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Bending-active reciprocal structures based on equilateral polyhedral geometries

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Abstract
As mutually supported beam structures, reciprocal frames limit the number of components that are joined at each connection to two. However, this system of intermediate connections introduces undesirable bending moments in the beam elements. By utilising elastic deformation to create curved geometries, bending-active structures show the potential of bending as a formation process. Moreover, the curved geometries showcase an increased resistance to bending. Despite the apparent potential, only a few geometric explorations of bending-active reciprocal structures exist. Therefore, we investigated the principle by developing a design methodology based on polyhedral shapes. As this work is part of a research on transformable, rapidly assembled structures, the focus lies on simplicity of the connections, uniformity of the components and reconfigurability. This paper discusses the development of a kit of parts of reciprocal bending-active components based on a selection of polyhedral dome types. To simplify the assembly of the structures and avoid the manual bending of the components on site, we introduce the concept of a double-layered, pre-bent component. Finally, this paper presents the development, fabrication and assembly of the ReciPlyDome, a full-scale prototype of a bending-active reciprocal dome with double-layered components. Preliminary analyses of the load-bearing behaviour show the potential of these systems for material-efficient, lightweight structures. The research presented in this paper contributes to the understanding of bending-active reciprocal frames as a structural principle for temporary and rapidly assembled structures.

Keywords: bending-active structures, reciprocal frames, kit-of-parts structures, transformable structures, prototyping

1. Introduction
Reciprocal frame structures consist of a grid of mutually supported beams (Popovic Larsen [10]). The organisation of these beam elements in a weave pattern invokes structural stability and limits the number of elements joined at each connection to two. The resulting reduced complexity of the nodes and the ability of creating grid structures with shorter, discontinuous elements are the main advantages of reciprocal frames. The induction of bending forces due to the intermediate connections is a significant drawback, often causing the disfavour of reciprocal structures towards other structural systems. On the other hand, recent advances in the design and analysis of bending-active structures show the potential of bending as a formation process for curved structures (Lienhard [11]). The elastic deformation of flexible components allows the assembly of curved structures from flat or linear elements and facilitates reuse, transport and reconfiguration thanks to its reversible nature. Thus, the potential of bending-active reciprocal structures lies in the beneficial manipulation of bending forces to further simplify fabrication and detailing. Figure 1 shows a reciprocal configuration of three bending-active elements with flat cross-section next to the commonly used rectangular and circular ones. Whereas the beam inclinations constitute a certain joint complexity with cutouts or inclined bolt holes for the latter, bending-active elements create the inclination by their curvature, enabling simple bolted connections between tangential planes.

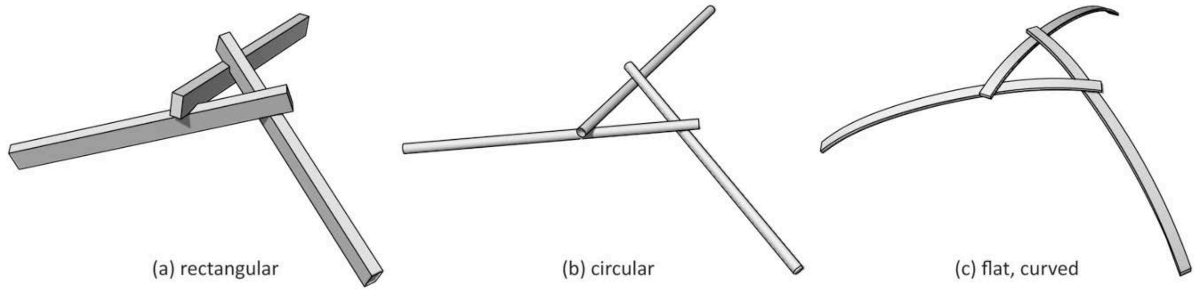


Figure 1: Reciprocal frames usually consist of straight beams with rectangular (a) or circular (b) cross-sections. Although the complexity of the nodes is limited by connecting only two elements at each joint, the inclination of the beam elements requires cutouts or inclined bolt holes. Using flexible, curved elements (c) avoids this, allowing simple bolted connections thanks to coinciding tangential planes.

