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$$\hat{\vec{R}}.\hat{\vec{L}} = \hat{X}\hat{L}_x + \hat{Y}\hat{L}_y + \hat{Z}\hat{L}_z = \hat{X}\left(\hat{Y}\hat{P}_z - \hat{Z}\hat{P}_y\right) + \hat{Y}\left(\hat{Z}\hat{P}_x - \hat{X}\hat{P}_z\right) + \hat{Z}\left(\hat{X}\hat{P}_y - \hat{Y}\hat{P}_x\right)$$
$$= \left(\hat{X}\hat{Y} - \hat{Y}\hat{X}\right)\hat{P}_z + \left(\hat{Z}\hat{X} - \hat{X}\hat{Z}\right)\hat{P}_y + \left(\hat{Y}\hat{Z} - \hat{Z}\hat{Y}\right)\hat{P}_x = \left[\hat{X},\hat{Y}\right]\hat{P}_z + \left[\hat{Z},\hat{X}\right]\hat{P}_y + \left[\hat{Y},\hat{Z}\right]\hat{P}_x = 0$$

$$\hat{\vec{P}}.\hat{\vec{L}} = \hat{P}_x\hat{L}_x + \hat{P}_y\hat{L}_y + \hat{P}_z\hat{L}_z = \hat{P}_x\left(\hat{Y}\hat{P}_z - \hat{Z}\hat{P}_y\right) + \hat{P}_y\left(\hat{Z}\hat{P}_x - \hat{X}\hat{P}_z\right) + \hat{P}_z\left(\hat{X}\hat{P}_y - \hat{Y}\hat{P}_x\right)$$
$$= \hat{P}_x\hat{Y}\hat{P}_z - \hat{P}_x\hat{Z}\hat{P}_y + \hat{P}_y\hat{Z}\hat{P}_x - \hat{P}_y\hat{X}\hat{P}_z + \hat{P}_z\hat{X}\hat{P}_y - \hat{P}_z\hat{Y}\hat{P}_x = 0$$

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