# Solving the model representation problem with broom

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Who am I?

Statistical models

The model representation problem

The broom package

Use cases

Open Questions & Hot Takes

tidymodels

#### These slides are available online!

https://tinyurl.com/broom-talk-sgsa

## Who am I?

#### **About Me**



- Just started the Ph.D. program here
- Interned at RStudio over the summer
- Current maintainer of the broom package
- Active on #rstats Twitter and Github

# Statistical models

#### What is a model?

Let x be values that live in some space  $\mathcal{X}$ , and let y be observations of interest that live in some space  $\mathcal{Y}$ . A **statistical model** is a *set* of probability distributions  $\mathcal{P}(y|x)$  indexed by parameters  $\theta \in \Theta^1$ .

<sup>&</sup>lt;sup>1</sup>In some cases we treat  $\theta$  as itself random, which means that our model is a class of probability distributions  $\mathcal{P}(y,\theta|x)$ .

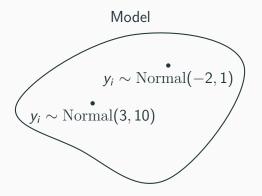
## **Example:** the normal model

$$y_i \stackrel{\text{iid}}{\sim} \text{Normal}(\mu, \sigma^2)$$

Here  $\theta = (\mu, \sigma^2)$  and the parameter space is  $\mathbb{R} \times \mathbb{R}^+$ .

## Visualizing the normal model

Again: a model is a set.



We call a single element of a model a **fit**. The distribution with  $\mu=-2,\sigma^2=1$  is a fit, for example.

## Another (parametric) example: the linear model

Given response y and predictor variables  $x_1$  and  $x_2$ , the linear model looks like:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$$
  $\varepsilon_i \stackrel{\text{iid}}{\sim} \text{Normal}(0, \sigma^2)$ 

This model says that y is i.i.d with a mean that depends on x and  $\vec{\beta}$ , and with fixed variance  $\sigma^2$ .

#### **Estimation**

An **estimator** is a way to calculate the parameters of a model from data<sup>2</sup>. They are many estimators for any given model, and which one we think is best depends on how we define "agrees best with the data."

<sup>&</sup>lt;sup>2</sup>This is equivalent to selecting the best fit!

#### Some estimators for the normal model

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  $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$ 

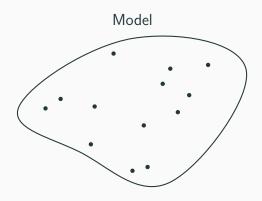
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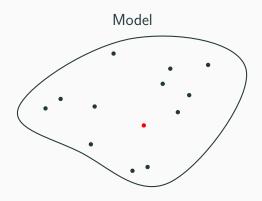
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$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$
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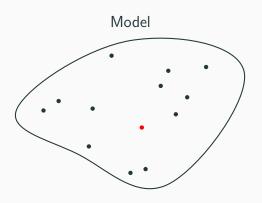
# Visualizing estimation



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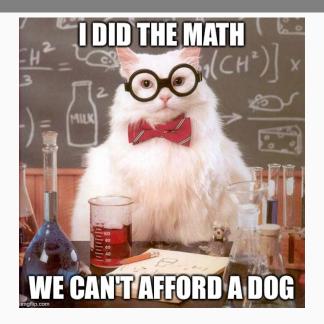


#### **Estimates found!**



Best fit:  $y_i \sim \text{Normal}(-2, 1)$ . Finally we can do inference!

#### Do all the inference



#### Some intuition about models, estimators and fits

A **model** corresponds to a set of possible truths about the world. A **fit** is a single truth about the world. An **estimator** is a way to choose the truth most suggested by the data from a set of many possible truths.

# The model representation problem

Hopefully it feels natural to describe models mathematically

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Key: shared notation and community standards

How does R represent models, estimators and fits?

