

Introduction to Computational Chemistry

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- **Computational Chemistry** is a branch of chemistry that uses computer science to assist in solving chemical problems.
- Incorporates the results of theoretical chemistry into efficient computer programs.
- Application to single molecule, groups of molecules, liquids or solids.
- Calculates the structure and properties of interest.
- Computational Chemistry Methods range from
 - 1 Highly accurate (*Ab-initio*, DFT) feasible for small systems
 - 2 Less accurate (semi-empirical)
 - 3 Very Approximate (Molecular Mechanics) large systems



Theoretical Chemistry: broadly can be divided into two main categories

1 Static Methods \Rightarrow Time-Independent Schrödinger Equation

- ◆ Quantum Chemical/*Ab Initio* /Electronic Structure Methods
- ◆ Molecular Mechanics

2 Dynamical Methods \Rightarrow Time-Dependent Schrödinger Equation

- ◆ Classical Molecular Dynamics
- ◆ Semi-classical and *Ab-Initio* Molecular Dynamics



- *Ab Initio* meaning "from first principles" methods solve the Schrödinger equation and does not rely on empirical or experimental data.
- Beginning with fundamental and physical properties, calculate how electrons and nuclei interact.
- The Schrödinger equation can be solved exactly only for a few systems
 - ◆ Particle in a Box
 - ◆ Rigid Rotor
 - ◆ Harmonic Oscillator
 - ◆ Hydrogen Atom
- For complex systems, *Ab Initio* methods make assumptions to obtain approximate solutions to the Schrödinger equations and solve it numerically.
- "Computational Cost" of calculations increases with the accuracy of the calculation and size of the system.



What can we predict with *Ab Initio* methods?

- Molecular Geometry: Equilibrium and Transition State
- Dipole and Quadrupole Moments and polarizabilities
- Thermochemical data like Free Energy, Energy of reaction.
- Potential Energy surfaces, Barrier heights
- Reaction Rates and cross sections
- Ionization potentials (photoelectron and X-ray spectra) and Electron affinities
- Frank-Condon factors (transition probabilities, vibronic intensities)
- Vibrational Frequencies, IR and Raman Spectra and Intensities
- Rotational spectra
- NMR Spectra
- Electronic excitations and UV-VIS spectra
- Electron density maps and population analyses
- Thermodynamic quantities like partition function



Ab Initio Theory

- **Born-Oppenheimer Approximation:** Nuclei are heavier than electrons and can be considered stationary with respect to electrons. Also known as "clamped nuclei" approximations and leads to idea of potential surface
- **Slater Determinants:** Expand the many electron wave function in terms of Slater determinants.
- **Basis Sets:** Represent Slater determinants by molecular orbitals, which are linear combination of atomic-like-orbital functions i.e. basis sets



Born-Oppenheimer Approximation

- Solve time-independent Schrödinger equation

$$\hat{H}\Psi = E\Psi$$

- For many electron system:

$$\hat{H} = \underbrace{-\frac{\hbar^2}{2} \sum_{\alpha} \frac{\nabla_{\alpha}^2}{M_{\alpha}}}_{\hat{T}_n} - \underbrace{\frac{\hbar^2}{2m_e} \sum_i \nabla_i^2}_{\hat{T}_e} + \underbrace{\sum_{\alpha > \beta} \frac{e^2 Z_{\alpha} Z_{\beta}}{4\pi\epsilon_0 R_{\alpha\beta}}}_{\hat{V}_{nn}} - \underbrace{\sum_{\alpha, i} \frac{e^2 Z_{\alpha}}{4\pi\epsilon_0 R_{\alpha i}}}_{\hat{V}_{en}} + \underbrace{\sum_{i > j} \frac{e^2}{4\pi\epsilon_0 r_{ij}}}_{\hat{V}_{ee}}$$

$\underbrace{\hspace{15em}}_{\hat{V}}$

- The wave function $\Psi(R, r)$ of the many electron molecule is a function of nuclear (R) and electronic (r) coordinates.
- Motion of nuclei and electrons are coupled.
- However, since nuclei are much heavier than electrons, the nuclei appear fixed or stationary.



- Born-Oppenheimer Approximation: Separate electronic and nuclear motion:

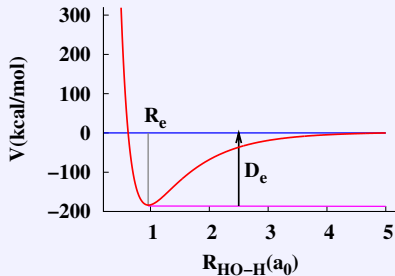
$$\Psi(R, r) = \psi_e(r; R)\psi_n(R)$$

- Solve electronic part of Schrödinger equation

$$\hat{H}_e\psi_e(r; R) = E_e\psi_e(r; R)$$

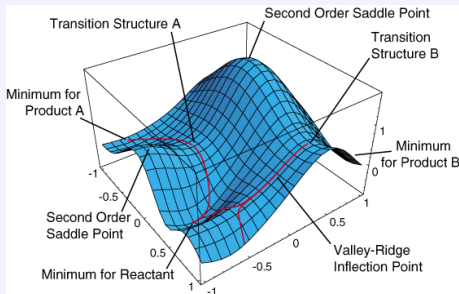
- BO approximation leads to the concept of potential energy surface

$$V(R) = E_e + V_{nn}$$



Potential Energy Surfaces

- The potential energy surface (PES) is multi-dimensional ($3N - 6$ for non-linear molecule and $3N - 5$ for linear molecule)
- The PES contains multiple minima and maxima.
- Geometry optimization search aims to find the global minimum of the potential surface.
- Transition state or saddle point search aims to find the maximum of this potential surface, usually along the reaction coordinate of interest.



Picture taken from Bernard Schlegel's course slide at <http://www.chem.wayne.edu/~hbs/chm6440/>



- The electronic Hamiltonian (in atomic units, $\hbar, m_e, 4\pi\epsilon_0, e = 1$) to be solved is

$$\hat{H}_e = -\frac{1}{2} \sum_i \nabla_i^2 - \sum_{\alpha,i} \frac{Z_\alpha}{R_{i\alpha}} + \sum_{i>j} \frac{1}{r_{ij}} + \sum_{\alpha>\beta} \frac{Z_\alpha Z_\beta}{R_{\alpha\beta}}$$

- Calculate electronic wave function and energy

$$E_e = \frac{\langle \psi_e | \hat{H}_e | \psi_e \rangle}{\langle \psi_e | \psi_e \rangle}$$

- The total electronic wave function is written as a Slater Determinant of the one electron functions, i.e. molecular orbitals, MO's

$$\psi_e = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_1(1) & \phi_2(1) & \cdots & \phi_N(1) \\ \phi_1(2) & \phi_2(2) & \cdots & \phi_N(2) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_1(N) & \phi_2(N) & \cdots & \phi_N(N) \end{vmatrix}$$



- MO's are written as a linear combination of one electron atomic functions or atomic orbitals (AO's)

$$\phi_i = \sum_{\mu=1}^N c_{\mu i} \chi_{\mu}$$

$c_{\mu i} \Rightarrow$ MO coefficients

$\chi_{\mu} \Rightarrow$ atomic basis functions.

