Digital Image Processing Laboratory

2-D Random Processes

Alexandre Olivé Pellicer

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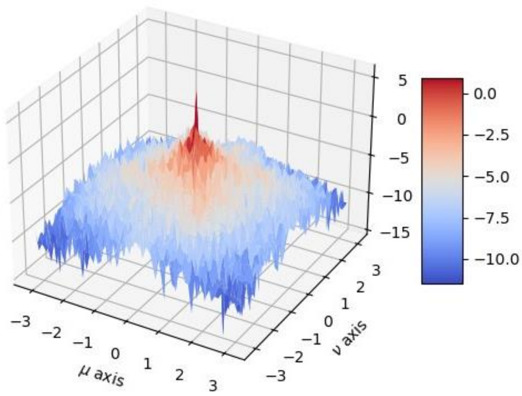
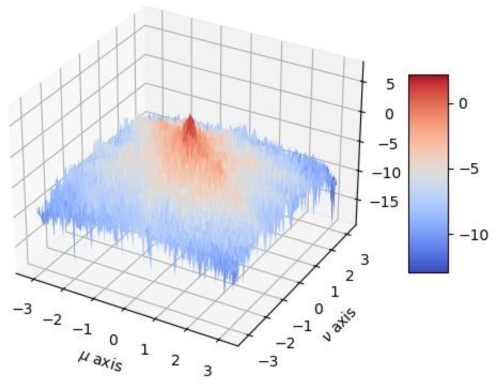
1. Power Spectral Density of an Image

1.1. The gray scale image *img04g.tif*

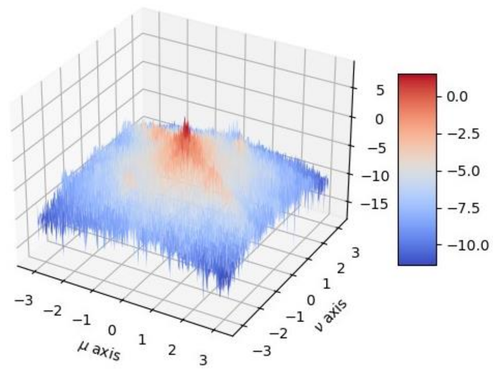
1.2. The power spectral density plots for block sizes of 64 x 64, 128 x 128, and 256 x 256.



