Solutions PYL101 Midterm

2020-21 - Sem II

Sol. 1

Q. Assume that the electric field of a point charge is given by:

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} (1-\sqrt{r})\,\hat{r}$$

Consider two concentric spherical shells S_1 & S_2 of radii 'r_0' and '$r_0+\delta$' respectively surrounding a point charge 'q' located at their centre. Find out the divergence of the electric field at $r = r_0$.

Sol.

For spherical shell S_1 $\quad d\vec{s} = ds\,\hat{r}$

$$\oint_{S_1} \vec{E}\cdot d\vec{s} = \oint_{S_1} E\, ds = E(r_0)\,4\pi r_0^2$$

$$= \frac{q}{\cancel{4\pi}\varepsilon_0}\frac{1}{r_0^2}(1-\sqrt{r_0})\cdot \cancel{4\pi} r_0^2$$

$$= \frac{q}{\varepsilon_0}(1-\sqrt{r_0})$$

Similarly, for spherical shell S_2 $\quad d\vec{s} = ds\,\hat{r}$

$$\oint_{S_2} \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}\left(1-\sqrt{r_0+\delta}\right)$$

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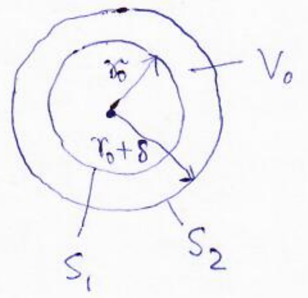
Now consider a volume V_0 bounded by S_1 & S_2

$$V_0 = \frac{4}{3}\pi\left((r_0+\delta)^3 - r_0^3\right)$$



In such a case

$$dS_1 = -ds\,\hat{r}$$

$$dS_2 = ds\,\hat{r}$$

$$\Rightarrow \quad \oint_{S_1+S_2} E \cdot ds = \int_{V_0} \vec{\nabla} \cdot \vec{E}\, dV$$

$$\frac{q}{\varepsilon_0}\left(1-\sqrt{r_0+\delta}\right) - \frac{q}{\varepsilon_0}\left(1-\sqrt{r_0}\right) = \int_{V_0} \vec{\nabla} \cdot \vec{E}\, dV$$

$$\frac{q}{\varepsilon_0}\left[\sqrt{r_0} - \sqrt{r_0+\delta}\right] = \int_{V_0} \vec{\nabla} \cdot \vec{E}\, dV$$

Assuming $\frac{\delta}{r_0} \ll 1$, V_0 is an infinitesimal volume

$$V_0 = \frac{4}{3}\pi r_0^3\left[\left(1+\frac{\delta}{r_0}\right)^3 - 1\right]$$

$$\simeq \frac{4}{3}\pi r_0^3\left[\cancel{1} + \frac{3\delta}{r_0} - \cancel{1}\right]$$

$$\simeq 4\pi r_0^2 \delta$$

2

& $\vec{\nabla} \cdot \vec{E}$ inside V_0 can be approximated by its value at r_0.

$$r_0^{1/2} \; \frac{q}{\varepsilon_0} \left[1 - \left(1 + \frac{\delta}{r_0} \right)^{1/2} \right] \simeq 4\pi r_0^{2} \, \delta \; \vec{\nabla} \cdot \vec{E} \Big|_{r = r_0}$$

$$r_0^{1/2} \; \frac{q}{\varepsilon_0} \left[\cancel{1} - \cancel{1} - \frac{\delta}{2 r_0} \right] \simeq 4\pi r_0^{2} \, \delta \; \vec{\nabla} \cdot \vec{E} \Big|_{r = r_0}$$

$$\Rightarrow \quad \boxed{ \vec{\nabla} \cdot \vec{E} \Big|_{r = r_0} = - \frac{q}{8\pi \varepsilon_0 \, r_0^{5/2}} }$$

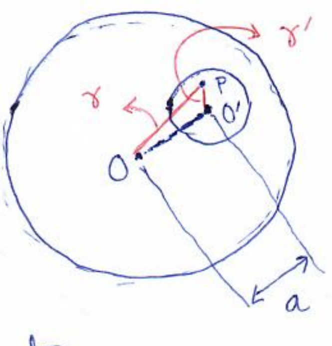
1

$\boxed{\text{Sol. 2}}$ Q. Consider a sphere of radius R_1 with uniform charge density ρ. Now if we remove a sphere of radius R_2 out of it such that the centres of the two spheres are at a distance 'a'. Find the following at the centre of the spherical cavity.

