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We have:

$$\frac{d^2\psi(x)}{dx^2}+\frac{2m}{\hbar^2}\big(E-V_0\delta(x-a)-V_0\delta(x+a)\big)\psi(x)=0$$

Except for $x\neq\pm a$ we have

$$\frac{d^2\psi(x)}{dx^2}+k^2\psi(x)=0\quad,\quad k=\sqrt{\frac{2mE}{\hbar^2}}$$

Thus

$$\psi(x)=\begin{cases}Ae^{ikx}+Be^{-ikx}; & x\leq -a\\ Ce^{ikx}+De^{-ikx}; & -a\leq x\leq a\\ Ee^{ikx} & ;-a\leq x\leq a\end{cases}$$

imposing continuity condition for $\psi(x)$ in $x=\pm a$ we have

$$Ae^{ikx}+Be^{-ikx}=Ce^{ikx}+De^{-ikx}\quad (3)$$

$$Ce^{ikx}+De^{-ikx}=Ee^{ikx}\quad (1)$$

and from discontinuity of its first derivative at $x=\pm a$ and integrating from Schrödinger equation we have:

$$\left.\frac{d\psi(x)}{dx}\right|_{x=-a+\varepsilon}-\left.\frac{d\psi(x)}{dx}\right|_{x=-a-\varepsilon}=\frac{2mV_0}{\hbar^2}\psi(-a)$$

$$\left.\frac{d\psi(x)}{dx}\right|_{x=a+\varepsilon}-\left.\frac{d\psi(x)}{dx}\right|_{x=a-\varepsilon}=\frac{2mV_0}{\hbar^2}\psi(a)$$

or

$$ik\big(Ce^{-ika}-De^{ika}\big)-ik\big(Ae^{-ika}-Be^{ika}\big)=\frac{2mV_0}{\hbar^2}\big(Ae^{-ika}+Be^{ika}\big)\quad (2)$$

$$ikEe^{ika}-ik\big(Ce^{ika}-De^{-ika}\big)=\frac{2mV_0}{\hbar^2}Ee^{ika}$$

$$\Rightarrow ik\big(Ce^{ika}-De^{-ika}\big)=Ee^{ika}\left(ik-\frac{2mV_0}{\hbar^2}\right)$$

On the other hand, from (1) we have

$$ik\big(Ce^{ika}+De^{-ika}\big)=ikEe^{ika}$$

By adding and subtracting these two equations we get

$$C=E\left(1+\frac{imV_0}{k\hbar^2}\right)\quad,\quad D=-E\frac{imV_0}{k\hbar^2}e^{2ika}$$

Thus (2) take this form

$$Be^{ika}\left(1+\frac{2imV_0}{k\hbar^2}\right)=Ae^{-ika}\left(1-\frac{2imV_0}{k\hbar^2}\right)-E\left(e^{-ika}+\frac{imV_0}{k\hbar^2}e^{-ika}+\frac{imV_0}{k\hbar^2}e^{3ika}\right)$$

By finding B from above Equation and impose it on (3) and doing some calculations

$$\frac{E}{A}=\left(1-\left(\frac{mV_0}{k\hbar^2}\right)^2+\left(\frac{mV_0}{k\hbar^2}\right)^2\cos(4ka)+i\frac{mV_0}{k\hbar^2}\left(2+\frac{mV_0}{k\hbar^2}\sin(4ka)\right)\right)^{-1}$$

$$T=\left|\frac{E}{A}\right|^2=\left(1+4\left(\frac{mV_0}{k\hbar^2}\right)^2\left[\frac{mV_0}{k\hbar^2}\sin(2ka)+\cos(2ka)\right]\right)^{-1}$$

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