146 a) $x^2-3x+5=0$

$$x=\frac{3\pm\sqrt{9-4.5}}{2}=\frac{3\pm\sqrt{-11}}{2}=\boxed{\frac{3\pm\sqrt{11}.i}{2}}$$

b) $x^4-4x^3+9x^2-10x+4=0$

Por inspeção $x=1$ é raiz, logo

$x^4-4x^3+9x^2-10x+4=(x-1)(x^3-3x^2+6x-4)$

Por inspeção $x=1$ é raiz de x^3-3x^2+6x-4

$\therefore x^4-4x^3+9x^2-10x+4=(x-1)(x-1)(x^2-2x+4)$

$=(x-1)^2((x-1)^2+3)$

Logo, as raízes de $x^4-4x^3+9x^2-10x+4$ é

$\boxed{x=1;\ x=1+i\sqrt{3},\ x=1-i\sqrt{3}}$

c) $2x^8+6x^7+9x^6+6x^5-6x^3-9x^2-6x-2=0$

Por inspeção, $x=1$ e $x=-1$ são raízes da equação.

Logo, $2x^8+6x^7+9x^6+6x^5-6x^3-9x^2-6x-2=$
$=(x-1)(x+1)(2x^6+6x^5+11x^4+12x^3+11x^2+6x+2)$

Além disso, i e $-i$ são raízes de $2x^6+6x^5+11x^4+12x^3+11x^2+6x+2$.

$$= (x-1)(x+1)(x+i)(x-i)(2x^4+6x^3+9x^2+6x+2)$$

$2x^4+6x^3+9x^2+6x+2$ é um polinômio recíproco.
dividindo por x^2:

$$2x^2+6x+9+6\cdot\frac{1}{x}+2\cdot\frac{1}{x^2} = 2\left(x^2+\frac{1}{x^2}\right)+6\left(x+\frac{1}{x}\right)+9$$

$$\left(x+\frac{1}{x}\right)^2-2 = x^2+\frac{1}{x^2} \Rightarrow 2\left[\left(x+\frac{1}{x}\right)^2-2\right]+6\left(x+\frac{1}{x}\right)+9$$

$$\Rightarrow x+1/x = y \Rightarrow 2y^2+6y+5=0$$

$$y = \frac{-6\pm\sqrt{36-40}}{4} = \boxed{\frac{-3\pm i}{2}}.$$

I) $x^2+1 = \left(\frac{-3+i}{2}\right)x \qquad 2x^2+(3-i)x+2=0.$

$$x = \frac{(i-3)\pm\sqrt{9-6i-1-4.2.2}}{4}$$

$$x = \frac{i-3\pm\sqrt{-8-6i}}{4} = \frac{i-3\pm i\sqrt{8+6i}}{4}$$

II) $x^2+1 = \frac{(-3-i)x}{2} \qquad 2x^2+(i+3).x+2=0$

$$x = \frac{-(i+3)\pm\sqrt{-1+6i+9-4.2.2}}{4} = \frac{-(i+3)\pm i\sqrt{8-6i}}{4}$$

$$\boxed{\text{Raízes são: } \pm 1, \pm i, -\frac{(3-i)\pm i\sqrt{8+6i}}{4}, -\frac{(3+i)\pm\sqrt{8-6i}}{4}}$$

d) $x^5 - 1 = 0 \Rightarrow x^5 = 1.$

Vamos usar a segunda Lei de D'Moivre

$$z = |z| \operatorname{cis}\theta \Rightarrow \sqrt[n]{z} = \sqrt[n]{|z|} \operatorname{cis}\left(\frac{\theta + 2k\pi}{n}\right) \quad \begin{cases} k \in \mathbb{Z} \\ 0 \leq k \leq n-1 \end{cases}$$

$$x^5 = 1 \qquad (\theta = 0; \ |x| = 1)$$

$$x = \sqrt[5]{1} \cdot \operatorname{cis}\left(\frac{2k\pi}{5}\right) = \boxed{\operatorname{cis}\frac{2k\pi}{5}, \ k = \{0,1,2,3,4\}}$$

e) $x^4 + 1 = 0$

$$x^4 - i^2 = 0$$

$$(x^2 - i)(x^2 + i) = 0$$

$$\boxed{x = \pm\sqrt{i} \ ; \ x = \pm i\sqrt{i}}$$

f) $x^3 = 8$ Podemos resolver fatorando ou usando a segunda Lei de D'Moivre

$$x^3 = 8 \quad (\theta = 0; \ |x| = 8)$$

$$x = \sqrt[3]{8} \cdot \operatorname{cis}\left(\frac{2k\pi}{3}\right) = \boxed{2\operatorname{cis}\left(\frac{2k\pi}{3}\right), \ k = \{0,1,2\}}$$

g) $x^4 = 81$ Novamente, podemos resolver fatorando ou pela segunda Lei de D' Moivre.

$$x^4 - 9^2 = (x^2-9)(x^2+9) = (x-3)(x+3)(x^2+9)$$
$$= (x-3)(x+3)(x+3i)(x-3i)$$

Raízes $\boxed{\pm 3 \text{ e } \pm 3i}$ //.

