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calibmsm: An R package for calibration plots of the transition probabilities in a multistate model

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Abstract

Multistate models, which allow the prediction of complex multistate survival processes such as multimorbidity, or recovery, relapse and death following treatment for cancer, are beginning to be used for clinical prediction. It is paramount to evaluate the calibration (as well as other metrics) of a risk prediction model before implementation of the model. Currently no software exists to aid in assessing the calibration of a multistate model. calibram has been developed to fill this gap, providing easy to use software for model developers. Calibration of the transition probabilities between given follow up times is made possible through three approaches. The first two utilise calibration techniques for binary and multinomial logistic regression models in combination with inverse probability of censoring weights, whereas the third utilises psuedo-values. All methods are implemented in conjunction with landmarking to allow calibration assessment of predictions made at any time beyond the start of follow up.

This article details the methodology and provides a comprehensive example on how to assess the calibration of a model developed to predict recovery, adverse events, relapse and survival in patients with blood cancer after a transplantation. This is an illustrative example to showcase how to use the package and motivate a discussion around which of the calibration methods is most appropriate.

Keywords: clinical prediction, calibration, validation, multistate, multi-state, R.

1. Introduction

Risk prediction models enable the prediction of clinical events in either diagnostic or prognostic settings (van Smeden et al. 2021) and are used widely to inform clinical practice. A multistate model (Putter et al. 2007) may be used when there are multiple outcomes of interest, or when a single outcome of interest may be reached via intermediate states. For example, prediction of death after local recurrence or distant metastasis in patients with breast cancer following surgery (Putter et al. 2006); prediction of death following progression of chronic kidney disease (Lintu et al. 2022); prediction of non-AIDS events and death in individuals living with HIV (Masia et al. 2017). Using a multistate model for prediction is important when the development of an intermediate condition occurring post index date may have an impact on the risk of future outcomes of interest. Risk prediction models developed for use in clinical practice should be evaluated in a relevant cohort, or preferably multiple settings/cohorts, prior to implementation (Steverberg and Harrell Jr 2016). If the intended use of this model is known, targeted validation in a specific setting may be preferred (Sperrin et al. 2022). A key part of the validation process is assessment of the calibration of the model (Van Calster et al. 2019). Calibration assesses whether the predicted risks match the observed event rates in the cohort of interest. Ideally calibration curves should be produced, which estimate observed event rates as a function the predicted risks over the entire distribution of predicted risk. This corresponds to a moderate assessment of calibration (Van Calster et al. 2016).

The R package **mstate** (de Wreede *et al.* 2011) provides a comprehensive set of tools to develop a multistate model for a continuously observed multistate survival process. However, currently no software exists to aid researchers in assessing the calibration of a multistate model that has been developed for the purposes of individual risk prediction. **calibmsm** has been developed to enable researchers to estimate calibration curves and scatter plots using three approaches outlined by Pate et al (XXXX REF project 6, may have to put on Arxiv), which focused on assessing the calibration of the transition probabilities out of the starting state. The work in this paper extends the framework to assess the calibration of transition probabilities out of any state j at any time s using landmarking (van Houwelingen 2007; Dafni 2011), provides more details on estimation of the inverse-probability of censoring weights (where relevant), and demonstrates the process for estimating confidence intervals. **calibmsm** is available from the Comprehensive R Archive Network at REF XXXX.

de Wreede et al. (2011) used data from the European Society for Blood and Marrow Transplantation (EBMT 2023) to showcase how to develop a multistate model for clinical prediction of outcomes after bone morrow transplantation in leukemia patients (Figure 1). In this study, we show how to assess the calibration of a model developed on the same EBMT data as a way of illustrating the syntax and workflows of calibmsm. This clinical example also highlights some important differences between the methods in how they deal with informative censoring and computational feasibility, which may impact future uptake of the methods. Details on the methodology are given in section 2. The clinical setting for our example and steps for data preparation and are described in section 3. In section 4 we show how to estimate calibration curves and scatter plots using calibmsm. Section 5 contains a discussion and summary.

2. Methodology

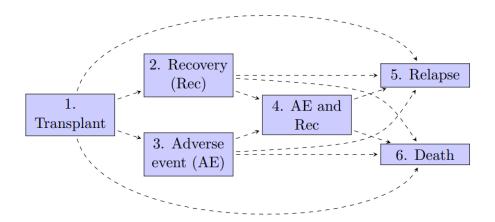


Figure 1: A six-state model for leukemia patients after bone marrow transplantation. Figure taken from (de Wreede *et al.* 2011).

Let $X(t) \in \{1, ..., K\}$ be a multistate survival process with K states. We assume a multistate model has already been developed and we want to assess the calibration of the predicted transition probabilities, $\hat{p}_{j,k}(s,t)$, in a cohort of interest. The transition probabilities are the probability of being in state k at time t, if in state j at time s, where s < t. To assess the calibration of the multistate model, we must estimate observed event probabilities:

$$o_{j,k}(s,t) = P[X(t) = k | X(s) = j, \hat{p}_{j,k}(s,t)].$$

In a well calibrated model, the transition probabilities will be equal to the observed event probabilities.

In the absence of censoring, $o_{j,k}(s,t)$ can be estimated using cross sectional calibration techniques in a landmark (van Houwelingen 2007; Dafni 2011) cohort of individuals who are in state j at time s (i.e. methods to assess the calibration of models predicting binary or multinomial outcomes). In the presence of censoring, calibration must be assessed in this landmark cohort of individuals either using these cross sectional techniques in combination with inverse probability of censoring weights, or through pseudo-values. These approaches are detailed in sections 2.2 - 2.6.

2.2. Binary logistic regression with inverse probability of censoring weights (BLR-IPCW) calibration curves

The first approach produces calibration curves using a framework for binary logistic regression models in conjunction with inverse probability of censoring weights to account for informative censoring. Let $I_k(t)$ be an indicator for whether an individual is in state k at time t. $I_k(t)$ is then modeled using a flexible approach with $\hat{p}_{j,k}(s,t)$ as the sole predictor. This model is fit in the landmark cohort (in state j at time s) of individuals who are also still uncensored at time t. This cohort is weighted using inverse probability of censoring weights (see section 2.4). We suggest using a loess smoother (Austin and Steyerberg 2014):

$$I_k(t) = \operatorname{loess}(\hat{p}_{i,k}(s,t)),\tag{1}$$

or a logistic regression model with restricted cubic splines (Harrell 2015):

$$logit(I_k(t)) = rcs(logit(\hat{p}_{i,k}(s,t))). \tag{2}$$

Any flexible model for binary outcomes could be used, but these are the most common and are implemented in this package. Observed event probabilities $\hat{o}_{j,k}(s,t)$ are then estimated as fitted values from these models. The calibration curve is plotted using the set of points $\{\hat{p}_{j,k}(s,t),\hat{o}_{j,k}(s,t)\}$. To obtain unbiased calibration curves, the assumption that each outcome $I_k(t)$ is independent from the censoring mechanism in the reweighted population must hold.

