EFB Takeoff Calculations

In the real world for calculating takeoff performance (V.Speeds, Flex Temp etc.) several RTOW charts are produced which are specific to that runway, and specific runway conditions. The Airbus FCOM does not directly state how to calculate V.Speeds, they are obtained from the RTOW charts for the specific runway.

For the EFB, we are not able to store every RTOW chart for every airport. Therefore we must try to calculate the V.Speeds ourselves in a similar way to how they would be determined for RTOW charts, based on the factors that affect takeoff performance (wind, runway length, runway condition, temperature, pressure etc.).

This document details the methods that can be used to gain accurate estimates for what data is likely to be published by the airport authority for given runway lengths and environmental conditions.

Stall speeds

Stall speeds contribute to the minimum values for V2 and VR. We already have stall speeds calculated which are used in the CDU we will use, determined from the QRH.

V2

V2 speed can be calculated as the maximum of the following values:

- Stall speed * 1.13
- Speed obtained from FCOM VMCG/VMCA table for the selected flap configuration and pressure altitude
- Speed obtained from FCOM VMU/VMCA table for the pressure altitude and takeoff weight

All of this data is available from the FCOM.

OPERATING SPEEDS

Ident.: OPS-00010999.0069001 / 21 MAR 17
Applicable to: D-AVVI, D-AXAJ, F-WWTQ, PR-OBD, PR-OBF, PR-OBH, PR-OBI, PR-OBJ

OPERATING SPEEDS (KT) CG ≥ 25 %					
Weight (1000 KG)	**F**	**S**	**Green dot FL < 200**(1)	**VLS CONF 3**	**VREF**
40	131	152	165	116	116
45	131	161	175	116	116
50	131	169	185	123	116
55	137	178	195	129	119
60	144	186	205	134	124
65	149	193	215	140	129
70	155	200	225	145	134
75	160	207	235	150	139
80	166	214	245	155	143

(1) Above FL 200 add 1 kt per additional 1 000 ft.

For CG <25 % add 2 kt to VLS and VREF.

Figure 1: QRH Operating Speeds

SPEEDS LIMITED BY VMCG/VMCA

Ident.: PER-TOF-TOD-25-10-00001754.0350001 / 08 JUL 15
Applicable to: PR-OBB, PR-OCA, PR-OCB, PR-OCD, PR-OCH, PR-OCI, PR-OCM, PR-OCN, PR-OCO, PR-OCP, PR-OCQ, PR-OCR, PR-OCT, PR-OCV, PR-OCW, PR-OCY, PR-ONS, PR-ONT, PR-ONW, PR-ONX, PR-ONY, PR-ONZ

All takeoff speeds have a minimum value limited by control. These minimum speeds are usually provided on each RTOW chart. If these speeds are not available, use the following conservative values. These speeds may be slightly higher than the minimum control speeds displayed on the RTOW chart.

MINIMUM V1 (KT IAS)													
CONF	PRESSURE ALTITUDE (FT)												
	-2000	**0**	**1000**	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9200**	**10000**	**14100**
1+F	116	116	115	114	114	114	114	113	112	110	108	106	100
2	114	114	113	112	112	112	112	111	110	108	106	104	98
3	114	113	113	111	111	111	111	110	109	108	106	104	98

MINIMUM VR (KT IAS)													
CONF	PRESSURE ALTITUDE (FT)												
	-2000	**0**	**1000**	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9200**	**10000**	**14100**
1+F	119	118	117	116	116	116	116	115	114	112	110	108	101
2	117	117	115	114	114	114	114	113	112	110	108	106	99
3	117	116	115	114	114	114	113	113	111	110	107	106	99

MINIMUM V2 (KT IAS)													
CONF	PRESSURE ALTITUDE (FT)												
	-2000	**0**	**1000**	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9200**	**10000**	**14100**
1+F	122	121	120	119	119	119	119	118	116	115	112	110	104
2	122	121	120	119	119	119	118	117	116	115	112	110	103
3	122	121	120	119	119	119	118	117	116	114	112	110	103

MINIMUM V2 LIMITED BY VMU/VMCA (KT IAS)

Ident.: PER-TOF-TOD-25-20-00001756.1105001 / 01 MAR 17
Applicable to: D-AVVI, D-AXAJ, F-WWTQ, PR-OBD, PR-OBF, PR-OBH, PR-OBI, PR-OBJ

