Nombre: TNTE Paguay Alex

NRC : 2823

1) Derive y simplifique la siguiente función

$$y=\frac{1}{2}\tanh(x)+\frac{\sqrt{2}}{8}\ln\left(\frac{1+\sqrt{2}\tanh(x)}{1-\sqrt{2}\tanh(x)}\right)$$

$$y=\frac{1}{2}\tanh(x)+\frac{\sqrt{2}}{8}\left(\ln(1+\sqrt{2}\tanh(x))-\ln(1-\sqrt{2}\tanh(x)\right)$$

$$y=\frac{1}{2}A+\frac{\sqrt{2}}{8}(B-C)$$

$$y'=\frac{1}{2}A'+\frac{\sqrt{2}}{8}(B'-C')$$

$A=\tanh(x)$ $\quad A'=\operatorname{Sech}^2(x)$

$B=\ln(1+\sqrt{2}\tanh(x))=\ln(D)$ $\quad C=\ln(1-\sqrt{2}\tanh(x))=\ln(E)$

$B'=\frac{D'}{D}$ $\quad C'=\frac{E'}{E}$

$D=1+\sqrt{2}\tanh(x)$ $\quad E=1-\sqrt{2}\tanh(x)$

$D'=1+\sqrt{2}A$ $\quad E=1-\sqrt{2}A$

$D'=\sqrt{2}A'$ $\quad E'=-\sqrt{2}A'$

$\boxed{D'=\sqrt{2}\operatorname{Sech}2(x)}$ $\quad \boxed{E'=-\sqrt{2}\operatorname{Sech}2(x)=-D'}$

$$B'-C'=\left(\frac{D'}{D}-\frac{E'}{E}\right)=\left(\frac{D'}{D}-\frac{(-D')}{E}\right)=D'\left(\frac{1}{D}+\frac{1}{E}\right)=D'\left(\frac{E+D}{DE}\right)$$

$E+D=1+\sqrt{2}\tanh(x)+1-\sqrt{2}\tanh(x)=2$ $\quad \boxed{=\frac{D'(2)}{DE}}$

$$y'=\frac{1}{2}(A')+\frac{\sqrt{2}}{8}\left(D'\left(\frac{2}{DE}\right)\right)$$

$$y'=\frac{1}{2}\operatorname{Sech}^2(x)+\frac{\cancel{\sqrt{2}}}{8}\left(\cancel{\sqrt{2}}\operatorname{Sech}^2(x)\left(\frac{\cancel{2}}{DE}\right)\right)$$

$$y'=\frac{1}{2}\operatorname{Sech}^2(x)+\frac{1}{2}\operatorname{Sech}^2(x)\left(\frac{1}{DE}\right)$$

$$y'=\frac{1}{2}\operatorname{Sech}^2(x)\left(1+\frac{1}{DE}\right)=$$

$DE=(1+\sqrt{2}\tanh(x))(1-\sqrt{2}\tanh(x))=1-2\tanh^2(x)$

$$1+\frac{1}{DE}=1+\frac{1}{1-2\tanh^2(x)}=\frac{1-2\tanh2(x)+1}{DE}=\boxed{\frac{2-2\tanh^2(x)}{DE}}$$

$$y' = \frac{1}{2}\operatorname{Sech}^2(x)\left(1 + \frac{1}{DE}\right)$$

$$y' = \frac{1}{2}\operatorname{Sech}^2(x)\left(\frac{\cancel{2}(1-\tanh^2(x))}{DE}\right)$$

$$1-\tanh^2(x) = \operatorname{Sech}^2(x)$$

$$y' = \frac{\operatorname{Sech}^2(x)(\operatorname{Sech}^2(x)}{DE}$$

$$\boxed{y' = \frac{\operatorname{Sech}^4(x)}{(1-2\tanh^2(x))}}$$

② Derive y simplifique la siguiente función

$$f(x) = \ln\left(\sqrt{\frac{1-\sin(x)}{1+\sin(x)}}\right) + \arcsin\left(\frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\right)$$

$$f(x) = \frac{1}{2}\ln\left(\frac{1-\sin(x)}{1+\sin(x)}\right) + \arcsin\left(\frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\right)$$

