Carleton University

ELEC 4700 - Assignment #4

Circuit Modeling

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supervised by

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1 Question #1

1.1 Part #1

The following circuit was realized with a set of matrices to represent the passive components present in the circuit.

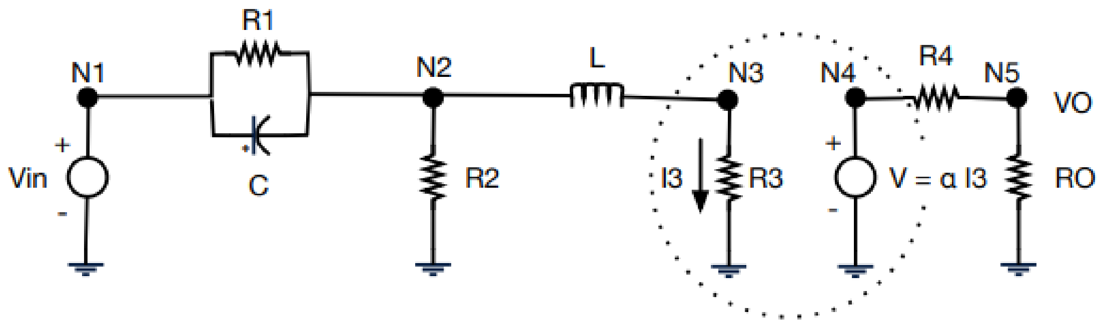

Figure 1: Circuit to be simulated

To solve the above circuit, the following matrix equation was implemented in code to with the use of modal analysis:

$$\hat{C}\frac{d\hat{V}}{dt} + \hat{G}\hat{V} = \hat{F}(t) \tag{1}$$

A G matrix containing the resistive components was constructed with many G_n terms to represent the inverse of the resistance of the components. The following snippet of code was used to form the G matrix:

```
%Set up equation
%V = [V1 V2 V3 V4 VO IL I3];
G = [1 0 0 0 0 0 0;
    -G2 G1+G2 0 0 0 -1 0;
    0 0 G3 0 0 -1 0;
    0 0 G3 0 0 0 -1;
    0 0 0 -G4 G4+GO 0 0;
    0 1 -1 0 0 0 0;
    0 0 0 1 0 0 -alpha];
```

The G matrix looks like the following in the console:

```
G =
    1.0000         0         0         0         0         0         0
   -0.5000    1.5000         0         0         0   -1.0000         0
         0         0    0.1000         0         0   -1.0000         0
         0         0    0.1000         0         0         0   -1.0000
         0         0         0  -10.0000   10.0010         0         0
         0    1.0000   -1.0000         0         0         0         0
         0         0         0    1.0000         0         0 -100.0000
```

The reactive components such as the capacitors and inductors was represented in the C matrix. The following snippet of code was used to form the C matrix:

```
%Set up equation
%V = [V1 V2 V3 V4 VO IL I3];
C = [0 0 0 0 0 0 0;
    -C1 C1 0 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 -L 0;
    0 0 0 0 0 0 0];
```

The C matrix looks like the following in the console:

```
C =
         0         0         0         0         0         0         0
   -0.2500    0.2500         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0   -0.2000         0
         0         0         0         0         0         0         0
```

1.2 Part #2

A DC sweep from -10 V to 10 V was performed on the circuit. The following plot represents the output voltage of the circuit with the above DC inputs.

