MEng Final Year Project

Distributed Optimization in Multi-agent Systems with Coupling Constraints

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Distributed Optimization

- General optimization problem:

$$\min_x f(x) \qquad \text{subject to} \quad \begin{cases} c^j(x) = 0, & j \in \mathcal{P} \\ c^j(x) \le 0, & j \in \mathcal{Q} \end{cases}$$

- General local problem P_i in distributed optimization scheme:

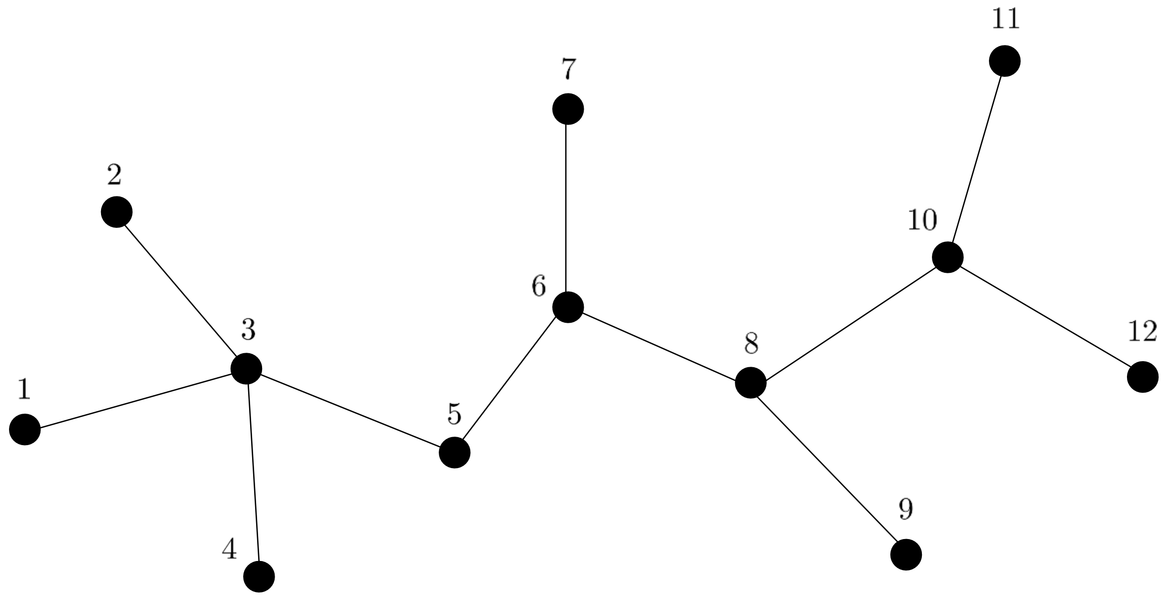
$$\begin{aligned} \min_{\{x_{ji},\, j \in \mathcal{S}_i\}} & \sum_{j \in \mathcal{S}_i} f_j(x_{ji}) \\ \text{s.t. } & c_i^l(\{x_{ji},\ j \in \mathcal{S}_i\}) = 0, \quad l \in \mathcal{P}_i \\ & c_i^l(\{x_{ji},\ j \in \mathcal{S}_i\}) \le 0, \quad l \in \mathcal{Q}_i \end{aligned}$$

- Each local problem extracts the solution $x_i^* = x_{ii}^*$ and the solution to the global problem can then be assembled:

$$x^* = (x_1^*, \ldots, x_N^*)$$

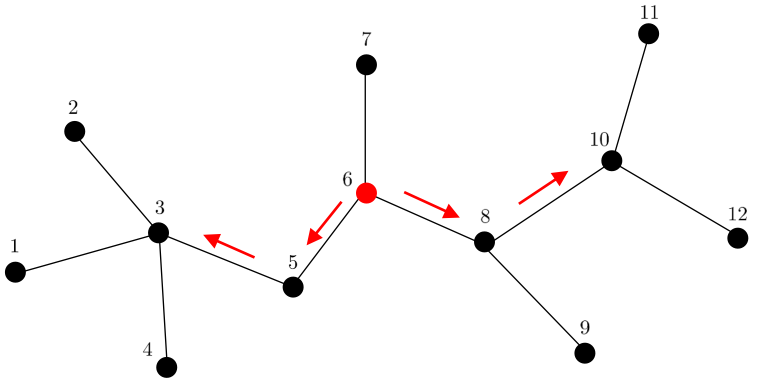
Optimization Over Graphs

- Graph: $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$
- Neighbours: $\mathcal{N}_i = \{j \mid (i,j) \in \mathcal{E}\}$
- Neighbourhood: $\mathcal{S}_i = \{i\} \cup \mathcal{N}_i$