MINIMUM V2 LIMITED BY VMU/VMCA (KT IAS)								
CONFIGURATION 1+F								
PRESSURE ALTITUDE (FT)	TAKE OFF WEIGHT (1000 KG)							
	45	**50**	**55**	**60**	**65**	**70**	**75**	**80**
-2000	127	127	127	132	137	142	146	151
-1000	126	126	127	132	137	142	147	151
0	126	126	127	132	137	142	147	151
1000	126	126	127	132	137	142	147	151
2000	125	125	127	132	137	142	147	151
3000	125	125	127	132	137	142	147	151
4000	124	124	127	132	137	142	147	152
5000	124	124	127	132	137	142	147	152
6000	123	123	127	132	137	142	147	152
7000	122	122	127	132	137	142	147	152
8000	120	121	127	132	137	143	148	152
9000	119	121	127	132	137	143	148	153
10000	117	121	127	132	137	143	148	153
11000	115	121	127	132	138	143	149	154
12000	115	121	127	132	138	143	149	154
13000	115	121	127	132	138	144	149	154
14100	115	121	127	132	138	144	150	155
15100	115	121	127	133	139	144	150	155

MINIMUM V2 LIMITED BY VMU/VMCA (KT IAS)								
CONFIGURATION 2								
PRESSURE ALTITUDE (FT)	TAKE OFF WEIGHT (1000 KG)							
	45	**50**	**55**	**60**	**65**	**70**	**75**	**80**
-2000	127	127	127	127	132	136	141	145
-1000	126	126	126	127	132	136	141	145
0	126	126	126	127	132	137	141	146
1000	126	126	126	127	132	137	141	146
2000	125	125	125	127	132	137	141	146

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MINIMUM V2 LIMITED BY VMU/VMCA (KT IAS)								
CONFIGURATION 2								
PRESSURE ALTITUDE (FT)	TAKE OFF WEIGHT (1000 KG)							
	45	50	55	60	65	70	75	80
3000	125	125	125	127	132	137	142	146
4000	124	124	124	127	132	137	142	146
5000	124	124	124	127	132	137	142	146
6000	123	123	123	127	132	137	142	146
7000	122	122	122	127	132	137	142	146
8000	120	120	122	127	132	137	142	146
9000	119	119	122	127	132	137	142	147
10000	117	117	122	127	132	137	142	147
11000	115	117	122	127	132	137	142	147
12000	113	117	122	127	132	138	143	147
13000	111	116	122	127	133	138	143	148
14100	111	116	122	127	133	138	143	148
15100	111	116	122	127	133	138	143	148

MINIMUM V2 LIMITED BY VMU/VMCA (KT IAS)								
CONFIGURATION 3								
PRESSURE ALTITUDE (FT)	TAKE OFF WEIGHT (1000 KG)							
	45	50	55	60	65	70	75	80
-2000	126	126	126	126	128	132	137	141
-1000	125	125	125	125	128	132	137	141
0	125	125	125	125	128	132	137	141
1000	125	125	125	125	128	132	137	141
2000	124	124	124	124	128	132	137	141
3000	124	124	124	124	128	133	137	141
4000	123	123	123	123	128	133	137	141
5000	123	123	123	123	128	133	137	142
6000	122	122	122	123	128	133	137	142
7000	121	121	121	123	128	133	138	142
8000	119	119	119	123	128	133	138	142
9000	118	118	118	123	128	133	138	142

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MINIMUM V2 LIMITED BY VMU/VMCA (KT IAS)								
CONFIGURATION 3								
PRESSURE ALTITUDE (FT)	TAKE OFF WEIGHT (1000 KG)							
	45	50	55	60	65	70	75	80
10000	116	116	118	123	128	133	138	142
11000	114	114	118	123	128	133	138	142
12000	112	113	118	123	128	133	138	143
13000	110	113	118	123	128	133	138	143
14100	108	113	118	123	128	134	139	143
15100	107	113	118	123	129	134	139	143

VR

VR must be calculated as the greater of:

- Stall speed * Safety Margin
- Speed obtained from FCOM VMCG/VMCA table for the selected flap configuration and pressure altitude

The safety margin is a % margin which the stall speed is multiplied by. This can be any arbitary value for our case, but should probably be under 10%. For example, 1.05 = 5% safety margin.

