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Evolutionary Finance for Multi-Asset Investors

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Abstract

Standard strategic asset allocation procedures usually neglect market interaction. However, returns are not generated in a vacuum but are the result of the market's price discovery mechanism which is driven by investors' investment strategies. Evolutionary finance accounts for this and endogenizes asset prices.

This paper develops a multi-asset evolutionary finance model. Requiring little more than dividend and interest rate data, it facilitates an interesting glimpse into the inner workings of financial markets and provides a valuable guide to this class of models. While traditional mean/variance optimization is static and concerned with finding the optimal asset allocation, evolutionary portfolio theory is dynamic and its focus is on finding the optimal investment strategy. This paper shows that yield-based strategies generate asset allocations that outperform competing alternatives. Therefore, strategic asset allocation approaches that rely on such an economic foundation are evolutionarily advantageous for multi-asset investors.

JEL classification: G10, G11, G17.
Keywords: Evolutionary finance, strategic asset allocation, multi-asset.

1 Introduction

Evolutionary finance studies financial markets using concepts rooted in biology[1]. Investment strategies compete for market share and only the best survives - finance's equivalent to the "survival of the fittest" popularized by Darwin (1869).

In nature, many interactions and dependencies exist. Foxes will decimate rabbits until there are too few of them to feed the many foxes at which point the population of rabbits will start growing again. A finance parallel might be momentum strategies that invest until prices have moved so far away from fundamentals that value investors' approach becomes profitable again and the market is pushed in the opposite direction.

Evolutionary portfolio theory puts these ideas to work in the realms of portfolio choice. Most importantly, it allows for prices to be determined *within* the model which is certainly one of its most exciting features. In the traditional asset allocation literature, the interaction between investment strategies is little studied. However, especially for big multi-asset investors such as sovereign wealth funds or large pension plans, their actions (e.g. asset allocation changes) have the potential to move prices. Moreover, even small occupational pension plans often follow very similar investment strategies and their collective impact can also affect market prices. Therefore, many investors are increasingly concerned with understanding their investment strategy's impact on financial markets - and whether their investment strategy is set up to be successful – i.e. survives – in the long run.

Whereas in classic mean/variance optimization, investors search for the optimal allocation, evolutionary portfolio theory's focus is on finding the optimal strategy (i.e. a sequence of asset allocations over time). In an evolutionary finance setting, investing according to mean/variance optimization is one of many possible strategies. This paper shows how the mathematical framework of evolutionay portfolio theory can be applied to the decisions faced by multi-asset investors. Moreover, contrary to most portfolio choice models which require an investor's utility function (which is still a much debated topic, see for example Sharpe (2007) or recently Warren (2019)) this is not the case for evolutionary portfolio theory. Using simulations that require little more than (real-life) payouts, this paper proofs that yield-based strategies outperform alternative allocation approaches for a portfolio comprised of bonds, REITs and equities.

2 Methodology

Evolutionary portfolio theory primarily studies relative wealth shares. The strategy that gathers the highest wealth will ultimately dominate the market. In order to get there, it must earn payouts and achieve capital gains larger than those of competing strategies. Contrary to backtests, returns are determined by the strategies' interaction within the model[2].

The key to endogenizing returns is to understand market capitalizations as prices. Suppose a stock currently has a market capitalization of USD 100m. Now, more investors get attracted to this company's shares and they (collectively) want to invest USD 10m while

[1]Research in this field includes Farmer (2002), Lo (2004), Evstigneev, Hens, and Schenk-Hoppé (2006), Evstigneev, Hens, and Schenk-Hoppé (2008), Evstigneev, Hens, and Schenk-Hoppé (2016) and Lo (2017).

[2]This distinction was exemplified in Hens, Schenk-Hoppé, and Woesthoff (2020).

the existing investors wish to retain their position. In order for the market to clear, the company's market capitalization must rise to USD 110m. Assuming no new issuances or redemptions[3], this (i.e. market clearing) can only be achieved if the share price rises by 10%.

2.1 Theory

In terms of notation and model setup, we closely follow Evstigneev et al. (2016). Evolutionary finance models of this type are based on the work of Lucas (1978) and feature three key equations.