Due to their low thickness, which is a defining parameter for the overall geometry of a reciprocal structure, other geometric constraints need to control the curvature of bending-active components. Geometric explorations by CODA [4], Alison Grace Martin [6] and Hiroshi Murata [8], as well as contributions to the field of rotegrity [5] show the potential of structural topology in defining curved reciprocal shapes. Baverel and Nooshin [1] performed similar work for rigid reciprocal structures with linear tubes. One of the main aims of this research is to investigate the use of bending-active reciprocal systems for temporary, rapidly assembled structures. As ease of fabrication and assembly, transportability, reuse and reconfiguration are key criteria for this, we focused on the development of kit-of-parts systems with all identical, or at least a limited set of different, components (Heschler et al. [7]). This paper therefore focusses on the geometric modelling of dome structures based on polyhedral shapes. Other geometries will be part of further work. Although elastic deformation is a very intuitive and simple way of creating curvature in a structure, dealing with the bending forces during assembly on site can be a cumbersome task. Therefore, we investigated the development of a pre-bent component, which we applied in a full-scale project: the ReciPlyDome.

This paper discusses the development of bending-active reciprocal dome structures based on polyhedral shapes. After presenting the geometric modelling approach and parameterisation, it describes the development of a kit-of-parts system to create six dome types. Next, we introduce the pre-bent, double-layered component and discuss its application in a full-scale dome prototype. The ReciPlyDome project shows the potential of the proposed reciprocal system for rapidly assembled structures, demonstrating a very fast and low-tech fabrication and assembly process.

2. Geometric and parametric modelling based on polyhedral shapes

Inspired by the topological explorations of Soriano et al. [12], we experimented with physical models in different geometric configurations. As this research is part of a wider investigation into the development of temporary, transformable and rapidly assembled structures using bending-active components (Brancart et al. [2]), we focused the geometric study on reconfigurable kit-of-parts structures, thus aiming for maximal uniformity among the components. A wide range of convex polyhedra with identical edge lengths exists, among which the commonly known Platonic and Archimedean solids (Pottmann et al. [9]). Figure 2 shows an approach to model spherical reciprocal configurations based on these geometries, the icosahedron in this case (figure 2a). Since all the vertices of the polyhedron lie on its circumscribed sphere, we can project the edges to create identical arcs on the sphere surface (figure 2b). Rotating them about the centreline through the midpoint of the arc explodes the vertices, in this case into five endpoints (figure 2c). Extending and intersecting the arcs generates a reciprocal relation by creating additional polygons around the original polyhedron's vertex, here a pentagon where five edges used to meet (figure 2d).

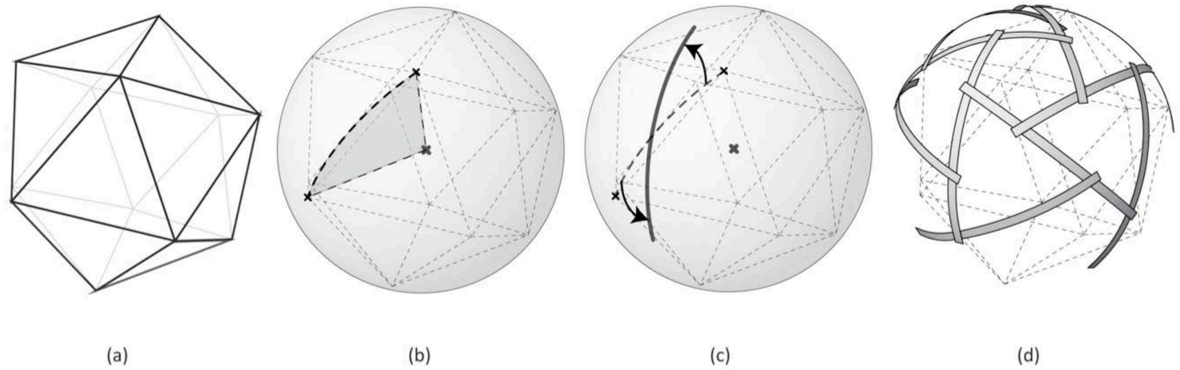


Figure 2: Based on a polyhedral shape, in this case an icosahedron (a), we can model a spherical reciprocal structure by projecting the edges on the circumscribed sphere (b), rotating them (c) and finding the four intersection points (d).

