How to Prove Program Equivalence with Matching Logic

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This presentation includes joint work with

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Workshop on Program Equivalence
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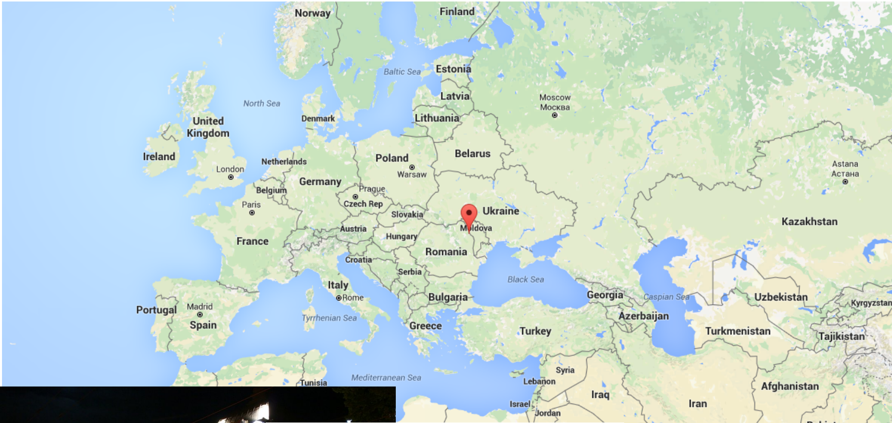
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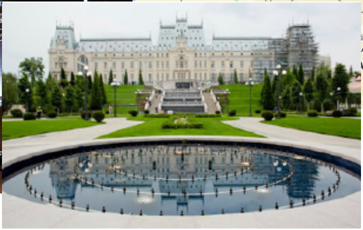


The Plan

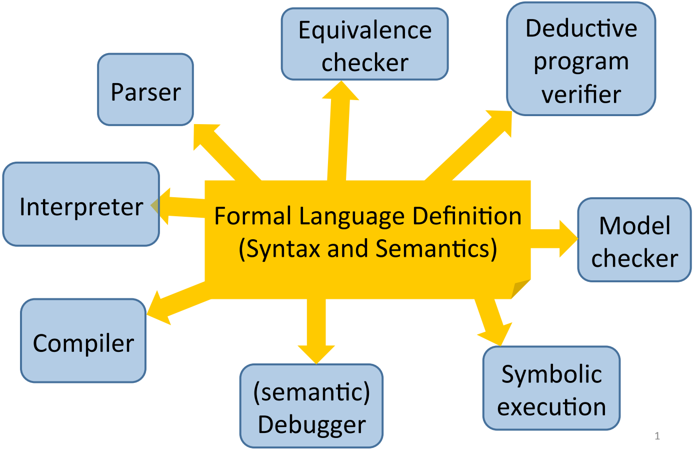
1. **Background**
2. Matching Logic
3. Reachability Logic
4. Language Aggregation
5. Proving Program Equivalence

Background - State-of-the-art in Program Verification

```
int main(void)
{
  int x = 0;
  return (x = 1) + (x = 2);
}
```

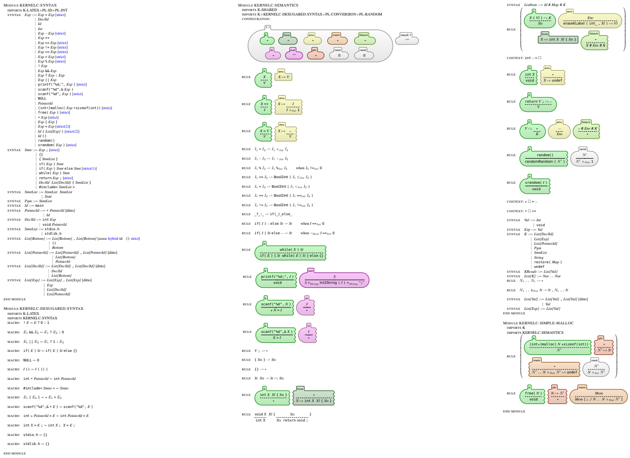
Jessie/Frama-C and Havoc prove the program returns. 4
GCC4, MSVC: the program returns. 4
GCC3, ICC, Clang: the program returns. 3
Reality: the program is undefined.