Energy Spectrum over a 64 x 64 window of the image

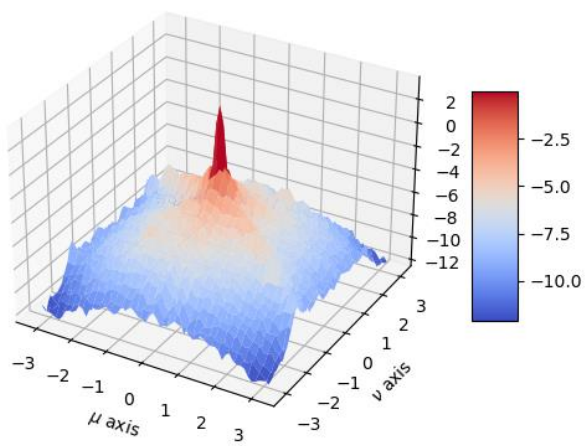


Energy Spectrum over a 128 x 128 window of the image



Energy Spectrum over a 256 x 256 window of the image

1.3. The improved power spectral density estimate

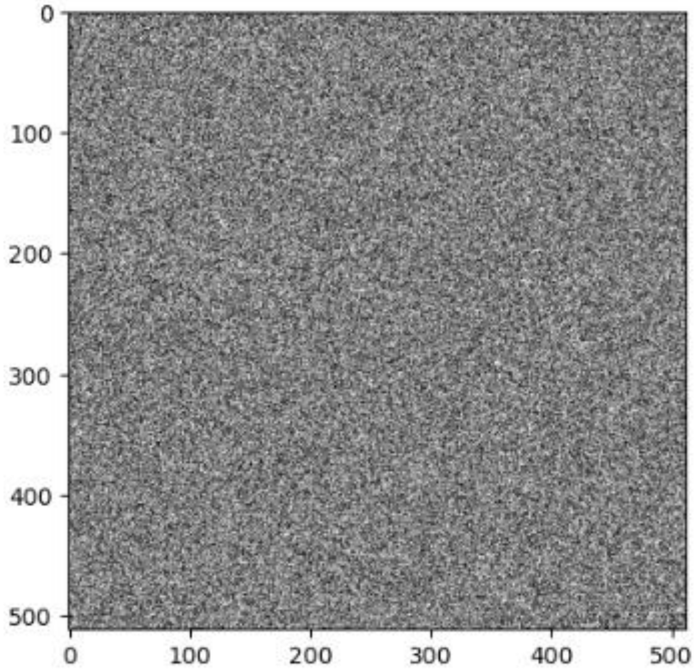


1.4. Your code for *BetterSpecAnal(x)* function.

```
def BetterSpecAnal(x):
    # We apply a square of 5x5 non-overlapping filters. The center of the centered filter is in the center of the image
    N = 64 #window size

    #Getting the coordinates of the pixel where the first filter will be applied
    height, width = x.shape
    i = int(height/2 - 5*N/2)
    j = int(width/2 - 5*N/2)

    W = np.outer(np.hamming(64), np.hamming(64))

    Z = np.zeros((N,N))

    for _ in range(5):
        for _ in range(5):
            z = x[i:N+i, j:N+j] * W
            # Compute the power spectrum for the NxN region.
            Z = Z + (1/N**2)*np.abs(np.fft.fft2(z))**2
            j = j + N

        i = i + N
        j = int(width/2 - 5*N/2) #reset j

    # Use fftshift to move the zero frequencies to the center of the plot.
    Z = np.fft.fftshift(Z)

    # Compute the average between the 25 filters
    Zabs_avg = Z / 25

    # Compute the logarithm of the Power Spectrum.
    Zabs_avg_log = np.log(Zabs_avg)

    return Zabs_avg_log
```

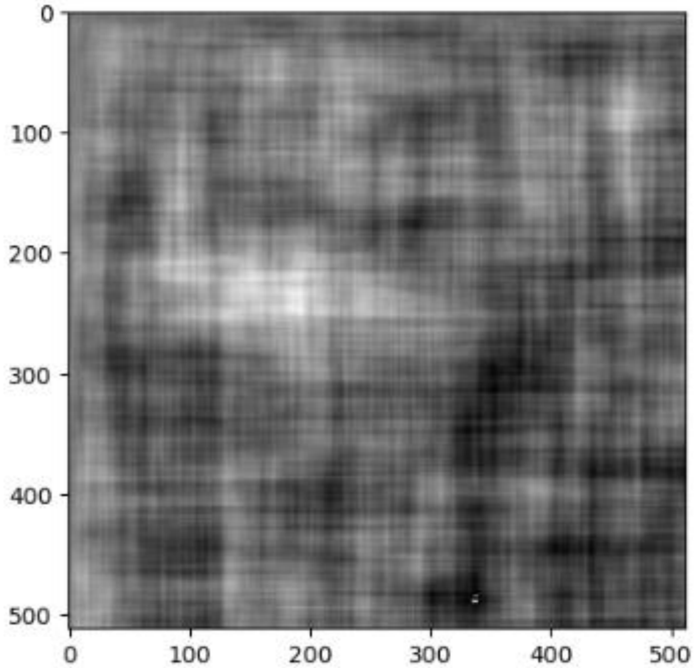
2. Power Spectral Density of a 2-D AR Process

2.1. The image 255 * (x + 0.5)



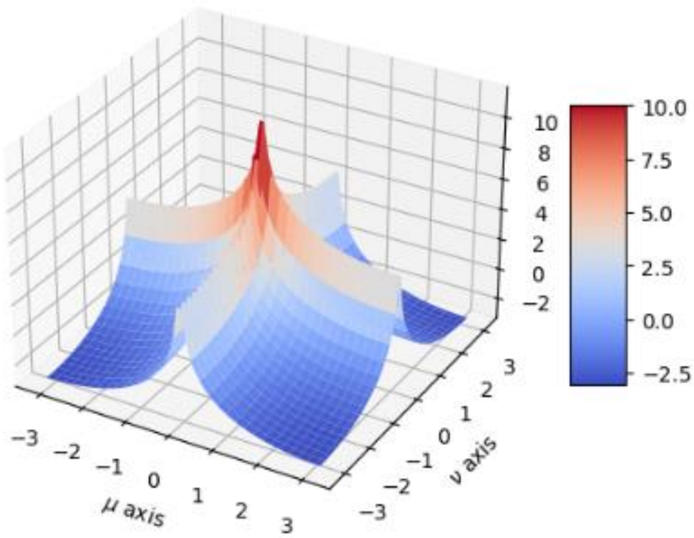

```
x = np.random.uniform(low = -0.5, high = 0.5, size = (512,512))
x_scaled = 255 * (x + 0.5)
x_scaled = x_scaled.astype(np.uint8)
plt.imshow(x_scaled, cmap=plt.cm.gray)
```

