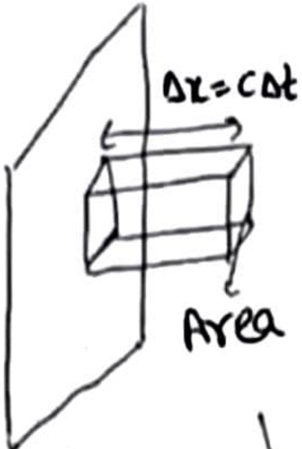


Energy E in this volume travelling in $+x$ direction will go through the area ΔA in time Δt.

θ is angle b/w x axis and Area vector

$$\frac{dE}{d\lambda} = \text{energy / unit length}$$

Radiated power/ unit area/ unit length $= dR/d\lambda$

So Energy E can be expressed in terms of R

$$\boxed{\text{Power} = \frac{E}{\text{Time}}}$$

$$\frac{dE}{d\lambda} = 2\left[\frac{dR}{d\lambda}\right]_{\theta=0} \Delta t \cdot \Delta A = \left[\frac{dR}{d\lambda}\right]_{\theta=0} \cdot \frac{2 \times \Delta x \times \Delta A}{c}$$

Since $\frac{1}{2}$ the power will be going in negative direction

$$\left[\frac{dR}{d\lambda}\right] = \frac{c}{2\,\underbrace{\Delta x\,\Delta A}_{\text{volume}}} \times \frac{dE}{d\lambda}$$

$$\boxed{\frac{dR}{d\lambda} = \frac{c}{2}\,\frac{dU}{d\lambda}}$$

where U = energy density

Now, If area vector makes angle θ with x-direction

$$\left[\frac{dR}{d\lambda}\right]_\theta = \frac{dU}{d\lambda} \cdot \frac{c}{2} \cdot \cos^2\theta$$

Taking average of angular part over all angles,

$$\boxed{\frac{dR}{d\lambda} = \frac{dU}{d\lambda}\frac{c}{4}}$$

Hence proved.

Question-2

Show that the total energy density $U = \dfrac{8\pi^5 k^4}{15 h^3 c^3} T^4$

Planck Distribution: $u(\lambda) = \dfrac{8\pi hc}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)}$

$$U = \int_0^\infty d\lambda \, u(\lambda) = 8\pi hc \int_0^\infty \frac{d\lambda}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)}$$

Put $\xi = \dfrac{hc}{\lambda kT}$; $d\xi = \dfrac{-hc}{\lambda^2 kT} d\lambda$

$$\frac{d\xi}{\xi} = \frac{-d\lambda}{\lambda}.$$

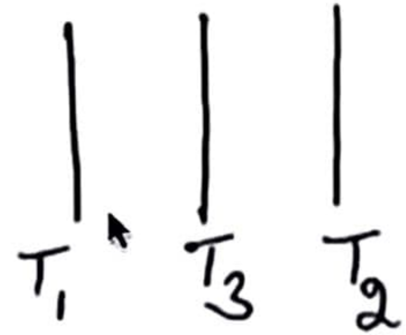
$$= \cancel{8\pi hc} \int_0^\infty \frac{8\pi (kT)^4}{\left(e^{\xi} - 1\right)(hc)^3} \xi^3 \, d\xi$$

$$= 8\pi (kT)^4 \int^\infty \xi^3 \, d\xi$$

$$\boxed{U \Rightarrow \frac{8 K^4 T^4}{h^3 c^3} \frac{\pi^5}{15}}$$

without the third plane
the energy flux / area is

$$w_0 = \sigma\left(T_1^4 - T_2^4\right)$$



The equilibrium temp. of the third plane can be found from the energy balance:

$$\sigma\left(T_1^4 - T_3^4\right) = \sigma\left(T_3^4 - T_2^4\right)$$

$$T_1^4 + T_2^4 = 2T_3^4$$

or $$\boxed{T_3 = \left[\frac{T_1^4 + T_2^4}{2}\right]^{1/4}}$$

The energy flux between the I^{st} and 2^{nd} planes in presence of third plane:

$$W = \sigma[T_1^4 - T_3^4] = \sigma\left[T_1^4 - \frac{T_1^4 + T_2^4}{2}\right]$$

$$= \frac{1}{2}\sigma[T_1^4 - T_2^4]$$

$$\boxed{W = \frac{1}{2}W_0}$$ cut in half.

because of 3rd plane we calculate

Plane block energy cutting to half.