(a) the electric field

(b) the electric potential.

Sol. For an arbitrary point P inside the spherical cavity



$$OP = \vec{r}$$

$$O'P = \vec{r}\,'$$

$$|OO'| = a$$

where O & O' are the centres of sphere (R_1) & cavity (R_2)

(a) First imagine that there is no cavity, so field at point P can be found using Gauss's law

$$E_1 \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\vec{E}_1 = \frac{\rho}{3\varepsilon_0}\vec{r}$$

Similarly, consider a sphere of radius R_2 with uniform charge density ρ and centre at O', the electric field due to such a sphere, again using Gauss's law, would be:

$$E_2\, 4\pi r'^2 = \frac{q_{enc}}{\varepsilon_0} = \frac{\frac{4}{3}\pi r'^3 \rho}{\varepsilon_0}$$

$$\vec{E}_2 = \frac{\rho}{3\varepsilon_0}\vec{r}'$$

Now using superposition principle, net electric field at point P :

$$\vec{E} = \vec{E}_1 - \vec{E}_2$$

$$= \frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{r}')$$

3

$$\boxed{\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{a}}$$

So the electric field at the centre of the cavity is $\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{a}$ which is uniform inside the cavity.

(b)

Now electric field outside the sphere of radius R with uniform charge density ρ, using Gauss's law:

$$E_r \cdot 4\pi r^2 = \frac{\rho \cdot \frac{4}{3}\pi R^3}{\varepsilon_0}$$

$$\vec{E}_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r} = \frac{\rho R^3}{3\varepsilon_0 r^3}\vec{r}$$

We just derived

$$\vec{E}_{in} = \frac{\rho}{3\varepsilon_0}\vec{r}$$

So potential at an arbitrary point inside the sphere:

$$\varphi = -\int_\infty^r \vec{E}\cdot d\vec{r}$$

$$= -\int_\infty^R \vec{E}_{out}\cdot d\vec{r} - \int_R^r \vec{E}_{in}\cdot d\vec{r}$$

$$= -\frac{\rho R^3}{3\varepsilon_0}\int_\infty^R \frac{1}{r^2}\,dr - \frac{\rho}{3\varepsilon_0}\int_R^r r\,dr$$

$$= +\frac{\rho R^3}{3\varepsilon_0}\left\{\frac{1}{r}\right\}_\infty^R - \frac{\rho}{3\varepsilon_0}\left[\frac{r^2}{2}\right]_R^r$$

$$= \frac{\rho R^3}{3\varepsilon_0}\cdot\frac{1}{R} - \frac{\rho}{3\varepsilon_0}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$= \frac{\rho}{3\varepsilon_0}\left[R^2 - \frac{r^2}{2} + \frac{R^2}{2}\right]$$

$$\boxed{\varphi = \frac{\rho}{6\varepsilon_0}\left[3R^2 - r^2\right]}$$

Now potential at O' due to a sphere of radius R_1 with uniform charge density:

$$\varphi_1 = \frac{\rho}{6\varepsilon_0}\left[3R_1^2 - a^2\right]$$

& potential at O' due to a sphere of radius R_2 with uniform charge density:

3

$$\varphi_2 = \frac{\rho}{6\varepsilon_0}\left[3R_2^2 - 0\right]$$

So net potential at O', using superposition principle:

$$\varphi_{\text{at } O'} = \varphi_1 - \varphi_2$$

$$= \frac{\rho}{6\varepsilon_0}\left[3R_1^2 - a^2 - 3R_2^2\right]$$

$$\boxed{\varphi_{\text{at } O'} = \frac{\rho}{6\varepsilon_0}\left[3(R_1^2 - R_2^2) - a^2\right]}$$

$\boxed{\text{Sol}^n\ 3}$ Q. Consider a sphere of radius R made up of linear dielectric material with dielectric constant ε_r. Now if a free charge density $\rho(r) = Ar^2$ is embedded in this sphere, find:

(a) the electric field inside and outside the sphere

(b) the potential at the centre of the sphere.

Sol.