- Obtain coefficients by minimizing the energy via Variational Theorem.
- Variational Theorem: Expectation value of the energy of a trial wavefunction is always greater than or equal to the true energy

$$E_e = \langle \psi_e | \hat{H}_e | \psi_e \rangle \geq \varepsilon_0$$

- Increasing $N \Rightarrow$ Higher quality of wavefunction \Rightarrow Higher computational cost



The most popular classes of ab initio electronic structure methods:

- Hartree-Fock methods
 - ◆ Hartree-Fock (HF)
 - Restricted Hartree-Fock (RHF): singlets
 - Unrestricted Hartree-Fock (UHF): higher multiplicities
 - Restricted open-shell Hartree-Fock (ROHF)
- Post Hartree-Fock methods
 - ◆ Møller-Plesset perturbation theory (MPn)
 - ◆ Configuration interaction (CI)
 - ◆ Coupled cluster (CC)
- Multi-reference methods
 - ◆ Multi-configurational self-consistent field (MCSCF)
 - ◆ Multi-reference configuration interaction (MRCI)
 - ◆ n-electron valence state perturbation theory (NEVPT)
 - ◆ Complete active space perturbation theory (CASPTn)



- 1 Wavefunction is written as a single determinant

$$\Psi = \det(\phi_1, \phi_2, \dots \phi_N)$$

- 2 The electronic Hamiltonian can be written as

$$\hat{H} = \sum_i h(i) + \sum_{i>j} v(i,j)$$

where $h(i) = -\frac{1}{2}\nabla_i^2 - \sum_{\alpha} \frac{Z_{\alpha}}{r_{i\alpha}}$ and $v(i,j) = \frac{1}{r_{ij}}$

- 3 The electronic energy of the system is given by:

$$E = \langle \Psi | \hat{H} | \Psi \rangle$$

- 4 The resulting HF equations from minimization of energy by applying of variational theorem:

$$\hat{f}(x_1)\chi_i(x_1) = \varepsilon_i\chi_i(x_1)$$

where ε_i is the energy of orbital χ_i and the Fock operator f , is defined as

$$\hat{f}(x_1) = \hat{h}(x_1) + \sum_j [\hat{J}_j(x_1) - \hat{K}_j(x_1)]$$



- 1 $\hat{J}_j \Rightarrow$ Coulomb operator \Rightarrow average potential at x due to charge distribution from electron in orbital χ_i defined as

$$\hat{J}_j(x_1)\chi_i(x_1) = \left[\int \frac{\chi_j^*(x_2)\chi_j(x_2)}{r_{12}} dx_2 \right] \chi_i(x_1)$$

- 2 $\hat{K}_j \Rightarrow$ Exchange operator \Rightarrow Energy associated with exchange of electrons \Rightarrow No classical interpretation for this term.

$$\hat{K}_j(x_1)\chi_i(x_1) = \left[\int \frac{\chi_j^*(x_2)\chi_i(x_2)}{r_{12}} dx_2 \right] \chi_j(x_1)$$

- 3 The Hartree-Fock equations are solved numerically or in a space spanned by a set of basis functions (Hartree-Fock-Roothan equations)

$$\begin{aligned}\chi_i &= \sum_{\mu=1}^K C_{\mu i} \tilde{\chi}_{\mu} & S_{\mu\nu} &= \int dx_1 \tilde{\chi}_{\mu}^*(x_1) \tilde{\chi}_{\nu}(x_1) \\ \sum_{\nu} F_{\mu\nu} C_{\nu i} &= \varepsilon_i \sum_{\nu} S_{\mu\nu} C_{\nu i} & F_{\mu\nu} &= \int dx_1 \tilde{\chi}_{\mu}^*(x_1) \hat{f}(x_1) \tilde{\chi}_{\nu}(x_1) \\ \mathbf{FC} &= \mathbf{SC}\varepsilon\end{aligned}$$



- 1 The Hartree-Fock-Roothan equation is a pseudo-eigenvalue equation
- 2 C 's are the expansion coefficients for each orbital expressed as a linear combination of the basis function.
- 3 Note: C depends on F which depends on $C \Rightarrow$ need to solve self-consistently.
- 4 Starting with an initial guess orbitals, the HF equations are solved iteratively or self consistently (Hence HF procedure is also known as self-consistent field or SCF approach) obtaining the best possible orbitals that minimize the energy.

SCF procedure

- 1 Specify molecule, basis functions and electronic state of interest
- 2 Form overlap matrix S
- 3 Guess initial MO coefficients C
- 4 Form Fock Matrix F
- 5 Solve $FC = SC\epsilon$
- 6 Use new MO coefficients C to build new Fock Matrix F
- 7 Repeat steps 5 and 6 until C no longer changes from one iteration to the next.



What are Post Hartree-Fock Methods

- 1 In Hartree-Fock theory, electron motions are independent of each other i.e. uncorrelated.
- 2 However, this is not true. For two electrons with same spin $|\Psi_1(r_1)\alpha(\omega_1)\Psi_2(r_2)\alpha(\omega_2)\rangle$, the probability of finding electron 1 at r_1 and electron 2 at r_2

$$P(r_1, r_2)dr_1dr_2 = \frac{1}{2} \left(|\Psi_1(r_1)|^2 |\Psi_2(r_2)|^2 + |\Psi_1(r_2)|^2 |\Psi_2(r_1)|^2 - [\Psi_1^*(r_1)\Psi_2(r_1)\Psi_2^*(r_2)\Psi_1(r_2) + \Psi_2^*(r_1)\Psi_1(r_1)\Psi_1^*(r_2)\Psi_2(r_2)] \right) dr_1dr_2$$

Now $P(r_1, r_1) = 0 \Rightarrow$ No two electrons with same spins can be at the same place \Rightarrow "Fermi hole"

- 3 Same-spin electrons are correlated while different spin electrons are not.
- 4 Energy difference between HF energy and the true energy is the correlation energy

$$E_{corr} = E_0 - E_{HF}$$



- ◆ Methods that improve the Hartree-Fock results by accounting for the correlation energy are known as **Post Hartree-Fock methods**
- ◆ The starting point for most Post HF methods is the Slater Determinant obtained from Hartree-Fock Methods.
- ◆ **Configuration Interaction (CI) methods:** Express the wavefunction as a linear combination of Slater Determinants with the coefficients obtained variationally

$$|\Psi\rangle = \sum_I c_I |\Psi_I\rangle$$

- ◆ **Many Body Perturbation Theory:** Treat the HF determinant as the zeroth order solution with the correlation energy as a perturbation to the HF equation.