V1

The maths for calculating V1 is complicated. V1 must meet the following criteria:

- Greater than the minimum speed obtained from the FCOM VMCG/VMCA table for the selected flap configuration and pressure altitude
- Less than than the maximum speed at which the aircraft is able accelarate to and then stop within the runway length

The distance traveled in the time to accelerate to V_1, $V1_{dist}$, plus the braking distance from V_1 speed, B_{dist}, must be less than the available runway length.

We can calculate the V_1 speed provided we know:

- The mass of the aircraft
  - Input by the user into the EFB
- Runway length
  - Input by the user into the EFB
- The thrust of the aircraft
  - The thrust of the engines used on the A320NEO, CFM International LEAP 1A26, is rated at 120.64kN takeoff thrust. We can simply multiply this by 2 to get the thrust

- The drag constant due to air resistance on the aircraft
  - This can be found based on FCOM data, which will be explained later

Overview

First, find the value of ρ for the air pressure and temperature:

$\rho = \frac{p}{287.058 \times \text{temp}}$

Where p is pressure **in Pascals** and temp is the temperature **in Kelvin**.

Then find b for the given flap configuration:

- $Cd_1 = 1671.16$ Flaps 1 + F
- $Cd_2 = 1825.18$ Flaps 2
- $Cd_3 = 1956.10$ Flaps 3

$b = Cd \times \rho$

Then find the thrust T per engine:

If $\rho >= 1.2985$ then $T = T_{\text{MAX}}$, otherwise $T = T_{\text{MAX}} \times \frac{\rho}{1.2985}$

Then the acceleration distance (D_{V_1}), braking distance (D_{BRAKE}) and total distance (D_{RTO}) for a rejected takeoff for a given candidate V_1 speed can be calculated with the following functions:

$D_{\text{RTO}} = D_{V_1}(V) + D_{\text{BRAKE}}(V)$

$D_{V_1}(V) = D(t(V))$

$D_{\text{BRAKE}}(V) = \frac{mV^2}{2B}$

$D(t) = 2T(\frac{me^{\frac{-bt}{m}} - m}{b^2} + \frac{t}{b})$

$t(V) = \frac{-m \ln(1 - \frac{Vb}{2T})}{b}$

Where:

- V is the candidate V_1 speed, in m/s
- B is the braking force of the brakes at max in Newtons. The real value of this is currently unknown, likely higher than T
- T is the thrust of a single engine in Newtons. The A320NEO CFM International LEAP 1A26, is rated at 120,640N takeoff thrust
- b is the drag coefficient
- m is the aircraft mass in KGs

The output of the distance functions is in meters.

Iteratively increase the candidate speed, starting from the stall speed, to find the highest value that D_{RTO} is not longer than the available runway length.

If V_R is a valid V_1 speed, then $V_1 = V_R$

If the RTO distance of stall speed $\times$ safety margin is longer than the available runway length, then safe takeoff is not possible.

Proof

Finding velocity by time equation

First, we need to calculate an equation for finding the velocity of the aircraft at a given point in time during the takeoff accelaration ($V \leq V_1$). Using Newton's second law, where:

- F is the combined force on the aircraft in the horizontal direction in Newtons
- m is the aircraft mass in Kgs
- V is the aircraft velocity in m/s
- t is elapsed time since thrust is applied
- T is the thrust provided by a single engine
- b is the drag constant for the aircraft (will be determined later)

- D is the displacement from start position

$$\begin{aligned}
\text{Newton's second law} \\
ma &= F \\
m\frac{dV}{dt} &= F && F = 2T - bV \\
\text{Substitute thrust minus air resistance} \\
m\frac{dV}{dt} &= 2T - dV \\
mdV &= dt(2T - bV) \\
\frac{dv}{2T - bV} &= \frac{dt}{m} \\
\text{Integrate to solve for } V \\
\int \frac{dV}{2T - bV} &= \int \frac{dt}{m} \\
&= -\frac{1}{b}\int \frac{-b}{2T - bV} dV \\
&= -\frac{1}{b}\ln(2T - bV) \\
-\frac{1}{b}\ln(2T - bV) &= \frac{t}{m} + C \\
-ln(2T - bV) &= \frac{-bt}{m} - bC \\
|2T - bV| &= e^{\frac{-bt}{m}} e^{-bc} \\
2T - bV &= \pm e^{\frac{-bt}{m}} e^{-bc} \\
2T - bV &= Ae^{\frac{-bt}{m}}
\end{aligned}$$