This modelling approach is similar to the one described by Baverel and Nooshin [1] for linear tubes. For rigid circular elements however, the relation between the geometrically defined inclination and the cross-section of the beam elements makes that some configurations, with large rotation and thus higher inclination, only work for very large cross-sections. This is not the case when using curved elements, as their curvature covers the inclination and the geometry is more or less independent from the cross-sectional thickness.

Figure 3 shows the development of four reciprocal sphere geometries starting from the same icosahedral base geometry, scaling the sphere to keep the length of the beam elements fixed. As a result, their components vary only in the location of their two intermediate connections. They slide towards the middle with increasing angle (from figure 3a to b), crossing in the middle (from figure 3b to c) and sliding back towards the endpoints (from figure 3c to d). By flipping in the middle, the base geometry of the structure changes to its dual polyhedron, in this case from icosahedron to dodecahedron. Therefore, each geometry has a dual configuration that consist of the same components, arranged in a different topological organisation (figure 3a and d, and 3b and c).

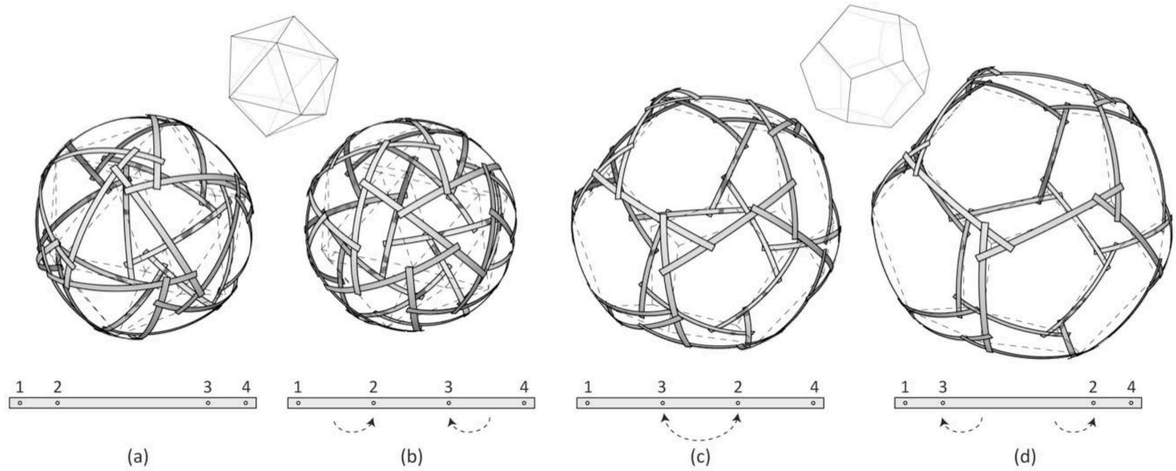


Figure 3: By rotating the beam elements on the spherical surface, the intermediate connections slide over the beam. In the middle they flip sides, creating the reciprocal configurations of the original polyhedron's dual (here the dodecahedron). Thus, each configuration has a dual consisting of the same components (a and d, b and c).

3. A kit-of-parts system: six dome types

Thanks to the reversibility of their elastic deformation and the independence of their cross-sections from the structure's geometry and connections, bending-active components are well-suited for the development of kit-of-parts systems for reciprocal structures. Yet, aside from the geometric possibilities, the reconfiguration of kit-of-parts structures is largely restricted by the structural behaviour in the different configurations. Out of the wide range of polyhedra, we therefore selected six geometries that resulted in reciprocal structures with a reasonable amount of beam elements and acceptable curvature and compared their behaviour: the icosahedron and its dual the dodecahedron, the icosidodecahedron and its dual the rhombic triacontahedron, and the truncated icosahedron and its dual the pentakis dodecahedron. Figure 4a shows the displacements of the six dome types for four different beam elements. It becomes apparent that the selection of the location of the intermediate hinges significantly defines the reconfiguration potential, as the spreading of the displacements becomes a lot higher when we move the connections towards the ends. When optimising reconfigurable systems, the aim is after all not to optimise for a single best solution, but rather for an optimal spread of configurations, where the most optimal component leads to the largest amount of structurally viable and efficient structures. Bracing (e.g. in the form of cladding) and double layering (as discussed in the following section) will be important parameters in optimising this spread. In the case of bending-active structures, the pre-bending is an important criterion for the dimensioning of the components. Figure 4b shows the evolution of the curvature, and consequently the pre-stress, along the beam elements for the different dome configurations. The variation of the curvature along the beam, which is constant in a zero-thickness model, compensates for the element thickness, analogue to a changing inclination and component intersection in rigid reciprocal frames.

