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(a) Normalization condition is

$$\int |\psi(\vec{r})|^2 dr = 1$$

ϕ_{nlm} s are complete set of orthonormal states

$$\Rightarrow |A|^2 + \left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1$$

$$\Rightarrow A = \sqrt{\frac{7}{15}}\, e^{i\delta} \qquad (\delta \text{ any real value})$$

(b) $\psi(\vec{r},t) = e^{-\frac{iHt}{\hbar}} \psi(\vec{r},0)$, $H\psi_{nlm}(\vec{r}) = E_n \psi_{nlm}(\vec{r})$

$$\Rightarrow \psi(\vec{r},t) = A e^{-\frac{iE_2 t}{\hbar}} \phi_{200}(\vec{r}) + \frac{1}{\sqrt{5}} e^{-\frac{iE_3 t}{\hbar}} \phi_{311}(\vec{r})$$

$$+ \frac{1}{\sqrt{3}} e^{-\frac{iE_4 t}{\hbar}} \phi_{422}(\vec{r})$$

(c) measurment of energy will yield E_2, E_3, and E_4

with Probabilities

$$P_{E_2} = |\langle 200|\psi\rangle|^2 = |A|^2 = \frac{7}{15}$$

$$P_{E_3} = |\langle 311|\psi\rangle|^2 = \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{1}{5}$$

$$P_{E_4} = |\langle 422|\psi\rangle|^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

(d) $\langle E \rangle = \sum_n P_{E_n} E_n = \frac{7}{15} E_2 + \frac{1}{5} E_3 + \frac{1}{3} E_4$

Knowing $E_n = E_1/n^2$

$\Rightarrow \langle E \rangle = \left(\frac{7}{15} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{9} + \frac{1}{3} \cdot \frac{1}{16} \right) E_1 = \frac{115}{720} E_1$

$$= \frac{-115 \times 13.6}{720} \, eV \simeq -2.17 \, eV$$

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