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \lambda \hat{H}'; \varepsilon_i = E_i^{(0)} + \lambda E_i^{(1)} + \lambda^2 E_i^{(2)} + \dots \\ |\Psi_i\rangle &= |\Psi_i^{(0)}\rangle + \lambda |\Psi_i^{(1)}\rangle + \lambda^2 |\Psi_i^{(2)}\rangle + \dots\end{aligned}$$

- ◆ **Coupled Cluster Theory:** The wavefunction is written as an exponential ansatz

$$|\Psi\rangle = e^{\hat{T}} |\Psi_0\rangle$$

where $|\Psi_0\rangle$ is a Slater determinant obtained from HF calculations and \hat{T} is an excitation operator which when acting on $|\Psi_0\rangle$ produces a linear combination of excited Slater determinants.

Scaling Behavior	Method(s)
N^4	HF
N^5	MP2
N^6	MP3, CISD, CCSD, QCISD
N^7	MP4, CCSD(T), QCISD(T)
N^8	MP5, CISDT, CCSDT
N^9	MP6
N^{10}	MP7, CISDTQ, CCSDTQ

- N = Number of Basis Functions



- Density Functional Theory (DFT) is an alternative to wavefunction based electronic structure methods of many-body systems such as Hartree-Fock and Post Hartree-Fock.
- In DFT, the ground state energy is expressed in terms of the total electron density.

$$\rho_0(r) = \langle \Psi_0 | \hat{\rho} | \Psi_0 \rangle$$

- We again start with Born-Oppenheimer approximation and write the electronic Hamiltonian as

$$\hat{H} = \hat{F} + \hat{V}_{ext}$$

where \hat{F} is the sum of the kinetic energy of electrons and the electron-electron interaction and \hat{V}_{ext} is some external potential.



- Modern DFT methods result from the Hohenberg-Kohn theorem

- 1 The external potential V_{ext} , and hence total energy is a unique functional of the electron density $\rho(r)$

$$\text{Energy} = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \equiv E[\rho]$$

- 2 The ground state energy can be obtained variationally, the density that minimizes the total energy is the exact ground state density

$$E[\rho] > E[\rho_0], \text{ if } \rho \neq \rho_0$$

- If density is known, then the total energy is:

$$E[\rho] = T[\rho] + V_{ne}[\rho] + J[\rho] + E_{nn} + E_{xc}[\rho]$$

where

$$E_{nn}[\rho] = \sum_{A>B} \frac{Z_A Z_B}{R_{AB}}$$

$$V_{ne}[\rho] = \int \rho(r) V_{ext}(r) dr$$

$$J[\rho] = \frac{1}{2} \int \frac{\rho(r_1) \rho(r_2)}{r_{12}} dr_1 dr_2$$



- If the density is known, the two unknowns in the energy expression are the kinetic energy functional $T[\rho]$ and the exchange-correlation functional $E_{xc}[\rho]$
- To calculate $T[\rho]$, Kohn and Sham introduced the concept of Kohn-Sham orbitals which are eigenvectors of the Kohn-Sham equation

$$\left(-\frac{1}{2}\nabla^2 + v_{\text{eff}}(r)\right) \phi_i(r) = \varepsilon_i \phi_i(r)$$

Here, ε_i is the orbital energy of the corresponding Kohn-Sham orbital, ϕ_i , and the density for an "N"-particle system is

$$\rho(r) = \sum_i^N |\phi_i(r)|^2$$

- The total energy of a system is

$$E[\rho] = T_s[\rho] + \int dr v_{\text{ext}}(r)\rho(r) + V_H[\rho] + E_{xc}[\rho]$$



- T_s is the Kohn-Sham kinetic energy which is expressed in terms of the Kohn-Sham orbitals as

$$T_s[\rho] = \sum_{i=1}^N \int dr \phi_i^*(r) \left(-\frac{1}{2} \nabla^2 \right) \phi_i(r)$$

v_{ext} is the external potential acting on the interacting system (at minimum, for a molecular system, the electron-nuclei interaction), V_H is the Hartree (or Coulomb) energy,

$$V_H = \frac{1}{2} \int dr dr' \frac{\rho(r)\rho(r')}{|r - r'|}$$

and E_{xc} is the exchange-correlation energy.

- The Kohn-Sham equations are found by varying the total energy expression with respect to a set of orbitals to yield the Kohn-Sham potential as

$$v_{\text{eff}}(r) = v_{\text{ext}}(r) + \int \frac{\rho(r')}{|r - r'|} dr' + \frac{\delta E_{xc}[\rho]}{\delta \rho(r)}$$

where the last term $v_{xc}(r) \equiv \frac{\delta E_{xc}[\rho]}{\delta \rho(r)}$ is the exchange-correlation potential.



- The exchange-correlation potential, and the corresponding energy expression, are the only unknowns in the Kohn-Sham approach to density functional theory.
- There are many ways to approximate this functional E_{xc} , generally divided into two separate terms

$$E_{xc}[\rho] = E_x[\rho] + E_c[\rho]$$

where the first term is the exchange functional while the second term is the correlation functional.

- Quite a few research groups have developed the exchange and correlation functionals which are fit to empirical data or data from explicitly correlated methods.
- Popular DFT functionals (according to a recent poll)
 - ◆ PBE0 (PBEPBE), B3LYP, PBE, BP86, M06-2X, B2PLYP, B3PW91, B97-D, M06-L, CAM-B3LYP
 - <http://www.marcelswart.eu/dft-poll/index.html>
 - <http://www.ccl.net/cgi-bin/ccl/message-new?2011+02+16+009>