2.2. The image y + 127



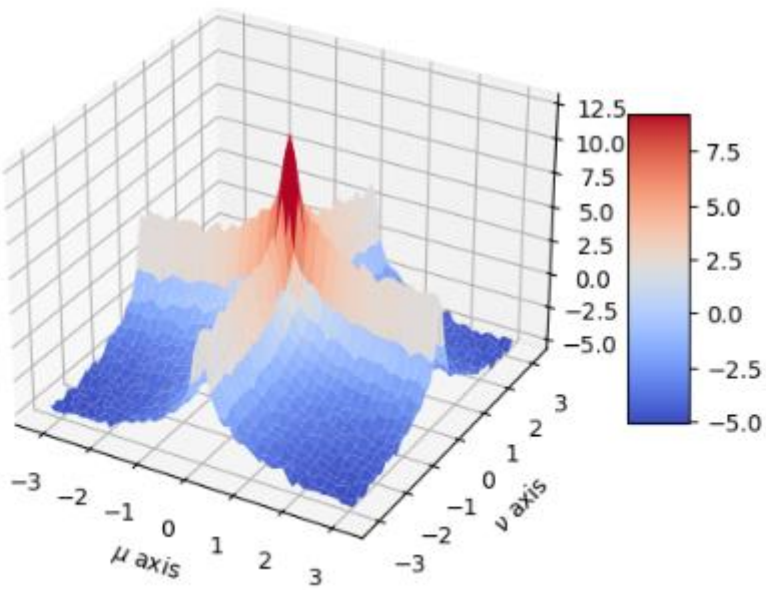
```
y = np.zeros((512,512))
height, width = x_scaled.shape
for i in range(height):
    for j in range(width):
        if (i == 0 and j == 0):
            y[i,j] = 3*x[i,j]
        if (i == 0 and j > 0):
            y[i,j] = 3*x[i,j] + 0.99*y[i, j-1]
        if (i > 0 and j == 0):
            y[i,j] = 3*x[i,j] + 0.99*y[i-1, j]
        if (i > 0 and j > 0):
            y[i,j] = 3*x[i,j] + (0.99*y[i-1, j]) + (0.99*y[i, j-1]) - (0.9801 * y[i-1,j-1])

plt.imshow((y + 127).astype(np.uint8), cmap=plt.cm.gray)
```

2.3. A mesh plot of the function $\log S_Y(e^{j\mu}, e^{jv})$




```
import matplotlib.pyplot as plt

N = 64

a = b = np.linspace(-np.pi, np.pi, num = N)
X, Y = np.meshgrid(a, b)

density = (np.abs((3 / (1 - 0.99 * np.exp(-1j * X) - 0.99* np.exp(-1j * Y) + 0.9801*np.exp(-1j * (X + Y))))))**2 * (1/12)
density_log = np.log(density)

fig = plt.figure()
ax = plt.axes(projection='3d')
surf = ax.plot_surface(X, Y, density_log, cmap=plt.cm.coolwarm)
ax.set_xlabel('$\mu$ axis')
ax.set_ylabel('$\\nu$ axis')

fig.colorbar(surf, shrink=0.5, aspect=5)
```

2.4. A mesh plot of the log of the estimated power spectral density of y using *BetterSpecAnal(y)*




```python
def BetterSpecAnal(x):
    # We apply a square of 5x5 non-overlapping filters. The center of the centered filter is in the center of the image
    N = 64 #window size

    #Getting the coordinates of the pixel where the first filter will be applied
    height, width = x.shape
    i = int(height/2 - 5*N/2)
    j = int(width/2 - 5*N/2)

    W = np.outer(np.hamming(64), np.hamming(64))

    Z = np.zeros((N,N))

    for _ in range(5):
        for _ in range(5):
            z = x[i:N+i, j:N+j] * W
            # Compute the power spectrum for the NxN region.
            Z = Z + (1/N**2)*np.abs(np.fft.fft2(z))**2
            j = j + N

        i = i + N
        j = int(width/2 - 5*N/2) #reset j

    # Use fftshift to move the zero frequencies to the center of the plot.
    Z = np.fft.fftshift(Z)

    # Compute the average between the 25 filters
    Zabs_avg = Z / 25

    # Compute the logarithm of the Power Spectrum.
    Zabs_avg_log = np.log(Zabs_avg)

    return Zabs_avg_log

Zabs = BetterSpecAnal(y)

N= 64
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
a = b = np.linspace(-np.pi, np.pi, num = N)
X, Y = np.meshgrid(a, b)

surf = ax.plot_surface(X, Y, Zabs, cmap=plt.cm.coolwarm)

ax.set_xlabel('$\mu$ axis')
ax.set_ylabel('$\\nu$ axis')
ax.set_zlabel('Z Label')

fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()
```