Investors $i = 1, ..., I$ determine their allocation[4] α^i to strategies $s = 1, ..., S$ at time t. Strategies allocate a fraction λ to asset classes $k = 1, ..., K$ according to their weighting scheme (e.g. market capitalization weights). Equation (1) describes an asset's (relative) market capitalization where ω expresses relative wealth either of investors $\omega_t^i = \frac{w_t^i}{\sum_{i=1}^{I} w_t^i}$ or – what we are ultimately interested in – of strategies $\omega_t^s = \frac{w_t^s}{\sum_{s=1}^{S} w_t^s}$.

$$\lambda_t^{M,k} = \sum_{s=1}^{S} \lambda_t^{s,k} \omega_t^s \text{ where } \omega_t^s = \sum_{i=1}^{I} \alpha_t^{i,s} \omega_t^i \tag{1}$$

The return for asset class k can be computed according to equation (2) with D denoting the dividend or interest payment.

$$r_{t+1}^k = \frac{\lambda_{t+1}^{M,k} + D_{t+1}^k}{\lambda_t^{M,k}} \tag{2}$$

The evolution of wealth for strategy s is then given by equation (3).

$$\omega_{t+1}^s = \sum_{i=k}^{K} r_{t+1}^k \lambda_t^{s,k} \omega_t^s \tag{3}$$

Equations (1) to (3) generate a dynamic stochastic model, i.e. a random dynamical system. This can be expressed in matrix notation as in equation (4) and solved using linear programming where Id is the identity matrix, Θ the matrix of asset holdings (i.e. $\left[\frac{\lambda_t^{s,k}\omega_t^s}{\sum_{s=1}^{S}\lambda_t^{s,k}\omega_t^s}\right]_s^k$), Λ the matrix of allocations and d the scaled payout matrix. The equation's first part captures (relative) wealth changes due to capital gains or losses and the second part those due to investment income.

$$\omega_{t+1} = [Id - \Theta_t \Lambda_t]^{-1} \Theta_t d_t \tag{4}$$

Hence, strategies determine asset demands. Summing the allocation of all strategies to

[3]This assumption can be challenging for indices but the robustness check using programatically fixed capital bases shows that the conclusions still hold (see section 4.1.1).

[4]In the multi-asset case, investors will usually only allocate to one strategy (i.e. $\alpha^{i,60/40} = 1$ meaning investor i employing the 60/40 asset allocation rule for example).

a particular asset class determines its price[5] at time t. At the end of year t, dividend and interest are paid and investors (through strategies) trade again. The return to a strategy is thus comprised of payouts and ultimately the capital gains or losses (the percentage difference between the summed demand at t and $t+1$) it receives on its holdings.

The dividend and interest rate processes were assumed to be exogenous. Moreover, we used payouts (coupons (shifted +1 year) times aggregate face value[6] or dividend yields times market cap (in USD)) which anchors assets to what they are conceivably able to produce in reality. Dividends and interest rate payments are made from cash earned through operating activities in the economy. Prices in financial markets however, are determined by supply and demand for financial assets. Financial markets and the real economy are thus viewed similar to two separate ecosystems. Nevertheless, even if dividends were allowed to increase with market value for example[7], an evolutionary stable investment strategy still exists (Amir, Evstigneev, Hens, Potapova, and Schenk-Hoppé, 2021).

2.2 Investment strategies

Contrary to stocks where the most popular strategies are those based on factors (e.g. value), the strategy set is more diverse in the multi-asset domain. On the one hand, it includes virtually every investor and asset class and on the other hand, allocation procedures are often less straightforward. Nevertheless, some strategies are very widely-followed and thus represent a good proxy for the power balance present in the markets.

Neither leverage nor negative weights were allowed. If a strategy were to have allocated a non-positive value to an asset class, that weight was set to zero (should all be zero, we resort to 1/n).

2.2.1 Basic strategies

These four strategies represent the most popular and time-tested approaches and lay the foundation for this paper's analysis.

1/n allocation Equal weights on all asset classes.

Market-cap weighted Using (endogenous) market capitalization weights after $t = 0$. Corresponds most closely to passive index investing.

Historical mean return Historical mean (total) returns are a popular source for expected returns in practice and this method allocates according to them. If we had only two assets with US equities having achieved a mean return of 10% and US Treasuries one of 5%, they

[5]This leads to an inconsistency at $t = 0$ where the summed asset demand in the simulation and the real-life market capitalizations may (and most probably will) not be the same which is adressed in a robustness check in section 4.1.1.

[6]The total face value was backed out from the index' average yield-to-maturity, coupon, remaining time to maturity and an assumed face value of 100 per bond. The resulting price was then utilized as a scaling factor to get from the aggregate market capitalization to the aggregate face value.

[7]An approximation for this is to use yields instead of payouts which was investigated in section 4.1.1.

would get an allocation for the next period of $\frac{10}{10+5} = 67\%$ and $\frac{5}{10+5} = 33\%$, respectively. Realized historical returns were used until the simulation started and from then onwards a blend of those and endogenous ones.

Yield The current yield for bonds and the dividend yield for equity and REIT indices. The same allocation procedure as for the historical mean return variant was used – just using yields instead of returns (i.e. if US Treasuries offered a yield-to-maturity of 5% and US Equities a dividend yield of 5%, the allocation at that point in time would be 50/50).

