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1) Derive y simplifique la siguiente función

$$y = \frac{1}{2}\tanh(x) + \frac{\sqrt{2}}{8}\ln\left(\frac{1+\sqrt{2}\tanh(x)}{1-\sqrt{2}\tanh(x)}\right)$$

$$y = \frac{1}{2}\tanh(x) + \frac{\sqrt{2}}{8}\left(\ln(1+\sqrt{2}\tanh(x)) - \ln(1-\sqrt{2}\tanh(x))\right)$$

$$y = \frac{1}{2}A + \frac{\sqrt{2}}{8}(B-c)$$

$$y' = \frac{1}{2}A' + \frac{\sqrt{2}}{8}(B'-c')$$

$A = \tanh(x)$ $\qquad A' = \text{Sech}^2(x)$

$B = \ln(1+\sqrt{2}\tanh(x)) = \ln(D)$ $\qquad C = \ln(1-\sqrt{2}\tanh(x)) = \ln(E)$

$B' = \dfrac{D'}{D}$ $\qquad\qquad\qquad\qquad C' = \dfrac{E'}{E}$

$D = 1+\sqrt{2}\tanh(x)$ $\qquad\qquad E = 1-\sqrt{2}\tanh(x)$

$D' = 1+\sqrt{2}A$ $\qquad\qquad\qquad E = 1-\sqrt{2}A$

$D' = \sqrt{2}A'$ $\qquad\qquad\qquad E' = -\sqrt{2}A'$

$\boxed{D' = \sqrt{2}\,\text{Sech}^2(x)}$ $\qquad\boxed{E' = -\sqrt{2}\,\text{Sech}^2(x) = -D'}$

$$B'-c' = \left(\frac{D'}{D} - \frac{E'}{E}\right) = \left(\frac{D'}{D} - \frac{(-D')}{E}\right) = D'\left(\frac{1}{D}+\frac{1}{E}\right) = D'\left(\frac{E+D}{DE}\right)$$

$$E+D = 1+\sqrt{2}\tanh(x) + 1-\sqrt{2}\tanh(x) = 2$$

$$\boxed{= \frac{D'(2)}{DE}}$$

$$y' = \frac{1}{2}(A') + \frac{\sqrt{2}}{8}\left(D'\left(\frac{2}{DE}\right)\right)$$

$$y' = \frac{1}{2}\text{Sech}^2(x) + \frac{\sqrt{2}}{8}\left(\sqrt{2}\,\text{Sech}^2(x)\left(\frac{2}{DE}\right)\right)$$

$$y' = \frac{1}{2}\text{Sech}^2(x) + \frac{1}{2}\text{Sech}^2(x)\left(\frac{1}{DE}\right)$$

$$y' = \frac{1}{2}\text{Sech}^2(x)\left(1 + \frac{1}{DE}\right) =$$

$$DE = (1+\sqrt{2}\tanh(x))(1-\sqrt{2}\tanh(x)) = 1-2\tanh^2(x)$$

$$1+\frac{1}{DE} = 1 + \frac{1}{1-2\tanh^2(x)} = \frac{1-2\tanh^2(x)+1}{DE} = \boxed{\frac{2-2\tanh^2(x)}{DE}}$$

$$y' = \frac{1}{2}\operatorname{sech}^2(x)\left(1 + \frac{1}{DE}\right)$$

$$y' = \frac{1}{2}\operatorname{sech}^2(x)\left(\frac{2(1 - \tanh^2(x))}{DE}\right)$$

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

$$y' = \operatorname{sech}^2(x)\frac{(\operatorname{sech}^2(x))}{DE}$$

$$\boxed{y' = \frac{\operatorname{sech}^4(x)}{(1 - 2\tanh^2(x))}}$$

② Derive y simplifique la siguiente función

$$f(x) = \ln\left(\sqrt{\frac{1 - \sin(x)}{1 + \sin(x)}}\right) + \operatorname{arcsin}\left(\frac{\sin(x)}{\sqrt{1 + \sin^2(x)}}\right)$$

$$f(x) = \frac{1}{2}\ln\left(\frac{1 - \sin(x)}{1 + \sin(x)}\right) + \operatorname{arcsin}\left(\frac{\sin(x)}{\sqrt{1 + \sin^2(x)}}\right)$$