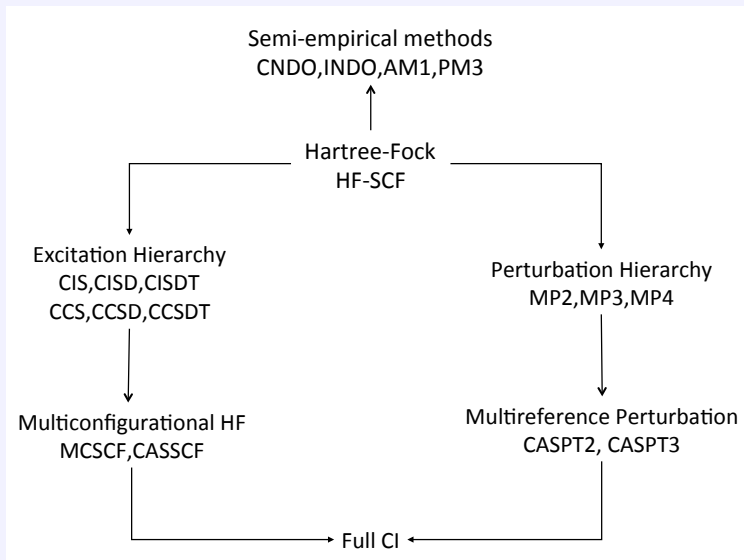


- Semi-empirical quantum methods:
 - ◆ Represents a middle road between the mostly qualitative results from molecular mechanics and the highly computationally demanding quantitative results from *ab initio* methods.
 - ◆ Address limitations of the Hartree-Fock calculations, such as speed and low accuracy, by omitting or parametrizing certain integrals
- integrals are either determined directly from experimental data or calculated from analytical formula with *ab initio* methods or from suitable parametric expressions.
- Integral approximations:
 - ◆ Complete Neglect of Differential Overlap (CNDO)
 - ◆ Intermediate Neglect of Differential Overlap (INDO)
 - ◆ Neglect of Diatomic Differential Overlap (NDDO) (Used by PM3, AM1, ...)

Semi-empirical methods are fast, very accurate when applied to molecules that are similar to those used for parametrization and are applicable to very large molecular systems.



Heirarchy of Methods

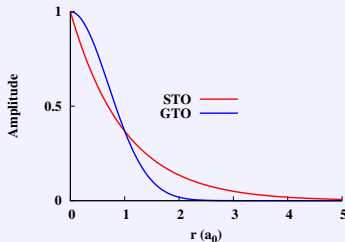


- Slater type orbital (STO) or Gaussian type orbital (GTO) to describe the AO's

$$\chi^{\text{STO}}(r) = x^l y^m z^n e^{-\zeta r}$$

$$\chi^{\text{GTO}}(r) = x^l y^m z^n e^{-\xi r^2}$$

where $L = l + m + n$ is the total angular momentum and ζ, ξ are orbital exponents.



Why STO

- Correct cusp at $r \rightarrow 0$
- Desired decay at $r \rightarrow \infty$
- Correctly mimics H orbitals
- Natural Choice for orbitals
- Computationally expensive to compute integrals and derivatives.

Why GTO

- Wrong behavior at $r \rightarrow 0$ and $r \rightarrow \infty$
- Gaussian \times Gaussian = Gaussian
- Analytical solutions for most integrals and derivatives.
- Computationally less expensive than STO's



Pople family basis set

- 1 Minimal Basis: STO-nG
 - ◆ Each atom optimized STO is fit with n GTO's
 - ◆ Minimum number of AO's needed
- 2 Split Valence Basis: 3-21G, 4-31G, 6-31G
 - ◆ Contracted GTO's optimized per atom.
 - ◆ Valence AO's represented by 2 contracted GTO's
- 3 Polarization: Add AO's with higher angular momentum (L)
 - ◆ 3-21G* or 3-21G(d), 6-31G* or 6-31G(d), 6-31G** or 6-31G(d,p)
- 4 Diffuse function: Add AO with very small exponents for systems with diffuse electron densities
 - ◆ 6-31+G*, 6-311++G(d,p)



Correlation consistent basis set

- ◆ Family of basis sets of increasing sizes.
- ◆ Can be used to extrapolate basis set limit.
- ◆ cc-pVDZ: Double Zeta(DZ) with d's on heavy atoms, p's on H
- ◆ cc-pVTZ: triple split valence with 2 sets of d's and 1 set of f's on heavy atom, 2 sets of p's and 1 set of d's on H
- ◆ cc-pVQZ, cc-pV5Z, cc-pV6Z
- ◆ can be augmented with diffuse functions: aug-cc-pVXZ (X=D,T,Q,5,6)



Pseudopotentials or Effective Core Potentials

- ◆ All Electron calculations are prohibitively expensive.
- ◆ Only valence electrons take part in bonding interaction leaving core electrons unaffected.
- ◆ Effective Core Potentials (ECP) a.k.a Pseudopotentials describe interactions between the core and valence electrons.
- ◆ Only valence electrons explicitly described using basis sets.
- ◆ Pseudopotentials commonly used
 - Los Alamos National Laboratory: LanL1MB and LanL2DZ
 - Stuttgart Dresden Pseudopotentials: SDDAll can be used.
 - Stevens/Basch/Krauss ECP's: CEP-4G, CEP-31G, CEP-121G
- ◆ Pseudopotential basis are "ALWAYS" read in pairs
 - Basis set for valence electrons
 - Parameters for core electrons

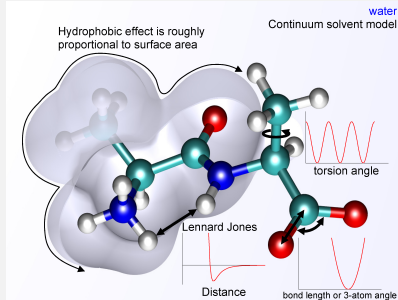


- The potential energy of all systems in molecular mechanics is calculated using force fields.
- Molecular mechanics can be used to study small molecules as well as large biological systems or material assemblies with many thousands to millions of atoms.
- All-atomistic molecular mechanics methods have the following properties:
 - ◆ Each atom is simulated as a single particle
 - ◆ Each particle is assigned a radius (typically the van der Waals radius), polarizability, and a constant net charge (generally derived from quantum calculations and/or experiment)
 - ◆ Bonded interactions are treated as "springs" with an equilibrium distance equal to the experimental or calculated bond length
- The exact functional form of the potential function, or force field, depends on the particular simulation program being used.