2.2.2 Additional strategies

Adding to the basic strategies and later applied to a more exhaustive asset sample, this section introduces a wider array of multi-asset allocation approaches.

60/40 allocation 60% of the wealth allocated to equities and 40% to bonds. Within the equity and bond portion, allocations were made according to relative (simulated) market capitalization. Although a very popular approach especially in the US, it was not included into the basic strategies as its weights were frequently very close to those of the market capitalization allocation. In order to see a dominant strategy emerge, strategies should arguably be as orthogonal as possible (i.e. not hold very similar portfolios) at least for the initial evaluation.

Trailing one-year return This is a proxy for time series momentum and allocates according to the previous year return for each asset class.

Risk-free rate plus historical excess return A sensible way to take the current interest rate environment[8] into account when forecasting returns.

Carry-based expected returns A strategic asset allocation variant using carry-based expected returns as in Schnetzer (2018). Expected returns for bonds were computed as the yield-to-worst minus (expanding window) historical credit losses plus the rolldown and for REITs and equities as the dividend yield plus the dividend growth rate. Two allocation procedures were tested. One variant allocated simply according to expected returns and the other one based on the expected Sharpe ratio.

Insurance A proxy for a typical (life) insurer's balance sheet. 50% government bonds, 35% corporate bonds and 7.5% each in real estate and equities (market capitalization-weighted within categories). Since REITs are usually more highly levered than the direct real estate holdings of most insurance companies they are a sensible but of course only imperfect proxy.

[8] The risk-free rate and yield curve were both assumed to be exogenous in this paper as they were not directly traded as an asset. However, they could for example be inferred from the yield on the US Treasuries index.

Mean/variance optimization Traditional mean/variance optimal weights based on the seminal work by Markowitz (1952) using historical (and simulated) mean returns and a shrunk covariance matrix.

3 Description of the data set

The base sample consisted of the Bloomberg Barclays US Treasury Index, the Bloomberg Barclays US Corporate Bond Index, the US Nareit Equity REIT index and the S&P 500 index. The extended sample added indices for government bonds, corporate bonds and equities in the UK, the Eurozone and Japan as well as US high-yield bonds and the MSCI emerging markets equity index.

The time period covers data from 1973 to 2020 with the simulation period starting when the necessary data was first available for all indices, namely in 1974 for the base sample and in 2002 for the extended sample with annual intervals.

All data were retrieved from Bloomberg except where stated otherwise. In particular, our source for credit loss data was Moody's Annual Default Study and data for US REITs were retrieved from the Nareit website[9].

4 Results

Every year in December, asset allocation was carried out and prices were determined within the system. The base evaluation was restricted to only few but very popular strategies. Figure 1 shows each strategy's wealth share over time.

The yield-based strategy outperformed the others in this multi-asset environment, thereby confirming the results for stocks analytically devised by Evstigneev et al. (2006). In particular, it ate away at the strategy that allocated according to historical returns and the one that allocated according to market capitalization weights. Simply using past returns was not evolutionarily advantageous. Interest and dividend payments comprise the stable component of total returns and in an evolutionary setting, they ultimately dominate since capital gains are more erratic. Although this sample period was too short for the yield-based strategy to achieve complete dominance (convergence is slow in real life), it is important to note that the more dominant a strategy becomes, the more it also determines prices.