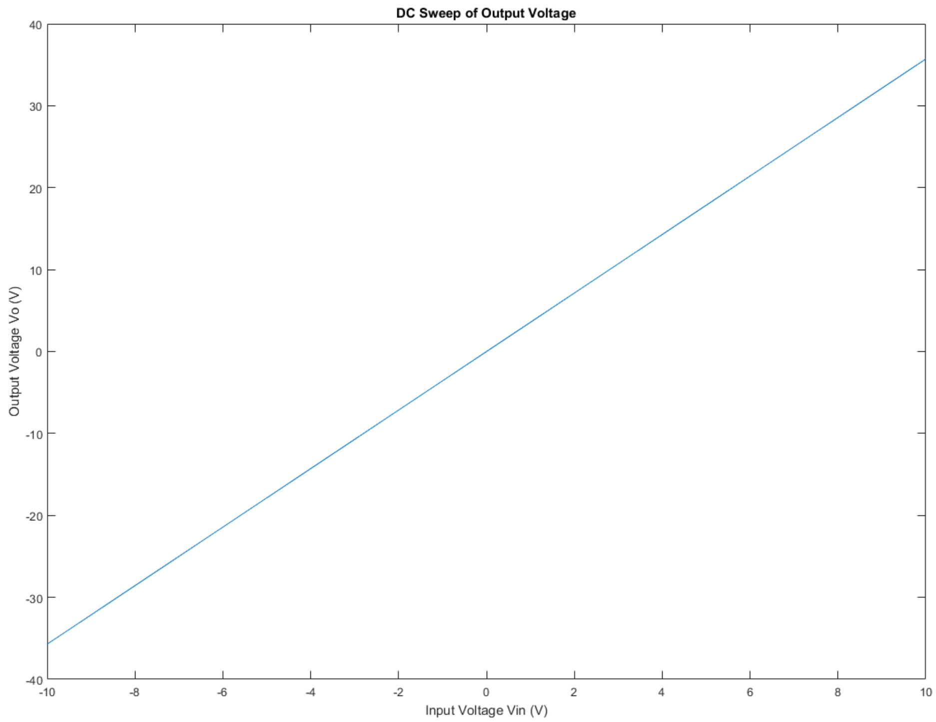

Figure 2: DC Sweep of Output Voltage from -10 V to 10 V

1.3 Part #3

The output voltage was analyzed over a variety of frequencies. Figure (3) shows the output voltage over frequencies 0 to 1000 kHz. The gain of the circuit $20log(\frac{V_{out}}{V_{in}})$ is plotted in Figure (4).

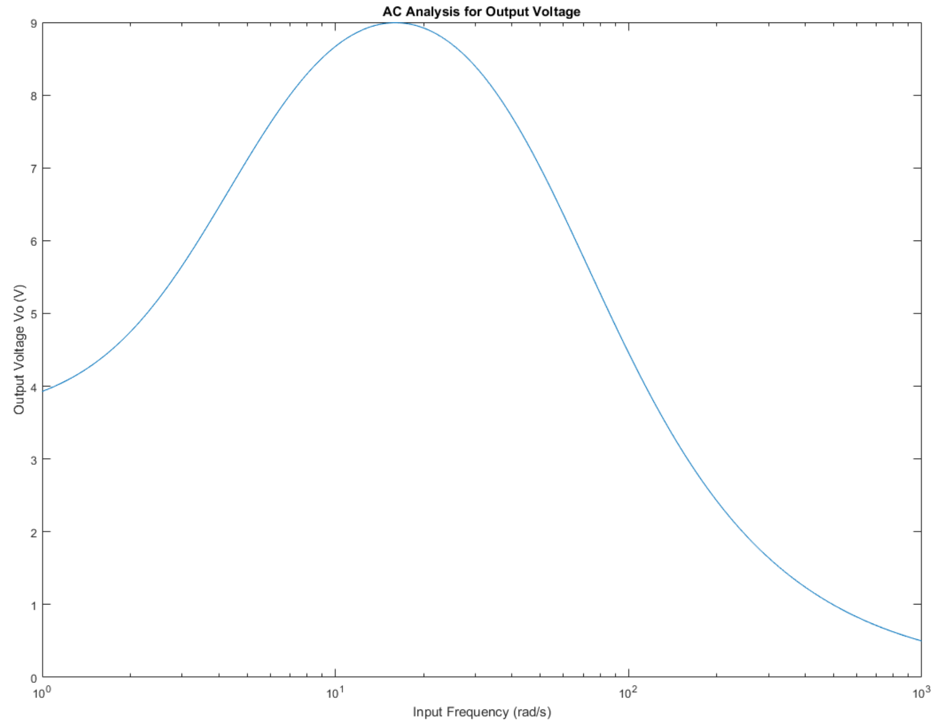

Figure 3: AC Sweep of Output Voltage from 0 to 1kHz

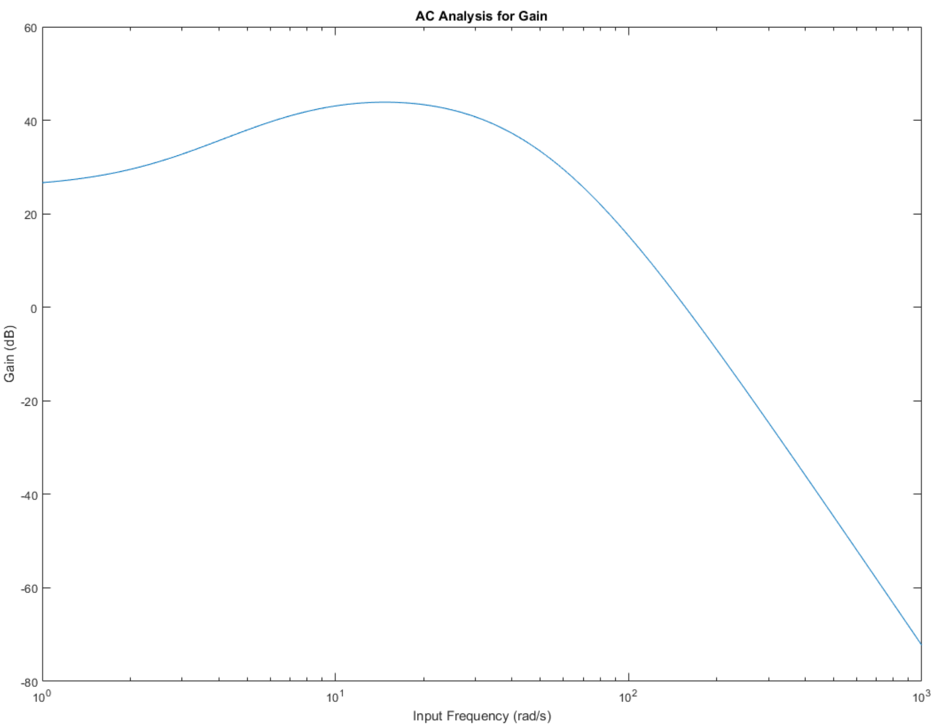

Figure 4: AC Sweep of Circuit Gain from 0 to 1kHz

2 Question #2

2.1 Part #1

The input voltage of the circuit was set to represent a step function that begins at 0.03s. Figure (5) shows the input and output signal of the circuit over a period of 1 second. Figure (6) is the Fourier transform of the input and output step signals represented in the frequency domain.