2.3. Multinomial logistic regression with inverse probability of censoring weights (MLR-IPCW) calibration scatter plots

The second approach produces calibration scatter plots using a framework for multinomial logistic regression models with inverse probability of censoring weights (MLR-IPCW). Let $I_X(t)$ be an multinomial indicator variable taking values $I_X(t) \in \{1, ..., K\}$ such that $I_X(t) = k$ if an individual is in state k at time t. The nominal recalibration framework of Van Hoorde et al. (2014, 2015) is then applied in the landmark cohort of individuals uncensored at time t, weighted using inverse probability of censoring weights (section 2.4). First calculate the log-ratios of the predicted transition probabilities:

$$\hat{LP}_k = ln\left(\frac{\hat{p}_{j,k}(s,t)}{\hat{p}_{j,k_{ref}}(s,t)}\right),$$

Then fit the following multinomial logistic regression model:

$$ln\left(\frac{P[I_X(t)=k]}{P[I_X(t)=k_{ref}]}\right) = \alpha_k + \sum_{h=2}^K \beta_{k,h} * s_k(\hat{LP}_h), \tag{3}$$

where k_{ref} is an arbitrary reference category which can be reached from state j, $k \neq k_{ref}$ takes values in the set of states that can be reached from state j, and where s is a vector spline smoother (Yee 2015). Observed event probabilities $\hat{o}_{j,k}(s,t)$ are then estimated as fitted values from this model. This results in a calibration scatter plot rather than a curve due to all states being modeled simultaneously, as opposed to BLR-IPCW, which is a "one vs all" approach. The scatter occurs because the observed event probabilities for state k vary depending on the predicted transition probabilities of the other states. This is a stronger (Van Calster et al. 2016) form of calibration than that evaluated by BLR-IPCW, and will also result in observed event probabilities which sum to 1. In future iterations of calibrasm functionality will be added to produce smoothed curves estimated from these scatter plots. To obtain unbiased calibration curves, the assumption that the outcome $I_X(t)$ is independent from the censoring mechanism in the reweighted population must hold.

2.4. Estimation of the inverse probability of censoring weights

The estimand for the weights is $w_j(s,t)$, the inverse of the probability of being uncensored at time t if in state j at time s:

$$w_j(s,t) = \frac{1}{P[t_{cens} > t | t > s, X(s) = j, \mathbf{Z}, \mathbf{X(t)}]},$$

where $\mathbf{X}(\mathbf{t})$ denotes the history of the multistate survival process up to time t, including the transition times, and \mathbf{Z} is a set of baseline predictor variables believed to be predictive of the censoring mechanism. Note that \mathbf{Z} may be the same as, but is not restricted to, the variables used for prediction when developing the multistate model. First the estimator $\hat{P}[t_{cens} > t | t > s, X(s) = j, \mathbf{Z}]$ is calculated by developing an appropriate survival model. The outcome in this model is the time until censoring occurs. Moving into an absorbing state prevents censoring from happening and is treated as a censoring mechanism in this model (i.e. a competing risks approach is not taken when fitting this model). $\mathbf{X}(\mathbf{t})$ is explicitly conditioned on when defining $w_j(s,t)$ because the weights must reflect that censoring can no longer be observed for an individual if they enter an absorbing state at some time $s < t_{abs} < t$. Therefore

$$\hat{P}[t_{cens} > t | t > s, X(s) = j, \mathbf{Z}, \mathbf{X(t)}] = \hat{P}[t_{cens} > min\{t, t_{abs}\} | t > s, X(s) = j, \mathbf{Z}]$$

In **calibmsm**, unless otherwise specified, $\hat{P}[t_{cens} > t | t > s, X(s) = j, \mathbf{Z}]$ is estimated using a cox proportional hazards model where all predictors \mathbf{Z} are assumed to have a linear effect on the log-hazard. This is highly restrictive, users can therefore also input their own vector of weights, which is strongly recommended. Given the BLR-IPCW and MLR-IPCW approaches are both reliant on correct estimation of the weights, we encourage users to take the time to carefully estimate the inverse probability of censoring weights using a well specified model. The limitations of using the **calibmsm** internal functions for estimating the weights in this clinical example (section ??) are discussed in more detail later, and explored in vignette-Evaluation-of-estimation-of-IPCWs.

Stabilised weights can be estimated by multiplying by the weights $w_j(s,t)$ by the mean probability of being uncensored:

$$w_j^{stab}(s,t) = \frac{P[t_{cens} > t | t > s, X(s) = j]}{P[t_{cens} > t | t > s, X(s) = j, \mathbf{Z}, \mathbf{X(t)}]}.$$

The numerator can be estimated using an intercept only model, and note there is no dependence on X(t).

Another option is to estimate w(s,t), which is the inverse of the probability of being uncersored at time t if uncensored at time s:

$$w(s,t) = \frac{1}{P[t_{cens} > t | t > s, \mathbf{Z}, \mathbf{X(t)}]}.$$

This can be estimated using the same approach as for $w_j(s,t)$, except there is no requirement to be in state j when landmarking at time s. If the censoring mechanism is non-informative after conditioning on \mathbf{Z} , then $w(s,t)=w_j(s,t)$, and any consistent estimator for w(s,t) will be a consistent estimator of $w_j(s,t)$. The advantage is that $\hat{w}(s,t)$ is calculated by developing a model in the cohort of individuals uncensored at time s, which is a larger cohort than those uncensored and in state j at time s. Therefore $\hat{w}(s,t)$ will be a more precise estimator than $\hat{w}_j(s,t)$. On the contrary, if the assumption of non-informative censoring after conditioning on

Z does not hold, there is a risk of bias in estimation of the weights. We therefore recommend using the estimator $w_j(s,t)$ unless sample size (number of individuals in state j at time s) is low, which may be assessed using sample size formula for prediction models with time-to-event outcomes (Riley et al. 2019). If the sample size is deemed insufficient, one may consider using w(s,t), but the risk of bias associated with this estimator must be carefully considered. Finally, we state the importance of using inverse probability of censoring weights, even if the censoring mechanism is believed to be completely non-informative (i.e. happens at random). All multi-state models must have an absorbing state, entry into which prevents censoring from happening. This induces a dependence between the outcome and the censoring mechanism which must be adjusted for using inverse probability of censoring weights. This is issue was highlighted in the supplementary material of previous work XXXX.