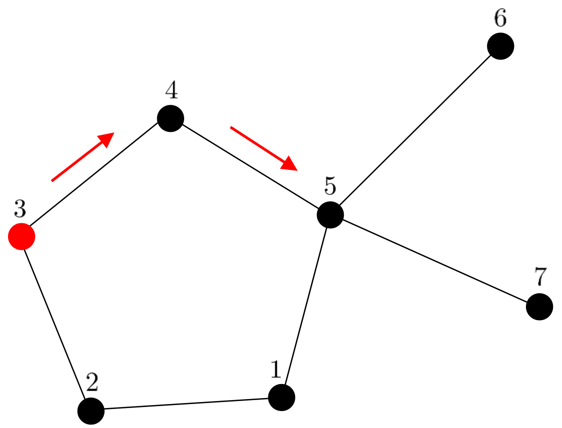
Staged Optimization Over Connected Dominating Sets

- Converging to consensus: $x_{ij}^* \to z_i^*,\ x_{ik}^* \to z_i^*$
- Instead, just copy: $x_{ik}^* \triangleq x_{ij}^*$



ROOT PROBLEM

$$\begin{aligned}\min_{\{x_{jv_0},\, j\in\mathcal{S}_{v_0}\}} \ & \sum_{j\in\mathcal{S}_{v_0}} f_j(x_{jv_0})\\ \text{s.t.}\ & c_{v_0}^l(\{x_{jv_0},\ j\in\mathcal{S}_{v_0}\}) = 0, \quad l \in \mathcal{P}_{v_0}\\ & c_{v_0}^l(\{x_{jv_0},\ j\in\mathcal{S}_{v_0}\}) \le 0, \quad l \in \mathcal{Q}_{v_0}\end{aligned}$$



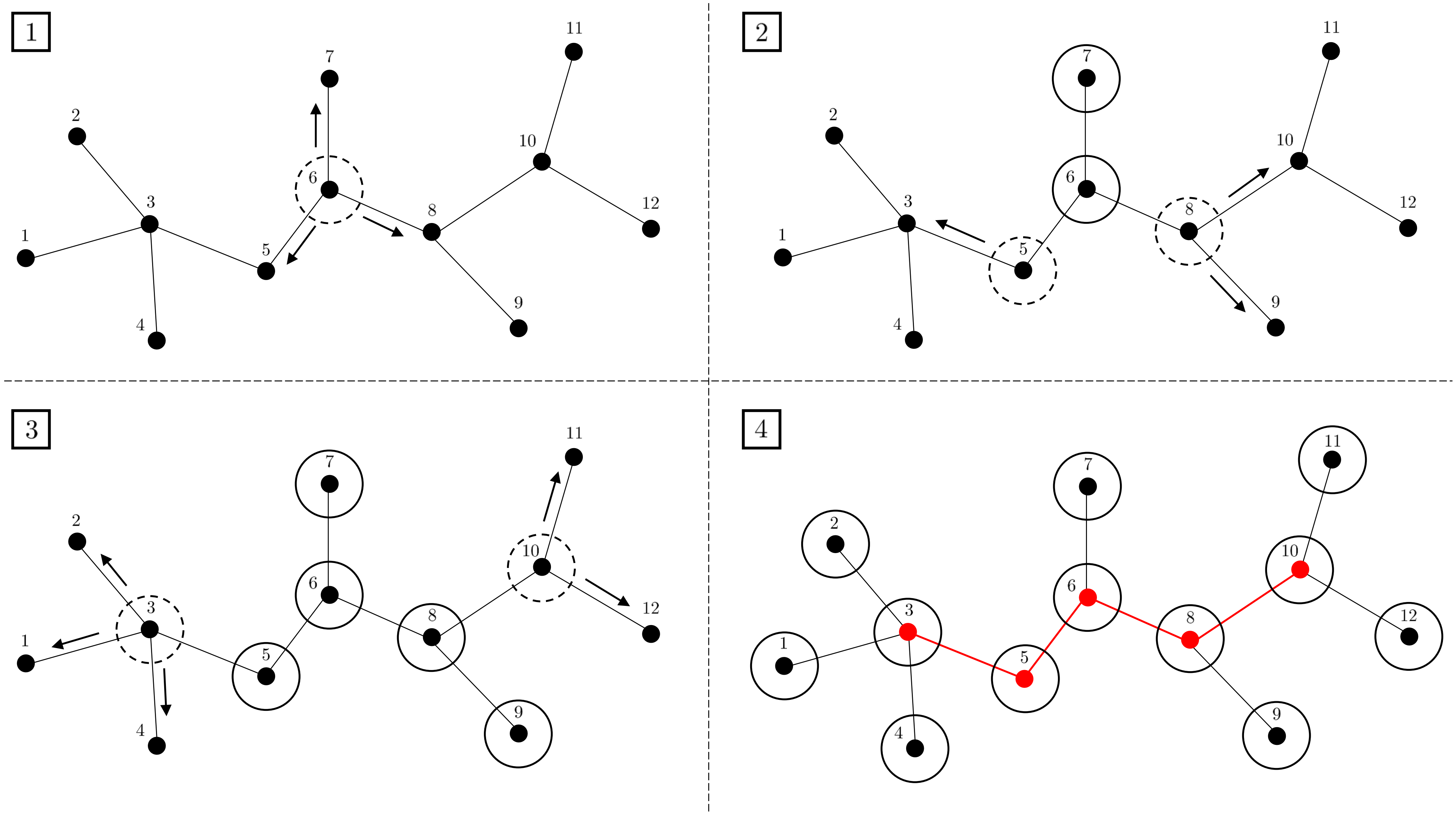
$$\begin{aligned}\min_{\{x_{ji},\, j\in\bar{\mathcal{S}}_i\}} \ & \sum_{j\in\bar{\mathcal{S}}_i} f_j(x_{ji})\\ \text{s.t.}\ & c_i^l(\{x_{ji},\ j\in\bar{\mathcal{S}}_i\}, x_i^*, x_{p_i i}^*) = 0, \quad l \in \bar{\mathcal{P}}_i\\ & c_i^l(\{x_{ji},\ j\in\bar{\mathcal{S}}_i\}, x_i^*, x_{p_i i}^*) \le 0, \quad l \in \bar{\mathcal{Q}}_i\end{aligned}$$

