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Understanding Asset Correlations

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UNDERSTANDING ASSET CORRELATIONS

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Abstract

We study low-frequency movements in US stock-bond correlations. Estimating a regime-switching model, we find an inverse relation between stock-bond correlations and correlations of growth and inflation. We document that inflation uncertainty predicts stock-bond correlations positively when inflation is countercyclical and negatively when inflation is procyclical. While a rise in inflation uncertainty always lowers stock prices, it lowers nominal bond prices in times of stagflation but raises them when inflation is procyclical. Our main point is to show how the time-varying correlation of growth and inflation affects asset prices and correlations. We rationalize our findings in a long-run risk model featuring non-neutral inflation shocks and regime shifts, allowing for countercyclical and procyclical inflation regimes. The model also provides a rational explanation for the comovement of dividend yields and nominal yields, often named the Fed-model. Furthermore, the model can produce an upward-sloping real yield curve. Finally, we document that inflation and monetary policy shocks were important drivers of stock-bond correlations during the countercyclical inflation period 1965-2000 while output shocks dominated during the procyclical inflation period, 2000-2011.

Keywords: fed-model, inflation, long-run risks, money illusion, regime-switching, stock-bond correlation

JEL Classification Number: E43, E44, G12.

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1 Introduction

The unconditional correlation between returns on US stocks and nominal government bonds has been close to zero in post-war data but has varied substantially over time as shown in Figure 1. From being positive throughout most of the pre-2000 period, correlations turned sharply negative in the early 2000's. We document that these changes in asset correlations coincide with changes in several other relations between inflation, inflation uncertainty, and asset prices. We provide new empirical evidence showing that these shifts are related to changes in the cyclical nature of inflation. The main point of our paper is to show how the correlation of growth and inflation varies over time and how this affects the relation between inflation and asset prices and stock-bond correlations.

We estimate a regime-switching model on real consumption growth, inflation, stock, and bond excess returns for the period 1965-2011 and find a significant inverse relation between stock-bond correlations and correlations of growth and inflation. Stock-bond correlations are positive in times of stagflation but negative when inflation and growth are positively correlated. In particular, the large shift around 2000 when stock-bond correlations turned sharply negative coincided with inflation turning procyclical after having been countercyclical for several decades. Figure 2 exemplifies this inverse relation by plotting stock-bond correlations against growth-inflation correlations.[1]

We also provide new evidence showing that the effect of inflation uncertainty on stock-bond correlations can be both positive and negative depending on the inflation regime. We find that inflation uncertainty predicted stock-bond correlations positively in the countercyclical inflation period 1965-2000, but negatively in the procyclical inflation period 2000-2011. While the effect of inflation uncertainty on stock prices was negative throughout our sample period, its impact on nominal bond prices switched sign from negative to positive around 2000. We believe this is a novel finding suggesting that inflation uncertainty lowers nominal bond prices when inflation is countercyclical but raises bond prices when inflation is procyclical. We do not claim that inflation uncertainty is the only driver of stock-bond correlations.[2] However, we believe it is an important

[1]Correlations are based on quarterly data and computed for non-overlapping five-year periods. Our inflation measure uses the price index for nondurables and services provided by the Bureau of Economic Analysis. Our results are robust to using the Consumer Price Index and the Core-Consumer Price Index which means our findings are not driven by volatile food or energy prices.

[2]For example, Baele et al. (2010) find that liquidity factors play an important role.

factor since it can produce both positive and negative stock-bond correlations.

We also contribute to the literature on the comovement of nominal yields and dividend yields, often named the Fed-model.[3] We find that also the correlation of dividend yields and nominal yields is inversely related to the correlation of growth and inflation. Figure 3 exemplifies this. From being positively correlated 1965-2000, correlations of dividend yields and nominal yields turned sharply negative in the early 2000's at the same time as inflation turned procyclical. Since nominal yields move with inflation, this suggests that the relation between equity valuations and inflation has changed considerably over time and seems to coincide with changes in the cyclical nature of inflation.

So is there a plausible economic story for the empirical observations described above? Yes. Theory suggests that nominal bonds are risky when inflation is countercyclical, since it implies procyclical returns, but provide a hedge against bad times when inflation is procyclical. An increase in inflation risk should therefore raise bond risk premia and yields in the first case while lowering risk premia and yields in the second case. Equity risk premia should load positively on inflation risk if inflation is bad for growth and equity returns, producing procyclical stock returns. The same holds if inflation is positively associated with growth and stock returns since high inflation then coincides with low marginal utility of investors and high stock returns. Equity risk premia should therefore load positively on inflation risk while nominal bond risk premia and yields can load negatively or positively depending on the cyclical state of inflation. We find empirical support for this in data.

Our empirical findings suggest that expected inflation and inflation volatility are important drivers of both equity and bonds. We therefore choose to rationalize our findings using the long-run risk framework since it emphasizes time-varying macroeconomic expectations and volatilities. The main setup follows Bansal and Yaron (2004) but we introduce two novel features that are crucial for our results and well-supported by data. First, inflation shocks are assumed to be non-neutral, affecting real growth. Inflation therefore has a direct impact on real asset prices and on equity and bond risk premia which both vary with inflation volatility. Second, we introduce a regime-

[3]See for example Ritter and Warr (2002), Campbell and Vuolteenaho (2004), Cohen et al. (2005), and Bekaert and Engstrom (2010).

switching mechanism that allows for countercyclical and procyclical inflation regimes. This implies that inflation can be associated with both bad and good economic times in the model. Both equity and bonds are risky in the countercyclical regime which makes equity and bond risk premia move positively with inflation volatility, producing positive stock-bond correlations. However, equity and bond risk premia are negatively related in the procyclical regime as nominal bonds act as a hedge while stocks are still risky. This generates negative stock-bond correlations. The same mechanism also allows the model to produce switching correlations between dividend yields and nominal yields. Overall, the model matches the changing asset and macro correlations we observe in data while being consistent with a range of unconditional macro and asset-price moments.

Interestingly, the theoretical model can produce an upward-sloping real term structure in contrast to traditional long-run risk models.[4] In a single-regime model, the average slope is determined by the average risk premium. With multiple regimes, however, expectations of future short rates also matter for the average slope. For example, standing in a low-short rate regime, the "expectations part" of the yield curve is positive as long as there is a non-zero probability of jumping to a high-short rate state. Short rates were low in the procyclical period 2000-2011 but high in the countercyclical period 1965-2000. The model therefore implies a positive (negative) expectations part of the real yield curve post 2000 (pre 2000). Risk premiums on real bonds are negative in both regimes but the positive expectation part outweighs the negative risk premium post 2000. Hence, the model generates a positively sloped real yield curve for the 2000-2011 period. This is different from existing long-run risk models which generate a downward-sloping real yield curve since the slope is exclusively determined by the negative real bond risk premia.[5]

Identifying the source behind changes in macro and asset correlations is an important question. We estimate a VAR model of inflation, real output growth, the Federal funds rate, and the stock-bond covariance for the two sub-periods pre 2000 and post 2000. We impose a simple recursive orthogonalization scheme using the Cholesky factor in order to interpret shocks to the Federal

[4]The real yield curve has been upward sloping in the US. However, real bonds only started trading in 1997 so the sample period is short and the market was initially highly illiquid. Long-term evidence from UK real bonds suggest a downward-sloping real yield curve (e.g., Piazzesi and Schneider, 2006).

[5]Eraker et al. (2011) generate an upward sloping real yield curve in a long-run risk framework through a different channel by introducing durable consumption good risk.

funds rate as structural monetary policy shocks. We find that shocks to inflation and monetary policy were important drivers of stock-bond covariances during the countercyclical period 1965:1-1999:4 while output shocks clearly dominated during the procyclical period 2000:1-2011:4. Hence, the relative magnitude of nominal versus real shocks in the economy seem to be an important determinant of the different regimes.

While early contributions focussed on unconditional stock-bond correlations (e.g., Shiller and Beltratti, 1992, and Campbell and Ammer, 1993), the focus has shifted towards understanding conditional correlations. Connolly et al. (2005) document that a rise in stock market uncertainty predicts stock-bond correlations negatively. Baele et al. (2010) find that macro factors have limited success in explaining the time-varying stock-bond correlation. Campbell et al. (2010) estimate a quadratic term-structure model with latent state variables of which one captures the covariance between inflation and the real pricing kernel. They identify this state variable through the observed stock-bond covariance and describe how it impacts bond risk premia. The authors provide a nice intuition for how the cyclicality of inflation might affect the riskiness of nominal bonds. However, it is never actually shown using fundamental data whether there exists different inflation regimes in data and whether such regimes line up with different regimes in stock-bond correlations. David and Veronesi (2009) explore the role of learning about inflation and real earnings for the variance and covariance of stock and bond returns. Their model is successful in predicting second moments of returns and emphasizes cash flow effects and uncertainty about the current state of the economy while keeping market prices of risk constant.[6]

In contrast to these papers, we provide several new pieces of empirical evidence that highlight the impact of a changing covariance between growth and inflation on asset prices. For example, we explicitly estimate changes in the cyclicality of inflation from data and link it to asset prices. We show how the different inflation regimes determine how inflation uncertainty impacts asset prices and how this can explain the changing asset correlations. Furthermore, we motivate our findings using a consumption-based equilibrium model which means asset prices are directly tied to fundamental macro factors such as consumption growth and inflation.

[6]See also Bekaert et al. (2010) who discuss stock-bond correlations in an external-habit framework.

The so called Fed-model refers to the, on average, positive relation between dividend yields and nominal yields in post-war data. The positive relation has puzzled many observers. Why should a real variable, like the dividend yield, move with nominal interest rates? Since nominal rates rise with inflation, inflation must be negatively related to real dividend growth rates and/or positively associated with expected equity returns in order to explain a positive dividend yield-nominal yield correlation. Virtually all papers in this literature rule out a rational explanation for the link between inflation and dividend yields and rely instead on inflation illusion in which irrational investors basically discount real cash flows with nominal discount rates (Modigliani and Cohn, 1979).[7] An increase in inflation raises discount rates and lowers stock prices, producing a positive correlation between dividend yields and nominal yields. However, this is inconsistent with the large negative correlations observed post 2000 and during the 1930's. This raises questions about the validity of the inflation illusion explanation. Instead, we provide a rational explanation based on the changing effects of inflation uncertainty on stocks and bonds stemming from the time-varying relation between growth and inflation. Our theoretical model matches the large shifts in correlations observed in data.

This paper is also related to the vast literature on stock returns and inflation. An incomplete list includes Fama (1981), Geske and Roll (1983), Stulz (1986), Kaul (1987), Marshall (1992), and Boudoukh (1993).[8] Recently, Bekaert and Wang (2010) study the relation between inflation and stock returns across a large number of countries and find evidence of predominantly negative inflation betas. More generally, we build on the literature of pricing stocks and bonds in equilibrium using the recursive preferences of Epstein and Zin (1989) and Weil (1989) (e.g., Campbell, 1993, 1996, 1999, Duffie et al., 1997, and Restoy and Weil, 1998). Related papers that use the long-run risk framework are Piazzesi and Schneider (2006), Eraker (2008), Bansal and Shaliastovich (2010), and Hasseltoft (2012).[9]

[7] A notable exception is Bekaert and Engstrom (2010) who argue that rational mechanisms are at work and ascribe the positive correlation between dividend yields and nominal yields to the large incidence of stagflation in US data. They document a positive relation between expected inflation and proxies for the equity risk premium.

[8] Note that Fama's so called proxy hypothesis is distinct from this paper since we stress the link between inflation and risk premiums rather than inflation and cash flows.

[9] Our paper is also related to the literature on regime-switching models in equity and bond markets. For example, Cecchetti et al. (1990), Ang and Chen (2002), Bibkov and Chernov (2008), Lettau et al. (2008), Constantinides and Ghosh (2011) study regime-switching models for equities while Hamilton (1988), Gray (1996), Evans (1998), Ang

Our article proceeds as follows. Section 2 describes our data and provides new empirical evidence. Section 3 rationalizes our empirical findings using a long-run risk model that incorporates non-neutral inflation shocks and a regime-switching mechanism allowing for both counter- and procyclical inflation regimes. Section 4 describes the calibration of the theoretical model and its implications for a range of unconditional macro and asset-price moments. Section 5 describes in detail implications for bond risk premia, slope of the term structure, equity risk premia, and asset correlations in the model. Section 6 estimates shocks to inflation, output, and monetary policy and studies how they impact stock-bond covariances. Section 7 concludes.

2 Empirical Analysis

2.1 Data

Quarterly aggregate US consumption data for the period 1965:1-2011:4 and annual data for the period 1930-2011 on nondurables and services is collected from the Bureau of Economic Analysis. Real consumption growth and inflation are computed as in Piazzesi and Schneider (2006) using the price index that corresponds to the consumption data. Value-weighted market returns (NYSE/AMEX) are retrieved from CRSP. Nominal interest rates and bond returns are collected from the Fama-Bliss files in CRSP. Price-dividend ratios are formed by imputing dividends from monthly CRSP returns that includes and excludes dividends (e.g., Bansal et al., 2005). Quarterly dividends D_t are formed by summing monthly dividends. Due to the strong seasonality of dividend payments, we use a four-quarter moving average of dividend payments, $\bar{D}_t = \frac{D_t+D_{t-1}+D_{t-2}+D_{t-3}}{4}$. Real dividend growth rates are found by taking the log first difference of $\bar{D}_t$ and deflating using the constructed inflation series. Data on industrial production and real GDP are obtained from the St. Louis FRED database and from GlobalFinancialData respectively.

and Bekaert (2002b, 2002c), Bansal and Zhou (2002), Bansal et al. (2004), Dai et al. (2007), and Ang et al. (2008) all study bond markets. Ang and Bekaert (2002a) and (2004) study asset allocation in a regime-switching framework.

2.2 Empirical Evidence

The introduction of the paper provided motivating examples for the inverse relation between asset and macro correlations found in data. Table 1 reports the actual correlation coefficients between growth, inflation, and asset returns. Motivated by the preceding figures, we choose to break the sample in two parts, prior to and after 2000.[10] The table shows that inflation and growth were negatively correlated prior to 2000, -0.42, but comoved positively thereafter, 0.30. The correlations between stock and bond returns also underwent a significant change, from 0.28 pre 2000 to -0.63 thereafter. Interestingly, the correlation between inflation and stock returns also changed from -0.20 to 0.27 after 2000. Overall, this suggests that several relations switched around year 2000; inflation turned procyclical, stock-bond correlations turned sharply negative and equity switched to act as an inflation hedge.