(a) Gauss's law for displacement $\vec{D}$

$$\oint \vec{D} \cdot d\vec{a} = Q_{f_{enc}} \quad \text{(total free charge enclosed)}$$

For $r < R$

$$D_r \, 4\pi r^2 = \int_0^r \rho(s) \, d^3s \qquad d^3s = 4\pi s^2 \, ds \text{ as } \varphi \,\&\, \theta \text{ symmetry is there.}$$

$$= 4\pi \int_0^r A s^2 \cdot s^2 ds$$

$$= 4\pi A \int_0^r s^4 \, ds$$

$$= 4\pi A \left[\frac{s^5}{5} \right]_0^r$$

$$D_r \, \cancel{4\pi} r^2 = \frac{\cancel{4\pi} A r^5}{5}$$

$$\varepsilon E_r = D_r = \frac{A}{5} r^3$$

$$E_r = \frac{A r^3}{5 \varepsilon}$$

OR $\boxed{\vec{E}_{in} = \frac{A r^3}{5 \varepsilon} \hat{r}}$

1.5

for $r > R$

Again from Gauss's law:

$$D_r \, 4\pi r^2 = 4\pi A \int_0^R s^4 \, ds$$

$$D_r \, \cancel{4\pi} r^2 = \frac{\cancel{4}\pi A R^5}{5}$$

$$\varepsilon_0 E_r = D_r = \frac{A R^5}{5 r^2}$$

$$E_r = \frac{A R^5}{5 r^2 \varepsilon_0}$$

1.5

$$\boxed{\vec{E}_{out} = \frac{A R^5}{5 \varepsilon_0 r^2} \hat{r}}$$

(b) Now potential at the centre of the sphere

$$V = - \int_\infty^0 \vec{E} \cdot d\vec{r}$$

$$= - \int_\infty^R \vec{E}_{out} \cdot d\vec{r} - \int_R^0 \vec{E}_{in} \cdot d\vec{r}$$

$$= - \int_\infty^R \frac{A R^5}{5 \varepsilon_0 r^2} \, dr - \int_R^0 \frac{A r^3}{5 \varepsilon} \, dr$$

$$= + \frac{A R^5}{5 \varepsilon_0} \int_\infty^R \left(-\frac{1}{r^2}\right) dr - \frac{A}{5\varepsilon} \int_R^0 r^3 \, dr$$

$$= \frac{A R^5}{5 \varepsilon_0} \left[\frac{1}{r}\right]_\infty^R - \frac{A}{5\varepsilon} \left[\frac{r^4}{4}\right]_R^0$$

$$= \frac{AR^5}{5\varepsilon_0}\left(\frac{1}{R} - 0\right) - \frac{A}{20\varepsilon}\left(r^4 - R^4\right)$$

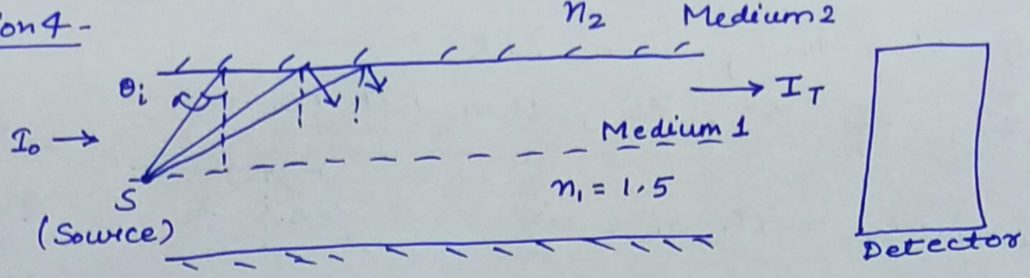
$$= \frac{AR^4}{5\varepsilon_0} + \frac{AR^4}{20\varepsilon}$$

$$\boxed{V(r=0) = \frac{AR^4}{5\varepsilon_0}\left(1 + \frac{1}{4\varepsilon_r}\right)}$$

3

IF ONLY FORMULA CORRECT 1 MARK IS GIVEN

Solution 4 -



If medium 1 and medium 2 are dielectric materials. Then, wave guiding will occur in medium 1 provided

$$n_1 > n_2 \qquad \boxed{\text{Given } n_1 = 1.5}$$

0.5

For medium 2 = air and a dielectric with $n_2 = 1.45$ both will act as wave guide, since, for guiding, total internal reflection (TIR) is necessary.

$\theta_i \rightarrow$ angle of incidence

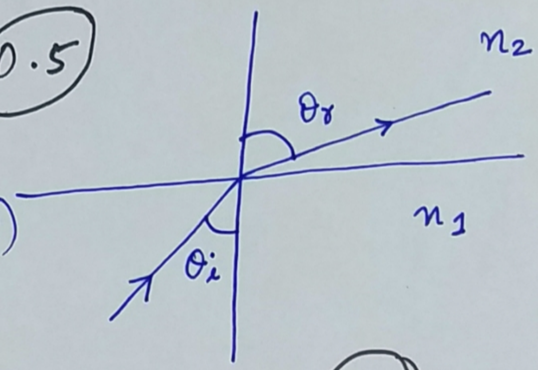
$\theta_r \rightarrow$ angle of refraction

Therefore, $\theta_i > \theta_c$ (condition for TIR)

and for $\theta_i = \theta_c$, the refracted wave goes parallel to the interface.

