

5.1

(a)

$$\begin{aligned}
 [\hat{Y}, \hat{L}_y] &= [\hat{Y}, \hat{z}\hat{p}_x - \hat{x}\hat{p}_z] = [\hat{Y}, \hat{z}\hat{p}_x] - [\hat{Y}, \hat{x}\hat{p}_z] \\
 &= [\hat{Y}, \hat{z}]\hat{p}_x + \hat{z}[\hat{Y}, \hat{p}_x] - [\hat{Y}, \hat{x}]\hat{p}_z - \hat{x}[\hat{Y}, \hat{p}_z] = 0
 \end{aligned}$$

$$[\hat{z}, \hat{L}_z] = 0$$

$$\begin{aligned}
 [\hat{Y}, \hat{L}_z] &= [\hat{Y}, \hat{x}\hat{p}_y - \hat{y}\hat{p}_x] = [\hat{Y}, \hat{x}\hat{p}_y] - [\hat{Y}, \hat{y}\hat{p}_x] \\
 &= [\hat{Y}, \hat{x}]\hat{p}_y + \hat{x}[\hat{Y}, \hat{p}_y] - [\hat{Y}, \hat{y}]\hat{p}_x - \hat{y}[\hat{Y}, \hat{p}_x] \\
 &= i\hbar \hat{x}
 \end{aligned}$$

similar to above

$$[\hat{z}, \hat{L}_x] = i\hbar \hat{Y}$$

$$[\hat{Y}, \hat{L}_x] = -i\hbar \hat{z}$$

$$[\hat{z}, \hat{L}_y] = -i\hbar \hat{x}$$

(b)

$$[\hat{x}, \hat{L}_x] = 0$$

$$[\hat{x}, \hat{L}_y] = i\hbar \hat{z}$$

$$[\hat{x}, \hat{L}_z] = -i\hbar \hat{Y}$$

$$\begin{aligned}
 (c) \quad [\hat{R}^2, \hat{L}_x] &= [\hat{x}^2 + \hat{y}^2 + \hat{z}^2, \hat{L}_x] = [\hat{y}^2, \hat{L}_x] + \hat{y}[\hat{y}, \hat{L}_x] \\
 &+ \hat{z}[\hat{z}, \hat{L}_x] + [\hat{z}, \hat{L}_x]\hat{z} = 0 \quad | \quad [\hat{R}^2, \hat{L}_y] = [\hat{R}^2, \hat{L}_z] = 0
 \end{aligned}$$