2.5. Pseudo-value calibration plots

The third approach produces calibration curves using pseudo-values (Andersen and Pohar Perme 2010; Andersen et al. 2022). Pseudo-values can be used in place of the outcome of interest in a regression model if some outcomes are not observed due to right censoring. This is the case in models (1) and (2). For certain estimators $\hat{\theta}$ (where θ estimates the expectation of the outcome it is replacing), the pseudo-value for individual i is defined as:

$$\hat{\theta}^i = n * \hat{\theta} - (n-1) * \hat{\theta}^{-i},$$

where $\hat{\theta}^{-i}$ is equal to $\hat{\theta}$ estimated in a cohort without individual i. One such estimator for the outcomes in models (1) and (2) given the underlying multistate survival process, is the Landmark Aalen-Johansen estimator (Putter and Spitoni 2018), which estimates the expectation of $I_k(t)$ in the landmark cohort of individuals in which calibration is being assessed. The resulting pseudo-values are a vector with K elements, one for each possible transition, for every individual i. These pseudo-values can replace the outcome $I_k(t)$ in equations (1) and (2) in order to estimate $o_{j,k}(s,t)$.

Pseudo-values are based on the same assumptions as the underlying estimator $\hat{\theta}$. The Landmark Aalen-Johansen estimator is valid for both Markov and non-Markov multistate models. However, it does make the assumption that the multistate survival process and the censoring distribution are independent (uninformative censoring). The approach to alleviate this is to estimate the pseudo-values within sub-groups of individuals, now making the assumption that censoring is non-informative within the specified subgroups. This can be done by calculating the pseudo-values within subgroups defined by baseline predictors, or subgroups defined by the predicted transition probabilities $\hat{p}_{j,k}(s,t)$. Both options are implemented in this package. When pseudo-values are calculated within subgroups, they are still used as the outcome in models (1) and (2) in the same way. Note that the pseudo-values $\hat{\theta}^i$ are continuous, as opposed to binary $I_k(t)$, but the link function in model (2) remains the same to ensure $\hat{o}_{j,k}(s,t)$ are between zero and one.

2.6. Estimation of confidence intervals

Confidence intervals for both BLR-IPCW and pseudo-value calibration curves can be estimated using bootstrapping. While theoretically feasible, it is currently unclear how to present confidence intervals for each data point in the calibration scatter plots produced by MLR-

IPCW, and therefore these are omitted. A process for estimating the confidence intervals around the BLR-IPCW calibration curves is as follows:

- 1. Resample validation dataset with replacement
- 2. Landmark the dataset for assessment of calibration
- 3. Calculate inverse probability of censoring weights
- 4. Fit the preferred calibration model in the landmarked dataset (restricted cubic splines or loess smoother)
- 5. Generate observed event probabilities for a fixed vector of predicted transition probabilities (specifically the predicted transition probabilities from the non-bootstrapped landmark validation dataset)

This will produce a number of bootstrapped calibration curves, all plotted over the same vectors of predicted transition probabilities. Taking the $\frac{\alpha}{2}$ and $(1-\frac{\alpha}{2})$ percentiles of the observed event probabilities for each predicted transition probability gives the required $1-\alpha$ confidence interval around the estimated calibration curve. To estimate confidence intervals for the pseudo-value calibration curves using bootstrapping, the same procedure is applied except the third step is replaced with 'calculate the pseudo-values within the landmarked bootstrapped dataset'. This will be highly computationally demanding as the pseudo-values must be estimated in every bootstrap dataset.

If using the pseudo-value method, confidence intervals can however be calculated using parametric estimates of the standard error when making predictions of the observed event probabilities (i.e. when obtaining fitted values from models (1) or (2)). We recommended this due to the computational burden of bootstrapping the confidence intervals around the pseudo-value calibration curves. There are a number of issues with estimating parametric confidence intervals for the BLR-IPCW calibration curves. Firstly, a robust sandwich-type estimator should be used to estimate the standard error Hernan and Robins (2020), which are known to result in conservative confidence intervals, i.e. too large Hernan and Robins (2020); Austin et al. (2020). On the contrary, the size of the confidence interval will be underestimated as uncertainty in estimation of the weights is not considered. Due to the impact of these two factors, we recommend using bootstrapping to estimate the confidence intervals for BLR-IPCW calibration curves.

3. Clinical example

3.1. Clinical setting and data preparation

We utilise data from the European Society for Blood and Marrow Transplantation (EBMT 2023), containing multistate survival data after a transplant for patients with blood cancer. The start of follow up is the day of the transplant and the initial state is alive and in remission. There are three intermediate events (2: recovery, 3: adverse event, or 4: recovery + adverse event), and two absorbing states (5: relapse and 6: death). This data is available from the mstate package (de Wreede et al. 2011). We assume the user of calibmsm has experience with

R> library("calibmsm")

R> data("msebmtcal")

handling the type of data used to develop a multistate model as outlined by de Wreede *et al.* (2011).

Four datasets are provided to enable assessment of a multistate model fitted to these data. The code for deriving all these datasets is provided in the source code for calibmsm. The first is ebmtcal, which is the same as the ebmt dataset provided in mstate, with two extra variables derived: time until censoring (dtcens) and an indicator for whether censoring was observed (dtcens.s = 1) or an absorbing state was entered (dtcens.s = 0). This dataset contains baseline information on year of transplant (year), age at transplant (age), prophylaxis given (proph), and whether the donor was gender matched (match). The second dataset provided is msebmtcal, which is the ebmt dataset converted into a dataset of class msdata using the processes and functions in the package mstate (de Wreede et al. 2011). It contains all transition times, an event indicator for each transition, as well as a trans attribute containing the transition matrix.

```
R> data("ebmtcal")
R> head(ebmtcal)
  id
                    ae ae.s recae recae.s
                                              rel rel.s
      rec rec.s
                                                           srv srv.s
   1
       22
                   995
                           0
                               995
                                           0
                                              995
                                                       0
                                                          995
                                                                    0
1
               1
   2
       29
                                                           579
2
               1
                    12
                           1
                                29
                                           1
                                              422
                                                       1
                                                                    1
3
   3 1264
               0
                    27
                           1
                              1264
                                           0 1264
                                                       0 1264
                                                                    0
4
   4
       50
               1
                    42
                           1
                                 50
                                           1
                                               84
                                                       1
                                                           117
                                                                    1
                                                                    0
5
   5
       22
                 1133
                           0
                              1133
                                           0
                                              114
                                                       1 1133
               1
6
       33
               1
                    27
                           1
                                33
                                           1 1427
                                                       0 1427
                                                                    0
       year agecl proph
                                         match dtcens dtcens.s
1 1995-1998 20-40
                       no no gender mismatch
                                                    995
                                                                1
2 1995-1998 20-40
                       no no gender mismatch
                                                    422
                                                                0
3 1995-1998 20-40
                                                                1
                          no gender mismatch
                                                   1264
4 1995-1998 20-40
                              gender mismatch
                                                     84
                                                                0
                       no
                                                                0
5 1995-1998
               >40
                              gender mismatch
                                                    114
                       no
6 1995-1998 20-40
                       no no gender mismatch
                                                   1427
                                                                1
```

```
id from to trans Tstart Tstop time status
    1
           1
               2
                       1
                                             22
1
                                0
                                       22
                                                        1
2
           1
               3
                       2
                                0
                                       22
                                             22
                                                        0
     1
3
     1
           1
               5
                       3
                                0
                                       22
                                             22
                                                        0
               6
                       4
                                0
                                       22
                                             22
                                                        0
4
    1
           1
5
     1
           2
               4
                       5
                               22
                                     995
                                            973
                                                        0
           2
               5
6
     1
                       6
                               22
                                     995
                                            973
                                                        0
           2
7
     1
               6
                       7
                               22
                                     995
                                            973
                                                        0
    2
           1
               2
                                                        0
8
                       1
                                0
                                       12
                                             12
    2
               3
                       2
                                0
                                       12
9
           1
                                             12
                                                        1
```

