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We have:

$$\frac{d^2\psi(x)}{dx^2}+\frac{2m}{\hbar^2}\big(E-V_0\delta(x)\big)\psi(x)=0$$

Supposing $x\neq 0$

$$\frac{d^2\psi(x)}{dx^2}+k^2\psi(x)=0 \quad , \quad k=\sqrt{\frac{2mE}{\hbar^2}}$$

Thus

$$\psi(x)=\begin{cases}Ae^{ikx}+Be^{-ikx} & ; x\leq 0\\ Ce^{ikx} & ; x\geq 0\end{cases}$$

From continuity of $\psi(x)$ we have $A+B=C$, and from discontinuity of its first derivative at $x=0$ and integrating from Schrödinger equation we have:

$$\int_{x=0^-}^{x=0^+}\frac{d}{dx}\left(\frac{d\psi(x)}{dx}\right)dx-\frac{2mE}{\hbar^2}\int_{x=0^-}^{x=0^+}dx-\frac{2mV_0}{\hbar^2}\int_{x=0^-}^{x=0^+}\delta(x)\psi(x)dx=0$$

$$\left.\frac{d\psi(x)}{dx}\right|_{x=0^+}-\left.\frac{d\psi(x)}{dx}\right|_{x=0^-}=\frac{2mV_0}{\hbar^2}\psi(0)$$

$$ikC-ik(A-B)=\frac{2mV_0}{\hbar^2}C$$

$$ik(A+B)-ik(A-B)=\frac{2mV_0}{\hbar^2}(A+B)$$

$$\Rightarrow\frac{B}{A}=\frac{1}{\frac{ik\hbar^2}{mV_0}-1}$$

Thus

$$R=\left|\frac{B}{A}\right|^2=\frac{1}{\left(\frac{k\hbar^2}{mV_0}\right)^2+1}=\frac{m^2{V_0}^2}{k^2\hbar^4+m^2{V_0}^2}$$

$$T=1-R=\frac{k^2\hbar^4}{k^2\hbar^4+m^2{V_0}^2}$$