4.1 More assets, more strategies

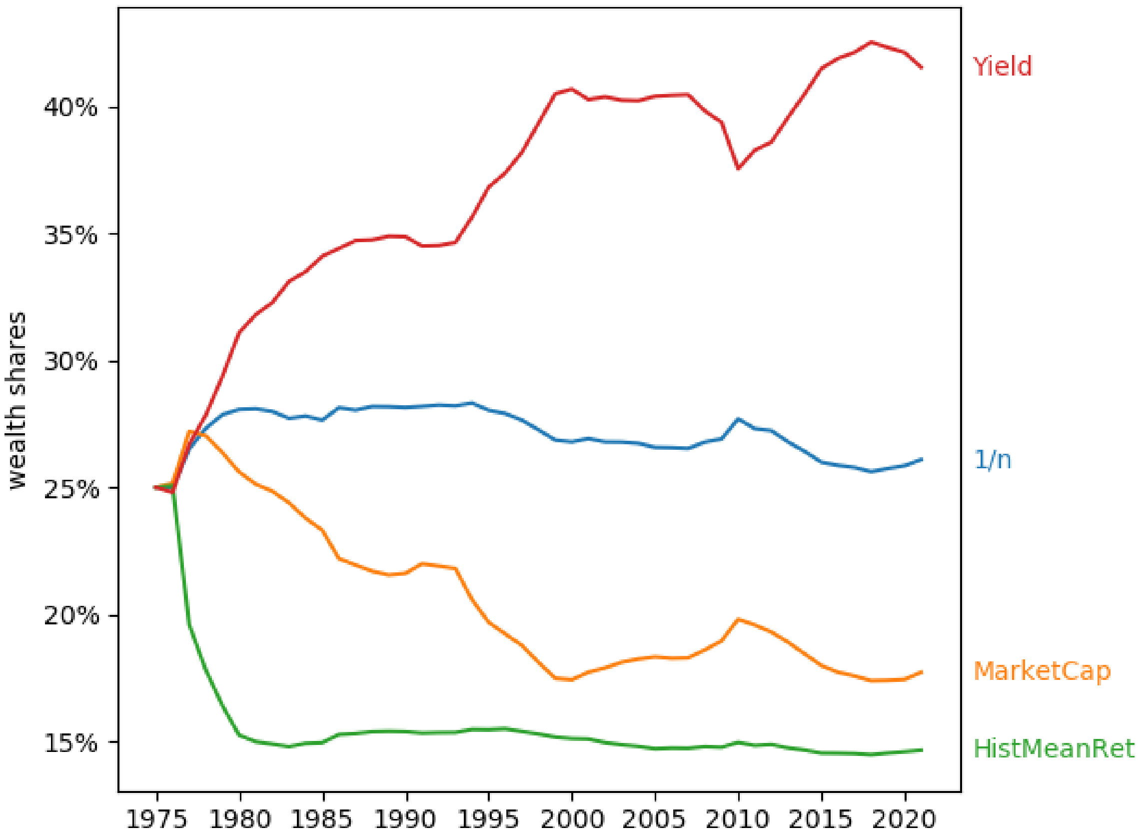
The main simulation delivered intriguing results but most multi-asset investors have to proof themselves in global markets and against a more diverse set of competing strategies. Hence, the sample was expanded to include a broader and more representative array of asset classes. Moreover, the strategy set was extended to more asset allocation schemes.

Figure 2 depicts the results of this expanded sample[10].

[9]https://www.reit.com/data-research [Accessed Oct. 5, 2021]

[10]This also meant a shorter sample period as many asset classes, in particular non-US fixed income indices, only had data available from the early 2000s onward.

