3-18

(a)

eigen values of $\hat{A}$ are $a_1 = a_2 = \sqrt{2}$ and $a_3 = 0$. Be cause of degeneracy eigen vectors of $\hat{A}$ doesn't form a complete set automatically but by a little concentration we can find it's orthonormal eigen vectors which are:

$$\left|a_1 = \sqrt{2}\right\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \left|a_2 = \sqrt{2}\right\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, |a_3 = 0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

And respectively orthonormal eigen vectors of $\hat{B}$ are:

$$|b_1 = -1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, |b_2 = 1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |b_3 = 1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

If by measuring $\hat{A}$ we get $\sqrt{2}$ system is either in two $|a_1\rangle$ or $|a_2\rangle$ states if $\hat{B}$ acts on each one of them we get 1 as the answer thus in second measuring -1 never happens. So, the probability of (a) is zero.

(b)

If by measuring $\hat{B}$ we get -1 then system is in $|b_1 = -1\rangle$ and acting $\hat{A}$ on it yields zero so in second measuring $\sqrt{2}$ never happens. So, the probability of (b) is zero as well.

(c)

the reason that (a) and (b) results are the same is that $\hat{A}$ and $\hat{B}$ are commutable and order of measuring is unimportant.

Mohammad Behtaj & Adel Sepehri