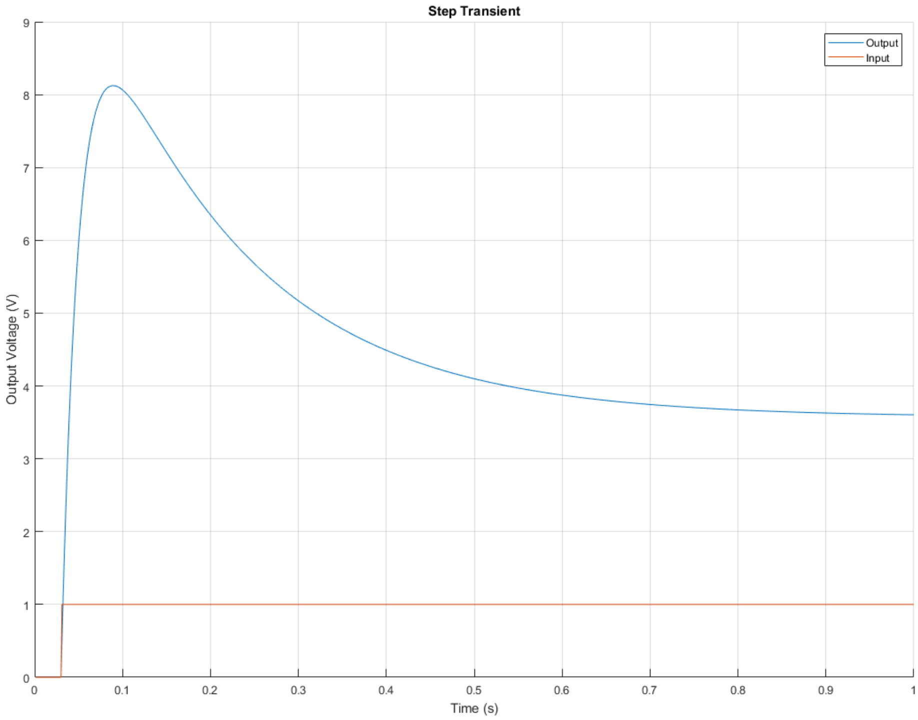

Figure 5: Input and Output Voltage Waveforms for a Step Function

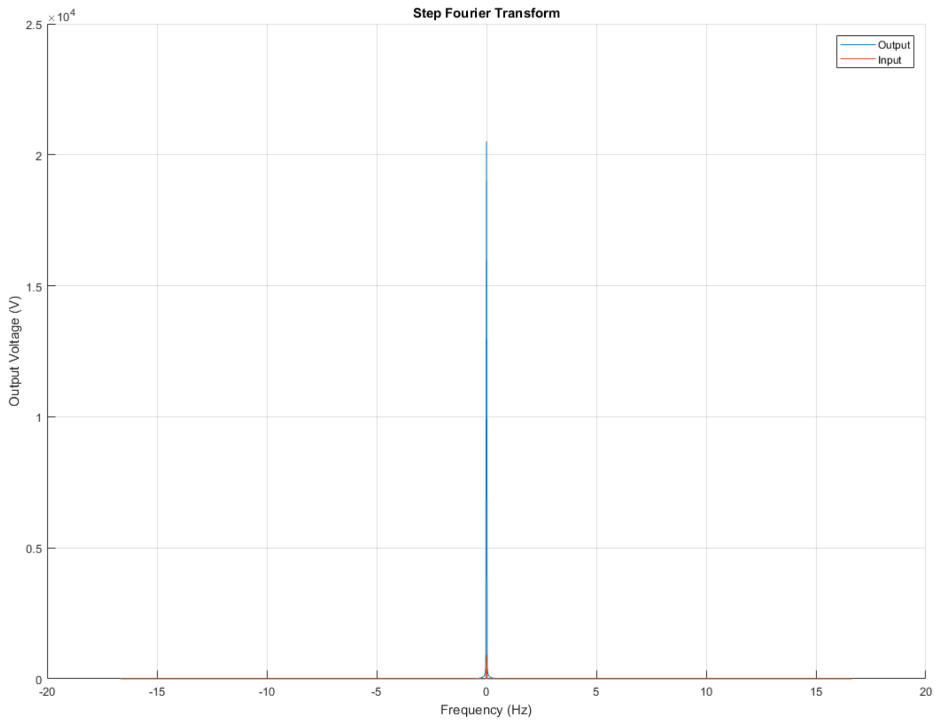

Figure 6: Fourier Transform of Step Function Input and Output Waveforms

2.2 Part #2

The input voltage of the circuit was set to represent a sine function with a frequency of $\frac{1}{0.03}Hz$. Figure (7) is the input and output signal of the circuit over a period of 1 second. Figure (8) is the Fourier transform of the input and output sine signals represented in the frequency domain.

