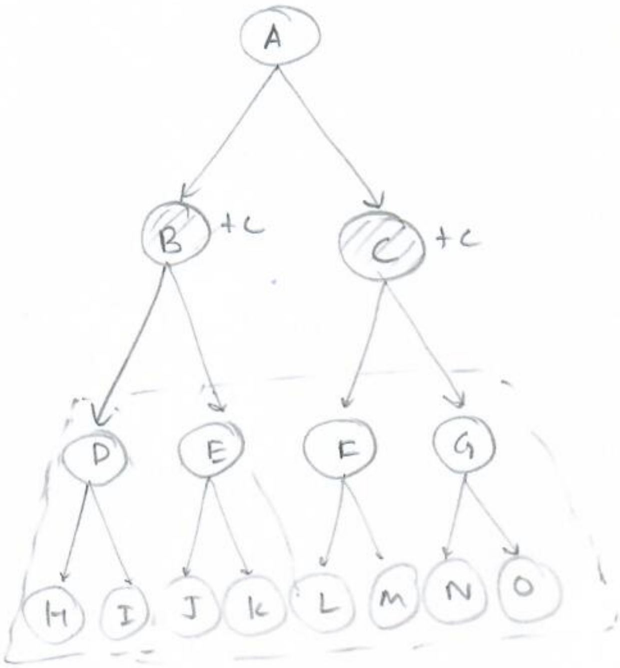
Bayes Net

(a) $P(A = +m \mid B = +m, C = +m)$



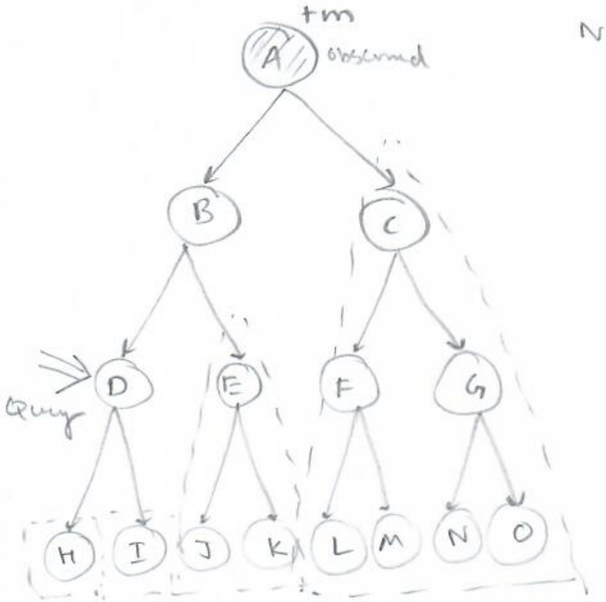
Note: If variables B and C are observed, then the variable nodes in the subtree below B and C do not affect the likelihood over variable A.

$$P(A = +m \mid B = +m, C = +m) = \frac{P(A = +m, B = +m, C = +m)}{\sum_{a \in \{+m, -m\}} P(A = a, B = +m, C = +m)}$$

$$= \left[\frac{P(A = +m)\, P(B = +m \mid A = +m)\, P(C = +m \mid A = +m)}{P(A = +m)\, P(B = +m \mid A = +m)\, P(C = +m \mid A = +m) + P(A = -m)\, P(B = +m \mid A = -m)\, P(C = +m \mid A = -m)} \right]$$

$$= \left[\frac{0.5 \times 0.9 \times 0.9}{0.5 \times 0.9 \times 0.9 + 0.5 \times 0.1 \times 0.1} \right] \approx 0.988$$

(b) $p(D=+m \mid A=+m)$



Note: For this query, the nodes in the subtree rooted at C, the subtree rooted at E and nodes below D do not influence the likelihood.

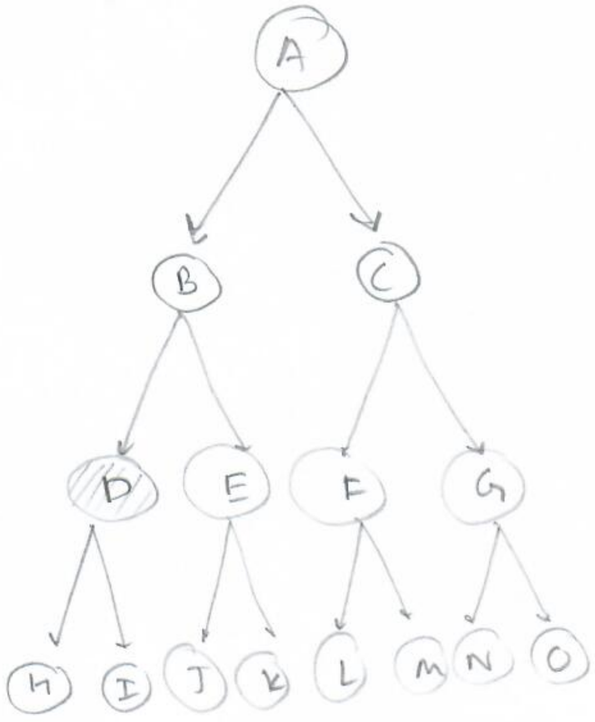
$$p(D=+m \mid A=+m) = \frac{\sum_{b \in \{+m,-m\}} p(D=+m, B=b, A=+m)}{\sum_{d,b \in \{+m,-m\}} p(D=d, B=b, A=+m)}$$

$$= \left[\frac{p(A=+m) \cdot p(B=+m \mid A=+m) \cdot p(D=+m \mid B=+m) + p(A=+m) \cdot p(B=-m \mid A=+m) \cdot p(D=+m \mid B=-m)}{(\cdots) + \left(p(A=+m) \cdot p(B=+m \mid A=+m) \cdot p(D=-m \mid B=+m) + p(A=+m) \cdot p(B=-m \mid A=+m) \cdot p(D=+m \mid B=-m) \right)} \right]$$

$$= \left[\frac{(0.5 \times 0.9 \times 0.9 + 0.5 \times 0.1 \times 0.1)}{(0.5 \times 0.9 \times 0.9 + 0.5 \times 0.1 \times 0.1) + (0.5 \times 0.9 \times 0.1 + 0.5 \times 0.1 \times 0.9)} \right]$$

$$= \frac{0.41}{0.41 + 0.09} = \frac{0.41}{0.5} = \boxed{0.82}$$

(c) Identify X such that $P(A \mid X, d) \neq P(A \mid D)$



Note:

- Any node that is not conditionally independent of A given D will change the posterior bel about variable A.

- Only nodes H and I will not change the bel about A given D. Hence, H or I cannot be the answer.

- Observations of the following variables will change the bel over A given D:

$X = \{B, C, E, F, G, J, K, L, M, N, O\}$