According to Snell's law

0.5



$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_2}{n_1}$$

$$\Rightarrow \frac{\sin \theta_c}{1} = \frac{n_2}{n_1} \quad (\theta_r = 90^\circ)$$

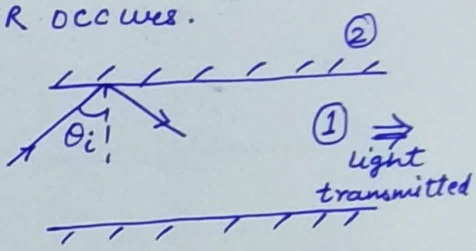
$$\Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

2

For air, $\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = \boxed{41.8^\circ}$

For medium 2 ($n_2 = 1.45$) $\theta_c = \sin^{-1}\left(\frac{1.45}{1.5}\right) = \boxed{75^\circ}$

For air, if $\theta_i > 41.8°$, then TIR occurs.
For $n_2 = 1.45$, if $\theta_i > 75°$, then TIR occurs.



Clearly, waveguide with n_2 = air will guide more light than that with $n_2 = 1.45$

Now, if the surrounding medium is metal coating. Then,
⇒ for every small angle light gets reflected from boundary. 1
⇒ highest light guided through.

Finally, the order of decreasing power detected on the power meter will be

$\boxed{\text{Metal coating} > \text{air} > n_2 = 1.45 \text{ medium}}$ 1

Q5. Express mathematically in Cartesian coordinates, a monochromatic EM wave that is propagating along a direction (1, −2, 1) in air and its polarization in the xy-plane. At any arbitrary time t and phase angle 0, what is the wave phase front of the above wave? Express mathematically and draw a sketch in the same xyz-axis system. [5 marks]

Solution.

EM wave in air ⇒ transverse wave with velocity c.

Let the angular frequency of wave be ω.

Now, direction of $\vec{k} = (1, -2, 1) \equiv \hat{x} - 2\hat{y} + \hat{z}$

$$\Rightarrow \hat{k} = \frac{1}{\sqrt{6}}(\hat{x} - 2\hat{y} + \hat{z})$$

$$\therefore \vec{k} = \frac{\omega}{c}\hat{k} = \frac{\omega}{c\sqrt{6}}(\hat{x} - 2\hat{y} + \hat{z}) \quad 1$$

It is given that polarization is in xy-plane.

Let the polarization vector $\hat{n} = \alpha\hat{x} + \beta\hat{y}$.

Also, since the wave is transverse, $\vec{k} \cdot \hat{n} = 0$

$$\Rightarrow \frac{1}{\sqrt{6}}(\alpha - 2\beta) = 0 \quad \Rightarrow \quad \alpha = 2\beta$$

For $\hat{n}$ to be unit vector, $|\hat{n}| = 1 \Rightarrow \alpha^2 + \beta^2 = 1$

$$\Rightarrow \alpha = \frac{2}{\sqrt{5}} \text{ and } \beta = \frac{1}{\sqrt{5}}$$

$$\therefore \hat{n} = \frac{2}{\sqrt{5}}\hat{x} + \frac{1}{\sqrt{5}}\hat{y} \quad 1$$

Let the maximum amplitude of $\vec{E}$ be E_0.

$$\vec{E} = E_0 \hat{n} \exp\{i(\vec{k}\cdot\vec{r} - \omega t + \phi_0)\}$$

where ϕ_0 is initial phase.

For $\phi_0 = 0$, substituting $\hat{n}$, $\vec{k}$ and $\vec{r} \equiv x\hat{x} + y\hat{y} + z\hat{z}$:

$$\vec{E} = E_0\left(\frac{2}{\sqrt{5}}\hat{x} + \frac{1}{\sqrt{5}}\hat{y}\right)\exp\left[i\left\{\frac{\omega}{c\sqrt{6}}(x - 2y + z) - \omega t\right\}\right]$$

or 1

$$\vec{E} = E_0\left(\frac{2}{\sqrt{5}}\hat{x} + \frac{1}{\sqrt{5}}\hat{y}\right)\cos\left\{\frac{\omega}{c\sqrt{6}}(x - 2y + z) - \omega t\right\}$$