R> subset(msebmtcal, id %in% c(1,2,3))

10	2	1	5		3	0		12		12	0
11	2	1	6		4	0		12		12	0
12	2	3	4		8	12		29		17	1
13	2	3	5		9	12		29		17	0
14	2	3	6	1	0	12		29		17	0
15	2	4	5	1	1	29		422	3	93	1
16	2	4	6	1	2	29		422	3	93	0
17	3	1	2		1	0		27		27	0
18	3	1	3		2	0		27		27	1
19	3	1	5		3	0		27		27	0
20	3	1	6		4	0		27		27	0
21	3	3	4		8	27	1	264	12	37	0
22	3	3	5		9	27	1	264	12	37	0
23	3	3	6	1	0	27	1	264	12	37	0

In the work of de Wreede $et\ al.\ (2011)$, the focus is on predicting transition probabilities made at times s=0 and s=100 days, across a range of follow up times t, and comparing prognosis for patients in different states j. In this study we also focus on assessing the calibration of the transition probabilities made at these times. We assess calibration of the transition probabilities at t=5 years, a common follow up time for cancer prognosis, but calibration of the model may vary for other values of t. We estimate transition probabilities for each individual by developing a model as demonstrated in de Wreede $et\ al.\ (2011)$, following the theory of Putter $et\ al.\ (2007)$.

The predicted transitions probabilities from each state j at times s=0 and s=100 are contained in stacked datasets tps0 and tps100 respectively. A leave-one-out approach was used when estimating these transition probabilities. This means each individual was removed from the development dataset when fitting the multistate model to estimate their transition probabilities. This approach allows validation to be assessed in the same dataset that the model was developed with minimal levels of in-sample optimism. Note that for tps100 the predicted probabilities for some states k are equal to 0. This is because no individuals in state j=1 at time s=100 transition into states 3 or 4. This may be due to the definition of an adverse event having to occur within a certain number of days post transplant.

```
R> data("tps0")
R> head(tps0)
```

```
pstate1
                 pstate2
                            pstate3
                                      pstate4
                                                pstate5
                                                          pstate6
  1 0.1139726 0.2295006 0.08450376 0.2326861 0.1504855 0.1888514
  2 0.1140189 0.2316569 0.08442692 0.2328398 0.1481977 0.1888598
  3 0.1136646 0.2317636 0.08274331 0.2325663 0.1504787 0.1887834
  4 0.1383878 0.1836189 0.07579429 0.2179331 0.1538475 0.2304185
  5 0.1233226 0.1609740 0.05508100 0.1828176 0.1425950 0.3352099
  6 0.1136646 0.2317636 0.08462424 0.2305854 0.1505534 0.1888087
                               se3
                                                                 se6 j
1 0.01291133 0.02369584 0.01257251 0.02323376 0.01648630 0.01601795 1
2 0.01291552 0.02374329 0.01256056 0.02324869 0.01632797 0.01603703 1
```

```
3 0.01289444 0.02375770 0.01245752 0.02322375 0.01647890 0.01601525 1 4 0.01857439 0.03004447 0.01462570 0.03018673 0.02124071 0.02416121 1 5 0.01944967 0.03419721 0.01367768 0.03423941 0.02329644 0.03688586 1 6 0.01289444 0.02375770 0.01257276 0.02317348 0.01649531 0.01602438 1
```

R> data("tps100")
R> head(tps100)

```
id
       pstate1
                  pstate2 pstate3 pstate4
                                             pstate5
                                                        pstate6
   1 0.7013881 0.05239271
                                         0 0.1408120 0.1054072
1
                                 0
   2 0.7012745 0.05261136
                                 0
                                         0 0.1407625 0.1053516
  3 0.7011368 0.05270176
                                 0
                                         0 0.1407628 0.1053987
                                         0 0.1700565 0.1045183
  4 0.6840325 0.04139266
                                 0
  5 0.6804049 0.04308434
                                         0 0.1500344 0.1264764
   6 0.7011368 0.05270176
                                 0
                                         0 0.1407628 0.1053987
                                                    se6 j
         se1
                    se2 se3 se4
                                        se5
1 0.04691168 0.02077138
                           0
                               0 0.03457006 0.03081258 1
2 0.04691218 0.02082871
                               0 0.03456448 0.03079617 1
3 0.04693068 0.02086917
                               0 0.03456101 0.03081033 1
4 0.05885230 0.02161973
                               0 0.04710517 0.03673242 1
                           0
5 0.06694739 0.02484634
                           0
                               0 0.04905043 0.04628088 1
6 0.04693068 0.02086917
                           0
                               0 0.03456101 0.03081033 1
```

The procedure for producing calibration plots requires the use of two functions. The first function, calib_blr, calib_pv or calib_mlr, calculates the data for the calibration plot using the methods described in section 2. The second function, plot, produces the plots. Plot is an S3 generic written for objects of class calib_blr, calib_mlr or calib_pv, and produces the calibration plots using ggplot2 (Wickham 2016). Separating these processes allows users to manually estimate bootstrapped calibration curves (see vignette-BLR-IPCW-manual-bootstrap) using the output from calib_blr, calib_pv or calib_mlr. It also allows users the flexibility of producing their own plots utilising the full functionality of ggplot2, rather than being reliant on the S3 generics provided.