Question-4

$$\text{Intensity} = \frac{\text{Power Radiated}}{\text{Area}}$$

$$\text{Radiation} = \text{surface area} \times \sigma T^4$$

$$= 4 \times 3.14 \times (7 \times 10^8)^2 \times 5.67 \times 10^{-8} \frac{W}{m^2} K^{-4} \times T^4$$

$$= 3.48 \times 10^{10}\ T^4\ W K^{-4}$$

= Solar constant × surface area of the sphere whose radius is equal to sun- earth distance

$$= 1.43 \times 10^3 \times 4 \times 3.14 \times (1.5 \times 10^{11})^2\ W$$

$$= 3.96 \times 10^{26}\ W$$

$$T^4 = \frac{3.96 \times 10^{26}}{3.48 \times 10^{10}}$$

$$\boxed{T = 5805\,K}$$

Question-5

Hole area : $\pi r^2 \Rightarrow 78.57 \times 10^{-6}\ m^2$ where

$r = 5\ mm$

$T = 6000\ K$; $\lambda = 550 \times 10^{-9}\ m$ $d\lambda = 1.0 \times 10^{-9}\ m$

Power Radiated/Area $\omega(\lambda)\,d\lambda = \frac{c}{4} u(\lambda)\,d\lambda$

$$\Rightarrow \frac{8\pi h c^2}{4\lambda^5 \exp\left(\frac{hc}{\lambda KT}\right) - 1}\, d\lambda$$

$$\frac{hc}{\lambda KT} = \frac{6.625 \times 10^{-34} \times 3.0 \times 10^{8}}{550 \times 10^{-9} \times 1.38 \times 10^{-23} \times 6000} = 4.36$$

$$\frac{8\pi h c^2}{4\lambda^5} d\lambda = \frac{8 \times 3.14 \times 6.625 \times 10^{-34} \times (3.0 \times 10^{8})^2 \times 1.0 \times 10^{-9}}{4 \times (550 \times 10^{-9})^5}$$

$$= 7.7 \times 10^{6}$$

$$\omega(\lambda)\, d\lambda = \frac{7.7 \times 10^{6}}{\exp(4.36) - 1} = 1 \times 10^{5} \frac{W}{m^2}$$

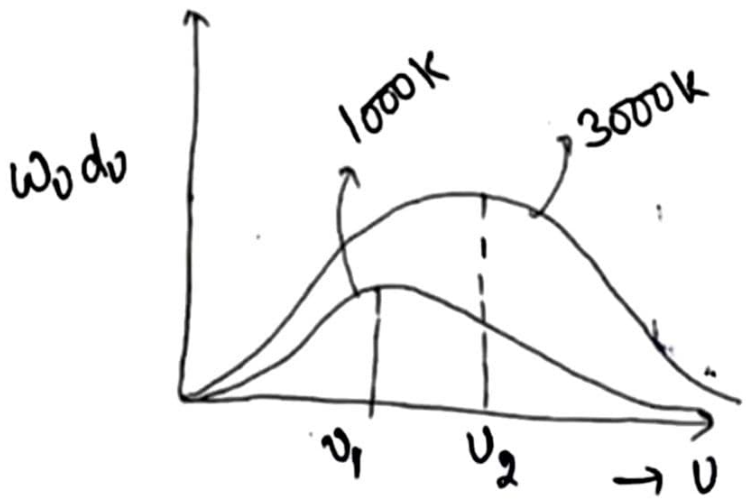
Power Radiated $= A \cdot W \Rightarrow 78.57 \times 10^{-6} \times 1 \times 10^{5}$

$$= \boxed{7.8\ W}$$

Question:- 6

(a)

(b)



$v_2 > v_1$

Question-7

Radiation by sun = surface area × Solar constant

$$= 1.43\times10^{3} \times 4\times3.14\times2.25\times10^{22}\ W$$

$$= 3.96\times10^{26}\ W$$

$$\Delta m c^2 = 3.96\times10^{26}\ W$$

$$\Delta m = \frac{4\times10^{26}}{(3\times10^{8})^{2}} \cdot \frac{W\ sec^2}{m^2} \qquad W = kg\ m^2 sec^{-3}$$

$$\Delta m = 4.4\times10^{9}\ kg/sec.$$

1 sec —— 4.4×10^{9}

1 year (3.154×10^{7} sec) —— $4.4\times10^{9}\times3.154\times10^{7}$

$$= 1.38\times10^{17}\ kg$$

So In one year $\Delta m = 1.38\times10^{17}$ kg.

Fractional change $\frac{\Delta m}{m} = \frac{1.38 \times 10^{17}}{2 \times 10^{30}}$

$$= \boxed{6.9 \times 10^{-14}}$$

Question -8

Radiation is exchanged between the source and detector

Radiation from white tape area = Radiation from the detector

Intensity = $\frac{\text{Power}}{\text{Area}}$

Power = Intensity × Area

$$\epsilon_w \cancel{A_w} \sigma T_{cup}^4 = \epsilon_d \cancel{A_d} \sigma T_d^4$$

$$\epsilon_w \times (89)^4 = 0.95\,(89)^4$$

$$\boxed{\epsilon_w = 0.95}$$

Similarly for black taped region. Note that black body simply means an object with emissivity 1 : by this definition, the black and white tape qualifies as "black bodies"

Now for steel region:

$$\epsilon_s \cancel{A_s} \sigma T_{cup}^4 = \epsilon_d \cancel{A_d} \sigma T_d^4$$

$$\epsilon_s (89+273)^4 = 0.95 (32+273)^4$$

$$\boxed{\epsilon_s = 0.47}$$

Emissivity :- Emissivity of the surface of material is its

effectiveness ...

