4-3

(a)

In general we have $\left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)\psi(x)=E\psi(x)$

thus
$$\begin{cases}\dfrac{-\hbar^2}{2m}\dfrac{d^2\psi_1(x)}{dx^2}=E\psi_1(x) & ; \quad x\leq 0\\[2ex] \dfrac{-\hbar^2}{2m}\dfrac{d^2\psi_2(x)}{dx^2}=(E-V_0)\psi_2(x) & ; \quad x>0\end{cases}$$

or
$$\begin{cases}\dfrac{d^2\psi_1(x)}{dx^2}+k_1^2\psi_1(x)=0 & ; \quad x\leq 0\\[2ex] \dfrac{d^2\psi_2(x)}{dx^2}-k_2'^2\psi_2(x)=0 & ; \quad x>0\end{cases}$$

$$k_1^2=\frac{2mE}{\hbar^2}\quad , \quad k_2'^2=\frac{2m}{\hbar^2}(V_0-E)$$

$$\Rightarrow\begin{cases}\psi_1(x)=Ae^{ik_1x}+Be^{-ik_1x}\\ \psi_2(x)=Ce^{-k_2'x}+De^{k_2'x}\end{cases}$$

wave function must be finite so D must be zero. $(x\to+\infty\Rightarrow e^{k_2'x}\to+\infty)$

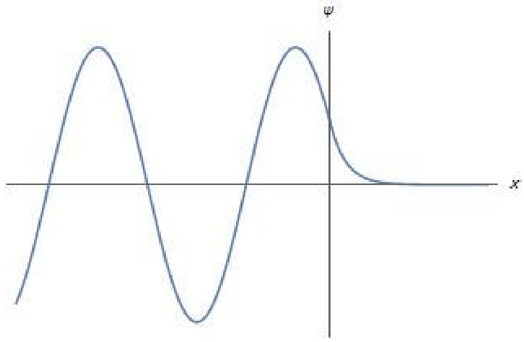
wave function must be continuous

$\Rightarrow\psi_1(0)=\psi_2(0)$ or $A+B=C$ (1)

from cntinuity of wave functions slope on $x=0$ we have $ik_1(A-B)=-k_2'C$ (2)

① & ② $\Rightarrow \begin{cases} B = \dfrac{k_1 - ik'_2}{k_1 + ik'_2} A \\ C = \dfrac{2k_1}{k_1 + ik'_2} A \end{cases}$

(b) or $\psi'_E(x) = \dfrac{k_1 + ik'_2}{2A}\psi_E(x) = \begin{cases} k_1 \cos k_1 x - k'_2 \sin k_1 x \; ; & x \le 0 \\ k_1 e^{-k'_2 x} \; ; & x > 0 \end{cases}$



(c)

for $-20m < x < -10m$ electrons weve length is:

$$\lambda_1 = \frac{2\pi}{k_1} = \frac{2\pi}{\left(\frac{2mE}{\hbar^2}\right)^{1/2}} = \frac{2\pi\hbar}{\sqrt{2mE}}$$

$$= \frac{2\pi \times 197.327 \, \frac{eV.nm}{c}}{\left(2 \times 0.511 \, \frac{MeV}{c^2} \times 10 \, eV\right)^{1/2}} \simeq 3.95 \times 10^{-10} \, m$$

for $x > 10$ wave function have exponential behavior so wave length is meaningless.

(e) $$\frac{P}{P'} = \frac{|\psi(x = 10^{-10} m)|^2}{|\psi(x=0)|^2} = e^{-2k'_2 \times 10^{-10} m}$$

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