General form of Molecular Mechanics equations

$$\begin{aligned}
 E &= E_{\text{bond}} + E_{\text{angle}} + E_{\text{torsion}} + E_{\text{vdW}} + E_{\text{elec}} \\
 &= \frac{1}{2} \sum_{\text{bonds}} K_b (b - b_0)^2 && \text{Bond} \\
 &+ \frac{1}{2} \sum_{\text{angles}} K_\theta (\theta - \theta_0)^2 && \text{Angle} \\
 &+ \frac{1}{2} \sum_{\text{dihedrals}} K_\phi [1 + \cos(n\phi)]^2 && \text{Torsion} \\
 &+ \sum_{\text{nonbonds}} \left\{ \begin{aligned} &\left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right] && \text{van der Waals} \\ &+ \frac{q_1 q_2}{Dr} && \text{Electrostatics} \end{aligned} \right.
 \end{aligned}$$



Picture taken from

http://en.wikipedia.org/wiki/Molecular_mechanics



- What do we do if we want simulate chemical reaction in large systems?
 - Quantum Mechanics(QM): Accurate, expensive ($\mathcal{O}(N^4)$), suitable for small systems.
 - Molecular Mechanics(MM): Approximate, does not treat electrons explicitly, suitable for large systems such as enzymes and proteins, cannot simulate bond breaking/forming
-
- Methods that combine QM and MM are the solution.
 - Such methods are called Hybrid QM/MM methods.
 - The basic idea is to partition the system into two (or more) parts
 - 1 The region of chemical interest is treated using accurate QM methods eg. active site of an enzyme.
 - 2 The rest of the system is treated using MM or less accurate QM methods such as semi-empirical methods or a combination of the two.

$$\hat{H}_{\text{Total}} = \hat{H}_{\text{QM}} + \hat{H}_{\text{MM}} + \hat{H}_{\text{QM-MM}}^{\text{int}}$$



ONIOM: Divide the system into a real (full) system and the model system. Treat the model system at high and low level. The total energy of the system is given by

$$E = E(\text{low}, \text{real}) + E(\text{high}, \text{model}) - E(\text{low}, \text{model})$$

Empirical Valence Bond: Treat any point on a reaction surface as a combination of two or more valence bond structures

$$H(\mathbf{R}, \mathbf{r}) = \begin{vmatrix} H_{11}(\mathbf{R}, \mathbf{r}) & H_{12}(\mathbf{R}, \mathbf{r}) \\ H_{21}(\mathbf{R}, \mathbf{r}) & H_{22}(\mathbf{R}, \mathbf{r}) \end{vmatrix}$$

Effective Fragment Potential: Divide a large system into fragments and perform *ab initio* or DFT calculations of fragments and their dimers and including the Coulomb field from the whole system.



Why Molecular Dynamics?

- Electronic Structure Methods are applicable to systems in gas phase under low pressure (vacuum).
- Majority of chemical reactions take place in solution at some temperature with biological reactions usually at specific pH's.
- Calculating molecular properties taking into account such environmental effects which can be dynamical in nature are not adequately described by electronic structure methods.

Molecular Dynamics

- Generate a series of time-correlated points in phase-space (a trajectory).
- Propagate the initial conditions, position and velocities in accordance with Newtonian Mechanics. $\mathbf{F} = m\mathbf{a} = -\nabla V$
- Fundamental Basis is the **Ergodic Hypothesis**: the average obtained by following a small number of particles over a long time is equivalent to averaging over a large number of particles for a short time.



- Solve the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{R}, \mathbf{r}, t) = \hat{H} \Psi(\mathbf{R}, \mathbf{r}, t)$$

with

$$\Psi(\mathbf{R}, \mathbf{r}, t) = \chi(\mathbf{R}, t) \Phi(\mathbf{r}, t)$$

and

$$\hat{H} = - \sum_I \frac{\hbar^2}{2M_I} \nabla_I^2 + \underbrace{\frac{-\hbar^2}{2m_e} \nabla_i^2 + V_{n-e}(\mathbf{r}, \mathbf{R})}_{H_e(\mathbf{r}, \mathbf{R})}$$

- Obtain coupled equations of motion for electrons and nuclei:
Time-Dependent Self-Consistent Field (TD-SCF) approach.

$$i\hbar \frac{\partial \Phi}{\partial t} = \left[- \sum_i \frac{\hbar^2}{2m_e} \nabla_i^2 + \langle \chi | V_{n-e} | \chi \rangle \right] \Phi$$
$$i\hbar \frac{\partial \chi}{\partial t} = \left[- \sum_I \frac{\hbar^2}{2M_I} \nabla_I^2 + \langle \Phi | H_e | \Phi \rangle \right] \chi$$



- Define nuclear wavefunction as

$$\chi(\mathbf{R}, t) = A(\mathbf{R}, t) \exp [iS(\mathbf{R}, t)/\hbar]$$

where A and S are real.

- Solve the time-dependent equation for nuclear wavefunction and take classical limit ($\hbar \rightarrow 0$) to obtain

$$\frac{\partial S}{\partial t} + \sum_I \frac{\hbar^2}{2M_I} (\nabla_I S)^2 + \langle \Phi | H_e | \Phi \rangle = 0$$

an equation that is isomorphic with the Hamilton-Jacobi equation with the classical Hamilton function given by

$$\mathcal{H}(\{\mathbf{R}_I\}, \{\mathbf{P}_I\}) = \sum_I \frac{\hbar^2}{2M_I} \mathbf{P}_I^2 + V(\{\mathbf{R}_I\})$$

where

$$\mathbf{P}_I \equiv \nabla_I S \quad \text{and} \quad V(\{\mathbf{R}_I\}) = \langle \Phi | H_e | \Phi \rangle$$

- Obtain equations of nuclear motion from Hamilton's equation

$$\begin{aligned} \frac{d\mathbf{P}_I}{dt} &= -\frac{d\mathcal{H}}{d\mathbf{R}_I} \Rightarrow M\ddot{\mathbf{R}}_I = -\nabla_I V \\ \frac{d\mathbf{R}_I}{dt} &= \frac{d\mathcal{H}}{d\mathbf{P}_I} \end{aligned}$$



- Replace nuclear wavefunction by delta functions centered on nuclear position to obtain

$$i\hbar \frac{\partial \Phi}{\partial t} = H_e(\mathbf{r}, \{\mathbf{R}_I\}) \Phi(\mathbf{r}; \{\mathbf{R}_I\}, t)$$

- This approach of simultaneously solving the electronic and nuclear degrees of freedom by incorporating feedback in both directions is known as **Ehrenfest Molecular Dynamics**.
- Expand Φ in terms of many electron wavefunctions or determinants

$$\Phi(\mathbf{r}; \{\mathbf{R}_I\}, t) = \sum_i c_i(t) \Phi_i(\mathbf{r}; \{\mathbf{R}_I\})$$

with matrix elements

$$H_{ij} = \langle \Phi_i | H_e | \Phi_j \rangle$$

- Inserting Φ in the TDSE above, we get

$$i\hbar \dot{c}_i(t) = c_i(t) H_{ii} - i\hbar \sum_{I,i} \dot{\mathbf{R}}_I \mathbf{d}_I^{ij}$$

with non-adiabatic coupling elements given by

$$\mathbf{d}_I^{ij}(\mathbf{R}_I) = \langle \Phi_i | \nabla_I | \Phi_j \rangle$$



- Upto this point, no restriction on the nature of Φ_i i.e. adiabatic or diabatic basis has been made.
- Ehrenfest method rigorously includes non-adiabtic transitions between electronic states within the framework of classical nuclear motion and mean field (TD-SCF) approximation to the electronic structure.
- Now suppose, we define $\{\Phi_i\}$ to be the adiabatic basis obtained from solving the time-independent Schrödinger equation,

$$H_e(\mathbf{r}, \{\mathbf{R}_I\})\Phi_i(\mathbf{r}; \{\mathbf{R}_I\}) = E_i(\{\mathbf{R}_I\})\Phi_i(\mathbf{r}; \{\mathbf{R}_I\})$$

