LISTA 5 (seção 3):

Seção 3.1: 11, 22, 24, 32, 33, 49, 55, 58, 68, 69, 72, 74, 75, 78, 82, 83

11 $f(x) = x^{3/2} + x^{-3}$

$f'(x) = \frac{3}{2} x^{3/2-1} + (-3).\, x^{-3-1} = \frac{3}{2} x^{1/2} - 3x^{-4} \boxed{= \frac{3}{2}\sqrt{x} - \frac{3}{x^4}}$

22 $S(R) = 4\pi R^2$

$S'(R) = 4\pi . 2R \boxed{= 8\pi R}$

24 $y(x) = \frac{\sqrt{x} + x}{x^2} = (x^{1/2} + x) . x^{-2} = x^{1/2-2} + x^{1-2} = x^{-3/2} + x^{-1}$

$y'(x) = -\frac{3}{2} . x^{-3/2-1} + (-1) . x^{-1-1} = -\frac{3}{2} x^{-5/2} - x^{-2} = -\left(\frac{3}{2\sqrt{x^5}} + \frac{1}{x^2}\right)$

$$\boxed{= -\frac{1}{x^2}\left(\frac{3}{2\sqrt{x}} + 1\right)}$$

32 $f(v) = \frac{\sqrt[3]{v} - 2ve^v}{v} = v^{1/3-1} - 2e^v = v^{-2/3} - 2e^v.$

$f'(v) = -\frac{2}{3} v^{-2/3-1} - 2e^v = -\frac{2}{3} v^{-5/3} - 2e^v = \boxed{-2\left(\frac{1}{3\sqrt[3]{v^5}} + e^v\right)}$

(33) $P(w) = \dfrac{2w^2 - w + 4}{\sqrt{w}} = 2w^{2-1/2} - w^{1-1/2} + 4.w^{-1/2}$

$P(w) = 2w^{3/2} - w^{1/2} + 4w^{-1/2}$

$P'(w) = 2.\frac{3}{2}.w^{3/2-1} - \frac{1}{2}.w^{1/2-1} + 4.\left(\frac{-1}{2}\right).w^{-1/2-1}$

$P'(w) = 3w^{1/2} - \frac{1}{2}w^{-1/2} - 2.w^{-3/2}$

$$\boxed{P'(w) = 3\sqrt{w} - \frac{1}{2\sqrt{w}} - \frac{2}{\sqrt{w^3}}}$$

(49) $f(x) = 0{,}001.x^5 - 0{,}02x^3 = \frac{1}{1000}x^5 - \frac{2}{100}x^3$

$f'(x) = \frac{1}{1000}.5x^4 - \frac{2}{100}.3x^2 = \boxed{\frac{1}{200}x^4 - \frac{3}{50}x^2.}$

$f''(x) = \frac{1}{200}.4x^3 - \frac{3}{50}.2x = \boxed{\frac{1}{50}x^3 - \frac{3}{25}x}$

(55) $L = 0{,}0390A^3 - 0{,}945A^2 + 10{,}03A + 3{,}07$

$\left.\frac{dL}{dA}\right|_{A=12}$

$\frac{dL}{dA} = 0{,}0390.3A^2 - 0{,}945.2A + 10{,}03.1 + 0$

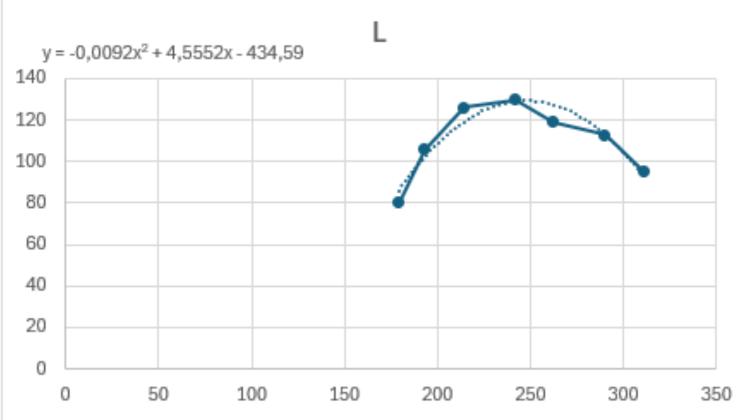
$= 0{,}117A^2 - 1{,}89A + 10{,}03.$

$\left.\frac{dL}{dA}\right|_{A=12} = 0{,}117.144 - 1{,}89.12 + 10{,}03 = 49{,}558.$

Esse resultado indica que o animal tem uma taxa de crescimento de 49,558 cm/ano, quando ele tiver 12 anos.

58

P	L
179	80
193	106
214	126
242	130
262	119
290	113
311	95



a) Eq: $L(P) = -0{,}0092P^2 + 4{,}5552P - 434{,}59$.

b) $L'(P) = -0{,}0184P + 4{,}5552$

- $L'(200) = 0{,}8752$
- $L'(300) = -0{,}9648$

A derivada é uma taxa de variação da vida do pneu pela pressão do pneu. A unidade é km/KPa. O sinal negativo é um decrescimento da vida útil.