Question :- 9

Total Energy density = $\int_0^\infty \frac{8\pi (KT)^4}{(hc)^3} \frac{\gamma^3}{(\exp\gamma - 1)} d\gamma$

$$\Rightarrow \frac{8\pi (KT)^4}{(hc)^3} \frac{\pi^4}{15} = 7.52 \times 10^{-16}\, T^4 \ J/m^3$$

$$= 7.52 \times 10^{-16} \times (2.7)^4$$

$$= \boxed{3.99 \times 10^{-14} \ J/m^3}$$

1 Joule = 6.2×10^{18} ev

So ~~[illegible]~~ $\boxed{2.4 \times 10^{5} \ ev/m^3}$

Planck's law:

$$\text{total Energy} = nh\nu$$

$$= \frac{nhc}{\lambda}$$

$$n = \frac{E \times \lambda}{hc}$$

$$= \frac{3.99 \times 10^{-14} \times 1 \times 10^{-3}}{6.625 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= \boxed{2 \times 10^{8}}$$

Question:-10

work function of metal is 5ev.

Power at a distance of 100 cm = 1W

Power per unit area = $\frac{1W}{4\pi(1)^2}$

Power absorbed by electron = incident power x area of atom

$$\frac{1}{4\pi(1)^2} \times \pi(10^{-10})^2 =$$

watt → $\frac{\text{Joule}}{\text{sec.}}$

$$= 2.5 \times 10^{-21} W$$

using relation: $1\,eV = 4.4 \times 10^{-23}$ wh

$5\,eV \rightarrow 22 \times 10^{-23}$ wh

22×10^{-23} —— 1 hour

1 —— $\frac{1}{22 \times 10^{-23}}$

2.5×10^{-21} —— $\frac{2.5 \times 10^{-21}}{22 \times 10^{-23}}$

$= \boxed{11.36}$ hours

$$\boxed{\text{Question :- 11}}$$

In first case only one surface is allowed to loose energy (neglect thickness)

Radiation $(A\sigma T^4)$ = Heat absorbed = Solar constant

$$T^4 = \frac{1.4 \times 10^3 \ \cancel{\text{watts per m}^2}}{5.67 \times 10^{-8} \ \cancel{K^4}}$$

$$\boxed{T = 396\ K}$$

In second case two surfaces loose energy (neglect thickness)

$2A\sigma T^4 =$ solar constant

$$T^4 = \frac{1.4 \times 10^3}{2 \times 5.67 \times 10^{-8}}$$

$$\boxed{T = 333 \text{ K}}$$

$\boxed{\text{Question :- 12}}$

$T_1 = 33.0\,^\circ C$ and $T_2 = 31.6\,^\circ C$

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{33+273}{31.6+273}\right)^2$$

$$\frac{P_1}{P_2} = 1.0185$$

So $$\frac{P_1 - P_2}{P_2} = 1.0185 - 1 = \boxed{0.018}$$

i.e. P_1 is about 2% greater than P_2

In order to distinguish between two regions the

Regarding error in solutions Tutorial-1 QM

Sir, Ma'am

The solution for Q10 of Tutorial-1 QM has some error. The final expression before deriving the answer is dimensionally incorrect as its dimension is hr^{-1}, however it should have been in hr. The correct answer should be the the reciprocal of the final answer obtained in the solution. Instead of $t = \frac{E}{P}$ the solution has used $t = \frac{P}{E}$. So, I request you to please correct any discrepancy which might have crept in this regard.

Thank You

quantum_mechanics

edit · good question | 0

Updated 1 day ago by Manas Choudhary

the students' answer, *where students collectively construct a single answer*

Actions

Yes, I too have same concern. I am getting answer as 320 sec. Could someone please confirm the answer?

Thank you.

edit · thanks! | 0

Updated 1 day ago by Anonymous Gear

the instructors' answer, *where instructors collectively construct a single answer*

Thanks Manas for noticing it! The conversion factor that should have been used is 1 eV = 4.4505E-23 Wh. first you calculate the total power at a distance 1 m (Classically).

Power absorbed by the electron is = incident power x area of the atom

Then in the last step the TA made the mistake - it should have been -

2.5X10^-21 W X time = 5eV (in Wh)

i.e., just the inverse of what she showed you in class!

I am suffering from CORONA, so couldnt keep up wih the posts... getting delayed in replying.

thanks! | 1

Updated 13 hours ago by Ratnamala Chatterjee