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### Chapter 1

### **Derivations**

#### 1.1 Theoretical Cross Section $\sigma$

#### 1.1.1 $\gamma - \gamma$

Define

$$\alpha = \frac{g_e^4}{(8\pi)^2 s} \sqrt{1 - 4\varepsilon^2}.$$

The differential cross section then becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \alpha(1 - \cos^2 \vartheta + 4\varepsilon^2 \sin^2 \vartheta).$$

Making the canonical substitution  $\sin^2 \vartheta = 1 - \cos^2 \vartheta$  and integrating over the solid angle  $d\Omega = d\phi dx$  (with implicit change of variable  $\cos \vartheta \to x$ ):

$$\sigma = \alpha \int_{0}^{2\pi} d\phi \int_{-1}^{1} 1 - x^{2} + 4\varepsilon^{2} (1 - x^{2}).$$

This is trivially evaluated to

$$\sigma = \frac{8\pi\alpha}{3}(1 + 4\varepsilon^2)$$

1.1.2 
$$Z - Z$$

Define

$$\begin{split} \alpha &= \frac{g_Z^4}{(32\pi)^2 s} \frac{\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2 + (\frac{\lambda \Gamma_Z}{\sqrt{s}})}, \\ \beta &= ((C_V^e)^2 + (C_A^e)^2)((C_V^\mu)^2), \\ \Gamma &= ((C_V^e)^2 + (C_A^e)^2)((C_A^\mu)^2)(1-4\varepsilon^2), \\ \Delta &= 8C_V^e C_A^e C_V^\mu C_A^\mu \sqrt{1-4\varepsilon^2}. \end{split}$$

These definitions produce the differential cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{Z-Z} = \alpha(\beta(1+\cos^2\vartheta + 4\varepsilon^2\sin^2\vartheta) + \Delta(1+\cos^2\vartheta) + \Gamma\cos\vartheta)$$

Under the substitution  $\sin^2 \vartheta \to 1 - \cos^2 \vartheta$  we see that the integral over the solid angle  $d\Omega = d\phi dx$  (with implicit change of variable  $\cos \vartheta \to x$ ) becomes

$$\sigma = \alpha \int_{0}^{2\pi} d\phi \int_{-1}^{1} \beta + \beta x^{2} + 4\varepsilon^{2} \beta (1 - x^{2}) + \Gamma + \Gamma x^{2} + \Delta x \, dx$$
$$= 2\pi \alpha \int_{-1}^{1} (1 + 4\varepsilon^{2}) \beta + \Gamma + ((1 - 4\varepsilon^{2})\beta + \Gamma) x^{2} \, dx.$$

The  $\Gamma$  term disappears as it is antisymmetric in the integration limits. After simplification, the integral is evaluated to

$$\sigma^{Z-Z} = \frac{16\pi\alpha}{3} (\Gamma + (1 + 2\varepsilon^2)\beta).$$

**1.1.3** 
$$\gamma - Z$$

Define

$$\begin{split} \alpha &= \frac{2g_e^2g_Z^2}{(16\pi)^2s}\frac{(1-\lambda^2)\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2+\left(\frac{\lambda\Gamma_Z}{\sqrt{s}}\right)},\\ \beta &= C_V^eC_V^\mu,\\ \Delta &= 2C_A^eC_A^\mu\sqrt{1-4\varepsilon^2}, \end{split}$$

These definitions lead to the differential cross section

$$\frac{\mathrm{d}\sigma^{\gamma - Z}}{\mathrm{d}\Omega} = \alpha(\beta(1 + \cos^2 \vartheta + 4\varepsilon^2 \sin^2 \vartheta) + \Delta \cos \vartheta)$$

Under the substitution  $\sin^2 \vartheta \to 1 - \cos^2 \vartheta$ , we see that the integral over the solid angle  $d\Omega = d\phi dx$  (with implicit change of variable  $\cos \vartheta \to x$ ) becomes

$$\sigma = \alpha \int_{0}^{2\pi} d\phi \int_{-1}^{1} \beta (1 + x^{2} + 4\varepsilon^{2} (1 - x^{2})) + \Delta x \, dx$$
$$= 2\pi \alpha \int_{-1}^{1} \beta (1 + 4\varepsilon^{2}) + \beta (1 - 4\varepsilon^{2}) x^{2} + \Delta x \, dx.$$

The antisymmetric  $\Delta x$  terms vanishes under integration, giving (after rearranging)s

$$\sigma^{\gamma-Z} = \frac{16\pi\alpha\beta}{3}(1+2\varepsilon^2).$$

## Chapter 2

# Misc.

### 2.1 Feynman Diagrams

Here are the Feynman diagrams for the scatterings.

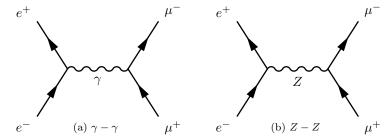


Figure 2.1:  $e^-e^+ \to \mu^-\mu^+$  scattering via different bosons.