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(a)

$$\hat{H} = \frac{\hat{P}^2}{2m}$$

$$\left[\hat{H}, \hat{X}\right] = -i\hbar\frac{\hat{P}}{m}$$

Thus

$$\hat{X}_H(t) = e^{\frac{i\hat{H}t}{\hbar}}\hat{X}e^{\frac{-i\hat{H}t}{\hbar}} = \hat{X} + \frac{it}{\hbar}\left[\hat{H}, \hat{X}\right] + \frac{1}{2!}\left(\frac{it}{\hbar}\right)^2\left[\hat{H}, \left[\hat{H}, \hat{X}\right]\right] + \cdots$$

$\left[\hat{H}, \left[\hat{H}, \hat{X}\right]\right] = 0$ so, we conclude that

$$\hat{X}_H(t) = \hat{X} + \frac{t\hat{P}}{m}$$

(b)

$$\left[\hat{X}_H(t), \hat{X}_H(0)\right] = \left[\hat{X} + \frac{t\hat{P}}{m}, \hat{X}_H(0)\right] = \left[\hat{X}, \hat{X}\right] + \frac{t}{m}\left[\hat{P}, \hat{X}\right] = \frac{-i\hbar t}{m}$$

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Translated by: