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- Aristotle's logic was the logic of syllogisms. He gave this lovely elegant and exhaustive theory of syllogisms which meant that there was nothing really to be added to it. The history of deductive logic began with Aristotle and stalled there, nowhere really to go. There was a little bit of extra movement in addition to the syllogism. The stoics had a lovely discussion of conditionals, but everything that they said was forgotten during the Dark Ages and so logic returned to being the theory of conditionals.
- people say with great assurance that all deductive reasoning comprises stringing together syllogisms. But you cannot do anything that Euclid did, for example, by saying that he was just stringing together syllogisms.
- The big leap forward largely came in the late 19th century with Frege. Aristotle had things like "All Greeks are humans", "Some greeks are humans"—these are the tools he used to develop his logic. What Frege did was to give an analysis of these things. He took these sentences to be: $(\forall x)(Gx \to Hx)$ and $(\exists x)(Gx.Hx)$. He then handle the other sentential connectives,
- If you want to say, "All cows are purple", it's $\forall x(Cx \to Px)$. If you want to treat "most" on the analogy to the way that Frege treated "all", then to formalize "Most cows are purple", then you get a result if you do Fregian analysis. Frege's analysis does not work well for other quantifiers, but it does work well for the quantifiers used in Aristotelian logic.
- We know the logic of $\wedge$ (what is the truth condition), and we know the truth conditions for $\neg$, Frege's analysis of the conditionals give us a truth-functional reading of the conditionals where the truth or falsity of the conditional depends on tthe truth or falsity of its components. Frege went on to develop a theory whose rules abided by his truth-functional reading (the truth table for the conditional).
- Frege has this analysis of the quantifiers and he ad a logical system that treated conditionals truth-functionally, and on the basis of that he developed modern mathematical logic. Why did he develop modern mathematical logic? There was a real need for it brought about by teh advent of non-Euclidean geometry. As long as you stick to Euclidean geometry, you can both derive conclusions from axioms and be guided from spatial intuitions. Those two things work together to direct you, but in non-Euclidean gemoetry you don't have the spatial intuitions and so you have to make sure that your conclusions actually do follow from the axioms. You have to make sure that spatial intutions are not creeping into your reasoning; you have to check that your conclusions are actually following from your premises.
- Good reasoning is what you find in Euclid's Elements. Well, it turns out that Euclid's proofs aren't rigorous; if you look at Euclid's proofs, there are axioms and derivations that look like they rely on the axioms, but *there are also pictures*. So what Euclid is doing is importing

information that isn't contained solely in the axioms from the pictures, and so there was an urgent need for what counts as good reasoning in mathematics. Frege's theory fulfilled it.

- You don't find a comparable need outside of mathematics. You almost never see rigorous deductions outside of a mathematical context. Scientists write down basic principles and draw consequences from the basic principles, but the conclusions are not purely logical but rather are making use of mathematics and counting on the mathematics being sound. It is the mathematicians who use logic. You seldom see an inference that is written out in full detail except in mathematics. In mathematics, the rules Frege gave us are exactly what is needed. In particular, the conditionals that you find in mathematical reasoning can all be understood as material conditionals.
- Sometimes they occur in the subjunctive mood ("If the square root of 2 were not irrational, then you would get a contradiction") for *reductio ad absurdum* arguments. But that's not really any different than the logic that Frege gave to us.
- Mathematicians rarely ever need any other quantifiers than the standard logical ones, if you do then you just implement them in set theory (to express things like "for uncountably many x", "for almost all", etc.). The base logic that people use throughout mathematics use the truth-functional connective for conditionals.
- But in the rest of the sciences, they have an urgent need for conditionals that go beyond this. We want to say that "if you apply a voltage to this thing, then a current will pass through it." If you understand this as the material conditional, then if you have something that isn't a conductor the conditional will still be true.
- When you talk about the theory of computable functions, then the functions that are uncomputable there are some functions that are more uncomputable than others. We can say things like the problem of testing whether a program computes a total function is undecidable, and we can ask whether there is going . But these fall into different layers of computability (relative computability), and so we're asking about whether we can solve such and such problem if we can solve a different class of problems, and this kind of thinking is not captured by the material conditional. Once you get the basic ideas of computability theory, then you can get computability theory just from ordinary math/logic. The logic of the partiuclar logic that articulates what is said in recursion theory was axiomized by Lemming. That's interested because we think of the counterfactual: "if this function were solvable by algorithm, then this other function would be solvable." But this isn't just counterfactual it is counterpossible—it is not possible for the antecedent to obtain.
- With a few exceptions, all we see in mathematics is the logic of the material conditional, and it's pretty much only in mathematics that we see fully written out arguments. It looks like there are exceptions for counterfactual conditionals. But for indicative conditionals, it looks as if there are no obvious exceptions.
- It was widely thought that the standard conditional with its truth table is perfectly fine for indicative but not subjective conditionals. Where people finally convinced themselves that they shouldn't be so complacent about accepting the truth table as completely characterizing the logic of conditionals came when people started looking at arguments that are validated by a Frege-type account but don't seem to quite be captured by it.
- **The Logic of Conditionals** - three rules:
  - Modus ponens - derive q from P and $P \to Q$.