- The classical nuclei now move along the adiabatic or Born-Oppenheimer potential surface. Such dynamics are commonly known as **Born-Oppenheimer Molecular Dynamics** or BOMD.
- If we restrict the dynamics to only the ground electronic state, then we obtain ground state BOMD.
- If the Ehrenfest potential $V(\{\mathbf{R}_I\})$ is approximated to a global potential surface in terms of many-body contributions $\{v_n\}$.

$$V(\{\mathbf{R}_I\}) \approx V_e^{approx}(\mathbf{R}) = \sum_{I=1}^N v_1(\mathbf{R}_I) + \sum_{I>J}^N v_2(\mathbf{R}_I, \mathbf{R}_J) + \sum_{I>J>K}^N v_3(\mathbf{R}_I, \mathbf{R}_J, \mathbf{R}_K) + \dots$$



- Thus the problem is reduced to purely classical mechanics once the $\{v_n\}$ are determined usually Molecular Mechanics Force Fields. This class of dynamics is most commonly known as **Classical Molecular Dynamics**.
- Another approach to obtain equations of motion for ab-initio molecular dynamics is to apply the Born-Oppenheimer approximation to the full wavefunction $\Psi(\mathbf{r}, \mathbf{R}, t)$

$$\Psi(\mathbf{r}, \mathbf{R}, t) = \sum_k \chi(\mathbf{R}, t) \Phi_k(\mathbf{r}; \mathbf{R}(t))$$

where

$$H_e \Phi_k(\mathbf{r}; \mathbf{R}(t)) = E_k(\mathbf{R}(t)) \Phi_k(\mathbf{r}; \mathbf{R}(t))$$

- Assuming that the nuclear dynamics doesn't change the electronic state, we arrive at the equation of motion for nuclear wavefunction

$$i\hbar \frac{\partial}{\partial t} \chi(\mathbf{R}, t) = \left[\sum_I -\frac{\hbar^2}{2M_I} \nabla_I^2 + E_k(\mathbf{R}) \right] \chi(\mathbf{R}, t)$$



- The Lagrangian for this system is given by.

$$\mathcal{L} = \hat{T} - \hat{V}$$

- Corresponding Newton's equation of motion are then obtained from the associated Euler-Lagrange equations,

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{R}}_I} = \frac{\partial \mathcal{L}}{\partial \mathbf{R}_I}$$

- The Lagrangian for ground state BOMD is

$$\mathcal{L}_{\text{BOMD}} = \sum_I \frac{1}{2} M_I \dot{\mathbf{R}}_I^2 - \min_{\Phi_0} \langle \Phi | H_e | \Phi \rangle$$

and equations of motions

$$M_I \ddot{\mathbf{R}}_I = \frac{d}{dt} \frac{\partial \mathcal{L}_{\text{BOMD}}}{\partial \dot{\mathbf{R}}_I} = \frac{\partial \mathcal{L}_{\text{BOMD}}}{\partial \mathbf{R}_I} = -\nabla_I \min_{\Phi_0} \langle \Phi | H_e | \Phi \rangle$$

Extended Lagrangian Molecular Dynamics (ELMD)

Extend the Lagrangian by adding kinetic energy of fictitious particles and obtain their equation of motions from Euler-Lagrange equations.

Molecular Orbitals: $\{\phi_i\}$
 Density Matrix: $P_{\mu\nu} = \sum_i c_{\mu i}^* c_{\nu i}$



Car-Parrinello Molecular Dynamics (CPMD)

CPMD and NWCHEM

$$\mathcal{L}_{\text{CPMD}} = \sum_I \frac{1}{2} M_I \dot{\mathbf{R}}_I^2 + \sum_i \frac{1}{2} \mu_i \langle \dot{\phi}_i | \dot{\phi}_i \rangle - \langle \Phi_0 | H_e | \Phi_0 \rangle + \text{constraints}$$

R. Car and M. Parrinello, Phys. Rev. Lett. 55 (22), 2471 (1985)

Atom centered Density Matrix Propagation (ADMP)

Gaussian 03/09

$$\mathcal{L}_{\text{ADMP}} = \frac{1}{2} \text{Tr}(\mathbf{V}^T \mathbf{M} \mathbf{V}) + \frac{1}{2} \mu \text{Tr}(\dot{\mathbf{P}} \dot{\mathbf{P}}) - E(\mathbf{R}, \mathbf{P}) - \text{Tr}[\mathbf{\Lambda}(\mathbf{P} \mathbf{P} - \mathbf{P})]$$

H. B. Schlegel, J. M. Millam, S. S. Iyengar, G. A. Voth, A. D. Daniels, G. E. Scuseria, M. J. Frisch, J. Chem. Phys. 114, 9758 (2001)

curvy-steps ELMD (csELMD)

Q-Chem

$$\mathcal{L}_{\text{csELMD}} = \sum_I \frac{1}{2} M_I \dot{\mathbf{R}}_I^2 + \frac{1}{2} \mu \sum_{i < j} \dot{\Delta}_{ij} - E(\mathbf{R}, \mathbf{P}); \quad \mathbf{P}(\lambda) = e^{\lambda \Delta} \mathbf{P}(0) e^{-\lambda \Delta}$$

J.M. Herbert and M. Head-Gordon, J. Chem. Phys. 121, 11542 (2004)



- Electronic energy obtained from
 - Molecular Mechanics \Rightarrow Classical Molecular Dynamics
 - 1 LAMMPS
 - 2 NAMD
 - 3 Amber
 - 4 Gromacs
 - Ab-Initio Methods \Rightarrow Quantum or Ab-Initio Molecular Dynamics
 - 1 Born-Oppenheimer Molecular Dynamics: Gaussian, GAMESS
 - 2 Extended Lagrangian Molecular Dynamics: VASP, CPMD, Gaussian (ADMP), NWCHEM(CPMD), QChem (curvy-steps ELMD)
 - 3 Time Dependent Hartree-Fock and Time Dependent Density Functional Theory: Gaussian, GAMESS, NWCHEM, QChem
 - 4 Multiconfiguration Time Dependent Hartree(-Fock), MCTDH(F)
 - 5 Non-Adiabatic and Ehrenfest Molecular Dynamics, Multiple Spawning, Trajectory Surface Hopping
 - 6 Quantum Nuclei: QWAIMD(Gaussian), NEO(GAMESS)