In Table 2, we formally test whether the difference in unconditional correlation coefficients pre and post 2000 are statistically significant using a Jennrich (1970) test. We consider four different correlation matrices; macro variables and asset returns jointly, only macro variables, only asset returns, and only stock returns and inflation. The table shows that we can clearly reject the null hypothesis of equal correlation coefficients across subsamples for all specifications. It has been argued in the literature that simply splitting the sample ex-post is not a perfectly reliable method for identifying regime switches as it might lead to a selection bias (e.g., Boyer et. al, 1999, and Chesnay and Jondeau, 2001). We therefore, later in this section, estimate a regime-switching model that explicitly allows us to formally identify breakpoints between countercyclical and procyclical inflation regimes.

To further validate the inverse relation between macro and asset correlations, we take a long-term perspective by considering annual consumption growth and inflation starting in 1930 in Figure 4. Visual inspection suggests that the comovement between inflation and growth has varied considerably. In particular, the 1930's experienced a strong positive comovement as The Great Depression was associated with low growth coupled with deflation. The positive correlation was further exacerbated by the strong rebound in growth and inflation starting in 1933. In contrast, the US economy

[10] We estimate a more formal breakpoint later in this section.

underwent a stagflationary period in the 1970's and early 1980's. Growth and inflation started to comove positively again around the year 2000.

Figure 5 computes 10-year correlations between growth and inflation and between dividend yields and inflation for the period 1930-2011.[11] First, the graph quantifies the considerable variation in the cyclicality of inflation. From a correlation of 0.80 in the 1930's, correlations reached -0.80 during the 1970's and were then close to 0.60 in the 2000's. Second, the graph indicates a clear inverse relation between macro correlations and correlations of dividend yields and inflation for this extended time period.

We also provide new evidence showing that not only macro and asset correlations switched sign in the early 2000's but also several other relations between inflation and asset prices. Figure 6 shows that the relation between the level of expected inflation and price-dividend ratios switched sharply from negative to positive around 2000.[12] This is in line with our earlier finding that stock returns were positively related to inflation post 2000. This stands in contrast to the common finding that stocks are a poor inflation hedge. Interestingly, the same figure shows that inflation risk has been negatively related to price-dividend ratios throughout the entire sample period.[13] Hence, stock prices seem to respond differently to changes in the level of inflation and changes in inflation risk. The model we present later provides a rational explanation for this phenomenon.

Table 3 reports results from regressing log price-dividend ratios onto expected inflation and inflation risk for the full sample and for the two subsamples. First, the two variables explain a large part of the variation in price-dividend ratios with R^2s around 50%. Second, the regression coefficient for expected inflation switches sign to positive in the procyclical state, albeit not statistically significant. Third, inflation risk is consistently negatively related to price-dividend ratios with a high statistical significance. Hence, the table suggests that while the relation between equity-valuation ratios and the level of inflation may change sign, an increase in inflation risk always

[11] Since interest rates were not market determined prior to the Treasury-Fed accord in 1951, we choose to focus on the relation between equity and inflation as opposed to using bond returns or interest rates.

[12] Expected inflation is measured as the fitted value from projecting quarterly inflation onto lagged growth, inflation, and yield spread.

[13] Inflation risk is measured as the dispersion of inflation forecasts based on the GDP price deflator. The online Appendix shows that our general results are robust to using the dispersion of inflation forecasts based on CPI or the conditional volatility of inflation estimated from a GARCH(1,1) model.

depresses equity prices.

Conventional wisdom suggests that an increase in inflation risk should raise nominal yields and affect bond returns negatively. Figure 7 shows that this is not always true. In fact, inflation risk and nominal interest rates were *negatively* correlated during the periods 1985-1990 and 2000-2011. We believe this is a novel finding and raises the question of what the underlying mechanism is. The answer is, as discussed above, that the riskiness of nominal bonds depends on whether inflation is counter- or procyclical. Since nominal bonds are risky assets in periods of countercyclical inflation, their returns will suffer in periods of high inflation risk. Conversely, nominal bonds provide a hedge against bad times when inflation is procyclical. This makes their returns positively related to inflation risk. Table 4 supports these findings by reporting results from regressing the 5-year nominal interest rate onto expected inflation and inflation risk. We find that the regression coefficient for inflation risk is positive in the countercyclical inflation regime but switches sign to negative in the procyclical state.[14]

Consistent with our finding that inflation risk affects asset prices differently depending on the inflation regime, we find that inflation risk predicts stock-bond correlations positively when inflation is countercyclical but negatively when inflation is procyclical. Table 5 reports results from regressing quarterly stock-bond covariances onto lagged inflation risk. While it has been documented elsewhere that inflation volatility predicts the stock-bond covariance positively (e.g. Viceira, 2010), we are not aware of any paper showing that this relation can switch sign.

To formally analyze the relation between macro variables and asset returns and to identify potential regime switches, we estimate a two-state Markov-switching (MS) model. We assume that quarterly real consumption growth, inflation, excess stock returns, and excess bond returns follow a one-lag vector autoregression:

$$Y_{t+1} = \mu(s_{t+1}) + \beta(s_{t+1})Y_t + \epsilon_{t+1}, \tag{1}$$

[14]This result does not depend on the maturity of the bond and is robust to using a different measure of expected inflation as control variable in the form of the long-run mean of inflation, as shown in the Appendix. Borrowing from the literature on adaptive learning, this measure is computed as $\frac{\sum_{i=0}^{t-1} v^i \pi_{t-i}}{\sum_{i=0}^{t-1} v^i}$ where v is set equal to 0.98 and where we consider a backward looking period of 40 quarters. Similar results are also obtained when using inflation expectations from Survey of Professional Forecasters.

where $Y_{t+1} = [\Delta c_{t+1}, \pi_{t+1}, r_{s,t+1}, r_{b,t+1}]'$, $\epsilon_{t+1} \sim N(0, \Omega(s_{t+1}))$ and where the regime s_{t+1} is presumed to follow a two-state Markov chain with transition probabilities $p_{ij} = P(s_{t+1} = j | s_t = i)$. The probability of ending up in tomorrow's regime $s_{t+1} = (0,1)$ given today's regime $s_t = (0,1)$ is governed by the transition probability matrix of a Markov chain:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix},$$

where $\sum_{j=0}^{1} p_{ij} = 1$ and $0 < p_{ij} < 1$.

Stock returns refer to the NYSE/AMEX portfolio available from CRSP and bond returns refer to the 5 to 10-year US Treasury Fama bond portfolio also obtained from CRSP. All variables are observed quarterly. The companion matrix β, the μ matrix and the variance-covariance matrix Ω are assumed to follow the same Markov chain process yielding two possible states in total.[15]

The estimation results are reported in Table 6. There are three key takeaways from this table. First, the effect of inflation on future growth (element (1,2) in β) and the covariance between growth and inflation shocks (element (1,2) in Ω) both switch from negative to positive in the second state. Although estimates for the second state are subject to rather large standard errors, the numbers are consistent with the fact that the correlation between growth and inflation turned positive in the second state. Second, the covariance between shocks to stock and bond returns changed even more dramatically from positive to negative in the second state (element (3,4) in Ω), where both covariance terms are statistically significant. This is consistent with our earlier evidence that correlations between stocks and bonds turned sharply negative in the early 2000's. Third, the covariance between shocks to inflation and stock returns (element (2,3) in Ω) switched from negative to positive in the second state, consistent with our earlier evidence that stocks seem to have provided an inflation hedge throughout the 2000's. We report in the Appendix that the unconditional correlation coefficients implied by our estimated Markov-switching model are close to data and that a Jennrich (1970) test of these implied correlations suggest we can reject the null

[15]The setup follows Hamilton (1989,1994) among others. We have estimated various MS-VAR specifications allowing for a larger number of states, more lags in the VAR, etc.. We have also elaborated with time-varying transition probabilities. They all yield similar qualitative conclusions.

hypothesis of equal correlations across subsamples.

The transition probabilities, p_{11} and p_{22}, are estimated to 0.977 and 0.939. This implies unconditional probabilities of 0.73 and 0.27 for being in the counter- versus procyclical state. The estimated probabilities imply a longer expected duration for the countercyclical state, 43 quarters, versus 16 quarters for the procyclical inflation state.[16] Next, we test the null hypothesis that conditional correlations are constant, $\rho(s_t = 0) = \rho(s_t = 1)$, across regimes using a Likelihood Ratio test. Our system with four variables yields six restricted correlation coefficients. Table 7 reports that we can clearly reject the null of constant conditional correlations across regimes.

Using our estimation results we compute the implied probability of being in the procyclical regime at each time t. We consider both filtered regime probabilities that use information up to time t and smoothed regime probabilities that use information from the entire sample. The probabilities are plotted in Figure 8. We find that a large part of the sample period is dominated by the countercyclical state. We find a temporary jump to the procyclical state in the late 1980's and then a more long-lasting move to a procyclical state in the early 2000's. Large opposite shocks to growth and inflation in the fourth quarter of 2007 explain the large drop in smoothed probabilities at the end of the sample. However, a large deflationary shock together with a sharp drop in growth in the fourth quarter of 2008 explains the reversal in probabilities. We find that the smoothed probability exceeds 0.50 in the second quarter of 2001 wherefore we treat this as the formally estimated breaking point between the two inflation regimes.

From the estimated regime-switching model we compute conditional correlations of growth and inflation and of stock and bond returns and plot them in Figure 9. First, the relative movements of the two lines suggest a clear inverse relation, particulary in the late 1980's, the early 2000's, and recently around the financial crisis. Second, the absolute movements of the correlations are also consistent with our earlier evidence that macro and asset correlations underwent a significant shift in the early 2000's.

Overall, we have shown that a number of relations between inflation and asset prices have switched sign over time. Our evidence suggests that the switch in signs all occurred at the same

[16] Considering two states, i and j, the unconditional probability of being in state j is computed as $\frac{1-p_{ii}}{2-p_{jj}-p_{ii}}$. The expected duration of state i is computed as $\frac{1}{1-p_{ii}}$.

time, namely around 2000. We have shown that these observations line up with a contemporaneous switch in the correlation of growth and inflation, from negative to positive.

3 A Long-Run Risk Model with Switching Inflation Regimes

This section presents a consumption-based model with a representative agent that provides a rational explanation for our empirical findings while matching a range of important macro and asset-price moments. The model builds on the so-called long-run risk literature which relies on Epstein and Zin (1989) and Weil (1989) recursive preferences, persistent macro shocks, and time-varying macroeconomic volatility. The original long-run risk model of Bansal and Yaron (2004) relies on persistent shocks to expected consumption growth which together with recursive preferences produces sizeable equity risk premia. Inflation plays no role in that model.

In contrast, this paper focusses on long-run shocks to inflation and their effect on growth and asset prices. The model contains two additional key features compared to the standard long-run risk model: First, long-run inflation shocks are non-neutral and impact real economic growth directly. This allows inflation to have a direct impact on the real pricing kernel and therefore on both equity and bond risk premia. As a result, expected returns on equity and bonds vary with the conditional variance of inflation. Second, we incorporate a Markov-switching regime mechanism that allows the relation between real growth and inflation to switch sign. This allows the model to produce both a counter and procyclical inflation regime. We find these ingredients to be crucial for explaining the switching relations between inflation and asset prices in general and between stock and bond returns in particular. In general, we find that these two main distinctions from the original long-run risk model open up a range of novel asset-pricing implications.

3.1 Macro Dynamics

Let $\Delta c_{t+1}, \pi_{t+1}$, Δd_{t+1}, and $\sigma^2_{\pi,t+1}$ denote the logarithmic consumption growth, inflation, dividend growth, and the conditional variance of inflation respectively. Let μ_c, μ_π, and μ_d denote the unconditional means and let x_c and x_π denote the time-varying parts of the conditional means. We consider an economy subject to regime-shifts between two possible states. Tomorrow's regime is denoted $s_{t+1} = (0,1)$ and the probability of ending up in tomorrow's regime given today's regime $s_t = (0,1)$ is governed by the transition probability matrix of a Markov chain:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix},$$

where $P(s_{t+1} = j | s_t = i) = p_{ij}, \sum_{j=0}^{1} p_{ij} = 1$ and $0 < p_{ij} < 1$. We assume that agents are able to observe the current regime.

We assume the following macro dynamics:

$$\Delta c_{t+1} = \mu_c(s_{t+1}) + x_{c,t} + \sigma_c \eta_{c,t+1}, \tag{2}$$

$$\pi_{t+1} = \mu_\pi(s_{t+1}) + x_{\pi,t} + \sigma_{\pi,t} \eta_{\pi,t+1}, \tag{3}$$

$$\Delta d_{t+1} = \mu_d + \phi x_{c,t} + \varphi \sigma_c \eta_{d,t+1}, \tag{4}$$

$$\sigma^2_{\pi,t+1} = \sigma^2_\pi + v_\pi \left(\sigma^2_{\pi,t} - \sigma^2_\pi \right) + \sigma_\nu w_{t+1}, \tag{5}$$

$$\begin{pmatrix} x_{c,t+1} \\ x_{\pi,t+1} \end{pmatrix} = \begin{pmatrix} \beta_1(s_{t+1}) & \beta_2(s_{t+1}) \\ 0 & \beta_4(s_{t+1}) \end{pmatrix} \begin{pmatrix} x_{c,t} \\ x_{\pi,t} \end{pmatrix} + \begin{pmatrix} \delta_1(s_{t+1}) & \delta_2(s_{t+1}) \\ \delta_3(s_{t+1}) & \delta_4(s_{t+1}) \end{pmatrix} \begin{pmatrix} \sigma_c \varepsilon_{c,t+1} \\ \sigma_{\pi,t} \varepsilon_{\pi,t+1} \end{pmatrix}, \tag{6}$$

where all shocks are uncorrelated, i.i.d. normally distributed with a mean of zero and a variance of one. The β and δ matrices plus μ_c and μ_π depend on tomorrow's regime $s_{t+1} = (0,1)$. We set element (2,1) in β equal to zero for parsimonious and tractability reasons. This does not affect our qualitative findings. We keep the volatility parameters and the dividend growth parameters constant across regimes since we are mainly interested in the interaction between growth and inflation.

The presence of regime-shifts, inflation, and inflation volatility are new compared to the specification in Bansal and Yaron (2004). While the process for dividend growth is identical to the original long-run risk model, the specification for realized and expected growth in Bansal and Yaron (2004) using our notation would be: $\Delta c_{t+1} = \mu_c + x_{c,t} + \sigma_c \eta_{c,t+1}$ and $x_{c,t+1} = \beta_1 x_{c,t} + \delta_1 \sigma_c \varepsilon_{c,t+1}$.