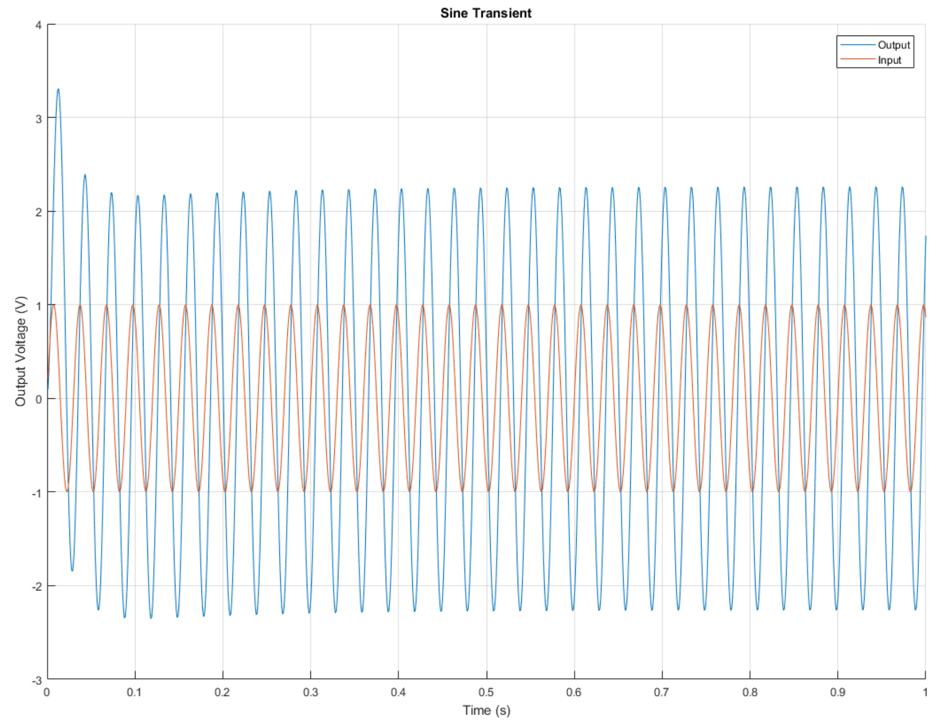

Figure 7: Input and Output Voltage Waveforms for a Sine Function

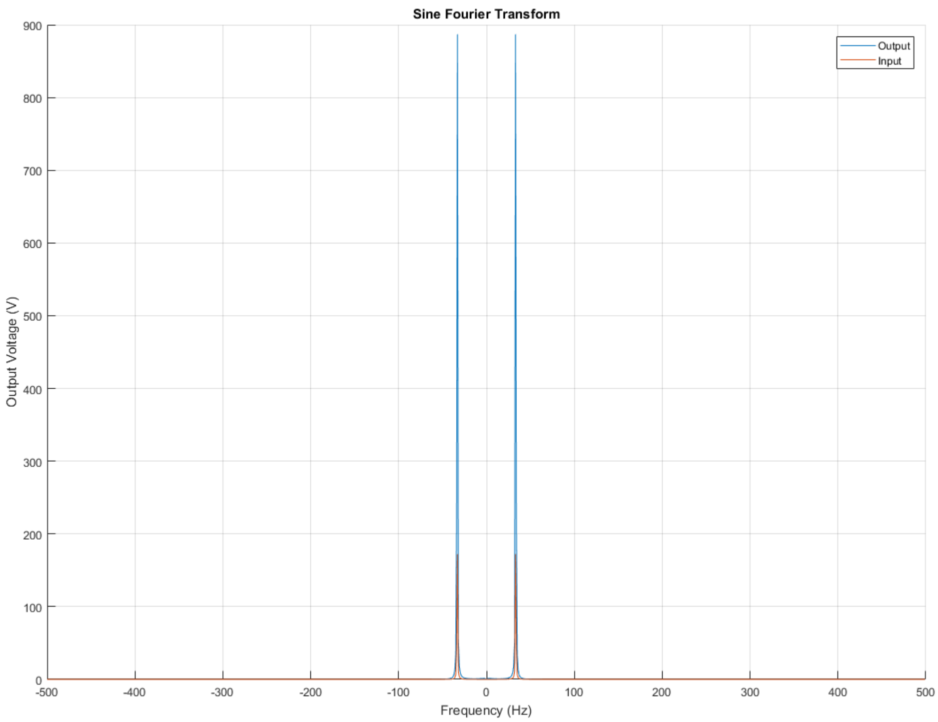

Figure 8: Fourier Transform of Sine Function Input and Output Waveforms

2.3 Part #3

The input voltage of the circuit was set to represent a Gaussian function with a magnitude of 1 and a standard deviation of 0.03s. There was a delay of 0.06s before the signal was introduced into the system. Figure (9) is the input and output signal of the circuit over a period of 1 second. Figure (10) is the Fourier transform of the input and output sine signals represented in the frequency domain.