(08) a) $y = x^2 + x \Rightarrow y' = 2x + 1 = m$

retas tangentes: $y = mx + p \Rightarrow -3 = 2m + p$

$p = -3 - 2m$

$\boxed{y = mx - 3 - 2m}$

$mx - 3 - 2m = x^2 + x$

$x^2 + (1-m)x + (3+2m) = 0$

$$x = \frac{-(1-m) \pm \sqrt{(1-m)^2 - 4(3+2m)}}{2}$$

Como é uma reta tangente, $(1-m)^2 - 4(3+2m) = 0$

$1 - 2m + m^2 - 12 - 8m = 0$

$m^2 - 10m - 11 = 0$

$m = 11$

$m = -1$

Logo, as retas são: $\boxed{\begin{array}{l} y = 11x - 25 \\ y = -x - 1 \end{array}}$ ponto $(5, 30)$ ponto $(-1, 0)$

b) Seguindo o mesmo raciocínio:

$y = mx + p$ (reta tangente a $x^2 + x$)

$7 = 2m + p$

$p = 7 - 2m \Rightarrow y = mx + 7 - 2m$

Igualando: $mx + 7 - 2m = x^2 + x$

$x^2 + (1-m)x + (2m - 7) = 0$

$$x = \frac{-(1-m) \pm \sqrt{(1-m)^2 - 4(2m-7)}}{2}$$

$$\Rightarrow (1-m)^2 - 4(2m-7) = 0$$

$$1 - 2m + m^2 - 8m + 28 = 0$$

$$m^2 - 10m + 29 = 0$$

$$m = \frac{10 \pm \sqrt{100 - 4 \cdot 29}}{2} \rightarrow < 0$$

$\therefore \nexists\, m \in \mathbb{R}$ que satisfaça as condições.

Pode fazer no geogebra e atribuir valores para m.

(69) Definição: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$.

$f(x) = \frac{1}{x}$. $\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{x - x - h}{(x+h)x \cdot h}$

$= \lim_{h \to 0} \frac{-\cancel{h}}{x(x+h) \cdot \cancel{h}} = \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)} = \boxed{\frac{-1}{x^2}}$.

72 $y'' + y' - 2y = x^2$

$$\begin{cases} y = Ax^2 + Bx + C \\ y' = 2Ax + B. \\ y'' = 2A. \end{cases}$$

$$2A + 2Ax + B - 2Ax^2 - 2Bx - 2C = x^2$$
$$-2Ax^2 + (2A - 2B)x + (2A + B - 2C) = x^2$$

$$\begin{cases} -2A = 1 & \Rightarrow A = -1/2. \\ 2A - 2B = 0 & \Rightarrow B = -1/2. \\ 2A + B - 2C = 0 & \Rightarrow C = -3/4 \end{cases}$$

$$\boxed{A = -\frac{1}{2};\ B = -\frac{1}{2};\ C = -\frac{3}{4}}$$

74 $y = ax^2 + bx + c$

$y'(1) = 4$ e $y'(-1) = -8$ e passa por $(2, 15)$

$$y' = 2ax + b. \Rightarrow \begin{cases} 4 = 2a + b \\ -8 = -2a + b. \end{cases} \Rightarrow 2b = -4 \quad \boxed{b = -2},\ \boxed{a = 3}.$$

$$y = 3x^2 - 2x + c \Rightarrow 15 = 3\cdot(2)^2 - 2\cdot(2) + c$$
$$\cancel{15}^{3} = \cancel{12} - 4 + c \Rightarrow \boxed{c = 7}.$$

$$\boxed{y = 3x^2 - 2x + 7}$$

75 $f(x) = \begin{cases} x^2 + 1, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$

I) f derivável em $x = 1$?

$$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$$ deve existir. Logo,

$$\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1}$$

$$\lim_{x\to 1^+} \frac{x+1-2}{x-1} = \lim_{x\to 1^-} \frac{x^2+1-2}{x-1}$$

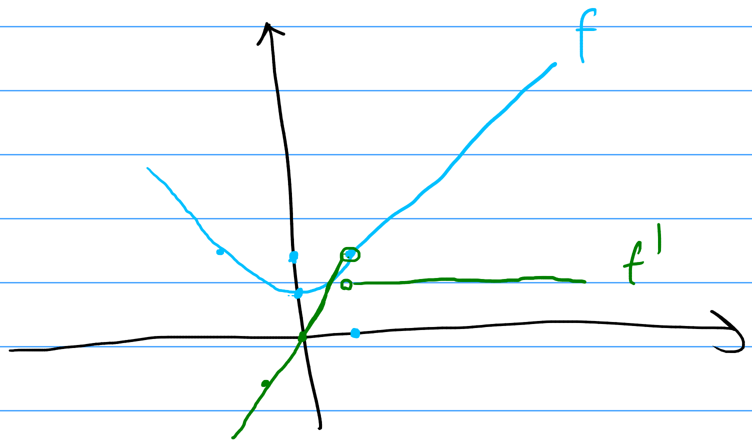
$$\lim_{x\to 1^+} \frac{\cancel{x-1}}{\cancel{x-1}} = \lim_{x\to 1^-} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$1 = \lim_{x\to 1^-} (x+1)$$

$1=2$ (Absurdo). Logo f não é derivável em $x=1$.

II)

$$f'(x) = \begin{cases} 2x, & x < 1 \\ 1, & x > 1. \end{cases}$$



78 $h(x) = |x-1| + |x+2|$

I) $x-1 \geq 0$ e $x+2 \geq 0$

$h(x) = 2x+1.$ $(x \geq 1)$.

II) $x-1 \leq 0$ e $x+2 \geq 0$

$h(x) = 1-x+x+2 = 3$ $(-2 \leq x \leq 1)$

III) $x-1 \geq 0$ e $x+2 \leq 0$

$h(x) = x-1-x+2 = 1$ (Absurdo)

IV) $x-1 \leq 0$ e $x+2 \leq 0$

$h(x) = -x+1-x-2 = -2x-1$ $(x \leq -2)$.

$$h(x) = \begin{cases} 2x+1, & x \geq 1 \\ 3, & -2 \leq x \leq 1 \\ -2x-1, & x \leq -2. \end{cases}$$

$2x+1$ é contínua em $x \geq 1$

3 é contínua em $-2 \leq x \leq 1$

$-2x-1$ é contínua em $x \leq -2$.

$h(x)$ é contínua em $x = 1$ (só fazer o limite) e em $x = -2$ (tbm só fazer o limite).