SIMPLIFIED PROBLEM (BRANCH)

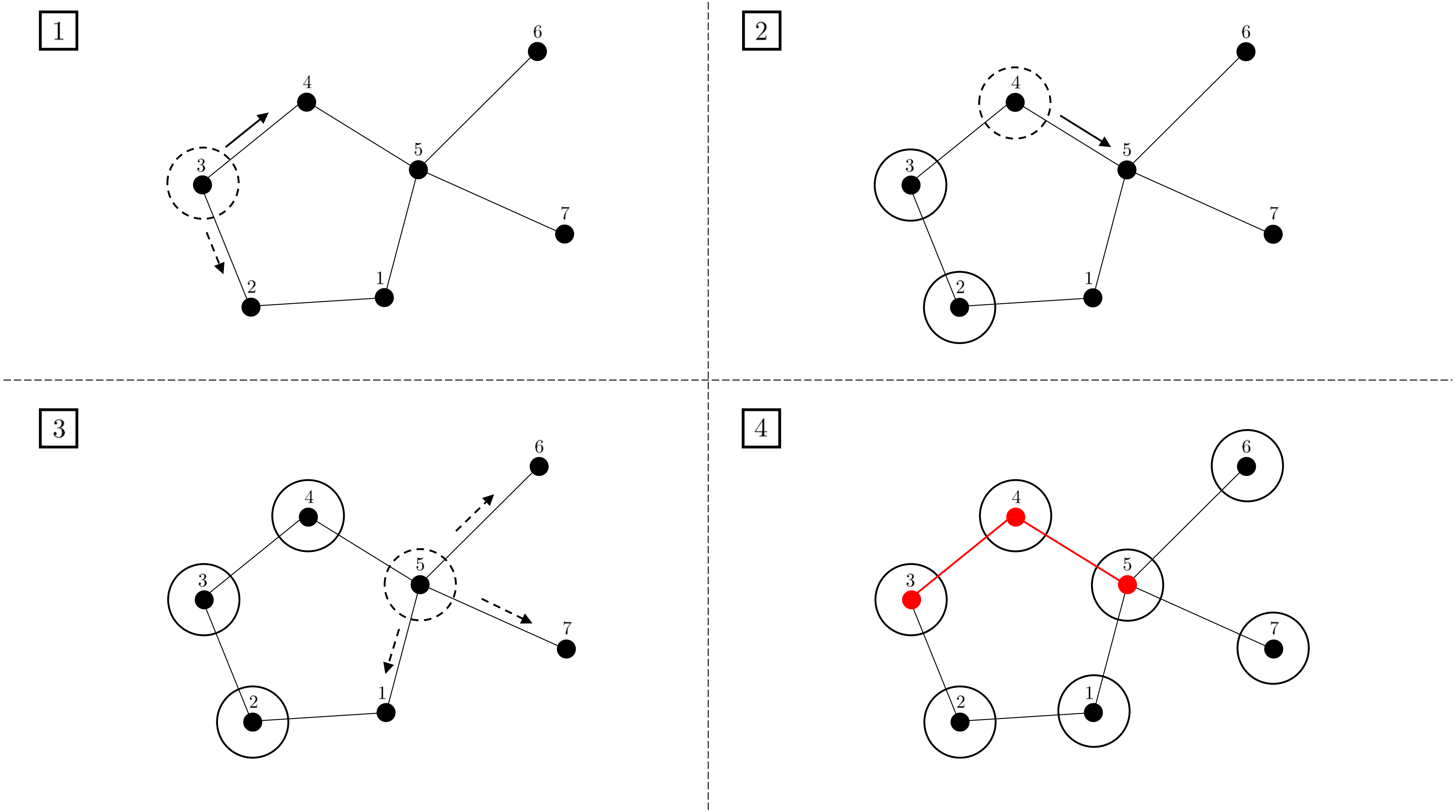
$$\begin{aligned}\min_{\{x_{jv_k},\, j\in\bar{\mathcal{S}}_{v_k}\}} \ & \sum_{j\in\bar{\mathcal{S}}_{v_k}} f_j(x_{jv_k})\\ \text{s.t.}\ & c_{v_k}^l(\{x_{jv_k},\ j\in\bar{\mathcal{S}}_{v_k}\}, \{x_{jv_k}^*,\ j\in\mathcal{F}_{k-1}\}) = 0, \quad l \in \bar{\mathcal{P}}_{v_k}\\ & c_{v_k}^l(\{x_{jv_k},\ j\in\bar{\mathcal{S}}_{v_k}\}, \{x_{jv_k}^*,\ j\in\mathcal{F}_{k-1}\}) \le 0, \quad l \in \bar{\mathcal{Q}}_{v_k}\end{aligned}$$

