Online Appendix to Recency, Records and Recaps: Learning and Non-equilibrium Behavior in a Simple Decision Problem

DREW FUDENBERG, Harvard University ALEXANDER PEYSAKHOVICH, Harvard University

Section 1: Supplemental Figures

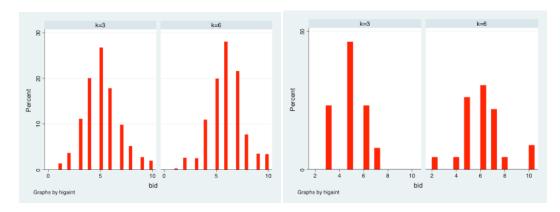


Figure A1: Histograms of bids in last 10 rounds of experiment 1. The first panel includes all bids, the second panel is restricted to individuals whose action does not change in the last 10 periods – this second panel shows that even individuals that have converged to some stationary strategy continue to overbid.

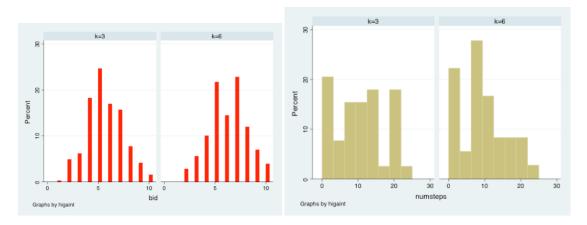


Figure A2: Bids in the last 10 rounds in experiment 2.

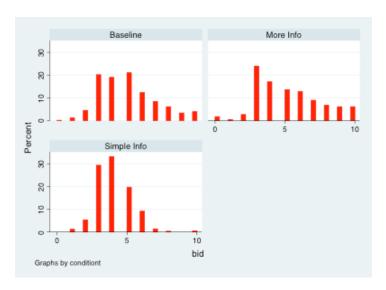


Figure A3: Bids in the last 10 rounds of the recaps experiment, by condition.

Section 2: Evidence for Corner Aversion

We noted before that we chose the ALP to avoid the extremeness of optimal offers that obtains in similar games such as the ACG. When optimal offers are on the boundary of the feasible set, censoring and corner aversion can each generate misleading inferences from the observed data.

Censoring is a mechanical confound present when optimal play is implemented with errors. Recall that in our experimental data we see that distributions of behavior appear to be symmetric and centered on an interior point of the strategy space. Thus, purely idiosyncratic noise should "wash out" in our estimations. When the optimal offer is on the lower boundary of the feasible set (as it is in the ACG) white noise in responses would push the observed mean of decisions higher than the optimum even if each agent's uncensored strategy was to pick the optimum plus a mean-zero noise term Similarly, when k=6 in the ALP, censoring would lead to apparent underbidding by optimizing agents if the highest allowed bid were close to 6.

A second potential confound is corner aversion. Existing work shows that individuals are prone to 'avoid the edges' in some strategic settings (Rubinstein et al. 1993). Such a bias would make optimal behavior in the ACG harder to learn than in the ALP. To test the effect of the feasible set on bids, we performed another experiment.

Design

We recruited an additional N=79 individuals to participate in 30 rounds of our baseline ALP conditions with offers restricted to the integers 0-6. This restriction makes the optimal offer in the k=6 condition on the edge of the interval and the optimal offer in the k=3 condition squarely in the middle. We see that this lowers offers in the final 10 rounds from 5.09 to 4.29 when k=3 (clustered s.e. = .209) and from 5.98 to 4.21 (clustered s.e. = .185) when k=6. Both of these changes are statistically significant (clustered t-test both p<.01).

We can test whether this decrease is driven purely by censoring of bids or if corner aversion also appears to play a role. To do so, we take combine this data with the results of experiment 1. We then manually censor the distributions of bids in experiment 1. If the difference in outcomes is driven purely by censoring, we should see that the manually censored experiment 1 distribution is identical to the one from the current experiment. If corner aversion plays a role, we should see a difference in distributions. Figure 12 shows the histogram produced by this exercise. The distributions are different across experiments (Kolmogorov-Smirnov tests p<.01 for both k conditions) and especially so in the k=6 condition where optimal bids lie on the border of the strategy space. This shows that both censoring and corner aversion can act as confounds when optimal bids are on the edge of the interval.