Now,

$$\vec{B} = \frac{E_0}{c}(\hat{k} \times \hat{n}) \exp\{i(\vec{k}\cdot\vec{r} - \omega t)\}$$

$$\hat{k} \times \hat{n} = \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{5}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & -2 & 1 \\ 2 & 1 & 0 \end{vmatrix} = \frac{1}{\sqrt{30}}(-\hat{x} + 2\hat{y} + 5\hat{z})$$

$$\therefore \vec{B} = \frac{E_0}{c}\left(\frac{-\hat{x} + 2\hat{y} + 5\hat{z}}{\sqrt{30}}\right) \exp\left[i\left\{(+x - 2y + z)\frac{\omega}{c\sqrt{6}} - \omega t\right\}\right]$$ 1

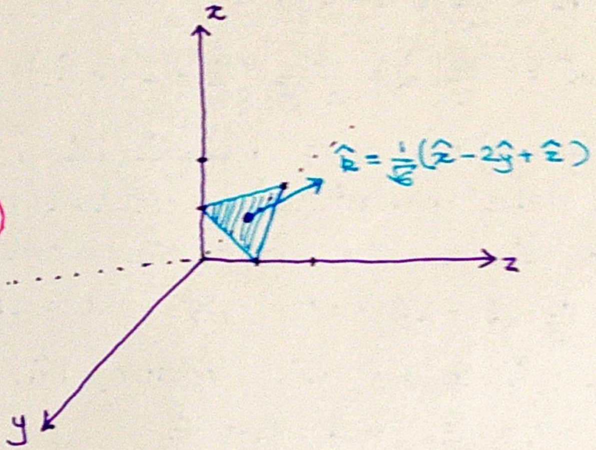
The wavefront / phase front for zero phase is given by the plane:

$$\vec{k}\cdot\vec{r} = 0$$

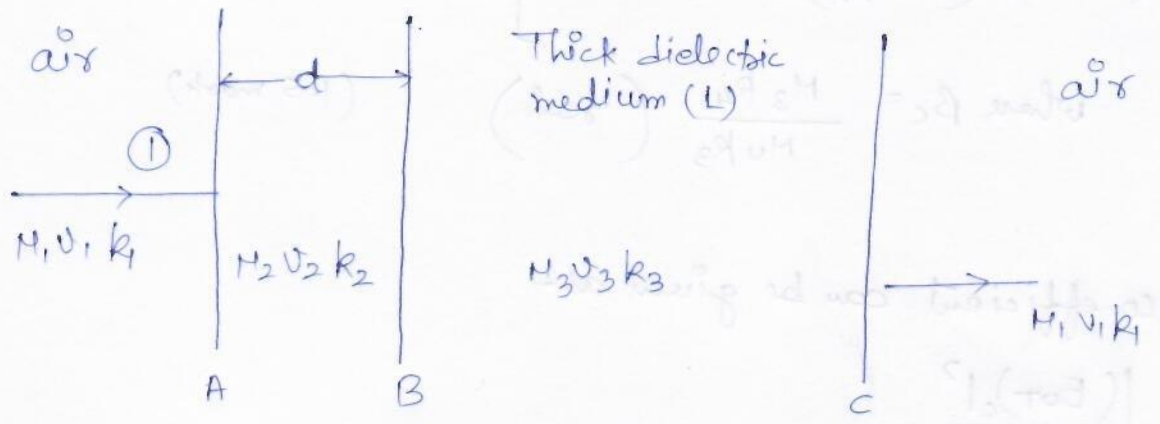
$$\Rightarrow \frac{1}{\sqrt{6}}(x - 2y + z)\frac{\omega}{c} = 0$$

$$\Rightarrow x - 2y + z = 0$$ 1

The wavefront is shown in the figure.



Question 6 ⇒ Solution ⇒



⇒ Normal Incidence, $\vec{E_I} = E_0 \cos(kz - \omega t)\hat{x}$; $E_{0I} = E_0$

⇒ There are three interfaces Ⓐ, Ⓑ and Ⓒ.