SIMPLIFIED PROBLEM (PATH)

CDS Branch Sweep (for trees with neighbourhood coupling)

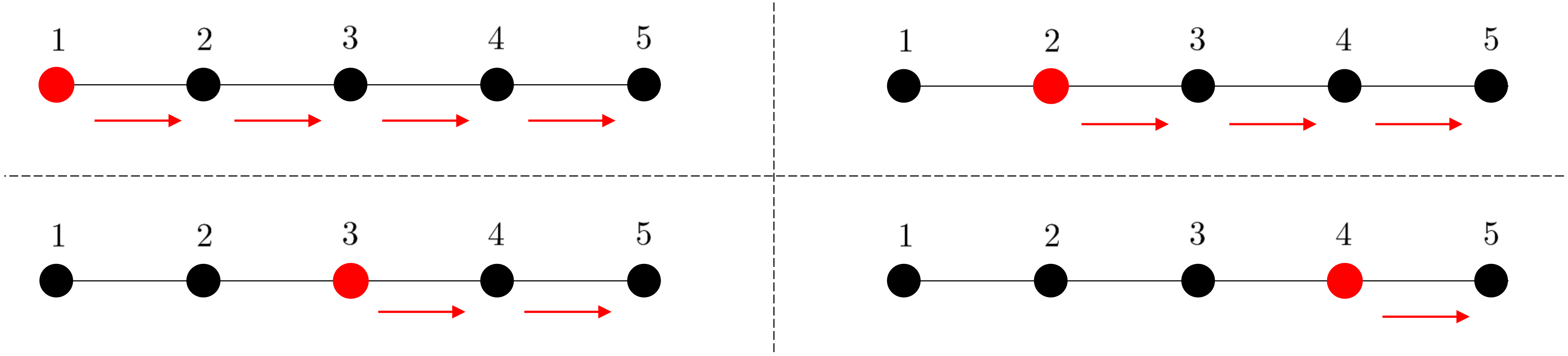


CDS Path Sweep (for graphs with a CDS-induced path)

