1. We have the $\phi = 4.73$ eV

$\therefore$ Cut off wavelength,

If K.E. = 0 , i.e. the photo electron is just ejected from the metal.

$$\therefore \quad h\nu = \underbrace{K.E.}_{0} + \phi$$

$$\Rightarrow \quad \frac{hc}{\lambda_c} = \phi \Rightarrow \lambda_c = \frac{hc}{\phi}$$

$$\lambda_c = \frac{1240 \text{ eV-nm}}{4.73} \Rightarrow \lambda = 262 \text{ nm.}$$

2. $\lambda = 180$ nm , $V_0 = -0.80$ Volt

(a) Work funcn of the metal:

K.E. from stopping Potential : 0.80 eV

$$\therefore \quad h\nu = K.E. + \phi$$

$$\Rightarrow \quad \phi = h\nu - K.E.$$

$$= \frac{1240 \text{ eV-nm}}{180} - 0.80 \text{ eV}$$

$$\Rightarrow \quad \phi = 6.08 \text{ eV.}$$

(b) Cut off frequency, $h\nu_0 = 6.08 \times 1.6 \times 10^{-19}$

Solving for ν_0 $\hookrightarrow$ $\nu_0 = 1.46 \times 10^{15}$ Hz.

4. Compton scattering conditions :

(i) $h\nu \sim mc^2$ (So the particles are treated relativistical

(ii) Energy of incoming photons > $(B.E.)_{electron}$

3. Relativistic correction for wavelength:

Solve for a particle of charge 'q'.

$K.E. \neq \frac{p^2}{2m}$ for relativistic case

$\therefore$

$$E = \sqrt{(pc)^2 + m_0^2 c^4} = K.E. + m_0c^2$$

$$(pc)^2 + (m_0c^2)^2 = (qV + m_0c^2)^2$$

$$\Rightarrow (pc)^2 + \cancel{(m_0c^2)^2} = (qV)^2 + \cancel{(m_0c^2)^2} + 2qVm_0c^2$$

$$\Rightarrow p^2 = \left(\frac{qV}{c}\right)^2 + 2qVm_0$$

$$p^2 = 2qVm_0\left(1 + \frac{qV}{2m_0c^2}\right)$$

$$p = \left[2qm_0V\left(1 + \frac{qV}{2m_0c^2}\right)\right]^{1/2}.$$

Now,

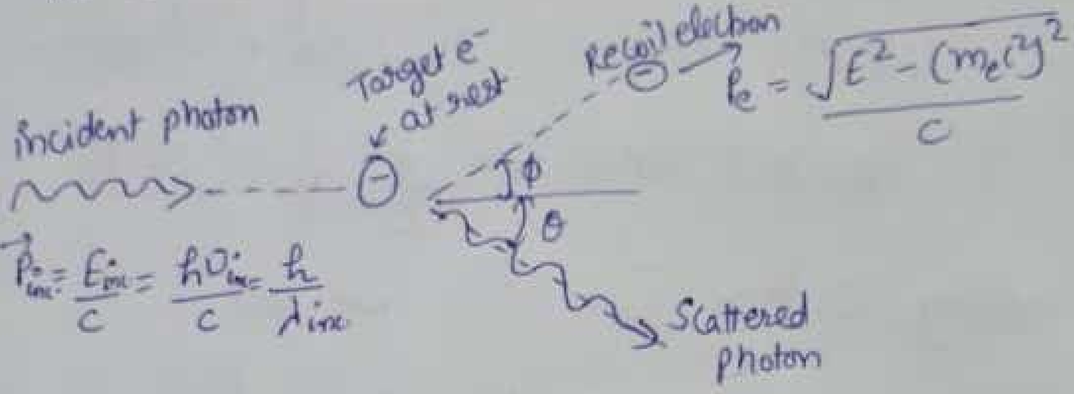
$$\lambda_{rel.} = \frac{h}{p}$$

$$\lambda_{rel.} = \frac{h}{\sqrt{2qm_0V\left(1 + \frac{qV}{2m_0c^2}\right)}}.$$ Ans.

Question:-5

In Compton scattering we consider the electron (scatterer) is at rest, but in reality the electron is not stationary, but has a momentum spread inside the atom.