⇒ At interface Ⓐ -

$$(E_{0T})_A = \left(\frac{2}{1+\beta_A}\right) E_{0I} \quad ; \quad \beta_A = \frac{\mu_1 \tilde{k}_2}{\mu_2 k_1} = \tilde{\beta}_A$$

⇒ Since it is conducting interface, so β is complex $= \tilde{\beta}$

for conducting film - $k_2 = \tilde{k}_2 = k_2 + i K_2 \rightarrow$ imaginary part which is responsible for decay.

$$\left[\therefore (E_{0T})_A = \left(\frac{2}{1+\tilde{\beta}_A}\right) E_0 \right]$$

k_2 ↑ real part propogation constant

(1 mark)

⇒ At interface Ⓑ -

$$(E_{0I})_B = e^{-K_2 d} (E_{0T})_A$$ (by ignoring the additional phase factor)

$$\Rightarrow \left[(E_{0T})_B = \left(\frac{2}{1+\tilde{\beta}_B}\right) e^{-K_2 d} \left(\frac{2}{1+\tilde{\beta}_A}\right) E_0 \right], \text{ where } \tilde{\beta}_B = \frac{\mu_2 k_3}{\mu_3 \tilde{k}_2}$$

(2 mark)

⇒ At interface Ⓒ -

$$(E_{0I})_C = \frac{k_1}{\quad} (E_{0T})_B$$

$$\therefore (E_{0T})_C = \left(\frac{2}{1+\beta}\right)(E_{0T})_B$$

$$\left[(E_{0T})_C = \left(\frac{2}{1+\beta_C}\right)\left(\frac{2}{1+\tilde{\beta}_B}\right)\left(\frac{2}{1+\tilde{\beta}_A}\right) e^{-\mathcal{K}_2 d} E_0 \right]$$

where $\beta_C = \frac{\mu_3 k_4}{\mu_4 k_3}$ (real) (1.5 mark)

Now transmission co-efficient can be given as-

$$T = \frac{I_T}{I_0} = \frac{|(E_{0T})_C|^2}{|(E_{0I})|^2}$$ (1 mark)

$$\Rightarrow T = \left| \left(\frac{2}{1+\beta_C}\right)\left(\frac{2}{1+\tilde{\beta}_B}\right)\left(\frac{2}{1+\tilde{\beta}_A}\right) \cdot e^{-\mathcal{K}_2 d} \right|^2$$

where
$$\left.\begin{aligned} \tilde{\beta}_A &= \frac{\tilde{k}_2}{k_1} = \frac{k_2 + i\mathcal{K}_2}{k_1} \\ \tilde{\beta}_B &= \frac{k_3}{\tilde{k}_2} = \frac{k_3}{k_2 + i\mathcal{K}_2} \end{aligned}\right\} \text{complex}$$

$$\left.\beta_C = \frac{k_4}{k_3}\right\} \text{real}$$

After solving these-

$$\boxed{T = \frac{64\, k_1^2 (k_2^2 + \mathcal{K}_2^2)\, k_3^2}{(k_3+k_4)^2 \cdot \left((k_2+k_3)^2 + \mathcal{K}_2^2\right) \cdot \left((k_1+k_2)^2 + \mathcal{K}_2^2\right)}}$$ (0.5 mark)

Solution Q.7.

Given $\vec{M} = \frac{\alpha^2}{2r}\hat{\phi}$

$\Rightarrow$ Bound volume current $\vec{J_b} = \vec{\nabla} \times \vec{M}$

$$= \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ 0 & r\cdot\frac{\alpha^2}{2r} & 0 \end{vmatrix}$$

$$= 0 \quad \text{--- } 1$$

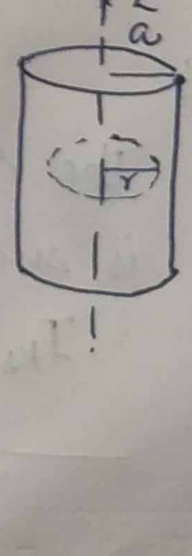
Bound surface current $\vec{K_b} = \vec{M} \times \hat{n}$

$$= \frac{\alpha^2}{2r}\hat{\phi}\times\hat{r}$$

$$= -\frac{\alpha^2}{2r}\hat{z} \quad \text{--- } 1$$

First way to calculate $\vec{B}$ (taking the given value of $\vec{M}$, which leads to non conserved total bound current as it is).

(a) Considering the Amper's Law on closed loop of radius r.



