Problem set 2: Operator Formalism

PYL101(April-May 2021)

1. Show that:

   (i) $[\hat{A}, a\hat{B}] = [a\hat{A}, \hat{B}] = a[\hat{A}, \hat{B}],\ where\ \ a = constant.$

   (ii) $[\hat{A} + \hat{B}, \hat{C}] = [\hat{A}, \hat{C}] + [\hat{B}, \hat{C}]$

   (iii) $[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$

   (iv) $[e^{\hat{A}}, \hat{A}] = 0$

2. Using the relation $\Delta A = A - \langle A \rangle$, show that $\langle (\Delta A)^2 \rangle = \langle A^2 \rangle - \langle A \rangle^2$.

3. Ehrenfest theorem (or principle) states that the fundamental equations (Newton's) of classical dynamics are satisfied by the expectation values of the corresponding operators in Quantum mechanics. Prove it for the following:

$$(i) \quad \frac{\partial x}{\partial t} = \frac{p_x}{m}, \qquad\qquad (ii) \quad \frac{\partial p_x}{\partial t} = -\frac{\partial V}{\partial x}$$

4. Show that the wavefunction, for which the product of uncertainties in position and momentum is minimum, is a gaussian wave packet.