$$
\begin{aligned}
\ln \mathcal{L}_i =& y_i \ln f(t_i) + (1-y_i)\ln S(t_i) - \ln S(e_i) \\
\frac{\partial \ln \mathcal{L}_i}{\partial \alpha} =& \frac{\partial y_i \ln f(t_i)}{\partial \alpha} + \frac{\partial (1-y_i)\ln S(t_i)}{\partial \alpha} - I(e_i > 0)\frac{\partial \ln S(e_i)}{\partial \alpha} \\
\frac{\partial y_i \ln f(t_i)}{\partial \alpha} =& y_i \frac{\partial \ln f(t_i)}{\partial \alpha} + \ln f(t_i)\frac{\partial y_i}{\partial \alpha} \\
=& y_i \frac{\partial \ln f(t_i)}{\partial \alpha} \\
\frac{\partial y_i \ln S(t_i)}{\partial \alpha} =& (1-y_i)\frac{\partial \ln S(t_i)}{\partial \alpha}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \ln \mathcal{L}}{\partial \alpha} =& \sum_i \left(y_i \frac{\partial \ln f(t_i)}{\partial \alpha} + (1-y_i)\frac{\partial \ln S(t_i)}{\partial \alpha} - I(e_i > 0)\frac{\partial \ln S(e_i)}{\partial \alpha} \right) \\
\frac{\partial \ln \mathcal{L}}{\partial \rho} =& \sum_i \left(y_i \frac{\partial \ln f(t_i)}{\partial \rho} + (1-y_i)\frac{\partial \ln S(t_i)}{\partial \rho} - I(e_i > 0)\frac{\partial \ln S(e_i)}{\partial \rho} \right) \\
\nabla \ln \mathcal{L} =& \left[\frac{\partial \ln \mathcal{L}}{\partial \alpha}, \frac{\partial \ln \mathcal{L}}{\partial \rho} \right] \\
\nabla \equiv \nabla \ln \mathcal{L} =& \left[\sum_i \left(y_i \frac{\partial \ln f(t_i)}{\partial \alpha} + (1-y_i)\frac{\partial \ln S(t_i)}{\partial \alpha} - I(e_i > 0)\frac{\partial \ln S(e_i)}{\partial \alpha} \right), \sum_i \left(y_i \frac{\partial \ln f(t_i)}{\partial \rho} + (1-y_i)\frac{\partial \ln S(t_i)}{\partial \rho} - I(e_i > 0)\frac{\partial \ln S(e_i)}{\partial \rho} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
H \equiv H \ln \mathcal{L} =& \begin{bmatrix} \frac{\partial^2 \ln \mathcal{L}}{\partial \alpha^2} & \frac{\partial^2 \ln \mathcal{L}}{\partial \alpha \partial \rho} \\ \frac{\partial^2 \ln \mathcal{L}}{\partial \rho \partial \alpha} & \frac{\partial^2 \ln \mathcal{L}}{\partial \alpha^2} \end{bmatrix} \\
\frac{\partial^2 \ln \mathcal{L}}{\partial \alpha \partial \rho} =& \sum_i \left(y_i \frac{\partial^2 \ln f(t_i)}{\partial \alpha \partial \rho} + (1-y_i)\frac{\partial^2 \ln S(t_i)}{\partial \alpha \partial \rho} - I(e_i > 0)\frac{\partial^2 \ln S(e_i)}{\partial \alpha \partial \rho} \right)
\end{aligned}
$$

Define $\theta = (\alpha, \rho)$

$$
\theta_{j+1} = \theta_j - H^{-1}\nabla \qquad \text{(Newton-Raphson step)}
$$