3-3

Eigen values and eigen functions of energy for a particle in the box of $0 < x < a$ are:

$$\phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} \ ; n = 1,2, \dots$$

(a)

$$\psi(x.0) = \sqrt{\frac{2}{7}}\phi_1(x) + \sqrt{\frac{1}{7}}\phi_2(x) + \sqrt{\frac{4}{7}}\phi_3(x)$$

According to $\psi(x.0)$ form, it is obvious that measuring Energy yields one of E_1, E_2 and E_3

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} \simeq \frac{5n^2\hbar^2}{ma^2}$$

$$E_1 \simeq \frac{5\hbar^2}{ma^2} \simeq 3.23\ MeV$$

$$E_2 = 4E_1 \simeq 12.92\ MeV$$

$$E_3 = 9E_1 \simeq 29.07\ MeV$$

(b)

$$E = \langle \hat{H} \rangle = P_1E_1 + P_2E_2 + P_3E_3 = \left(\sqrt{\frac{2}{7}}\right)^2 \times 3.23 + \left(\sqrt{\frac{1}{7}}\right)^2 \times 12.92 + \left(\sqrt{\frac{4}{7}}\right)^2 \times 29.07 = 19.38 MeV$$

(c)

Uncertainty of position is in the order of a.

$$\Delta x \simeq a$$

Then according to uncertainty principle:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2\Delta x}$$

Then if momentum of the particle is in the order of Δp then we have

$$p \simeq \frac{\hbar}{2\Delta x} \simeq \frac{\hbar}{2a} = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v^2 = \frac{c^2}{1 + \frac{4a^2 {m_0}^2 c^2}{\hbar^2}} \simeq 2 \times 10^{-4} c^2$$

$$v = 10^{-2}\sqrt{2}c = 0.014c$$

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