$$f(x) = \frac{1}{2}\left(\ln(1 - \sin(x)) - \ln(1 + \sin(x))\right) + \operatorname{arcsin}\left(\frac{\sin(x)}{\sqrt{1 + \sin^2(x)}}\right)$$

$$f(x) = \frac{1}{2}(\ln A - \ln B) + C$$

$$y' = \frac{1}{2}\left(\frac{A'}{A} - \frac{B'}{B}\right) + c'$$

$$A = 1 - \sin(x) \qquad B = 1 + \sin(x)$$
$$A' = -\cos(x) \qquad \boxed{B' = \cos(x) = -A'}$$

$$\left(\frac{A'}{A} - \frac{B'}{B}\right) = \left(\frac{A'}{A} - \left(-\frac{A'}{B}\right)\right) = A'\left(\frac{1}{A} + \frac{1}{B}\right) = A'\left(\frac{B + A}{AB}\right)$$

$$B + A = 1 + \sin(x) + 1 - \sin(x) = 2 \implies A'\frac{(2)}{AB}$$

$$AB = (1 - \sin(x))(1 + \sin(x)) = 1 - \sin^2(x) = \boxed{\cos^2(x)}$$

$$A'\left(\frac{B+A}{AB}\right) = A'\left(\frac{2}{\cos^2(x)}\right) = \frac{-\cos(x)(2)}{\cos^2(x)} = \boxed{-2\sec(x)}$$

$$C = \text{arcsen}\left(\frac{\sin x}{\sqrt{1 + \sin^2(x)}}\right) = \text{arcsen}(D)$$

$$C' = \frac{D'}{\sqrt{1 - D^2}}$$

$$D = \frac{\sin(x)}{\sqrt{1 + \sin^2(x)}} = \frac{E}{F^{1/2}}$$

$$\ln D = \ln\left(\frac{\sin(x)}{\sqrt{1 + \sin^2(x)}}\right)$$

$$\ln D = \ln \sin(x) - \ln(1 + \sin^2(x))^{1/2}$$

$$\ln D = \ln E - \ln F^{1/2}$$

$$\ln D = \ln E - \frac{1}{2}\ln F$$

$$\frac{D'}{D} = \frac{E'}{E} - \frac{1}{2}\frac{F'}{F}$$

$$E = \sin(x)$$
$$\boxed{E' = \cos(x)}$$

$$F = 1 + \sin^2(x)$$
$$F' = 2\sin(x)\cos(x)$$
$$\boxed{F' = 2\sin(x)E'}$$

$$\frac{D'}{D} = \frac{E'}{E} - \frac{1}{2}\frac{F'}{F} = \frac{E'}{E} - \frac{E'(2\sin(x))}{2F} = E'\left(\frac{1}{E} - \frac{\sin x}{F}\right)$$

$$\frac{D'}{D} = E'\left(\frac{F - E\sin(x)}{EF}\right)$$

$$D' = \frac{E}{F^{1/2}}\left(E'\left(\frac{F - E\sin(x)}{EF}\right)\right) = \frac{E'(F - E\sin(x))}{F^{3/2}}$$

$$F - E\sin(x) = (1 + \sin^2(x)) - \sin(x)(\sin(x))$$
$$= 1 + \sin^2(x) - \sin^2(x) = 1$$

$$\boxed{D' = \frac{E'}{F^{3/2}}} = \boxed{D' = \frac{\cos(x)}{F^{3/2}}}$$

$$1 - D^2 = 1 - \left(\frac{\sin(x)}{\sqrt{F}}\right)^2 = 1 - \frac{\sin^2(x)}{F} = \frac{F - \text{sen}^2(x)}{F}$$

$$F - \text{sen}^2(x) = 1 + \text{sen}^2(x) - \text{sen}^2(x) = \frac{1}{F}$$

$$\sqrt{1 - D^2} = \sqrt{\frac{1}{F}} = \frac{1}{F^{1/2}}$$

$$C' = \frac{\frac{\cos(x)}{F^{3/2}}}{\frac{1}{F^{1/2}}} = \boxed{\frac{\cos(x)}{F}} = \frac{\cos(x)}{1 + \text{sen}^2(x)} \quad \boxed{C' = \frac{\cos(x)}{1 + \text{sen}^2(x)}}$$