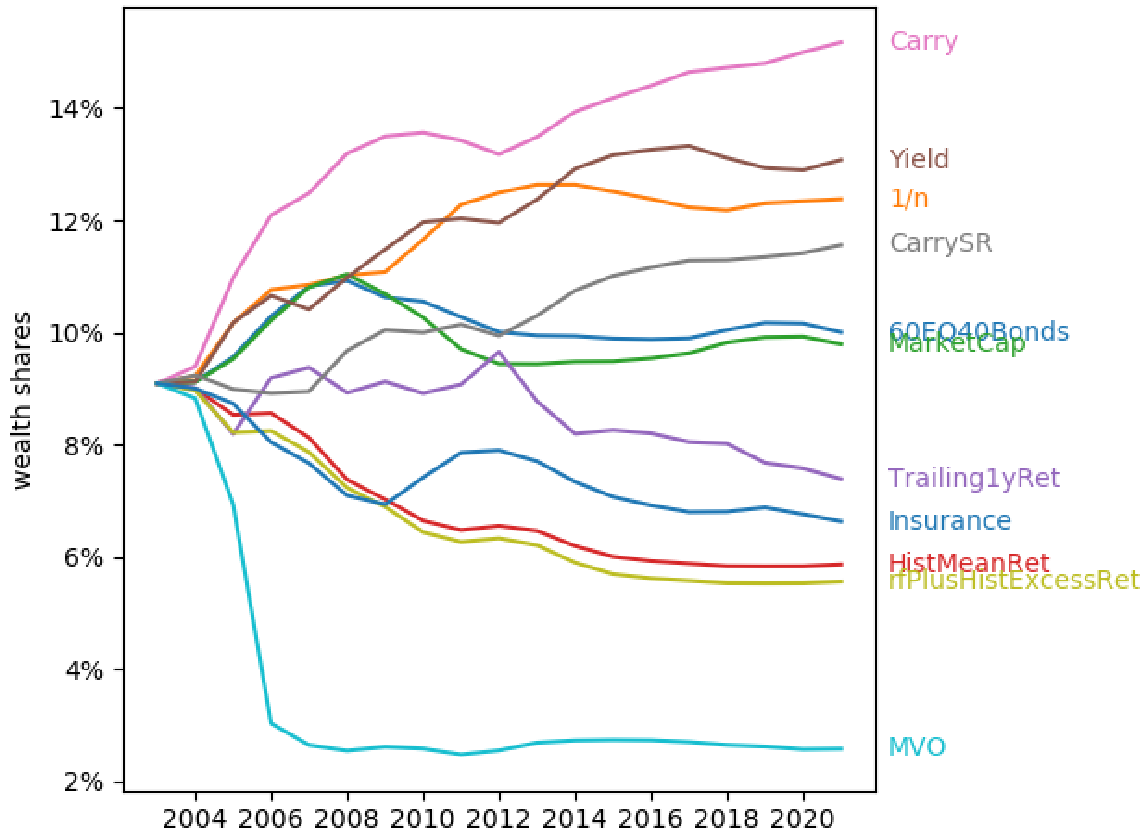
Figure 1: Wealth shares



Again, the yield-based strategy achieved a high relative wealth share but was now surpassed by the carry-based strategy. Contrary, to the yield-based variant, the carry approach employs a more comprehensive estimate of expected returns. In particular, it extends the equation for equities and REITs by a growth estimate. Also for this expanded strategy set, capital gains-focused approaches were among those that were dominated. A strategy's relevant factor in a random dynamical system is its growth rate. The intuition behind yield-based strategies' success can thus be seen as the carry representing the stable component of wealth accumulation. Capital gains on the other hand are noisy and thus evolutionarily fragile. Hence, there always exists a mutant strategy that will able to dominate a capital gains-based strategy.

The biggest disappointment might be the low market share achieved by the mean-variance optimization. However, this might be a consequence of the relatively short estimation period that was available to estimate the covariance matrix which will be examined in section 4.1.1. Moreover, note that even though it was dominated, this need not mean that it lost its investors value in absolute terms. Even the mean/variance optimization achieved a terminal wealth slightly above its starting one (and thus generated a positive annualized return) but it just grew much slower than more evolutionary successful approaches.

Figure 2: Wealth shares (more assets, more strategies)



4.1.1 Robustness checks and variations

Constant market capitalization In order for market capitalizations to correspond to prices, the capital base must stay the same. However, this need not necessarily be the case for indices. For example the market capitalization of the REIT index has grown substantially over the past decades also because more qualifying securities could be included. In this paper, the payout was determined as yield times (point-in-time) market capitalization. If market capitalization increases simply due to an increasing number of securities, the approximation might not be accurate anymore. This robustness check fixed market capitalizations to the value they had in 2002 (the simulation's start date)[11] and also calculated payouts based on that value. Figure 3a shows that this even increased the most successful payout-based strategy's lead while it narrowed the distance between the other yield-based ones, the market cap-oriented allocation approaches and the 1/n allocation method.

Yields instead of USD payouts This variant is related to the previous one. Another cure for market cap-agnostic behavior is the use of fixed percentage yields instead of fixed payouts. This is also how most analytical work in evolutionary finance has been done. Although an

[11] Using the indices' average market capitalization over the sample period lead to virtually identical results.

appealing alternative, its drawback is that an asset class' yield will not decrease at higher prices. Figure 3b depicts the results. Little change was observable to the initial specification.

Initial demand market cap-based The main simulation starts by endowing USD 1 to every strategy and letting each one choose the weights according to its specific asset allocation algorithm. However, an alternative would be to assume that the whole world currently invests passively according to market capitalization weights and use those as the starting weights at $t = 0$. The early points in figure 3c already show why this solution is not preferrable. Such a setup leads to a big price change at $t = 1$ which then plagues asset allocation approaches that rely on historical returns and/or volatilities over the whole sample period. On the other hand, this is the only variant where initial (real-life) market capitalizations and strategies' asset demand proportional. Anyway, the same strategies as in the previous evaluations achieved the highest wealth shares and were therefore evolutionarily superior.

Full sample for correlation estimation The mean-variance optimization's dismal performance certainly warrants a closer look given its popularity in practice. It was certainly handicapped by the comparatively short history (3 years) available to estimate correlations at sample start. Hence, in this variation, all strategies that required standard deviation or correlation inputs for their asset allocation were allowed to use the entire sample of real-life prices (note that this creates a forward-looking bias). The resulting wealth shares in figure 3d indeed show that this improved the performance of the mean-variance optimization allocation a bit and that of the Sharpe ratio carry-based approach a lot. However, even despite them having the benefit of using some forward-looking information, they still could not outperform the dominant strategy.

Transparent auction In this paper, strategies' asset demand works similar to them submitting market orders. This can potentially generate large price movements (i.e. slippage) at which investors would maybe no longer be interested in transacting. This variant takes that into account and introduces a price discovery mechanism similar to a transparent (opening) auction. Before actual trading and price determination takes place, investors (i.e. through their strategies) go through five rounds of price negotiation and get the opportunity to adjust their allocation and orders based on the expected clearing price.