Lets say p_i is the initial momentum of e's.



$$\vec{p}_{inc} = \frac{E_{inc}}{c} = \frac{h\nu_{inc}}{c} = \frac{h}{\lambda_{inc}}$$

So, momentum conservation:

$$\vec{P}_{initial} = \vec{P}_{final} + \vec{P}_{electron}$$

So $\boxed{\cos}$ component that is in the direction of incident photon

$$\frac{h\nu_0}{c} = \frac{h\nu}{c}\cos\theta + (p_f - p_i)\cos\phi \longrightarrow 1$$

and $\boxed{\sin}$ component is in $\perp$ direction of incident photon

$$0 = \frac{h\nu}{c}\sin\theta - (p_f - p_i)\sin\phi \longrightarrow 2$$

$$\Rightarrow \frac{h\nu}{c}\sin\theta = (p_f - p_i)\sin\phi \longrightarrow 2$$

Energy Conservation:-

$$h\nu_0 + (\underbrace{E_0 - B.E}_{\text{negligible}}) = h\nu + \sqrt{p_f^2 c^2 + m_0^2 c^4} \longrightarrow 3$$

Also, we know B.E of e^- is usually very small $\approx$ usually tens of eV

So, from equation (1) and (2)

$$(p_f - p_i)\cos\phi = \frac{h\nu_0}{c} - \frac{h\nu}{c}\cos\theta$$

$$(p_f - p_i)\sin\phi = \frac{h\nu}{c}\sin\theta$$

Squaring above equations we get

$$(p_f - p_i)^2\cos^2\phi = \frac{h^2}{c^2}\left[\nu_0^2 + \nu^2\cos^2\theta - 2\nu_0\nu\cos\theta\right] \rightarrow (4)$$

$$(p_f - p_i)^2\sin^2\phi = \frac{h^2}{c^2}\left[\nu^2\sin^2\theta\right] \rightarrow (5)$$

adding equation (4) & (5) we get

$$(p_f - p_i)^2 = \frac{h^2}{c^2}\left[\nu_0^2 + \nu^2 - 2\nu_0\nu\cos\theta\right] \rightarrow (6)$$

Now squaring equation (3)

$$\left[h(\nu_0 - \nu) + m_0c^2\right]^2 = p_f^2c^2 + m_0^2c^4$$

$$h^2(\nu_0-\nu)^2 + \cancel{m_0^2c^4} + 2hm_0c^2(\nu_0-\nu) = p_f^2c^2 + \cancel{m_0^2c^4}$$

$$\boxed{\frac{h^2}{c^2}(\nu_0-\nu)^2 + 2hm_0(\nu_0-\nu) = p_f^2} \rightarrow (7)$$

using equation (6) find value of p_f^2 we get

$$p_f^2 = 2p_fp_i - p_i^2 + \frac{h^2}{c^2}\left[\nu_0^2 + \nu^2 - 2\nu_0\nu\cos\theta\right] \rightarrow (8)$$

Equating equation (7) and (8) we get

$$\frac{h^2}{c^2}(\nu_0-\nu)^2 + 2hm_0(\nu_0-\nu) = 2p_fp_i - p_i^2 + \frac{h^2}{c^2}\left[\nu_0^2 + \nu^2 - 2\nu_0\nu\cos\theta\right]$$

$$2hm_0(\nu_0-\nu) = \frac{h^2}{c^2}\left[\cancel{\nu_0^2} + \cancel{\nu^2} - 2\nu_0\nu\cos\theta - \cancel{\nu_0^2} - \cancel{\nu^2} + 2\nu_0\nu\right] + 2p_fp_i - p_i^2$$

$$\cancel{2} h m_0 (\nu_0 - \nu) = \cancel{2}\frac{h^2}{c^2}\nu_0 \nu [1-\cos\theta] + \cancel{2}\left[p_f p_i - \frac{p_i^2}{2}\right]$$

$$\left(\frac{c}{\lambda_0} - \frac{c}{\lambda}\right) h m_0 = \frac{h^2}{c^2}\frac{c}{\lambda_0}\frac{c}{\lambda}[1-\cos\theta] + \left[p_f p_i - \frac{p_i^2}{2}\right]$$

$$\Rightarrow c h m_0 \left[\frac{\lambda - \lambda_0}{\lambda \lambda_0}\right] = \frac{h^2}{\lambda\lambda_0}[1-\cos\theta] + \frac{\lambda\lambda_0}{\lambda\lambda_0}\left[p_f p_i - \frac{p_i^2}{2}\right]$$

$$(\lambda - \lambda_0) = \frac{h}{m_0 c}[1-\cos\theta] + \frac{\lambda\lambda_0}{h m_0 c}\left[p_f p_i - \frac{p_i^2}{2}\right]$$

for x-rays of high energy second term is smaller than e first term, because of variation of p_i, this $p_f p_i$ gives ise to the spread.