Formal Language Definition
(Syntax and Semantics)



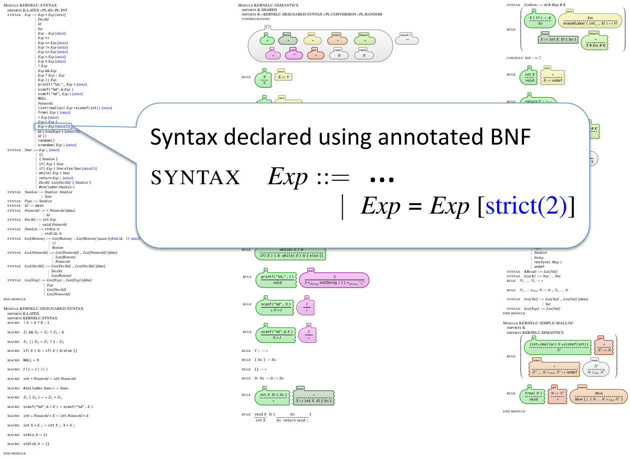
2005: Jose Meseguer and Grigore Roșu
The Rewriting Logic Semantics Project

Complete K Definition of KernelC



[Roșu, 2016]

Complete K Definition of KernelC

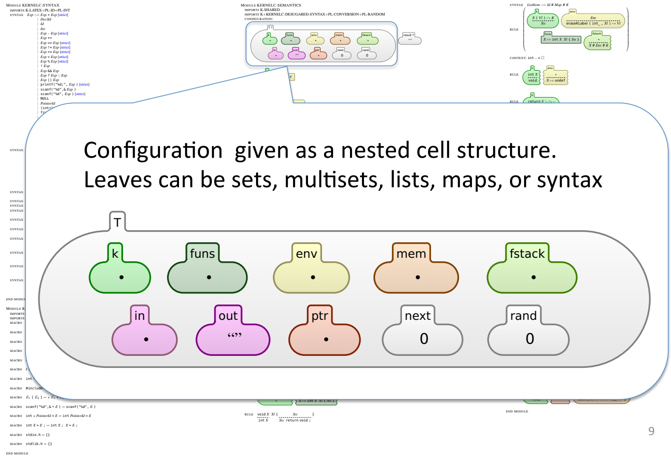


Syntax declared using annotated BNF

$$\textsc{syntax} \quad Exp ::= \ \ldots \qquad \mid \ Exp = Exp \ [\text{strict}(2)]$$

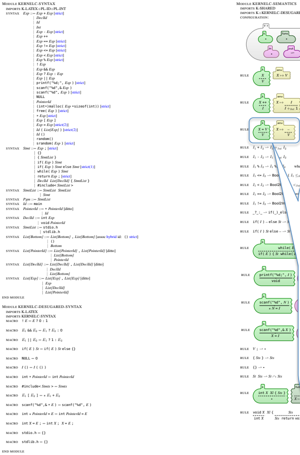
[Roșu, 2016]

Complete K Definition of KernelC

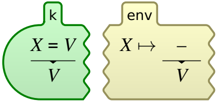


[Roșu, 2016]

Complete K Definition of KernelC



Semantic rules given contextually




```
rule
  <k> X = V => V ...</k>
  <env>... X |-> (_ => V) ...</env>
```


[Roșu, 2016]

Real Languages Defined in K

- Ellison et. al ([POPL 2012, PLDI 2015]) - C
- Bogdănaș et. al ([POPL 2015]) - Java 1.4
- Park et. al ([PLDI 2015]) - JavaScript

