Bayesian Classification

(a) Class that the Maximum Likelihood (ML) classifier will predict for the new data point.

$$h_{ML} = \underset{h_i}{\text{argmax}} \; P(h_i \mid D) = \frac{1}{Z_{ML}} \prod_0 P(d_j \mid h_i)$$

$$P(X \mid d) \simeq P(X \mid h_{ML})$$

Now,

$$P(h_1 \mid D) = \frac{1}{Z_{ML}} \times P(D \mid h_1) = \frac{1}{Z_{ML}} \times \left[1 - \frac{1}{1+e^{-0.1}}\right]\left[\frac{1}{1+e^{-0.8}}\right]\left[\frac{1}{1+e^{-1.3}}\right]$$

$$= \boxed{0.38}$$

$$P(h_2 \mid D) = \frac{1}{Z_{ML}} \times \left[1 - \frac{1}{1+e^{+0.4}}\right]\left[\frac{1}{1+e^{-0.3}}\right]\left[\frac{1}{1+e^{-0.8}}\right]$$
$$= \frac{1}{Z_{ML}} P(D \mid h_2)$$

$$= \boxed{0.35}$$

$$P(h_3 \mid D) = \frac{1}{Z_{ML}} \times P(D \mid h_3)$$

$$= \frac{1}{Z_{ML}}\left[1 - \frac{1}{1+e^{+0.9}}\right]\left[\frac{1}{1+e^{+0.2}}\right]\left[\frac{1}{1+e^{-0.3}}\right]$$

$$= \boxed{0.27}$$

Where $Z_{ML} = P(D \mid h_1) + P(D \mid h_2) + P(D \mid h_3) = 0.679$

h_1 is the ML classifier

Prediction of point is $P((0.3, C_1) \mid h_1) = 0.57$

Estimated class is $\boxed{C_1}$.

(b) MAP classifier

$$h_{MAP} = \underset{h_i}{\arg\max}\, P(h_i|D) = \underset{h_i}{\arg\max}\left(\frac{1}{Z_{MAP}} \prod_d P(d_j|h_i)\, P(h_i)\right)$$

Prediction of the MAP classifier $P(x|d) \sim P(x|h_{MAP})$

$$P(h_1|D) = \frac{1}{Z_{MAP}} P(D|h_1)\, P(h_1) = \frac{1}{Z_{MAP}} P(D|h_1) \times 0.2 = \boxed{0.23}$$

$$P(h_2|D) = \frac{1}{Z_{MAP}} P(D|h_2)\, P(h_2) = \frac{1}{Z_{MAP}} P(D|h_2) \times 0.4 = \boxed{0.43}$$

$$P(h_3|D) = \frac{1}{Z_{MAP}} P(D|h_3)\, P(h_3) = \frac{1}{Z_{MAP}} P(D|h_3) \times 0.4 = \boxed{0.33}$$

$$Z_{MAP} = P(D|h_1)P(h_1) + P(D|h_2)P(h_2) + P(D|h_3)p(h_3)$$
$$= 0.22$$

h_2 is the MAP classifier

Prediction of h_2 for point is:

$$P((0.3, c_1)|h_2) = 0.45$$
$$\Rightarrow P(0.3, c_2|h_2) = 0.55$$
$$\Rightarrow \text{Estimated class is } \boxed{c_2}$$

Note: The likelihoods $P(D|h_1)$, $P(D|h_2)$, $P(D|h_3)$ were computed in (a).

(c) Bayesian classifier

Takes all hypothesis into consideration in making a classification decision.

$$P((0.3, c_1)|D) = P((0.3, c_1)|h_1) \, P(h_1|D) +$$
$$P((0.3, c_1)|h_2) \, P(h_2|D) +$$
$$P((0.3, c_1)|h_3) \, P(h_3|D)$$

$$= \left[\frac{1}{1+e^{-0.3}}\right] \times 0.23 + \left[\frac{1}{1+e^{+0.2}}\right] \times 0.43 +$$

$$\left[\frac{1}{1+e^{+0.7}}\right] \times 0.33$$

$$= \boxed{0.44}$$

$$P((0.3, c_2)|D) = 1 - 0.44 = 0.56$$

$\Rightarrow$ Predicted class is $\boxed{c_2}$.

Note: The posterior likelihoods $P(h_1|D)$, $P(h_2|D)$ & $P(h_3|D)$ were calculated in the previous part.