The volatility of inflation $\sigma^2_{\pi,t+1}$ varies over time and gives rise to time-varying risk premiums as shown in Section 5.[17] The notion of heteroscedasticity in inflation is a well established empirical fact; early contributions include Engle (1982) and Bollerslev (1986). The state variables of the model are consequently $x_{c,t}$, $x_{\pi,t}$, and $\sigma^2_{\pi,t}$. The conditional means of consumption growth and inflation are related through β_2 which means that expected inflation feeds into future expectations of growth. β_2 allows real bonds and price-dividend ratios, in addition to nominal bonds, to be functions of expected inflation. Furthermore, β_2 creates a direct link between expected inflation and the real pricing kernel which means that inflation affects both bond and equity risk premiums. Since β_2 is subject to regime shifts, it can take on both positive and negative values which is important for matching the changing growth-inflation and asset correlations we observe in data.

This is the most parsimonious specification that allows the model to match both unconditional and conditional macro and asset-price moments. By restricting a number of parameters to be constant across regimes we make it harder for the model to match the data. One could of course relax several of the restrictions. For example, we could allow for a non-zero β_3, i.e. an interaction between $x_{c,t}$ and $x_{\pi,t+1}$. We could allow dividend growth parameters, volatility parameters, or the entire variance-covariance matrix of growth and inflation to vary with time. Hasseltoft (2009) contains some of these relaxations. Overall, giving more flexibility to the model does not change our main qualitative results.

3.2 Investor Preferences

The representative agent in the economy has Epstein and Zin (1989) and Weil (1989) recursive preferences:

$$U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \tag{7}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, $\gamma \geq 0$ denotes the risk aversion coefficient and $\psi \geq 0$ the elasticity of intertemporal substitution (EIS). The discount factor is represented by δ. This preference specification allows time preferences to be separated from risk preferences. This stands in contrast to time-separable

[17] The typical long-run risk specification relies on time-varying volatility of consumption growth. For parsimonious reasons, we have restricted time-variation in second moments to inflation since our focus is mainly on the impact of inflation on asset prices.

expected utility in which the desire to smooth consumption over states and over time are interlinked. The agent prefers early (late) resolution of risk when the risk aversion is larger (smaller) than the reciprocal of the EIS. A preference for early resolution and an EIS above one imply that $\theta < 1$. This specification nests the time-separable power utility model for $\gamma = \frac{1}{\psi}$ (i.e., $\theta = 1$).

The agent is subject to the budget constraint $W_{t+1} = R_{c,t+1}(W_t - C_t)$, where the agent's total wealth is denoted W_t, $W_t - C_t$ is the amount of wealth invested in asset markets and $R_{c,t+1}$ denotes the gross return on the agents total wealth portfolio. This asset delivers aggregate consumption as its dividends each period.

Following Epstein and Zin (1989) plus acknowledging the presence of regime-shifts, the logarithm of the stochastic discount factor (SDF) can be written as:

$$m_{t+1}(s_{t+1}) = \theta \ln(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} - (1-\theta)\, r_{c,t+1}(s_{t+1}), \tag{8}$$

where $\ln R_{c,t+1} = r_{c,t+1}$. Note that the SDF depends on both consumption growth and on the return from the total wealth portfolio. Recall that $\theta = 1$ under power utility, which brings us back to the standard time-separable SDF.

3.3 Solving the Model

Returns on the aggregate wealth portfolio and the market portfolio are approximated as in Campbell and Shiller (1988):

$$r_{c,t+1}(s_{t+1}) = k_{c,0} + k_{c,1} pc_{t+1}(s_{t+1}) - pc_t(s_t) + \Delta c_{t+1}, \tag{9}$$

$$r_{m,t+1}(s_{t+1}) = k_{d,0} + k_{d,1} pd_{t+1}(s_{t+1}) - pd_t(s_t) + \Delta d_{t+1}, \tag{10}$$

where pc_t and pd_t denote the log price-consumption ratio and the log price-dividend ratio.[18] The constants k_c and k_d are functions of the average level of pc_t and pd_t, denoted $\bar{pc}$ and $\bar{pd}$.[19]

[18] Bansal et al. (2007a) show that the approximate analytical solutions for the returns are close to the numerical solutions and deliver similar model implications.

[19] Specifically, the constants are $k_{c,1} = \frac{\exp(\bar{pc})}{1+\exp(\bar{pc})}$ and $k_{c,0} = \ln(1 + \exp(\bar{pc})) - k_{c,1}\bar{pc}$ and similarly for the k_d coefficients.

3.4 Solving for Equity

All asset prices and valuation ratios are conjectured to be functions of the time-varying conditional means of consumption growth and inflation plus the time-varying conditional variance of inflation. Starting with the log price-consumption ratio, it is conjectured to be a linear function of the state variables as follows:

$$pc_t(s_t) = A_{c,0}(s_t) + A_{c,1}(s_t)x_{c,t} + A_{c,2}(s_t)x_{\pi,t} + A_{c,3}(s_t)\sigma^2_{\pi,t}. \tag{11}$$

The regime dependence of the coefficients plus the existence of $A_{c,2}$ and $A_{c,3}$ are new compared to Bansal and Yaron (2004). $A_{c,2}(s_t)$ and $A_{c,3}(s_t)$ arise from the fact that inflation has a direct impact on real economic growth through $\beta_2(s_t)$ and $\delta_2(s_t)$. In order to analytically solve for the A-coefficients we make use of the macro dynamics, the law of iterated expectations, and of the Euler equation for the consumption asset. The Appendix describes in detail how we solve the model and contains analytical expressions for the A-coefficients. The expression for $A_{c,1}(s_t)$ collapses to the same expression as in Bansal and Yaron (2004) should we restrict the model to a single regime.

$A_{c,1}(s_t)$ is positive whenever the elasticity of intertemporal substitution (EIS) is greater than one, implying a positive relation between expected growth and asset prices in both regimes. $A_{c,2}(s_t)$ represents the loading of the price-consumption ratio on expected inflation. Its sign depends mainly on $\beta_2(s_t)$ and on the EIS. For $\beta_2(s_t) < 0$, high inflation signals negative growth and will therefore depress the price-consumption ratio, i.e., $A_{c,2}(s_t) < 0$. The opposite holds when $\beta_2(s_t) > 0$ as high inflation then signals positive growth and therefore higher price-consumption ratios, $A_{c,2}(s_t) > 0$. This only holds when the EIS exceeds one meaning that the intertemporal substitution effect dominates the wealth effect. Therefore, higher expected inflation lowers price-consumption ratios in times of stagflation but raises them when inflation is procyclical. In the case of expected utility ($\frac{1}{\psi} = \gamma$), a risk aversion coefficient above one implies that the wealth effect dominates which results in a positive value of $A_{c,2}(s_t)$ when $\beta_2(s_t) < 0$. This is counterfactual since it implies rising asset prices in times of stagflation which is opposite to what we observe in data. $A_{c,3}(s_t)$ is negative in both regimes given a high value of the EIS, indicating a negative relation between price-consumption ratios and inflation volatility. As uncertainty about growth is bad for stock prices in Bansal and Yaron (2004), uncertainty about inflation is bad for stocks in our specification.

In order to understand how macro shocks affect asset prices we consider innovations to the real pricing kernel. To do so, we take expectations using the information set $I_t = \{x_{c,t}, x_{\pi,t}, \sigma^2_{\pi,t}, s_t\}$ which means that the state tomorrow s_{t+1} is uncertain:

$$\begin{aligned} m_{t+1}(s_{t+1}) - E[m_{t+1}(s_{t+1})|I_t] &= -\lambda_{\eta_c}\sigma_c\eta_{c,t+1} - \lambda_{\varepsilon_c}(s_{t+1})\sigma_c\varepsilon_{c,t+1} - \lambda_{\nu}(s_{t+1})\sigma_{\upsilon}w_{t+1} - \lambda_{\varepsilon_\pi}(s_{t+1})\sigma_{\pi,t}\varepsilon_{\pi,t+1} \\ &\quad +V(s_t, s_{t+1}) \qquad (12) \\ \lambda_{\eta_c} &= \gamma \\ \lambda_{\varepsilon_c}(s_{t+1}) &= (1-\theta)[k_{c,1}A_{c,1}(s_{t+1})\delta_1(s_{t+1}) + k_{c,1}A_{c,2}(s_{t+1})\delta_3(s_{t+1})] \\ \lambda_{\nu}(s_{t+1}) &= (1-\theta)k_{c,1}A_{c,3}(s_{t+1}) \\ \lambda_{\varepsilon_\pi}(s_{t+1}) &= (1-\theta)[k_{c,1}A_{c,1}(s_{t+1})\delta_2(s_{t+1}) + k_{c,1}A_{c,2}(s_{t+1})\delta_4(s_{t+1})], \end{aligned}$$

where the λ's represent market prices of risk and where the expression for $V(s_t, s_{t+1})$ arises because tomorrow's state is uncertain as of time t. We report the actual expression for $V(s_t, s_{t+1})$ in the Appendix.

The first two sources of risk are the same as in Bansal and Yaron (2004), namely short run consumption risk and long-run consumption risk which both are positive across regimes. The third shock term reflects shocks to inflation volatility which have a negative price of risk ($\lambda_\nu(s_{t+1}) < 0$) regardless of economic regime. Hence, the representative agent dislikes higher inflation uncertainty in all states of the world. In contrast to existing long-run risk specifications, long-run inflation shocks are priced which makes up the last shock term in Equation (12). A negative value of $\lambda_{\varepsilon_\pi}(s_t)$ implies that the representative agent dislikes positive shocks to expected inflation and therefore requires a positive risk premium on assets that perform badly in periods of high inflation. Note that $\lambda_{\varepsilon_\pi}(s_t)$ can switch sign depending on the regime. It is negative in times of stagflation and positive when inflation is procyclical. Recall that $\theta = 1$ under power utility, which means that long-run inflation risk is not priced and that the only sources of priced risk left are short-run consumption risk λ_{η_c}. Overall, long-run inflation shocks represent an additional risk premium part in the economy compared to models in which only consumption shocks are priced.

The log price-dividend ratio is also conjectured to be a linear function of the three state variables:

$$pd_t(s_t) = A_{d,0}(s_t) + A_{d,1}(s_t)x_{c,t} + A_{d,2}(s_t)x_{\pi,t} + A_{d,3}(s_t)\sigma^2_{\pi,t}. \tag{13}$$

The coefficients are solved for in an analogous manner to the A_c coefficients. The Appendix describes the derivations in detail and reports the full analytical expressions. Again, the expression for $A_{d,1}(s_t)$ collapses to the expression in Bansal and Yaron (2004) should we restrict the model to a single-regime economy. $A_{d,2}(s_t)$ and $A_{d,3}(s_t)$ are new compared to existing long-run risk models and determine the impact of the level of inflation and inflation volatility on equity prices.

Expected consumption growth and price-dividend ratios are positively associated for high values of the EIS meaning that $A_{d,1}(s_t) > 0$ in both regimes. $A_{d,2}(s_t)$ is in general negative when the EIS is above one and when inflation is bad for future growth $\beta_2(s_t) < 0$. This means that high expected inflation depresses equity valuation ratios in periods of countercyclical inflation. In periods of procyclical inflation, $\beta_2(s_t)$ is positive which switches the sign of $A_{d,2}(s_t)$ to positive. This mechanism allows the model to match the switching relation between price-dividend ratios and inflation levels found in data, as was shown in Figure 6 and Table 3.

A rise in inflation volatility has a negative impact on price-dividend ratios ($A_{d,3}(s_t) < 0$) in both regimes provided a high value of the EIS. While the relation between price-dividend ratios and the level of inflation can switch sign in the model, a rise in inflation uncertainty is always bad news for equity valuations. This arises since inflation shocks always contribute to procyclical stock returns in the model regardless of the inflation regime. Hence, equity is always risky with respect to inflation shocks. This is consistent with Figure 6 and Table 3 showing a negative relation between price-dividend ratios and inflation risk in data throughout the entire sample period.

3.5 Solving for Real Bonds

The log price of a real bond with a maturity of n periods is conjectured to be a function of the same state variables as before:

$$q_{t,n}(s_t) \quad = \quad D_{0,n}(s_t) + D_{1,n}(s_t)x_{c,t} + D_{2,n}(s_t)x_{\pi,t} + D_{3,n}(s_t)\sigma^2_{\pi,t}. \tag{14}$$

Let $y_{t,n} = -\frac{1}{n}q_{t,n}$ denote the n-period continuously compounded real yield. Then:

$$y_{t,n}(s_t) \quad = \quad -\frac{1}{n}(D_{0,n}(s_t) + D_{1,n}(s_t)x_{c,t} + D_{2,n}(s_t)x_{\pi,t} + D_{3,n}(s_t)\sigma^2_{\pi,t}), \tag{15}$$

where the D-coefficients determine how yields respond to changes in expected consumption growth, expected inflation, and inflation volatility. The Appendix shows how we solve for the coefficients and reports the expressions.

For plausible parameter values, real yields increase in response to higher expected consumption growth ($D_{1,n}(s_t) < 0$). Consumption shocks therefore generate countercyclical bond returns and contribute to negative risk premiums on real bonds. Real yields decrease in response to higher inflation when inflation is bad news for growth, resulting in a positive $D_{2,n}(s_t)$ coefficient. In this case, inflation shocks also contribute to negative expected returns since they generate positive bond returns in bad inflationary times. This is consistent with earlier studies such as Fama and Gibbons (1982), Pennacchi (1991), and Boudoukh (1993). Ang et al. (2008) also document a negative relation between real rates and expected inflation but find the correlation to be positive for longer horizons. In the procyclical inflation regime, real yields instead move positively with inflation ($D_{2,n}(s_t) < 0$). Hence, the model can accommodate changes in the relation between real interest rates and inflation by accounting for changes in the cyclical nature of inflation.

An increase in inflation uncertainty lowers real yields ($D_{3,n}(s_t) > 0$) with long rates dropping more than short rates. This occurs because inflation risk moves real bonds through a discount-rate channel. When inflation is considered bad news for growth, inflation shocks lower real yields as discussed above and therefore generate high bond returns in bad times. If inflation instead is positively related to growth, inflation shocks raise real yields, generating poor bond returns in good times. In both cases, inflation shocks contribute to countercyclical returns and therefore to a negative risk premium on real bonds. Hence, a rise in inflation volatility is always associated with lower real yields regardless of regime.