This gives the general formula of velocity:

$$V = \frac{2T - Ae^{\frac{-bt}{m}}}{b}$$

Now substitude our initial $V_0 = 0$ to find the equation specific to starting from rest

$$\begin{aligned} A &= 2T - V_0 b && V_0 = 0 \\ A &= 2T \\ V &= \frac{2T}{b} - \frac{2T}{b} e^{\frac{-bt}{m}} \end{aligned}$$

Getting values for air resistance constant

The limit of the velocity equation is the maximum speed the plane can reach with the given thrust, which we can use to obtain the air resistance contant b. We could calculate the limit mathematically to determine this, however it is much easier to use a graphing tool (Desmos was used for this example) to plot the velocity and vary the b value until the limit lands on our target value. Mass has no effect on the limit of the curve, only the initial gradient.

The operating speed limits for the A320NEO are published in the FCOM Limitations.

MAXIMUM FLAPS/SLATS SPEEDS

Ident.: LIM-AG-SPD-00020087.0001001 / 17 MAR 17
Applicable to: **ALL**

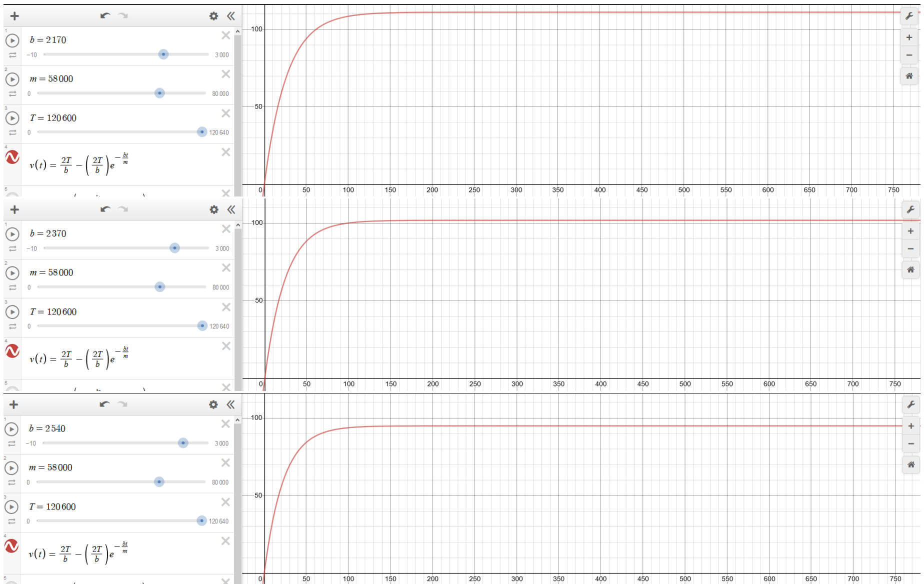
Flaps Lever Position	Configuration on Slat/Flap Display	Max Speed	Flight Phase
0		VMO/MMO	CRUISE
1	1	230 kt	HOLDING
	1 + F	215 kt	TAKEOFF
2	2	200 kt	TAKEOFF/APPROACH
3	3	185 kt	TAKEOFF/APPROACH/LANDING
FULL	FULL	177 kt	LANDING

Figure 2: FCOM Speed Maximums

Note that the chart is in knots but the speed unit for the equation is m/s. Convert knots to m/s by multiplying by $0.51\overline{4}$. This gives us maximum speeds of ~111 m/s for 1 + F, ~103 m/s for 2 and ~95 m/s for 3.

Varying b to find the correct limit gives the following values:

- $b_1 = 2170$ Flaps 1 + F
- $b_2 = 2370$ Flaps 2
- $b_3 = 2540$ Flaps 3



Finding displacement by time equation

We now need to find the equation for displacement by time, which will be used to work out how far we travel before hitting V_1. This is simply the integral of the velocity equation.

$$V = \frac{dD}{dt} = \frac{2T}{b} - \frac{2T}{b}e^{-bt}m$$
$$D = \int (\frac{2T}{b} - \frac{2T}{b}e^{-bt}m)dt$$
$$D = \frac{2Tt}{b} - \frac{\frac{2Tt}{b}e^{-bt}m}{(\frac{-bt}{m})} + C$$
$$D = \frac{2Tt}{b} - \frac{2TMe^{-bt}m}{b^2} + C$$

