3.16

(a)

Eigenvalues of Hamiltonian are what we get when we measure the energy so

$\det(H-h)=0$ $\qquad h' = \frac{h}{\mathcal{E}_0}$ $\rightsquigarrow$ change of variable

$$\Rightarrow \begin{vmatrix} -h' & -i & 0 \\ i & -h' & 2i \\ 0 & -2i & -h' \end{vmatrix} = 0$$

$$\Rightarrow h_1 = 0 \quad , \quad h_2 = \sqrt{5}\,\mathcal{E}_0 \quad , \quad h_3 = -\sqrt{5}\,\mathcal{E}_0$$

if $|h_1 = 0\rangle = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\Rightarrow \mathcal{E}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 2i \\ 0 & -2i & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\Rightarrow y = 0 \, , \, -x = 2z$$

$$\Rightarrow |h_1 = 0\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$ similar to h_1

$$|h_2 = \sqrt{5}\,\mathcal{E}_0\rangle = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ i\sqrt{5} \\ 2 \end{pmatrix}$$

$$|h_3 = -\sqrt{5}\,\mathcal{E}_0\rangle = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ -i\sqrt{5} \\ 2 \end{pmatrix}$$

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Similar method applying to A we get

$$|a_1 = 0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-i\end{pmatrix}, \quad |a_2 = -a_0\rangle = \begin{pmatrix}1\\-i\\i\end{pmatrix}\frac{1}{\sqrt{3}}$$

$$, \quad |a_3 = 2a_0\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\2i\\i\end{pmatrix}$$

(b) Probabilities corresponds to a_1, a_2, a_3 are

$$P_1 = |\langle a_1|h_2\rangle|^2 = \left|\frac{1}{\sqrt{2}}(1 \;\; 0 \;\; i)\frac{1}{\sqrt{10}}\begin{pmatrix}1\\i\sqrt{5}\\2\end{pmatrix}\right|^2$$

$$= \frac{1}{20}|1+2i|^2 = \frac{1}{4}$$

$$P_2 = |\langle a_2|h_2\rangle|^2 = \frac{10-2\sqrt{5}}{30}$$

$$P_3 = 1-(P_1+P_2) = \frac{4\sqrt{2}+25}{60}$$

when we measure Energy and we get $\sqrt{5}\,\varepsilon_0$ system goes to $|h_2\rangle$ state.

(c) $$\langle A\rangle = \sum_i P_i a_i = \frac{1}{4}\times 0 + \frac{10-2\sqrt{5}}{30}(-a_0)$$

$$+\frac{4\sqrt{5}+25}{60}(2a_0) = \left(\frac{6\sqrt{5}+15}{30}\right)a_0$$

$$= \boxed{\frac{2\sqrt{5}+5}{10}\,a_0}$$

Mohammad Behtaj & Adel Sepehri