- Conditional proof - If $p \cup \{P\}$
- Peirce's Law - from $(P \to Q) \to P$ infer P.

This is good enough to capture material conditionals. The reason people become dissatisfied with the logic given to us by Frege is by coming up with examples that are validated by Frege's conditionals but don't at all fit the way people use it in English.

- Occassionally, the shaving lather winds up in Vann's coffee because he brings his coffee into the bathroom when he gets ready in the morning. Vann believes today that "Tomorrow morning, I'll enjoy my cup of coffee", but he does not believe that "Tomorrow morning, my coffee will taste good if it has my shaving lather in it". A good argument is one such that if you are certain of the premises, then you're entitled to be 100 percent sure of the conclusion. By that standard, the above is a good argument. But Vann just believes that tomorrow morning it will taste good, but he is not completely sure of it because he could get shaving cream in it. If he were certain of the premise that he won't get shaving cream in his coffee, he could assert the conclusion. But real-life reasoning is never certain of anything,
- A good relaxed standard of reasoning is that if you are certain of the premise then you can be certain of the conclusion, and so if you just think that the premise is highly likely (you have good reason to believe it), then you have good reason to believe the conclusion, as well. Ramsey made this distinction, distinguishing between the logic of certainty from the logic of truth, which is the way that human beings reason because we're almost never able to reason on the basis of things about which we're completely certain.
- The challenge to the material conditional as an account of indicative conditionals in English came from looking at arguments that are legitimated by the classical rules of deduction but for which English speakers are willing to accept the premises but unwilling to accept the conclusions. Seeing this mismatch between wht the theory tells us what good reasoning is and the way that people actually reason puts pressure on the classical account. That English speakers don't make inferences that we tell them they ought to make is reason to believe that the classical theory is mistaken.
- Consider: "If I put milk in my coffee, then it will taste good" and "If I put milk and shaving cream in my coffee, it will taste good". The second follows from the first by classical reasonsing. One of Dorothy Edginton's examples is: "I'll be home tonight before 6, therefore if I'm not home before 6, the queen will worry".
- There's an example from Ernie Adams: "if p then q and if q then r, then you can derive if p then r". So you can just assume p and derive r and then use conditional proof to derive "if p then r". "If Smith wins the election, Jones will go back to practicing law. If Jones dies before the election, Smith will win the election. Therefore, if Jones dies before the election, he'll go back to practicing law." An example involving negation is: from "if p then q" you can derive "if not q, then not p". But consider: "if I buy a car, I won't buy a Buick" yields "If I buy a Buick, then I won't buy a car." Or, from "it's not the case that if God exists, then we're free to do whatever we like" infer "God exists". Edgington's proof of the existence of God: "If God exists then it's not the case that if I pray then my prayers will be answered. I do not pray, and so therefore God exists". Or, from "if switch A and B are pulled, then the engine will start" infer "Either the engine will start if switch A is pulled or the engine will start if switch B is pulled". In order for the conclusion to be false, then it's true unless both disjuncts to be false; in order for both disjuncts to be false, A has to be true and S has to be false and

B has to be true and S has to be false. So then A and B are both true, but then the premise is true.

- One thing that was going on with some of these examples is they are talking about the future. Is this a problem with talking about the future, or a problem with talking about conditionals? Or a problem with both?
- Another thing with some of these examples (coffee) is that if I'm completely sure of the consequent, then you are justified in being compeltely sure of the conditional. If you're not fully sure of the consequent, then you're doubt of the conditional will get passed down through to the premises (??).
- One thing that is at work here is the idea of conversational issues, that there are cases where people make an effort to be evasive and uninformative (like in a deposition). In general, you expect people to not saying something week when they could've said something strong. We wouldn't say "if the wind is blowing from the Northeast, then Theresa May is writing her resignation letter" rather than "Theresa May is writing her resignation letter" even though the truth values are the same becuase it violates the norms of good conversation to not be extraneous. These things sound very odd but they are in fact good inferences. They are just pragmatically infelicitous.
- Vann doesn't know what conclusion to draw from these examples, but it seems like at least we can get this much: it seems like the classical account is under pressure, maybe we have reason to be dissatisfied by this account. There maybe aren't reasons to reject the classical account, but there are reasons to be dissatisfied with it and so there are reasons for us to look for alternatives to the classical account.