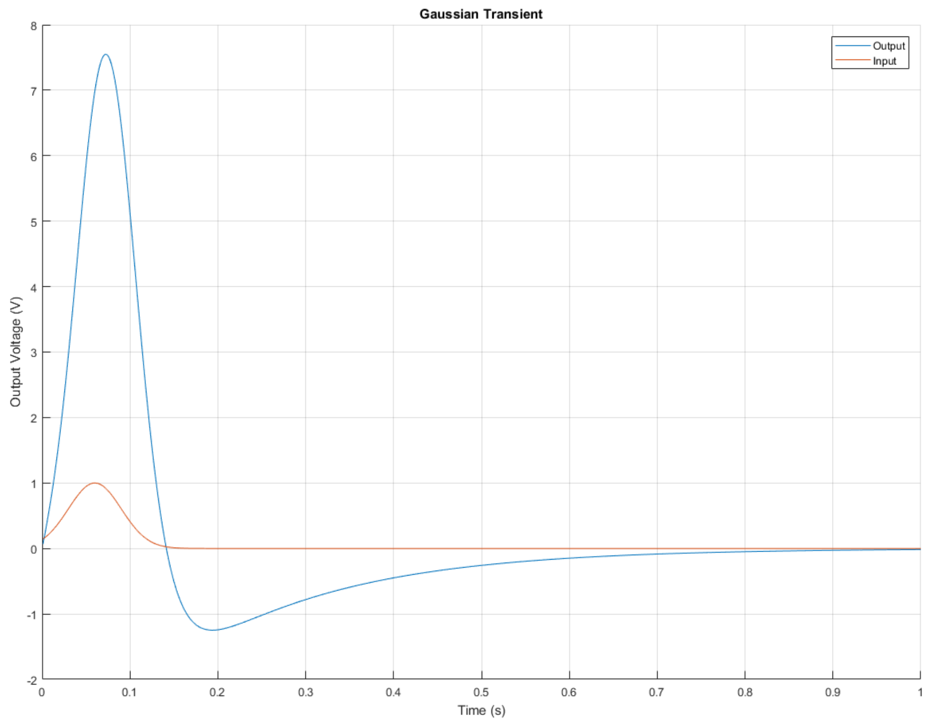

Figure 9: Input and Output Voltage Waveforms for a Gaussian Function

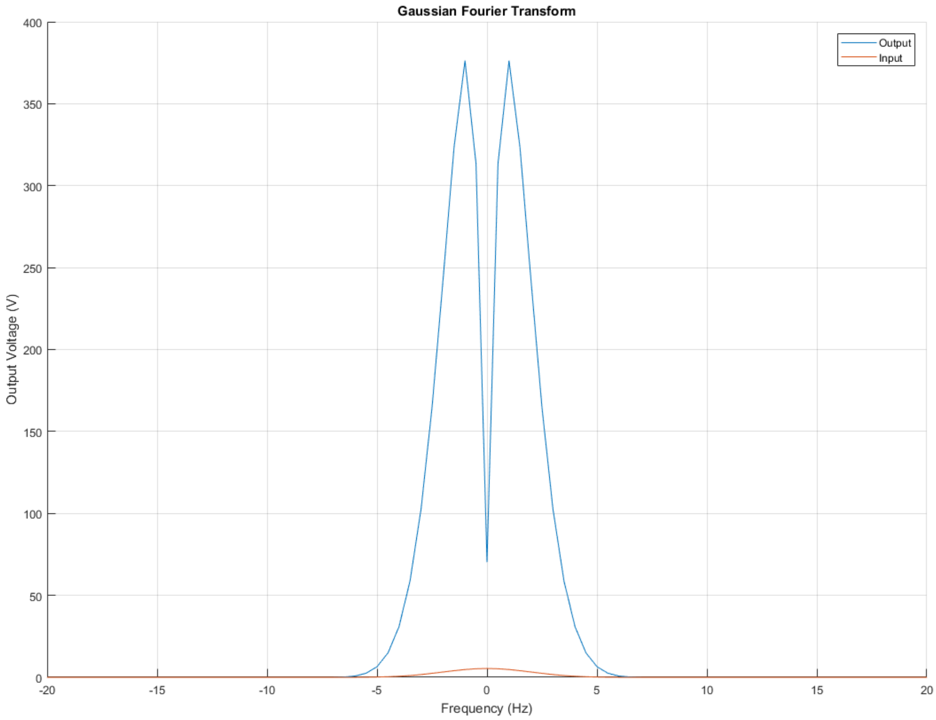

Figure 10: Fourier Transform of Gaussian Function Input and Output Waveforms

3 Question #3

3.1 Part #1

The circuit was modified to add a noise source.