Figure 3e shows that this particularly improved the approaches that heavily rely on market capitalizations for their asset allocation. Nevertheless, the carry method as a representative of payout-focused strategies remained in the top group and even started to gain back market share from those strategies after some time. The 60/40 and the market cap allocation also benefited here from their comparable investments. Due to their similar allocation styles, they together have (almost) twice the market power of more orthogonal approaches and can thus quicker reach a state where they can potentially determine prices easier.

Lengthened sample period Many of the assets in the extended sample have a relatively short data history (∼20 years). As convergence to strategy dominance is relatively slow it would be nice to have a (much) longer evaluation period. This variant simply stacks the same data set 5 times to the original one (always reversed to avoid jumps at joints). This extended version confirmed the main results as is evident from figure 3f.

However, in particular the Sharpe ratio-based carry version lost a lot of market share towards the middle of the simulated period. One reason was that the Sharpe ratio indeed turned negative for most equity asset classes as dividend growth approached zero (an index with increasing payouts in the main sample will become one with negative dividend growth in a reversed period) over time and the risk-free rate was higher than the dividend yield leading to no allocation to those assets. A (block) bootstrap analysis will certainly be able to present a more realistic picture.

Bootstrap analysis A way to "expand" a data set and investigate it from another perspective is to employ bootstrap analysis. We used the circular block bootstrap of Politis and Romano (1992) and the Politis and White (2004) algorithm[12] to determine the optimal block size. Then, we ran 10 bootstrap iterations and took the average resulting wealth to determine the wealth shares in figure 3g. This analysis again confirmed the main results.

Dominant starting strategy It could be the case that a dominant strategy can never be pushed out of the market. Hence, this variant endows the market capitalization-based strategic asset allocation which is very popular in practice, an initial relative wealth of 50% (with the other strategies sharing the other half). Figure 3h illustrates that indeed more evolutionary successful strategies will start to eat into the (artificially-set) dominant strategy. However, convergence, as always, is slow.

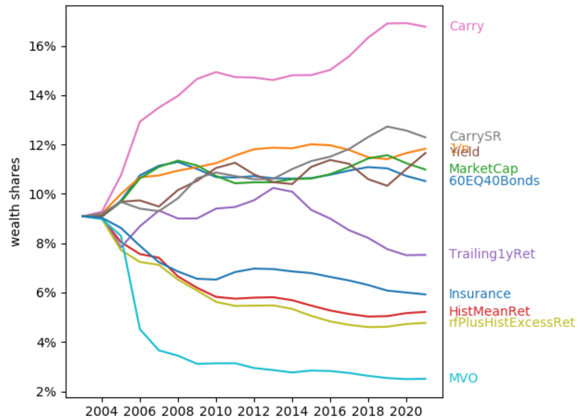
Taxing payouts Yield-based strategies might favour assets with high payout yields which bear a tax disadvantage in most jurisdictions. In this robustness check, payouts were taxed at a (marginal) tax rate of 20%. However, as can be seen in figure 3i, this did not materially affect the results.

Enthusiastic retail traders (inflows) So far, we have not assumed any in- or outflows. This variant introduces an outside capital stock (cash) held by retail investors. These investors monitor the strategy that allocates according to trailing one-year returns and if that strategy generated a positive return (i.e. similar to having observed longer-term time series momentum), invest in that strategy. Their collective contribution is 10c in that case (as a reminder, USD 1 was endowed to each strategy at $t = 0$). Figure 3j shows that these inflows naturally increased the trailing one-year return strategy's wealth share but without being able to challenge the most evolutionary successful strategies' dominance.

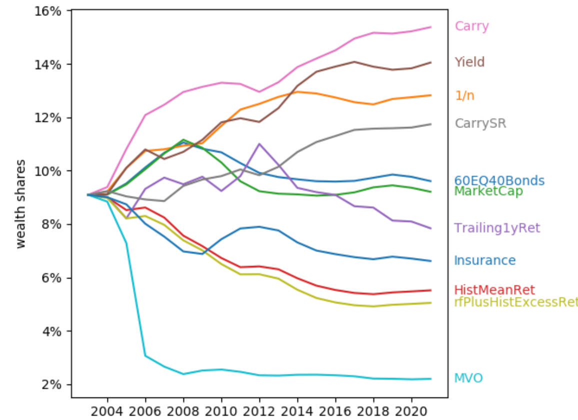
[12] See also the correction in Patton, Politis, and White (2009).

