Tutorial 8B: Type inference

Exercise 1: In the lambda-calculus, the **values** or **answers** of a computation are the **normal forms** of reduction: those λ-terms without redexes.

a) Convince yourself that if a term is in normal form, it has the shape

$$\lambda x_1. \ldots \lambda x_n. x\, N_1 \ldots N_k$$

where each N_i is in normal form. (Either n or k may be zero, that is, a term might just look like $x N_1 \ldots N_k$ or like $\lambda x_1.\lambda x_2.x$, for instance.)

b) Using this characterization, write down all the closed normal forms (i.e. normal forms with no free variables) of the following types:

- $o \to o$
- $o \to o \to o$
- $(o \to o) \to o \to o$
- $(o \to o) \to o$

Exercise 2: Recall the following standard terms in the lambda-calculus:

$$\begin{aligned}
\mathsf{true} &\triangleq \lambda x.\lambda y.\, x \\
\mathsf{false} &\triangleq \lambda x.\lambda y.\, y \\
\mathsf{iszero} &\triangleq \lambda n.\, n\, (\lambda f.\, \mathsf{false})\, \mathsf{true} \\
\mathsf{add} &\triangleq \lambda m.\lambda n.\lambda f.\lambda x.\, m\, f\, (n\, f\, x)
\end{aligned}$$

Find the most general types of the following terms, or show that none exists.

a) the numerals 2, 1, and 0

b) the terms true and false

c) add

d) $\mathsf{add}\; 0\; 1$

e) iszero

f) $\mathsf{iszero}\; 2$

g) $\lambda x.xx$

h) $\lambda x.\lambda f.\, x\, (f\, x)$

i) $\lambda f.\lambda x.\lambda y.\, f\, (x\, y)\, (y\, x)$

j) $\lambda f.\, f\, (\lambda x.\, f\, (\lambda y.\, x))$