3.6 Solving for Nominal Bonds

Nominal log bond prices are conjectured to be functions of the same state variables:

$$q^{\$}_{t,n}(s_t) \quad = \quad D^{\$}_{0,n}(s_t) + D^{\$}_{1,n}(s_t)x_{c,t} + D^{\$}_{2,n}(s_t)x_{\pi,t} + D^{\$}_{3,n}(s_t)\sigma^2_{\pi,t}. \tag{16}$$

Let $y^{\$}_{t,n} = -\frac{1}{n}q^{\$}_{t,n}$ denote the n-period continuously compounded nominal yield. Then:

$$y^{\$}_{t,n}(s_t) \quad = \quad -\frac{1}{n}(D^{\$}_{0,n}(s_t) + D^{\$}_{1,n}(s_t)x_{c,t} + D^{\$}_{2,n}(s_t)x_{\pi,t} + D^{\$}_{3,n}(s_t)\sigma^2_{\pi,t}), \tag{17}$$

where the $D^{\$}$-coefficients determine how nominal yields respond to changes in expected consumption growth, inflation, and inflation volatility. Solving for nominal log bond prices requires the use of the nominal log pricing kernel which is determined by the difference between the real log pricing kernel and the inflation rate:

$$m^{\$}_{t+1}(s_{t+1}) = m_{t+1}(s_{t+1}) - \pi_{t+1}.$$

The Appendix shows how to solve for the coefficients and reports the detailed expressions.

The response of nominal yields to changes in expected growth is the same as for real yields meaning that $D^{\$}_{1,n}(s_t) < 0$ in both states for reasonable parameter values. This means that shocks to consumption growth contribute to negative risk premiums also for nominal bonds. As expected, nominal yields move positively with expected inflation implying a negative value of $D^{\$}_{2,n}(s_t)$. This holds regardless of the economic state and reflects a cash-flow effect on nominal bonds. Hence, while real yields may decrease or increase in response to expected inflation depending on the current cyclical state, nominal yields always rise with the level of inflation.

The effect of inflation volatility on yields depends on whether inflation is counter- or procyclical and reflects a discount-rate channel stemming from inflation risk. When inflation is negatively (positively) correlated with growth, higher inflation will raise yields through the cash-flow channel and generate poor bond returns in bad (good) times. Hence, nominal bonds may be subject to either countercyclical or procyclical returns and can therefore constitute both a risky asset or a hedging asset with respect to inflation risk. A rise in inflation risk can therefore be associated with both higher or lower yields through its different effect on bond risk premia. This means that $D^{\$}_{3,n}(s_t)$ can be either negative or positive depending on the inflation regime. This is the key mechanism that allows the model to match the switching relation between nominal interest rates and inflation uncertainty documented in data. We elaborate further on the link between inflation risk and bond risk premia in Section 5.

4 Calibration of Model

Motivated by the estimated breaking point in our empirical regime-switching model, namely 2001:2, we calibrate the model for the periods pre and post the second quarter of 2001. We target a range of unconditional macro and asset-pricing moments based on consumption growth, inflation, dividend growth, stock returns, and bond returns. We assume that the quarterly frequency of the model coincides with the decision interval of the agent.[20]

We first describe how we calibrate the model and then we discuss the implied macro and asset-pricing moments. All calibrated parameters are reported in Table 8. The mean parameters for growth and inflation, $\mu_c(s_{t+1})$ and $\mu_\pi(s_{t+1})$, are set equal to their sample values for each sub-period. The mean of dividend growth, μ_d, is set equal to the full sample mean. The persistence of shocks to consumption growth, β_1, is set to 0.951 and 0.995 for the two periods. These values translate into 0.983 and 0.998 on a monthly frequency. The first value is in line with the existing long-run risk literature while the second value is higher since we find the persistence of consumption growth to be substantially higher in the second period compared to the first.

The β_2 parameter is set to -0.012 and 0.012 respectively, which means that inflation expecta-

[20]This means we abstract away from issues related to time-aggregation of consumption growth. See for example Bansal et al., (2007a), Bansal et al., (2007b), and Hasseltoft (2012) who all estimate long-run risk models using simulation estimators, taking into account time-aggregation of consumption growth.

tions have a negative impact on expected growth in the countercyclical state but a positive impact in the procyclical period. This creates a negative (positive) correlation between growth and inflation in the countercyclical (procyclical) period. The different signs of β_2 for the two states also allow the model to match the switching relation between price-dividend ratios and expected inflation, from negative to positive, that we documented earlier. More specifically, the sign of β_2 determines the sign of $A_{d,2}$ in Equation (13). The last of the β parameters is β_4 which governs the persistence of inflation. We find that the persistence of inflation was substantially higher during the countercyclical period than in the recent procyclical period. We therefore calibrate β_4 to 0.90 and 0.40 respectively.

The next set of parameters refer to the δ matrix which governs the size of long-run shocks to growth and inflation. The parameters governing shocks to expected growth, δ_1 is set to 0.12 and 0.15 for both periods respectively. The parameters δ_2 and δ_3 affect the correlation between growth and inflation and are set equal to -0.12 and 0.20 for δ_2 and to -0.10 and 0.9 for δ_3. This helps the model to match the switch from negative to positive macro correlations. Lastly, δ_4 governs the size of long-run shocks to inflation and is calibrated to 0.90 for both states.

Overall, the main difference in parameter values across regimes stems from matching the significant shift in the growth-inflation correlation from negative to positive plus matching the large increase in growth persistence and the large drop in inflation persistence that occurred during the post-2000 period.

The persistence of volatility shocks v_π and their volatility σ_ν are calibrated to standard values in the long-run risk literature, 0.98 and $1 * 10^{-6}$ respectively. Dividend parameters and the preference parameters are also calibrated to standard values in the literature. The risk aversion is set to 10 and the EIS to 2. It is well-known that long-run risk models need an EIS above one in order to generate plausible asset pricing implications. The value of the EIS is subject to controversy. While for example Hall (1988), Campbell (1999), and Beeler and Campbell (2011) estimate the EIS to be close to zero, Attanasio and Weber (1993), Attanasio and Vissing-Jorgensen (2003), Chen et al. (2008), and Hasseltoft (2012) among others find the EIS to be above one. Lastly we need to calibrate the transition probabilities. We set them equal to the estimated values from the empirical regime-switching model, namely $p_{00} = 0.98$ and $p_{11} = 0.94$.

Having calibrated the model, we simulate the model 150000 quarters and evaluate the implied macro and asset-price moments. Table 9 reports macro moments. We report moments for the countercyclical period, the procyclical period, and for the full sample. The unconditional means of growth and inflation are matched perfectly by construction. Volatility of the macroeconomic variables all lie close to their sample values. The table shows that the persistence of growth increased sharply post 2000 with a first-order autocorrelation coefficient of 0.77 versus 0.39 for the countercyclical period. The persistence of inflation also changed markedly but in the opposite direction with a large drop in the first-order autocorrelation coefficient from 0.84 in the countercyclical regime to 0.27 in the procyclical regime. Our calibration matches these sharp changes in persistence. We report the fourth-order autocorrelation coefficient for dividend growth since the moving-average procedure described above automatically induces positive autocorrelation for up to three lags.

Table 10 contains unconditional asset-price moments. The calibration generates model moments that are broadly in line with data. While the level of the equity risk premium and price-dividend ratios are broadly in line with data, the volatilities are lower than in data. The level of the nominal short rate is matched well and the model generates an upward sloping nominal yield curve in both regimes. However the model-implied slope of roughly 50 basis points is lower than observed in data. The model-implied real yield curve is downward sloping in the countercyclical period but upward sloping in the procyclical period. The fact that our regime-switching model can produce an upward sloping real term structure is interesting since standard long-run risk models can only produce downward sloping real term structures. We discuss this more in detail in Section 5.2.

Table 11 reports various macro and asset correlations. First, the correlation between real growth and inflation changed from negative to positive in data, -0.42 versus 0.30. The model is able to match this shift. Second, the correlation between dividend yields and nominal yields changed sharply in data from 0.68 to -0.65. The model is able to generate a similar shift from positive to negative correlations, albeit the correlation in the procyclical is more negative than in data. Third, the correlation between stock and bond returns changed substantially from positive to negative in data, 0.28 versus −0.63. The model matches this transition well.

Overall, we believe the model does a good job in matching key moments. In particular, the model is able to match the large shifts in growth-inflation correlations and asset correlations. The model could of course do an even better job if we relaxed the many imposed restrictions. Our objective, however, has been to match the broad change in macro and asset-prices across the two regimes while keeping the model as parsimonious as possible.

5 Asset Pricing Implications

Given our calibrated parameters, we here discuss in detail the model-implications for bond risk premia, equity risk premia, slopes of the real and nominal term structures, and asset correlations.

5.1 Real and Nominal Bond Risk Premia

Let $h_{t+1,n}(s_{t+1}) = q_{t+1,n-1}(s_{t+1}) - q_{t,n}(s_t)$ denote the one period log holding period return on a real bond with a maturity of n periods. The risk premium can then be written as:

$$\begin{aligned} E[h_{t+1,n}(s_{t+1}) - r_{f,t}|I_t] &+ \frac{1}{2}\sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} Var[h_{t+1,n}(s_{t+1})|I_{t+1}] \qquad (18) \\ &= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}}[A(s_{t+1}) + B(s_{t+1})\sigma^2_{\pi,t}], \\ B(s_{t+1}) &= [D_{1,n-1}(s_{t+1})\delta_2(s_{t+1}) + D_{2,n-1}(s_{t+1})\delta_4(s_{t+1})]\lambda_{\varepsilon_\pi}(s_{t+1}). \end{aligned}$$

The same expression holds for nominal bond returns $h^{\$}_{t+1,n}(s_{t+1})$ by replacing the A and B-coefficients by $A^{\$}$ and $B^{\$}$. Derivations and expressions for A and $A^{\$}$ are reported in the Appendix. Time-varying volatility of inflation gives rise to time-varying bond risk premiums. The $B(s_{t+1})$ coefficient is determined by the market price of long-run inflation risk $\lambda_{\varepsilon_\pi}(s_{t+1})$ times the response of bond prices to inflation shocks. Positive inflation shocks raise nominal yields in both regimes while they lower (raise) real yields in the countercyclical (procyclical) regime. As discussed earlier, the market price of inflation risk is negative in the countercyclical regime and positive in the procyclical regime. This implies that nominal bond risk premia loads positively on inflation volatility when inflation is countercyclical but negatively when inflation is procyclical. The model suggests the loading was positive for the pre-2000 period but negative post 2000. Real bond risk premia, however, load negatively on inflation volatility in both regimes since real bonds always provide a hedge against bad times.

Recall the empirical evidence reported earlier in Table 4 which showed how the relation between nominal yields and inflation uncertainty switches sign between the inflation regimes. The model can explain this effect by allowing for changes in the riskiness of nominal bonds. Note that the effect of inflation uncertainty on nominal yields is distinct from the level of inflation. An increase in the level of expected inflation represents a cash-flow effect on nominal bonds and is always bad for nominal bond returns and raises yields while an increase in inflation uncertainty can either increase or decrease nominal yields through a discount-rate channel.

Table 12 shows results from regressing the level of 5-year nominal rates onto expected inflation and inflation uncertainty inside the model. We simulate 150000 quarters and run regressions for the full simulated sample, for the countercyclical period of the sample, and for the procyclical period of the sample. First of all, simulated yields load positively on the level of expected inflation for all three samples which is expected. The slope coefficients for inflation uncertainty are more interesting. They suggest that model-implied rates load positively on inflation uncertainty for the full sample and for the countercyclical period. However, the coefficient on inflation uncertainty switches sign in the procyclical state. An increase in inflation uncertainty *lowers* nominal bond yields through its negative impact on bond risk premia.

5.2 Slope of the Real and Nominal Term Structure

A conventional way of expressing the slope of the term structure is to write it as the sum of expected future short rates plus a risk premium (term premium) part. In the case of a single regime, this can be written as:

$$y_{t,n} - y_{t,1} = \frac{1}{n}E(y_{t,1} + y_{t+1,1} + ... + y_{t+n-1,1}|I_t) - y_{t,1} + RPT_{t,n}. \tag{19}$$

Taking the unconditional expectation of this expression and using the law of iterated expectations yields that the first part equals zero. This implies that the average slope depends on the average risk premium:

$$E(y_{t,n} - y_{t,1}) = E(RPT_{t,n}), \tag{20}$$

which of course holds for both real and nominal bonds. Typically, long-run risk models generate negative risk premiums on real bonds which implies a downward sloping real yield curve. As mentioned earlier, this is in contrast to US data which indicates a positively sloped real yield curve using available data from 1997. The nominal yield curve, however, has been upward sloping on average in post-war data and for both regimes as was shown in Table 10. To generate an upward-sloping nominal yield curve, consumption-based models typically rely on a negative unconditional covariance between growth and inflation observed in post-war data. A negative covariance implies that nominal bonds are risky which in turn implies a positive inflation risk premium and a positive slope. We find, however, that the covariance of growth and inflation turned positive around 2000. Does this necessarily imply a downward-sloping nominal yield curve in the model, in contrast to data? No.

In the case of multiple regimes, the average slope in a given regime also depends on expected future short rates. To see this, write the slope given today's regime s_t as:

$$y_{t,n}(s_t) - y_{t,1}(s_t) = \frac{1}{n}E[y_{t,1}(s_t) + y_{t+1,1}(s_{t+1}) + ... + y_{t+n-1,1}(s_{t+n-1})|I_t] - y_{t,1}(s_t) + RPT(s_t)_{t,n}, \tag{21}$$

which holds for both real and nominal yields. For simplicity, consider a two-period bond where $n = 2$. The Appendix shows that the average yield-curve slope given regime $s_t = s$ then can be written as:

$$\begin{aligned} E[y_{t,2}(s_t) - y_{t,1}(s_t)|s_t = s] = & \quad \frac{1}{2}\{p_{s,0}E[y_{t+1,1}|s_t = s, s_{t+1} = 0] + p_{s,1}E[y_{t+1,1}|s_t = s, s_{t+1} = 1]\} \\ & - \quad \frac{1}{2}E[y_{t,1}(s_t)|s_t = s] \\ & + \quad E[RPT_{t,2}(s_t)|s_t = s]. \end{aligned} \tag{22}$$

The average slope for a given regime depends on both the risk premium part and on the expectations part.[21] Consider two regimes with low versus high short rates and suppose we stand in the low rate regime. Then the expectations part is positive as long as there is a non-zero probability of jumping to the high short rate state. And vice versa if we stand in the high short rate regime. Table 10 reported that the average nominal 3-month rate was 6.47 during the countercyclical pre-2000 period but only 1.78 in the procyclical post-2000 period. Consequently, our model implies that the average value of the expectations part in Equation (21) is positive for the post-2000 period and negative for the pre-2000 period.