Classical Molecular Dynamics

- Advantages
 - 1 Large Biological Systems
 - 2 Long time dynamics
- Disadvantages
 - 1 Cannot describe Quantum Nuclear Effects

Ab Initio and Quantum Dynamics

- Advantages
 - 1 Quantum Nuclear Effects
- Disadvantages
 - 1 ~ 100 atoms
 - 2 Full Quantum Dynamics ie treating nuclei quantum mechanically: less than 10 atoms
 - 3 Picosecond dynamics at best



Software	QB	Eric	Louie	Oliver	Painter	Poseidon	Philip	Tezpur
Amber	✓	✓	✓	✓	✓	✓	✓	✓
Desmond	✓							
DL_Poly	✓	✓	✓	✓	✓	✓	✓	✓
Gromacs	✓	✓	✓	✓	✓	✓	✓	✓
LAMMPS	✓	✓	✓	✓	✓	✓	✓	✓
NAMD	✓	✓	✓	✓	✓	✓		✓
OpenEye	✓	✓	✓	✓	✓	✓	✓	✓
CPMD	✓	✓	✓	✓	✓	✓		✓
GAMESS	✓	✓	✓	✓	✓	✓	✓	✓
Gaussian		✓	✓	✓	✓		✓	✓
NWCHEM	✓	✓	✓	✓	✓	✓		✓
Piny_MD	✓	✓	✓	✓	✓	✓	✓	✓

Software	Bluedawg	Ducky	Lacumba	Neptune	Zeke	Pelican	Pandora
Amber		✓	✓			✓	
Gromacs	✓	✓	✓	✓	✓	✓	
LAMMPS	✓	✓	✓	✓	✓	✓	
NAMD	✓	✓	✓	✓	✓		
CPMD	✓	✓	✓	✓	✓		
Gaussian	✓	✓	✓		✓	✓	
NWCHEM	✓	✓	✓	✓	✓	✓	
Piny_MD	✓	✓	✓	✓	✓	✓	



- Commercial Software: Q-Chem, Jaguar, CHARMM
- GPL/Free Software: ACES, ABINIT, Octopus
- http://en.wikipedia.org/wiki/Quantum_chemistry_computer_programs
- <http://www.ccl.net/chemistry/links/software/index.shtml>
- <http://www.redbrick.dcu.ie/~noel/linux4chemistry/>



Job Types and Keywords

Job Type	Gaussian	GAMESS	NWCHEM
	# keyword	runtyp=	task
Energy	sp	energy	energy
Force	force	gradient	gradient
Geometry optimization	opt	optimize	optimize
Transition State	opt=ts	sadpoint	saddle
Frequency	freq	hessian	frequencies, freq
Potential Energy Scan	scan	surface	✓
Excited State	✓	✓	✓
Reaction path following	irc	irc	✓
Molecular Dynamics	admp, bomd	drc	dynamics, Car-Parrinello
Population Analysis	pop	pop	✓
Electrostatic Properties	prop	✓	✓
Molecular Mechanics	✓	✓	✓
Solvation Models	✓	✓	✓
QM/MM	oniom	✓	qmmm



Molecular Dynamics Calculations

- Gaussian:
 - BOMD: Born-Oppenheimer Molecular Dynamics
 - ADMP: Atom centered Density Matrix Propagation (an extended Lagrangian Molecular Dynamics similar to CPMD) and ground state BOMD
- GAMESS:
 - DRC: Direct Dynamics, a classical trajectory method based on "on-the-fly" ab-initio or semi-empirical potential energy surfaces
- NWCHEM:
 - Car-Parrinello: Car Parrinello Molecular Dynamics (CPMD)
 - DIRDYVTST: Direct Dynamics Calculations using POLYRATE with electronic structure from NWCHEM



- Fall Semester

Introduction to Gaussian/Electronic Structure Methods

- Spring Semester

Introduction to Computational Chemistry: Molecular Dynamics

April 27th



Useful Links

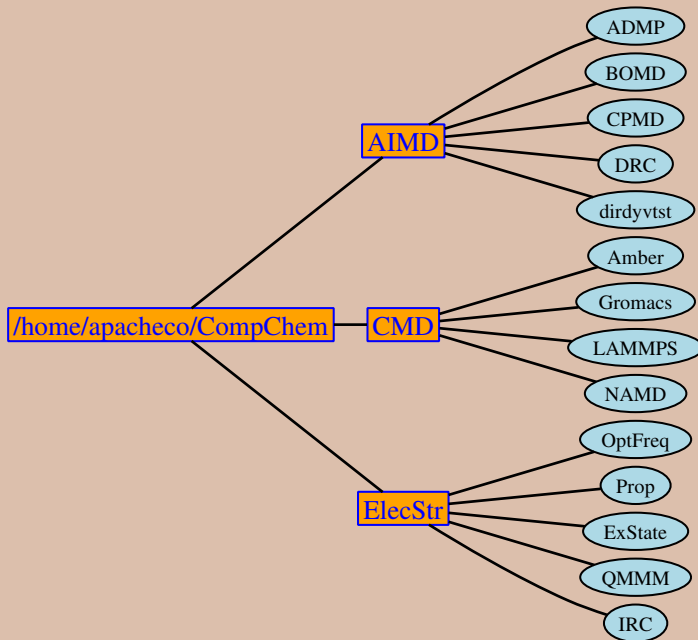
- Amber: <http://ambermd.org>
- Desmond: http://www.deshawresearch.com/resources_desmond.html
- DL_POLY: http://www.cse.scitech.ac.uk/ccg/software/DL_POLY
- Gromacs: <http://www.gromacs.org>
- LAMMPS: <http://lammps.sandia.gov>
- NAMD: <http://www.ks.uiuc.edu/Research/namd>
- CPMD: <http://www.cpmd.org>
- GAMESS: <http://www.msg.chem.iastate.edu/gamess>
- Gaussian: <http://www.gaussian.com>
- NWCHEM: <http://www.nwchem-sw.org>
- PINY_MD: http://homepages.nyu.edu/~mt33/PINY_MD/PINY.html
- Basis Set: <https://bse.pnl.gov/bse/portal>



