4t + 5 = v(t)$$

a) $t^2/2 + 4t + 5 = v(t)$

b) $v(t) > 0 \Rightarrow$ distância percorrido:

$$\int_0^{10} v(t) = \left(\frac{t^3}{6} + 2t^2 + 5t\right)\Big|_0^{10} = \frac{1000}{6} + 200 + 50 = \boxed{\frac{1250m}{3}}$$