Tutorial 6B: Inductive proofs

Exercise 1: Below is an inductive definition of the Church numerals.

$$N'_0 = x \qquad N'_{m+1} = f\ N'_m \qquad N_m = \lambda f.\lambda x.\ N'_m$$

Using this definition:

a) Prove that succ $N_m \to^*_\beta N_{m+1}$. You will not need induction.

b) Prove by induction on m that $N'_m[N'_k/x] = N'_{m+k}$.

c) Use this to prove that add $N_m\ N_k \to^*_\beta N_{m+k}$.

d) Prove by induction on m that $N'_m[\lambda x.\ N'_k/f] \to^*_\beta N'_{m\times k}$, using (b) above.

e) Use this to prove that mul $N_m\ N_k \to^*_\beta N_{m\times k}$.