Using the analytical expressions provided in the Appendix, we decompose the slope of real and nominal term structures into the two components. Table 13 reports the decompositions. Starting with nominal bonds, the model implies a negative expectations part and a positive risk premium part for the countercyclical period. This is consistent with this regime being a high rate regime in which nominal bonds are risky assets. The net effect is a positive slope of 44 basis points. The procyclical period looks different. The average slope is still positive but now the expectations part is highly positive while the risk premium part is negative. This is consistent with the procyclical regime being a low-rate regime in which nominal bonds provide a hedge against bad times.

Turning to real bonds, Table 13 reports a negative slope in the countercyclical period of 46 basis points. Both components of the slope are negative as the pre-2000 period was characterized by high short rates and real bonds acting as hedges against bad times. Real bond risk premiums are also negative in the procyclical period but the expectations part is now positive since the regime is subject to low short rates. The net effect is a positive real slope. This effect is not present in standard long-run risk models since the slope is exclusively determined by the risk premium part.

[21] The Appendix contains derivations of the expectations part and the risk premium part in our model for any maturity n.

5.3 Equity Risk Premia

Let $r_{m,t+1}(s_{t+1})$ denote the one period log market return. The equity risk premium can then be written as:

$$
\begin{aligned}
E[r_{m,t+1}(s_{t+1}) - r_{f,t}|I_t] &+ \frac{1}{2}\sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} Var[r_{m,t+1}(s_{t+1})|I_{t+1}] \\
&= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}}[A(s_{t+1}) + B(s_{t+1})\sigma^2_{\pi,t}], \\
B(s_{t+1}) &= [k_{d,1}A_{d,1}(s_{t+1})\delta_2(s_{t+1}) + k_{d,1}A_{d,2}(s_{t+1})\delta_4(s_{t+1})]\lambda_{\varepsilon_\pi}(s_{t+1}),
\end{aligned} \tag{23}
$$

and where the $A(s_{t+1})$ coefficient is reported in the Appendix for brevity. As with bonds, risk premiums on equity vary with inflation volatility. The $B(s_{t+1})$ coefficient is determined by the market price of long-run inflation risk $\lambda_{\varepsilon_\pi}(s_{t+1})$ times the impact of inflation shocks on real equity returns. In the countercyclical state, the market price of inflation is negative while inflation is bad for stock returns. Low returns in bad times implies that the equity risk premia moves positively with inflation uncertainty.

In the procyclical state, the market price of inflation is positive while stock returns are positively related to inflation. High stock returns in good times means that the equity risk premia moves positively with inflation uncertainty also in this regime. Using the price-dividend ratio as a proxy for expected excess stock returns, we find this to be consistent with data.

Table 12 shows results from regressing the log price-dividend ratio onto expected inflation and inflation uncertainty on simulated data inside the model. Results show that price-dividend ratios load negatively on both expected inflation and inflation uncertainty for the full sample period. This is consistent with the voluminous literature that documents equity being a poor inflation hedge. Coefficients are similar for the countercyclical sample period. However, the coefficient on expected inflation switches sign in the procyclical state. The model suggests that equity prices respond differently to changes in the level of inflation as opposed to changes in inflation uncertainty. A procyclical inflation induces a positive relation between inflation levels and stock prices. This can interpreted as a cash-flow effect on stocks since higher growth in the model feeds into higher dividend growth. On the other hand, an increase in inflation uncertainty is always bad for stock prices since stock returns tend to always be procyclical. The model qualitatively matches the empirical regression results that were reported in Table 3.

5.4 Asset Correlations

This section describes model implications for the correlation between stock and bond returns and for the correlation between dividend yields and nominal yields. Both of these correlations changed sign dramatically in the early 2000's as was shown in Figures 2 and 3. We show below that the model can account for this switch by accounting for the changing cyclicality of inflation.

5.4.1 Stock and Bond Returns

We show in the Appendix that the conditional covariance between stock and bond returns can be written as:

$$\begin{aligned} Cov[r^{\$}_{m,t+1}(s_{t+1}), h^{\$}_{t+1,n}(s_{t+1})|I_t] &= M_t + A + B\sigma^2_{\pi,t}, \\ B &= p_{s_t,0}[k_{d,1}A^0_{d,1}\delta^0_2 + k_{d,1}A^0_{d,2}\delta^0_4][D^{\$,0}_{1,n-1}\delta^0_2 + D^{\$,0}_{2,n-1}\delta^0_4] \\ &+ p_{s_t,1}[k_{d,1}A^1_{d,1}\delta^1_2 + k_{d,1}A^1_{d,2}\delta^1_4][D^{\$,1}_{1,n-1}\delta^1_2 + D^{\$,1}_{2,n-1}\delta^1_4], \end{aligned} \tag{24}$$

and where the M_t and A coefficients are reported in the Appendix. We choose to focus on the B coefficient since it is largest in magnitude and determines how the stock-bond covariance moves with inflation uncertainty. The B-coefficient is basically a probability weighted measure of the impact of inflation shocks on price-dividend ratios and nominal bonds in the two regimes. Based on the earlier discussion, we know that inflation shocks impact price-dividend ratios negatively in the countercyclical state and positively in the procyclical state. For nominal bonds we know that positive inflation shocks lower bond prices regardless of economic state. It is then evident from Equation (24) that the conditional covariance moves positively with inflation uncertainty when inflation is bad for economic growth and negatively with inflation uncertainty when inflation is procyclical. One way to interpret this comovement is to think about how inflation uncertainty moves equity and bond risk premia. Higher inflation uncertainty always raises expected returns on equity but move expected bond returns either up or down depending on the inflation regime.

We saw in Table 5 that inflation uncertainty predicted quarterly stock-bond covariances positively during the countercyclical state but negatively during the procyclical period. We would like to simulate our model and run the same regressions inside the model. However we cannot generate realized quarterly covariances in the model since the model is formulated on a quarterly frequency. Instead we report the value of the analytical B coefficient above. Table 12 reports that B is positive in the countercyclical state but switches to negative in the procyclical state. The model can match the switching behavior in data due to changes in the cyclical nature of inflation

Next we would like to plot the model-implied conditional correlation between stock and bond returns. In order to do so we need empirical proxies for our state variables. We construct expected growth and inflation by projecting realized values onto a set of instruments and treat the fitted values as our state variables. Conditional variance of inflation is constructed by estimating an AR(1)-GARCH(1,1) on expected inflation.

Figure 10 plots the model-implied conditional correlations. Consistent with data, the model implies highly positive correlations throughout the 1970's and early 1980's but a sharp drop in correlations in the late 1980's which coincided with inflation briefly entering a procyclical period. As we approach the end of the 1990's correlations start to turn lower and drops sharply in the early 2000 as we enter the procyclical region. Model correlations then stay highly negative throughout the 2000's except for a sharp spike towards the end of the sample. Overall, conditional correlations implied by the theoretical model share very similar dynamics to the rolling and estimated correlations based on data which are presented in Figures 1 and 9.

5.4.2 Dividend Yields and Nominal Yields

Since equity returns are closely related to changes in dividend yields and bond returns to changes in yields, the same mechanism can be used to explain why dividend yields and nominal yields comove. The existing literature focuses on the highly positive correlation between these two variables between 1960 and 2000. As mentioned earlier, this observation is often dubbed the Fed-model. However, it is rarely mentioned in the literature that this correlation changed dramatically during the last 10 years, from a correlation of 0.64 during 1965:1-2001:2 to a correlation of -0.57 during 2001:3-2011:4. The behavioral concept of inflation illusion has been extensively used in the literature to explain the positive comovement. However, the inflation illusion story cannot explain the significant shift to negative correlations. We find that our model can provide a rational explanation for why the comovement of dividend yields and nominal yields changes over time.

Consider the expression for the conditional covariance between dividend yields and nominal yields:

$$Cov[pd_{t+1}(s_{t+1}), y^{\$}_{t+1,n}(s_{t+1})|I_t] = M_t + A + B\sigma^2_{\pi,t}, \tag{25}$$

$$\begin{aligned} B &= -\frac{1}{n}\Big[p_{s_t,0}[A^0_{d,1}\delta^0_2 + A^0_{d,2}\delta^0_4][D^{\$,0}_{1,n-1}\delta^0_2 + D^{\$,0}_{2,n-1}\delta^0_4] \\ &+ p_{s_t,1}[A^1_{d,1}\delta^1_2 + A^1_{d,2}\delta^1_4][D^{\$,1}_{1,n-1}\delta^1_2 + D^{\$,1}_{2,n-1}\delta^1_4]\Big], \end{aligned}$$

and where M_t and A are reported in the Appendix. We focus on the B-coefficient since it is largest in magnitude and provide intuition of how the covariance moves with inflation uncertainty. Comparing the B-coefficient with the one in Equation (24), it is evident that they are very similar. The same argument as for the stock-bond covariance goes through for explaining the so called Fed-model. Changes in inflation uncertainty moves equity and bond risk premiums in the same direction when inflation is countercyclical. However inflation uncertainty moves stock and bond risk premia in opposite directions when inflation is procyclical since nominal bonds then provide insurance against bad times while equity is still risky. As a result, we find that movements in inflation risk together with changes in the cyclical nature of inflation can provide a plausible explanation for why dividend yields and nominal yields sometimes comove positively and sometimes diverge.

6 Understanding the Regimes

We have so far documented the existence of two distinct regimes in which macro and asset correlations are inversely related and have opposite signs. A natural question to ask is what characterizes these different regimes. One potential answer is that the two regimes differ in the magnitudes of output and inflation shocks. For example, negative output shocks should lower equity prices while the effect on nominal bonds depends on how output and inflation interact. If the negative output shock is associated with lower inflation, (e.g., a negative aggregate demand shock), then nominal rates will decrease through both the real rate component of nominal interest rates (assuming procyclical real interest rates) and through lower inflation. Such a scenario produces poor stock returns but good bond returns, yielding a negative stock-bond correlation. Alternatively, a large positive shock to inflation that is detrimental to growth (e.g., an adverse supply shock) should raise nominal interest rates and lower equity prices. Such a shock would produce poor stock and bond returns, yielding a positive stock-bond correlation.

Another potential explanation is monetary policy and its impact on asset prices. For example, Kuttner (2001), Rigobon and Sack (2004), and Bernanke and Kuttner (2005) document that a lower (higher) Federal funds rate raises (lowers) equity and nominal bond prices.[22] This implies that changes in the Federal funds rate should produce positive stock-bond correlations. Furthermore, several papers have documented that the impact of monetary policy on the economy has changed over time. In particular, the impact of monetary policy on macroeconomic variables has decreased in more recent times.[23] It is therefore likely that the impact of monetary policy on stock-bond correlations also has varied over time.

With this as a background, we identify shocks to inflation, output, and monetary policy for the countercyclical period 1965:1-1999:4 and for the procyclical period 2000:1-2011:4 and study how they impact the stock-bond covariance. We do so by formulating a simple VAR consisting of inflation as measured by the GDP price deflator (π), real GDP growth ($\Delta(y)$), the Federal funds rate (ff), and the stock-bond covariance (cov).[24] The ordering is consequently $X_t = [\pi_t, \Delta(y)_t, ff_t, cov_t]$. All variables are observed quarterly. We consider a standard VAR(1) for the vector of observables $X_{t+1} = \mu + \beta X_t + \epsilon_{t+1}$, where $\epsilon_{t+1} \sim N(0, \Sigma)$. The VAR disturbances are

[22] A potential explanation for these effects is the link between monetary policy and risk aversion as documented by Bekaert et al. (2012).

[23] See for example Kuttner and Mosser (2002) and Boivin and Giannoni (2006).

[24] Inflation, GDP growth rate, and the Federal funds rate are obtained from the Federal Reserve Bank of St. Louis. The stock-bond covariance is computed as described earlier.

assumed to be related to the underlying structural economic shocks, η, as follows: $\epsilon_{t+1} = A\eta_{t+1}$, where A is a lower triangular matrix and where $\eta_{t+1} \sim N(0, I)$.

In order to treat shocks to the Federal funds rate as structural monetary policy shocks, we need to impose some identification restrictions. We choose a simple recursive identification scheme where the matrix A is assumed to equal the Cholesky factor of Σ. The ordering of our variables implies that shocks to output and inflation affect the Federal funds rate contemporaneously while monetary policy shocks affect output and inflation with a one-period lag. These assumptions can of course be debated but have often been used in the literature on identifying monetary policy shocks.[25] The equation for the Federal funds rate can be viewed as a "Taylor rule" in which the Federal Reserve sets interest rates based on current inflation and output growth (Taylor, 1993). Our main interest, however, lies in the stock-bond covariance which is ordered last. Being ordered last implies that macro shocks and monetary policy shocks all have a contemporaneous effect on the stock-bond covariance.

We estimate the VAR with OLS, apply the Cholesky factorization, and then compute impulse response functions and variance decompositions. We estimate the VAR over two sub-periods, the countercyclical inflation period 1965:1-1999:4 and the procyclical inflation period 2000:1-2011:4. Impulse response functions for the countercyclical period are displayed in Figure 11 and show the impact of a one-standard deviation shock to each variable. First, the figure shows that shocks to inflation were substantially more persistent than output shocks in the countercyclical period. Second, we find that inflation shocks had a negative impact on output, consistent with the negative correlation between growth and inflation that prevailed during the period. Furthermore, the results suggest that a one-standard deviation shock to the Federal funds rate cause a drop of around 0.15% in quarterly output growth, after which it recovers.[26] Our main interest, however, lies in the impact of shocks on the stock-bond covariance. The last graph indicates a large positive effect on the stock-bond covariance from a positive shock to the Federal funds rate. Hence, a contractionary monetary policy is associated with a positive comovement of stock and bond returns. This is in line with

[25] See for example Christiano et al., (1998), Boivin and Giannoni (2006), and Olivei and Tenreyro (2007).

[26] The positive response of inflation to a positive shock to the Federal Funds rate is a version of the "price puzzle", documented by Sims (1992). It has been shown that this effect can be reduced by including commodity prices in the VAR. However, we refrain from doing so as we are mainly interested in stock-bond covariances.

papers showing that a tightening in monetary conditions is typically bad for both stock and bond returns. The graph also shows a negative immediate impact on stock-bond covariances stemming from inflation shocks. Interestingly, the effect of monetary policy shocks on covariances is much stronger than output or inflation shocks.