Further Reading

- David Sherill's Notes at Ga Tech:
<http://vergil.chemistry.gatech.edu/notes/index.html>
- Mark Tuckerman's Notes at NYU:
<http://www.nyu.edu/classes/tuckerman/quant.mech/index.html>
- Modern Quantum Chemistry: Introduction to Advanced Electronic Structure Theory, A. Szabo and N. Ostlund
- Introduction to Computational Chemistry, F. Jensen
- Essentials of Computational Chemistry - Theories and Models, C. J. Cramer
- Exploring Chemistry with Electronic Structure Methods, J. B. Foresman and A. Frisch
- Ab Initio Molecular Dynamics: Theory and Implementation, D. Marx and J. Hutter
<http://www.theochem.ruhr-uni-bochum.de/research/marx/marx.pdf>
- Molecular Modeling - Principles and Applications, A. R. Leach
- Computer Simulation of Liquids, M. P. Allen and D. J. Tildesley
- ◆ Modern Electronic Structure Theory, T. Helgaker, P. Jorgensen and J. Olsen
(Highly advanced text, second quantization approach to electronic structure theory)





- Site specific license
 - ① Gaussian 03 and 09
 - LSU Users: Eric
 - Latech Users: Painter, Bluedawg
 - ② Gaussian 03
 - ULL Users: Oliver, Zeke
 - Tulane Users: Louie, Ducky
 - Southern Users: Lacumba
 - ③ UNO Users: No License
- Add +gaussian-03/+gaussian-09 to your .soft file and resoft
- If your institution has license to both G03 and G09, have only one active at a given time.



Example Job submission script on Intel x86

```
#!/bin/tcsh
#PBS -A your_allocation
# specify the allocation. Change it to your allocation
#PBS -q checkpt
# the queue to be used.
#PBS -l nodes=1:ppn=4
# Number of nodes and processors
#PBS -l walltime=1:00:00
# requested Wall-clock time.
#PBS -o g03_output
# name of the standard out file to be "output-file".
#PBS -j oe
# standard error output merge to the standard output file.
#PBS -N g03test
# name of the job (that will appear on executing the qstat command).

# setup g03 variables
source $g03root/g03/bsd/g03.login
set NPROCS='wc -l $PBS_NODEFILE |gawk '{print $1}''
setenv GAUSS_SCRDIR /scratch/$USER
# cd to the directory with Your input file
cd ~apacheco/g03test
# Change this line to reflect your input file and output file
g03 < g03job.inp > g03job.out
```

Linda Access

```
set Nodelist = ( -vv -nodelist "" `cat $PBS_NODEFILE` "" -mp 4)
setenv GAUSS_LFLAGS " $Nodelist "
g03l < g03job.inp > g03job.out
```



Example Job submission script on P5

```
#!/bin/tcsh
# @ account_no = your_allocation
# @ requirements = (Arch == "Power5")
# @ environment = LL_JOB=TRUE ; MP_PULSE=1200
# @ job_type = parallel
# @ node_usage = shared
# @ wall_clock_limit = 12:00:00
# @ initialdir = /home/apacheco/g03test
# @ class = checkpt
# @ error = g03_$(jobid).err
# @ queue

# setup g03 variables
source $g03root/g03/bsd/g03.login
# setup and create Gaussian scratch directory
setenv GAUSS_SCRDIR /scratch/default/$USER
mkdir -p $GAUSS_SCRDIR
# cd to the directory with Your input file
cd ~apacheco/g03test
# Change this line to reflect your input file and output file
g03 < g03job.inp > g03job.out
```



Sample Input

```
%chk=h2o-opt-freq.chk
%mem=512mb
%NProcShared=4

#p b3lyp/6-31G opt freq

H2O OPT FREQ B3LYP

0 1
O
H 1 r1
H 1 r1 2 a1

r1 1.05
a1 104.5
```

Input Description

checkpoint file
amount of memory
number of smp processors
blank line
Job description
blank line
Job Title
blank line
Charge Multiplicity
Molecule Description
Z-matrix format
with variables
blank line
variable value
blank line



- Add +gamess-12Jan2009R1-intel-11.1 (on Queenbee) to your .soft and resoft

Job submission script

```
#!/bin/bash
#PBS -A your_allocation
#PBS -q checkpt
#PBS -l nodes=1:ppn=4
#PBS -l walltime=00:10:00
#PBS -j oe
#PBS -N gamess-exam1

export WORKDIR=$PBS_O_WORKDIR
export NPROCS=`wc -l $PBS_NODEFILE | gawk '{print $1}'`
export SCRDIR=/work/$USER/scr
if [ ! -e $SCRDIR ]; then mkdir -p $SCRDIR; fi
rm -f $SCRDIR/*

cd $WORKDIR
rungms h2o-opt-freq 01 $NPROCS h2o-opt-freq.out $SCRDIR
cp -p $SCRDIR/$OUTPUT $WORKDIR/
```



Sample Input

```
$CONTRL SCFTYP=RHF RUNTYP=OPTIMIZE  
COORD=ZMT NZVAR=0 $END  
$STATPT OPTTOL=1.0E-5 HSSEND=.T. $END  
$BASIS GBASIS=N31 NGAUSS=6  
NDFUNC=1 NPFUNC=1 $END  
$DATA  
H2O OPT  
Cnv 2  
  
O  
H 1 rOH  
H 1 rOH 2 aHOH  
  
rOH=1.05  
aHOH=104.5  
$END
```

Input Description

Job control data

geometry search control
6-31G** basis set

molecular data control
Title
Symmetry group and axis

molecule description in
z-matrix

variables

end molecular data control



- Add +nwchem-5.1.1-intel-11.1-mvapich-1.1 (on Queenbee) to your .soft and resoft

Job submission script

```
#!/bin/sh
#
#PBS -q checkpt
#PBS -M apacheco@cct.lsu.edu
#PBS -l nodes=1:ppn=4
#PBS -l walltime=0:30:00
#PBS -V
#PBS -o nwchem_h2o.out
#PBS -e nwchem_h2o.err
#PBS -N nwchem_h2o

export EXEC=nwchem
export EXEC_DIR=/usr/local/packages/nwchem-5.1-mvapich-1.0-intel-10.1/bin/LINUX64/
export WORK_DIR=$PBS_O_WORKDIR
export NPROCS='wc -l $PBS_NODEFILE |gawk '{print $1}''

cd $WORK_DIR
mpirun_rsh -machinefile $PBS_NODEFILE -np $NPROCS $EXEC_DIR/$EXEC \
  $WORK_DIR/h2o-opt-freq.nw >& $WORK_DIR/h2o-opt-freq.nwo
```



Sample Input

```
title "H2O"  
echo  
charge 0  
geometry  
zmatrix  
O  
H 1 r1  
H 1 r1 2 a1  
variables  
r1 1.05  
a1 104.5  
end  
end  
basis noprint  
  * library 6-31G  
end  
dft  
  XC b3lyp  
  mult 1  
end  
task dft optimize  
task dft energy  
task dft freq
```

Input Description

Job title
echo contents of input file
charge of molecule
geometry description in
z-matrix format

variables used with values

end z-matrix block
end geometry block
basis description

dft calculation options

job type

