5-38

Knowing

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad |\psi_x\rangle_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} \quad , \quad |\psi_y\rangle_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

so, we have

$$\hat{S}_x|\psi_x\rangle_\pm = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} = \frac{\hbar}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} \pm 1 \\ 1 \end{pmatrix} = \pm\frac{\hbar}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} = \pm\frac{\hbar}{2}|\psi_x\rangle_\pm$$

$$\hat{S}_x|\psi_x\rangle_\pm = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix} = \frac{\hbar}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} \pm 1 \\ i \end{pmatrix} = \pm\frac{\hbar}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix} = \pm\frac{\hbar}{2}|\psi_y\rangle_\pm$$

to finding eigen values of $\hat{\vec{L}}^2$and $\hat{\vec{L}}_z$ we act them on given states

$$\hat{\vec{L}}^2 Y_{21}(\theta,\varphi) = 2(2+1)\hbar^2 Y_{21}(\theta,\varphi) = 6\hbar^2 Y_{21}(\theta,\varphi)$$

$$\hat{\vec{L}}_z Y_{21}(\theta,\varphi) = 1\hbar Y_{21}(\theta,\varphi) = \hbar Y_{21}(\theta,\varphi)$$

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Translated by: @PhysicsDirectory