Now we substitute $D = 0$ at $t = 0$ to find C, and simplify

$$\frac{2Tme^0}{b^2} + C = 0$$
$$C = \frac{-2Tm}{b^2}$$

$$\therefore D = \frac{2mTe^{\frac{-bt}{m}}}{b^2} + \frac{2Tt}{b} - \frac{2Tm}{b^2}$$
$$D = 2T(\frac{me^{\frac{-bt}{m}} - m}{b^2} + \frac{t}{b})$$

Finding time by velocity

We need to know what time we hit V_1 at so that we can use that time in our displacement equation. We can find an equation for time given velocity by re-arranging the velocity equation to make t the subject.

$$V = \frac{2T}{b} - \frac{2T}{b}e^{\frac{-bt}{m}}$$
$$\frac{2T}{b}e^{\frac{-bt}{m}} = \frac{2T}{b} - V$$
$$2Te^{\frac{-bt}{m}} = 2T - Vb$$
$$e^{\frac{-bt}{m}} = 1 - \frac{Vb}{2T}$$
$$\frac{-bt}{m} = \ln(1 - \frac{Vb}{2T})$$
$$t = \frac{-m\ln(1 - \frac{Vb}{2T})}{b}$$

Finding braking distance

The kinetic energy of the plane at v_1 is $\frac{1}{2}m{v_1}^2$ where m is the mass of the plane in KGs. The energy required by the brakes bringing the kinetic energy to 0 is Bd, where B is the braking force applied and d is the distance the force was applied over (the stopping distance)

$$\frac{1}{2}m{v_1}^2 = Bd$$
$$d = \frac{mv^2}{2B}$$

The exact braking force B of the A320 is not known, but an estimate will suffice here.

Our final function for braking distance for a given velocity is

$D_{\text{BRAKE}}(V) = \frac{mV^2}{2B}$

Computing candidate speeds to find V_1

We now have all of the functions we need:

- $D(t)$ Gives the distance travelled while accelerating for a given time
- $t(V)$ Gives the time we reach a given velocity
- $D_{\text{BRAKE}}(V)$ Gives braking distance for a given velocity

From this we can find the rejected takeoff distance D_{RTO} for a candidate V_1 speed.

$D_{\text{RTO}}(V) = D(t(V)) + D_{\text{BRAKE}}(V)$

We can then iteratively compute RTO distances for different speeds.

Begin at the stall speed * safety margin. Compute D_{RTO} for this speed. If D_{RTO} of stall speed * safety margin is longer than runway, then safe takeoff is not possible.

Gradually increase the candidate speed until the RTO distance is longer than the available runway length, or we reach the calculated V_R speed, whichever is sooner.

If V_R is a valid V_1 speed, then $V_1 = V_R$. Otherwise, the last valid candidate speed is V_1

Effect of pressure and temperature on V1

Up until now we have ignored the effect of pressure and temperature - these are included in the constant b. However the value for b should really be $b = C_d * \rho$ where C_d is a constant specific to the aircraft/flaps configuration and ρ is air density.

Air density ρ is given as $\rho = \frac{p}{R_{\text{specific}}T}$ where p is pressure in Pa, R_{specific} for dry air is 287.058 J/(kg K) and T is temperature in Kelvin.

Pressure in millibars can be converted to Pa by by multiplying by 100. Temperature in Celcius can be converted to Kelvin by adding 273.15.

Our previous values where for at the International Standard Atmosphere, which has a ρ of 1.2985. We can find C_d by dividing by this constant:

- $Cd_1 = 2170/1.2985 = 1671.16$ Flaps 1 + F
- $Cd_2 = 2370/1.2985 = 1825.18$ Flaps 2
- $Cd_3 = 2540/1.2985 = 1956.10$ Flaps 3

Then $b = Cd(\frac{p}{287.058 \times \text{temp}})$

The air density will also affect the performance of the engines. A linear approximation of this effect will be sufficient.

If $\rho < 1.2985$ then:

$\text{Thrust} = \text{Thrust}_{\text{MAX}} \times \frac{\rho}{1.2985}$

https://en.wikipedia.org/wiki/Density_of_air

https://en.wikipedia.org/wiki/International_Standard_Atmosphere

Raise to minimum chart value

If the calculated possible V_1 value is less than indicated in the FCOM VMCG/VMCA charts listed at the start of this document, V_1 should be raised to this minimum.

Adjustments after calculation

Once V_1, V_R and V_2 have been independently calculated, they should be adjusted as follows:

- If $V_R < V_1$: $V_R = V_1$
- If $V_2 < V_R$: $V_2 = V_R$

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