Contents

Chapter 1

Derivations

1.1 Theoretical Cross Section σ

1.1.1 $\gamma - \gamma$

Define

$$\alpha = \frac{g_e^4}{(8\pi)^2 s}\sqrt{1 - 4\varepsilon^2}.$$

The differential cross section then becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \alpha(1 - \cos^2\vartheta + 4\varepsilon^2 \sin^2\vartheta).$$

Making the canonical substitution $\sin^2\vartheta = 1 - \cos^2\vartheta$ and integrating over the solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\, \mathrm{d}x$ (with implicit change of variable $\cos\vartheta \to x$):

$$\sigma = \alpha \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} 1 - x^2 + 4\varepsilon^2(1 - x^2).$$

This is trivially evaluated to

$$\sigma = \frac{8\pi\alpha}{3}(1 + 4\varepsilon^2)$$

1.1.2 $Z - Z$

Define

$$
\begin{aligned}
\alpha &= \frac{g_Z^4}{(32\pi)^2 s} \frac{\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2 + (\frac{\lambda\Gamma_Z}{\sqrt{s}})},\\
\beta &= ((C_V^e)^2 + (C_A^e)^2)((C_V^\mu)^2),\\
\Delta &= ((C_V^e)^2 + (C_A^e)^2)((C_A^\mu)^2),\\
\Gamma &= 8C_V^e C_A^e C_V^\mu C_A^\mu.
\end{aligned}
$$

These definitions produce the differential cross section:

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{Z-Z} = \alpha(\beta(1+\cos^2\vartheta+4\varepsilon^2\sin^2\vartheta)+\Delta(1+\cos^2\vartheta)(1-4\varepsilon^2)+\Gamma\sqrt{1-4\varepsilon^2}\cos\vartheta)
$$

Under the substitutions

$$
\begin{aligned}
\sin^2\vartheta &\to 1-\cos^2\vartheta \quad \text{and}\\
\varepsilon_\pm &= 1 \pm 4\varepsilon^2,
\end{aligned}
$$

we see that the integral over the solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\,\mathrm{d}x$ (with implicit change of variable $\cos\vartheta \to x$) becomes

$$
\begin{aligned}
\sigma &= \alpha \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \beta\varepsilon_+ + \beta\varepsilon_- x^2 + \Delta\varepsilon_- + \Delta\varepsilon_- x^2 + \Gamma\sqrt{\varepsilon_-}x\ \mathrm{d}x\\
&= 2\pi\alpha \int_{-1}^{1} \beta\varepsilon_+ + \Delta\varepsilon_- + (\beta+\Delta)\varepsilon_- x^2 + \Gamma\sqrt{\varepsilon_-}x\ \mathrm{d}x.
\end{aligned}
$$

The Γ term will disappear as it is antisymmetric in the integration limits. After simplification, the trivial integral is evaluated to

$$
\sigma^{Z-Z} = 4\pi\alpha(\beta(\varepsilon_+ + \frac{1}{3}\varepsilon_-) + \frac{4}{3}\Delta\varepsilon_-).
$$

1.1.3 $\gamma - Z$

Define

$$
\begin{aligned}
\alpha &= \frac{2g_e^2 g_Z^2}{(16\pi)^2 s} \frac{(1-\lambda^2)\sqrt{1-4\varepsilon^2}}{(1-\lambda^2)^2 + (\frac{\lambda\Gamma_Z}{\sqrt{s}})},\\
\beta &= C_V^e C_V^\mu,\\
\Delta &= 2C_A^e C_A^\mu \sqrt{1-4\varepsilon^2},
\end{aligned}
$$

These definitions lead to the differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{\gamma-Z} = \alpha(\beta(1+\cos^2\vartheta + 4\varepsilon^2\sin^2\vartheta) + \Delta\cos\vartheta)$$

Under the substitution $\sin^2\vartheta \to 1-\cos^2\vartheta$, we see that the integral over the solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\ \mathrm{d}x$ (with implicit change of variable $\cos\vartheta \to x$) becomes

$$\begin{aligned}\sigma &= \alpha\int_0^{2\pi}\mathrm{d}\phi\int_{-1}^{1}\beta(1+x^2+4\varepsilon^2(1-x^2)) + \Delta x\ \mathrm{d}x \\ &= 2\pi\alpha\int_{-1}^{1}\beta(1+4\varepsilon^2) + \beta(1-4\varepsilon^2)x^2 + \Delta x\ \mathrm{d}x.\end{aligned}$$

The antisymmetric Δx terms vanishes under integration, giving (after rearranging)s

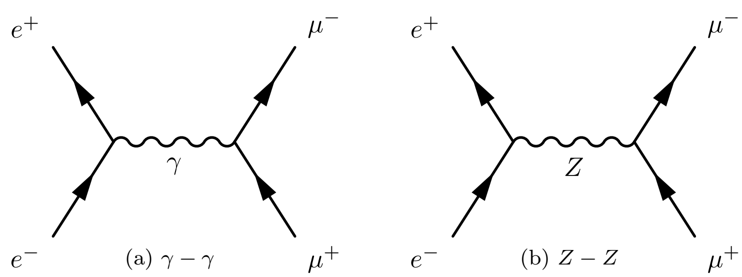
$$\sigma = \frac{16\pi\alpha\beta}{3}(1+2\varepsilon^2).$$

Chapter 2

Misc.

2.1 Feynman Diagrams

Here are the Feynman diagrams for the scatterings.



(a) $\gamma - \gamma$

(b) $Z - Z$

Figure 2.1: $e^-e^+ \to \mu^-\mu^+$ scattering via different bosons.