$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (I' + \int \vec{J_b}\cdot d\vec{a}) \text{ --- } 1$$

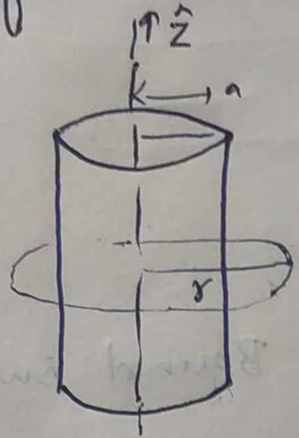
I' — Current enclosed in closed loop of radius r

$$\Rightarrow I' = \int \frac{I}{\pi a^2}\cdot 2\pi r\, dr$$

Therefore, $|\vec{B}| 2\pi r = \mu_0 \left[\frac{I}{\pi a^2}\cdot \pi r^2\right] \quad (\because J_b = 0)$

$$\Rightarrow \boxed{\vec{B}_{inside} = \frac{\mu_0}{2\pi} \frac{Ir}{a^2} \hat{\phi}}$$ ①

(b) Considering amperian loop of radius 'r' as shown in fig.



$$\oint \vec{B}_{outside} \cdot \vec{dl} = \mu_0 I_{enclosed}$$

$$= \mu_0 \left[I + \int_0^a \vec{J_b} \cdot d\vec{a} + \int_0^a K_b \cdot dl \right]$$

$$= \mu_0 \left[I + 0 + K_b \cdot 2\pi a \right]$$

$$= \mu_0 I + \mu_0 \left(\frac{-\alpha^2}{2a} \right) \cdot 2\pi a$$

$$\Rightarrow B_{out} \cdot 2\pi r = \mu_0 \left[I - \alpha^2 \pi \right]$$

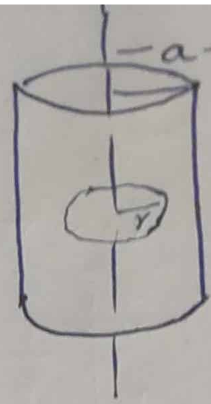
$$\boxed{\vec{B}_{out} = \frac{\mu_0}{2\pi r} \left[I - \alpha^2 \pi \right] \hat{\phi}}$$ ③

The second way to calculate $\vec{B}$ (assuming $\vec{M}$ is such that total bound current is zero ie. $I_{J_b}|_{r>a} + I_{K_b}|_{r>a} = 0$ ie. $\int \vec{J_b} \cdot d\vec{a} + \int K_b \cdot d\vec{l}$ for $r>a$ $= 0$

(a) field inside

$$\oint \vec{H}\cdot d\vec{l} = I_{f\,enclosed}$$

$$H\cdot 2\pi r = \frac{I}{\pi a^2}\cdot \pi r^2$$



$$\Rightarrow \boxed{\vec{H} = \frac{I r}{2\pi a^2}\,\hat{\phi}}$$

$$\because \vec{H} = \frac{B}{\mu_0} - \vec{M} \Rightarrow \vec{B} = \mu_0(\vec{H}+\vec{M})$$

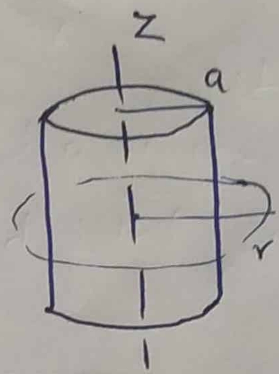
2

$$\boxed{\vec{B} = \mu_0\left[\frac{I r}{2\pi a^2} + \frac{\alpha^2}{2r}\right]\hat{\phi}}$$

(b) field outside $(\vec{M}=0)$

$$\oint_c \vec{H}\cdot d\vec{l} = I_{f\,enclosed} = I$$

$$H = I/2\pi r\ \hat{\phi}$$



$$\Rightarrow \vec{B} = \mu_0(\vec{H}+\vec{m})$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r}\,\hat{\phi}}$$

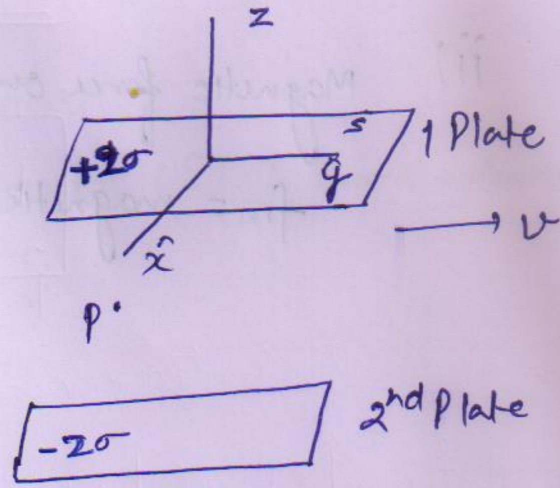
2

Note: i). You have to calculate both $\vec{J}_b$ & $\vec{K}_b$ (irrespect of whether their sum is zero or not) with correct direction and magnitude.

