Model selection for estimating CEF derivatives

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3rd year

June 7, 2022

Fixing objects

From the *all causes* framework, the object of interest:

$$\left\{ \frac{\partial}{\partial x} \phi(X, u) \right\}_{u \in supp\ U} \tag{1}$$

The average that's considered to be a more feasible target:

$$\frac{\partial}{\partial x} \mathbb{E}\left[\phi(X, U) | X = x\right] \tag{2}$$

Weight derivatives at different values of the domain with the density of those values:

$$\mathbb{E}\left[\frac{\partial}{\partial x} \mathbb{E}\left[\phi(X, U) | X = x\right]\right] \tag{3}$$

Can we do model selection in this context – 1

Even if you have a decent predictor for levels $\hat{f}(x)$, it might not work as well for predicting $f(x') - f(x)$. Think of the average treatment effect as a result of moving from x to x'.

- Recall Yitzhaki weights – β_{BLP} estimates a 'weird' average of derivatives with weights
- Banerjee (2007): Directly target the average derivative – local OLS is offered as a solution

To me this looks like a b/v issue but with an interesting twist – both supervised and unsupervised ?? How much can we gain in terms of variance by imposing the bias of the sort that β_{BLP} does?
What could do for the proj with reasonable prob: Take a context, simulate (wiemann style) and see if BLP can do better for some dgps?

Can we do model selection in this context – 2

- What is the bias like for β_{BLP} in this setting? Can we obtain the parameter that β_{BLP} estimates as a result of some penalisation? So it would be a penalisation that leads to the bandwidth being the complete domain, right?
- When we do lasso: we chose a particular 'space' with respect to which to penalise the model, right? What can we choose as a 'space' in this case of estimating derivatives?
- It seems weird to impose a minimum distance between the set of weights and the pdf of X, right? Maybe penalise directly the distance between the set of Yitzhaki weights and the true pdf of X?

Set of weights $\omega : \mathit{supp}\, X \to \mathbf{R}_+$.

$$\min d(\omega, f_X) \tag{4}$$
$$\text{s.t. } d(\omega, \omega_z) \leq \lambda \tag{5}$$

monte carlo

$$
\begin{aligned}
X &\sim U[0,1]^n \\
\varepsilon \sim \mathcal{N}(0,1) \quad X &\perp \varepsilon \\
Y &= X^2 + \varepsilon
\end{aligned}
\tag{6}
$$

Average derivative: 1

Vapnik kernel:

$$J[f] = \sum_{i=1}^{N} (y_i - f(x_i))^2 + \lambda ||f||_{\mathcal{H}}^2 \tag{7}$$

$$K(x, y) = 1 + 2xy + x^2 y^2 \tag{8}$$

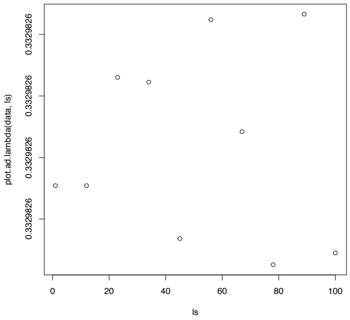


Figure 1: deviation from average derivative

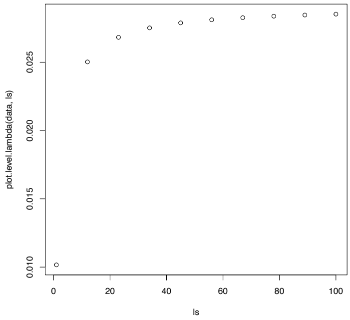
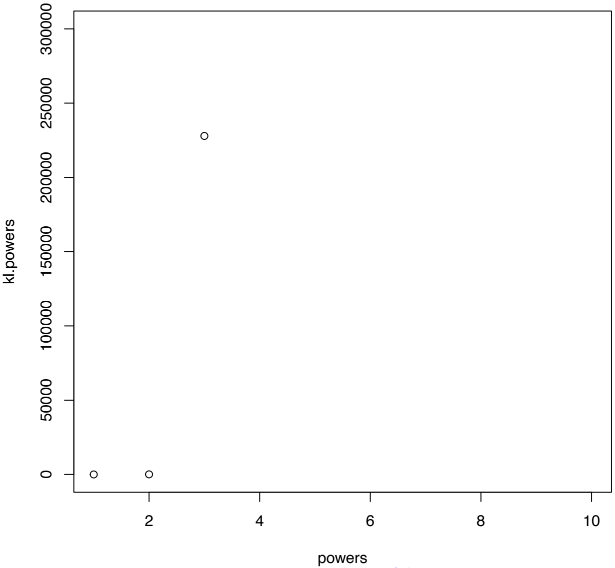


Figure 2: mse

using yitzhaki weights

choose λ based on the distance between yitzhaki weights and the true density

kl distance as a function of the power



References