

0.1 Theoretical $Z - Z$ Cross Section σ

Define

$$\begin{aligned}\alpha &= \frac{g^4}{(32\pi)^2 s} \frac{\sqrt{1-4\epsilon^2}}{(1-\lambda^2)^2 + (\frac{\lambda\Gamma_Z}{\sqrt{s}})}, \\ \beta &= ((C_V^e)^2 + (C_A^e)^2)((C_V^\mu)^2), \\ \Delta &= ((C_V^e)^2 + (C_A^e)^2)((C_A^\mu)^2), \\ \Gamma &= 8C_V^e C_A^e C_V^\mu C_A^\mu.\end{aligned}$$

These definitions produce the differential cross section:

$$\frac{d\sigma}{d\Omega}^{Z-Z} = \alpha(\beta(1+\cos^2\vartheta+4\epsilon^2\sin^2\vartheta)+\Delta(1+\cos^2\vartheta)(1-4\epsilon^2)+\Gamma\sqrt{1-4\epsilon^2}\cos\vartheta)$$

Under the substitutions

$$\begin{aligned}\sin^2\vartheta &\rightarrow 1 - \cos^2\vartheta \quad \text{and} \\ \epsilon_\pm &= 1 \pm 4\epsilon^2,\end{aligned}$$

we see that the integral over the solid angle $d\Omega = d\phi dx$ (with implicit change of variable $\cos\vartheta \rightarrow x$) becomes

$$\begin{aligned}\sigma &= \alpha \int_0^{2\pi} d\phi \int_{-1}^1 \beta\epsilon_+ + \beta\epsilon_-x^2 + \Delta\epsilon_- + \Delta\epsilon_-x^2 + \Gamma\sqrt{\epsilon_-}x \, dx \\ &= 2\pi\alpha \int_{-1}^1 \beta\epsilon_+ + \Delta\epsilon_- + (\beta + \Delta)\epsilon_-x^2 + \Gamma\sqrt{\epsilon_-}x \, dx.\end{aligned}$$

The Γ term will disappear as it is symmetric in the integration limits. After simplification, the trivial integral is evaluated to

$$\sigma^{Z-Z} = 4\pi\alpha(\beta(\epsilon_+ + \frac{1}{3}\epsilon_-) + \frac{4}{3}\Delta\epsilon_-).$$