Computing Derivatives Numerically

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Theory

We are using a set of methods to numerically approximate function derivatives. Here are the methods used for approximations (you can use this list as a transcript of plots shown later):

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{0}$$

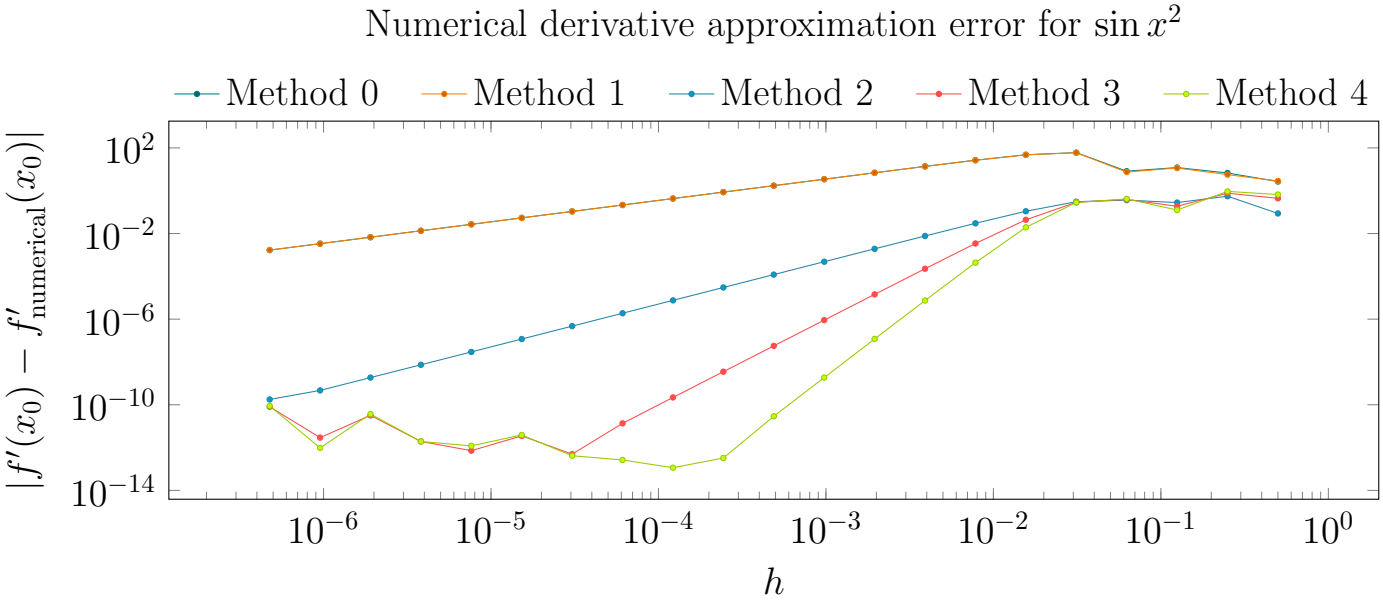
$$f'(x) = \frac{f(x) - f(x-h)}{h} \tag{1}$$

