Contents

Chapter 1

Derivations

1.1 Theoretical Cross Section σ

1.1.1 $\gamma - \gamma$

Define

$$\alpha = \frac{g_e^4}{(8\pi)^2 s}\sqrt{1-4\varepsilon^2}.$$

The differential cross section then becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \alpha(1+\cos^2\vartheta + 4\varepsilon^2\sin^2\vartheta).$$

Making the canonical substitution $\sin^2\vartheta = 1-\cos^2\vartheta$ and integrating over the solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\,\mathrm{d}x$ (with implicit change of variable $\cos\vartheta \to x$):

$$\sigma = \alpha \int\limits_0^{2\pi} \mathrm{d}\phi \int\limits_{-1}^{1} 1 + x^2 + 4\varepsilon^2(1-x^2).$$

This is trivially evaluated to

$$\sigma = \frac{16\pi\alpha}{3}(1+2\varepsilon^2)$$

1.1.2 $Z - Z$

Define

$$
\begin{aligned}
\alpha &= \frac{g_Z^4}{(32\pi)^2 s}\frac{\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2+(\frac{\lambda\Gamma_Z}{\sqrt{s}})},\\
\beta &= ((C_V^e)^2+(C_A^e)^2)((C_V^\mu)^2),\\
\Gamma &= ((C_V^e)^2+(C_A^e)^2)((C_A^\mu)^2)(1-4\varepsilon^2),\\
\Delta &= 8C_V^e C_A^e C_V^\mu C_A^\mu\sqrt{1-4\varepsilon^2}.
\end{aligned}
$$

These definitions produce the differential cross section:

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{Z-Z} = \alpha(\beta(1+\cos^2\vartheta+4\varepsilon^2\sin^2\vartheta)+\Delta(1+\cos^2\vartheta)+\Gamma\cos\vartheta)
$$

Under the substitution $\sin^2\vartheta\to 1-\cos^2\vartheta$ we see that the integral over the solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\ \mathrm{d}x$ (with implicit change of variable $\cos\vartheta\to x$) becomes

$$
\begin{aligned}
\sigma &= \alpha\int_0^{2\pi}\mathrm{d}\phi\int_{-1}^{1}\beta+\beta x^2+4\varepsilon^2\beta(1-x^2)+\Gamma+\Gamma x^2+\Delta x\ \mathrm{d}x\\
&= 2\pi\alpha\int_{-1}^{1}(1+4\varepsilon^2)\beta+\Gamma+((1-4\varepsilon^2)\beta+\Gamma)x^2\ \mathrm{d}x.
\end{aligned}
$$

The Γ term disappears as it is antisymmetric in the integration limits. After simplification, the integral is evaluated to

$$
\sigma^{Z-Z} = \frac{16\pi\alpha}{3}(\Gamma+(1+2\varepsilon^2)\beta).
$$

1.1.3 $\gamma - Z$

Define

$$
\begin{aligned}
\alpha &= \frac{2g_e^2 g_Z^2}{(16\pi)^2 s}\frac{(1-\lambda^2)\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2+(\frac{\lambda\Gamma_Z}{\sqrt{s}})},\\
\beta &= C_V^e C_V^\mu,\\
\Delta &= 2C_A^e C_A^\mu\sqrt{1-4\varepsilon^2},
\end{aligned}
$$

These definitions lead to the differential cross section

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{\gamma-Z} = \alpha(\beta(1+\cos^2\vartheta+4\varepsilon^2\sin^2\vartheta)+\Delta\cos\vartheta)
$$

Under the substitution $\sin^2\vartheta \to 1-\cos^2\vartheta$, we see that the integral over the solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\ \mathrm{d}x$ (with implicit change of variable $\cos\vartheta \to x$) becomes

$$\begin{aligned}\sigma &= \alpha \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \beta(1+x^2+4\varepsilon^2(1-x^2)) + \Delta x\ \mathrm{d}x \\ &= 2\pi\alpha \int_{-1}^{1} \beta(1+4\varepsilon^2) + \beta(1-4\varepsilon^2)x^2 + \Delta x\ \mathrm{d}x.\end{aligned}$$

The antisymmetric Δx terms vanishes under integration, giving (after rearranging)s

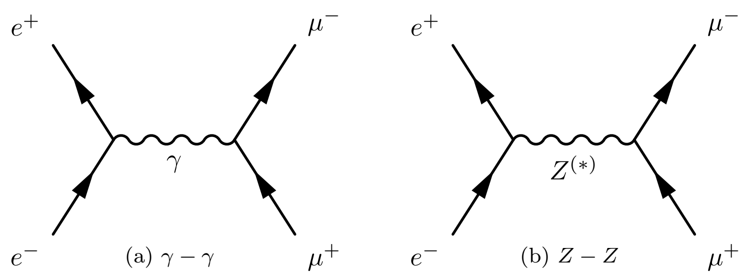
$$\sigma^{\gamma-Z} = \frac{16\pi\alpha\beta}{3}(1+2\varepsilon^2).$$

Chapter 2

Misc.

2.1 Feynman Diagrams

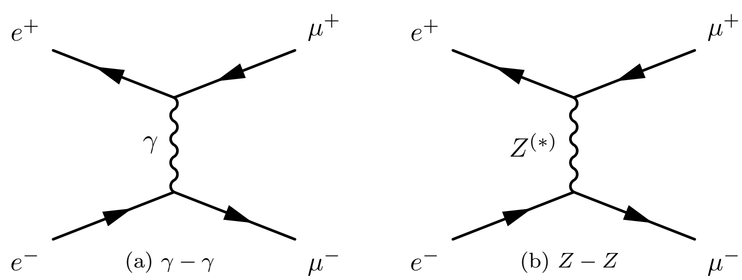
Feynman diagrams for the s-channel scatterings.



(a) $\gamma - \gamma$ (b) $Z - Z$

Figure 2.1: s-channel $e^-e^+ \to \mu^-\mu^+$ scattering via different bosons.

Feynman diagrams for the t-channel scatterings.



(a) $\gamma - \gamma$ (b) $Z - Z$

Figure 2.2: s-channel $e^-e^+ \to \mu^-\mu^+$ scattering via different bosons.