$$A = \frac{|\psi\rangle\langle\psi|}{\langle\psi|\psi\rangle}$$

$$A^{\dagger} = \frac{|\psi\rangle\langle\psi|}{\langle\psi|\psi\rangle} = A$$

$$A^2 = \frac{|\psi\rangle\langle\psi|}{\langle\psi|\psi\rangle}\frac{|\psi\rangle\langle\psi|}{\langle\psi|\psi\rangle} = \frac{|\psi\rangle\langle\psi|\psi\rangle\langle\psi|}{(\langle\psi|\psi\rangle)^2} = \frac{\langle\psi|\psi\rangle}{(\langle\psi|\psi\rangle)^2}|\psi\rangle\langle\psi| = \frac{|\psi\rangle\langle\psi|}{\langle\psi|\psi\rangle} = A$$

Thus $A$ is Hermitian as well as a projection operator