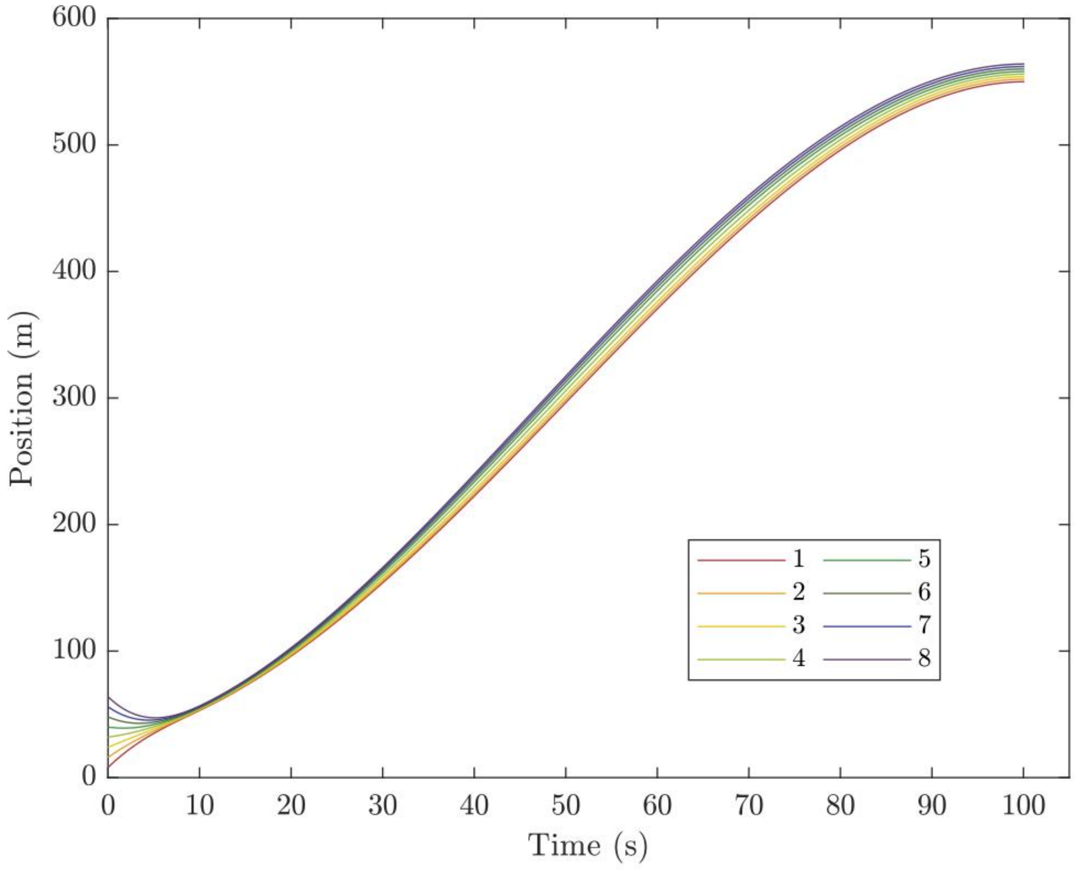


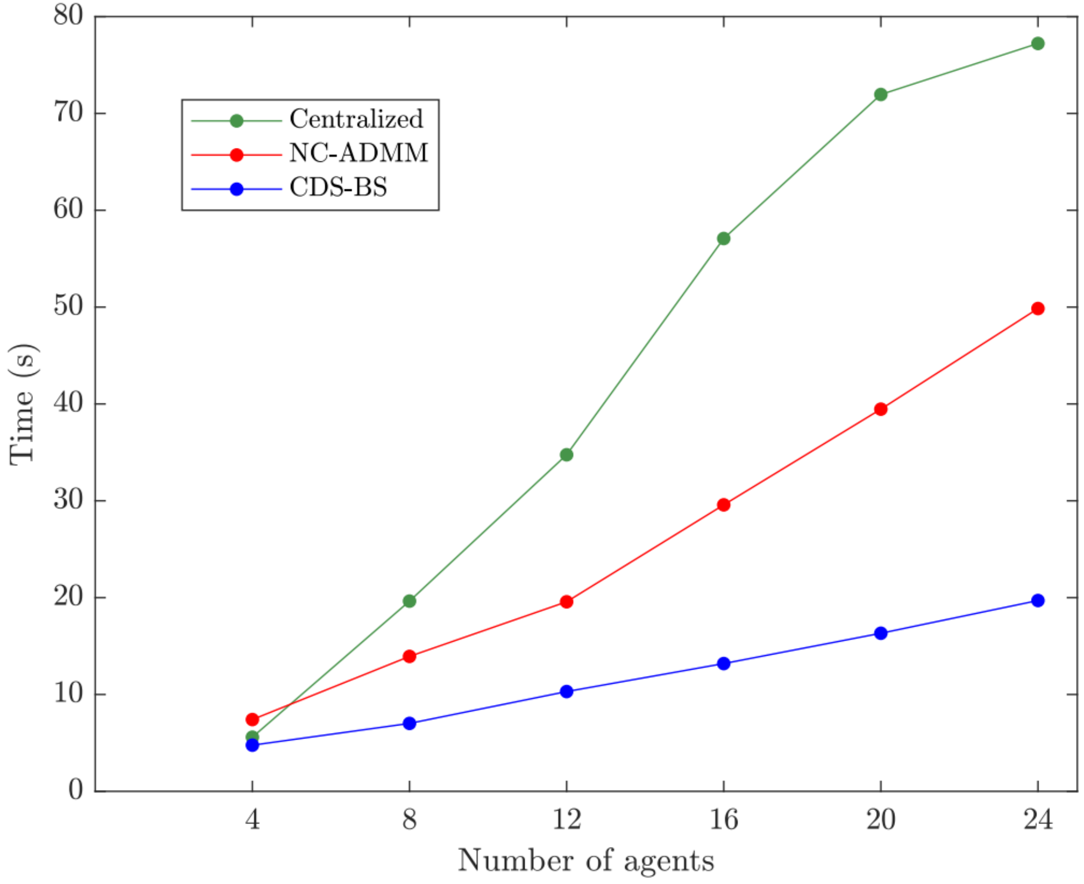
Characteristics of the sweeping algorithms