The validation cohort must be provided to functions calib_blr, calib_pv and calib_mlr in two different formats. The data.raw argument requires a data.frame (one observation per individual) and is used to fit the calibration models. For methods BLR-IPCW and MLR-IPCW, data.raw should contain variables dtcens (censoring time) and dtcens.s (censoring indicator, dtcens.s = 1 if the individual is censored at time dtcens, dtcens.s = 0 otherwise), plus any baseline predictors \mathbf{Z} used to estimate the weights. For the pseudo-value approach, this dataset should contain any baseline predictors \mathbf{Z} which variables will be grouped by before calculating the pseudo-values. The data.mstate argument requires a dataset of class msdata, which is used to implement the landmarking and estimate the Aalen-Johansen etimator for the pseudo-value approach. A dataset of this class must be produced using the package mstate (de Wreede et al. 2011). Both data.mstate and data.raw should contain corresponding patient ID variables id. The predicted transition probabilities out of state j at time s must then be specified through the tp.pred argument, which must contain a column for each transition k, even if the transition from j to k has zero probability. The rows in

tp.pred must be ordered in the same way as those in data.raw. The datasets described in section 3.1 meet these criteria.

3.2. Calibration plots for the transition probabilities out of state j = 1 at time s = 0

We start by producing calibration curves for the predicted transition probabilities out of state j=1 at time s=0. Given all individuals start in state 1, there is no need to consider the transition probabilities out of states $j\neq 1$ at s=0. Calibration is assessed at follow up time (t=1826 days). We start by extracting the predicted transition probabilities from state j=1 at time s=0 from the object tps0. These are the transition probabilities we aim to assess the calibration of.

We first evaluate calibration using the BLR-IPCW approach, implemented through the function calib_blr. We choose to estimate the calibration curves using restricted cubic splines, although the use of loess smoothers would be equally valid. When using restricted cubic splines the number of knots must always be specified by the user, and 3 knots are chosen here given the reasonably small size of the dataset. Calibration curves could Weights are estimated using the internal estimation procedure and the predictor variables year, agecl, proph and match. The w.landmark.type argument assigns whether weights are estimated using all individuals uncensored at time s, or only those uncensored and in state j at time s, as discussed in section 2.4. The maximum weight (w.max = 10) and stabilisation of weights (stabilised = TRUE) are left as default. Weights can also be manually specified using the weights argument. We request 95% confidence intervals for the calibration curves calculated through bootstrapping with 200 bootstrap replicates.

The first element of dat.calib.blr (named plotdata) contains 6 data frames for the calibration curves of the transition probabilities into each of the six states, $k \in \{1, 2, 3, 4, 5, 6\}$. Each data frame contains three columns, id: the identifier of each individual; pred: the

predicted transition probabilities; obs: the observed event probabilities. The second element (named metadata) is a metadata argument containing information about the data and chosen calibration analysis.

R> str(dat.calib.blr[["plotdata"]])

```
List of 6
 $ state1:'data.frame':
                               1778 obs. of 5 variables:
               : int [1:1778] 2 4 5 7 10 13 14 16 18 19 ...
  ..$ id
               : num [1:1778] 0.114 0.1384 0.1233 0.0974 0.1137 ...
  ..$ pred
  ..$ obs
               : num [1:1778] 0.11 0.104 0.105 0.124 0.11 ...
  ..$ obs.lower: num [1:1778] 0.0882 0.0793 0.0841 0.0847 0.0882 ...
  ..$ obs.upper: num [1:1778] 0.136 0.128 0.129 0.176 0.137 ...
 $ state2:'data.frame':
                               1778 obs. of 5 variables:
               : int [1:1778] 2 4 5 7 10 13 14 16 18 19 ...
  ..$ id
               : num [1:1778] 0.232 0.184 0.161 0.212 0.232 ...
  ..$ pred
               : num [1:1778] 0.17 0.186 0.176 0.179 0.17 ...
  ..$ obs
  ..$ obs.lower: num [1:1778] 0.119 0.157 0.144 0.144 0.119 ...
  ..$ obs.upper: num [1:1778] 0.225 0.216 0.208 0.211 0.225 ...
 $ state3:'data.frame':
                               1778 obs. of 5 variables:
               : int [1:1778] 2 4 5 7 10 13 14 16 18 19 ...
               : num [1:1778] 0.0844 0.0758 0.0551 0.0615 0.0844 ...
  ..$ pred
               : num [1:1778] 0.1249 0.1167 0.0919 0.1001 0.1248 ...
  ..$ obs.lower: num [1:1778] 0.0966 0.0878 0.0493 0.0648 0.0966 ...
  ..$ obs.upper: num [1:1778] 0.158 0.147 0.142 0.14 0.158 ...
 $ state4:'data.frame':
                               1778 obs. of 5 variables:
  ..$ id
               : int [1:1778] 2 4 5 7 10 13 14 16 18 19 ...
               : num [1:1778] 0.233 0.218 0.183 0.221 0.233 ...
  ..$ pred
               : num [1:1778] 0.243 0.224 0.185 0.228 0.243 ...
  ..$ obs.lower: num [1:1778] 0.197 0.191 0.158 0.193 0.197 ...
  ..$ obs.upper: num [1:1778] 0.287 0.262 0.217 0.267 0.286 ...
 $ state5:'data.frame':
                               1778 obs. of 5 variables:
               : int [1:1778] 2 4 5 7 10 13 14 16 18 19 ...
  ..$ id
               : num [1:1778] 0.148 0.154 0.143 0.144 0.149 ...
  ..$ pred
               : num [1:1778] 0.191 0.165 0.222 0.212 0.188 ...
  ..$ obs.lower: num [1:1778] 0.165 0.146 0.182 0.177 0.163 ...
  ..$ obs.upper: num [1:1778] 0.219 0.181 0.269 0.255 0.213 ...
 $ state6:'data.frame':
                               1778 obs. of 5 variables:
               : int [1:1778] 2 4 5 7 10 13 14 16 18 19 ...
  ..$ id
  ..$ pred
               : num [1:1778] 0.189 0.23 0.335 0.264 0.189 ...
               : num [1:1778] 0.207 0.254 0.316 0.28 0.207 ...
  ..$ obs.lower: num [1:1778] 0.186 0.23 0.272 0.258 0.186 ...
  ..$ obs.upper: num [1:1778] 0.23 0.281 0.35 0.31 0.23 ...
R> str(dat.calib.blr[["metadata"]])
List of 8
 $ valid.transitions : num [1:6] 1 2 3 4 5 6
```

R> plot(dat.calib.blr, combine = TRUE, nrow = 2, ncol = 3)