Question-6

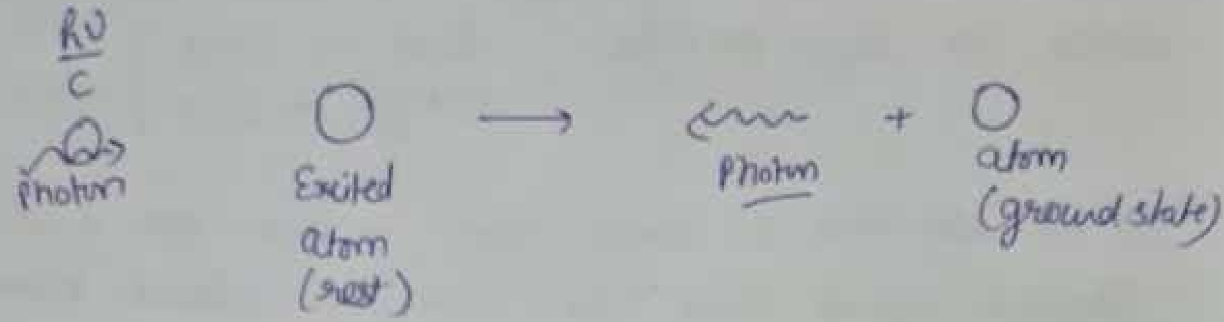
$$\Delta\lambda = \frac{h}{m_e c}[1-\cos\theta] \qquad \theta = 90^\circ, \quad \cos\theta = 0$$

$$\Delta\lambda = \frac{h}{m_e c} \Rightarrow \frac{6.62\times 10^{-34}\ Js^2}{1.2\times 10^5 \times 9.1\times 10^{-31}\times 3\times 10^8\ kg\,m}$$

$$= \boxed{0.02\times 10^{-15}\ m} \qquad \boxed{J = \frac{kg\,m^2}{sec^2}}$$

e^- is vibrating around mean, i.e,

Question-7



Momentum conservation:-

$$\frac{h\nu}{c} + 0 = -\frac{h\nu}{c} + (P_a)_f$$

$$(P_a)_f = \frac{2h\nu}{c}$$

Energy Conservation :

$$E_i = E_{ex} + m_a c^2$$

$$= E_0 + E + m_a c^2$$

$E_0 \rightarrow$ Photon energy ~~ground state~~

E = excited energy

$$E_f = E_0 + \sqrt{p_a^2 c^2 + m_a^2 c^4}$$

But $\quad E_i = E_f$

$$E = \sqrt{p_a^2 c^2 + m_a^2 c^4} - m_a c^2$$

$$= m_a c^2 \left[1 + \frac{p_a^2}{m_a^2 c^4} \right]^{+1/2} - m_a c^2$$

neglecting higher term

$$= m_a c^2 \left[1 + \frac{1}{2}\frac{p_a^2}{m_a^2 c^4} - - \cancel{\frac{1}{8}} - - \right] - m_a c^2$$

$$\Rightarrow m_a c^2 \left[\cancel{1} + \frac{1}{2} \times \frac{1}{m_a^2 c^4} \times \frac{4h^2\nu^2}{c^2} - \cancel{1} \right]$$

$$E \Rightarrow m_a \cancel{c^2} \times \frac{4h^2\nu^2}{2m_a^2 c^4 \times \cancel{c^2}} \Rightarrow \boxed{\frac{2h^2\nu^2}{m_a c^4}}$$

we says that when e^- is moving in A direction so e^- moves towards the photon, with speed v (say) & when moving in B direction so it moves away from the photon.

so from Relativistic Doppler Effect that states that

$$\boxed{\nu_{observed} = \nu_{source}\left[\frac{\sqrt{1-v^2/c^2}}{1-v/c}\right]}$$

$$\nu_{approach} = \nu_0\sqrt{\frac{1+v/c}{1-v/c}}$$

$$\therefore \quad \lambda_{app} = \frac{c}{\nu_{app}} = \frac{c}{\nu_0\sqrt{\frac{1+v/c}{1-v/c}}} = \lambda_0\left[\frac{1}{\left[\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)^{-1}\right]}\right]^{\frac{1}{2}}$$

$$\lambda_{app} = \frac{\lambda_0}{\left[\left(1+\frac{v}{c}\right)\left(1+v/c\right)\right]^{1/2}} \qquad \text{as } [v << c]$$

$$\text{so} \quad \lambda_{app} = \frac{\lambda_0}{(1+v/c)} = \lambda_0\left[1-v/c\right] \left\{\begin{array}{l}\text{as} \\ (1+v/c)^{-1} \\ = (1-v/c)\end{array}\right\}$$

similarly

$$\lambda_{opposite} = \lambda_0\left[1+\frac{v}{c}\right]$$

$$\boxed{\lambda' = \lambda_0 + \lambda_0\left[1-\frac{v}{c}\right]} \quad \boxed{\text{will appear shorter}}$$