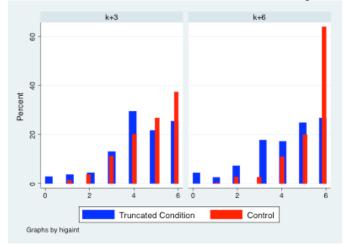


Figure A4: Histogram of bids in last 10 rounds of the restricted strategy space experiment overlaid with experiment 1 bids censored at 6.

Section 3: Model Selection in Experiment 2

We use the Bayesian Information Criterion (BIC) to select the number of lags to include. The BIC of a fixed model is given by

$$-2\ln(L) + m*\ln(n)$$

where L is the likelihood of the model given the data, n is the number of observations (which we treat as number of subjects) and m is the number of model parameters. Thus, the BIC decreases in variance explained (model fit) and increases in number of free parameters (m*ln(n)).

Table 5 shows the results of this analysis restricted to the second half of the experimental rounds.² We use bids pooled across conditions as our dependent variable, add subject level fixed effects and use BIC to select the optimal number of lagged experiences. To account for the fact that the same computer value v may have different effects on behavior in

¹ See Hastie et al. (2009) for a description and derivation of the BIC.

² Here we use the last 15 rounds instead of the last 10 as in the rest of our analyses to increase power. Restricting to the last 10 rounds gives qualitatively similar (but less significant) results and BIC continues to select a single lag.

k=3 and k=6 conditions we also include lagged interaction terms. As predicted by the recency hypothesis, the linear regression model selected by the BIC involves only a single lag.³

Table 5: BIC-based model selection. Regressions include subject-level dummies.

| | bid | bid | bid | bid |
|------------|------------|------------|------------|------------|
| L1.v | 0.078 | 0.075 | 0.078 | 0.079 |
| | (0.042)* | (0.041)* | (0.040)* | (0.038)** |
| L1.(k=6)Xv | -0.092 | -0.090 | -0.093 | -0.096 |
| | (0.046)** | (0.045)** | (0.044)** | (0.043)** |
| L2.v | | 0.017 | 0.012 | 0.012 |
| | | (0.022) | (0.022) | (0.022) |
| L2.(k=6)Xv | | -0.013 | -0.008 | -0.009 |
| | | (0.029) | (0.030) | (0.030) |
| L3.v | | | 0.030 | 0.029 |
| | | | (0.019) | (0.018) |
| L3.(k=6)Xv | | | -0.030 | -0.027 |
| | | | (0.028) | (0.027) |
| L4.v | | | | 0.004 |
| | | | | (0.028) |
| L4.(k=6)Xv | | | | -0.017 |
| | | | | (0.037) |
| constant | 5.615 | 5.568 | 5.495 | 5.519 |
| | (0.123)*** | (0.161)*** | (0.159)*** | (0.174)*** |
| R^2 | 0.49 | 0.49 | 0.49 | 0.49 |
| N | 1,200 | 1,200 | 1,200 | 1,200 |
| BIC | 4063.22 | 4071.17 | 4077.78 | 4086.01 |

Coefficients from OLS regressions.

Standard errors in parentheses clustered at participant level.

³ Note that this does not mean that individuals literally discount all information beyond the last outcome. Rather this result means that the additional predictive power from looking at longer individual-level histories is small.

Section 4: Experimental Materials

Experimental Instructions (ALP)

General Info Screen

Welcome, you are eligible to participate! Thank you for signing up!

You will be asked to play rounds of a game during which you can earn points. This game will take approximately 10 minutes and you can earn a bonus of up to \$1.50 for playing.

This money will be paid to you via the bonus system on Mechanical Turk within a few days of the completion of this HIT.