Figure 12 shows impulse response functions for the procyclical period. The first observation is that output shocks were more persistent than inflation shocks, in contrast to the earlier period. Secondly, output shocks have a strong impact on all variables, and considerably stronger effects than during the first sample period. For example, a shock to output increases quarterly inflation by almost 0.10% after 2 quarters, producing a positive comovement between growth and inflation. Interestingly, the impact of monetary policy shocks on output and inflation are smaller compared to the first sample period. This is line with earlier studies cited above. The figure shows that stock-bond covariances were strongly impacted by output shocks during the post-2000 period. Even though inflation and monetary policy shocks still had sizeable effects on stock-bond covariances, their effect is dwarfed by output shocks. Overall, the impulse response functions suggest that shocks to inflation and monetary policy had a sizeable impact on stock-bond covariances during the countercyclical period while output shocks dominated during the procyclical period.

Next, we consider variance decompositions of forecast errors. Table 14 reports the fraction of variance accounted for by each shock. Our main interest lies at the stock-bond covariance. Considering first the countercyclical period, the table reports that shocks to inflation and monetary policy accounted for larger fractions compared to output shocks. Over a one-quarter horizon, inflation and monetary policy shocks accounted for 1% and 9% respectively while rising to 6% and 18% over a 20-quarter horizon. This can be compared with the 1% and 2% accounted for by output shocks. Turning to the procyclical period, the results differ strongly. Our results suggest that output shocks played an important role during the post-2000 period. In fact, we find that output shocks accounted for 25% of the variance in stock-bond covariance over one quarter and 56% over 20 quarters. In contrast, shocks to inflation and monetary policy played less of a role during this period.

Overall, our results suggest that inflation and monetary policy shocks had a sizeable impact on stock-bond covariances during the countercyclical period 1965:1-1999:4 while output shocks clearly dominated during the procyclical period 2000:1-2011:4. Hence, in order to understand why growth-inflation and stock-bond correlations change over time, our findings suggest that it depends on the relative magnitudes of nominal shocks versus real output shocks.

7 Conclusion

The correlation between returns on US stocks and Treasury bonds and the relation between inflation and asset prices have varied substantially over time. For example, the 1970-1980's was characterized by a highly positive correlation between stock and bond returns and a strong negative relation between inflation and price-dividend ratios. In contrast, the period 2000-2011 experienced the exact opposite with strongly negative asset correlations and a positive relation between inflation and equity valuations. We show that these observations line up remarkably well with the time-varying correlation between consumption growth and inflation going back to the 1930's. While the 1970-1980's was characterized by stagflation, we show that inflation switched to a procyclical state in the early 2000's.

We document in data that inflation risk is always negatively related to stock prices but can either decrease or increase bond prices depending on whether inflation is counter- or procyclical. In countercyclical inflation regimes, nominal bonds are risky assets and therefore perform badly as inflation risk increases. However, nominal bonds provide a hedge against bad times when inflation is procyclical. This produces a drop in nominal rates as inflation risk increases, generating positive bond returns. We find that this asymmetry in how inflation risk impacts asset prices helps explain why the stock-bond correlation switches sign over time.

We calibrate a long-run risk model that rationalizes our empirical findings and illustrates the connection between the cyclicality of inflation and the joint movements of bond and equity risk premia and inflation and asset prices. Persistent inflation shocks have real effects and affects both equity and bond risk premia. We introduce a Markov-switching regime mechanism into the model which allows the relation between real growth and inflation to switch sign. Equity and bond risk premia are both functions of inflation volatility in the model but the loading of bond risk premia on inflation uncertainty depends on the cyclicality of inflation and can therefore switch sign. Hence, inflation uncertainty can produce both positive and negative stock-bond correlations depending on inflation regime.

Results from a VAR analysis suggest that the positive stock-bond correlations during the countercyclical inflation period of 1965:1-1999:4 were dominated by nominal shocks in the form of inflation and monetary policy shocks. In contrast, we find that the period 2000:1-2011:4 with positive growth-inflation correlations and negative stock-bond correlations was dominated by real output shocks. Hence, in order to understand why growth-inflation and stock-bond correlations change over time, it seems important to consider the relative magnitudes of nominal shocks versus real shocks.

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Table 1: **Unconditional Correlation Matrices and Variances over Subperiods**

	Correlation Matrix				Variance
	Cg	Inf	Stock	Bond	
1965-2011					
Cg	1.000	–0.161	0.191	-0.128	0.227
Inf		1.000	–0.086	–0.241	0.415
Stock			1.000	0.073	72.54
Bond				1.000	12.12
1965-2000					
Cg	1.000	–0.420	0.154	–0.083	0.212
Inf		1.000	–0.201	–0.202	0.412
Stock			1.000	0.283	66.70
Bond				1.000	12.77
2001-2011					
Cg	1.000	0.298	0.349	–0.177	0.144
Inf		1.000	0.273	–0.329	0.263
Stock			1.000	–0.631	94.70
Bond				1.000	9.583

This table presents unconditional correlation matrices and variances based on quarterly data over the full sample period 1965-2011 and the considered subperiods 1965-2000 (countercyclical period) and 2001-2011 (procyclical period). Stock and bond returns are excess returns.

Table 2: **Jennrich (1970) Test of Equality of Correlation Matrices over Subperiods**

Model	Degree of freedom	1965-2000 compared to 2001-2011	
		Statistics	p-value
Cg-Inf-Stock-Bond	6	50.344	0.000
Cg-Inf	1	19.368	0.000
Stock-Bond	1	27.577	0.000
Stock-Inflation	1	7.446	0.006

This table presents results from a Jennrich (1970) test of constant unconditional correlations. The test statistic is asymptotically distributed as a Chi-square with the degree of freedom equal to the number of correlation coefficients.

Table 3: **Regressing Price-Dividend Ratios onto Inflation and Inflation Risk**

	Data				
	β_π	t-stat	$\beta_{\sigma^2_\pi}$	t-stat	R^2
Full sample	–0.16	–2.22	–47.76	–5.10	0.48
Countercyclical state	–0.20	–3.34	–30.20	–3.86	0.44
Procyclical state	0.10	1.09	–84.58	–6.35	0.51

This table presents results from regressing log price-dividend ratios onto expected inflation (β_π) and inflation risk ($\beta_{\sigma^2_\pi}$): $pd_t = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma^2_\pi}\sigma^2_{\pi,t} + \epsilon_t$. Inflation risk is measured as the cross-sectional variance of individual forecasters of the GDP price deflator (PGDP), taken from Survey of Professional Forecasters. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4.

Table 4: **Regressing Nominal Yields onto Inflation and Inflation Risk**

	Data				
	β_π	t-stat	$\beta_{\sigma^2_\pi}$	t-stat	R^2
Full sample	1.61	2.86	244.68	3.19	0.34
Countercyclical state	0.42	0.78	223.89	3.04	0.25
Procyclical state	0.50	1.40	–382.72	–2.49	0.27

This table presents results from regressing 5-year nominal interest rates onto expected inflation (β_π) and inflation risk ($\beta_{\sigma^2_\pi}$): $y^{\$}_{t,5y} = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma^2_\pi}\sigma^2_{\pi,t} + \epsilon_t$. Inflation risk is measured as the cross-sectional variance of individual forecasters of the GDP price deflator (PGDP), taken from Survey of Professional Forecasters. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4

Table 5: **Predicting Covariance of Stock and Bond Returns**

	Data		
	$\beta_{\sigma^2_\pi}$	t-stat	R^2
Full sample	11.33	4.02	0.08
Countercyclical state	5.16	2.30	0.08
Procyclical state	–40.64	–2.33	0.06

This table presents results from predicting quarterly covariances between returns on US stocks and Treasury bonds using inflation risk: $\sigma(r_{stock,t+1}, r_{bond,t+1}) = \alpha + \beta_{\sigma^2_\pi}\sigma^2_{\pi,t} + \epsilon_t$. Dependent variable is the realized quarterly covariance between stock and bond returns computed using daily returns. Independent variable is the cross-sectional variance of individual forecasters of the GDP price deflator (PGDP), taken from Survey of Professional Forecasters. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4.

Table 6: **Estimation Results for Markov-Switching Model**

	Countercyclical State				Procyclical State			
	Cg	Inf	Stock	Bond	Cg	Inf	Stock	Bond
β Matrix								
Cg	0.347	–0.122	0.012	0.015	0.675	0.033	0.006	0.006
	(0.084)	(0.062)	(0.005)	(0.011)	(0.087)	(0.067)	(0.004)	(0.011)
Inf	0.138	0.866	–0.004	–0.008	0.667	–0.164	0.007	–0.047
	(0.067)	(0.050)	(0.004)	(0.009)	(0.170)	(0.127)	(0.007)	(0.020)
Stock	–1.274	–1.251	0.001	0.315	4.812	–2.826	0.150	0.388
	(1.627)	(1.221)	(0.051)	(0.197)	(3.983)	(2.812)	(0.205)	(0.6163)
Bond	–0.426	–1.167	–0.060	–0.047	–2.605	2.181	–0.070	–0.068
	(0.714)	(0.532)	(0.038)	(0.089)	(1.228)	(0.950)	(0.056)	(0.177)
Ω Matrix								
Cg	0.174	–0.040	0.671	–0.182	0.036	0.007	0.277	0.013
	(0.021)	(0.012)	(0.277)	(0.124)	(0.009)	(0.011)	(0.291)	(0.067)
Inf		0.110	–0.614	–0.202		0.122	1.127	–0.257
		(0.013)	(0.218)	(0.098)		(0.026)	(0.556)	(0.155)
Stock			59.012	8.865			99.156	–16.286
			(7.106)	(2.417)			(21.473)	(4.940)
Bond				12.248				7.762
				(1.466)				(1.658)
μ Vector								
μ	0.638	0.060	3.268	2.101	0.139	0.468	–0.042	0.722
	(0.123)	(0.099)	(2.468)	(1.056)	(0.057)	(0.104)	(0.162)	(0.697)
Probs								
p_{11}	0.977							
	(0.014)							
p_{22}	0.939							
	(0.039)							
fval	1259.7							

This table presents results from estimating a two-regime MS-VAR model using maximum likelihood. Sample period is 1965:1 to 2011:4 and the model is formulated as indicated in Equation (1). Stock and bond returns are excess returns. Standard errors in parentheses are computed using the Hessian. Tomorrow's state s_{t+1} is presumed to follow a two-state Markov chain with transition probabilities $p_{ij} = P(s_{t+1} = j|s_t = i)$ where $\sum_{j=1}^{N} p_{ij} = 1$ and $0 < p_{ij} < 1$. The probability of ending up in tomorrow's state $s_{t+1} = (0, 1)$ given today's state $s_t = (0, 1)$ is governed by the transition probability matrix:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix}.$$

Table 7: **LR test statistic for regime-independent correlations**

Model	Degree of freedom	LR test statistic H_0: $\rho(S_t) = \rho$	
		Statistics	p-value
Cg-Inf-Stock-Bond	6	25.387	0.0003

This table presents a likelihood ratio (LR) test for the null hypothesis of a constant conditional correlation matrix across regimes. The LR test statistic is 2(ln L(θ) - ln L(θ_0)), with θ_0 corresponding to the parameter vector resulting under the null hypothesis.

Table 8: **Calibrated Model Parameters**

	Countercyclical state	Procyclical state
μ_c	0.0081	0.0039
μ_π	0.0114	0.0067
μ_d	0.0035	0.0035
β_1	0.951	0.995
β_2	–0.012	0.012
β_4	0.90	0.40
δ_1	0.12	0.15
δ_2	–0.12	0.2
δ_3	–0.1	0.9
δ_4	0.9	0.9
σ_π	0.00375	0.00375
v_π	0.98	0.98
$\sigma_\nu * 10^{-6}$	1	1
σ_c	0.0031	0.0031
ϕ	2	2
φ	5	5
γ	10	10
ψ	2	2
δ	0.998	0.998
p_{00}	0.98	
p_{11}	0.94	

This table presents calibrated parameters for the two economic states. Parameters are calibrated as to match both standard macro and asset pricing moments as well as the various relations between stocks and bonds and between inflation and asset prices. The transition probabilities are the ones we estimated in the empirical regime-switching model. The countercyclical state refers to 1965:1-2001:2 and the procyclical state to 2001:3-2011:4.

Table 9: **Macro Moments**

	Countercyclical State			Procyclical State			Full Sample		
	Model	Data	SE	Model	Data	SE	Model	Data	SE
Consumption growth, Δc									
Mean	0.81	0.81	(0.06)	0.38	0.39	(0.11)	0.70	0.71	(0.06)
Std.Dev.	0.43	0.46	(0.04)	0.54	0.38	(0.09)	0.50	0.48	(0.04)
AC(1)	0.48	0.39	(0.05)	0.65	0.77	(0.12)	0.59	0.52	(0.08)
Inflation, π									
Mean	1.14	1.14	(0.11)	0.67	0.67	(0.08)	1.03	1.04	(0.09)
Std.Dev.	0.76	0.64	(0.08)	0.51	0.51	(0.13)	0.73	0.65	(0.08)
AC(1)	0.90	0.84	(0.05)	0.42	0.27	(0.14)	0.85	0.77	(0.07)
Dividend growth, Δd									
Mean	0.35	0.22	(0.18)	0.32	0.82	(0.75)	0.34	0.35	(0.22)
Std.Dev.	1.66	1.46	(0.15)	1.78	2.82	(0.66)	1.69	1.86	(0.24)
AC(4)	0.12	-0.02	(0.13)	0.23	0.06	(0.17)	0.15	0.07	(0.12)

This table presents unconditional quarterly moments of the macro variables. Sample statistics refer to the countercyclical period 1965:1-2001:2, to the procyclical period 2001:3-2011:4, and to the full sample period 1965:1-2011:4. Model statistics are based on a simulation of 150000 quarters. AC(k) denotes the autocorrelation for k lags. Standard errors for the observed data, denoted SE, are computed as in Newey West (1987), using four lags.