$$y' = \frac{1}{2}\left(\frac{A'}{A} - \frac{B'}{B}\right) + c'$$

$$y' = \frac{1}{2}(-2\sec(x)) + \frac{\cos(x)}{1+sco^2(x)}$$

$$\boxed{y' = -\sec h(x) + \frac{\cos(x)}{1+sen^2(x)}}$$

③ Derive y simplifique la siguiente fncma

$$y = \frac{(1-2x)^3 \cdot \sqrt[4]{3x+1}}{\sqrt{x^5(x+1)^3}}$$

$$y = \frac{(1-2x)^3 (3x+1)^{1/4}}{x^{5/2}(x+1)^{3/2}} = \frac{A^3 B^{1/4}}{C^{5/2} D^{3/2}}$$

$$\ln y = \ln\left(\frac{A^3 B^{1/4}}{C^{5/2}D^{3/2}}\right)$$

$$\ln y = \ln A^3 + \ln B^{1/4} - \ln C^{5/2} - \ln D^{3/2}$$

$$\ln y = 3\ln A + \frac{1}{4}\ln B - \frac{5}{2}\ln C - \frac{3}{2}\ln D$$

$$\frac{y'}{y} = 3\frac{A'}{A} + \frac{1}{4}\frac{B'}{B} - \frac{5}{2}\frac{C'}{C} - \frac{3}{2}\frac{D'}{D}$$

$$\boxed{\begin{array}{l} A = 1-2x = \quad A' = -2 \\ B = 3x+1 \quad B' = 3 \\ C = x \quad C' = 1 \\ D = x+1 \quad D' = 1 \end{array}}$$

$$\frac{y'}{y} = \frac{3(-2)}{A} + \frac{1(3)}{4B} - \frac{5}{2C} - \frac{3}{2D}$$

$$\frac{y'}{y} = -\frac{6}{A} + \frac{3}{4B} - \frac{5}{2C} - \frac{3}{2D}$$

$$\frac{Y'}{Y} = -3\left(\frac{2}{A} - \frac{1}{4B}\right) - \frac{1}{2}\left(\frac{5}{C} + \frac{3}{D}\right)$$

$$\frac{Y'}{Y} = -3\left(\frac{8B - A}{4AB}\right) - \frac{1}{2}\left(\frac{5D + 3C}{CD}\right)$$

$$-24B + 3A = -24(3x+1) + 3(1-2x) = -72x - 24 + 3 - 6x$$
$$= -78x - 21$$
$$= -3(26x + 7)$$

$$5D + 3C = 5(x+1) + 3(x) = 8x + 5$$

$$\frac{Y'}{Y} = \frac{1}{2}\left(\frac{-78x - 21}{2AB} - \frac{8x+5}{CD}\right)$$

$$\frac{Y'}{Y} = \frac{1}{2}\left(\frac{-(78x+21)(x)(x+1) - (8x+5)(2)(1-2x)(3x+1)}{2\,ABCD}\right)$$

$$(78x+21)(x^2+x) = 78x^3 + 78x^2 + 21x^2 + 21x$$
$$\boxed{E = 78x^3 + 99x^2 + 21x}$$

$$(8x+5)(2x-4)(3x+1) = (8x+5)(6x^2 + 2x - 12x - 4)$$
$$= (8x+5)(6x^2 - 10x - 4)$$
$$= 48x^3 - 80x - 32x + 30x^2 - 50x - 20$$
$$\boxed{F = 48x^3 - 55x^2 - 82x - 20}$$

$$\boxed{E + F = (126x^3 + 49x^2 - 61x - 20)}$$

$$Y' = -Y\,\frac{(126x^3 + 49x^2 - 61x - 20)}{4ABCD}$$

$$Y' = -\frac{A^3 B^{1/4}}{4\,C^{5/2} D^{3/2}}\left(\frac{126x^3 + 49x^2 - 61x - 20}{ABCD}\right)$$

$$Y' = -\frac{A^2}{4 B^{3/4} C^{7/2} D^{5/2}}\left(126x^3 + 49x^2 - 61x - 20\right)$$

$$\boxed{Y' = -\frac{(1-2x)^2(126x^3 - 49x^2 - 61x - 20)}{4(3x+1)^{3/4}\,x^{7/2}(x+1)^{5/2}}}$$