This is in addition to the money you earn just for taking the HIT!

We are interested in studying how individuals make decisions and how/if they change them when they get more information.

We are not trying to "trick" you in any way. This also means that if you think that something odd is happening, please let us know via e-mail as it is most likely a bug in the software.

In this study you earn points during rounds of a game. You will start with 10 points [100 points for multiple info] in your account, you can earn or lose points during each round. These points will be converted into USD at the end of the game at a rate of 15 points [150 points in multiple info] per dollar.

Sample Instructions (k=+3, With counterfactuals)

In this study you will be asked to play several *rounds* of a game with a computer player.

Each round proceeds as follows:

1) The computer gets a chip of value V, this value can be anywhere between 0 and 10 points. These values will be determined randomly.

- a. The way in which the value will be determined does not change from round to round. Additionally, the randomization does not depend on what happened in earlier rounds.
- 2) You will not know the value of any of the chips at the start of the round and you will be allowed to make a bid to the computer player. This bid can be any whole number between 0 and 10 points.
- 3) The computer player will look at your bid and compare it to their value V.
 - a. If your bid is higher than the value V, that computer will *accept* your bid and *sell* you the chip. That is, you will get the chip and pay the computer an amount equal to your bid.
 - b. If your bid is lower than the value V, that computer will reject your bid
- 4) Chips are worth more to you than to the computer. Specifically, if you get a chip that had value V to the computer it will have a value of V+3 points to you at the end of the round.
- 5) At the end of a round you will be given some information:
 - a. What bid you made
 - b. The value of the computer player's chip
 - c. Whether your bid was accepted
 - d. The value to you of the chip you got

Examples

Since the value to you is 3 higher than the value to the computer, you make a profit on any sale where your bid is less than V+3.

Suppose, the computer has a chip of value 6 and you make a bid of 7.

Then you will get the chip (worth 9 to you), pay 7 points, and end up with a profit of 2.

However, suppose the computer has a chip of value 1 and you make a bid of 5.

Then you will get the chip (worth 4 to you), pay 5 points and end up with a loss of 1.

You will play 30 rounds of this game.

Before you start, you will be asked to take a short quiz on the rules of the game. You must answer these questions correctly to receive bonuses.

Quiz

Question 1) True or False: Each round, the computer player will *accept* your bid if the bid is *less* than the computer's value V.

Question 2) True or False: You will know the computer player's value at the start of every round.

Question 3) If a computer player has a chip of value 3, how much is it worth to you?

Question 4) True or False: Each computer's value of a chip is determined randomly in each round in a way that does *not* depend on what happens in other rounds.

Sample Bid Screen

New Round

The computer has a new chip with value, ν between 0 and 10 points. If you have the chip at the end of the round, it will be worth $\nu+3$ points to you. How much do you want to offer the computer?



Sample Feedback Screen (With CF)

You bid: 4

Your bid was accepted.

The value to the computer was: 0.3
The value to you was: 3.3
You paid your bid.

More/Simple Info Bid Screen

New Round

Each of the 10 computers has a chip of value v between 0 and 10 points. Each chip you have at the end of the round will be worth v+3 points to you. How much do you want to offer each computer?



Simple Info

You bid 5.

| Computer Players' Values | 3.8 3.2 9.9 7.2 4.1 1 7.3 6.4 0.7 1.2 | |
|-----------------------------------------|---------------------------------------|--|
| Number of Chips You Got (out of 10) | 6 | |
| Average Value (to you) of Chips You Got | 5.3 | |
| Total Profits | 2 | |

More Info

You bid 5.

| Computer Players' Values | 7.9 9.1 5.3 8 5.6 7.5 0.1 4.8 2.5 3.1 | |
|-------------------------------------|---------------------------------------|--|
| Number of Chips You Got (out of 10) | 4 | |
| Values (to you) of Chips You Got | 3.1 7.8 5.5 6.1 | |
| Profits Of Each Transaction | 0 0 0 0 0 0 -1.9 2.8 0.5 1.1 | |