Fazendo $h'(x) = \begin{cases} 2, & x \geq 1 \\ 0, & -2 \leq x \leq 1 \\ -2, & x \leq -2. \end{cases}$

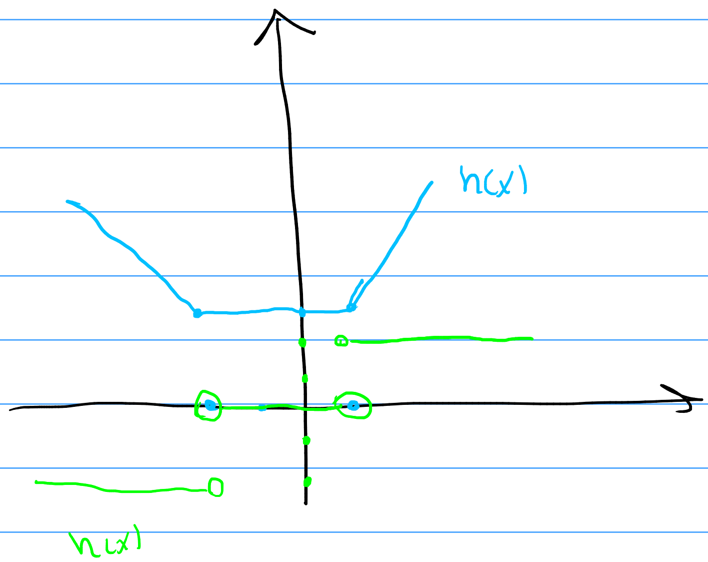
Perceba agora que $h(x)$ não é derivável em $x=1$ e $x=-2$, pois:

$$\lim_{x \to 1^+} \frac{h(x)-h(1)}{x-1} \neq \lim_{x \to 1^-} \frac{h(x)-h(1)}{x-1}$$

e

$$\lim_{x \to -2^+} \frac{h(x)-h(-2)}{x+2} \neq \lim_{x \to -2^-} \frac{h(x)-h(-2)}{x+2}$$

Logo, $h(x)$ é derivável em $\mathbb{R} - \{-2, 1\}$.



(82) $y = \frac{3}{2}x + 6$ é tangente a $y = c\sqrt{x}$

$y' = (c\sqrt{x})' = \dfrac{c}{2\sqrt{x}} \quad \therefore \quad \dfrac{3}{2} = \dfrac{c}{2\sqrt{x}} \Rightarrow \boxed{c = 3\sqrt{x}}$.

$\frac{3}{2}x + 6 = 3\sqrt{x}.\sqrt{x} \Rightarrow \frac{3}{2}x + 6 = 3x.$

$3x + 12 = 6x \Rightarrow \boxed{x = 4}$

Logo, $c = 3\sqrt{4} = \boxed{6}$

(83) $y = 2x + 3$ é tangente a $y = cx^2$.

$y' = (cx^2)' = 2cx. \quad \therefore \quad 2cx = 2 \Rightarrow \boxed{c = \dfrac{1}{x}}$.

$2x + 3 = \frac{1}{x} \cdot x^2 \Rightarrow 2x + 3 = x$

$\boxed{x = -3}$

Logo, $\boxed{c = -\dfrac{1}{3}}$

Seção 3.2: 1, 15, 35, 46, 49, 51, 52, 53, 57, 62

1) $f(x) = (1+2x^2)(x-x^2)$

I) Regra do produto:

$$f'(x) = 4x(x-x^2) + (1+2x^2)(1-2x).$$

$$f'(x) = 4x^2 - 4x^3 + 1 - 2x + 2x^2 - 4x^3$$

$$\boxed{f'(x) = -8x^3 + 6x^2 - 2x + 1}$$

II) Desenvolvendo:

$$f(x) = x - x^2 + 2x^3 - 2x^4$$

$$\boxed{f'(x) = 1 - 2x + 6x^2 - 8x^3}$$

15) $y = \frac{s-\sqrt{s}}{s^2} = (s - s^{1/2}) \cdot s^{-2}$

$$y' = (s - s^{1/2})(-2) \cdot s^{-3} + (1 - 1/2 \cdot s^{-1/2}) \cdot s^{-2}$$

$$y' = -2s^{-3}(s - s^{1/2}) + s^{-2}\left(1 - \frac{1}{2} s^{-1/2}\right)$$

$$y' = -\frac{2(s-\sqrt{s})}{s^3} + \frac{(2\sqrt{s}-1)}{2s^2 \cdot \sqrt{s}}.$$

$$y' = -\frac{2\left(1-\frac{\sqrt{s}}{s}\right) \cdot \sqrt{s} \cdot 2}{2s^2 \cdot \sqrt{s}} + \frac{(2\sqrt{s}-1)}{2s^2\sqrt{s}}$$

$$y' = \frac{-4\sqrt{s}\left(1-\frac{\sqrt{s}}{s}\right) + 2\sqrt{s} - 1}{2s^{5/2}}$$

$$\boxed{y' = \frac{3 - 2\sqrt{s}}{2s^{5/2}}}$$

(35) $y = \dfrac{x^2}{1+x}$ reta tangente no ponto $(1, 1/2)$

$$y = x^2(1+x)^{-1}$$
$$y' = 2x(1+x)^{-1} + x^2 \cdot (-1)(1+x)^{-2} \cdot 1$$
$$y' = \frac{2x}{1+x} - \frac{x^2}{(1+x)^2}$$
$$y' = \frac{2x(1+x) - x^2}{(1+x)^2}$$
$$y' = \frac{x^2+2x}{(x+1)^2} \Rightarrow y'(1) = \frac{3}{4}.$$

reta: $\dfrac{1}{2} = \dfrac{3}{4} + b \Rightarrow b = -\dfrac{1}{4} \Rightarrow \boxed{y = \dfrac{3}{4}x - \dfrac{1}{4}}$