K Team

UIUC, USA

- Grigore Rosu (started K in 2003)
- Brandon Moore
- Daejun Park
- Lucas Pena
- Cosmin Radoi
- Yuwen Shijiao
- Andrei Stefanescu
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- Manasvi Saxena

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  Former Members: Irina Asavoae, Mihai Asavoae, Denis Bogdanas, Gheorghe Grigoras

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Reasoning About Program Configurations - Matching Logic

- used to specify static properties of program configurations

$$\exists l.(\langle\langle \texttt{skip}\rangle_{code}, \langle \texttt{x} \mapsto l, \ldots\rangle_{env}, \langle l \mapsto i, \ldots\rangle_{heap}\rangle) \wedge i > 0$$

$$\langle\langle \texttt{if v then } s_1 \texttt{ else } s_2\rangle_{code}, \langle \texttt{v} \mapsto i\rangle_{env}, \langle \ldots\rangle_{heap}\rangle \wedge i \neq 0$$

Matching Logic = FOL + terms as atomic formulae

Reasoning about Configurations - Matching Logic

$$\varphi ::= \exists x.\varphi, \varphi \wedge \varphi, \neg\varphi, P(t_1, \ldots, t_n), \pi$$

$$\pi \in T_{Cfg}(Var)$$

- $(\gamma, \rho) \models \pi$, where π is a basic pattern if $\rho(\pi) = \gamma$.
- $(\gamma, \rho) \models \varphi_1 \wedge \varphi_2$ if $(\gamma, \rho) \models \varphi_1$ and $(\gamma, \rho) \models \varphi_2$
- $(\gamma, \rho) \models \neg\varphi'$ if it is not true that $(\gamma, \rho) \models \varphi'$
- $(\gamma, \rho) \models \exists x.\varphi'$, where x is of sort s, if there exists $e \in [\![s]\!]_{\mathcal{T}}$ such that $(\gamma, \rho[e/x]) \models \varphi'$ (where $\rho[e/x]$ is the valuation obtained from ρ by updating the value of x to be e).

$$\varphi = \langle \texttt{skip,}\ \ \texttt{s} \mapsto s, \texttt{n} \mapsto n \rangle \wedge s = n * (n+1)/2$$

$$(\langle \texttt{skip,}\ \ \texttt{s} \mapsto 6, \texttt{n} \mapsto 3 \rangle, \rho = (s/6, n/3)) \models \varphi$$

$$(\langle \texttt{skip,}\ \ \texttt{s} \mapsto 6, \texttt{n} \mapsto 3 \rangle, \rho = (s/10, n/4)) \not\models \varphi$$

$$(\langle \texttt{skip,}\ \ \texttt{s} \mapsto 6, \texttt{n} \mapsto 0 \rangle, \rho = (s/6, n/0)) \not\models \varphi$$

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Reachability Logic

RL formula: $\varphi \Rightarrow \varphi'$

Semantics: any *terminating* $\gamma \in [\![\mathit{Cfg}]\!]_{\mathcal{T}}$ *that matches* φ *moves into* some $\gamma' \in [\![\mathit{Cfg}]\!]_{\mathcal{T}}$ matching φ'

- $\langle \texttt{skip;}s\texttt{,}\ \ \mathit{env} \rangle \Rightarrow \langle s\texttt{,}\ \ \mathit{env} \rangle$
- $\langle \texttt{if}\ \mathrm{i}\ \texttt{then}\ s_1\ \texttt{else}\ s_2\texttt{,}\ \ \mathit{env} \rangle \land i \neq 0 \Rightarrow \langle s_1\texttt{,}\ \ \mathit{env} \rangle$
- $\langle \texttt{SUM,}\ \ \texttt{s} \mapsto 0, \texttt{n} \mapsto n \rangle \Rightarrow \langle \texttt{skip,}\ \ \texttt{s} \mapsto n * (n+1)/2, \texttt{n} \mapsto n \rangle$

```
SUM = (s := 0
       i := 1
       while i <= n do
         s := s + i
         i := i + 1)
```