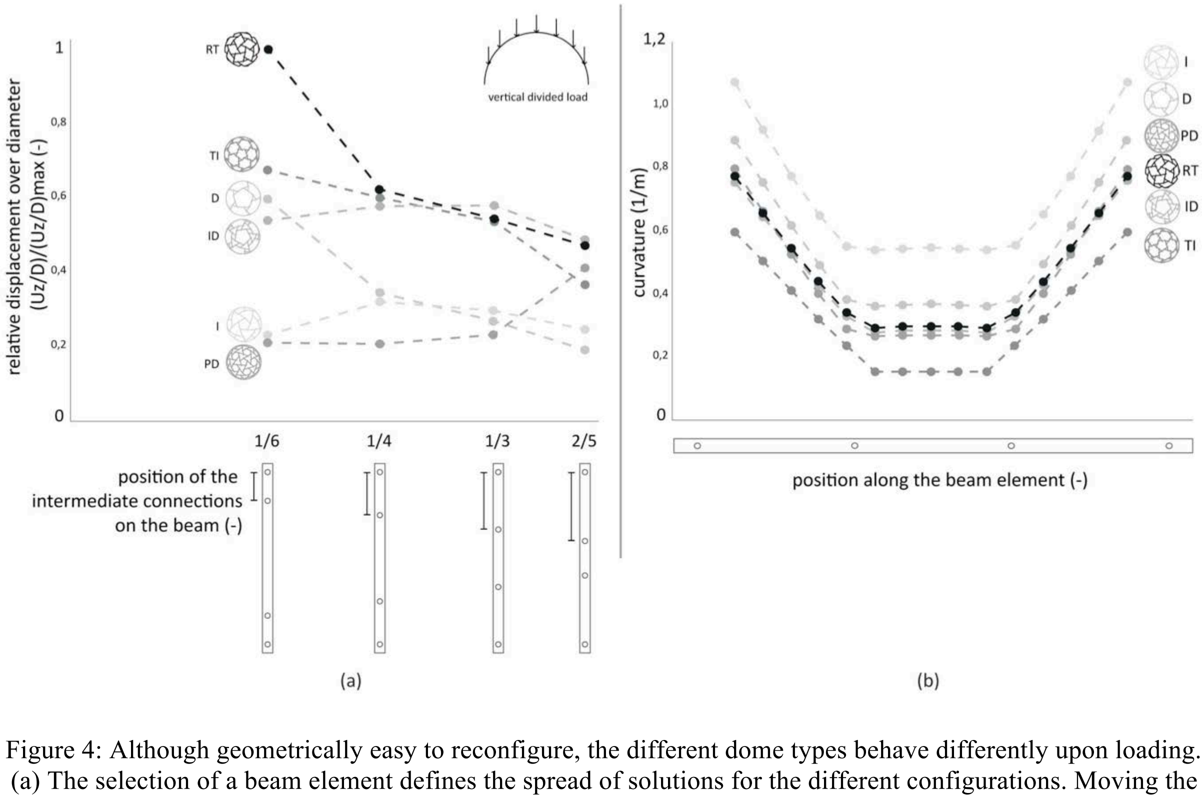


Figure 4: Although geometrically easy to reconfigure, the different dome types behave differently upon loading. (a) The selection of a beam element defines the spread of solutions for the different configurations. Moving the intermediate connections towards the ends enlarges this spread, which is unfavourable. (b) The variation in the curvature, and as a result the pre-bending, for different dome types complicates the dimensioning of the flexible beams, as the required flexibility reduces the stiffness of the assembled structure.