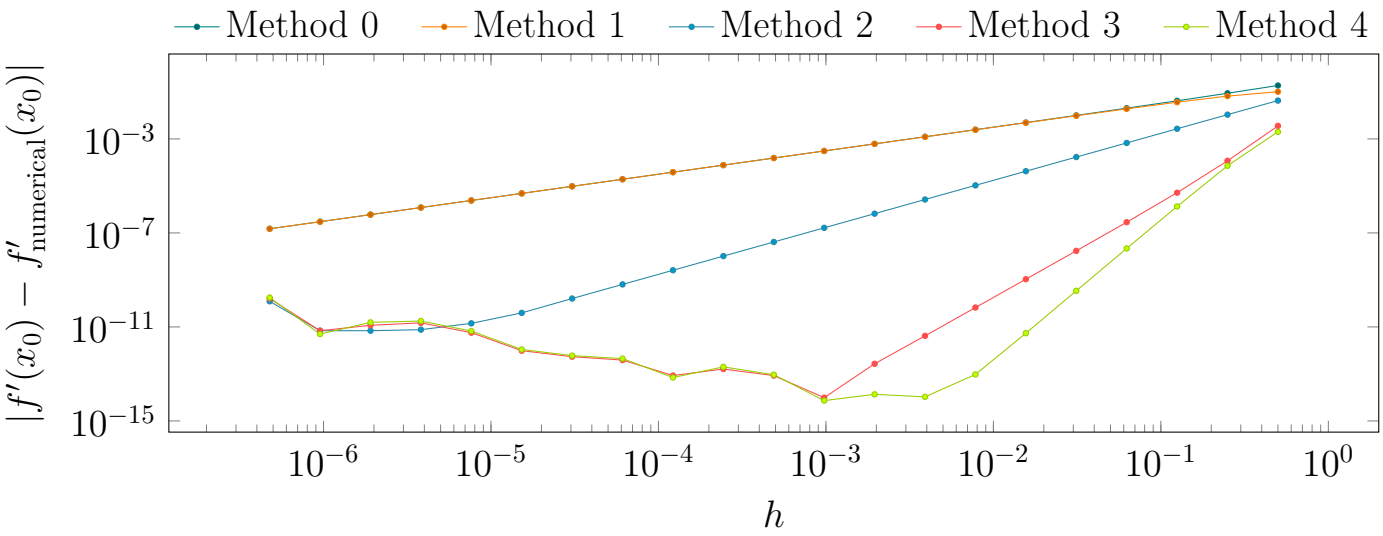
$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \tag{2}$$

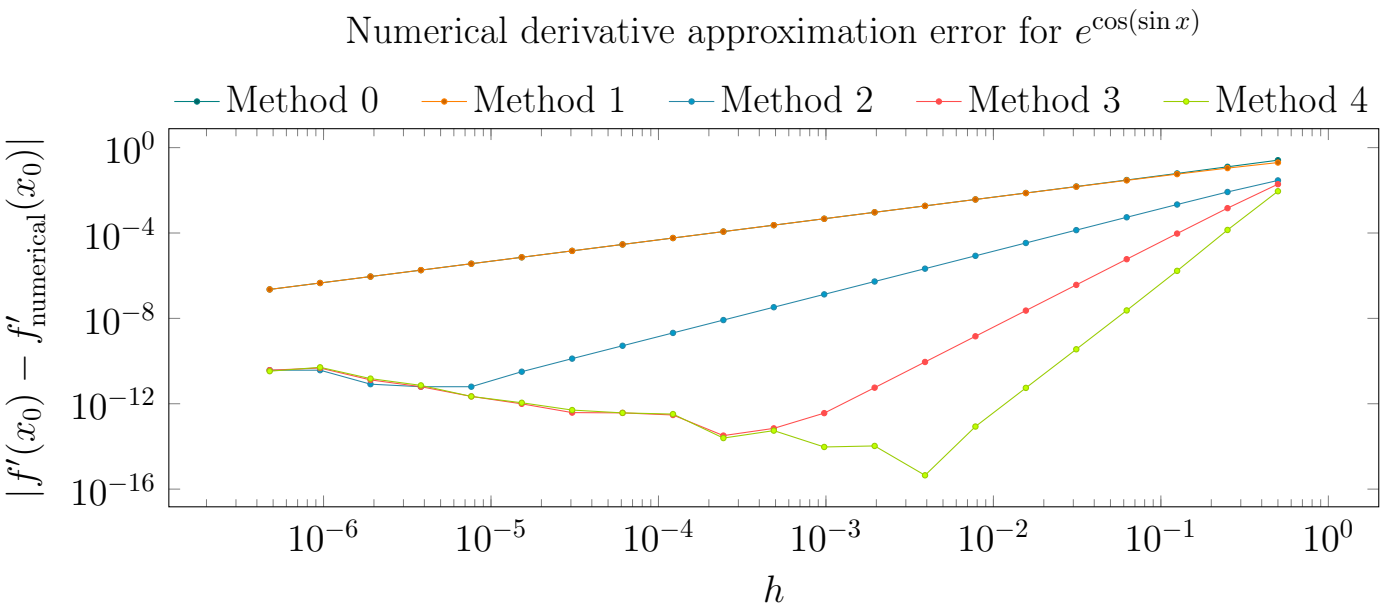
$$f'(x) = \frac{4}{3}\frac{f(x+h) - f(x-h)}{2h} - \frac{1}{3}\frac{f(x+2h) - f(x-2h)}{4h} \tag{3}$$

