

Please note that there will be zero tolerance for dishonest means like copying solutions from others, and even letting others copy your solution, in any assignment/quiz/exam. If you are found indulging in such an activity, your answer-paper/code will not be evaluated and your participation/submission will not be counted. Second-time offenders will be summarily awarded an F grade. The onus will be on the supposed offender to prove his or her innocence.

1. [~~1 marks~~ **Diwali offer: 2 marks**] Formalise the following as sentences of first order logic. Use $B(x)$ for “ x is a barber”, and $S(x, y)$ for “ x shaves y ”.

- (a) Every barber shaves all persons who do not shave themselves.
- (b) No barber shaves any person who shaves himself.

Convert your answers to Skolem form and use ground resolution to show that (c), given below, is a consequence of (a) and (b).

- (c) There are no barbers.

2. [~~1.5 marks~~ **Diwali offer: 3 marks**] Fix a signature σ . Consider a relation \sim on σ -assignments that satisfies the following two properties.

- If $\mathcal{A} \sim \mathcal{B}$ then for every atomic formula F we have $\mathcal{A} \models F$ iff $\mathcal{B} \models F$.
- If $\mathcal{A} \sim \mathcal{B}$ then for each variable x we have (i) for each $a \in U_{\mathcal{A}}$ there exists $b \in U_{\mathcal{B}}$ such that $\mathcal{A}_{[x \mapsto a]} \sim \mathcal{B}_{[x \mapsto b]}$, and (ii) for all $b \in U_{\mathcal{B}}$ there exists $a \in U_{\mathcal{A}}$ such that $\mathcal{A}_{[x \mapsto a]} \sim \mathcal{B}_{[x \mapsto b]}$.

Prove that if $\mathcal{A} \sim \mathcal{B}$ then for any formula F , $\mathcal{A} \models F$ iff $\mathcal{B} \models F$. You may assume that F is built from atomic formulas using the connectives \wedge and \neg , and the quantifier \exists .

3. [~~1 marks~~ **Diwali offer: 2 marks**] Express the following by formulas of first-order logic, using predicates $H(x)$ for “ x is happy”, $R(x)$ for “ x is rich”, $G(x)$ for “ x is a graduate”, and $C(x, y)$ for “ y is a child of x ”.

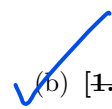
- (a) Any person is happy if all their children are rich.
- (b) All graduates are rich.
- (c) Someone is a graduate if they are a child of a graduate.
- (d) All graduates are happy.

Use first-order resolution to show that (d) is entailed by (a), (b), and (c). Indicate the substitutions in each resolution step.

4. [~~1 marks~~ **Diwali offer: 2 marks**] Let $A(x_1, \dots, x_n)$ be a formula with no quantifiers and no function symbols. Prove that $\forall x_1 \dots \forall x_n A(x_1, \dots, x_n)$ is satisfiable if and only if it is satisfiable in an interpretation with there being just one element in the universe.

5. In this question, we work with first-order logic without equality.

- (a) [~~1 marks~~ **Diwali offer: 2 marks**] Consider a signature σ containing only a binary relation R . For each positive integer n show that there is a satisfiable σ -formula F_n such that every model \mathcal{A} of F_n has at least n elements.



- (b) [~~1.5 marks~~ *Diwali offer: 3 marks*] Consider a signature σ containing only unary predicate symbols P_1, \dots, P_k . Using the question 2 (above), or otherwise, show that any satisfiable σ -formula has a model where the universe has at most 2^k elements.