147 $i^{8n+3} + i^{4n+1} = i^3\cdot(i^8)^n + i(i^4)^n = -i\cdot 1^n + i\cdot 1^n$
$= -i + i = \boxed{0}$ //

148 I) $(i+1)^{2011} \Rightarrow (i+1)^2 = 2i \Rightarrow (i+1)^4 = -4.$
$(1+1)^{2011} = (i+1)^{2008}\cdot(i+1)^2\cdot(i+1) = (-4)^{502}\cdot 2i(i+1).$
$= 4^{502}(2i-2) \Rightarrow \boxed{2^{1005}(i-1)}$ //

II) $(i-1)^{2012} \Rightarrow (i-1)^2 = -2i \Rightarrow (i-1)^4 = -4$
$(i-1)^{2011} = (i-1)^{2008}\cdot(i-1)^2\cdot(i-1) = (-4) = (-4)^{502}\cdot(-2i)(i-1)$
$= 4^{502}(2i+2i) \Rightarrow 2^{1005}(i+1)\cdot(1-i) = \boxed{2^{1000}}.$

III) $(i+1)^{2013} = 2^{1005}(i-1)\cdot 2i = 2^{1005}(-2-2i)$
$\boxed{= -2^{1006}(i+1)}$ //

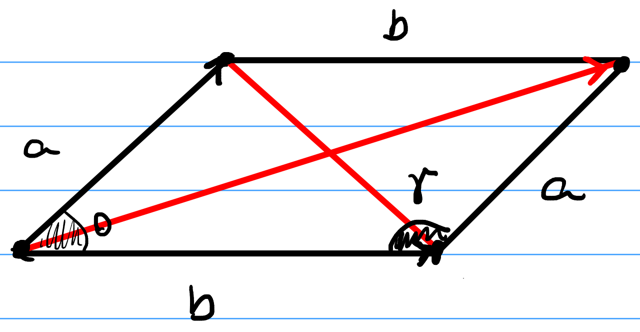
149 $|1+iz| = |1-iz| \Leftrightarrow z \in \mathbb{R}$.

$z = a+bi$.

$$|1+i(a+bi)| = |1-i(a+bi)|$$
$$|(1-b)+ai| = |(1+b)-ai|$$
$$\sqrt{(1-b)^2+a^2} = \sqrt{(1+b)^2+a^2}$$
$$\cancel{1}-2b+\cancel{b^2}+\cancel{a^2} = \cancel{1}+2b+\cancel{b^2}+\cancel{a^2}$$
$$4b=0$$
$$\boxed{b=0}$$ Parte imaginário $=0$

$\boxed{\text{Logo } z \in \mathbb{R}}$.

150



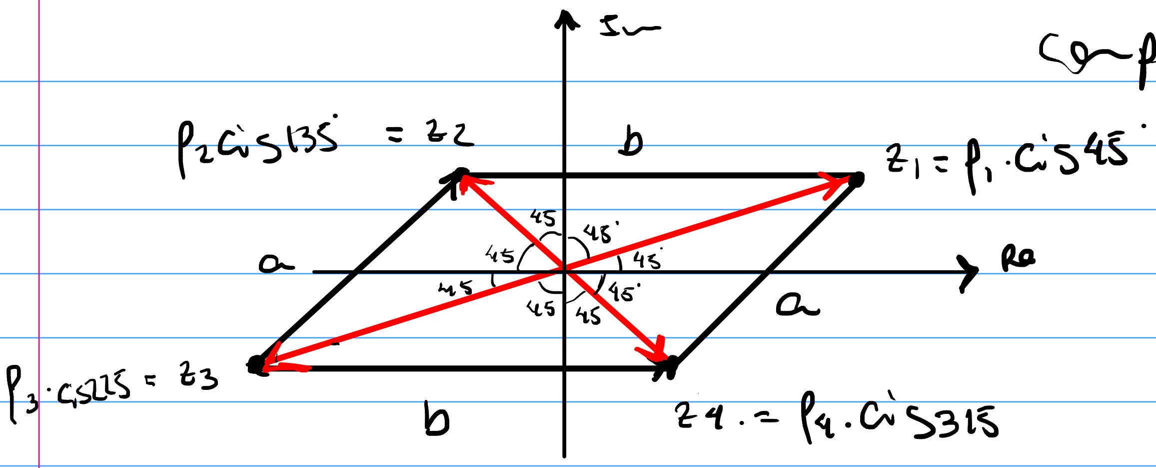
Geo. Plana

$$d = a^2+b^2-2ab\cos\theta$$
$$d = a^2+b^2-2ab\cos\gamma$$

$\therefore \cos\theta = \cos\gamma$, mas $\theta+\gamma = 180°$

como $0 < \theta, \gamma < 180$, se $\cos\theta = \cos\gamma$, então $\theta = \gamma$
Logo, $\boxed{\theta = \gamma = 90°}$.

