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Right Tail Hedging: *Managing Risk When Markets Melt Up*

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Equity markets do jump up. As a matter of fact, the historical data show that *draw-ups* in markets are at least as numerous and severe as drawdowns. Even for the S&P 500, the maximum drawdown since 1950 has only been 50%, whereas the draw-ups have been many multiples larger (we define drawdowns and draw-ups up to a time in the conventional way). Since 2015, the S&P has observed a maximum drawdown of approximately 12%, and it is still in a draw-up that has reached 50% and continuing. Despite these facts, losses are usually sharper, and because they are more damaging to investor portfolios, they usually gather more public attention. Thus, large losses become more salient in investors' psyches than large gains, and this fact is reflected in the high premiums of put options on equity markets relative to call options, which is termed the implied volatility *skew* in the index options markets.

When there is a potential for large right tails, can call options provide investment value that exceeds simply buying and holding? This question has perplexed both academics and practitioners for years. Much ink has flowed in academic journals on the naiveté of investors who buy options to protect their downside risk via puts or replace outright purchases of securities with call options. The usual arguments are supported by backward-looking analyses of options markets where the options are passively held over some fixed horizon, usually refer to the S&P 500, and, in most cases, conclude that *on average* it does not make sense to buy any options (calls, in particular). They "prove" that empirically buying and holding options is a negative expected return strategy over a given fixed horizon. Despite all these studies, which are well known to most institutional investors, the options markets continue to grow, and investors happily (and profitably) part with the premium to obtain the value of risk transfer to parties who are willing to assume it for a price. The empirical analyses are usually backed up with the truism that "markets melt down, but don't melt up," which, as discussed, is not entirely consistent with the data. Furthermore, in such studies, it is also argued that because it is possible to replicate a call option using a delta equivalent equity exposure on an almost continuous basis, call options are largely redundant. This article challenges many of these conclusions.

To be clear, this article is not about the debate over whether it makes more sense to buy or sell options as a speculative strategy; it would take volumes to present all points of view, and this controversy lies at the heart of normative (generally utility-based) theories of investing and descriptive (generally behavioral) theories. The gist of this article is that under certain—not necessarily abnormal—economic and market conditions, the purchase of both put and call options can be optimal for investors, and the additional convexity that only options markets can provide results in superior portfolio outcomes. Indeed, today's market environment is one such environment; the spectacular performance of equity markets since the 2016 U.S. presidential election has demonstrated the benefit of having upside convexity in portfolios. We believe that one reason why most studies have ignored this value added from the purchase of call options is that most empirical backtests of option strategies have relied on a sample data set that has not experienced major market melt-ups such as the one being witnessed recently. Just as the vanilla put option on the S&P 500 could be assailed as consistently one of the most expensive securities in the markets following the surge in downside protection, a call option on the same index has possibly performed as one of its least expensive counterparts.

Readers who have read my article in this journal (Bhansali [2008]) and my book on left tail hedging (see, e.g., Bhansali [2015]) may think of the present discussion as an extension of that work to upside or right tail hedging. The left tail risk article was published in this journal a few months before the financial crisis of 2008, and this author believes that market conditions make the current environment one that is ripe for tail hedging of the other kind: to the upside. It is possible that by the time this article goes to press, the right tail might have already passed and morphed into a left tail event. I believe, however, that in a world of tightly coupled markets, instant news feeds, and technologically advanced trading bots, both left and right tails will become a permanent fixture of investment markets. Thus, I hope that this article and the discussion will have a long shelf life that will outlive the most recently observed set of manias, panics, and crashes, because the next one is likely not too far behind. The value from both active left and right tail hedging of portfolio risks is being realized by astute investors, just as the value from actively managing bond portfolios was realized a few decades ago and has now become standard fare in the investment industry.

MOTIVATING NEED FOR UPSIDE TAIL RISK HEDGING

Unhedged upside risk in equity markets is a reality today because economics, politics, bond yields, interest rates, and equity markets are all in new regimes rarely experienced since real-time options data have been collected. Because of the significant amount of excess liquidity in the financial markets, the prevalence of indexed and passive strategies, and coordinated global macroeconomic growth, asset prices have rallied significantly from their financial crisis lows. For instance, the S&P 500 has increased over 400% during this period. Along with the performance of equity markets, the liquidity drove bond market yields, asset return volatilities, and credit spreads to almost all-time lows.