Figure 3: Robustness checks

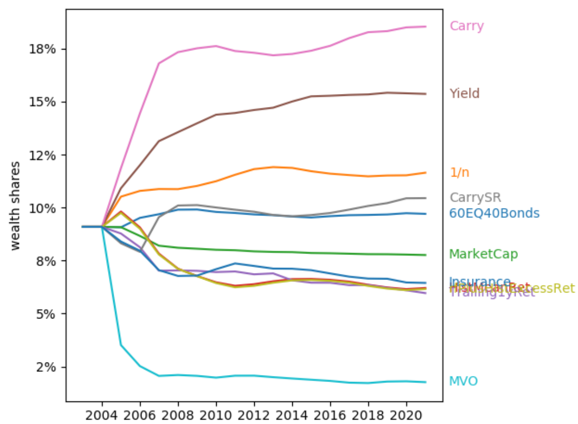
(a) Constant market capitalization



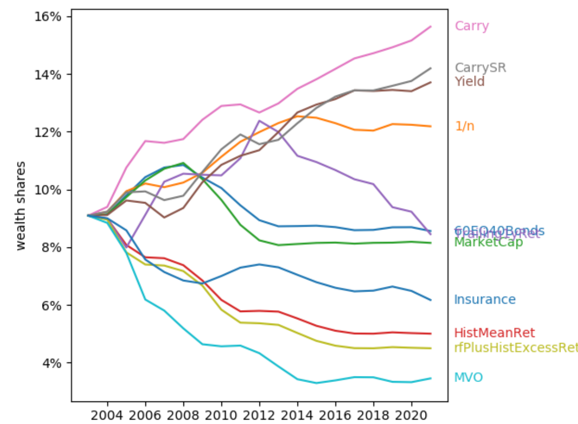
(b) Yields instead of payouts



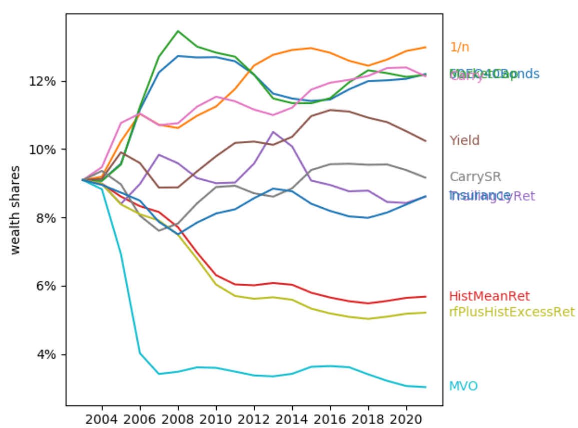
(c) Initial demand market cap-based



(d) Full sample for correlation estimation



(e) Transparent auction



(f) Lengthened sample

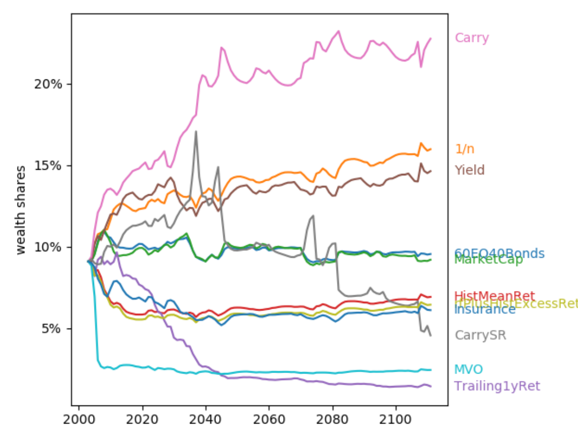
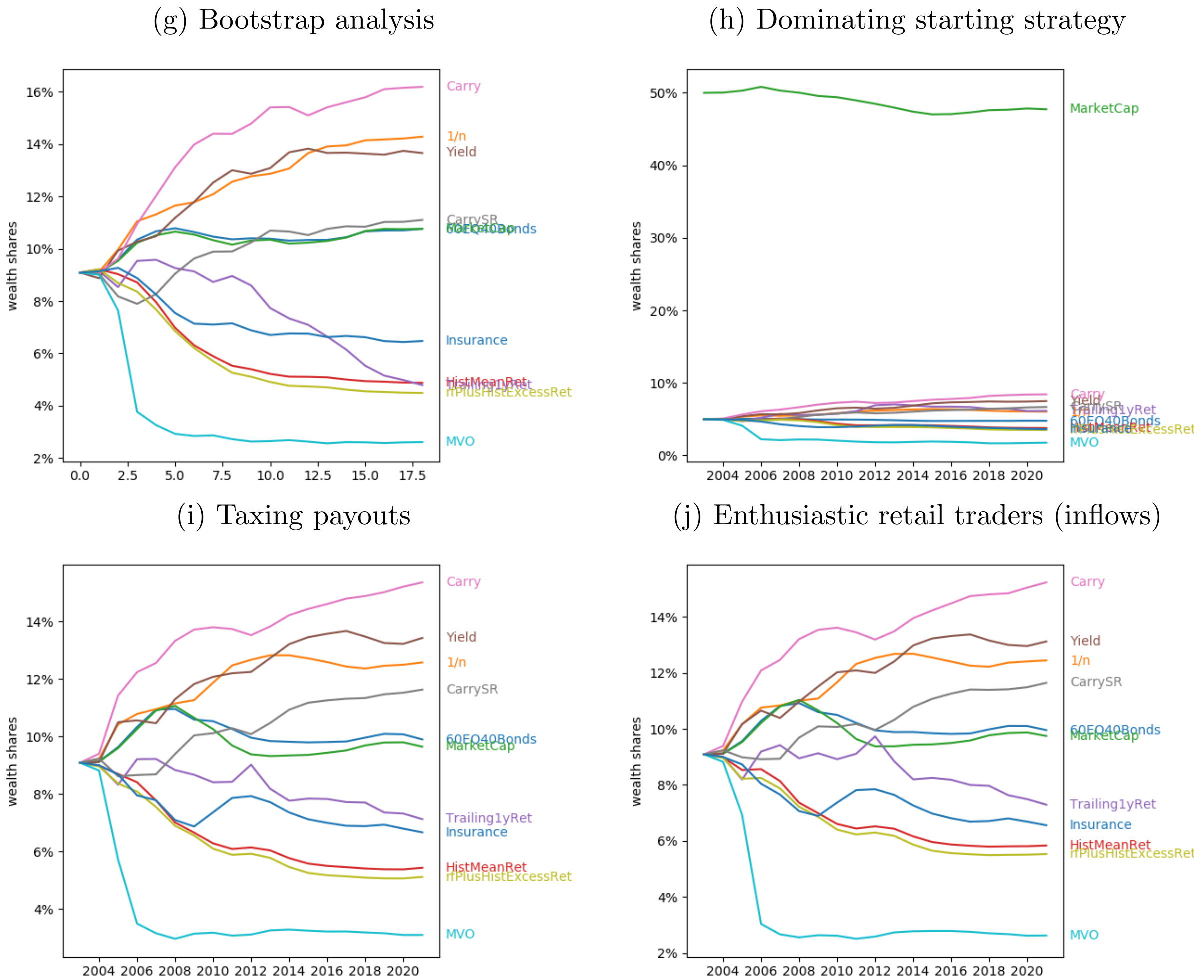


Figure 3: Robustness checks, continued



Removing one strategy Ultimately, including the relevant strategies is crucial. Although the strategies were carefully chosen with maximum representativeness in mind, it is important to examine how strongly the results depended on the strategy universe.

Table 1 shows that especially the winning (and losing) few strategies did not change substantially if a formerly present asset allocation variant was removed from the sample. Strategic asset allocations based on yields and payouts clearly remained dominant.

5 Conclusion

Evolutionary portfolio theory offers a fascinating solution to one of the most prolific problems in asset allocation. By allowing for interaction between investment strategies, it endogenizes prices and thereby facilitates a close look at the inner workings of financial markets. Moreover, it presents investors with a tool to test their strategic asset allocation's prowess in a dynamic and competitive environment.

Table 1: Removing one strategy

