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triplet spin state is symmetrical and noticing that wave function of a system consisting similar fermions is antisymmetric then the spacial part of wave function must be symmetric. Like this

$$\psi(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(x_1)\psi_{n_2}(x_2) - \psi_{n_1}(x_2)\psi_{n_2}(x_1)] \chi_{triplet}(\vec{S}_1, \vec{S}_2) \quad (*)$$

$\psi_n(x)$  is the nth harmonic wave function. so, the energy for the system is:

$$E_{n_1 n_2} = \sum_{i=1}^2 \left[ \hbar \omega \left( n_i + \frac{1}{2} \right) \right] = \hbar \omega (n_1 + n_2 + 1)$$

Now let's investigate the ground, first excitation, and second excitation states. Before continue according to (\*) it is obvious that  $n_1$  and  $n_2$  must have different values.

If we show wave function and ground state energy by  $\psi^{(0)}$  and  $E^{(0)}$  respectively, knowing that ground state corresponds to the case which one particle is in each  $\psi_0$  and  $\psi_1$  then we can say that:

$$\psi^{(0)}(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_0(x_1)\psi_1(x_2) - \psi_1(x_1)\psi_0(x_2)] \chi_{triplet}(\vec{S}_1, \vec{S}_2)$$

As  $\chi_{triplet}(\vec{S}_1, \vec{S}_2)$  has three independent states, the ground state does have triplet degeneracy and energy equivalent to that can obtain from this relation:

$$E^{(0)} = E_{10} = E_{01} = \hbar \omega (0 + 1 + 1) = 2\hbar \omega$$

Similar to this if we show wave function and the first excitation state energy by  $\psi^{(1)}$  and  $E^{(1)}$  respectively, knowing that the first excitation state corresponds to the case which one particle is in each  $\psi_0$  and  $\psi_2$  then we can say that:

$$\psi^{(1)}(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_0(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_0(x_2)] \chi_{triplet}(\vec{S}_1, \vec{S}_2)$$

$$E^{(1)} = \hbar \omega (0 + 2 + 1) = 3\hbar \omega$$

We have triplet degeneracy in here also.

There is something different in second excitation state. If we show the energy of this state by  $E^{(2)}$  we have

$$E^{(2)} = 4\hbar \omega$$

This happens in two cases:

1. One of two particle is in  $\psi_0$  state and the other in  $\psi_3$  in which we have

$$\psi^{(2)}_1 = \frac{1}{\sqrt{2}} [\psi_0(x_1)\psi_3(x_2) - \psi_3(x_1)\psi_0(x_2)] \chi_{triplet}(\vec{S}_1, \vec{S}_2)$$

$\psi^{(2)}_1$  consists from three independent states.

2. One of two particle is in  $\psi_1$  state and the other in  $\psi_2$  in which we have

$$\psi^{(2)}_2 = \frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)] \chi_{triplet}(\vec{S}_1, \vec{S}_2)$$

$\psi^{(2)}_2$  consists from three independent states.

Then according to explanations above second excitation state with eigen functions  $\psi^{(2)}_1$  and  $\psi^{(2)}_2$  and energy equal to  $E^{(2)}$  does have sextuplet degeneracy.