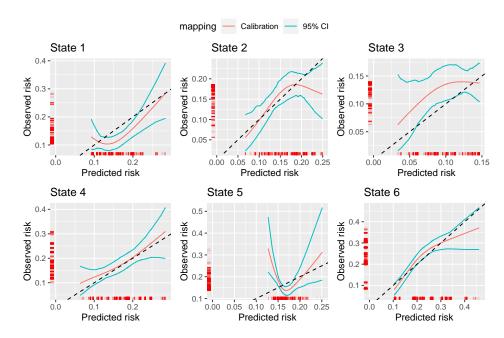


Figure 2: BLR-IPCW calibration curves out of state j = 1 at time s = 0.

```
$ assessed.transitions: num [1:6] 1 2 3 4 5 6
$ CI
                         num 95
$ CI.R.boot
                          num 200
$ curve.type
                              "rcs"
                          chr
$
 j
                         num
                              1
$ s
                         num 0
$ t
                         num 1826
```

Calibration curves can then be generated using plot. The calibration curves (Figure 2) indicate the level of calibration is different for the transition probabilities into each of the different states. The calibration into states 4 and 6 looks the best. State 2 has good calibration over the majority of the predicted risks but over predicts for individuals with the highest predicted risks. Transition probabilities into states 1 and 3 are over and under predicted respectively over most of the range of predicted risks. Importantly the calibration of the transition probabilities into state 5 (Relapse), a key clinical outcome in this clinical setting, is extremely poor. This could be driven by errors in any of the intermediate competing risks models out of states 1, 2, 3 and 4, which all contribute to the predicted transition probabilities into state 5. Further methodological development is required in order to pin down which of the competing risk sub-models may be driving poor calibration in the transition probabilities from a multistate model.

Next we use the pseudo-value approach to assess calibration, implemented through the function calib_pv. Instead of specifying how the weights are estimated, we now specify variables to define groups within which pseudo-values will be calculated (see section 2.5). The goal is to induce uninformative censoring within the chosen subgroups. We chose to calculate

pseudo-values in individuals with the same year of transplant (group.vars = c("year")), and then split individuals into a further three groups defined by their predicted risk (n.pctls = 3). The number of percentiles should be increased in bigger validation datasets, although guidance on specific numbers is currently lacking. Year of transplant was identified as a subgrouping variable because a later transplant resulted in a shorter possible follow up, an earlier administrative censoring time, and it was therefore highly predictive of being censored. Your data should be explored to identify appropriate variables for subgrouping (see vignette-Evaluation-of-estimation-of-IPCWs). A parametric confidence interval is estimated as recommended in section 2.6.

Calibration curves were then generated using plot. The pseudo-value calibration curves (Figure 3) are largely similar to the BLR-IPCW calibration curves (Figure 2). The agreement in the calibration curves from two completely distinct methods provides reassurance the assessment of calibration is correct. This is with the exception of state k=3, where the pseudo-value calibration plot indicates the transition probabilities are well calibrated, but the BLR-IPCW calibration plot indicates the transition probabilities under predict. In a situation like this, we recommend testing the assumptions made by each of the methods to try and diagnose which are most likely to hold, and what may be driving the difference, and . In this particular example, we hypothesised that the model for estimating the inverse probability of censoring weights may be misspecified due to the strong effect of year of transplant on the censoring mechanism. We explored this theory in more detail (see vignette-Evaluation-of-estimation-of-IPCWs), and concluded that the BLR-IPCW calibration curves may be biased in this particular clinical example due to incorrect estimation of the weights.

Next we use the MLR-IPCW to evaluate calibration which produces a calibration scatter plot. This is done using the calib_mlr function, which has the same inputs as calib_blr.

```
R> dat.calib.mlr <-
+ calib_mlr(data.mstate = msebmtcal,
+ data.raw = ebmtcal,
+ j=1,
+ s=0,
+ t = 1826,</pre>
```

R> plot(dat.calib.pv, combine = TRUE, nrow = 2, ncol = 3)

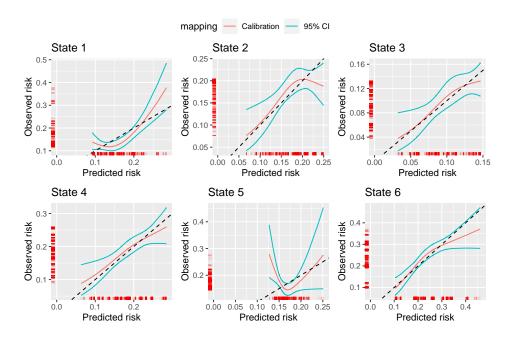


Figure 3: Pseudo-value calibration curves out of state j = 1 at time s = 0.

```
tp.pred = tp.pred.s0,

+ w.covs = c("year", "agecl", "proph", "match"))
```

The MLR-IPCW calibration scatter plots, produced using plot are contained in Figure 4. Within each plot for state k, there is a large amount of variation in calibration of the transition probabilities depending on the predicted transition probabilities into states $\neq k$. One valuable insight from these plots is that the variance in the calibration of the transition probabilities into state 6, is considerably smaller than that of state 4, despite these two states both having good calibration according to the BLR-IPCW plots (arguably state 4 looked better calibrated). This means the calibration of the transition probabilities into state 6 remains reasonably consistent, irrespective of the risks of the other states. On the contrary, the calibration of the predicted transition probabilities into state 4 is more highly dependent on the predicted transition probabilities of the other states. This insight can be gained because MLR-IPCW is a stronger (Van Calster *et al.* 2016) form of calibration assessment than the BLR-IPCW and pseudo-value approaches.

3.3. Calibration plots for the transition probabilities out of states j = 1 and 3 at time s = 100

In the work of de Wreede *et al.* (2011) focus then shifts to comparing transition probabilities when s = 100 depending on whether an individual has had an adverse event (state 3) or remains in state 1 (post transplant). Our focus therefore now shifts to assessing the calibration of these transition probabilities. This is done through landmarking as described in section 2. We start by extracting the predicted transition probabilities from state j = 1 and 3 at time s = 100 from the object tps100. These are the transition probabilities we aim to assess the

R> plot(dat.calib.mlr, combine = TRUE)

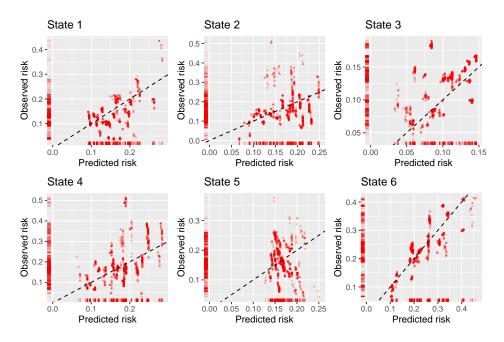


Figure 4: MLR-IPCW calibration scatter plots out of state j = 1 at time s = 0.

calibration of.