$\boxed{\text{Question-8}}$

wavelength of photon $\lambda = 4000 A^\circ$

energy $= h\upsilon = \frac{hc}{\lambda} = \frac{12400 \text{ ev } A^\circ}{4000 \ A^\circ}$

$= \boxed{3.1 \text{ ev}}$

momentum of photon $= \frac{E}{c} = \boxed{3.1 \text{ ev}/c = P(\text{say})}$

Photon + atom $\longrightarrow \overset{\leftarrow}{e^-} + \overset{\rightarrow}{\text{nucleus}}$

Conservation of momentum

$P + 0 \longrightarrow P_e + P_n$

$\boxed{P = P_n - P_e}$ as both moves in opposite direction.

Energy conservation:

$h\upsilon = \phi + (K.E)_e$

$(K.E)_e = 3.1 - 1.6 = 1.5$

momentum of e^- = $K.E = \frac{1}{2}\frac{m^2v^2}{m} \Rightarrow P = \sqrt{2mE}$

$P_e = \sqrt{2 \times \frac{0.5 \text{ Mev}}{c^2} \times 1.5}$

$= \boxed{1.22 \times 10^3 \text{ ev}/c}$

[rest mass energy = 0.5 Mev
$m_0c^2 = 0.5$ Mev

$P_n = P + P_e \Rightarrow 3.1 + 1220 \Rightarrow \boxed{1223.1} \text{ ev}/c$

9. Wavelength of the incident photons,

$$\lambda_d = h/\sqrt{2mE}$$

$$\Rightarrow \quad \lambda_d = 2.45\ \text{Å}$$

$\therefore$ λ_d is of the order of d, inter atomic distance.
Thus, Diffaracn effects can be observed.

10. Given $v = 3\times10^7$ m/s ,

$\therefore$ $\Delta v \sim 3\times10^7$ m/s

From HUP, $\Delta x\, \Delta p \geq \hbar/2 \Rightarrow \Delta x \sim \dfrac{\hbar}{2.m\,\Delta v}$

$$\Delta x = \frac{1.05\times10^{-34}}{2\times9.1\times10^{-31}\times3\times10^{7}} = 0.019\times10^{-10}$$

$$\Rightarrow \quad \Delta x = 0.019\ \text{Å}.$$

11.
(a)

(b) Ans. is NO.

Momentum conservan: $(p_i)_{\gamma} = (p_f)_{e^-} \Rightarrow p_f = \dfrac{h\nu}{c} \rightarrow ①$

Energy conservan: $E_f = \sqrt{p_f^2c^2 + m_0^2c^4}$

$$E_f = \sqrt{h^2\nu^2 + m_0^2c^4} = E_i$$

True only if $h\nu = 0$.

12. (a) The cross section of Compton scattering is proportional to the inverse of mass of the scattering particle

$$\frac{\text{Cross section of photon-electron scattering}}{\text{Cross sec}^n \text{ of photon-nucleus scattering}} = \frac{(m_N)^2}{(m_e)^2} >> 1.$$

(b) $\lambda|_{\text{Compton for } e^-} = 2.4 \text{ Å}.$

(c) Wien's displacement law: $\lambda_m \cdot T = 2.898 \times 10^{-3}$ m.K

$$\lambda_m = \frac{2.898 \times 10^{-3}}{3} \Rightarrow \lambda_m = 9.66 \times 10^{-4} \text{ m.}$$

(d) De-broglie wavelength of 10keV proton:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{(2 \times 1.67 \times 10^{-27} \times 10 \times 10^3 \times 1.6 \times 10^{-19})^{1/2}} \Rightarrow \boxed{\lambda = 2.86 \times 10^{-13} \text{ m}}$$

13. (b) Compton effect.

14. (a) is correct.

W: Power radiated per unit area
U: Energy density.

15. $\Delta x \cdot \Delta p \sim \hbar$

$$\Rightarrow \Delta x . m \Delta v \sim \frac{h}{2\pi} \Rightarrow \Delta v \sim \frac{h}{2\pi} \cdot \frac{1}{m \Delta x}$$

$$\boxed{\Delta v \sim \frac{v}{2\pi}.}$$ Ans.