complexos



- $|\rho_1 cis 45° - \rho_4 cis 315| = |\rho_2 cis 135 - \rho_3 cis 225| = a$
- $|\rho_1 cis 45 - \rho_2 cis 135| = |\rho_3 cis 225 - \rho_4 cis 315| = b$

$$\Rightarrow \left| \rho_1 \frac{\sqrt{2}}{2}(1+i) - \rho_4 \frac{\sqrt{2}}{2}(1-i) \right| = \left| \rho_2 \cdot \frac{\sqrt{2}}{2}(-1+i) - \rho_3 \frac{\sqrt{2}}{2}(-1-i) \right| = a$$

$$\Rightarrow \left| \rho_1 \frac{\sqrt{2}}{2}(1+i) - \rho_2 \frac{\sqrt{2}}{2}(-1+i) \right| = \left| \rho_3 \cdot \frac{\sqrt{2}}{2}(-1-i) - \rho_4 \frac{\sqrt{2}}{2}(1-i) \right| = b$$

$$\Rightarrow \begin{cases} |\rho_1(1+i) + \rho_4(i-1)| = |\rho_2(i-1) + \rho_3(i+1)| = a \\ |\rho_1(1+i) + \rho_2(1-i)| = |\rho_3(1+i) + \rho_4(1-i)| = b \end{cases}$$

$$\Rightarrow \begin{cases} \sqrt{(\rho_1-\rho_4)^2 + (\rho_1+\rho_4)^2} = \sqrt{(\rho_3-\rho_2)^2 + (\rho_3+\rho_2)^2} = a \\ \sqrt{(\rho_1+\rho_2)^2 + (\rho_1-\rho_2)^2} = \sqrt{(\rho_3+\rho_4)^2 + (\rho_3-\rho_4)^2} = b \end{cases}$$

$$\Rightarrow \begin{cases} \rho_1^2 + \rho_4^2 = \rho_2^2 + \rho_3^2 \\ \rho_1^2 + \rho_2^2 = \rho_3^2 + \rho_4^2 \end{cases} \quad \begin{cases} \rho_1 = \rho_3 \\ \rho_2 = \rho_4 \end{cases} \quad \boxed{\text{logo } a = b}$$

151 $a, b, n \in \mathbb{Z}_+$

Provar que $\exists x, y \in \mathbb{Z} \mid (a^2+b^2)^n = x^2+y^2$

Seja $z = a+bi$ e $w = x+yi$.

$$|z|^2 = a^2+b^2 = z\cdot\bar{z}$$
$$|w|^2 = x^2+y^2 = w\bar{w}$$

$$(|z|^n)^2 = |w|^2.$$

$$(|z|^n - |w|)(|z|^n + |w|) = 0$$

$\hookrightarrow \boxed{|w| = |z|^n}$

ou

$\boxed{|z| = |w| = 0.}$ (Não é possível, $a, b, n \in \mathbb{Z}_+$)

$\therefore |w| = |z|^n$

Se $w = z^n$. $\quad x = |z|^n \cos(n\alpha)$, $y = |z|^n \operatorname{sen}(n\alpha)$ $\quad \left\{ \begin{array}{l} a = |z|\cos\alpha \\ b = |z|\operatorname{sen}\alpha \end{array} \right.$

$\boxed{2ab = |z|^2 . \operatorname{sen}2\alpha}$.

$\boxed{a^2-b^2 = |z|^2 . \cos 2\alpha}$.

Fazendo $n=2$, podemos escrever x, y como relação dos inteiros a^2-b^2 e $2ab$, respectivamente, logo, existem $x, y \in \mathbb{Z}$ tais que

$(a^2+b^2)^n = x^2+y^2$ dados $a, b, n \in \mathbb{Z}_+$.

152 a) $\dfrac{z+i}{1+iz} = 2$ $\qquad z = a+bi$.

$\boxed{iz \neq -1} \Rightarrow -b+ai \neq -1 \Rightarrow \boxed{b \neq 1 \text{ e } a \neq 0}$.

$$a+bi+i = 2(1-b+ai)$$
$$a+(b+1)i = 2(1-b)+2ai$$

$$\begin{cases} a = 2(1-b) \Rightarrow a = 2(1+1-2a) \\ b+1 = 2a \Rightarrow 1-2a = -b. \end{cases}$$

$$a = 4(1-a)$$
$$a = 4-4a \qquad \boxed{a = 4/5} \checkmark$$

$b = 8/5 - 1 \boxed{= 3/5.} \checkmark \qquad \boxed{z = \dfrac{4}{5} + \dfrac{3}{5}i}$

b) $\dfrac{z+i}{1+iz} \in \mathbb{R}$, $\operatorname{Im} = 0$. $\qquad \boxed{z = a+bi.}$

$$\frac{a+(b+1)i}{(1-b)+ai} \Rightarrow \frac{(a+(b+1)i)((1-b)-ai)}{(1-b)^2+a^2}$$

$$\frac{a(1-b)-a^2i+(b+1)(1-b)i+(b+1)a}{a^2+(1-b)^2}$$

$$=\frac{a-a\cancel{b}-a^2i+(1-b^2)i+a+a\cancel{b}}{a^2+(1-b)^2}$$

$$=\frac{2a+(1-a^2-b^2)i}{a^2+(1-b)^2}$$

$$Re=\frac{2a}{a^2+(1-b)^2} \quad ; \quad Im=\frac{1-a^2-b^2}{a^2+(1-b)^2}$$

condições: $a^2+b^2=1 \wedge a\neq 0 \wedge b\neq 1.$

$\therefore |z|^2=1$; $Re(z)\neq 0$; $Im(z)\neq 1.$

Para ser real: $\boxed{|z|=1 \,;\, Re(z)\neq 0 \text{ e } Im(z)\neq 1}$.