The process for estimating the calibration curves remains the same, changing the inputted values j and s, and specifying the appropriate predicted transition probabilities to the argument tp.pred. We start by producing the calibration plots for j=1 and s=100 using the BLR-IPCW (Figure 5) and pseudo-value (Figure 6) methods. Given the small number of data points in this analysis induced by landmarking, we do not produce calibration scatter plots using MLR-IPCW, which may be misleading given the lack of confidence intervals.

```
+
                     curve.type = "rcs",
+
                     rcs.nk = 3,
                     w.covs = c("year", "agecl", "proph", "match"),
                     CI = 95,
                     CI.R.boot = 200)
R> ### Calibration using pseudo-values
  dat.calib.pv.j1.s100 <-</pre>
     calib_pv(data.mstate = msebmtcal,
+
                    data.raw = ebmtcal,
                    j=1,
                    s=100,
                    t = t.eval,
                    tp.pred = tp.pred.j1s100,
                    curve.type = "rcs",
                    rcs.nk = 3,
                    group.vars = c("year"),
                    CI = 95,
                    CI.type = "parametric")
```

There are only four calibration plots because no individuals in state j=1 at time s=100 are in states k=3 (adverse event) or k=4 (recovery + adverse event) after t=1826 days. We believe this is due to the definition of an adverse event occurring within 100 days, but as secondary users of the data, cannot be sure about this. The calibration of the predicted transition probabilities is very poor. Only for state k=6 is the observed risk a monotonically increasing function of the predicted transition probabilities. We follow this up with the pseudo-value calibration plots (Figure 6) which leads to similar conclusions, as again only state k=6 has a monotonically increasing calibration curve. The confidence intervals are very large. For states k=2 and k=5, we cannot rule out that the poor calibration is a result of sampling variation as opposed to a poorly performing prediction model. A larger validation dataset would be required to get to the bottom of this. There is a major issue with the calibration of the transition probabilities of staying in state 1, as the predicted risk is inversely proportional to the observed event rate.

Next we produce calibration plots for j=3 and s=100 using the BLR-IPCW (Figure 7) and pseudo-value (Figure 8) methods.



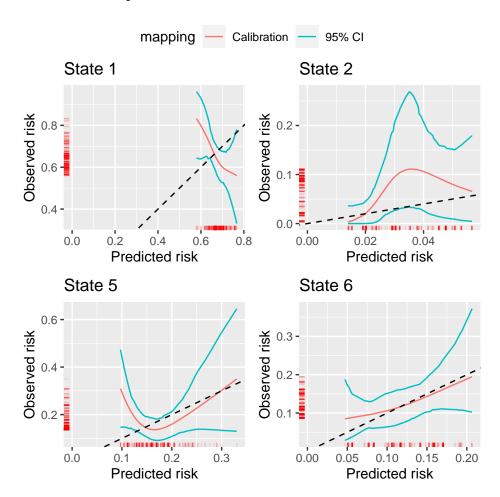


Figure 5: BLR-IPCW calibration curves out of state j=1 at time s=100.

R > plot(dat.calib.pv.j1.s100, combine = TRUE, nrow = 2, ncol = 2)

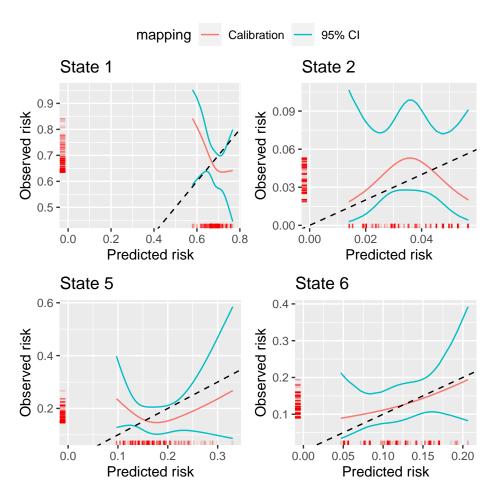


Figure 6: Pseudo-value calibration curves out of state j=1 at time s=100.

```
+ CI.R.boot = 200)
R> ### Calibration using pseudo-values
R> dat.calib.pv.j3.s100 <-
+ calib_pv(data.mstate = msebmtcal,
+ data.raw = ebmtcal,
+ j=3,
+ s=100,
+ t = t.eval,
+ tp.pred = tp.pred.j3s100,
+ curve.type = "rcs",
+ rcs.nk = 3,
+ group.vars = c("year"),
+ CI = 95,
+ CI.type = "parametric")</pre>
```

Again there are only four possible states that an individual may transition into, although this includes states 3 (adverse event) and 4 (recovery + adverse event), instead of 1 (post transplant) and 2 (recovery). This is because once an individual has entered state 3, they cannot move backwards into states 1 or 2. The calibration plots are better than for j=1. For transitions into states k=3,4 and 6, the calibration curves are monotonically increasing and comparatively close to the line of perfect calibration, although the confidence intervals are still quite large. This is true when calibration is assessed using BLR-IPCW or pseudo-values. Again the calibration of state 5 is very poor. This makes it difficult to base any clinical decisions on the predicted transition probabilities for relapse out of states j=1 or 3 at time s=100, whereas making clinical decisions based on the risk of death (k=6) after survival for 100 days is more viable, as this was well calibrated for both j=1 and j=3. With the exception of the transition probabilities from j=1 into state k=3 made at time s=0, there has been broad agreement between the calibration curves estimated using the BLR-IPCW and pseudo-value approaches. This provides some reassurance about the assessment of calibration, and that the assumptions on which each method is based are satisfied.

4. Discussion

Multistate models are a unique tool for prediction, handling both competing risks and the occurence of intermediate health states in the same model. Development of multistate models for prediction is becoming more common, yet validation of such models is still very uncommon. A major barrier to implementation of statistical techniques is often the availability of software (Pullenayegum et al. 2016). calibmsm has been developed to aid in the implementation of techniques to assess the calibration of the transition probabilities from a multistate model. This paper has extended previously proposed methods for assessing the calibration of the transition probabilities out of the initial state (XXXX REF PROJECT 6), to the transition probabilities out of any state j at any time s. While package development has focused on multistate models, calibmsm could be used to assess the calibration of predicted risks from a range of other models, including: any model which utilises information post baseline to update predictions (Bull et al. 2020), dynamic models (van Houwelingen 2007; ?; Grand et al. 2018), competing risks models (Putter et al. 2006) and standard single outcome

R > plot(dat.calib.blr.j3.s100, combine = TRUE, nrow = 2, ncol = 2)

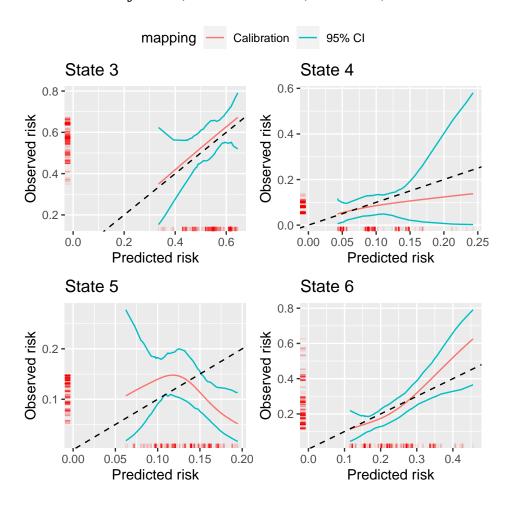


Figure 7: BLR-IPCW calibration curves out of state j=3 at time s=100.