$$\frac{\begin{array}{c}\varphi \Rightarrow \varphi' \ \text{ if } \ \varphi_1 \Rightarrow \varphi'_1 \bigwedge \cdots \bigwedge \varphi_n \Rightarrow \varphi'_n \ \in \ \mathcal{A} \\ \psi \text{ is a structureless pattern} \\ \mathcal{A} \cup C \ \vdash \varphi_1 \wedge \psi \Rightarrow \varphi'_1 \quad \cdots \quad \mathcal{A} \cup C \ \vdash \varphi_n \wedge \psi \Rightarrow \varphi'_n\end{array}}{\mathcal{A} \ \vdash_C \varphi \wedge \psi \Rightarrow \varphi' \wedge \psi} \quad \textbf{Axiom :}$$

Reflexivity : $\mathcal{A} \ \vdash \varphi \Rightarrow \varphi$

Transitivity : $$\frac{\mathcal{A} \ \vdash_C \varphi_1 \Rightarrow \varphi_2 \qquad \mathcal{A} \cup C \ \vdash \varphi_2 \Rightarrow \varphi_3}{\mathcal{A} \ \vdash_C \varphi_1 \Rightarrow \varphi_3}$$

Consequence : $$\frac{\models \varphi_1 \rightarrow \varphi'_1 \qquad \mathcal{A} \ \vdash_C \varphi'_1 \Rightarrow \varphi'_2 \qquad \models \varphi'_2 \rightarrow \varphi_2}{\mathcal{A} \ \vdash_C \varphi_1 \Rightarrow \varphi_2}$$

Case Analysis : $$\frac{\mathcal{A} \ \vdash_C \varphi_1 \Rightarrow \varphi \qquad \mathcal{A} \ \vdash_C \varphi_2 \Rightarrow \varphi}{\mathcal{A} \ \vdash_C \varphi_1 \vee \varphi_2 \Rightarrow \varphi}$$

Abstraction : $$\frac{\mathcal{A} \ \vdash_C \varphi \Rightarrow \varphi' \qquad \text{where } X \cap FV(\varphi') = \emptyset}{\mathcal{A} \ \vdash_C \exists X\, \varphi \Rightarrow \varphi'}$$

Circularity : $$\frac{\mathcal{A} \ \vdash_{C \cup \{\varphi \Rightarrow \varphi'\}} \varphi \Rightarrow \varphi'}{\mathcal{A} \ \vdash_C \varphi \Rightarrow \varphi'}$$

(LICS 2013)

Proving Equivalence of Programs

Can we construct a program equivalence prover from the formal semantics of languages?

```
s := 0
i := 1
while i <= n do
  s := s + i
  i := i + 1
return s
```

P

```
let f n a =
  if n = 0 then a
  else f (n - 1) (a + n)
in
  f n 0
```

Q

Idea: reason directly about pair-programs $\langle P, Q \rangle$.

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The Aggregation of Two PL Signatures

$$
\begin{tikzcd}
 & \boxed{(S_0, \Sigma_0)} \arrow[dl, "h_1"] \arrow[dr, "h_2"'] & \\
\boxed{(S_1, \Sigma_1)} \arrow[dr, "h'_1"] & & \boxed{(S_2, \Sigma_2)} \arrow[dl, "h'_2"'] \\
 & \boxed{(S', \Sigma')} \arrow[d, "\text{add } \langle \_, \_ \rangle"] & \\
 & \boxed{(S, \Sigma)} &
\end{tikzcd}
$$

