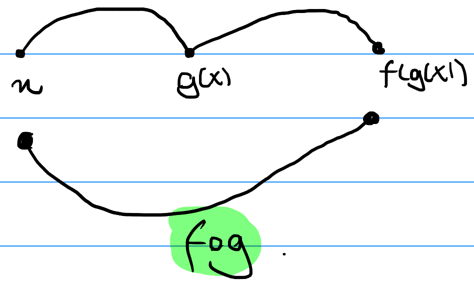
REGRA DA CADEIA:

Composição de funções: $f \circ g = f(g(x))$.

Condições: $\mathrm{Im}(g) \subset \mathrm{dom}(f)$.

⊛ Se f e g são contínuas, então $f(g(x))$ também é contínua.



Exemplos: $h(x) = \mathrm{sen}(x^2+1)$.

$f(x) = \mathrm{sen}\, x$

$g(x) = x^2+1 \Rightarrow f(g(x)) = \mathrm{sen}(x^2+1)$.

Diferenciabilidade:

Suponha g diferenciável em x_0 e f derivável em $y_0 = g(x_0)$.

- $h(x) = f(g(x))$.
- Existe $(h(x_0))'$?

$$h'(x_0) = \lim_{x \to x_0} \frac{h(x) - h(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{f(g(x)) - f(g_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \left[\frac{f(g(x)) - f(g_0)}{g(x) - g_0} \cdot \frac{g(x) - g_0}{x - x_0} \right] = \lim_{g(x) \to g_0} \frac{f(g(x)) - f(g_0)}{g(x) - g_0} \cdot$$

$$\lim_{g(x) \to g_0} \frac{f(g(x)) - f(g_0)}{g(x) - g_0} \cdot \lim_{x \to x_0} \frac{g(x) - g_0}{x - x_0} = \boxed{f'(g_0) \cdot g'(x_0)}$$

$$= f'(g_0) \cdot (g_0)'.$$

Escrevendo em termos gerais:

$$h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x)$$

Exercício: $h(x) = sen(x^2+1) \Rightarrow f(x) = sen\, x \wedge g(x) = x^2+1.$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$\boxed{h'(x) = \cos(x^2+1) \cdot 2x}$$

$h(x) = (x^3 + 3x^2 - 2)^{10}$ $\quad f(x) = x^{10}$; $g(x) = x^3 + 3x^2 - 2$

$$\boxed{h'(x) = 10(x^3 + 3x^2 - 2)^9 \cdot (3x^2 + 6x).}$$

$h(x) = arc\,tg(x^3 - x + 1)$. $\quad f(x) = arctg\, x$; $g(x) = x^3 - x + 1$

$$h'(x) = \frac{1}{1 + (x^3 - x + 1)^2} \cdot (3x^2 - 1) = \boxed{\frac{3x^2 - 1}{1 + (x^3 - x + 1)^2}}$$

$h(x) = |\operatorname{sen} x| \qquad f(x) = |x|, \quad g(x) = \operatorname{sen} x.$

$h'(x) = f'(g(x)) \cdot g'(x) = f'(\operatorname{sen} \pi/2) \cdot \cos \pi/2 = f'(0) \cdot 0$

$= 1 \cdot 0 = 0.$

$$\lim_{x \to a} \frac{|x| - |a|}{x - a} =$$

se $x > 0 \Rightarrow (|x|)' = 1.$

se $x < 0 \Rightarrow (|x|)' = -1$

Se $x = 0 \Rightarrow \nexists\ (|x|)'.$

como $f(g(\pi/2)) = 1$, então a derivada existe.

$f'(g(\pi/2)) = 1$

$g'(\pi/2) = \cos \pi/2 = 0.$

$h(x) = \sqrt[3]{\operatorname{sen}(x^2+1)} \qquad f(x) = \sqrt[3]{x} \qquad g(x) = \operatorname{sen}(x^2+1).$

$\frac{1}{3}(\operatorname{sen}(x^2+1))^{-2/3} \cdot (\operatorname{sen}(x^2+1))'$

$\frac{1}{3}(\operatorname{sen}(x^2+1))^{-2/3} \cdot \cos(x^2+1) \cdot 2x.$

$$\boxed{= \frac{2}{3} x \cdot \cos(x^2+1) \cdot [\operatorname{sen}(x^2+1)]^{-2/3}}$$

$h(x) = \sqrt{x^2 + \sqrt{3x^2+x}} = (x^2 + \sqrt{3x^2+x})^{1/2}$

$h'(x) = \frac{1}{2}(x^2 + \sqrt{3x^2+x})^{-1/2} (x^2 + \sqrt{3x^2+x})'$

$$\boxed{= \frac{1}{2}(x^2+\sqrt{3x^2+x})^{-1/2} \cdot \left(2x + \frac{1}{2}(3x^2+x)^{-1/2} \cdot (6x+1)\right)}$$

Derivada da Inversa pela Regra da Cadeia.

$$(f \circ f^{-1})(x) = x$$

$$f'(f^{-1}(x)) \cdot (f^{-1}(x))' = 1.$$

$$\boxed{(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}}$$

$f(x) = x^3 + 4x + 6$. Calcule $(f^{-1})'(6)$.

$$(f^{-1})'(6) = \frac{1}{f'(f^{-1}(6))} = \frac{1}{f'(0)} = \frac{1}{3 \cdot 0^2 + 4} = \boxed{\frac{1}{4}}$$

$$\boxed{\begin{aligned} f(x) = x^3 + 4x + \cancel{6} &= \cancel{6} \\ x(x^2+4) &= 0 \\ x &= 0. \end{aligned}}$$

$f^{-1}(a) = b$

$\therefore f(b) = a$

$$y = mx + p$$

$$0 = \frac{1}{\cancel{4}\,2} \cdot \cancel{6}^{3} + p \qquad p = -3/2$$

$$\boxed{r: y = \frac{1}{4}x - \frac{3}{2}}$$