5.21

(a) if $j=1$ then J_y s eigen values can be $0, \pm\hbar$

(b) first we obtain the state $|j_y=\hbar\rangle$

$$\Rightarrow |j_y=\hbar\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$J_y |j_y=\hbar\rangle = \hbar |j_y=\hbar\rangle$$

$$\Rightarrow \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \hbar \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$\Rightarrow -ib = a\sqrt{2}$

$ia - ic = b\sqrt{2}$ together with $\langle j_y=\hbar | j_y=\hbar\rangle = 1$

$ib = c\sqrt{2}$

$$\Rightarrow |j_y=\hbar\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix}$$

$$\langle \hat{J}_z \rangle = \langle j_y=\hbar | \hat{J}_z | j_y=\hbar\rangle = \frac{1}{2}\begin{pmatrix} 1 & -i\sqrt{2} & -1 \end{pmatrix}\hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\times \frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix} = 0$$

$$\langle \hat{J}_z^2 \rangle = \frac{1}{2}\begin{pmatrix} 1 & -i\sqrt{2} & -1 \end{pmatrix}\hbar^2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix} = \frac{\hbar^2}{2}$$

(c) like above $\langle J_x \rangle = 0$, $\langle J_x^2 \rangle = \frac{\hbar^2}{2}$

$$\Delta J_x = \sqrt{\langle J_x^2\rangle - \langle J_x\rangle^2} = \frac{\hbar}{\sqrt{2}}$$