Tutorial 7B: Simple types for the lambda-calculus

Exercise 1: What are the types of the following typed λ-terms? In each case, give a typing derivation to demonstrate that the term really is well-typed.

a) $\lambda x^o.x$

b) $\lambda x^o.\lambda y^o.\lambda z^{o\to o}.zx$

c) $\lambda f^{(o\to o)\to o}.\lambda g^{o\to o\to o}.f(\lambda x^o.gxx)$

Exercise 2: Can the term $(\lambda x.x)(\lambda y.y)$ be typed? That is, are there types with which one can label the variables x and y such that the term is well-typed? If not, why not?

Exercise 3: In the lecture, we said that the Church numerals could be typed, with type

$$(o \to o) \to o \to o.$$

(Let us call this type N for short.)

a) Give a type derivation to show that the successor operator

$$\lambda n^N.\lambda f^{o\to o}.\lambda x^o.f\ (n\ f\ x)$$

can be given type $N \to N$.

b) Write down the type you would expect the addition operator to have.

c) What does the term for addition look like when its bound variables are labelled appropriately?

d) Give a type derivation for the addition term.

e) Now consider the term for exponentiation: can that be similarly typed?