When an investor thinks of buying an option, he or she is making a conscious trade-off between limited loss and the possibility of time decay on the one hand, and unlimited gain as well as benefit from rising perceptions of risk on the other hand. In other words, purchase of an option instead of the underlying is an implicit bet on volatility mispricing, which tends to grow when there are jumps, uncertainty, and the possibility of new regimes. To set the stage, an example from our experience from last year will be helpful.

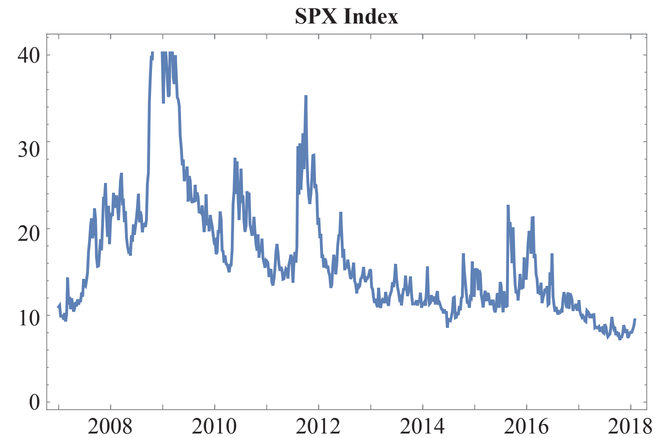
On September 1, 2017, with the S&P 500 trading at 2,471, the price of a 5% out-of-the money call, with a strike at 2,594 and with one year left to expiration, was 2.67%. This option had a theoretical Black–Scholes delta of 0.33, and the implied volatility for that strike at that time was 12.14%. Compare this to an outright exposure to the underlying at the same time. To equalize the linear exposures, approximately three times the options would need to be bought. On January 2, 2018, the price of the same option, after accounting for time decay, was 6.45%, with a reference S&P 500 Index value of 2,691 and an implied volatility of 14%. Even as the market rallied, the implied volatility of the fixed strike put increased because of the put–call skew; by put–call parity, in-the-money calls inherit the volatility of the out-of-the-money puts of the same strike. The index thus returned 8.91% over the four-month period. The option lost four months of time value, but despite the time decay, on an equal delta basis it delivered 10.88% marked-to-market return, handily beating the linear equivalent on a risk-adjusted basis.

Clearly, the reason the call option in this cherry-picked example outperformed the underlying index was because the ex ante probability distribution as implied by the options markets was incorrect in pricing the probability of such a large move in the underlying. When large market moves happen, the inherent nonlinearity of an option magnifies returns.

One of the key thrusts of this article is that when major economic and market forces are at work and there is a possibility of large, nonlinear jumps, using the implied probability distribution from traded options prices can be erroneous. Our view is that major policy changes, such as the U.S. tax reform, have the potential to create such nonlinearity. There are also some other reasons why upside hedging might be more relevant today than at any time in recent memory:

- *Influence of the trend toward passive products:* The move from active to passive investment management makes it likely that significant amounts of investment capital will continue to flow into low-cost exchange-traded funds (ETFs) and mutual funds that will be more price and valuation insensitive.
- *Low cost of capital:* Capital is cheap in a low (and negative in Europe) yield environment and, seeking return, it will flow into risk assets where the prospect of a possible loss looms less risky than a certain loss. In such an environment, it makes sense for corporations to issue debt to buy back equity.
- *Extremely low call-option volatility:* As discussed briefly in the following and in great detail in another article (Bhansali and Harris [2018]), the need for yield in a yield-starved environment has resulted in a proliferation of short volatility strategies. Many of these volatility selling strategies are symmetric in their exposure to calls and puts. As discussed later, a lower implied volatility of call options requires a larger notional sale of call options, making the call option volatility depressed. (see Exhibit 1).
- *Elevated volatility skew:* The cost of equity replacement using call options is at a multiyear low (see Exhibit 1). There are two reasons for this. First, the level of implied volatilities is at an all-time low; and second, the skew (the difference in the volatility spread between put options and call options) is still elevated (see Exhibit 3), which makes call options relatively cheaper than puts on a volatility-normalized basis. One driver of the elevated skew is costless collar strategies, in which investors sell call options to finance the purchase of out-of-the-money put options. As upside risk becomes more visible, it is likely that investors will increasingly look to finance the purchase of upside calls from the sales of put options.

EXHIBIT 1
Implied Volatility of 5% Out-of-the-Money Call Options in the S&P 500 Index



Sources: Bloomberg and LongTail Alpha.