(46) $f(4) = 2$
$g(4) = 5$ Encontre $h'(4)$.
$f'(4) = 6$
$g'(4) = -3$

a) $h(x) = 3f(x) + 8g(x)$
$$h'(x) = 3f'(x) + 8g'(x)$$
$$h'(4) = 3 \cdot 6 + 8(-3) = 18 - 24 = \boxed{-6.}$$

b) $h(x) = f(x) \cdot g(x)$
$$h'(x) = f'(x)g(x) + f(x) \cdot g'(x)$$
$$h'(4) = f'(4) \cdot g(4) + f(4) \cdot g'(4)$$
$$h'(4) = 6 \cdot 5 + 2 \cdot (-3)$$
$$h'(4) = 30 - 6 = \boxed{24}$$

c) $h(x) = \frac{f(x)}{g(x)} = f(x)\cdot(g(x))^{-1}$

$h'(x) = f'(x)\cdot(g(x))^{-1} + f(x)\cdot(-1)\cdot g(x)^{-2}\cdot g'(x).$

$h'(x) = \frac{f'(x)\cdot g(x)}{(g(x))^2} - \frac{f(x)\cdot g'(x)}{(g(x))^2}$

$$h'(x) = \frac{g(x)\cdot f'(x) - g'(x)\cdot f(x)}{(g(x))^2}.$$

$$h'(4) = \frac{g(4)\, f'(4) - g'(4)\cdot f(4)}{(g(4))^2}$$

$$h'(4) = \frac{5\cdot 6 - (-3)\cdot 2}{5^2} \boxed{= \frac{36}{25}}$$

d) $h(x) = \frac{g(x)}{f(x)+g(x)} = g(x)\cdot(f(x)+g(x))^{-1}$

$$h'(x) = \frac{g'(x)}{f(x)+g(x)} + \frac{g(x)\cdot(-1)(f'(x)+g'(x))}{(f(x)+g(x))^2}$$

$$h'(x) = \frac{(f(x)+g(x))\, g'(x) - g(x)(f'(x)+g'(x))}{(f(x)+g(x))^2}$$

$$h'(x) = \frac{g'(x)f(x) - f'(x)g(x)}{(f(x)+g(x))^2}$$

$$h'(4) = \frac{g'(4)\cdot f(4) - f'(4)\cdot g(4)}{(f(4)+g(4))^2}$$

$h'(x) = \dfrac{(-3).2 - 6.5}{(2+5)^2} = \boxed{-\dfrac{36}{49}}$

49) $g(x) = x\,f(x)$ $\quad f(3)=4\ ,\ f'(3)=-2$

reta tangente de g no ponto onde $x=3$.

$g'(x) = x.f'(x) + f(x)$

$g'(3) = 3.f'(3) + f(3) = 3.(-2)+4 = -2.$

$g(3) = 3.f(3) = 3.4 = 12 \Rightarrow (3,12)$

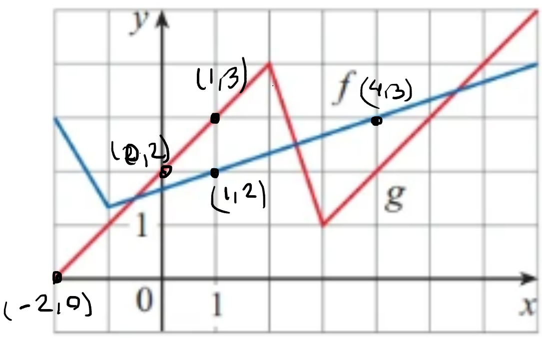
reta tangente: $12 = 3.(-2)+p \Rightarrow p = 18$

$\boxed{y = -2x+18}$

51) a) $u(x) = f(x).g(x)$

$u'(x) = f'(x)\,g(x) + g'(x)\,f(x)$

$u'(1) = f'(1)\,g(1) + g'(1)\,f(1)$



entre $-1 \le x \le 8$.

$f(x) = 1/3\,x + 5/3 \Rightarrow f'(x) = \frac{1}{3}$

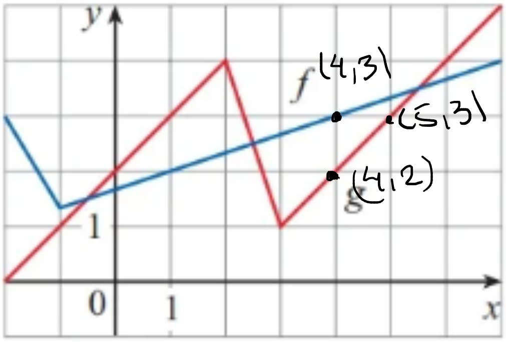
entre $-2 \le x \le 2$

$g(x) = x+2 \Rightarrow g'(x) = 1.$

$\therefore\ v'(1) = \frac{1}{3} \cdot 3 + 1 \cdot 2 = \boxed{3}$ //.

b) $v(x) = \dfrac{f(x)}{g(x)} \Rightarrow v'(x) = \dfrac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$.

$v'(4) = \dfrac{g(4)f'(4) - g'(4)f(4)}{(g(4))^2}$.

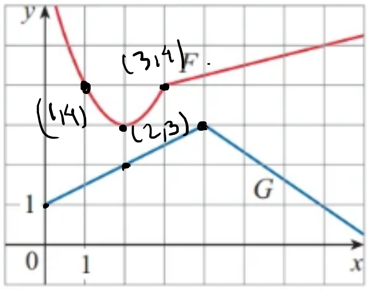


entre $3 \leq x \leq 8$
$g(x) = x - 2 \Rightarrow g'(x) = 1$

entre $-1 \leq x \leq 8$.
$f(x) = 1/3\,x + 5/3 \Rightarrow f'(x) = \frac{1}{3}$

$\therefore\ v'(4) = \dfrac{2 \cdot \frac{1}{3} - 1 \cdot 3}{2^2} = \dfrac{\frac{2}{3} - 3}{4} = \boxed{-\frac{7}{12}}$ //.