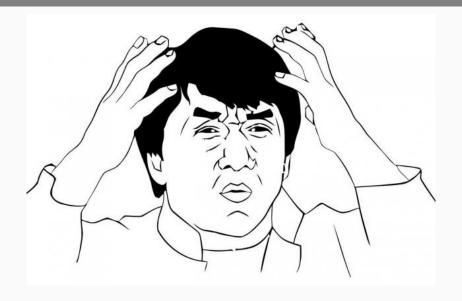


## A taste of the pain

Suppose you want class probabilities from a fit called obj:

Object	Code
lda	predict(obj)
glm	<pre>predict(obj, type = "response")</pre>
gbm	<pre>predict(obj, type = "response", n.trees)</pre>
mda	<pre>predict(obj, type = "posterior")</pre>
rpart	<pre>predict(obj, type = "prob")</pre>
Weka	<pre>predict(obj, type = "probability")</pre>
logitboost	<pre>predict(obj, type = "raw", nIter)</pre>
pamr.train	<pre>pamr.predict(obj, type = "posterior")</pre>

## How am I supposed to keep track of all this!?



## The model representation problem

We have no shared framework or understanding of how to represent statistical models, estimation methods and fits with R objects.

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 People who write and share code to enable data analysis are awesome

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#### **Aside**

- People who write and share code to enable data analysis are awesome
- This isn't anyone's fault
- Trying to get things done can still be frustrating

# The broom package

#### The scene

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- 1. pick a model,
- 2. pick an estimator for that model, and then
- 3. get an R object fit using that estimator.
- The fit object is different for every model.
- The fit object could be great to work with, or awful.

broom treats fits as having three parts:

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- 2. A table of information about each observation used to estimate the fit
- 3. A table of overall goodness-of-fit measures
- Each of table reported as a tidy tibble.
- Together these constitute a tidy representation of a fit.

#### Aside: What is a tibble

A tibble is a more consistent data.frame. tibbles are used widely throughout the tidyverse.

# Aside: What does it mean to be tidy?

I highly recommend reading Hadley Wickham's Tidy Data paper! The short version: a **tidy dataset** is one where:

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- 1. Each variable forms a column.
- 2. Each observation forms a row.
- 3. Differents kinds of data live in different tables.

# The broom generics

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So how do we turn fits into tidy tibbles?

- tidy() summarizes information about fit components
- glance() reports information about the entire fit
- augment() adds information about observations to a dataset

# Continuing with the normal model example

```
# simulate Normal(-2, 1) data
x \leftarrow rnorm(5000, -2, 1)
# create a fit object using
# MLE estimator and normal model
normal_fit <- MASS::fitdistr(</pre>
  x,
  dnorm, # use the normal model!
  start = list(mean = 0, sd = 1)
```

## What is normal\_fit?

```
str(normal fit)
## List of 5
## $ estimate: Named num [1:2] -2.03 1.01
## ..- attr(*, "names")= chr [1:2] "mean" "sd"
## $ sd : Named num [1:2] 0.0143 0.0101
## ..- attr(*, "names")= chr [1:2] "mean" "sd"
## $ vcov : num [1:2, 1:2] 2.06e-04 -4.62e-13 -4.62e-13 1.03
## ..- attr(*, "dimnames") = List of 2
## ....$ : chr [1:2] "mean" "sd"
## ....$ : chr [1:2] "mean" "sd"
##
   $ loglik : num -7165
## $ n : int 5000
## - attr(*, "class")= chr "fitdistr"
```

# What is the tidy representation of normal\_fit?

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There's no augment() method defined for univariate distributions at the moment.

## Another example: the linear model

```
# create a fit object using the
# OLS estimator for the linear model
ols_fit <- lm(hp ~ mpg + cyl, mtcars)
# try the following for yourself:
str(ols_fit)</pre>
```

### The tidy representation of lm objects

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### The tidy representation of lm objects

# Notes about the tidy representation

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- Some information in the original R object is lost