153 a) $2 = \boxed{2 \operatorname{cis} 2k\pi}$, $k \in \mathbb{Z}$

b) $3i = \boxed{3 \operatorname{cis}\left(\frac{\pi}{2} + 2k\pi\right)}$, $k \in \mathbb{Z}$

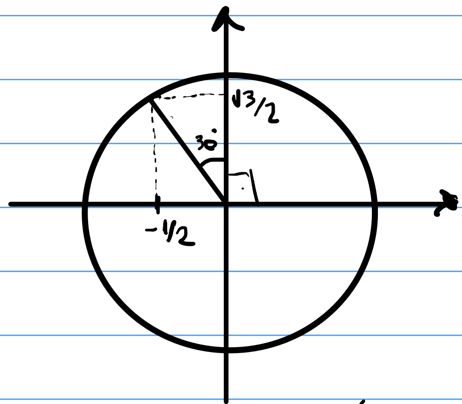
c) $1+i \Rightarrow \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i = \frac{\sqrt{2}}{2}(1+i)$

$= \operatorname{cis}(\pi/4 + 2k\pi)$. $\Rightarrow$ Logo, $(1+i) = \boxed{\sqrt{2} \operatorname{cis}(\pi/4 + 2k\pi) \quad k \in \mathbb{Z}}$

d) $1 + i\sqrt{3} = 2 \cdot \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\boxed{= 2 \cdot \operatorname{cis}(\pi/3 + 2k\pi) \text{ , } k \in \mathbb{Z}}$

154 $\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{100}$



$-\frac{1}{2} + i\frac{\sqrt{3}}{2} = \operatorname{cis} 2\pi/3$

$\Rightarrow \operatorname{cis} 200\pi/3 = \operatorname{cis}\left(\frac{198\pi}{3} + \frac{2\pi}{3}\right) = \operatorname{cis}\left(66\pi + \frac{2\pi}{3}\right)$

$= \operatorname{cis}\left(2.33\pi + \frac{2\pi}{3}\right) = \operatorname{cis} 2\pi/3 \boxed{= -\frac{1}{2} + i\frac{\sqrt{3}}{2}}$

155 $|z_1+z_2|=\sqrt{3}$ $\qquad z_1=a+bi$

$|z_1|=|z_2|=1$ $\qquad z_2=x+yi.$

$|z_1-z_2|=A$

$\sqrt{(a+x)^2+(b+y)^2}=\sqrt{3}$

$\sqrt{a^2+b^2}=\sqrt{b^2+y^2}=1.$

$\Rightarrow a^2+2ax+x^2+b^2+2by+y^2=3$

$\Rightarrow a^2+b^2=b^2+y^2=1.$

$2+2ax+2by=3$

$\boxed{2ax+2by=1}$

$A=\sqrt{(a-x)^2+(b-y)^2}=\sqrt{a^2-2ax+x^2+b^2-2by+y^2}$

$A=\sqrt{2-(2ax+2by)}=\sqrt{2-1}=\sqrt{1}\ \boxed{=1}$

156 $\left(\frac{\sqrt{2}}{1+i}\right)^{a3}$ $\qquad \left(\frac{\sqrt{2}(1-i)}{2}\right)^{a3}$

$=(cis(7\pi/4))^{a3}=cis\left(\frac{651\pi}{4}\right)=cis\left(\frac{162\pi+3\pi}{4}\right)$

$=cis(2.81\pi+3\pi/4)\ \boxed{=cis(3\pi/4)}=\boxed{-\sqrt{2}/2+\sqrt{2}/2i}$

157 a) x^5+x^4+1

Vamos observar as raízes cúbicas da unidade: $\omega^3=1 \Rightarrow (\omega-1)(\omega^2+\omega+1)=0$

$\therefore \omega^3=1$ e $\omega^2+\omega+1=0$.

Vamos trocar x^5+x^4+1 por $\omega^5+\omega^4+1$:

$\omega^5=\omega^3\cdot\omega^2=\omega^2$
$\omega^4=\omega^3\cdot\omega \quad =\omega$

$\therefore \omega^5+\omega^4+1=\omega^2+\omega+1=0$.
Logo, $\omega^2+\omega+1 \mid \omega^5+\omega^4+1$ e, por consequência, $x^2+x+1 \mid x^5+x^4+1$

$$\boxed{\therefore x^5+x^4+1=(x^2+x+1)(x^3-x+1)}$$

b) $x^{10}+x^5+1$.
Usando o mesmo raciocínio

$\omega^3=1$
$\omega^2+\omega+1=0$

$x^{10}+x^5+1 \Rightarrow \omega^{10}+\omega^5+1=\omega+\omega^2+1=0$
$\omega^{10}=(\omega^3)^3\cdot\omega=\omega$
$\omega^5=\omega^3\cdot\omega^2=\omega^2$

