9-1

We know that energy levels and Eigen functions related to unperturbed hamiltonian H_0 are

$$E_n^{(0)} = \frac{n^2\pi^2\hbar^2}{8mL^2} \quad , \quad \phi_n(x) = \begin{cases} \sqrt{\frac{1}{L}}\cos\left(\frac{n\pi x}{2L}\right) & ; n=1,3,5,\ldots \\ \sqrt{\frac{1}{L}}\sin\left(\frac{n\pi x}{2L}\right) & ; n=2,4,6,\ldots \end{cases}$$

According to Nondegenerate purturbation theory up to first order of nth energy state is

$$E_n = \frac{n^2\pi^2\hbar^2}{8mL^2} + E_n^{(1)} \quad , \quad E_n^{(1)} = \langle \phi_n | V_p(x) | \phi_n \rangle$$

So, (a)

$$E_n^{(1)} = \begin{cases} -V_0 \int_{-L}^{+L} \frac{1}{L}\cos^2\left(\frac{n\pi x}{2L}\right) dx & ; n=1,3,5,\ldots \\ -V_0 \int_{-L}^{+L} \frac{1}{L}\sin^2\left(\frac{n\pi x}{2L}\right) dx & ; n=2,4,6,\ldots \end{cases}$$

$$\Rightarrow E_n^{(1)} = -V_0 \quad , \quad n=1,2,3,\ldots$$

(b)

$$E_n^{(1)} = \begin{cases} -V_0 \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{L}\cos^2\left(\frac{n\pi x}{2L}\right) dx & ; n=1,3,5,\ldots \\ -V_0 \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{L}\sin^2\left(\frac{n\pi x}{2L}\right) dx & ; n=2,4,6,\ldots \end{cases}$$

$$\Rightarrow E_n^{(1)} = \begin{cases} 0 & ; n=2,4,6,\ldots \\ -V_0\left(\frac{1}{2}+\frac{2}{n\pi}\right) & ; n=1,5,9,\ldots \\ -V_0\left(\frac{1}{2}-\frac{2}{n\pi}\right) & ; n=3,7,11,\ldots \end{cases}$$

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