- For both algorithms, a single exact sweep limits class of problems for which feasibility is guaranteed.
- The increasing complexity in CDS-PS problems can be alleviated with an external active set strategy.
- Ensure efficient communication protocol with a more continuous flow of information.

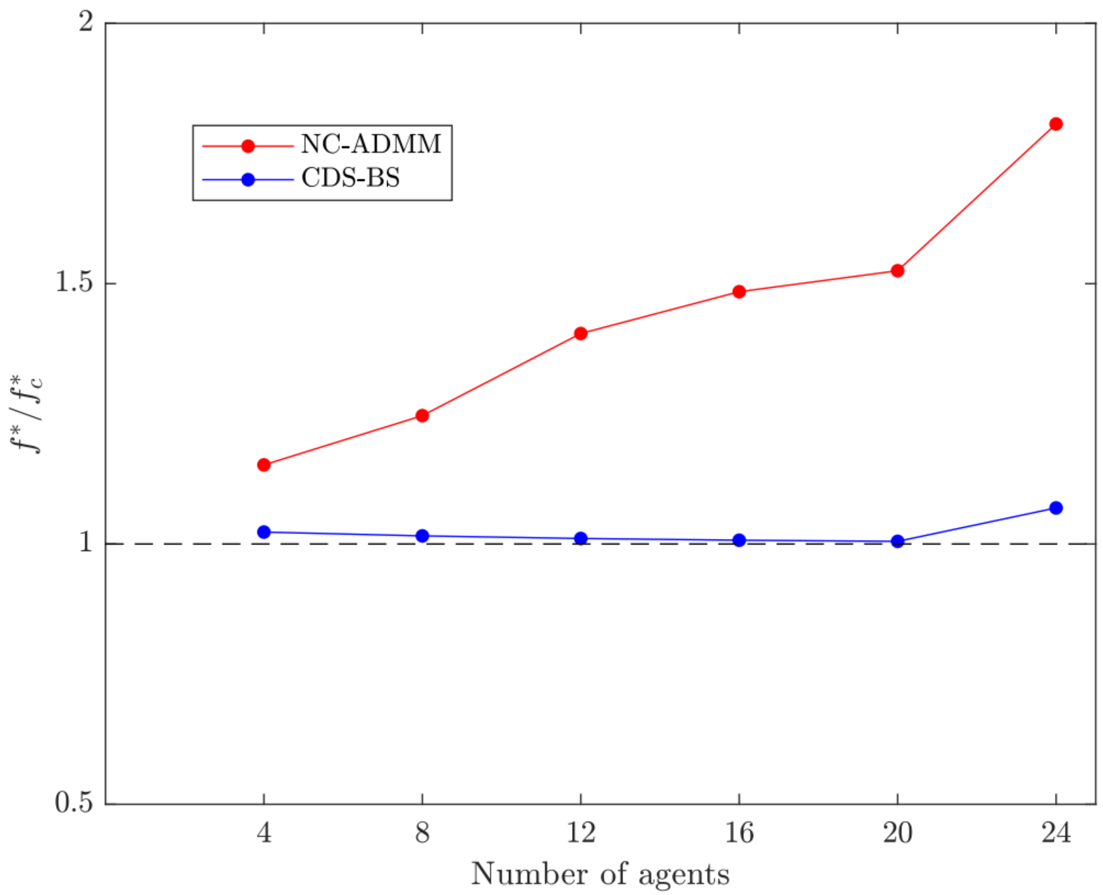


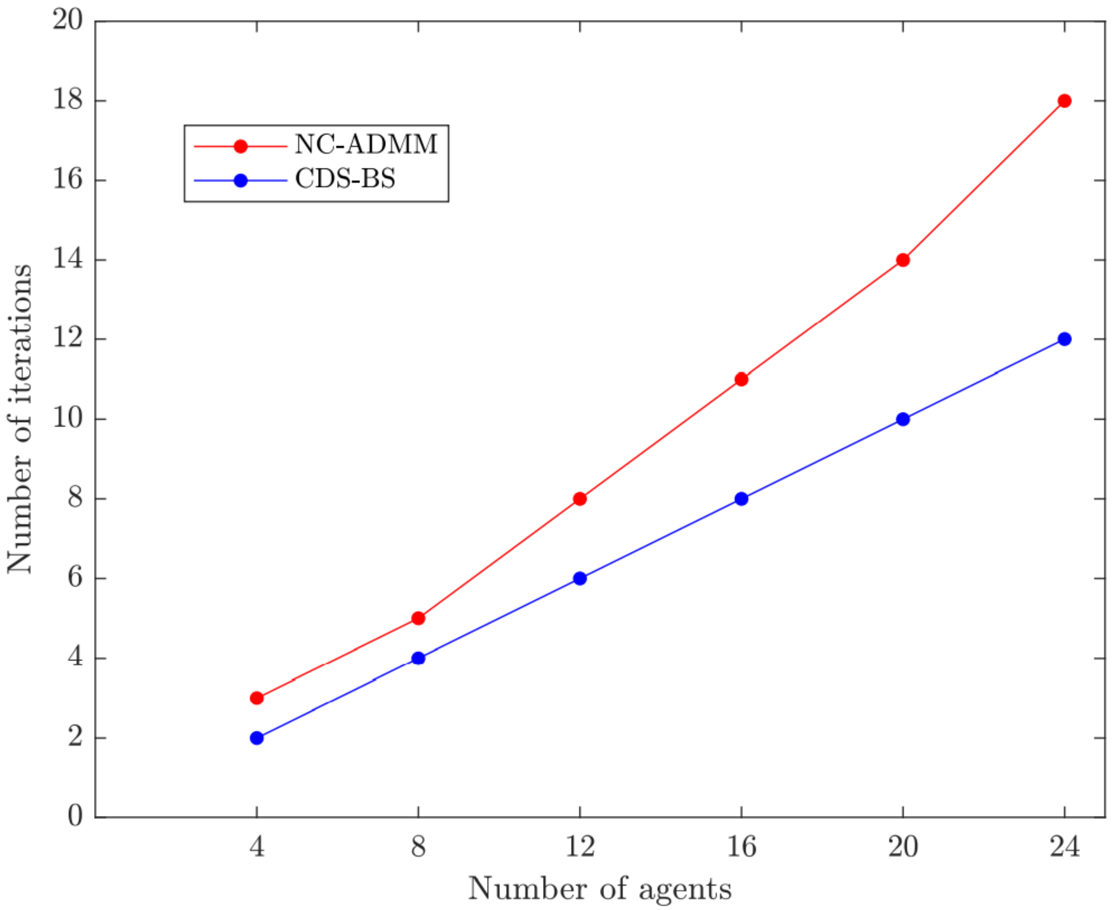
Results: CDS-BS



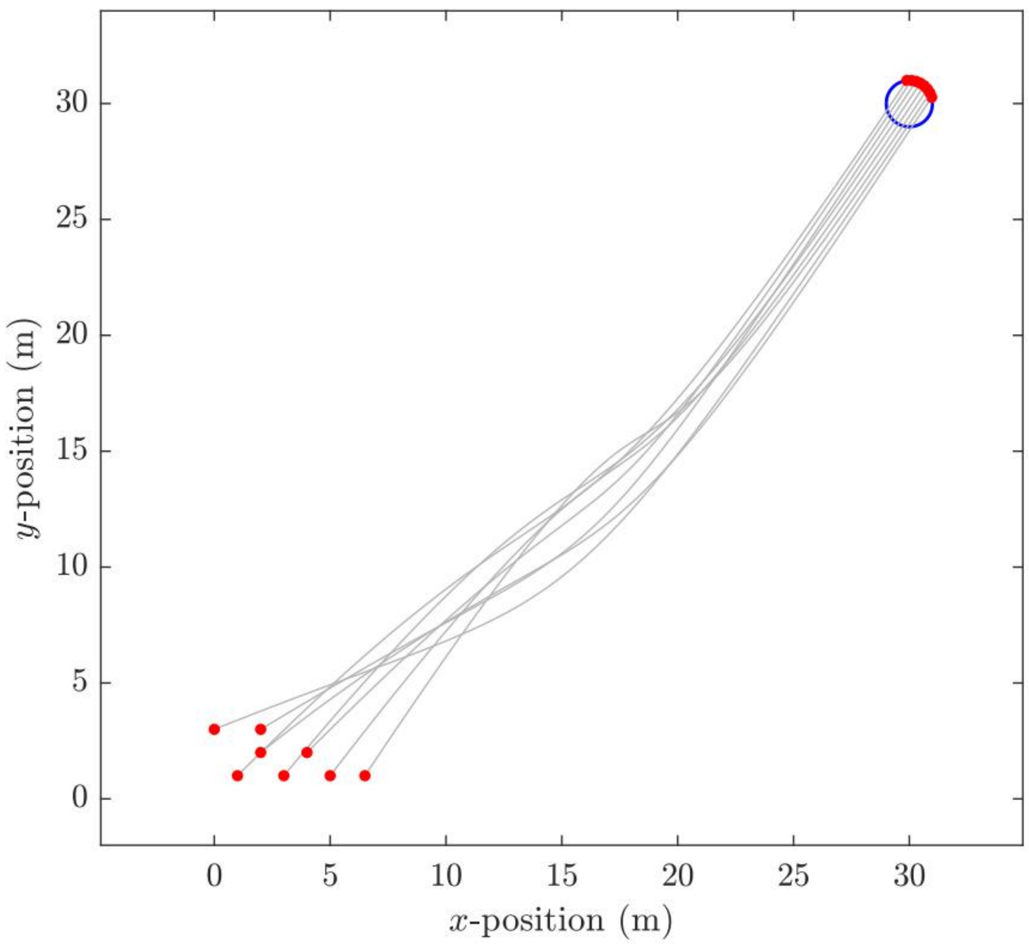


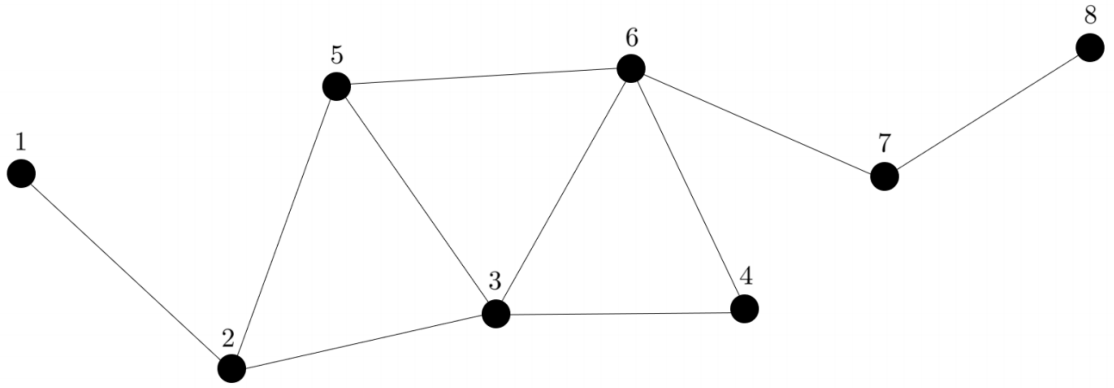
Results: CDS-BS





Results: CDS-PS





Algorithm	f^*	Time (s)
Centralized	13.277	160.4
CDS-PS	13.631	199.0
CDS-PS, EAS	13.294	24.40

Node	n_k	$\lvert\mathcal{Q}\rvert$	$\lvert\mathcal{W}\rvert$	$\lvert\mathcal{A}\rvert$	t(s)	$\%t_c$
2	3	7010	331	21	7.04	5.86
3	4	6511	515	53	7.68	17.7
6	5	4007	390	9	5.45	32.7
7	4	4508	172	4	4.08	22.3