Logo $x^2+x+1 \mid x^{10}+x^5+1$

$$\boxed{x^{10}+x^5+1=(x^2+x+1)(x^8-x^7+x^5-x^4+x^3-x+1)}$$

158 $\cos 2\pi/7+\cos 4\pi/7+\cos 6\pi/7+1/2=0$

$z=\rho \operatorname{cis}\theta.$
$\bar{z}=\rho \operatorname{cis}(-\theta)$

Fazendo $\rho=1$ $\quad z\cdot\bar{z}=|z|^2=1$
$\bar{z}=\frac{1}{z}.$

$z^n+\bar{z}^n=\rho^n\cdot(\operatorname{cis}(n\theta)+\operatorname{cis}(-n\theta))$

$$\boxed{z^n+\frac{1}{z^n}=2\cos n\theta}$$

Fazendo $z=\operatorname{cis}\pi/7$. (raízes sétimas da unidade).

$z^7=1$; $z^6+z^5+z^4+z^3+z^2+z+1=0$

$\cos 2\pi/7+\cos 4\pi/7+\cos 6\pi/7=\frac{1}{2}\left(z^2+\frac{1}{z^2}\right)+\frac{1}{2}\left(z^4+\frac{1}{z^4}\right)+\frac{1}{2}\left(z^6+\frac{1}{z^6}\right)$

$$= \frac{1}{2}\left(z^2 + z^4 + z^6 + \frac{1}{z^2} + \frac{1}{z^4} + \frac{1}{z^6}\right)$$

$$= \frac{1}{2}\left(\frac{z^8 + z^{10} + z^{12} + z^4 + z^2 + 1}{z^6}\right)$$

$$= \frac{1}{2}\left(\frac{z^7 \cdot z + z^7 \cdot z^3 + z^7 \cdot z^5 + z^4 + z^2 + 1}{z^6}\right)$$

$$= \frac{1}{2}\left(\frac{z + z^2 + z^3 + z^4 + z^5 + 1}{z^6}\right) = -\frac{1}{2}\,\frac{z^6}{z^6} = -\frac{1}{2}$$

$$e \quad \boxed{-\frac{1}{2} + \frac{1}{2} = 0}$$

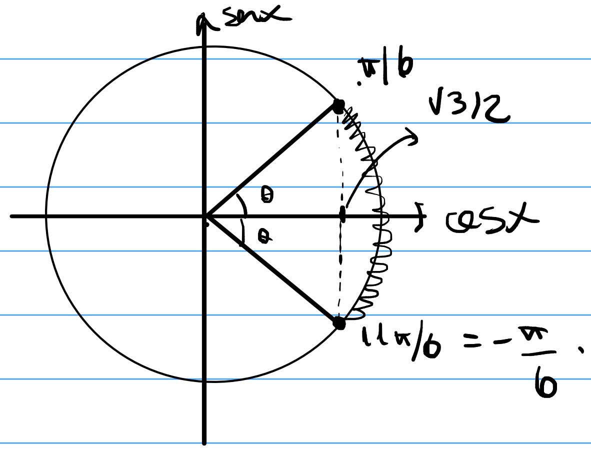
159

$$\cos x \geq \sqrt{3}\,\text{sen}\,x + \sqrt{3}.$$

$$\cos x - \sqrt{3}\,\text{sen}\,x \geq \sqrt{3}$$

$$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\,\text{sen}\,x \geq \frac{\sqrt{3}}{2}.$$

$$\cos(x + \pi/3) \geq \sqrt{3}/2$$

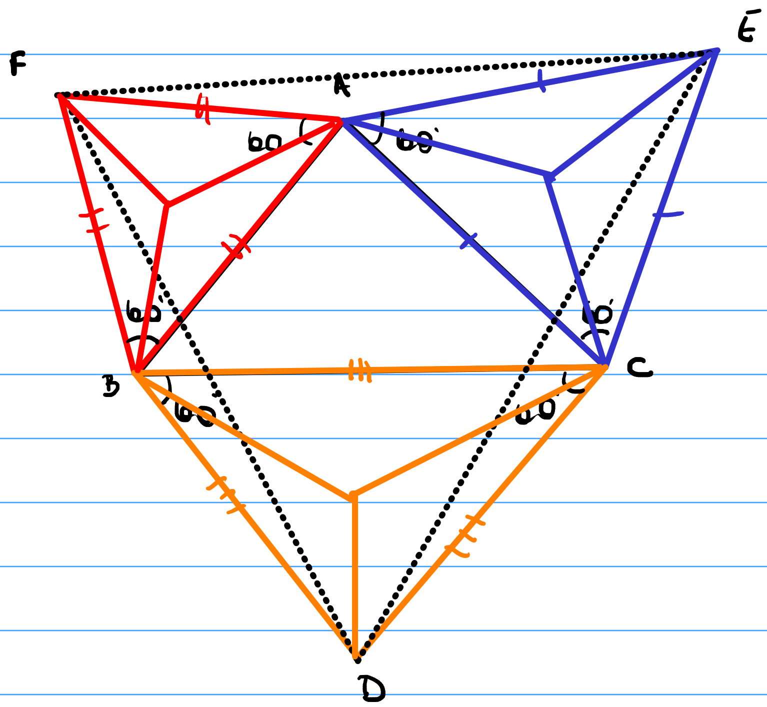


$$-\pi/6 + 2k\pi \leq x + \pi/3 \leq \pi/6 + 2k\pi$$

$$\boxed{\frac{-\pi}{2} \leq x \leq \frac{-\pi}{6}.}$$

$$\boxed{3\pi/2 \leq x \leq 11\pi/6.}$$ (+ suas reflexões)

160



Se BCD, ABF, CAE são Δ equiláteros prove que o ΔDEF também é equilátero.