ii). You have to calculate field $\vec{B}$ using either of two ways and mention the direction correctly.

iii). If you mix both type of solutions together, then that will not be considered as correct and you will not get marks for that.

ution 8:

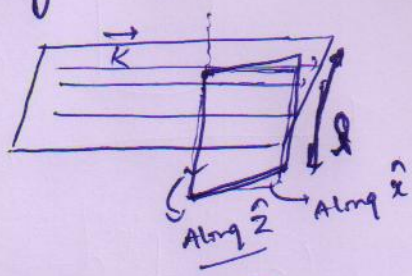


The Surface bound current due to upper & lower plate respectively

$\vec{K_1} = 2\sigma v\, \hat{y}$

$\vec{K_2} = 2\sigma v\, (-\hat{y})$

Magnetic field due to plate ① at P, applying Ampere's Law



$\oint \vec{B_1}\cdot d\vec{l} = \mu_0 I_{enclosed}$

$\Rightarrow 2B_1 l = \mu_0 K l$

$\Rightarrow \vec{B_1} = \frac{\mu_0 K}{2} (\hat{x})$ (into the page) 1

Similiarly due to plate ② at point P

$\oint \vec{B_2}\cdot d\vec{l} = \mu_0 I_{enclosed}$

$\Rightarrow B_2 = \frac{\mu_0 K}{2} (\hat{x})$ (into the page) +1

Therefore, B_{total} at P

$\Rightarrow \vec{B}_{total} = \mu_0 K (\hat{x})$

$\Rightarrow \vec{B}_{total} = 2\mu_0 \sigma V (\hat{x})$ OR (into the page)

+1

ii) Magnetic force on upper plate $= \oint (\vec{K_1} \times \vec{B_2}) \cdot d\vec{a}$

f_m = magnetic force per unit area $= |\vec{K_1} \times \vec{B_2}|$

$$= 2\sigma v \times \frac{\mu_0}{2} \cdot 2\sigma v$$

$$= 2\sigma^2 v^2 \cdot \mu_0$$

(upwards) $\frac{(\text{or } \hat{z})}{-i)}$

1

f_e = electric force per unit area on upper plate due to $\vec{E_2}$

$$= 2\sigma \vec{E_2} \qquad \left(\begin{array}{l} F = qE \\ = \int \vec{\sigma} \cdot d\vec{a} \cdot \vec{E} \\ \Rightarrow f_e = \sigma E \end{array} \right)$$

Here $\sigma \to 2\sigma$

$$= 2\sigma \cdot \frac{2\sigma}{2\epsilon_0}$$

$$= \frac{2\sigma^2}{\epsilon_0} \qquad (-\hat{z}) \quad (\text{down}) - ii)$$

+1

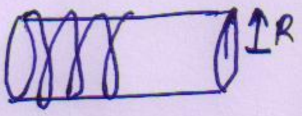
for balancing condition on upper plate due to these forces

$\Rightarrow \quad f_e = f_m$

$\Rightarrow \quad \frac{2\sigma^2}{\epsilon_0} = 2\sigma^2 v^2 \cdot \mu_0 \Rightarrow \boxed{v = 1/\sqrt{\mu_0 \epsilon_0} = c}$

+1

-tion 9:



given that

$I = 4 I_0 \sin \omega t$

$\Rightarrow \vec{B}_{inside} = \mu_0 N I \ (\hat{z})$ | Due to Solenoid

$\vec{B}_{out} = 0 \ (\hat{z})$

1

According to Faraday's law

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{a}$$

Also, $e = \oint \vec{E}\cdot d\vec{l}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{a} \quad -(1)$$

(a) Using eq^n (1) to calculate E_{inside}

$$E_{inside}\cdot 2\pi r = -\mu_0 N \frac{dI}{dt}\cdot \pi r^2$$

$$\Rightarrow \vec{E}_{inside} = -2\mu_0 N \omega r I_0 \cos \omega t \ (\hat{\phi})$$

+2

(b) Similiarly using equation (1) to calculate $E_{outside}$

$$\Rightarrow E_{out}\cdot 2\pi r = -\mu_0 N \frac{dI}{dt}\cdot \pi R^2$$

[$\because B_{out} = 0$ the change in flux only due to $\vec{B}_{inside}$ at the solenoid surface $r = R$)

$$\Rightarrow E_{out} = \frac{-2\mu_0 N R^2 \omega I_0 \cos(\omega t)}{r} (\hat{\phi})$$

2