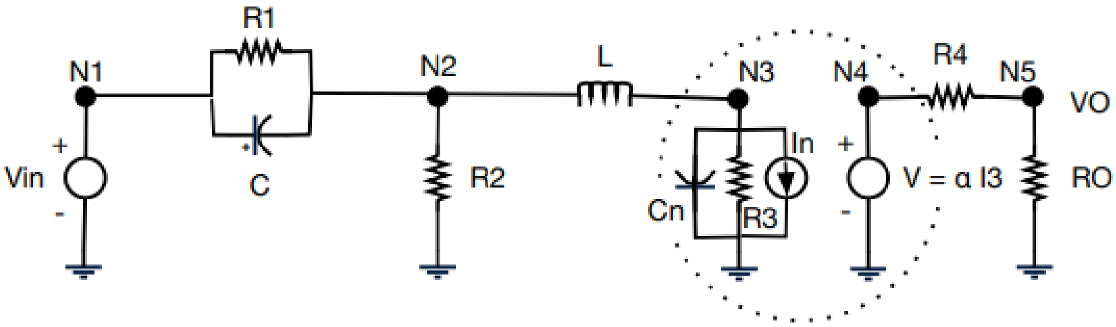

Figure 11: Simulated circuit with Noise Source

The C matrix was modified in the following manner.

```
C = [0 0 0 0 0 0 0 0;
    -C1 C1 0 0 0 0 0 0;
    0 0 Cn 0 0 0 0 0;
    0 0 0 0 0 0 0 0;
    0 0 0 0 0 0 0 0;
    0 0 0 0 0 -L 0 0;
    0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0];
```

The C matrix looks like the following in the console.

```
C =
        0         0         0         0         0         0         0         0
  -0.2500    0.2500         0         0         0         0         0         0
        0         0    0.0000         0         0         0         0         0
        0         0         0         0         0         0         0         0
        0         0         0         0         0         0         0         0
        0         0         0         0         0   -0.2000         0         0
        0         0         0         0         0         0         0         0
        0         0         0         0         0         0         0         0
```

3.2 Part #2

Using the circuit with a noise source capacitance of 10 μF and a time step value of 1 ms, the output voltage is shown in Figure (12).

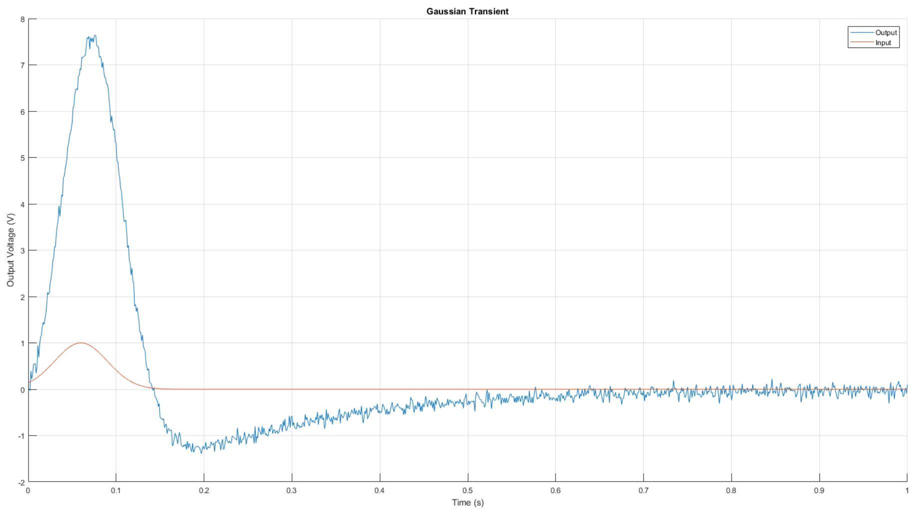

Figure 12: Simulated circuit Output Voltage with Noise Source

3.3 Part #3

The Fourier transform of the noisy circuit voltage form is shown in Figure (13).

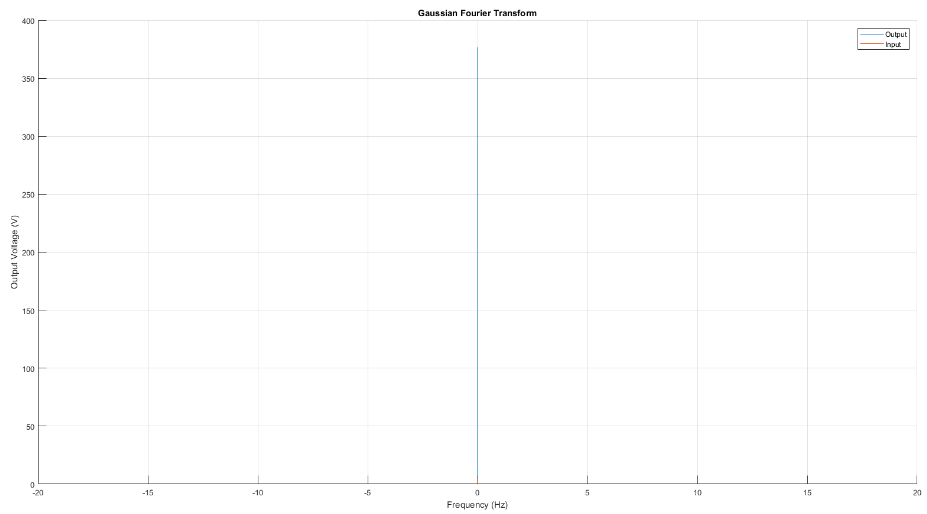

Figure 13: Fourier Transform of Simulated Circuit Output Voltage with Noise Source