Perceba que as soluções de $w^3 = 1$. formam um triângulo equilátero no plano complexo:

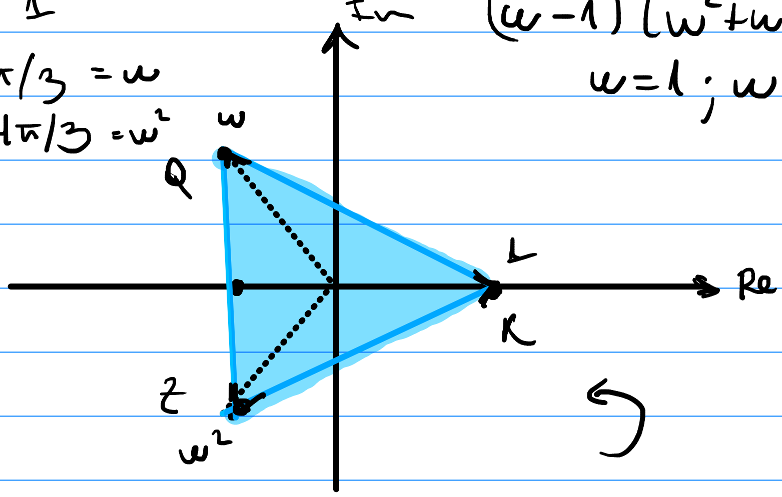
$w = 1 \quad = 1$

$w = cis\, 2\pi/3 = w$

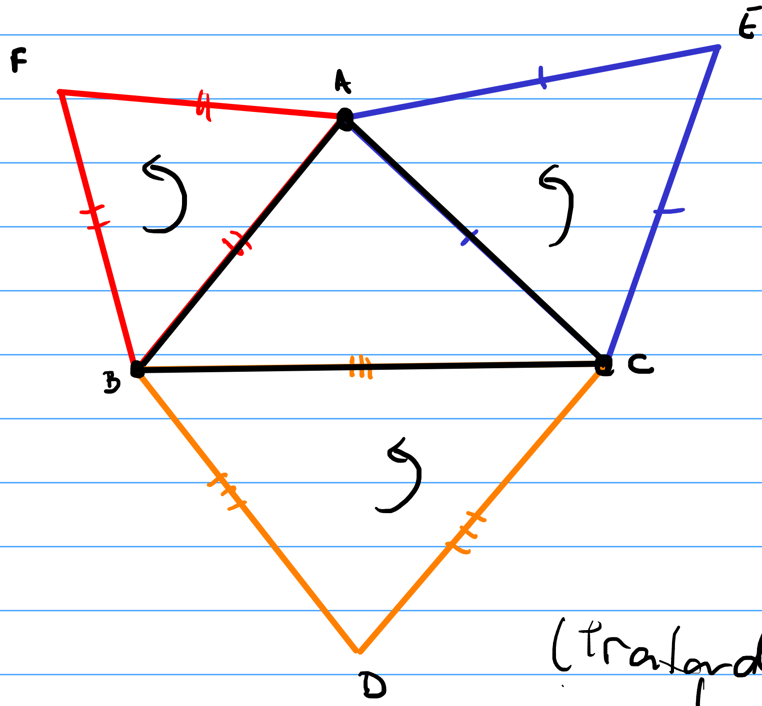
$w = cis\, 4\pi/3 = w^2$

$(w-1)(w^2+w+1) = 0$

$w = 1; \; w = -\frac{1}{2} + \frac{i\sqrt{3}}{2}; \; w = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$



$\therefore \; w^2 + w + 1 = 0 \quad e \quad K + Q w + Z w^2 = 0$



(tratando as arestas como vetores)

Como os triângulos ΔABF, ΔBCD, ΔACE são equiláteros:

$$F + Bw + Aw^2 = 0 \Rightarrow Fw + Bw^2 + A = Fw^2 + B + Aw = 0$$
$$E + Aw + Cw^2 = 0 \Rightarrow Ew + Aw^2 + C = Ew^2 + A + Cw = 0$$
$$D + Cw + Bw^2 = 0 \Rightarrow Dw + Cw^2 + B = Dw^2 + C + Bw = 0$$

Observando as equações chegamos que ΔDEF só será equilátero se o ΔABC for equilátero. Agora, se forem os baricentros do BCD, ABF e ACE, o triângulo será equilátero (esse resultado é conhecido como teorema de Napoleão).

161 $\sum_{k\geq 0}\binom{n}{3k}=\binom{n}{0}+\binom{n}{3}+\binom{n}{6}+\cdots+\binom{n}{3k}$.