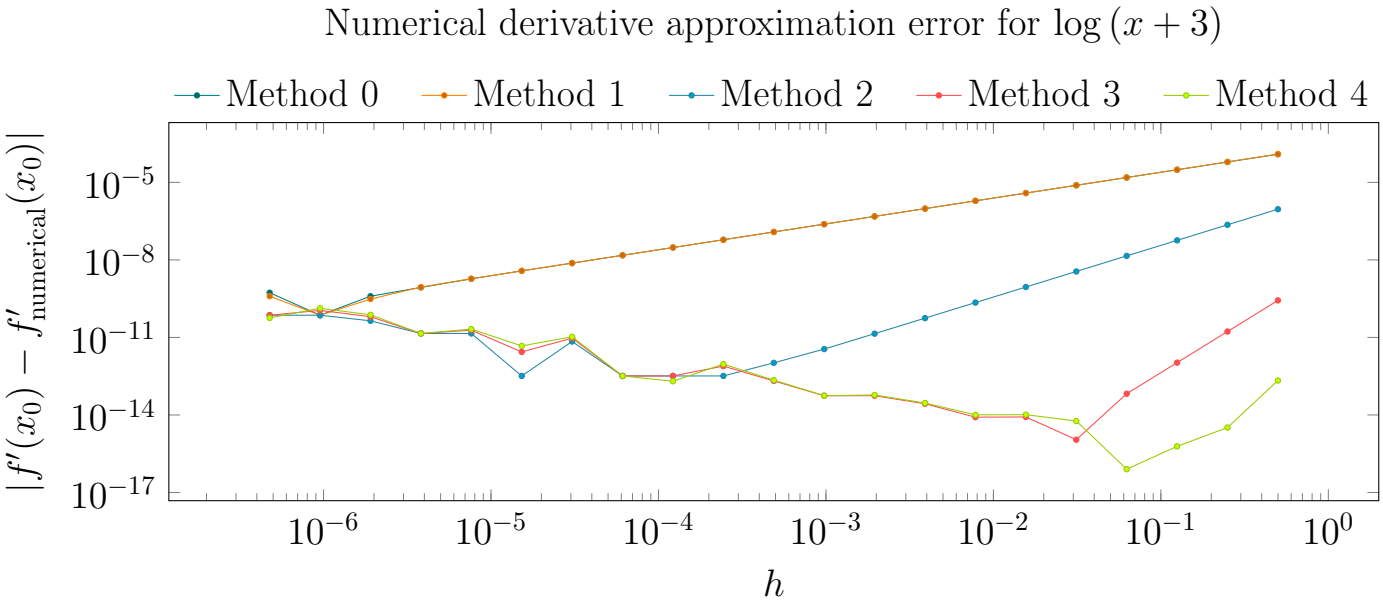
$$f'(x) = \frac{3}{2}\frac{f(x+h) - f(x-h)}{2h} - \frac{3}{5}\frac{f(x+2h) - f(x-2h)}{4h} + \frac{1}{10}\frac{f(x+3h) - f(x-3h)}{6h} \tag{4}$$