- *Cross-market demand for convexity:* Credit products, such as high yield, cannot keep up with large rallies in the equity markets. Because many credit investors track popular benchmarks, as credit lags equity they will likely look to enhance total return by using upside convexity strategies. Similarly, alpha strategies such as long–short equity hedge funds are unable to keep up with the rally in equity markets, and the only way to recapture the beta is via synthetic long positions.

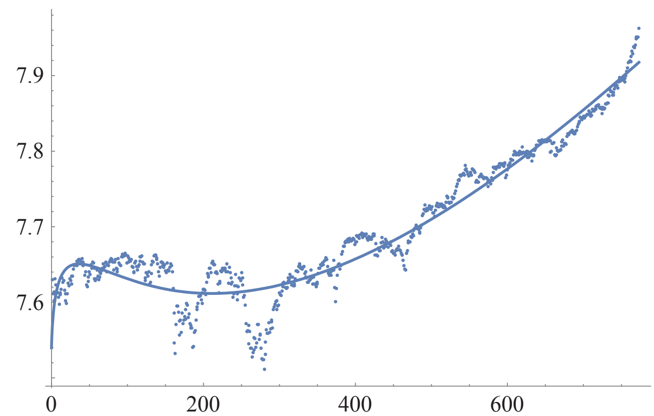
Let us illustrate the last point further. Consider an investor who is invested in high-yield credit. Credit risk as measured by credit spreads is negatively correlated with the equity of the issuing firm and positively correlated with the asset volatility of the firm. In other words, one can locally (but not globally) replicate a long position in the corporate bonds of a company by buying an appropriate amount of equity in the company. Replication of larger moves in the corporate bonds requires the purchase of options. As the value of the underlying assets rises, credit spreads compress because the implicit put option in the bond price is now worth less. If the uncertainty or volatility in the company's financial prospects decreases, the value of the put also falls, compressing spreads. This suggests that in addition to the indirect exposure to the equity price, an investor holding credit also has exposure to the volatility of the underlying equity, which reflects the uncertainty in the asset prices of the company. When spreads are neither too large nor too small, and the volatility of the underlying assets is not too high, credit can thus locally be replicated using only the underlying equity. When asset prices fall (which is accompanied by rising volatility) by larger magnitudes, or vice versa, the replication of credit requires that the investor supplement the equity with explicit options. Clearly, to participate in the upside, call options overlaid on credit are required.

Why would a credit investor need to keep up with the equity markets? To answer this, we have to think of modern financial markets as a competitive ecosystem in which each participant is seeking outperformance over the benchmark and also against other types of investors in the market. A credit investor can thus perform better relative to his or her own peer group by owning a small amount of additional convexity that, contingent on a large event, can result in outperformance. Such out-of-index bets are fairly common in the asset management industry and have become easier to implement because of the availability of liquid derivatives and ETFs. For instance, an equity investor can purchase an ETF, which trades on a stock exchange, to obtain exposure to interest rates, bonds, and even alternatives, such as the volatility and inverse-volatility ETFs that have recently become popular. By adding closely related sources of convexity that are not available from within credit, the credit investor is thus exploiting the mispricing of the fundamental risk factors across the credit and equity markets, which we know in the limit are closely related.

Derivatives such as call options are risky because of inherent leverage. Why the additional, finite loss (limited to premium) leverage risk is acceptable can be explained within an evolutionary model of markets (see McDermott, Fowler, and Smirnov [2008]). We assume that each credit market participant can choose between a safe choice and a risky choice. For survival in the competitive game of the markets, each participant has to meet or exceed a minimum absolute and relative return threshold. This, according to this model, requires investors to choose the risky proposition under four conditions: (1) when the difference between the required threshold return and the safe return rises, (2) when the difference between the risky return and the safe return rises, (3) when the variance in the risky asset falls, and (4) when the prospective variance of the environment

EXHIBIT 2

A Fit of the Log of the S&P 500 (January 1, 2015 to January 15, 2018)



Notes: This exhibit uses the log-periodic model of Sornette [2003]. The power-law dependence on time is illustrated.

Source: LongTail Alpha.

rises. When option volatilities are low, call options can provide the ability to achieve all four objectives in one package, with a finite risk of loss limited to the premium spent, ensuring survival in conditions in which survival is important.

