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(a)

Total angular momentum can have these values

$$|j_1 - j_2| \leq j \leq |j_1 + j_2|$$

Here we have $j_1 = l = 2$ and $j_2 = s = \frac{1}{2}$ thus

$$j = \frac{5}{2}, \frac{3}{2}$$

(b)

Eigen Values of H are in fact the same $|j, m\rangle$s because

$$\vec{J} = \vec{L} + \vec{S}$$

$$[\vec{L}, \vec{S}] = 0$$

$$\Rightarrow J^2 = L^2 + S^2 + \vec{L}.\vec{S} + \vec{S}.\vec{L} = L^2 + S^2 + 2\vec{L}.\vec{S}$$

$$\Rightarrow \vec{L}.\vec{S} = \frac{J^2 - L^2 + S^2}{2}$$

Now we act H on $|j, m\rangle$

$$H|j,m\rangle = \left(a + \frac{b}{2}(J^2 - L^2 + S^2) + cL^2\right)|j,m\rangle$$
$$= \left[a + \frac{b\hbar^2}{2}(j(j+1) - l(l+1) + s(s+1)) + cl(l+1)\hbar^2\right]|j,m\rangle$$

$$\Rightarrow E_j = \left[a + \frac{b\hbar^2}{2}\left(j(j+1) - 2(2+1) + \frac{1}{2}(\frac{1}{2}+1)\right) + 2c(2+1)\hbar^2\right]$$

$$E_{\frac{5}{2}} = \left[a + \frac{b\hbar^2}{2}\left(\frac{5}{2}(\frac{5}{2}+1) - 2(2+1) + \frac{1}{2}(\frac{1}{2}+1)\right) + 2c(2+1)\hbar^2\right] = a + \hbar^2(b + 6c)$$

$$E_{\frac{3}{2}} = \left[a + \frac{b\hbar^2}{2}\left(\frac{3}{2}(\frac{3}{2}+1) - 2(2+1) + \frac{1}{2}(\frac{1}{2}+1)\right) + 2c(2+1)\hbar^2\right] = a + \frac{3\hbar^2}{2}(-b + 4c)$$

Mohammad Behtaj
Adel Sepehri

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