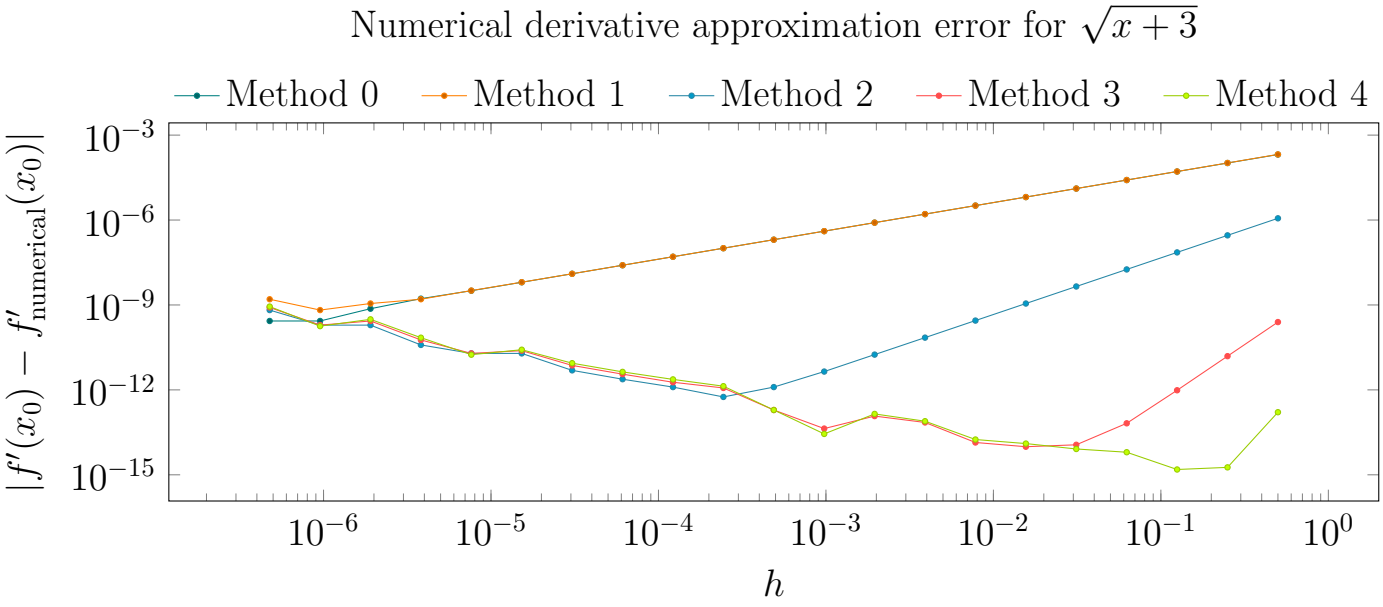
Results


Numerical derivative approximation error for $\sin x^2$
Method 0
Method 1
Method 2
Method 3
Method 4
$|f'(x_0) - f'_{\text{numerical}}(x_0)|$
10^{2}
10^{-2}
10^{-6}
10^{-10}
10^{-14}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
h


Numerical derivative approximation error for $\cos(\sin x)$
Method 0
Method 1
Method 2
Method 3
Method 4
$|f'(x_0) - f'_{\text{numerical}}(x_0)|$
10^{-3}
10^{-7}
10^{-11}
10^{-15}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
h


Numerical derivative approximation error for $e^{\cos(\sin x)}$
Method 0
Method 1
Method 2
Method 3
Method 4
$|f'(x_0) - f'_{\text{numerical}}(x_0)|$
10^{0}
10^{-4}
10^{-8}
10^{-12}
10^{-16}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
h


Numerical derivative approximation error for $\log(x+3)$
Method 0
Method 1
Method 2
Method 3
Method 4
$|f'(x_0) - f'_{\text{numerical}}(x_0)|$
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
h


Numerical derivative approximation error for $\sqrt{x+3}$
Method 0
Method 1
Method 2
Method 3
Method 4
$|f'(x_0) - f'_{\text{numerical}}(x_0)|$
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
h