Figure 5 shows a potential set of six dome types based on beam elements with intermediate connections in a one-third division. To create a finished dome structure with stable connections to the ground, each dome requires several shortened components. Moreover, the presence of two different edge lengths in the geometry of the pentakis dodecahedron results in an additional beam type for its resultant reciprocal dome and that of its dual, the truncated icosahedron. Despite this, several kit-of-parts systems remain possible. Since only three beam types are needed to create the six domes, it would even be possible to develop a universal component that combines all the necessary bolt holes in one element (see figure 5, beam type U). Since these bending-active reciprocal structures consist of simple linear, curved elements, the potential for reconfiguration seems quite high. Future research will broaden the scope in a geometric exploration that goes beyond the spherical geometries. Furthermore, mass customisation of a set of unique components can create additional geometric freedom. While this is not part of the scope of this work, where the focus is on rapid manual fabrication, reconfiguration and ease of assembly, it can be interesting in other cases for which a parametric design environment and CNC fabrication are available, like the Undulatus project (Brancart et al. [3]).

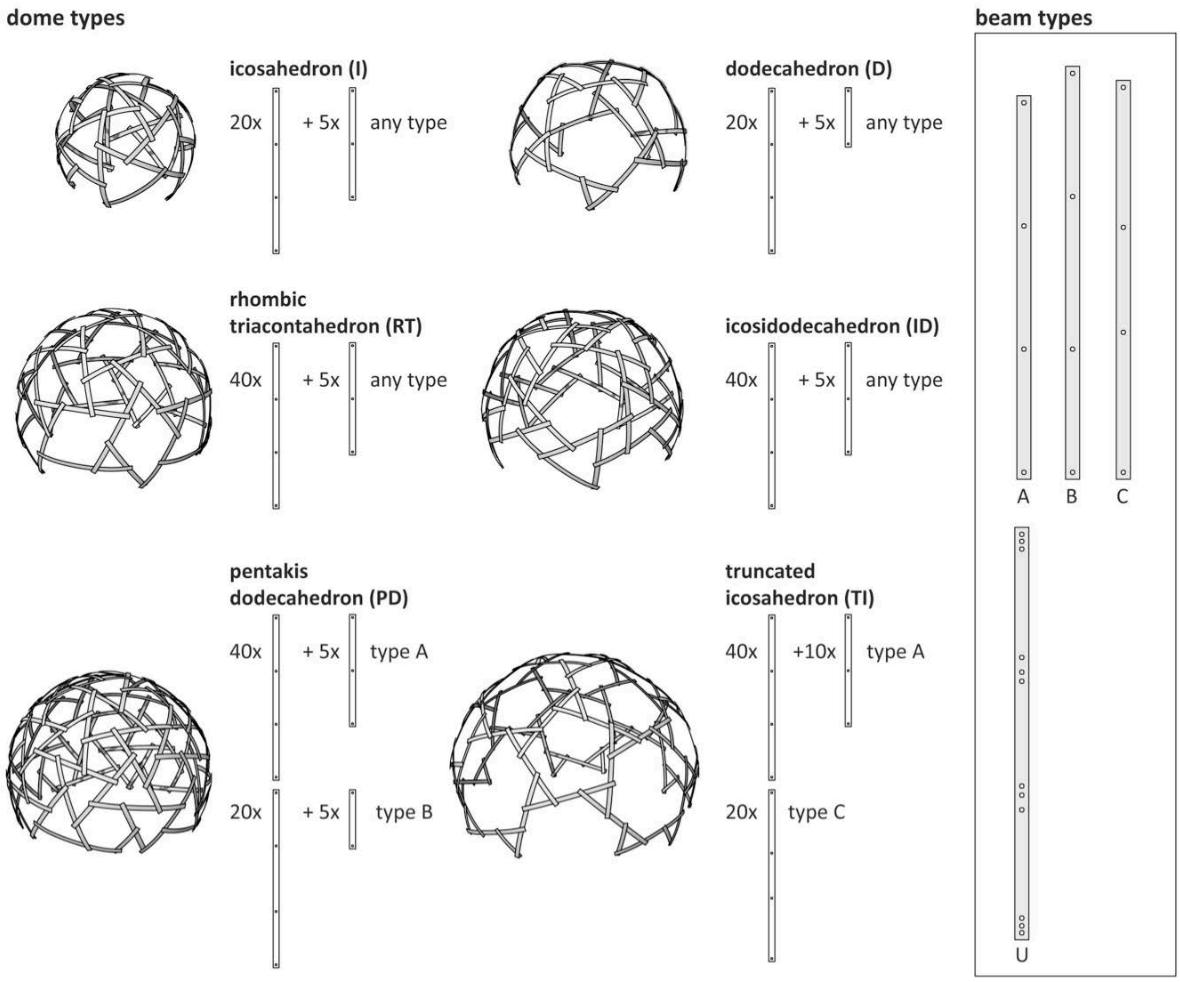
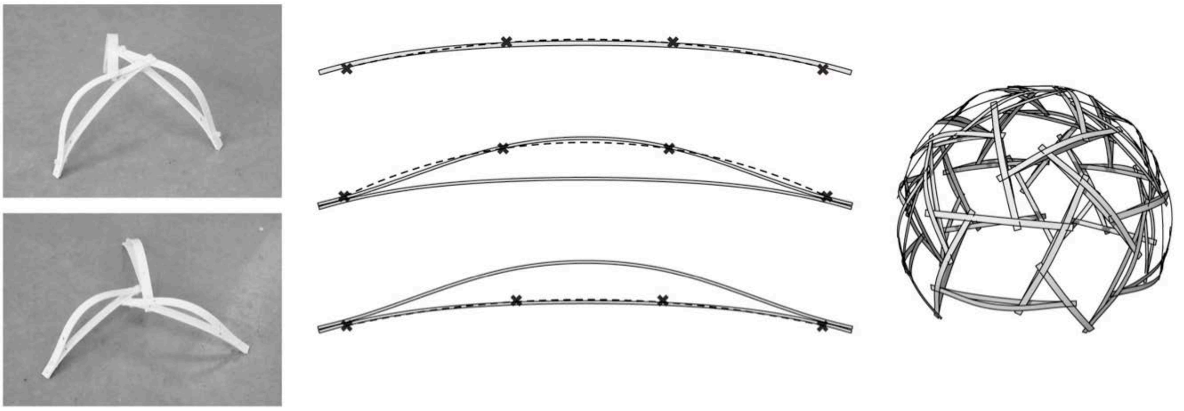