Constructing the Aggregated Model

$$
\begin{tikzcd}
 & \boxed{M_0} \arrow[r, dash, dashed] & \boxed{(S_0, \Sigma_0)} \arrow[dl, "h_1"] \arrow[dr, "h_2"] & \\
\boxed{M_1} \arrow[ur, "h_1"] \arrow[r, dash, dashed] & \boxed{(S_1, \Sigma_1)} \arrow[dr, "h'_1"] & \boxed{M_2} \arrow[ul, "h_2"'] \arrow[r, dash, dashed] & \boxed{(S_2, \Sigma_2)} \arrow[dl, "h'_2"'] \\
 & \boxed{M'} \arrow[ul, "h'_1"'] \arrow[ur, "h'_2"] \arrow[r, dash, dashed] & \boxed{(S', \Sigma')} \arrow[d, "\text{add new sort } s \text{ and } \langle \_, \_ \rangle"] & \\
 & \boxed{M} \arrow[u] \arrow[r, dash, dashed] & \boxed{(S, \Sigma)} &
\end{tikzcd}
$$

Constructing the Aggregated Semantics

- $$\mathcal{A}_1 \otimes_a \mathcal{A}_2 = \bigcup_{\varphi \Rightarrow \varphi' \in \mathcal{A}_1} \{\langle h_1'(\varphi), y\rangle \Rightarrow \langle h_1'(\varphi'), y\rangle\} \cup \bigcup_{\varphi \Rightarrow \varphi' \in \mathcal{A}_2} \{\langle x, h_2'(\varphi)\rangle \Rightarrow \langle x, h_2'(\varphi')\rangle\}$$

- $$\mathcal{A}_1 \otimes_p \mathcal{A}_2 = \bigcup_{\substack{\varphi_1 \Rightarrow \varphi_1' \in \mathcal{A}_1 \\ \varphi_2 \Rightarrow \varphi_2' \in \mathcal{A}_2}} \{\langle h_1'(\varphi_1), h_2'(\varphi_2)\rangle \Rightarrow \langle h_1'(\varphi_1'), h_2'(\varphi_2')\rangle$$

- $$A_1 \otimes A_2 = A_1 \otimes_a A_2 \cup A_1 \otimes_p A_2.$$

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Proving Partial/Total Equivalence

```
s := 0
i := 1
while i <= n do
  s := s + i
  i := i + 1
return s
```

P

```
let f n a =
  if n = 0 then a
  else f (n - 1) (a + n)
in
  f n 0
```

Q

$$\left\langle P, Q \right\rangle \Rightarrow \left\langle \langle \mathtt{skip}, \mathtt{s} \mapsto x \rangle, \langle x \rangle \right\rangle$$

Proving Partial/Total Equivalence

How to prove partial/total equivalence of P and Q?

1. Construct aggregated language (syntax, model, semantics);
2. Construct pair program $\langle P, Q \rangle$;
3. Show (e.g. using reachability logic) that $\langle P, Q \rangle$ reduces (partially/totally) to desired configuration;
4. Conclude that P and Q are partially/totally equivalent.

[SYNASC 2014]

Proving Full Equivalence

Two programs are fully equivalent if:

- both terminate on the same set of inputs and
- they produce the same result for the same input.

It seems that full equivalence cannot be reduced to partial correctness or total correctness.

Proving Full Equivalence ([ICFEM 2014], [FAOC 2016])

$$\textsc{Axiom}\ \frac{\varphi \in E}{\vdash \varphi \Downarrow^{\infty} E} \textit{ and}$$

$$\textsc{Conseq}\ \frac{\models \varphi \rightarrow \exists \tilde{x}.\varphi' \qquad \vdash \varphi' \Downarrow^{\infty} E}{\vdash \varphi \Downarrow^{\infty} E} \textit{ and}$$

$$\textsc{Case Analysis}\ \frac{\vdash \varphi \Downarrow^{\infty} E \qquad \vdash \varphi' \Downarrow^{\infty} E}{\vdash \varphi \vee \varphi' \Downarrow^{\infty} E} \textit{ and}$$

$$\textsc{Step}\ \frac{\models \varphi_1 \Rightarrow_1^{*} \varphi_1' \qquad \models \varphi_2 \Rightarrow_2^{*} \varphi_2' \qquad \vdash \langle \varphi_1', \varphi_2' \rangle \Downarrow^{\infty} E}{\vdash \langle \varphi_1, \varphi_2 \rangle \Downarrow^{\infty} E} \textit{ and}$$