Another perspective on this effect arises from the dynamics of imitation between traders, which is also well documented in speculative markets (see Sornette [2003]). Under this model, prices exhibit characteristic log-periodic behavior, as shown for the S&P 500 (see our fit in Exhibit 2):

$$\ln[S(t)] = A - mt^{\alpha}[1 + Cos[\omega \ln(t)]$$

This formula illustrates amplification and nonlinear acceleration in the price as a function of time as more and more participants begin to agree on the fundamentals. Markets then make new highs in succession, with the time intervals between new highs decreasing rapidly (before reaching a critical point that results in a crash). For the present purpose, the key parameter is α, which determines the rate of growth of S with time. Taking derivatives with respect to time:

$$\frac{dS}{dt} \sim \frac{e^{t^{\alpha}}}{t^{\alpha-1}}$$

Fitting the model to the logarithm of the S&P 500 since 2015 as shown in Exhibit 2 demonstrates that $\alpha = 0.72$—that is, the index level grows as $S \sim e^{t^{\alpha}} \sim e^{t^{0.72}}$. Compare this to a Black–Scholes world, where because of the assumption of the ability to create a continuous hedge using the underlying, the rate of change of the market is a linear function of time proportional to the difference between the interest rate and the dividend rate. For a market in a melt-up phase, the pricing of call options will then be attractive if linear drift is replaced by the power-law drift.

One final motivation for upside tail hedging emerges from the role that options markets play in enforcing investment discipline and time consistency for risk-management purposes. The well-known *disposition effect* has been studied extensively (Shefrin and Statman [1985]) in the literature and documents that unless there is a mechanism to enforce time consistency in investment decisions, investors who initially plan to let their profits run in the event of a large, low probability gain are very likely to change their mind midway to the gain being realized (Barberis [2012]). On the other hand, even though the investor plans to stop out of the trade when faced with losses, once confronted with small losses, investors tend to change their mind and stay with the losing position in contradiction to their initial plan. Thus, even in the case of a 50/50 bet, it is possible to create an ex ante strategy in which the stop-loss rule enables the investor to create a positively skewed investment plan. Although initially the investor plans to stay invested while profitable and exit as soon as he or she has a loss, because of the combination of time inconsistency and the absence of a commitment device, he or she ends up violating this initial plan and exits too early when winning and stays too long while losing. Entering a long position through call options does not completely eliminate the tendency to take profits too early, but building in a finite loss right at the inception of the trade, which is limited to the maximum premium paid, provides one mechanism by which large losses cannot accumulate. This is similar to the motivation discussed by Bhansali [2008] for pre-committing using left tail hedges when faced with the problem of keeping losses contained. Call options thus provide a pre-commitment device to overcome the disposition effect, but more importantly to contain the maximum loss when bull markets inevitably reverse.

DO CALL OPTIONS COST TOO MUCH?

However valuable upside optionality might be from the perspective of a real-world investor, what can we say about the pricing and value in call options? In this section we take a fresh look at the pricing of call options using a simple and well-known model that allows for "up jumps" in the price process. Note that in the presence of large jumps, it is not possible to create a perfect local hedge—that is, many of the assumptions of Black–Scholes are violated ab initio, so option prices should be expected to be higher than they would be in the absence of jumps.

Assume that returns of the equity market follow a jump diffusion:

$$\frac{dS}{S} = \mu \; dt + \sigma \; dZ + J \; dq$$

The expected log return after a time Δt is $(\mu + J\lambda)$ Δt, where λ is the density of jumps in a unit time interval (e.g., the number of jumps per year). For simplicity, we assume that the jumps are of a constant size J. In the limit that the time interval goes to zero, the variance of the jump diffusion for one large jump is then $\sigma^2_{JD} = \sigma^2 + J^2\lambda$; that is, the jumps increase the volatility of the underlying process. In the jump-diffusion framework (Merton [1976]), the price of a call option is the weighted sum of call options with zero to many jumps, with the weighting equal to the Poisson probability of observing that many jumps. In other words, the price of the call option C_{JD} is the expected payoff weighted by the number of jumps:

$$C_{JD} = e^{-rT} \sum_{n=0} \frac{(\lambda T)^n}{n!} e^{-\lambda T} E[\max[S^n_T - K, 0]]$$

The jump formula for a call option can then be written in terms of Black–Scholes options prices C_{BS} as

$$C_{JD} = e^{-\lambda t} \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} C_{BS}(S, K, t, \sigma, r_n)$$