Table 10: **Asset Price Moments**

	Countercyclical State			Procyclical State			Full Sample		
	Model	Data	SE	Model	Data	SE	Model	Data	SE
Equity									
$E(r_m - r_f)$	0.79	1.05	(0.62)	0.74	0.45	(1.54)	0.78	0.91	(0.60)
$\sigma(r_m - r_f)$	3.50	8.12	(0.75)	4.93	9.73	(1.40)	3.88	8.52	(0.67)
$E(pd)$	3.44	3.44	(0.06)	3.38	3.83	(0.05)	3.43	3.52	(0.06)
$\sigma(pd)$	0.12	0.34	(0.04)	0.21	0.19	(0.04)	0.15	0.35	(0.03)
Nominal Bonds									
$E(y^{\$}_{3m})$	6.36	6.47	(0.44)	3.12	1.78	(0.54)	5.59	5.43	(0.48)
$E(y^{\$}_{5y} - y^{\$}_{3m})$	0.44	0.98	(0.18)	0.47	1.40	(0.27)	0.44	1.07	(0.15)
$\sigma(y^{\$}_{3m})$	2.74	2.57	(0.40)	2.35	1.68	(0.27)	2.99	3.09	(0.38)
$\sigma(y^{\$}_{5y} - y^{\$}_{3m})$	1.81	1.16	(0.12)	1.74	0.94	(0.13)	1.79	1.13	(0.10)
Real Bonds									
$E(y_{3m})$	1.83			0.34			1.48		
$E(y_{5y})$	1.37			0.40			1.14		
$\sigma(y_{3m})$	0.62			0.93			0.94		
$\sigma(y_{5y})$	0.48			0.81			0.71		

This table presents unconditional asset-price moments. All results are reported on an annualized basis. Sample statistics refer to the countercyclical period 1965:1-2001:2, to the procyclical period 2001:3-2011:4 and to the full sample period 1965:1-2011:4. Model statistics are based on a simulation of 150000 quarters. Standard errors for the observed data, denoted SE, are computed as in Newey West (1987), using four lags.

Table 11: **Macro and Asset Correlations**

	Countercyclical State			Procyclical State			Full Sample		
	Model	Data	SE	Model	Data	SE	Model	Data	SE
$Corr(\Delta c, \pi)$	-0.41	-0.42	(0.10)	0.18	0.30	(0.19)	-0.15	-0.16	(0.15)
$Corr(dp, y^{\$}_{5y})$	0.46	0.68	(0.05)	-0.97	-0.65	(0.06)	-0.13	0.70	(0.06)
$Corr(r_{stock}, r_{bond})$	0.37	0.28	(0.08)	-0.59	-0.63	(0.08)	0.01	0.07	(0.11)
$Corr(\Delta c, \Delta d)$	0.26	0.19	(0.07)	0.40	0.49	(0.07)	0.28	0.19	(0.09)
$Corr(\Delta d, \pi)$	-0.20	-0.13	(0.08)	0.11	0.06	(0.06)	-0.14	-0.10	(0.08)

This table presents unconditional correlations of macro and asset-price data. $Corr(\Delta c, \pi)$ refers to the correlation between consumption growth and inflation, $Corr(dp, y^{\$}_{5y})$ refers to the correlation between dividend yields and nominal yields, $Corr(r_{stock}, r_{bond})$ refers to the correlation between excess stock and bond returns, $Corr(\Delta c, \Delta d)$ is the correlation between consumption growth and dividend growth and $Corr(\Delta d, \pi)$ refers to the correlation between dividend growth and inflation. Model statistics are based on a simulation of 150000 quarters. The countercyclical state refers to 1965:1-2001:2 and the procyclical state to 2001:3-2011:4. Standard errors for the observed data, denoted SE, are computed as in Newey West (1987), using four lags.

Table 12: **Inflation and Asset Prices - Model Regressions**

	Price-Dividend Ratios			Nominal Interest Rates			Stock-Bond Covariance
	β_{π}	$\beta_{\sigma^2_{\pi}}$	R^2	β_{π}	$\beta_{\sigma^2_{\pi}}$	R^2	$\beta_{\sigma^2_{\pi}}$
Full Sample	–0.08	–1.02	0.25	1.17	0.73	0.24	
Countercyclical state	–0.10	–1.01	0.60	1.19	2.89	0.89	36.16
Procyclical state	0.12	–1.03	0.13	0.91	–5.43	0.32	–32.15

This table presents results from running regressions using simulated asset prices and inflation inside the model. The first regression regresses log price-dividend ratios onto expected inflation and the conditional inflation variance, i.e. the two state variables: $pd_t = \alpha + \beta_{\pi} x_{\pi,t} + \beta_{\sigma^2_{\pi}} \sigma^2_{\pi,t} + \epsilon_t$. The second regression regresses the nominal 5-year interest rate onto expected inflation and the conditional inflation variance: $y^{\$}_{t,5y} = \alpha + \beta_{\pi} x_{\pi,t} + \beta_{\sigma^2_{\pi}} \sigma^2_{\pi,t} + \epsilon_t$. Finally, we report the analytical coefficient governing the relation between the conditional stock-bond covariance and the conditional inflation variance from Equation (24). We report the analytical coefficient since the model does not allow for simulation of realized covariances based on daily returns, as in data. Model statistics are based on a simulation of 150000 quarters.

Table 13: **Slope Components**

	Countercyclical State			Procyclical State			Full Sample		
	Slope	EH-part	RP-part	Slope	EH-part	RP-part	Slope	EH-part	RP-part
<u>Nominal Bonds</u>									
$E(y^{\$}_{5y} - y^{\$}_{3m})$	0.44	–0.38	0.82	0.47	1.20	–0.74	0.44	0.00	0.44
<u>Real Bonds</u>									
$E(y_{5y} - y_{3m})$	–0.46	–0.17	–0.29	0.06	0.55	–0.49	–0.34	0.00	–0.34

This table presents the different components of the slopes of real and nominal yield curves within our model. All results are reported on an annualized basis. Model statistics are based on a simulation of 150000 quarters.

Table 14: **Variance decompositions**

	Variable	Horizon (quarters)	Inflation shock	Output Shock	Fed shock	Cov shock
Countercyclical period 1965:1-1999:4	Inflation	1	1.00	0.00	0.00	0.00
		20	0.96	0.01	0.01	0.02
	Output	1	0.01	0.99	0.00	0.00
		20	0.08	0.82	0.11	0.01
	Fed rate	1	0.03	0.96	0.01	0.00
		20	0.35	0.09	0.55	0.01
	Stock-Bond cov	1	0.01	0.01	0.09	0.89
		20	0.06	0.02	0.18	0.74
Procyclical period 2000:1-2011:4	Inflation	1	1.00	0.00	0.00	0.00
		20	0.73	0.23	0.01	0.03
	Output	1	0.00	1.00	0.00	0.00
		20	0.01	0.93	0.02	0.04
	Fed rate	1	0.01	0.03	0.96	0.00
		20	0.06	0.59	0.31	0.04
	Stock-Bond cov	1	0.02	0.25	0.07	0.66
		20	0.01	0.56	0.04	0.39

This table presents variance decompositions of forecast errors stemming from one standard deviation shocks to inflation, real output growth, Federal funds rate, and stock-bond covariance. The shocks are retrieved from a VAR(1) with the ordering $X_t = [inflation, output, Fed\,funds\,rate, covariance]$ and where an orthogononalization is done using the Cholesky factor.

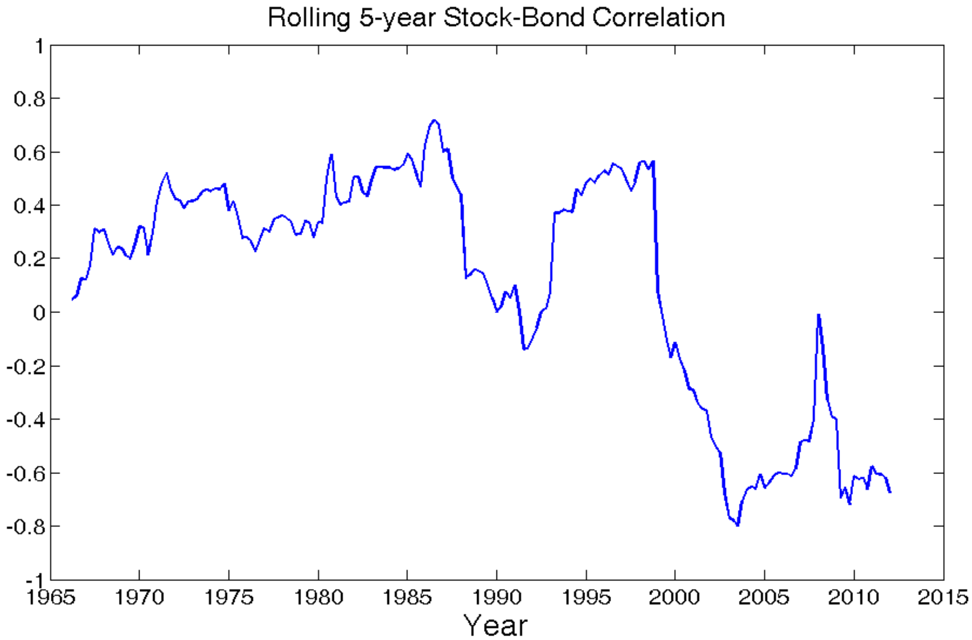
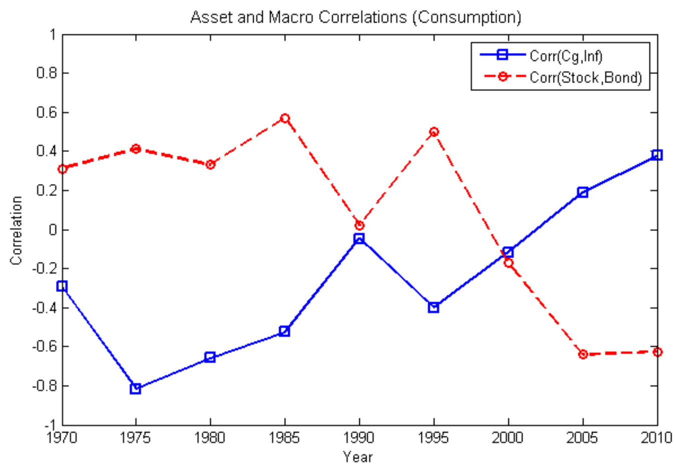
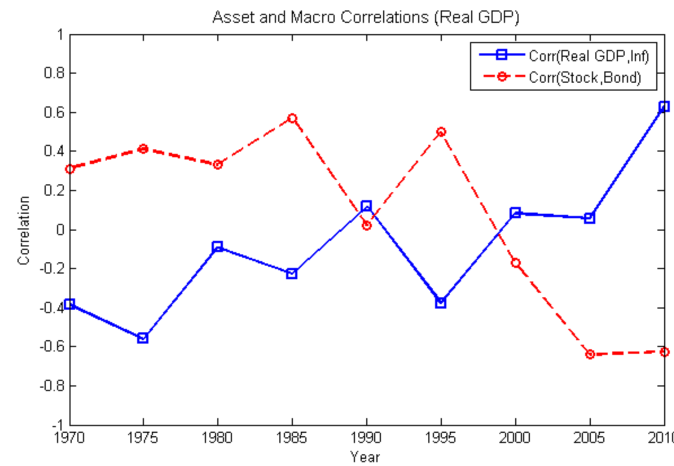


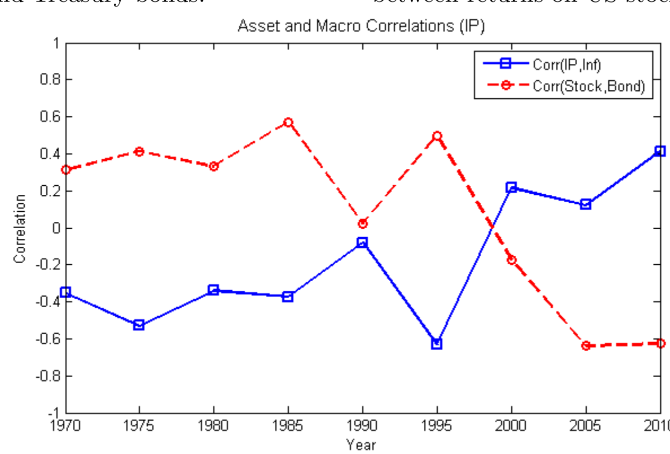
Figure 1: Rolling 5-year correlations between quarterly excess returns on US stocks and nominal 5-year Treasury bonds for the period Mar 1961-Dec 2011.



(a) Correlation between quarterly real consumption growth and inflation and between returns on US stocks and Treasury bonds.



(b) Correlation between quarterly real GDP growth and inflation and between returns on US stocks and Treasury bonds.



(c) Correlation between quarterly US industrial production growth and inflation and between returns on US stocks and Treasury bonds.

Figure 2: Asset and Macro Correlations computed for non-overlapping 5-year intervals over the period 1965:1-2011:4.

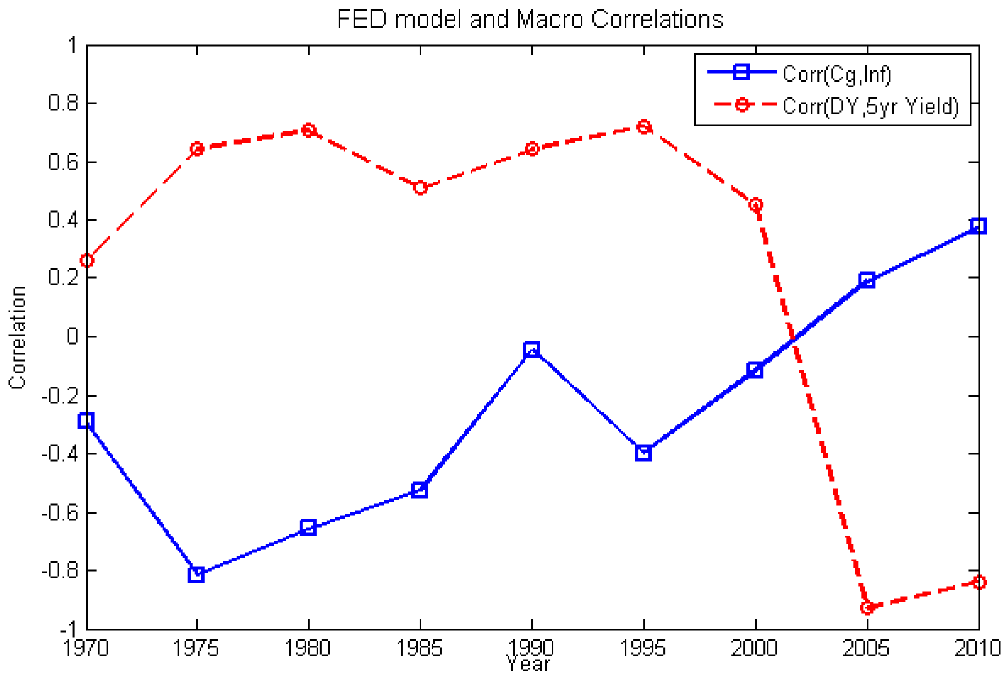


Figure 3: Correlation between quarterly real consumption growth and inflation and between the dividend yield on US stocks and 5-year nominal yields on US Treasury bonds. Correlations are computed for non-overlapping 5-year intervals over the period 1965:1-2011:4.