# Use cases

# Report model coefficients with tidy()

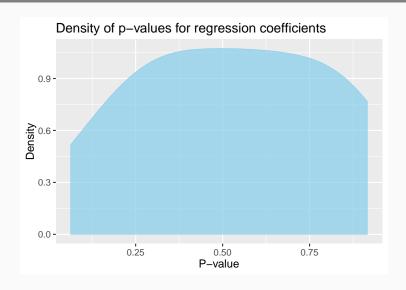
```
kable2 <- function(data)
  knitr::kable(mutate_if(data, is.numeric, round, 2))
tidy(ols_fit) %>%
  kable2()
```

term	estimate	std.error	statistic	p.value
(Intercept)	54.07	86.09	0.63	0.53
mpg	-2.77	2.18	-1.27	0.21
cyl	23.98	7.35	3.26	0.00

## Plot histograms/densities of p-values

```
fit <- lm(mpg ~ ., mtcars)</pre>
td <- tidy(fit)
p <- ggplot(td, aes(p.value)) +</pre>
  geom density(
    fill = "skyblue", color = "skyblue", alpha = 0.7
  ) +
  labs(
    title = "Density of p-values for regression coefficients",
    x = "P-value".
    y = "Density"
```

# Plot histograms/densities of p-values



# Comparing models by goodness of fit measures

```
fits <- list(
  fit1 = lm(hp ~ cyl, mtcars),
  fit2 = lm(hp ~ cyl + mpg, mtcars),
  fit3 = lm(hp ~ ., mtcars)
)

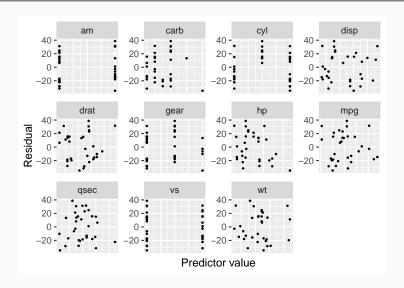
gof <- map_df(fits, glance, .id = "model") %>%
  arrange(AIC)
```

## Comparing models by goodness of fit measures

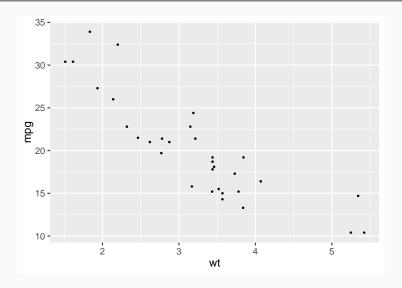
### Inspecting residuals from multiple linear regression

```
fit <- lm(hp ~ ., mtcars)</pre>
au <- broom::augment(fit)</pre>
p <- au %>%
  gather(x, val, -contains(".")) %>%
  ggplot(aes(val, .resid)) +
  geom_point() +
  facet_wrap(~x, scales = "free") +
  labs(x = "Predictor value", y = "Residual") +
  theme(axis.text.x = element blank(),
        axis.ticks.x = element blank())
```

# Inspecting residuals from multiple linear regression



# **Bootstrapping**



# **Bootstrapping**

Consider a model:

$$\mathtt{mpg} = \frac{k}{\mathtt{wt}} + b + \varepsilon, \quad \varepsilon \sim \mathrm{Normal}(0, \sigma^2)$$

Suppose we want to know the sampling distributions of k and b via bootstrapping

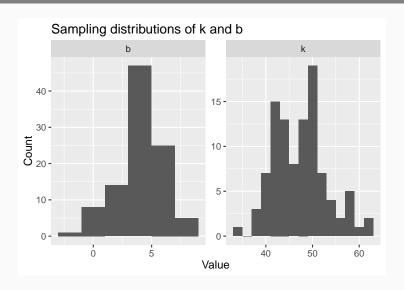
```
library(rsample)
boots <- bootstraps(mtcars, times = 100)</pre>
boots
## # Bootstrap sampling
## # A tibble: 100 x 2
## splits id
## <list> <chr>
## 1 <split [32/10] > Bootstrap001
## 2 <split [32/7]> Bootstrap002
## 3 <split [32/13] > Bootstrap003
## # ... with 97 more rows
```

```
fit_nls_on_bootstrap <- function(split) {
    nls(
        mpg ~ k / wt + b,
        analysis(split),
        start = list(k = 1, b = 0)
    )
}</pre>
```

```
boot fits <- boots %>%
            mutate(fit = map(splits, fit_nls_on_bootstrap),
                                  coef_info = map(fit, tidy))
boot fits
## # Bootstrap sampling
## # A tibble: 100 x 4
## splits id fit coef_info
## * <list> <chr> *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** 
## 1 <split [32/10] > Bootstrap001 <S3: nls > <tibble [2 x 5] >
## 2 <split [32/7]> Bootstrap002 <S3: nls> <tibble [2 x 5]>
## 3 <split [32/13] > Bootstrap003 <S3: nls > <tibble [2 x 5] >
## # ... with 97 more rows
```