3.4 Part #4

The capacitance of the noisy sources was modified to show the effect of capacitance on the overall noise of the circuit. Figure (13) shows a noise capacitor value of 100 μF, Figure (14) a value of 1 mF and Figure (15) a value of 10 mF.

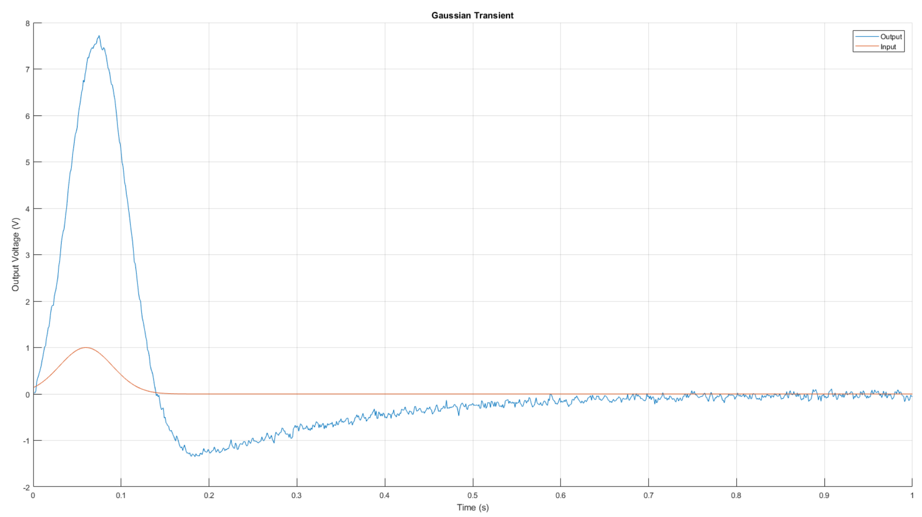

Figure 14: Simulated circuit with noise capacitor value of 100 μF

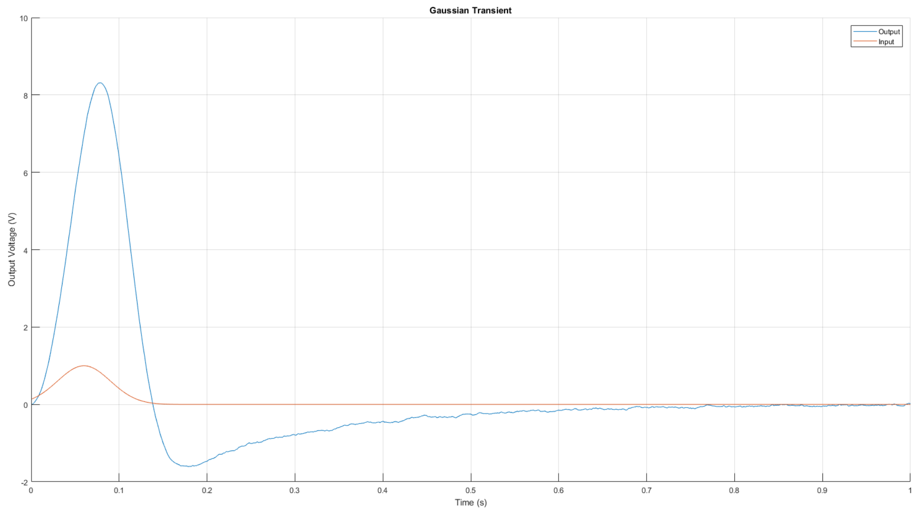

Figure 15: Simulated circuit with noise capacitor value of 1 mF

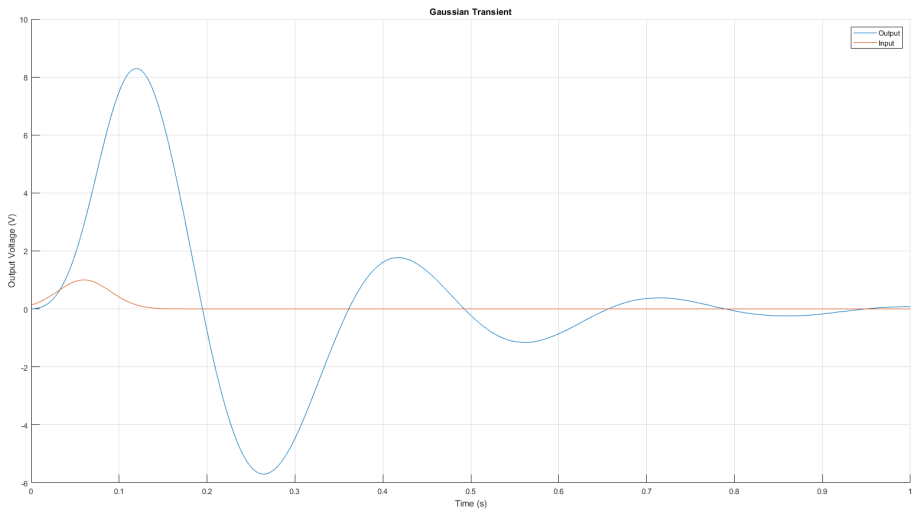

Figure 16: Simulated circuit with noise capacitor value of 10 mF

3.5 Part #5

Two difference time steps were chosen to demonstrate the effect of the noise on the output signal. Figure (16) demonstrates an output voltage with a time step of 10 ms and Figure (17) shows a time step of 1 μs.

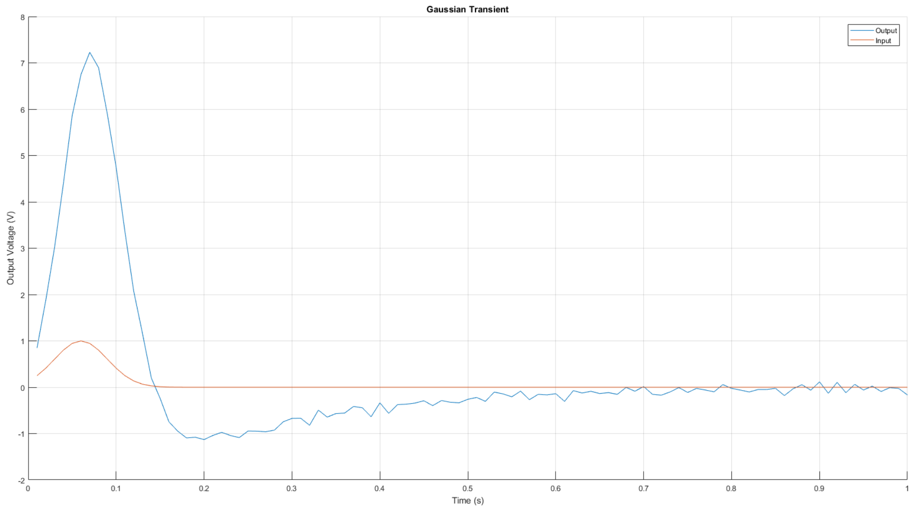

Figure 17: Simulated circuit with Noise and time step of 10 ms