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1) 60/40		8	6	3	5	9	10	7	2	1	4
(2) 1/n	7		6	3	5	9	10	8	2	1	4
(3) MarketCap	6	8		3	5	9	10	7	2	1	4
(4) HistMeanRet	6	8	5		2	9	10	7	3	1	4
(5) Trailing1yRet	8	9	7	5		6	10	3	4	1	2
(6) Yield	7	8	6	3	5		10	9	2	1	4
(7) Carry	7	8	6	3	5	10		9	2	1	4
(8) CarrySR	7	8	6	3	5	9	10		2	1	4
(9) rfPlusHistExcessRet	6	8	5	3	2	9	10	7		1	4
(10) MVO	6	8	5	2	4	9	10	7	1		3
(11) Insurance	6	8	5	3	4	9	10	7	2	1	

Highest rank corresponds to highest terminal wealth.

This paper has shown that value matters also for strategic asset allocation. Be it yields or carry-based metrics, a fundamental anchor is conducive to an investment strategy. The paper thereby also confirmed a theoretical result in evolutionary portfolio theory. The surving strategy in Evstigneev et al. (2006) for example allocated according to expected relative dividends. Similar allocation rules performed best in the multi-asset case studied here.

Using only observable market data (i.e. no assumptions on utility functions or investor's beliefs), this paper provides a novel and valuable vantage point for asset allocation decisions. Although complex mathematics and game theoretical concepts underpin evolutionary portfolio theory's analytical framework, it can be modelled relatively straightforward and in an easy to understand fashion which lends itself to applications in practice.

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