5.1

(a)

$$[\hat{Y},\hat{L}_y]=[\hat{Y},\hat{Z}\hat{P}_x-\hat{X}\hat{P}_z]=[\hat{Y},\hat{Z}\hat{P}_x]-[\hat{Y},\hat{X}\hat{P}_z]$$

$$=[\hat{Y},\hat{Z}]\hat{P}_x+\hat{Z}[\hat{Y},\hat{P}_x]-[\hat{Y},\hat{X}]\hat{P}_z-\hat{X}[\hat{Y},\hat{P}_z]=0$$

$$[\hat{Z},\hat{L}_z]=0$$

$$[\hat{Y},\hat{L}_z]=[\hat{Y},\hat{X}\hat{P}_y-\hat{Y}\hat{P}_x]=[\hat{Y},\hat{X}\hat{P}_y]-[\hat{Y},\hat{Y}\hat{P}_x]$$

$$=[\hat{Y},\hat{X}]\hat{P}_y+\hat{X}[\hat{Y},\hat{P}_y]-[\hat{Y},\hat{Y}]\hat{P}_x-\hat{Y}[\hat{Y},\hat{P}_x]$$

$$=i\hbar\hat{X}$$

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similar to above

$$[\hat{Z},\hat{L}_x]=i\hbar\hat{Y}$$

$$[\hat{Y},\hat{L}_x]=-i\hbar\hat{Z}$$

$$[\hat{Z},\hat{L}_y]=-i\hbar\hat{X}$$

(b)

$$[\hat{X},\hat{L}_x]=0$$

$$[\hat{X},\hat{L}_y]=i\hbar\hat{Z}$$

$$[\hat{X},\hat{L}_z]=-i\hbar\hat{Y}$$

(c) $$[\hat{R}^2,\hat{L}_x]=[\hat{X}^2+\hat{Y}^2+\hat{Z}^2,\hat{L}_x]=[\hat{Y},\hat{L}_x]\hat{Y}+\hat{Y}[\hat{Y},\hat{L}_x]$$

$$+\hat{Z}[\hat{Z},\hat{L}_x]+[\hat{Z},\hat{L}_x]\hat{Z}=0$$

$$[\hat{R}^2,\hat{L}_y]=[\hat{R}^2,\hat{L}_z]=0$$