Figure 5: Six polyhedra are well-suited for the development of bending-active reciprocal domes. As we can construct all these configurations with only a limited number of different beam types, several possibilities for developing a kit of parts exist. Moreover, a universal component (beam type U) could combine all the necessary bolt holes in one single beam element.

4. A pre-bent, double-layered component

Although active bending can significantly simplify the fabrication of curved structures, handling the bending forces in the components during on site assembly mostly complicates the erection process. Therefore, we investigated the use of pre-bent components. Essentially, all geometries that maintain the position and tangential planes of the connections are fit for assembly in the desired dome geometry. We worked with a simple double-layered component, illustrated in figure 6b. The curvature of this component is controlled by a difference in length of the two interconnected layers. The clamped connections at the endpoints assure that both layers are bent. Due to the difference in curvature of both layers, locating the connections on one or the other changes the inclination of the reciprocal configuration (see figure 6a).



(a) (b) (c)

Figure 6: A double-layered component allows to pre-bend the flexible elements in the desired shape, avoiding on-site bending during the assembly. (a) Both layers allow intermediate connections, reducing the inclination when connecting to the least curved bottom one. (b) The only requirement is that the component contains the correct connection locations for the desired geometry, and allows planar connections between the end and intermediate connections. (c) One specific component curvature corresponds to the desired dome geometry, here the rhombic triacontahedron.

Aside from simplifying the connections, the curvature of the arched components also increases the load-bearing capacity of the elements in bending. Figure 7a illustrates the effect on the displacements for three component types: a straight component as used in conventional reciprocal frame structures (12mm an double thickness 24mm), a single-layered, bent component (12mm and 24mm) and a double-layered component as introduced before (2x12mm). For the latter a distinction is made between loading the bottom or top layer. The results clearly show the effect of the curvature on the behaviour of the components, also indicating a large difference between loading the less curved bottom layer instead of the top layer of the double-layered components. Figure 7b shows the displacements of the reciprocal dome structure with rhombic triacontahedral geometry for three components: two single-layered components of 12 and 24mm and a double-layered one with two layers of 12mm. Although the two layers of the double-layered component are only connected at the ends and the intermediate connections within the dome are located only at the top layer, the behaviour is quite close to the single-layered double-thickness one. This shows the potential of the double layered component for increasing the structural efficiency, which will be investigated in more detail during further work.