(52) a) $P(x) = F(x) \cdot G(x)$
$P'(x) = F'(x)G(x) + G'(x)F(x)$
$P'(2) = F'(2)G(2) + G'(2)F(2)$



entre $0 \leq x \leq 3$

$$F(x) = \frac{7}{2}x^2 - \frac{23}{2}x + 12$$

$$F'(x) = 7x - \frac{23}{2}$$

entre $0 \leq x \leq 4$

$$G(x) = \frac{1}{2}x + 1 \Rightarrow G'(x) = \frac{1}{2}$$

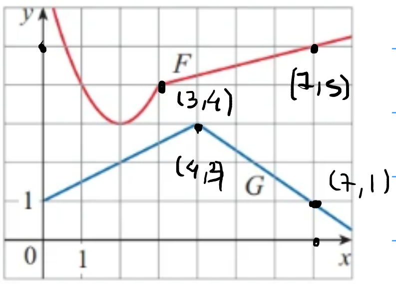
$$P'(2) = F'(2)G(2) + G'(2)F(2)$$

$$P'(2) = \left(14 - \frac{23}{2}\right) \cdot 3 + \frac{1}{2} \cdot 3$$

$$P'(2) = \left(\frac{5}{2} + \frac{1}{2}\right) \cdot 3 = \boxed{9}$$

b) $$Q(x) = \frac{F(x)}{G(x)} \Rightarrow Q'(x) = \frac{G(x)F'(x) - G'(x)F(x)}{(G(x))^2}$$

$$Q'(7) = \frac{G(7) \cdot F'(7) - G'(7)F(7)}{(G(7))^2}$$



entre $3 \leq x \leq 8$

$$F(x) = \frac{1}{4}x + \frac{13}{4} \Rightarrow F'(x) = 1/4$$

entre $4 \leq x \leq 8$

$$G(x) = -\frac{2}{3}x + \frac{17}{3} \Rightarrow G'(x) = -\frac{2}{3}$$

$$Q'(7) = \frac{G(7)\cdot F'(7) - G'(7)F(7)}{(G(7))^2}$$

$$Q'(7) = \frac{1\cdot\frac{1}{4} + 2\cdot\frac{2}{3}\cdot 5}{1} = \frac{1}{4} + \frac{10}{3} \boxed{= \frac{43}{12}}$$

53 a) $y = xg(x)$

$$\boxed{y' = xg'(x) + g(x)}$$

b) $y = \dfrac{x}{g(x)}$

$$\boxed{y' = \frac{g(x) - xg'(x)}{(g(x))^2}}$$

c) $y = \dfrac{g(x)}{x}$

$$\boxed{y' = \frac{xg'(x) - g(x)}{x^2}}$$

(57) $R(x) = \dfrac{x - 3x^3 + 5x^5}{1 + 3x^3 + 6x^6 + 9x^9} = \dfrac{f(x)}{g(x)}$

$$R'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$$

- $f'(x) = 1 - 9x^2 + 25x^4 \Rightarrow f'(0) = 1$
- $g'(x) = 9x^2 + 36x^5 + 81x^8 \Rightarrow g'(0) = 0$

$$R'(0) = \frac{1.1 - 0.0}{1^2} = \boxed{1}$$

(62) $B(t) = N(t).M(t)$

$N(4) = 820\ b \Rightarrow N'(4) = 50\ b/\text{semana}$
$M(4) = 1{,}2\ g \Rightarrow M'(4) = 0{,}14\ g/\text{semana}$.

$B'(4) = N'(4)M(4) + M'(4)N(4)$
$B'(4) = 50.1{,}2 + 0{,}14.820$
$\boxed{B'(4) = 174{,}8\ bg/\text{semana}}$

Seção 3.3: 5, 13, 21, 33, 36, 39, 41, 48

(5) $h(\theta) = \theta^2 \operatorname{sen}\theta$

$h'(\theta) = 2\theta \operatorname{sen}\theta + \theta^2 \cos\theta = \boxed{\theta(2\operatorname{sen}\theta + \theta\cos\theta)}$

(13) $f(\theta) = \dfrac{\operatorname{sen}\theta}{1+\cos\theta}$

$f'(\theta) = \dfrac{(1+\cos\theta)\cos\theta - \operatorname{sen}\theta(-\operatorname{sen}\theta)}{(1+\cos\theta)^2} = \dfrac{1+\cos\theta}{(1+\cos\theta)^2} = \boxed{\dfrac{1}{1+\cos\theta}}$

(21) Usando a regra do produto estendida:

$h(x) = fgp(x)$

$h'(x) = f'gp + fg'p + fgp'$

$h(x) = \theta\cos\theta\operatorname{sen}\theta$

$h'(\theta) = \cos\theta\operatorname{sen}\theta - \theta\operatorname{sen}^2\theta + \theta\cos^2\theta$

$\boxed{h'(\theta) = \dfrac{\operatorname{sen}2\theta}{2} + \theta\cos2\theta}$

(33) a) $f(x) = \sec x - x = (\cos x)^{-1} - x$

$f'(x) = (-1)(\cos x)^{-2}(-\operatorname{sen} x) - 1.$

$f'(x) = \dfrac{\operatorname{sen}x}{\cos^2 x} - 1$

$\boxed{f'(x) = \sec x \operatorname{tg} x - 1}$

b) $-\pi/2 < x < \pi/2$.

Os gráficos são razoáveis pois não englobam casos em que $\cos x = 0$.

