The Road To Majors PYL101 Part-2

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1 Introduction

Hello everyone. Hope all are doing well. We would be continuing with our discussion of the previous year questions. I got suggestions from many of you and I thought that it is a nice idea to include some references to the relevant chapters from standard books regarding different type of questions. I would strongly recommend that you go through the chapters for better understanding of the various aspects of QM.

2 Previous Year Questions

As in the first part, the question papers are readily available on the BSW site.

1. Minor 2 2017-18 Sem2-1 The questions are easy enough. Do not refer to the solutions written along with the questions.
2. Minor 2 2019-20 Sem2-1
   - Q8 This is a very nice question and the easiest way to do this is to shift the equation for particle in a box to the region described. You will need two different equations shifted to two different regions as asked by the question and then multiply them by the square root of probabilities. The solution structure is like this
   $$\psi = \sqrt{\frac{1}{3}}\psi_1 + \sqrt{\frac{2}{3}}\psi_2$$
   - Q9 Hefty calculations but straightforward. Remember to normalize first.
3. Major 2015-16 Sem1
   - Q1 (a) Direct Substitution (b) Check which of the operator has the given wavefunction as eigenfunction
   - Q2 Yet again direct substitution is the key
   - Q3 (a) Do not forget the factor of $\sqrt{\frac{2}{a}}$
   (b) Normalize using standard procedure
   (c) The possible energy values are E_1, E_3 and E_5 in the standard formula for energy in a 1-D box. The probabilities can be calculated using the square of the coefficients. Warning$\rightarrow$ Do not include the factor of $\sqrt{\frac{2}{a}}$ in the squaring procedure because that is a part of the eigenstate and not the coefficient. So, the coefficient method can be applied only when you write the wave function in this form
   $$\psi = \sum c_n \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$
   and now you can use the square of the coefficient method i.e the probability of being in a state is $|c_n|^2$
   - Q5 Write the equation for ground state and substitute the values.
   - Q6 Standard derivation of the expression for tunneling. Refer to lecture notes

3 References to Books

Here are some important chapters and relevant sections from the standard books which can help strengthen you concepts

3.1 Introduction to Quantum Mechanics: David J. Griffiths

The initial lectures on QM which are related to the birth of QM are not given in much detail in this book. However, other sections are quite informative and in my opinion are a must read. As all might not have the same edition, the names of different chapters have been written alongside

- Chapter-1 The Wave Function→ All sections are relevant
- Chapter-2 The Time- Independent Schrodinger Equation→ The Harmonic Oscillator, Scattering matrix and Delta Function Potential can be skipped. Rest are quite useful
- Chapter-3 Formalism → The entire chapter is relevant however the depth of study depends on you. Some sections are quite rich in Mathematics.

3.2 Quantum Mechanics: Zettili

The book is good except for one fact that you might get into some unknown waters while reading it because some portions are irrelevant to this course. Also reading Chapter-2 is purely your decision as there is a lot of extra stuff popping out in the chapter.

- Chapter-1 Origins of Quantum Physics→ All sections are worth reading
- Chapter-3 Postulates of Quantum Mechanics→ All sections
- Chapter-4 One Dimensional Problems→ You can skip Harmonic Oscillators. Rest are worth reading.

4 Important Side Points

The following points though not directly but indirectly help out in many questions

1. The normalization constants for any wave normalized at $t = 0$ will remain as is for all future time instants. This means that if you manage to normalize the wavefunction at $t = 0$ then you need not normalize it again if you are asked to find the state of the particle at a later time instant. This can be derived by proving this

$$\frac{d}{dt}\int_{-\infty}^{\infty} |\psi|^2 dx = 0$$

2. Be sure to read about some formulas related to Fourier Transform as it might come in handy for some questions.
3. Try proving the symmetry theorem of Quantum mechanics which we used for particle in a finite square well. The proof has two steps
   (a) First prove that there cannot be any degeneracy corresponding to any energy eigen value. This means that we need to prove that the eigen space has $dimension = 1$. Take two wavefunctions ψ_1 and ψ_2. Now both of them satisfy the TISE. Manipulate accordingly and derive that $\psi_1 = a\psi_2$. Thus they are linearly dependent
   (b) For the second part note that if the potential is an even function then $\psi(x)$ is a solution $\Rightarrow \psi(-x)$ is a solution corresponding to the same eigen value. By the first part, this means that $\psi(-x) = a\psi(x)$. Normalization gives $a = \pm 1$, which is what we wanted to prove.
4. Always try to keep an inventory of all the formulas and methods which are available to you. Whether it is needed or not is a different issue. Quantum Physics is highly Mathematically inclined. So the best way to solve a question is to bring it down to one of the simpler mathematical problems