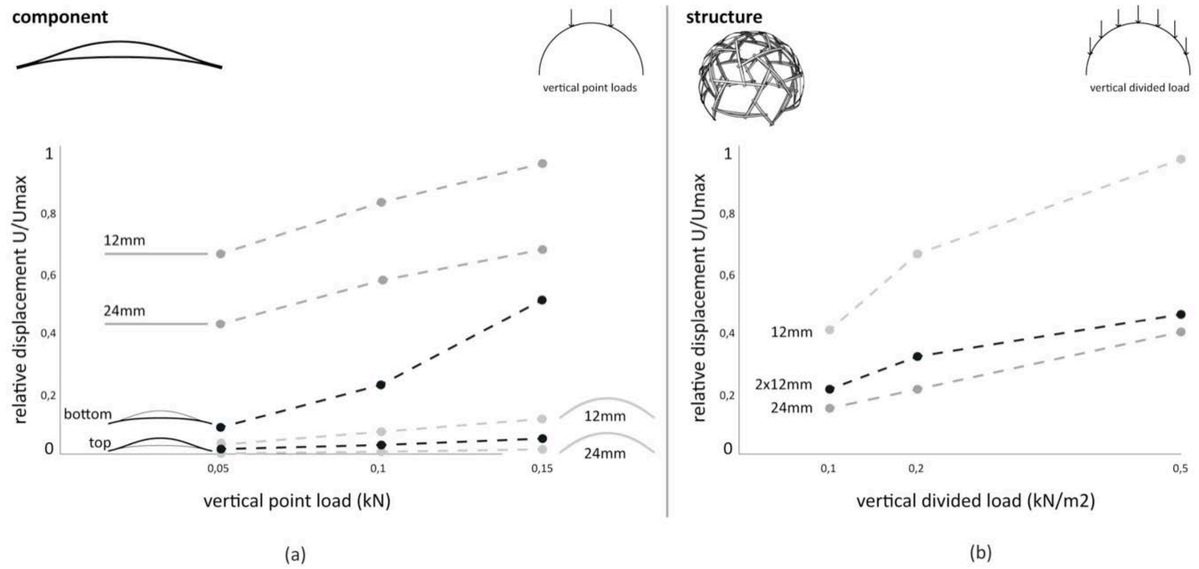


Figure 7: The curvature of the bending-active components influences their load-bearing behaviour in the reciprocal geometry. (a) The curved components have a higher resistance to the bending imposed on them by the reciprocal organisation of the elements. Although the layers are only connected at the ends, the double layering has a significant effect on the structural performance. (b) This is also the case when applying the double-layered component to a dome, as its performance is quite close to that of a single-layered component with double thickness.

5. Fabrication and assembly: the ReciPlyDome

To validate the geometric and structural principles, and test the fabrication and assembly of bending-active reciprocal structures, we developed a full-scale prototype: the ReciPlyDome. The structure is based on a rhombic triacontahedral base shape and consists of a set of double-layered elements. The dome has a diameter of about five meters and four meters of height. The five almost vertical rhombs at the bottom provide the supports of the structure and can be used to anchor it to the ground. They could also serve as the entrances when using the structural system for temporary architectural applications. The grid structure can easily be adapted to contain cladding and thus serve as a pavilion or shelter for a festival or other events. Notwithstanding the simplicity and uniformity of the components, the structure expresses an apparent complexity due to the varying orientation of the double-layered components and the combination of three polygonal shapes, triangle, rhombs and pentagons: uniformity generating complexity.