36) $f(t) = \sec t$ $\quad f''(t)$

$$f'(t) = (-1)\cdot(\cos t)^{-2}(-\text{sen}\, t) = \frac{\text{sen}\, t}{\cos^2 t} = \text{tg}\, t \cdot \sec t$$

$$f''(t) = \sec^2 t \cdot \sec t + \text{tg}\, t \cdot \text{tg}\, t \sec t$$

$$f''(t) = \sec t(\text{tg}^2 t + \sec^2 t).$$

$$\boxed{f''(t) = \sec t(2\sec^2 t - 1).}$$

39) $f(x) = x + 2\,\text{sen}\, x$

$$f'(x) = 1 + 2\cos x.$$

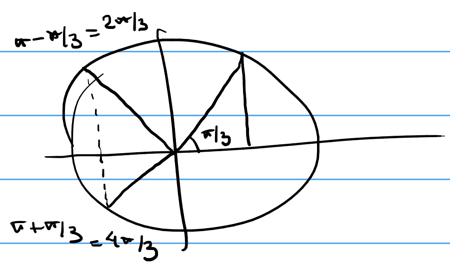
Tangente horizontal: $f'(x) = 0$

$$\cos x = -1/2$$

$$x = \frac{2\pi}{3} + 2K\pi$$

ou

$$x = \frac{4\pi}{3} + 2K\pi$$



(41) a) $x(t) = 8\operatorname{sen} t$

$$\boxed{\begin{aligned} v(t) &= 8\cos t \\ a(t) &= -8\operatorname{sen} t \end{aligned}}\ \text{m.}$$

b) $x(2\pi/3) = 8 \cdot \operatorname{sen}(2\pi/3) = 8 \cdot \dfrac{\sqrt{3}}{2} = 4\sqrt{3}$ m.

$v(2\pi/3) = 8\cos\left(\frac{2\pi}{3}\right) = -4$ m/s

$a(2\pi/3) = -8\operatorname{sen}\left(\frac{2\pi}{3}\right) = -4\sqrt{3}$ m/s^2

Para a esquerda.

(48) $\displaystyle\lim_{x\to 0} \frac{\operatorname{sen}^2 3x}{x} = \lim_{x\to 0}\left(3 \cdot \operatorname{sen} 3x \cdot \frac{\operatorname{sen} 3x}{3x}\right)$

$\displaystyle\lim_{x\to 0} 3 \cdot \lim_{x\to 0} \operatorname{sen} 3x \cdot \lim_{x\to 0}\left(\frac{\operatorname{sen} 3x}{3x}\right) = 3 \cdot 0 \cdot 1 = \boxed{0}$.

Seção 3.4: 4, 7, 9, 11, 21, 28, 34, 41, 50, 55, 63, 65, 70, 74, 77, 80, 95, 97, 102

(4) $y = tg(x^2)$ $\quad f(x) = tg\,x \; ; \; g(x) = x^2$

$\boxed{y' = \sec^2 x^2 \cdot 2x}$

Regra da cadeia:
$(f(g(x)))' = f'(g(x)) \cdot g'(x)$.

(7) $f(x) = (2x^3 - 5x^2 + 4)^5$ $\quad f(x) = x^5 \; ; \; g(x) = 2x^3 - 5x^2 + 4$

$f'(x) = 5(2x^3 - 5x^2 + 4)^4 \cdot (6x^2 - 10x)$

$\boxed{f'(x) = 10x(3x-5)(2x^3 - 5x^2 + 4)^4}$ ////.

(9) $f(x) = \sqrt{5x+1}$ $\quad f(x) = \sqrt{x} \quad g(x) = 5x+1$

$f'(x) = \dfrac{1}{2\sqrt{5x+1}} \cdot 5 = \boxed{\dfrac{5}{2} \cdot \dfrac{1}{\sqrt{5x+1}}}$ ////.

(11) $g(t) = \dfrac{1}{(2t+1)^2} = (2t+1)^{-2}$ $\quad f(t) = t^{-2} \; ; \; g(2t+1)$

$g'(t) = \dfrac{-2}{(2t+1)^3} \cdot 2 \boxed{= \dfrac{-4}{(2t+1)^3}}$ ////.

21) $F(x) = (4x+5)^3 (x^2-2x+5)^4$

$$F'(x) = 3(4x+5)^2 \cdot 4 (x^2-2x+5)^4 + (4x+5)^3 \cdot 4(x^2-2x+5)^3 \cdot (2x-2)$$

$$f'(x) = 12(4x+5)^2 \cdot (x^2-2x+5)^4 + 8(4x+5)^3 (x^2-2x+5)^3 \cdot (x-1)$$

$$F'(x) = 4(4x+5)^2 \cdot (x^2-2x+5)^3 \left(3(x^2-2x+5) + 2(4x+5)(x-1)\right)$$

$$\boxed{F'(x) = 4(4x+5)^2 (x^2-2x+5)^3 \cdot (11x^2-4x+5)}$$

28) $s(t) = \sqrt{\dfrac{1+\operatorname{sen} t}{1+\cos t}}$ $\qquad f(t) = \sqrt{t}$; $g(t) = \dfrac{1+\operatorname{sen} t}{1+\cos t}$

$$s'(t) = \frac{1}{2\sqrt{\dfrac{1+\operatorname{sen} t}{1+\cos t}}} \cdot \frac{(1+\cos t) \cdot \cos t + (1+\operatorname{sen} t) \cdot \operatorname{sen} t}{(1+\cos t)^2}$$

$$\boxed{s'(t) = \frac{(1+\operatorname{sen} t + \cos t)\sqrt{1+\cos t}}{2(1+\cos t)^2 \sqrt{1+\operatorname{sen} t}}}$$

34) $F(t) = \dfrac{t^2}{\sqrt{t^3+1}}$

$$F'(t) = \frac{\sqrt{t^3+1} \cdot 2t + t^2 \cdot \dfrac{1}{2\sqrt{t^3+1}} \cdot 3t^2}{t^3+1}$$

$$F'(t) = \frac{t(4(t^3+1) + 3t^3)}{2(t^3+1)\sqrt{t^3+1}} \boxed{= \frac{t(7t^3+4)}{2(t^3+1)^{3/2}}}$$

