

(a)

$$E_n^{(1)} = \begin{cases} -V_0 \int_{-\frac{L}{2}}^0 \frac{1}{L} \cos^2\left(\frac{n\pi x}{2L}\right) dx & ; n=1,3,5,\dots \\ -V_0 \int_{-\frac{L}{2}}^0 \frac{1}{L} \sin^2\left(\frac{n\pi x}{2L}\right) dx & ; n=2,4,6,\dots \end{cases}$$

$$\Rightarrow E_n^{(1)} = \begin{cases} -V_0 \left(\frac{1}{4} + \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right) \right) & ; n=1,3,5,\dots \\ -V_0 \left(\frac{1}{4} - \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right) \right) & ; n=2,4,6,\dots \end{cases}$$

(b)

$$E_n^{(1)} = \begin{cases} V_0 \int_0^{\frac{L}{2}} \frac{1}{L} \cos^2\left(\frac{n\pi x}{2L}\right) dx & ; n=1,3,5,\dots \\ V_0 \int_0^{\frac{L}{2}} \frac{1}{L} \sin^2\left(\frac{n\pi x}{2L}\right) dx & ; n=2,4,6,\dots \end{cases}$$

$$\Rightarrow E_n^{(1)} = \begin{cases} V_0 \left(\frac{1}{4} + \frac{1}{n\pi} \right) & ; n=1,5,9,\dots \\ V_0 \left(\frac{1}{4} - \frac{1}{n\pi} \right) & ; n=3,7,11,\dots \\ V_0/4 & ; n=2,4,6,\dots \end{cases}$$

$n=1,3,5,\dots$
 $n=3,7,11,\dots$
 $n=2,4,6,\dots$

$$(c) \quad E_n^{(1)} = \begin{cases} -V_0 \int_{-\frac{L}{2}}^0 \frac{1}{L} \cos^2\left(\frac{n\pi x}{2L}\right) dx + V_0 \int_0^{\frac{L}{2}} \frac{1}{L} \cos^2\left(\frac{n\pi x}{2L}\right) dx \\ -V_0 \int_{-\frac{L}{2}}^0 \frac{1}{L} \sin^2\left(\frac{n\pi x}{2L}\right) dx + V_0 \int_0^{\frac{L}{2}} \frac{1}{L} \sin^2\left(\frac{n\pi x}{2L}\right) dx \end{cases}$$

$$\Rightarrow E_n^{(1)} = 0 \quad ; n=1,2,3,\dots$$

$n=2,4,6,\dots$

