5-37

(a)

To finding eigen values of $\hat{\vec{L}}^2$and $\hat{\vec{L}}_z$ we act them on given states

$$\hat{\vec{L}}^2 Y_{21}(\theta,\varphi) = 2(2+1)\hbar^2 Y_{21}(\theta,\varphi) = 6\hbar^2 Y_{21}(\theta,\varphi)$$

$$\hat{\vec{L}}_z Y_{21}(\theta,\varphi) = 1\hbar Y_{21}(\theta,\varphi) = \hbar Y_{21}(\theta,\varphi)$$

(b)

For this state eigen value for $\hat{\vec{L}}^2$ is $12\hbar^2$and for $\hat{\vec{L}}_z$ is $-2\hbar$.

(c)

$$\hat{\vec{L}}^2 \frac{1}{\sqrt{2}}\Big(Y_{33}(\theta,\varphi) + Y_{3,-3}(\theta,\varphi)\Big) = \frac{1}{\sqrt{2}}\hat{\vec{L}}^2 Y_{33}(\theta,\varphi) + \frac{1}{\sqrt{2}}\hat{\vec{L}}^2 Y_{3,-3}(\theta,\varphi)$$
$$= 12\hbar^2\left(\frac{1}{\sqrt{2}}Y_{33}(\theta,\varphi) + \frac{1}{\sqrt{2}}Y_{3,-3}(\theta,\varphi)\right)$$

$$\hat{\vec{L}}_z \frac{1}{\sqrt{2}}\Big(Y_{33}(\theta,\varphi) + Y_{3,-3}(\theta,\varphi)\Big) = 3\hbar\left(\frac{1}{\sqrt{2}}Y_{33}(\theta,\varphi) - \frac{1}{\sqrt{2}}Y_{3,-3}(\theta,\varphi)\right)$$

So, this state is eigen state of $\hat{\vec{L}}^2$with eigen value $12\hbar^2$but not eigen state of $\hat{\vec{L}}_z$

(d)

For this state eigen value for $\hat{\vec{L}}^2$ is $20\hbar^2$and for $\hat{\vec{L}}_z$ is 0.

Mohammad Behtaj & Adel Sepehri

Translated by: @PhysicsDirectory