41 $y = sen^2(x^2+1)$. $f(x) = sen^2 x$ $g(x) = x^2+1$

$$y' = 2\,sen(x^2+1)\cdot\cos(x^2+1)\cdot(2x).$$

$$\boxed{y' = sen(2(x^2+1))\cdot 2x}$$

50 $y = sen(\theta + tg(\theta + \cos\theta))$

$f(\theta) = sen\,\theta$ $g(\theta) = \theta + tg(\theta + \cos\theta)$.

$$y' = \cos(\theta + tg(\theta+\cos\theta))\cdot(1 + (tg(\theta + \cos\theta))')$$

$$\boxed{y' = \cos(\theta + tg(\theta+\cos\theta))\cdot(1+(sec^2(\theta+\cos\theta))\cdot(1-sen\,\theta))}$$

55 $y = \sqrt{\cos x}$.

$$y' = \frac{1}{2\sqrt{\cos x}}\cdot(-sen\,x) = \boxed{-\frac{sen\,x}{2\sqrt{\cos x}}}$$

$$y'' = \frac{(2\sqrt{\cos x})(-\cos x) - (-sen\,x)\cdot\frac{2\cdot 1}{2\sqrt{\cos x}}\cdot(-sen\,x)}{4\cos x}$$

$$y'' = \frac{-\frac{sen^2 x}{\sqrt{\cos x}} - \frac{2\cos^2 x}{\sqrt{\cos x}}}{4\cos x} = \boxed{\frac{-sen^2 x - 2\cos^2 x}{4(\cos x)^{3/2}}}$$

63) $f(x) = x\sqrt{2-x^2}$

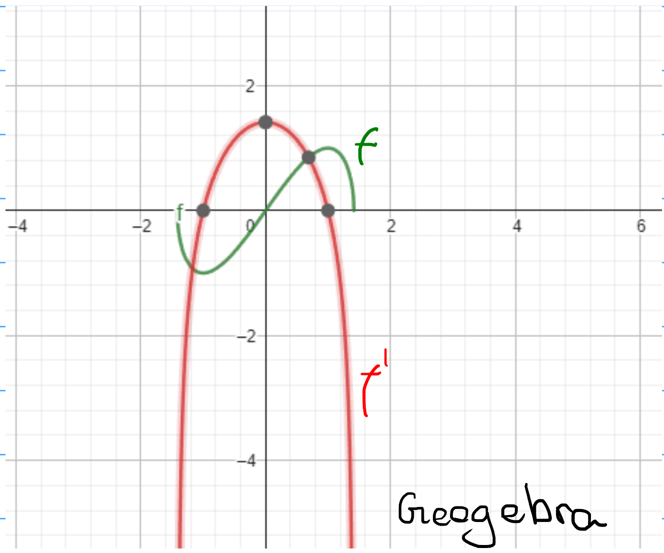
a) $f'(x) = \sqrt{2-x^2} + x\left(\sqrt{2-x^2}\right)'$

$$f'(x) = \sqrt{2-x^2} + x \cdot \frac{1}{2\sqrt{2-x^2}} \cdot (-2x)$$

$$f'(x) = \frac{2(2-x^2)}{2\sqrt{2-x^2}} - \frac{2x^2}{2\sqrt{2-x^2}}$$

$$f'(x) = \frac{4-4x^2}{2\sqrt{2-x^2}} = \boxed{\frac{2-2x^2}{\sqrt{2-x^2}}}$$

b)



Érazoával

(65) $f(x) = 2\operatorname{sen}x + \operatorname{sen}^2 x$.

$f'(x) = 2\cos x + 2\operatorname{sen}x\cos x$.
$= 2\cos x(1+\operatorname{sen}x)$

tangentes horizontais: $f'(x) = 0$

$2\cos x(1+\operatorname{sen}x) = 0$

$\cos x = 0$ ou $\operatorname{sen}x = -1$

$$\boxed{x = \frac{\pi}{2} + k\pi, k\in\mathbb{Z} \quad \text{ou} \quad x = \frac{3\pi}{2} + 2k\pi, k\in\mathbb{Z}.}$$

$$\boxed{\text{Pontos: } \left(\frac{\pi}{2} + k\pi, 3\right) \text{ ou } \left(\frac{3\pi}{2} + 2k\pi, -1\right), k\in\mathbb{Z}}$$

(70)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a) $F(x) = f(f(x)) \Rightarrow F'(2)$.
$F'(x) = f'(f(x)) \cdot f'(x)$
$F'(2) = f'(f(2)) \cdot f'(2) = f'(1) \cdot 5 = 20$.

b) $G(x) = g(g(x)) \Rightarrow G'(3)$

$G'(x) = g'(g(x)) \cdot g'(x)$

$G'(3) = g'(g(3)) \cdot g'(3)$

$G'(3) = g'(2) \cdot 9 = 7 \cdot 9 \boxed{= 63}$

74) $F(x) = f(x^{\alpha})$

$G(x) = (f(x))^{\alpha}$

a) $F'(x) = ?$ $h(x) = f(x)$ $g(x) = x^{\alpha}$

$\boxed{F'(x) = f'(x^{\alpha}) \cdot \alpha \cdot x^{\alpha - 1}}$

b) $G'(x) = ?$ $h(x) = x^{\alpha}$ $g(x) = f(x)$

$\boxed{G'(x) = \alpha \cdot (f(x))^{\alpha - 1} \cdot f'(x).}$

77) $r(x) = f(g(h(x)))$

$h(1) = 2$; $g(2) = 3$; $h'(1) = 4$; $g'(2) = 5$; $f'(3) = 6$.