Pensando nas raízes cúbicas da unidade:

$\omega^3=1$; $\omega^2+\omega+1=0$.

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}.b^k$$

$$(1+1)^n=\sum_{k=0}^{n}\binom{n}{k}=\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}$$

$$(1+\omega)^n=\sum_{k=0}^{n}\binom{n}{k}\omega^k=\binom{n}{0}+\binom{n}{1}\omega+\binom{n}{2}\omega^2+\binom{n}{3}\omega^3+\cdots+\binom{n}{n}\omega^n$$

$$(1+\omega^2)^n=\sum_{k=0}^{n}\binom{n}{k}\omega^{2k}=\binom{n}{0}+\binom{n}{1}\omega^2+\binom{n}{2}\omega^4+\binom{n}{3}\omega^6+\cdots+\binom{n}{n}\omega^{2n}$$

$$(1+1)^n+(1+\omega)^n+(1+\omega^2)^n=3\left[\binom{n}{0}+\binom{n}{3}+\cdots+\binom{n}{3k}\right]+\cancel{\binom{n}{1}\overset{0}{(1+\omega+\omega^2)}}+$$

$$\cancel{\binom{n}{2}\overset{0}{(1+\omega+\omega^2)}}+\cdots+\cancel{\binom{n}{3k-2}\overset{0}{(1+\omega+\omega^2)}}+\cancel{\binom{n}{3k-1}\overset{0}{(1+\omega+\omega^2)}}$$

$$(1+1)^n+(1+\omega)^n+(1+\omega^2)^n=3\sum_{k\geq 0}\binom{n}{3k}$$

$$\frac{2^n+(1+\omega)^n+(1+\omega^2)^n}{3} = \sum_{k\geq 0}\binom{n}{3k}.$$

$\omega = cis\, 2\pi/3$; $\omega^2 = cis\, 4\pi/3$.

$$\sum_{k\geq 0}\binom{n}{3k} = \frac{2^n+(1+cis\,2\pi/3)^n+(1+cis\,4\pi/3)^n}{3}$$

- $1+cis\,\theta = 2\cos\theta/2\; cis\,\theta/2$.

$$\sum_{k\geq 0}\binom{n}{3k} = \frac{2^n\left(1+(\cos\pi/3)^n(cis\,\pi/3)^n+(\cos 2\pi/3)^n(cis\,2\pi/3)^n\right)}{3}$$

$$= \frac{2^n}{3}\left(1+\frac{cis\,n\pi/3}{2^n}+\frac{cis\,2n\pi/3}{(-2)^n}\right)$$

$$= \boxed{\frac{2^n+cis\,n\pi/3+(-1)^n cis\,2n\pi/3}{3}}.$$

162 $z^n = 1$ $\Rightarrow |z| = 1$ $z = a + bi$

$(z+1)^n = 1.$ $|z+1| = 1$

I) $\therefore \sqrt{a^2+b^2} = 1$; $\sqrt{(a+1)^2+b^2} = 1$

$\cancel{a^2} + \cancel{b^2} = \cancel{a^2} + 2a + 1 + \cancel{b^2}$

$$\boxed{\begin{matrix} a = -1/2 \\ b = \pm\sqrt{3}/2 \end{matrix}} \to z = -\frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

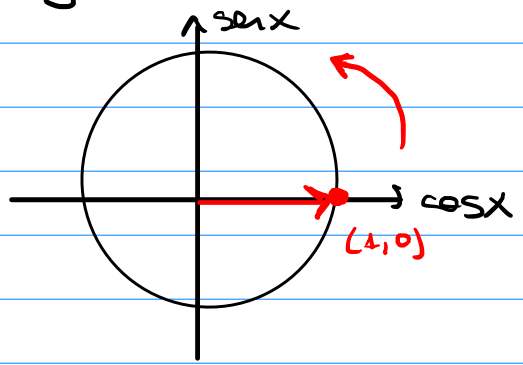
$$\boxed{z = cis^{2\pi/3} \text{ ou } cis^{4\pi/3} = z}$$ ///.

Agora, tanto $z = cis^{2\pi/3}$, quanto $z = cis^{4\pi/3}$, quando elevados a 3, resultam em 1. Logo $z^3 = 1$.

II) $z^n = 1 \Rightarrow (cis\, 2\pi/3)^n = 1$ ou $(cis\, {}^{4\pi}/3)^n = 1$

$(z+1)^n = 1 \Rightarrow (cis\, {}^{\pi}/3)^n = 1$ ou $(cis\, {}^{5\pi}/3)^n = 1$

n deve ser múltiplo de 6 por conta dos casos $(cis^{\pi/3})^n = 1$ e $(cis^{5\pi/3})^n = 1$, pois seria então na forma $cis(2K\pi)$ e isso é igual a 1.



