In this problem we use this relation

$$\begin{split} \left| l \pm \frac{1}{2}, m \right\rangle &= \pm \sqrt{\frac{l \pm m + \frac{1}{2}}{2l + 1}} \left| l, \frac{1}{2}; m - \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{l \mp m + \frac{1}{2}}{2l + 1}} \left| l, \frac{1}{2}; m + \frac{1}{2}, \frac{-1}{2} \right\rangle \\ \left| 2 + \frac{1}{2}, m \right\rangle &= \sqrt{\frac{m + \frac{5}{2}}{5}} \left| 2, \frac{1}{2}; m - \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{-m + \frac{5}{2}}{5}} \left| 2, \frac{1}{2}; m + \frac{1}{2}, \frac{-1}{2} \right\rangle \\ \left| \frac{5}{2}, \frac{5}{2} \right\rangle &= \left| 2, \frac{1}{2}; 2, \frac{1}{2} \right\rangle \\ \left| \frac{5}{2}, \frac{3}{2} \right\rangle &= \sqrt{\frac{8}{10}} \left| 2, \frac{1}{2}; 1, \frac{1}{2} \right\rangle + \sqrt{\frac{4}{10}} \left| 2, \frac{1}{2}; 2, \frac{-1}{2} \right\rangle \\ \left| \frac{5}{2}, \frac{-1}{2} \right\rangle &= \sqrt{\frac{4}{10}} \left| 2, \frac{1}{2}; 0, \frac{1}{2} \right\rangle + \sqrt{\frac{4}{10}} \left| 2, \frac{1}{2}; 1, \frac{-1}{2} \right\rangle \\ \left| \frac{5}{2}, \frac{-3}{2} \right\rangle &= \sqrt{\frac{2}{10}} \left| 2, \frac{1}{2}; -2, \frac{1}{2} \right\rangle + \sqrt{\frac{8}{10}} \left| 2, \frac{1}{2}; -1, \frac{-1}{2} \right\rangle \\ \left| \frac{5}{2}, \frac{-3}{2} \right\rangle &= \sqrt{\frac{2}{10}} \left| 2, \frac{1}{2}; m - \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{m + \frac{5}{2}}{5}} \left| 2, \frac{1}{2}; m + \frac{1}{2}, \frac{-1}{2} \right\rangle \\ \left| \frac{3}{2}, \frac{3}{2} \right\rangle &= -\sqrt{\frac{2}{10}} \left| 2, \frac{1}{2}; 0, \frac{1}{2} \right\rangle + \sqrt{\frac{8}{10}} \left| 2, \frac{1}{2}; 2, \frac{-1}{2} \right\rangle \\ \left| \frac{3}{2}, \frac{-1}{2} \right\rangle &= -\sqrt{\frac{4}{10}} \left| 2, \frac{1}{2}; 0, \frac{1}{2} \right\rangle + \sqrt{\frac{4}{10}} \left| 2, \frac{1}{2}; 1, \frac{-1}{2} \right\rangle \\ \left| \frac{3}{2}, \frac{-1}{2} \right\rangle &= -\sqrt{\frac{6}{10}} \left| 2, \frac{1}{2}; -1, \frac{1}{2} \right\rangle + \sqrt{\frac{4}{10}} \left| 2, \frac{1}{2}; 0, \frac{-1}{2} \right\rangle \\ \left| \frac{3}{2}, \frac{-3}{2} \right\rangle &= -\sqrt{\frac{8}{10}} \left| 2, \frac{1}{2}; -1, \frac{1}{2} \right\rangle + \sqrt{\frac{2}{10}} \left| 2, \frac{1}{2}; 0, \frac{-1}{2} \right\rangle \\ \left| \frac{3}{2}, \frac{-3}{2} \right\rangle &= -\sqrt{\frac{8}{10}} \left| 2, \frac{1}{2}; -2, \frac{1}{2} \right\rangle + \sqrt{\frac{2}{10}} \left| 2, \frac{1}{2}; -1, \frac{-1}{2} \right\rangle \end{aligned}$$

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