COL703 Quiz 3

Sreemanti Dey

TOTAL POINTS

2 / 2

QUESTION 1

1 Q1 **1 / 1**

- ✓ **+ 1 pts** Both parts correct
- **+ 0.5 pts** only part(a) correct
- **+ 0.5 pts** only part(b) correct
- **+ 0 pts** incorrect or unattempted

QUESTION 2

2 Q3 **1 / 1**

- ✓ **+ 1 pts** Correct
- **+ 0.5 pts** Partially correct
- **+ 0 pts** Incorrect/Not attempted

Quiz 3 (*Sep 8, Marks: 3, Duration: 30 mins*) **COL703, Aug-Nov 2022**

Name: Sreemanti Dey **Entry No.:** 2020CS10393

1. Recall that α is said to be *consistent* if $\nvdash \neg\alpha$. Suppose that $\vdash \alpha \to \beta$. For the following statements, answer whether they are true or not, and provide an explanation. Answers with missing or inadequate explanations will not get any marks.

   (a) **[0.5 marks]** If α is consistent then β is consistent.

   (b) **[0.5 marks]** If β is consistent then α is consistent.

2. **[1 marks]** Prove, in Hilbert's proof system, that $(\alpha \to \neg\neg\alpha)$.

3. **[1 marks]** Prove, in Hilbert's proof system, that $(\alpha \to \beta) \to ((\delta \to \gamma) \to ((\alpha \vee \delta) \to (\beta \vee \gamma)))$. Feel free to rewrite $\vee$ is terms of $\neg$ and $\to$ if you need to.

1) a) $\vdash \alpha \Rightarrow \beta$

To prove: ~~If~~ α is consistent then β is consistent.

If $\neg\alpha$ is not derivable then $\neg\beta$ is not derivable. } using contrapositive (proved in class)

If $\neg\beta$ is derivable then $\neg\alpha$ is derivable

$\vdash \neg\beta \Rightarrow \neg\alpha$

~~which is derived from~~

Sol^n $(\alpha \Rightarrow \beta) \Rightarrow (\neg\beta \Rightarrow \neg\alpha)$ ——— (2) [proved in next page]

hence $(\neg\beta \Rightarrow \neg\alpha)$ proved.

b)

To prove: If $\neg\alpha$ is derivable then $\neg\beta$ is derivable

$\neg\alpha \Rightarrow \neg\beta$.

Soln ~~$\alpha \Rightarrow \beta$~~.

This is false.

~~take~~ $\vdash \alpha \Rightarrow \beta$.

$\therefore \vDash \alpha \Rightarrow \beta$. using soundness and completeness of Hilbert's axiom.

hence if α is F and β is T.

Then $\alpha \Rightarrow \beta$ gives true

but $\neg\alpha \Rightarrow \neg\beta$ gives false

hence not valid.

(3) $(\alpha \Rightarrow \beta) \Rightarrow ((\delta \Rightarrow \gamma) \Rightarrow ((\alpha \vee \delta) \Rightarrow (\beta \vee \gamma))$

Deduction Thm

$(\alpha \Rightarrow \beta), (\delta \Rightarrow \gamma), (\alpha \vee \delta) \vdash (\beta \vee \gamma)$

$(\alpha \Rightarrow \beta), (\delta \Rightarrow \gamma),$ ~~$(\alpha \vee \delta)$~~ $(\neg\alpha \Rightarrow \delta) \vdash (\neg\beta \Rightarrow \gamma)$

using

$\alpha \Rightarrow \beta, \beta \Rightarrow \gamma \vdash \alpha \Rightarrow \gamma$ (in clam) — (1)

So $(\neg\alpha \Rightarrow \gamma), (\alpha \Rightarrow \beta) \vdash (\neg\beta \Rightarrow \gamma)$

using

$(\alpha \Rightarrow \beta) \Rightarrow (\neg\beta \Rightarrow \neg\alpha)$ (~~in clam~~ (2)

$(\neg\beta \Rightarrow \neg\alpha), (\neg\alpha \Rightarrow \gamma) \vdash (\neg\beta \Rightarrow \gamma)$

$(\neg\beta \Rightarrow \gamma)$ using — (1)

proved.

The 3

Proof of for Q1 & 3 — (2)

$(\alpha \Rightarrow \beta) \Rightarrow (\neg\beta \Rightarrow \neg\alpha)$

using

$(\neg\beta \Rightarrow \neg\alpha) \Rightarrow (\alpha \Rightarrow \beta)$ uni clam

$(\neg\neg\alpha \Rightarrow \neg\neg\beta) \Rightarrow (\neg\beta \Rightarrow \neg\alpha)$

using $\neg\neg\alpha \Rightarrow \alpha$ (in clam)

$(\alpha \Rightarrow \beta) \Rightarrow (\neg\beta \Rightarrow \neg\alpha)$ proved.