$$f(x) = \frac{1}{2}(\ln(1-\sin(x)) - \ln(1+\sin(x))) + \arcsin\left(\frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\right)$$

$$f(x) = \frac{1}{2}(\ln A - \ln B) + C$$

$$y' = \frac{1}{2}\left(\frac{A'}{A} - \frac{B'}{B}\right) + C'$$

$A = 1 - \sin(x)$ $\quad B = 1 + \sin(x)$

$A' = -\cos(x)$ $\quad \boxed{B' = \cos(x) = -A'}$

$$\left(\frac{A'}{A} - \frac{B'}{B}\right) = \left(\frac{A'}{A} - \left(-\frac{A'}{B}\right)\right) = A'\left(\frac{1}{A} + \frac{1}{B}\right) = A'\left(\frac{B+A}{AB}\right)$$

$$B+A = 1+\sin(x)+1-\sin(x) = 2 \Rightarrow A'\frac{(2)}{AB}$$

$$AB = (1-\sin(x))(1+\operatorname{Sen}(x)) = 1-\operatorname{Sen}^2(x) = \boxed{\cos^2(x)}$$

$$A'\left(\frac{B+A}{AB}\right) = A'\left(\frac{2}{\cos^2(x)}\right) = -\frac{\cos(x)(2)}{\cos^2(x)} = \boxed{-2\operatorname{Sec}(x)}$$

Escaneado con CamScanner

$$C = \arcsen\left(\frac{\sin x}{\sqrt{1+\sin^2(x)}}\right) = \arcsen(D)$$

$$C' = \frac{D'}{\sqrt{1-D^2}}$$

$$D = \frac{\sin(x)}{\sqrt{1+\sin^2(x)}} = \frac{E}{F^{1/2}}$$

$$\ln D = \ln\left(\frac{\sin(x}{\sqrt{1+\sin^2(x)}}\right)$$

$$\ln D = \ln \sin(x) - \ln(1+\sin^2(x))^{1/2}$$

$$\ln D = \ln E - \ln F^{1/2}$$

$$\ln D = \ln E - \frac{1}{2}\ln F$$

$$\frac{D'}{D} = \frac{E'}{E} - \frac{1}{2}\frac{F'}{F}$$

$$E = \sin(x)$$

$$\boxed{E' = \cos(x)}$$

$$F = 1+\sin^2(x)$$

$$F' = 2\sin(x)\cos(x)$$

$$\boxed{F' = 2\sin(x)E'}$$

$$\frac{D'}{D} = \frac{E'}{E} - \frac{1}{2}\frac{F'}{F} = \frac{E'}{E} - \frac{E'(\cancel{2}\sin(x))}{\cancel{2}F} = E'\left(\frac{1}{E} - \frac{\sin x}{F}\right)$$

$$\frac{D'}{D} = E'\left(\frac{F-E\sin(x)}{EF}\right)$$

$$D' = \frac{\cancel{E}}{F^{1/2}}\left(E'\left(\frac{F-E\sin(x)}{\cancel{E}F}\right)\right) = \frac{E'(F-E\sin(x))}{F^{3/2}}$$

$$F - E\sin(x) = (1+\sin^2(x)) - \sin(x)(\sin(x))$$

$$= 1 + \cancel{\sin^2(x)} - \cancel{\sin^2(x)} = 1$$

$$\boxed{D' = \frac{E'}{F^{3/2}}} = \boxed{D' = \frac{\cos(x)}{F^{3/2}}}$$

$$1 - D^2 = 1 - \left(\frac{\sin(x)}{\sqrt{F}}\right)^2 = 1 - \frac{\sin^2(x)}{F} = \frac{F-\sen^2(x)}{F}$$

$$F - \sen^2(x) = 1 + \cancel{\sen^2(x)} - \cancel{\sen^2(x)} = \frac{1}{F}$$

$$\sqrt{1-D^2} = \sqrt{\frac{1}{F}} = \frac{1}{F^{1/2}}$$

$$C' = \frac{\frac{\cos(x)}{F^{3/2}}}{\frac{1}{F^{1/2}}} = \boxed{\frac{\cos(x)}{F}} = \frac{\cos(x)}{1+\sen^2(x)} \quad \boxed{C' = \frac{\cos(x)}{1+\sen^2(x)}}$$

