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To compute $\left[\hat{L}_z, cos\varphi\right]$ we obtain the act of it on a well-behaved wave function $\psi(r,\theta,\phi)$

$$\left[\hat{L}_z, cos\varphi\right]\psi(r,\theta,\phi) = \hat{L}_z\big(cos\varphi\,\psi(r,\theta,\phi)\big) - cos\varphi\left(\hat{L}_z\,\psi(r,\theta,\phi)\right)$$
$$= \frac{\hbar}{i}\frac{\partial}{\partial\phi}\big(cos\varphi\,\psi(r,\theta,\phi)\big) - cos\varphi\,\frac{\hbar}{i}\frac{\partial\psi(r,\theta,\phi)}{\partial\phi}$$
$$= \frac{\hbar}{i}\left(-sin\varphi\,\psi(r,\theta,\phi) + cos\varphi\frac{\partial\psi(r,\theta,\phi)}{\partial\phi} - cos\varphi\,\frac{\partial\psi(r,\theta,\phi)}{\partial\phi}\right) = i\hbar sin\varphi\,\psi(r,\theta,\phi)$$

$\psi(r,\theta,\phi)$ is arbitrary this means $\left[\hat{L}_z, cos\varphi\right] = i\hbar sin\varphi$

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