PYL QM: Operators and Postulates Continued

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Not only is the Universe stranger than we think, it is stranger than we can think.

Werner Heisenberg

1 Introduction

The following notes are based upon the lectures uploaded on impartus. The content is much more mathematical as compared to the previous ones. So, it requires more rigorous treatment which has been avoided here for sake of minimizing the notes to the bare minimum. Those who are more interested can refer to the references and some notes released separately. Some of the content is in continuation to the last released notes.

2 Postulates Continued

The set of eigenfunctions of an eigenvalue equation of the form

$$\hat{Q}\psi_j = q_j\psi_j$$

is in general an infinite set of linearly independent functions. If ψ is a physically accepted solution then it can be expanded in eigenfunctions of any observable of the system.

3 Definitions

1. A set is complete set of linearly independent eigenfunctions for eigenvalue q if for all eigenfunctions of the same eigenvalue q the set combined with the new eigenfunction forms a linearly dependent set.

2. If two operators(say $\hat{R}$ and $\hat{S}$) are associated with two physical observables and there exists a complete set of ψ_j such that ψ_j is an eigenfunction for both $\hat{R}$ and $\hat{S}$ then the observables are said to be compatible.

3. The commutator of two operators $\hat{R}$ and $\hat{S}$ is denoted and defined as

$$[\hat{R}, \hat{S}] = \hat{R}\hat{S} - \hat{S}\hat{R}$$

. Two operators are said to commute if their commutator is zero

4. The Correspondence Principle

   Quantum mechanics when applied to large systems should give results identical to that obtained from classical mechanics.
   Expectation values of observable $\iff$ Position and momentum in classical mechanics

5. The square of uncertainty of a particular quantity is defined to be the mean of the square of deviations from the mean. In easy terms the uncertainty is the standard deviation.

4 Theorems

1. The complete set of eigenfunctions form a basis for any wavefunction having the same eigenvalue.

2. If two operators are compatible then they commute.

3. If two operators commute then there exists a complete set of functions which are simultaneous eigenfunctions of both the operators.

4. If $\hat{Q}$ is any operator then we have the following relation is true

$$\frac{d}{dt} < Q >= \frac{i}{\hbar} < [\hat{H}, \hat{Q}] > + < \frac{\partial Q}{\partial t} >$$

5 Important things to lookout in lecture slides

1. Proof of the above theorems

2. Derivation of Heisenberg's uncertainty relation.

6 Extra references

- Griffiths Introduction To QM Chapter-3 Formalism
- A follow up document would be provided for the more interested and curious people to see how all this works in the Dirac's notation

Hope all of you are keeping fine in the stressful times of the pandemic. Together we will defeat all the darkness of the world and emerge as a better world