6.10

(a) using $\int |\psi_{2,1,-1}|^2 dV = 1$

$$\to 1 = |N|^2 \int_0^\infty r^2 e^{-r/a_0} r^2 dr \int |Y_{1,-1}(\theta, \varphi)|^2 d\Omega$$

$$= |N|^2 \int_0^\infty r^4 e^{-r/a_0} dr$$

$$= |N|^2 \left(-a_0 e^{-r/a_0} \left[24a_0^4 + 24a_0^3 r + 12a_0^2 r^2 + 4a_0 r^3 + r^4 \right] \right) \Big|_0^\infty$$

$$= 24a_0^5 |N|^2 \rightarrow N = \frac{1}{2a_0^2 \sqrt{6a_0}}$$

(b)

$$\rho(r = a_0, \theta = 45^\circ, \varphi = 60^\circ)$$

$$= |\psi_{2,1,-1}|^2 = \left[N^2 r^2 e^{-\frac{r}{a_0}} |Y_{1,-1}|^2 \right] \Bigg|_{\substack{r = a_0 \\ \theta = 45^\circ \\ \varphi = 60^\circ}}$$

$$= \frac{1}{24a_0^5} a_0^2 e^{-1} \left(\sqrt{\frac{3}{8\pi}} \sin 45^\circ \right)^2$$

$$= \frac{1}{128 \pi e a_0^3}$$

(c) According to relation (6.178) of the book

$$P_{n,\ell}(r) = |R_{n\ell}(r)|^2 r^2 = N^2 r^4 e^{-\frac{r}{a_0}}$$

$$\rightarrow P_{2,1}(2a_0) = \frac{(2a_0)^4}{24 a_0^5} e^{-\frac{2a_0}{a_0}} = \frac{2}{3 a_0 e^2}$$

(d) According to relations (6.50)-(6.51)

$$\hat{L}^2 \psi_{2,1,-1} = 1(1+1)\hbar^2 \psi_{2,1,-1} = 2\hbar^2 \psi_{2,1,-1}$$

$$\hat{L}_z \psi_{2,1,-1} = -\hbar \psi_{2,1,-1}$$

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