$$\textsc{Circ}\ \frac{\models \varphi_1 \Rightarrow_1^{+} \varphi_1' \qquad \models \varphi_2 \Rightarrow_2^{+} \varphi_2' \qquad \vdash \langle \varphi_1', \varphi_2' \rangle \Downarrow^{\infty} E \cup \{\langle \varphi_1, \varphi_2 \rangle\}}{\vdash \langle \varphi_1, \varphi_2 \rangle \Downarrow^{\infty} E}$$

Proving Full Equivalence

```
 PGM₁  ≡  c := n; n := 1; LOOP₁
LOOP₁  ≡  while (c != 1)
            n := n + 1;
            if (c % 2 != 0)
              then c := 3 * c + 1
              else c := c / 2
```

```
 PGM₂  ≡  μf.λn.λa.LOOP₂
LOOP₂  ≡  if (n != 1)
             then if (n % 2 != 0)
              then f (3 * n + 1) (a + 1)
              else f (n / 2) (a + 1)
             else a
```

$$6, 3, 10, 5, 16, 8, 4, 2, 1$$

Proving Full Equivalence

$$\varphi := n \;\texttt{!=}_{Int}\; 1 \wedge \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; i\rangle\rangle$$

1.	$\vdash \langle\langle \texttt{skip,}\;\; \texttt{n} \mapsto i, \_\rangle, \langle i\rangle\rangle$	$\Downarrow^{\infty}$	E
2.	$\vdash \langle\langle \texttt{skip,}\;\; \texttt{n} \mapsto i, \_\rangle, \langle i\rangle\rangle$	$\Downarrow^{\infty}$	$E \cup \{\varphi\}$
3.	$\vdash \langle\langle \texttt{skip,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto 1\rangle, \langle i\rangle\rangle$	$\Downarrow^{\infty}$	E
4.	$\vdash \langle\langle \texttt{skip,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto 1\rangle, \langle i\rangle\rangle$	$\Downarrow^{\infty}$	$E \cup \{\varphi\}$
5.	$\vdash n \;\texttt{=}_{Int}\; 1 \wedge \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; i\rangle\rangle$	$\Downarrow^{\infty}$	E
6.	$\vdash n \;\texttt{=}_{Int}\; 1 \wedge \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; i\rangle\rangle$	$\Downarrow^{\infty}$	$E \cup \{\varphi\}$
7.	$\vdash n \;\texttt{!=}_{Int}\; 1 \wedge \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; i\rangle\rangle$	$\Downarrow^{\infty}$	$E \cup \{\varphi\}$
8.	$\vdash \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; i\rangle)$	$\Downarrow^{\infty}$	$E \cup \{\varphi\}$
9.	$\vdash n \;\texttt{!=}_{Int}\; 1 \wedge \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; i\rangle\rangle$	$\Downarrow^{\infty}$	E
10.	$\vdash \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto i, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; i\rangle\rangle$	$\Downarrow^{\infty}$	E
11.	$\vdash \langle\langle \texttt{LOOP}_1\texttt{,}\;\; \texttt{n} \mapsto 1, \texttt{c} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; 1\rangle\rangle$	$\Downarrow^{\infty}$	E
12.	$\vdash \langle\langle \texttt{PGM}_1\texttt{,}\;\; \texttt{n} \mapsto n\rangle, \langle \texttt{PGM}_2\; n\; 1\rangle\rangle$	$\Downarrow^{\infty}$	E

Conclusion

- Matching Logic/Reachability Logic enable reasoning about programs in a language-independent manner.
- Language Aggregation can be used to reduce partial/total equivalence to partial/full correctness.
- For full equivalence, a special proof system seems to be needed.

Future work:

- implementation
- other equivalences
- compiler correctness

Thank you!

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