Escaneado con CamScanner

$$y' = \frac{1}{2}\left(\frac{A'}{A} - \frac{B'}{B}\right) + c'$$

$$y' = \frac{1}{2}\left(-\cancel{2}\,\mathrm{Sec}(x)\right) + \frac{\cos(x)}{\cancel{1+}\,\mathrm{Sen}^2(x)}$$

$$\boxed{y' = -\mathrm{Sech}(x) + \frac{\cos(x)}{1+\mathrm{Sen}^2(x)}}$$

3. Derive y simplifique la siguiente forma

$$y = \frac{(1-2x)^3 \cdot \sqrt[4]{3x+1}}{\sqrt{x^5 (x+1)^3}}$$

$$y = \frac{(1-2x)^3 (3x+1)^{1/4}}{x^{5/2}(x+1)^{3/2}} = \frac{A^3 B^{1/4}}{C^{5/2} D^{3/2}}$$

$$\ln y = \ln\left(\frac{A^3 B^{1/4}}{C^{5/2} D^{3/2}}\right)$$

$$\ln y = \ln A^3 + \ln B^{1/4} - \ln C^{5/2} - \ln D^{3/2}$$

$$\ln y = 3\ln A + \frac{1}{4}\ln B - \frac{5}{2}\ln C - \frac{3}{2}\ln D$$

$$\frac{y'}{y} = \frac{3A'}{A} + \frac{1}{4}\frac{B'}{B} - \frac{5}{2}\frac{C'}{C} - \frac{3}{2}\frac{D'}{D}$$

$$\boxed{\begin{array}{l} A = 1-2x = A' = -2 \\ B = 3x+1 \quad B' = 3 \\ C = x \quad C' = 1 \\ D = x+1 \quad D' = 1 \end{array}}$$

$$\frac{y'}{y} = \frac{3(-2)}{A} + \frac{1(3)}{4B} - \frac{5}{2C} - \frac{3}{2D}$$

$$\frac{y'}{y} = -\frac{6}{A} + \frac{3}{4B} - \frac{5}{2C} - \frac{3}{2D}$$

$$\frac{y'}{y} = -3\left(\frac{2}{A} - \frac{1}{4B}\right) - \frac{1}{2}\left(\frac{5}{C} + \frac{3}{D}\right).$$

$$\frac{y'}{y} = -3\left(\frac{8B-A}{4AB}\right) - \frac{1}{2}\left(\frac{5D+3C}{CD}\right)$$

$$-24B+3A = -24(3x+1)+3(1-2x) = -72x-24+3-6x$$
$$= -78x-21$$
$$= -3(26x+7)$$

$$5D+3C = 5(x+1)+3(x) = 8x+5$$

$$\frac{y'}{y} = \frac{1}{2}\left(\frac{-78x-21}{2AB} - \frac{8x+5}{CD}\right).$$

$$\frac{y'}{y} = \frac{1}{2}\left(\frac{-(78x+21)(x)(x+1) - (8x+5)(2)(1-2x)(3x+1)}{2ABCD}\right)$$

$$(78x+21)(x^2+x) = 78x^3+78x^2+21x^2+21x$$
$$= 78x^3+99x^2+21x$$

$$(8x+5)(2x-4)(3x+1) = (8x+5)(6x^2+2x-12x-4)$$
$$= (8x+5)(6x^2-10x-4)$$
$$= 48x^3-80x-32x+30x^2-50x-20$$
$$= 48x^3-50x^2-82x-20$$
$$= (126x^3+49x^2-61x-20)$$

$$y' = -y\,\frac{(126x^3+49x^2-61x-20)}{4ABCD}$$

$$y' = -\frac{A^3B^{1/2}}{4C^{5/2}D^{3/2}}\left(\frac{(126x^3+49x^2-61x-20}{ABCD}\right)$$

$$y' = -\frac{A^2\,(126x^3+49x^2-61x-20)}{4B^{3/4}C^{7/2}D^{5/2}}$$

$$\boxed{y' = -\frac{(1-2x)^2(126x^3-49x^2-61x-20)}{4(3x+1)^{3/4}\,x^{7/2}(x+1)^{5/2}}}$$