The Road To Majors PYL101 Part-1

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May 2021

1 Introduction

Hello everyone. Hope all are doing well and preparations are going well for the final showdown. This document has been made with the intentions to clear up some common misconceptions and to highlight some important concepts needed to solve questions. Lastly, we would be taking a look at some of the previous year questions which could be instrumental in handling similar problems.

2 Important Points to Remember

Let us see some important points that are extremely important to take down any question at hand

2.1 The wavefunction need not be an eigenfunction of the Hamiltonian

Set this idea rock hard. If you are given a random function about which you do not know whether it is an eigenfunction or not, do not try doing $\hat{H}\psi = E\psi$ because this is pointless unless you are determining if the wavefunction is an eigenfunction or not. The wavefunction is valid if and only if it satisfies the time dependent Schrodinger equation. I repeat and stress as well time dependent and not time independent.

2.2 Operator and Measurement are different

The values of an observable are always the eigenvalues of the operator corresponding to that observable. Hence, doing $\hat{p}\psi$ at random gives us practically nothing about the momentum. When the particle is not in the eigenstate of the observable, at most we can determine the expectation value of that observable.

2.3 Boundary Conditions

Always remember that for a wave function to be an acceptable solution it must satisfy the relevant boundary conditions. One of them is quite obvious and universally applicable and that is continuity of ψ. The second boundary condition that is the derivative of ψ is continuous for all cases where the potential jump is finite but does not hold whenever the potential barrier is like a delta function i.e goes to infinity.

2.4 Always remember to Normalize properly

This can cost you an entire question, so always keep this in your mind. Whenever you are given a wavefunction for calculating the expectation value of any operator, always check if the function is normalized, else even after 10-15 min of intense mental stress trying to calculate the answers from an extremely long set of integrals, you end up getting the wrong answer.

2.5 Probability of being in an eigenstate when the particle is in a superposition state

Let $\psi = c_1\psi_1 + c_2\psi_2$......... and that we need to find the probability that the particle is in a state ψ_1. Then the required probability is $|c_1|^2$.

2.6 Use the concept of Orthogonality to the fullest

Whenever you are integrating the product of different wavefunctions and you know that they are orthogonal then

$$\int_{-\infty}^{\infty} \psi_1^* \psi_2 dx = 0$$

This concept is especially useful when you need to normalize a superposition state.

3 Previous Year Questions-PART-1

The previous year papers can be readily found out on the BSW site. Here we would be discussing some general ideas related to solving questions. Please note that this will be continued in Part-2 of this document(to be released after a few days).

1. Major 2019-20 Sem2
   - Q3 Just Apply the time-dependent Schrodinger equation by simple substitution. For the second part, as said earlier, normalize first. The rest is just formula for probability.
   - Q4 I would strongly recommend that you go through the lecture notes for this. We had this problem in the lectures as well. The solution (no spoilers please) can be written as

     $$\psi = \sum c_n \psi_n(x) e^{-\frac{iE_n t}{\hbar}}$$

     However, there are some dimensional inconsistencies in the question's initial data.
   - Q5 Running on similar lines the question is also easy enough if you take a suitable linear combination.
   - Q6 Hefty calculations. Remember to normalize first. Rest is standard procedure for finding standard deviation and expectation values.
2. Major 2019-20 Sem2-1
   - Q4 Data inconsistent. Probabilities are not adding up to 1. Leave this question or try changing one of the denominators from 6 to 3. The general procedure to solve (b) is to multiply each of the terms independently by the corresponding time factor which would have come out if would have solved the Schrodinger's equation. The solutions (no spoilers intended)

     $$\psi = \sum_{n=1}^{3} \psi_n(x) e^{-\frac{iE_n t}{\hbar}}$$

   - Q5 Find the solutions of $\hat{p}\psi = p_0\psi$ in the three regions created due to the infinite square well.
     Hint$\rightarrow$ Remember that wave function is zero outside the well.
   - Q6 Calculation Overdose. When I solved this question I could not believe that this question fetches just 6 marks. Anyway, the method is easy. Just find the standard deviation values.
   - Q7 Wave function needs to be explicitly solved just like we did for finite well, potential barrier and other questions. Important feature to note, the wave should be zero on the positive side of the x-axis
3. Major 2018-19 Sem1 The questions are easy enough. Just that there might be some inaccuracies and inconsistencies at some places in the statement of the question.
4. Major 2016-17 Sem1
   - Q2 Refer to the lecture notes for finite square well. This question is very much formula oriented
   - Q3 $E \propto n^2$. You just need to find the values for n using the number of nodes in the wave diagram.
   - Q5 (a) Try thinking in terms of probability, transmission and reflection coefficients
     (b) Just another day in the office with standard formulas