R > plot(dat.calib.pv.j3.s100, combine = TRUE, nrow = 2, ncol = 2)

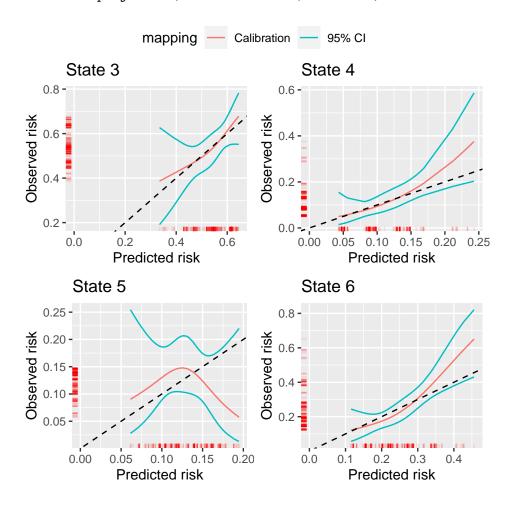


Figure 8: Pseudo-value calibration curves out of state j=3 at time s=100.

survival models, where predictions can be made at any landmark time.

All three methods (BLR-IPCW, MLR-IPCW and pseudo-value) have been shown to give an unbiased assessment of calibration under non-informative censoring mechanisms, and a predominately unbiased assessment of calibration under strongly informative censoring **XXXX REF PROJECT 6**. This paper found broadly similar evaluation of calibration when using the BLR-IPCW and pseudo-value methods, however there were discrepancies in the evaluation of calibration of the transition probabilities into state k = 3. In situations like this, we recommend testing the assumptions of each method as was done in vignette-Evaluation-of-estimation-of-IPCWs. While we concluded that the BLR-IPCW was likely to be biased in this particular example, this is not a general finding. Further research evaluating each methods performance in a wider range of simulation scenarios, and by a different research group (Boulesteix et al. 2013), would be highly valuable (Heinze et al. 2022).

Given it is possible to use **calibmsm** to validate a standard competing risks model, we carried out a sensitivity analysis to compare the approaches described in this paper with the 'graphical calibration curves' of Austin $et\ al.\ (2022)$, which already exist for this purpose vignette-Comparison-with-graphical-calibration-curves-in-competing-risks-setting. BLR-IPCW, pseudo-values, and graphical calibration curves (MLR-IPCW excluded for not producing a calibration curve) all resulted in similar calibration curves. This is with the exception of BLR-IPCW for state k=3, which has been previously discussed. The three methods take completely different approaches to assessing the calibration of a competing risks model. Therefore finding agreement between these assessments of calibration can provide reassurance that the calibration plots are correct, and is an exercise that could be repeated in practice. Despite this, the relative performance of each method in a wider range of competing risks scenarios remains unknown. A comparison of these methods in a simulation when the assumptions of each method do and do not hold, and under a range of sample sizes and multistate model structures, would be therefore valuable (Heinze $et\ al.\ 2022$).

The BLR-IPCW, MLR-IPCW and pseudo-value approaches have different computational burdens. A calibration curve can be obtained reasonably quickly using the BLR-IPCW or MLR-IPCW approaches, however estimation of confidence intervals for BLR-IPCW using bootstrapping (the recommend method in section 2.6) will result in a high computational time in large validation datasets. On the contrary, obtaining the calibration curve itself using the pseudo-value approach has a high computational burden due to estimation of the pseudo-values. Once these have been calculated, a calibration curve and confidence interval can be estimated quickly using parametric techniques, meaning estimation of the confidence interval adds minimal computational burden. We plan to extend the package to allow users to estimate the pseudo-values for each individual seperately before estimating the calibration curve. This will allow the first part of the process to be parallelised and will make estimation of calibration curves using the pseudo-value approach more feasible in large datasets.

Estimation of the weights is clearly of high importance for the BLR-IPCW and MLR-IPCW approaches. If the model to do so is misspecified, this could lead to incorrect evaluation of the calibration. It is possible this is what is causing the difference between the BLR-IPCW and pseudo-value approaches for the calibration of transition probabilities from state j=1 at time s=0 into state k=3, as was explored in vignette-Evaluation-of-estimation-of-IPCWs. This package is focused on creation of calibration curves, but is not a dedicated package for estimating inverse probability of censoring weights. We encourage users to create a well specified model for the weights. Custom functions for estimating the weights can be spec-

ified through the w.function argument in both calib_blr and calib_mlr. Alternatively, weights can be estimated externally and then specified though the weights argument. In this latter case, the internal bootstrapping procedure will not work, as the weights need to be re-estimated in each bootstrap dataset. We have provided a more detailed vignette about how to estimate calibration curves and confidence intervals using bootstrapping when defining your own function to estimate the weights (vignette-BLR-IPCW-manual-bootstrap).

In summary, **calibmsm** provides tools to assess the calibration of the transition probabilities of a multistate model or competing risks model using three approaches (BLR-IPCW, MLR-IPCW and pseudo-values). Further comparison of these approaches in targeted simulations to establish their performance under different censoring mechanisms and assumptions would be valuable.

Computational details

WILL UPDATE THIS SECTION ONCE PACKAGE HAS BEEN FINALISED AND UPLOADED TO CRAN

The results in this paper were obtained using R 4.3.1 with the **dplyr** 1.1.2, **tidyr** 1.3.0, **gg-plot2** 3.4.2, **ggpubr** 0.6.0, **Hmisc** 5.1.0, **rms** 6.7.0, **VGAM** 1.1.8, **boot** 1.3.28.1, **survival** 3.5.5, **stats** 4.3.1, **magrittr** 2.0.3. R itself and all packages used are available from the Comprehensive R Archive Network (CRAN) at https://cran.r-project.org/.

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