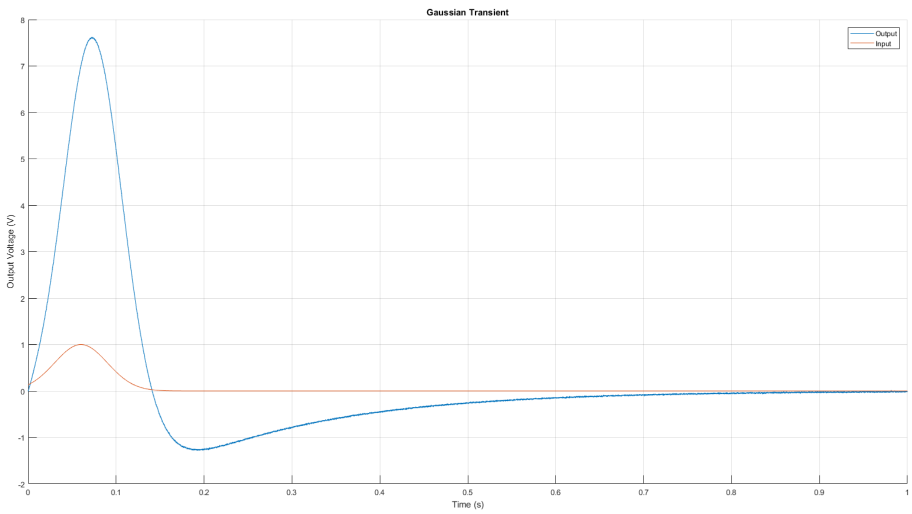

Figure 18: Simulated circuit with Noise and time step of 1 μs

4 Question #4

Supposing that the ideal current source were to have non-linearities such as $V = \alpha I_3 + \beta I_3^2 + \gamma I_3^3$, the equation used to solve the circuit would have to be modified to the following form.

$$\hat{C}\frac{d\hat{V}}{dt} + \hat{G}\hat{V} + \hat{B}(\hat{V}) = \hat{F}(t) \tag{2}$$

The $\hat{B}(\hat{V})$ term takes into account the non-linearities of the circuit. If the $\hat{B}(\hat{V})$ were to be implemented in code, it would have the following form:

```
B = [0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 (-ALPHA - 2*BETA*I3 - 3*GAMMA*I3^2), 0];
```

The derivative of the current source equation would then be iterated over time using a Newton-Raphson method to convert the non-linear terms into a linear form to be used with the rest of the equation (2).