$$\begin{split} |\psi_{i}\rangle &= \sum_{i} |\phi_{i}\rangle\langle\phi_{i}|\psi_{i}\rangle = |\phi_{1}\rangle\langle\phi_{1}|\psi_{i}\rangle + |\phi_{2}\rangle\langle\phi_{2}|\psi_{i}\rangle + |\phi_{3}\rangle\langle\phi_{3}|\psi_{i}\rangle = \frac{i}{\sqrt{3}}|\phi_{1}\rangle + \sqrt{\frac{2}{3}}|\phi_{2}\rangle \\ |\psi_{f}\rangle &= \sum_{i} |\phi_{i}\rangle\langle\phi_{i}|\psi_{f}\rangle = |\phi_{1}\rangle\langle\phi_{1}|\psi_{f}\rangle + |\phi_{2}\rangle\langle\phi_{2}|\psi_{f}\rangle + |\phi_{3}\rangle\langle\phi_{3}|\psi_{f}\rangle \\ &= \frac{1+i}{\sqrt{3}}|\phi_{1}\rangle + \sqrt{\frac{1}{6}}|\phi_{2}\rangle + \sqrt{\frac{1}{6}}|\phi_{3}\rangle \\ P &= \left|\langle\psi_{f}|\psi_{i}\rangle\right|^{2} = \left(\frac{1-i}{\sqrt{3}}\langle\phi_{1}| + \sqrt{\frac{1}{6}}\langle\phi_{2}| + \sqrt{\frac{1}{6}}\langle\phi_{3}|\right)\left(\frac{i}{\sqrt{3}}|\phi_{1}\rangle + \sqrt{\frac{2}{3}}|\phi_{2}\rangle\right) = \left|\frac{1-i}{\sqrt{3}}\frac{i}{\sqrt{3}} + \sqrt{\frac{1}{6}}\sqrt{\frac{2}{3}}\right|^{2} \\ &= \left|\frac{1+i}{3} + \frac{1}{3}\right|^{2} = \left|\frac{2+i}{3}\right|^{2} = \frac{2^{2}+1^{2}}{3^{2}} = \frac{5}{9} \end{split}$$

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