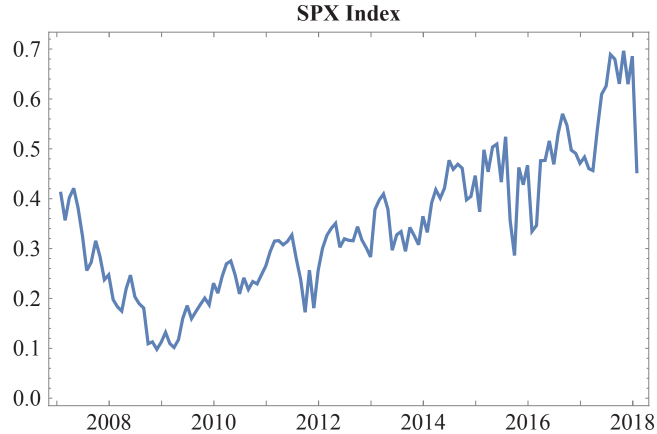
Here $r_n = r - \lambda(e^J - 1) + \frac{nJ}{t}$ is the compensated drift to account for risk neutrality.[1]

As an example, let us assume that the volatility of the traditional diffusion process is 12%, which is close to the realized volatility in the S&P 500 over the last few years. Now faced with a significant change in fiscal policy (e.g., tax laws, protectionism, or any number of macroeconomic surprises), let us assume that the stock market has a 100% probability of one jump of 10%. Using just the diffusion-based Black–Scholes formula, the price of a one-year option, 5% out of the money, is computed to be approximately 2.9%. With the 10% jump, the price of this same option is computed to be 4.23%, or approximately 50% higher. As another example, for a one-month horizon, the 5% out-of-the-money call option according to a jump model would cost almost four times as much if the jumps were appropriately priced. None of this is new knowledge: It is well known that the smile in the option volatility surface is very steep for shorter dated options. Nevertheless, the focus of the investment industry has been on down, rather than up, jumps.

If up-jumps are not priced in the market price of options, market participants may erroneously believe that they can hedge their upside exposure by trading continuously. Although the assumptions of the ability to dynamically hedge were found to be seriously flawed in the market crash of 1987, the dearth of equity market melt-ups still provides many participants with the comfort that they can hedge their upside risk by continuous trading, which has resulted in the asymmetric index option skew of Exhibit 3. Anecdotally, in the last few years we have observed up-jumps in international markets, such as in Shanghai (2015) and in Japan (late 2017). The put–call pricing asymmetry is reflected in the relative pricing of options, and hence in the implied volatilities corresponding to different strikes. For example, Credit Suisse First Boston's Fear Barometer

Exhibit 3

Normalized Volatility Skew for S&P 500 Three-Month Index Options



Notes: The difference of implied volatility for the 5% out-of-the-money put and 5% out-of-the-money call divided by the at-the-money volatility is displayed. A rising skew means that equidistant call options are priced at a lower volatility than the same distance put options.

Sources: Bloomberg, LongTail Alpha.

measures investor sentiment for a three-month horizon by pricing a "zero-cost" collar (selling an upside call to purchase a downside put). The Fear Barometer represents the strike of an out-of-the-money put that can be bought by selling a 10% out-of-the-money call. If the value of the Fear Barometer falls, this implies that the strike of the put that can be purchased is closer to at-the-money—that is, the market "feels" less fearful. In other words, the volatility skew can be interpreted as the extra premium a seller of the option requires to mitigate the risk that he or she might not be able to hedge the risk by trading in the underlying.

To address how up-jumps affect the volatility skew, let us assume that the risk-free rate is zero and that there is only one large jump of size J. Then, with a current index price of S and a small probability p of an up-jump and probability $1 - p$ of a small downward move D, risk neutrality requires

$$S = p(S + J) + (1 - p)(S - D)$$

That is:

$$D = \frac{p}{1-p} J \sim pJ$$

[1] In the more general case, which is not relevant to the current discussion, the jump size itself can be random. If the jump distribution is normally distributed with a jump volatility σ_J, then the formula for the call option becomes

$$C_{JD} = e^{-\lambda t} \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} C_{BS}(S, K, t, \sigma_{JD}, r_n)$$

where $\sigma_{JD} = \sigma^2 + n\frac{\sigma_J^2}{t}$ is the volatility adjusted for jumps and is strictly higher than the diffusion volatility.

