

Contents

1	Derivations	2
1.1	Theoretical Cross Section σ	2
1.1.1	$\gamma - \gamma$	2
1.1.2	$Z - Z$	2
1.1.3	$\gamma - Z$	3
2	Misc.	5
2.1	Feynman Diagrams	5

Chapter 1

Derivations

1.1 Theoretical Cross Section σ

1.1.1 $\gamma - \gamma$

Define

$$\alpha = \frac{g_e^4}{(8\pi)^2 s} \sqrt{1 - 4\varepsilon^2}.$$

The differential cross section then becomes

$$\frac{d\sigma}{d\Omega} = \alpha(1 + \cos^2 \vartheta + 4\varepsilon^2 \sin^2 \vartheta).$$

Making the canonical substitution $\sin^2 \vartheta = 1 - \cos^2 \vartheta$ and integrating over the solid angle $d\Omega = d\phi dx$ (with implicit change of variable $\cos \vartheta \rightarrow x$):

$$\sigma = \alpha \int_0^{2\pi} d\phi \int_{-1}^1 1 + x^2 + 4\varepsilon^2(1 - x^2).$$

This is trivially evaluated to

$$\sigma = \frac{16\pi\alpha}{3}(1 + 2\varepsilon^2)$$

1.1.2 $Z - Z$

Define

$$\begin{aligned}
\alpha &= \frac{g_Z^4}{(32\pi)^2 s} \frac{\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2 + (\frac{\lambda\Gamma_Z}{\sqrt{s}})}, \\
\beta &= ((C_V^e)^2 + (C_A^e)^2)((C_V^\mu)^2), \\
\Gamma &= ((C_V^e)^2 + (C_A^e)^2)((C_A^\mu)^2)(1-4\varepsilon^2), \\
\Delta &= 8C_V^e C_A^e C_V^\mu C_A^\mu \sqrt{1-4\varepsilon^2}.
\end{aligned}$$

These definitions produce the differential cross section:

$$\frac{d\sigma^{Z-Z}}{d\Omega} = \alpha(\beta(1 + \cos^2 \vartheta + 4\varepsilon^2 \sin^2 \vartheta) + \Delta(1 + \cos^2 \vartheta) + \Gamma \cos \vartheta)$$

Under the substitution $\sin^2 \vartheta \rightarrow 1 - \cos^2 \vartheta$ we see that the integral over the solid angle $d\Omega = d\phi dx$ (with implicit change of variable $\cos \vartheta \rightarrow x$) becomes

$$\begin{aligned}
\sigma &= \alpha \int_0^{2\pi} d\phi \int_{-1}^1 \beta + \beta x^2 + 4\varepsilon^2 \beta(1 - x^2) + \Gamma + \Gamma x^2 + \Delta x \, dx \\
&= 2\pi\alpha \int_{-1}^1 (1 + 4\varepsilon^2)\beta + \Gamma + ((1 - 4\varepsilon^2)\beta + \Gamma)x^2 \, dx.
\end{aligned}$$

The Γ term disappears as it is antisymmetric in the integration limits. After simplification, the integral is evaluated to

$$\sigma^{Z-Z} = \frac{16\pi\alpha}{3}(\Gamma + (1 + 2\varepsilon^2)\beta).$$

1.1.3 $\gamma - Z$

Define

$$\begin{aligned}
\alpha &= \frac{2g_e^2 g_Z^2}{(16\pi)^2 s} \frac{(1-\lambda^2)\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2 + (\frac{\lambda\Gamma_Z}{\sqrt{s}})}, \\
\beta &= C_V^e C_V^\mu, \\
\Delta &= 2C_A^e C_A^\mu \sqrt{1-4\varepsilon^2},
\end{aligned}$$

These definitions lead to the differential cross section

$$\frac{d\sigma^{\gamma-Z}}{d\Omega} = \alpha(\beta(1 + \cos^2 \vartheta + 4\varepsilon^2 \sin^2 \vartheta) + \Delta \cos \vartheta)$$

Under the substitution $\sin^2 \vartheta \rightarrow 1 - \cos^2 \vartheta$, we see that the integral over the solid angle $d\Omega = d\phi dx$ (with implicit change of variable $\cos \vartheta \rightarrow x$) becomes

$$\begin{aligned}\sigma &= \alpha \int_0^{2\pi} d\phi \int_{-1}^1 \beta(1 + x^2 + 4\varepsilon^2(1 - x^2)) + \Delta x \, dx \\ &= 2\pi\alpha \int_{-1}^1 \beta(1 + 4\varepsilon^2) + \beta(1 - 4\varepsilon^2)x^2 + \Delta x \, dx.\end{aligned}$$

The antisymmetric Δx terms vanishes under integration, giving (after rearranging)s

$$\sigma^{\gamma-Z} = \frac{16\pi\alpha\beta}{3}(1 + 2\varepsilon^2).$$

Chapter 2

Misc.

2.1 Feynman Diagrams

Here are the Feynman diagrams for the scatterings.

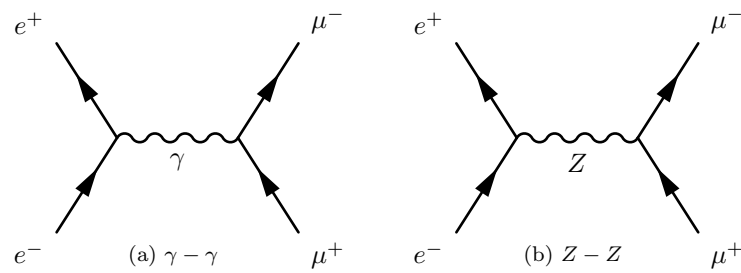


Figure 2.1: $e^-e^+ \rightarrow \mu^-\mu^+$ scattering via different bosons.