$r'(x) = f'(g(h(x))) \cdot (g(h(x)))'$

$r'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$

$r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1)$

$r'(1) = f'(g(2)) \cdot g'(2) \cdot 4$

$r'(1) = f'(3) \cdot 5 \cdot 4$

$r'(1) = 6 \cdot 5 \cdot 4 = 30 \cdot 4 = \boxed{120}$ //

80) $F(x) = f(xf(xf(x)))$

$f(1) = 2$; $f(2) = 3$; $f'(1) = 4$; $f'(2) = 5$; $f'(3) = 6$

$F'(x) = f'(xf(xf(x))) \cdot (x \cdot f(xf(x)))'$

$F'(x) = f'(xf(xf(x)))(f(xf(x)) + x \cdot (f(xf(x))'))$

$F'(x) = f'(xf(xf(x)))(f(xf(x)) + x \cdot [f'(xf(x)) \cdot (xf(x))'])$

$F'(x) = f'(xf(xf(x)))(f(xf(x)) + x \cdot [f'(xf(x)) \cdot (f(x) + x \cdot f'(x))])$

$F'(1) = f'(1 \cdot f(1 \cdot f(1))) \cdot (f(1 \cdot f(1)) + 1 \cdot [f'(1 \cdot f(x)) \cdot (f(1) + 1 \cdot f'(1))]$

$F'(1) = f'(f(2)) \cdot f(2) + (f'(2) \cdot (2+4))$

$F'(1) = f'(3) \cdot 3 + 5 \cdot 6 = 6 \cdot 3 + 5 \cdot 6 = 18 + 30 = \boxed{48}$ //

95 a) derivada de uma par é uma impar

$f(x) = f(-x)$.
$\therefore f'(x) = f'(-x)$.

$\Rightarrow f(g(x)) = f(-x) = f(x)$ $f(x)$ e $g(x) = -x$.

$f'(-x) \cdot (-1) = -f'(-x) = f'(-x) = f'(x)$

$$\boxed{\Rightarrow f'(x) = -f'(-x) \quad \text{(função impar).}}$$

b) derivada de uma impar é uma par.

$f(x) = -f(-x) \Rightarrow -f(x) = f(-x)$
$\therefore f'(x) = -f'(-x) \Rightarrow -f'(x) = f'(-x)$

$f(g(x)) = f(-x)$ $f(x)$ e $g(x) = -x$.

$f'(-x) = f'(-x) \cdot (-1)$
$-f'(x) = -f'(-x)$

$$\boxed{\Rightarrow f'(x) = f'(-x) \quad \text{(função par)}}$$

97 θ está em graus

$$\frac{d}{d\theta}\operatorname{sen}\theta = \frac{\pi}{180}\cdot\cos\theta$$

$$\pi \;—\; 180$$
$$x \;—\; \theta$$

$$\frac{180x}{\pi} = \theta.$$

$$\boxed{x = \theta\cdot\frac{\pi}{180}}$$

- $\operatorname{sen}' x = \cos x \Rightarrow$

$$\operatorname{sen}'\left(\frac{\pi\theta}{180}\right) = \cos\left(\frac{\pi\theta}{180}\right)\cdot\frac{\pi}{180}.$$

Substituindo a variável, sem perda de generalidade:

$$\operatorname{sen}'\theta = \cos\theta\cdot\frac{\pi}{180} \quad (\theta \text{ em graus}).$$

102

$$F''(x) = f''(g(x)) \cdot [g'(x)]^2 + f'(g(x)) \cdot g''(x)$$

$F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F''(x) = g''(x) \cdot f'(g(x)) + g'(x) \cdot f''(g(x)) \cdot g'(x)$

$$\boxed{F''(x) = [g'(x)]^2 \cdot f''(g(x)) + g''(x) \cdot f'(g(x)).}$$

Exercício

Seja $f: \mathbb{R} \to \mathbb{R}$, $f(x) > 0$, $f(0) = 1$
que satisfaz $f(x+y) = f(x) \cdot f(y)$.
Suponha que $\exists\, f'(0)$

Calcule $f'(x)$.

Sol:

Pela definição: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x) \cdot f(h) - f(x)}{h} = \lim_{h \to 0} f(x) \left(\frac{f(h) - 1}{h} \right)$$

$$= \lim_{h \to 0} f(x) \cdot \lim_{h \to 0} \left(\frac{f(h) - f(0)}{h} \right) = f(x) \cdot f'(0)$$

$$\therefore f'(x) = f(x) \cdot f'(0).$$

Se $\exists\, f'(0)$, então $\exists\, f'(x)$ | $f(x+y) = f(x) \cdot f(y)$, $f(0) = 1$
$\wedge$ f contínua $(\mathbb{R} \to \mathbb{R})$.

OBS: A função é uma função exponencial com $a = f(1)$.

$f(n) = f(1+1+1+\dots+1) = f(1) \cdot f(1) \cdot f(1) \dots f(1) = [f(1)]^n = a^n$

$f(-n) \Rightarrow f(0) = f(n + (-n)) = f(n) \cdot f(-n)$

$1 = f(n) \cdot f(-n) \Rightarrow f(-n) = \dfrac{1}{f(n)} \Rightarrow \boxed{a^{-n} = \dfrac{1}{a^n}}$ //

Logo, vale $\forall n \in \mathbb{R}$. (pois mexemos com limite).

$$f\left(\frac{n}{m}\right) = \left(f\left(\frac{1}{m}\right)\right)^n$$

$$1 = m \cdot \frac{1}{m} = \underbrace{\frac{1}{m} + \frac{1}{m} + \dots + \frac{1}{m}}_{m \text{ vezes}}$$

$$f(1) = a = \left(f(1/m)\right)^m \therefore \boxed{f(1/m) = \sqrt[m]{a}}$$ //.

$$\boxed{\therefore f\left(\frac{n}{m}\right) = \left(\sqrt[m]{a}\right)^n}$$ //