If we assume that $p \ll \sigma\sqrt{t} \ll J/S$ (i.e., the jump probability is much smaller than the diffusion volatility and the percentage size of the up-jump), then we can show that close to at-the-money, the price of a call option with strike K under jump diffusion with only one jump is given by:

$$C_{JD} \approx C_{BS}(S,\sigma) + pJ\left(\frac{1}{2} - \frac{1}{\sigma\sqrt{(2\pi t)}}\ln\left(\frac{S}{K}\right)\right)$$

Note that the risk to an options-market maker emanates from the mistake in estimating the gamma of an option between the jump-based model and the Black–Scholes model. This is derived by taking the second derivative of the option prices with respect to the underlying and equals:

$$\frac{\partial^2 C_{JD}}{\partial S^2} - \frac{\partial^2 C_{BS}}{\partial S^2} = \frac{Jp}{\sqrt{(2\pi t)}S^2\sigma}$$

Thus, when volatility is low, the gamma or the rate of change of delta of the option priced under jumps will be much larger than the option priced under Black–Scholes. If the product of the jump magnitude and probability increases, the gamma increases rapidly.

This equation illustrates why melt-ups in the markets may result in a feedback loop from the destabilizing influence of options-based hedging. In particular, because call option volatility has been very low for an extended period of time, it is likely that the destabilizing influence of call options could be significant. If many participants are hedging their call options using Black–Scholes and there is a sharp increase in the likelihood of a large upward jump, the need for extra hedging can propel the underlying market rapidly higher, as has been observed recently in the S&P 500. Note that by put–call parity, at-the-money put options would suffer the same risk of a hedging mismatch if the market were to suddenly jump down.

To translate the result into Black–Scholes volatility (which is market convention), we could set the jump price of the call to its Black–Scholes price with a volatility S; that is:

$$C_{JD} = C_{BS}(S,\Sigma) \approx C_{BS}(S,\sigma) + (\partial C_{BS}/\partial\sigma)(\Sigma - \sigma)$$

Solving this equation, we find that the adjusted volatility that needs to be plugged into Black–Scholes to recover the effect of the skew is:

$$\sum = \sigma + \frac{pJ}{S\sqrt{t}}\left(\sqrt{\frac{\pi}{2}} + \frac{1}{\sigma\sqrt{t}}\ln\left(\frac{K}{S}\right)\right)$$

Some observations from this result include the following:

- *Spot up, vol up*: As the probability of an up-jump or the size of a jump increases, the volatility increases as a function of the product of the jump probability and the jump magnitude. Hence, if a large positive shock is expected because of an increased probability of shocks, volatility will increase very rapidly if p and J are positively correlated. This phenomenon is typically called *spot up, volatility up* in trader's parlance because it implies a rise in implied volatility as the markets rally, which has been considered anomalous since the 1987 crash. We see that with large positive jumps, the result is not all that surprising.
- *Impact on term structure of skew*: As S increases, or the time t to expiration increases, the jump-adjusted volatility falls. Thus, the jump risk is a major risk for shorter dated options and decreases as the horizon increases. As an insurance policy, shorter dated call options are therefore likely to be more responsive to jump risks. Sudden unexpected positive shocks can then create rapid short-term changes in the volatility smile.
- *Effect of low-volatility environment*: When the volatility is low (i.e., σ is small), the correction to the Black–Scholes volatility from jumps is larger as compared to when volatility is already at a high level. In other words, the potential for upside dislocation and the need for upside hedges are more pronounced when starting from low-volatility levels, which are prevalent today. This last point is important because the combination of low volatility and high skew in index options has induced volatility selling strategies that have a large risk asymmetry to the upside. For example, because the volatility of the call options is much lower than the volatility of the put options, to make a three-month zero-cost strangle using 5% out-of-the-money options,

more than twice the number of calls than puts have to be sold. The equation for negative gamma then highlights the fact that delta hedging risk is more acute on the upside.

CONCLUSION

I discuss both the empirical and theoretical need for upside tail risk hedging. Under the current environment of large potential positive economic and political shocks that can affect markets, right tail hedging is a risk-management strategy whose importance is likely to increase. When markets have the potential for up-jumps, right tail hedging using call options can provide investors with an approach to manage their exposure. In a previous discussion of left tail hedging (Bhansali [2008]), I discussed some macro features of tail hedging. First, left tail risk is a systematic risk, and it increases as correlations increase. Because the variance of an index is the weighted average of the sum of variances and covariances, left tail risk increases if either the variances or the correlations increase. Correlations increase for large downward shocks generally because of the impact of deleveraging. It is generally assumed that for large positive increases in equity prices, correlations do not rise as much. These assumptions are likely on less solid ground today, both because of major macroeconomic shocks that can result in positive correlations and the low levels of volatility that can be potentially destabilizing on the upside.

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