Bending-active structures require flexible materials with sufficient elastic strength. After some iterations with different thicknesses and types of plywood, we selected a 12mm birch that can be bent manually into the desired shape while still resulting in a sufficiently rigid component. Fibre-reinforced polymers or other composite materials would be a good alternative, further increasing the structural performance of the structure, but also the fabrication time and cost. To ease the fabrication, we developed a jig system onto which to push and fix the top layer at the required curvature, enabling one person to manually assemble the components (figure 8, top left). The structure contains only two different elements, 40 basic ones and 5 that are shortened to complete the bottom ring (figure 8, right). Even when assembled, the whole set of components is quite compact, the basic elements being only about 2,20 meters long (figure 8, bottom left). The fabrication of the kit of parts requires only some bolts and simple tools to drill the holes and cut the plate elements.

Figure 8: The double layered components consist of two plywood laths of 12mm, connected at the ends. Pushing and fixing the top layer onto a jig simplifies the fabrication process, allows manual assembly by one person (top left). The kit of parts for the structure consists of only two different pieces (right). Thanks to the discretisation of the elements in the reciprocal organisation, the elements remain fairly short and the full set of components very compact (bottom left). (pictures by Jack Cripps)

Instead of assembling the dome piece by piece, we pre-assembled six pentagonal modules, maximising the assembly to be done on the ground (figure 9, top left). With 30 elements, they make up most of the structure, only to be finished by five sets of three components at the bottom edge. Thanks to the pre-bent components, the simplicity of the connections and the uniformity of the structure, we could assemble the dome in only a couple of hours, using no technical tools or cranes (figure 9, bottom left). Since the tangential planes of the components do not coincide completely at the connections, some torsion is present in the structure (figure 9, centre). The components allowed just enough deformation to manually induce this torsion by aligning the elements at the connections, with only a wrench to fasten the bolts. Weighing only 160 kilograms (foundations not included) the structure can be considered lightweight. Yet, it does show an efficient shell behaviour, maintaining the rigidity of the components in the fully assembled configuration (figure 9, bottom right). Further research should investigate the structural behaviour over a longer period of time, when creep occurs and the bending pre-stress decreases.

As a first validation of the application of bending-active reciprocal systems for rapidly assembled structures, the ReciPlyDome project shows how a thorough consideration for constructability, uniformity and ease of fabrication and assembly can significantly lower the production and construction time. In doing so the complexity of the project largely shifts to the design and modelling phase, supported by advances in the development of parametric design tools.

Figure 9: We constructed the ReciPlyDome from six pre-assembled pentagonal modules, optimising the assembly time on the ground (top left). Thanks to the very simple connections (top right), the assembly required only basic hand tools (bottom left). The components allowed just enough deformation to manually induce the torsion needed to align the components at the connections (centre). The fully assembled structure is lightweight but shows efficient shell behaviour (bottom right). (pictures by Jack Cripps)

4. Conclusions

This paper presents the development of bending-active reciprocal structures based on polyhedral geometries. Since this research is framed in the wider scope of investigating the use of bending-active elements for the design and fabrication of rapidly assembled structures, the presented approach focuses on ease of fabrication and assembly, uniformity of design and complexity, and reconfigurability of the structure. Using elastically curved beam elements turns the negatively-perceived bending forces in a reciprocal configuration into an advantage for the fabrication and formation process, reduces the complexity of the nodes and improves the load-bearing behaviour of the structure. The modelling approach illustrates how to generate a range of reciprocal dome geometries with identical sets of bending-active components based on polyhedral geometries. Since the elastic, adaptable curvature of the components is the main parameter for the geometric development of the bending-active dome configurations, instead of the inclination and element thickness in conventional reciprocal frame structures, reconfiguration of the (single-layered) components becomes possible. Further research can expand the geometric potential of the concept by including non-spherical geometries, reducing the uniformity by introducing more variation in the components, or even investigating the possibilities of mass-customised components. Although fixing the curvature of the components reduces the capacity for reconfiguration, the presented work shows the potential of pre-bent, double-layered components. Apart from simplifying the assembly process, the double-layered component significantly improves the load-bearing capacity of the reciprocal dome structures, without compromising the flexibility of the comprising parts. Studies on how to further manipulate this behaviour, e.g. by increasing the interaction between both layers, are part of future research. The successful construction of the ReciPlyDome illustrates the potential application of the presented structural principle in temporary, rapidly assembled kit-of-parts structures. The addition of cladding, alternative materialisation and the exploration of architectural uses can help optimise these systems for future applications.

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