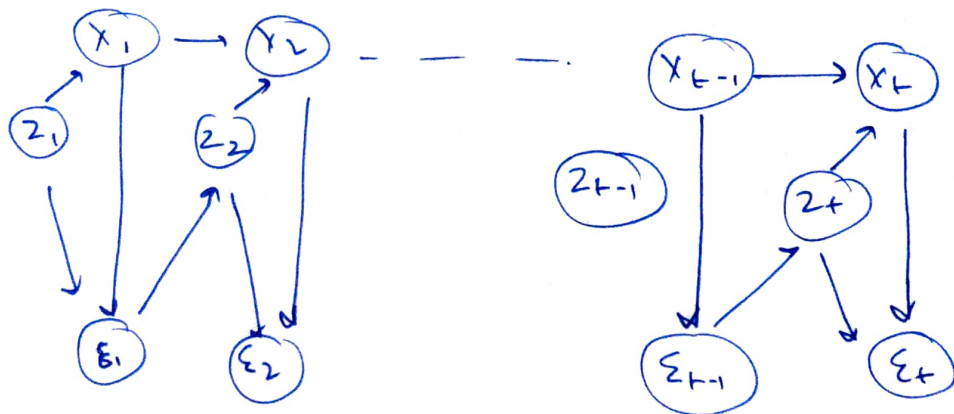


Question: Modified HMM
(a) modified HMM



Passage of time:

$$\begin{aligned}
 P(x_t, z_t | e_{1:t-1}) &= \sum_{x_{t-1}, z_{t-1}} P(x_t, z_t, x_{t-1}, z_{t-1} | e_{1:t-1}) \\
 &\quad \xrightarrow{\text{marginalisation}} \\
 &= \sum_{x_{t-1}, z_{t-1}} P(x_t, z_t | x_{t-1}, z_{t-1}, e_{1:t-1}) P(x_{t-1}, z_{t-1} | e_{1:t-1}) \\
 &\quad \xrightarrow{\text{chain rule}} \\
 &= \sum_{x_{t-1}, z_{t-1}} P(x_t | z_t, x_{t-1}, z_{t-1}, e_{1:t-1}) P(z_t | x_{t-1}, z_{t-1}, e_{1:t-1}) \\
 &\quad \quad P(x_{t-1}, z_{t-1} | e_{1:t-1}) \\
 &\quad \quad \xrightarrow{\text{chain rule}} \\
 &= \sum_{x_{t-1}, z_{t-1}} P(x_t | x_{t-1}, z_t) P(z_t | e_{t-1}) P(x_{t-1}, z_{t-1} | e_{1:t-1}) \\
 &\quad \quad \xrightarrow{\text{conditional indep.}}
 \end{aligned}$$

note: $P(x_t | z_t, x_{t-1}, z_{t-1}, e_{1:t-1})$

$x_t \perp\!\!\!\perp \{z_{t-1}, e_{1:t-1}\} | \{z_t, x_{t-1}\}$ ←

$P(z_t | x_{t-1}, z_{t-1}, e_{1:t-1})$

$z_t \perp\!\!\!\perp \{x_{t-1}, z_{t-1}, e_{1:t-2}\} | e_{t-1}$ ←

knowledge of z_t &
 x_{t-1} decouples
 x_t from other
variables

knowledge of
 e_{t-1} decouples
 z_t from previous
variables

Observation update

$$p(x_t, z_t | e_{1:t}) \propto p(x_t, z_t, e_t | e_{1:t-1}) \leftarrow \text{cond. prob rule}$$

$$\propto p(x_t, z_t | e_{1:t-1}) p(e_t | x_t, z_t, e_{1:t-1})$$

chain rule

$$\propto p(e_t | x_t, z_t) \underbrace{p(x_t, z_t | e_{1:t-1})}_{\text{computed previously}}$$

↓
use of conditional indep.
 $e_t \perp\!\!\!\perp \{e_{1:t-1}\} | x_t, z_t$

computed
previously