163 $x^{999}+x^{888}+\dots+x^{111}+1$ é divisível por $x^9+x^8+\dots+x+1$

Fazendo as raízes décimas da unidade:

$w^{10}=1 \Rightarrow (w-1)(w^9+w^8+w^7+w^6+w^5+w^4+w^3+w^2+w+1)=0$

$x^{999}+x^{888}+\dots+x^{111}+1 \to w^{999}+w^{888}+\dots+w^{111}+1$

$(w^{10})^{99}\cdot w^9+(w^{10})^{88}\cdot w^8+\dots+(w^{10})^{11}\cdot w+1$

$=1\cdot w^9+1\cdot w^8+\dots+1\cdot w+1=w^9+w^8+\dots+w+1=0$

$$\boxed{\therefore\ x^9+x^8+\dots+x+1 \mid x^{999}+x^{888}+\dots+x^{111}+1}$$

164

105

166 α, β, γ raízes de $x^3 - x - 1 = 0$

$$\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma} \Rightarrow$$

$$\frac{(1+\alpha)(1-\beta)(1-\gamma) + (1+\beta)(1-\alpha)(1-\gamma) + (1+\gamma)(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta)(1-\gamma)}$$

$$= \frac{(1-\beta+\alpha-\alpha\beta)(1-\gamma) + (1-\alpha+\beta-\alpha\beta)(1-\gamma) + (1-\alpha+\gamma-\alpha\gamma)(1-\beta)}{(1-\beta-\alpha+\alpha\beta)(1-\gamma)}$$

$$= \frac{(\cancel{1}-\cancel{\beta}+\cancel{\alpha}-\cancel{\alpha\beta}-\cancel{\gamma}+\cancel{\beta\gamma}-\cancel{\alpha\gamma}+\cancel{\alpha\beta\gamma}) + (\cancel{1}-\cancel{\alpha}+\cancel{\beta}-\alpha\beta-\cancel{\gamma}+\cancel{\alpha\gamma}-\cancel{\beta\gamma}+\cancel{\alpha\beta\gamma}) + (\cancel{1}-\cancel{\alpha}+\cancel{\gamma}-\alpha\gamma-\cancel{\beta}+\cancel{\alpha\beta}-\cancel{\beta\gamma}+\cancel{\alpha\beta\gamma})}{(1-\beta-\alpha+\alpha\beta-\gamma+\beta\gamma+\alpha\gamma-\alpha\beta\gamma)}$$

$$= \frac{3+3\alpha\beta\gamma-(\alpha+\beta+\gamma)-(\alpha\beta+\alpha\gamma+\beta\gamma)}{1-(\alpha+\beta+\gamma)+(\alpha\beta+\alpha\gamma+\beta\gamma)-\alpha\beta\gamma}$$

Pelas relações de Girard:

$\alpha+\beta+\gamma = -\frac{b}{a} = 0$; $\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a} = -1$;

$\alpha\beta\gamma = -\frac{d}{a} = 1.$

$$\therefore \Rightarrow \frac{3+3.1-0-(-1)}{1-0+(-1)-1} = \frac{7}{-1} = \boxed{-7}$$

167) $p(x) = x^6 - x^5 - x^3 - x^2 - x$
$q(x) = x^4 - x^3 - x^2 - 1$ (raízes z_1, z_2, z_3, z_4)

$p(z_1) + p(z_2) + (pz_3) + p(z_4) = ?$

- $q(z_1) = z_1^4 - z_1^3 - z_1^2 - 1 = 0 \Rightarrow z_1^6 - z_1^5 - z_2^4 - z_2^2 = 0$
- $q(z_2) = z_2^4 - z_2^3 - z_2^2 - 1 = 0 \Rightarrow z_2^6 - z_2^5 - z_2^4 - z_2^2 = 0$
- $q(z_3) = z_3^4 - z_3^3 - z_3^2 - 1 = 0 \Rightarrow z_3^6 - z_3^5 - z_3^4 - z_3^2 = 0$
- $q(z_4) = z_4^4 - z_4^3 - z_4^2 - 1 = 0 \Rightarrow z_4^6 - z_4^5 - z_4^4 - z_4^2 = 0$

$\hookrightarrow z_1^6 - z_1^5 - z_1^3 - z_1^2 - 1 - z_1^2 = 0$

$$\boxed{\therefore\ z_1^6 - z_1^5 - z_1^3 - z_1^2 = z_1^2 + 1}$$

$p(z_1) + p(z_2) + p(z_3) + p(z_4) = z_1^2 + z_2^2 + z_3^2 + z_4^2 + 4 - (z_1 + z_2 + z_3 + z_4)$

$\rightarrow (a+b+c+d)^2 = (a+b+c+d)(a+b+c+d)$
$= a^2 + \cancel{ab} + \cancel{ac} + \cancel{ad} + \cancel{ab} + b^2 + \cancel{bc} + \cancel{bd} + \cancel{ac} + \cancel{bc} + c^2 + \cancel{cd} + \cancel{ad} + \cancel{bd} + \cancel{cd} + d^2$

$\therefore\ a^2 + b^2 + c^2 + d^2 = (a+b+c+d)^2 - 2(ab + ac + ad + bc + bd + cd)$

$\Rightarrow p(z_1) + p(z_2) + p(z_3) + p(z_4) = (z_1 + z_2 + z_3 + z_4)^2 - (z_1 + z_2 + z_3 + z_4) - 2(ab + ac + ad + bc + bd + cd) + 4$

Pelas relações de Girard:

$(1)^2 - 1 - 2(-1) + 4 = \boxed{6}$