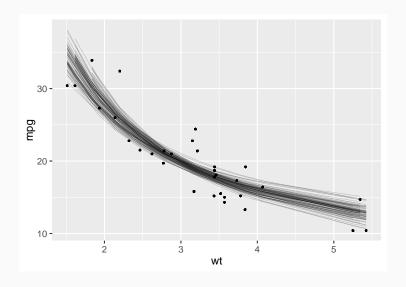
```
boot_coefs <- boot_fits %>%
   unnest(coef_info)
boot coefs
## # A tibble: 200 x 6
## id term estimate std.error statistic p.value
\#\# < chr> <math>< chr> <math>< dbl> <math>< dbl> <math>< dbl> <math><
## 1 Bootstrap001 k 41.8 4.05 10.3 2.18e-11
## 2 Bootstrap001 b 5.96 1.64 3.64 1.01e- 3
## 3 Bootstrap002 k 50.6 3.96 12.8 1.16e-13
## # ... with 197 more rows
```

```
p <- ggplot(boot_coefs, aes(estimate)) +
  geom_histogram(binwidth = 2) +
  facet_wrap(~ term, scales = "free") +
  labs(
    title = "Sampling distributions of k and b",
    y = "Count",
    x = "Value"
)</pre>
```



```
boot_aug <- boot_fits %>%
    mutate(augmented = map(fit, augment)) %>%
    unnest(augmented)

p <- ggplot(boot_aug, aes(wt, mpg)) +
    geom_point() +
    geom_line(aes(y = .fitted, group = id), alpha = 0.2)</pre>
```



### **General strategy**

broom provides tidying methods for 100+ classes!

1. Put fits in a list, or a list-column of a tibble

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- 1. Put fits in a list, or a list-column of a tibble
- 2. Use purrr::map() to apply tidy(), glance() or augment() to each fit
- 3. Use tidyverse tools to manipulate and visualize resulting data!

# **Open Questions & Hot Takes**

### How should we represent the following objects?

Sets of fits are particularly interesting to think about:

- Path algorithms (i.e. LASSO fits)
  - Can do inference from a single fit
  - Can do inference from the entire LASSO path
- Ensembles (i.e. random forests)

Are there different types of sets of fits?

## Generics should operate on fits, not models

Consider the following methods, all for the linear model:

plot\_lasso\_path() for OLS vs LASSO fits

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- coef\_standard\_errors() for OLS vs LASSO fits

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Consider the following methods, all for the linear model:

- plot lasso path() for OLS vs LASSO fits
- coef standard errors() for OLS vs LASSO fits
- interpret\_coefficients() for OLS vs GEE fits

### What operations should we be able to perform?

Things pretty much everybody agrees on:

- fit(model, estimator, data)
- predict(model, new\_data)

Actions that are begging for S3 generics:

- Diagnostics
  - Convergence
  - Assumption checking
- Model visualization
- ...

If you have thoughts about these, please talk to me!

# tidymodels

### tidymodels



#### https://github.com/tidymodels/tidymodels

A general attempt to make modeling in R more consistent and tidy. Lots of big projects. Highlights:

- best practices for developing modeling packages
- parnsip: standardized modeling interfaces (now on CRAN!!)
- recipes: data pre-processing
- rsample: infrastructure for resampling

### Questions?

Read about the recent broom release on the tidyverse blog.



https://broom.tidyverse.org



 ${\tt https://github.com/tidymodels/broom/}$ 





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