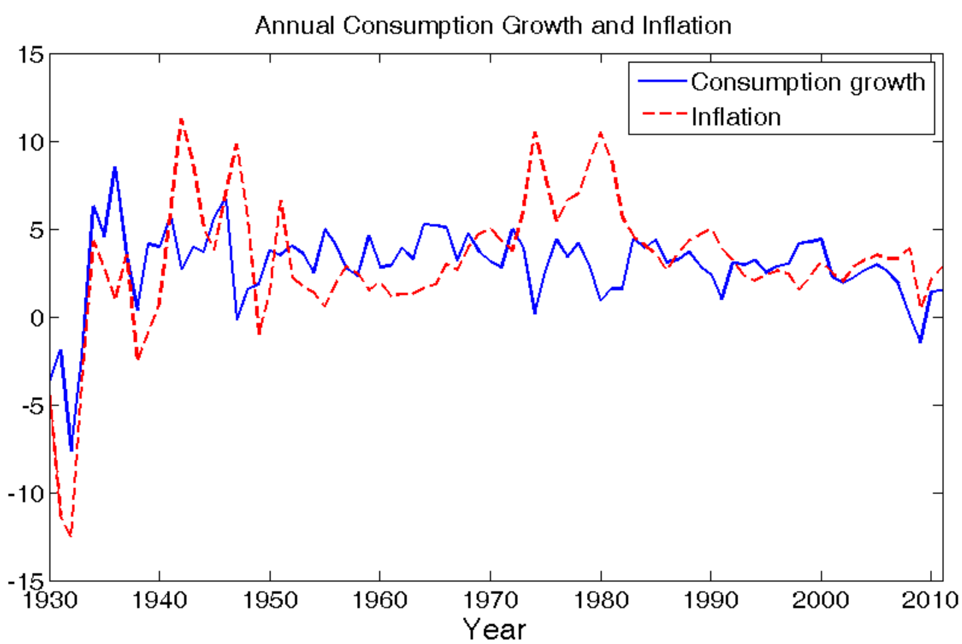


Figure 4: Annual real consumption growth and inflation over the period 1930-2011.

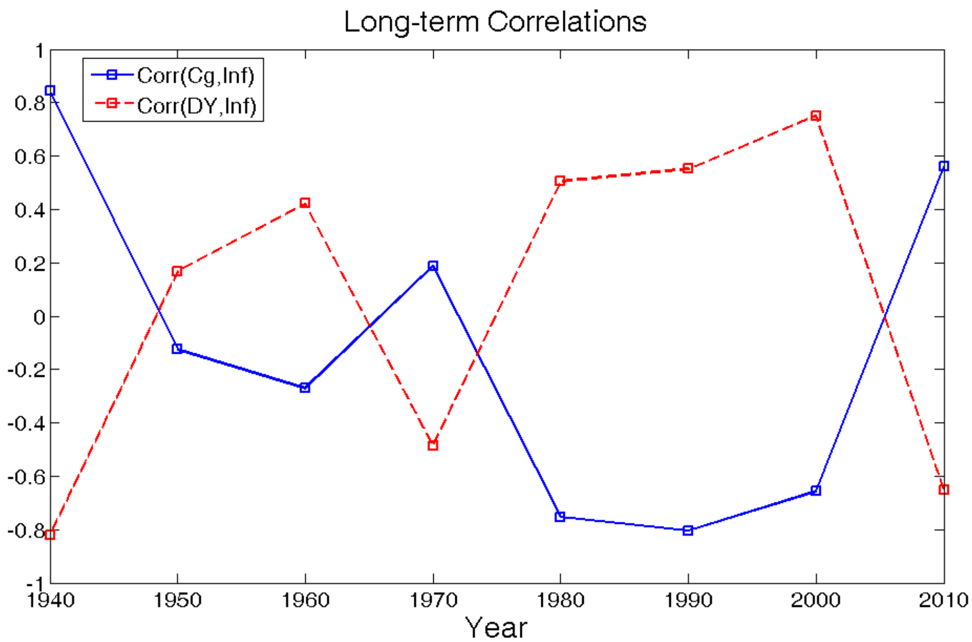


Figure 5: Correlations between annual real consumption growth and inflation and between dividend yields and inflation. Correlations are computed for non-overlapping 10-year intervals over the period 1930-2011.

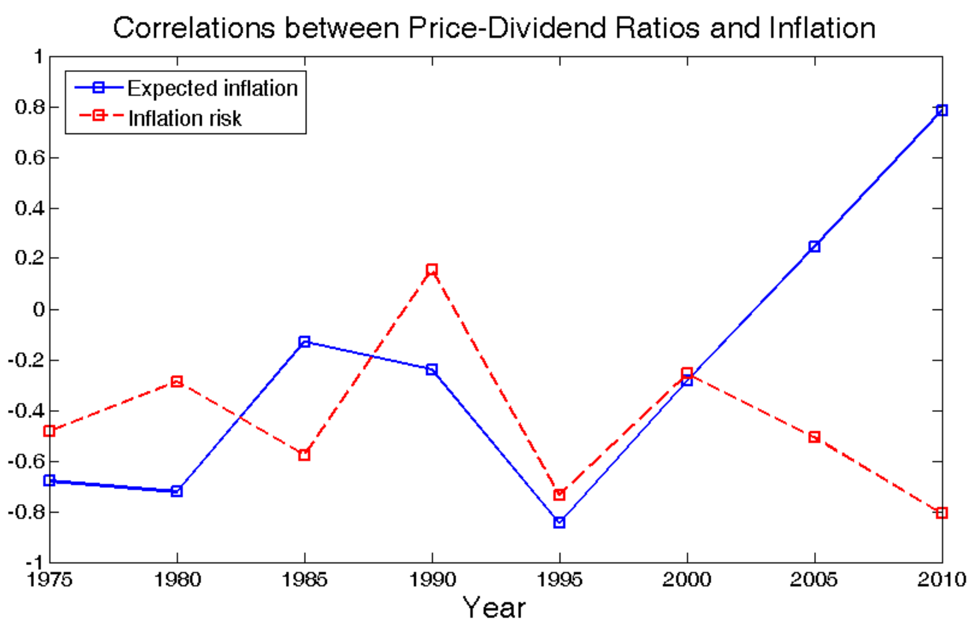


Figure 6: Correlations between log price-dividend ratios and expected inflation and inflation risk. Correlations are based on quarterly data and computed for non-overlapping 5-year intervals over the period 1970-2011. Inflation expectations are created by projecting quarterly inflation onto lagged growth, inflation, and yield spread. Inflation risk is measured as dispersion of inflation forecasts (PGDP) taken from Survey of Professional Forecasters.

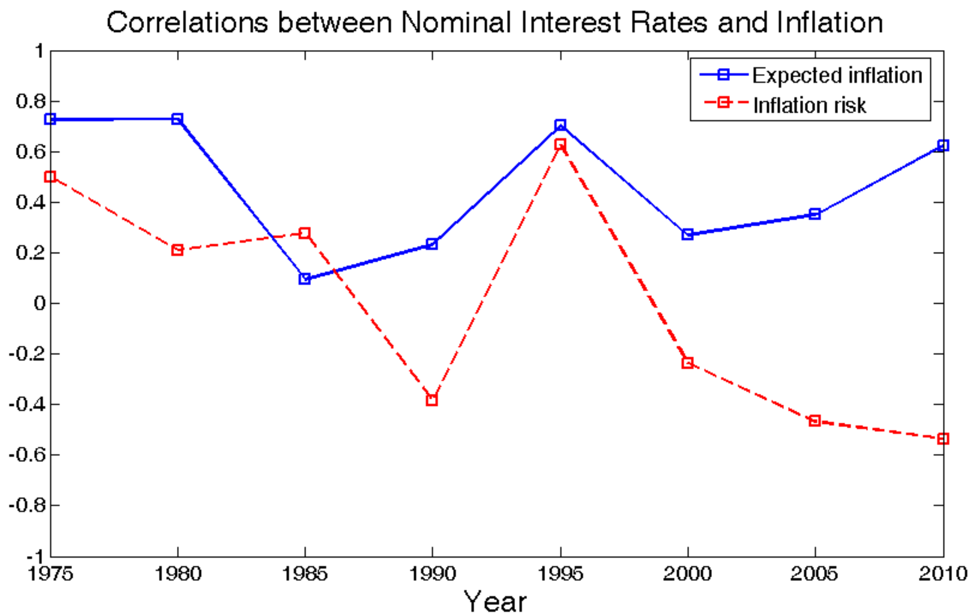


Figure 7: Correlations between the 5-year nominal interest rate and expected inflation and inflation risk. Correlations are based on quarterly data and computed for non-overlapping 5-year intervals over the period 1970-2011. Inflation expectations are created by projecting quarterly inflation onto lagged growth, inflation, and yield spread. Inflation risk is measured as dispersion of inflation forecasts (PGDP) taken from Survey of Professional Forecasters.

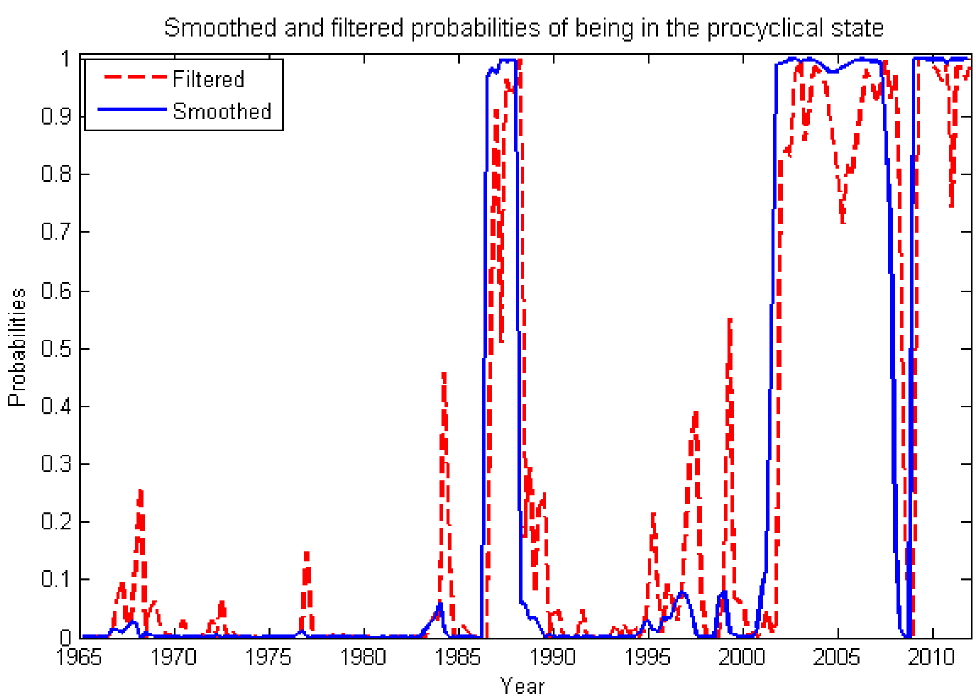


Figure 8: Filtered and smoothed probabilities of being in the procyclical inflation state. Probabilities are based on the estimated Markov-switching model described in Equation (1).

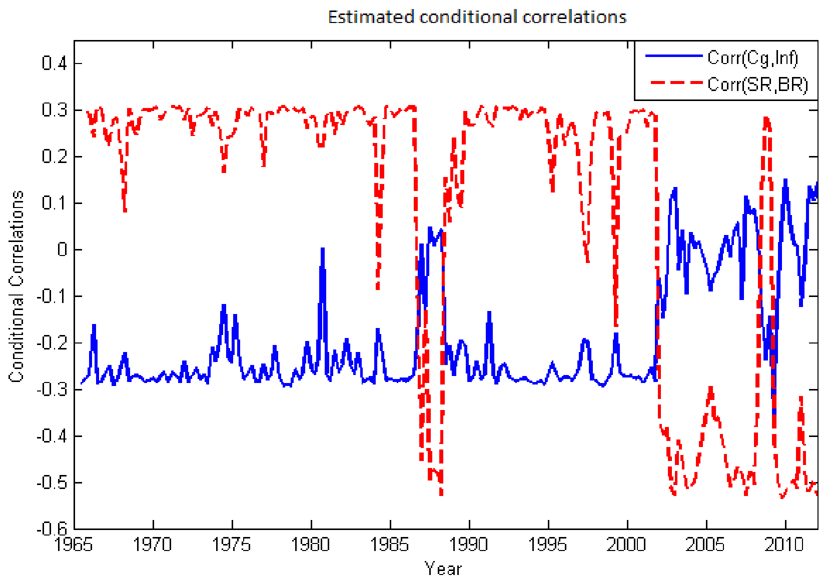


Figure 9: Empirically estimated quarterly conditional correlation between consumption growth and inflation and between stock and bond returns based on the estimated Markov-switching model described in Equation (1).

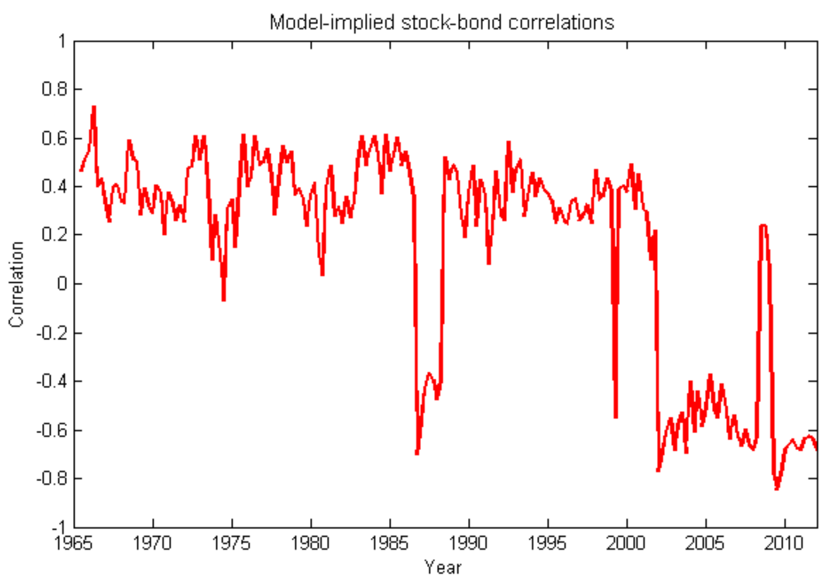


Figure 10: Model-implied quarterly conditional correlation between stock and 5-yr bond returns. Empirical proxies for the state variables, expected inflation and inflation volatility, are used to compute correlations. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. Inflation risk is measured as conditional inflation volatility based on an AR(1)-GARCH(1,1) on expected inflation. Correlations are computed using Equation (24) for the covariances together with analytical expressions for the volatility of stock and bond returns.

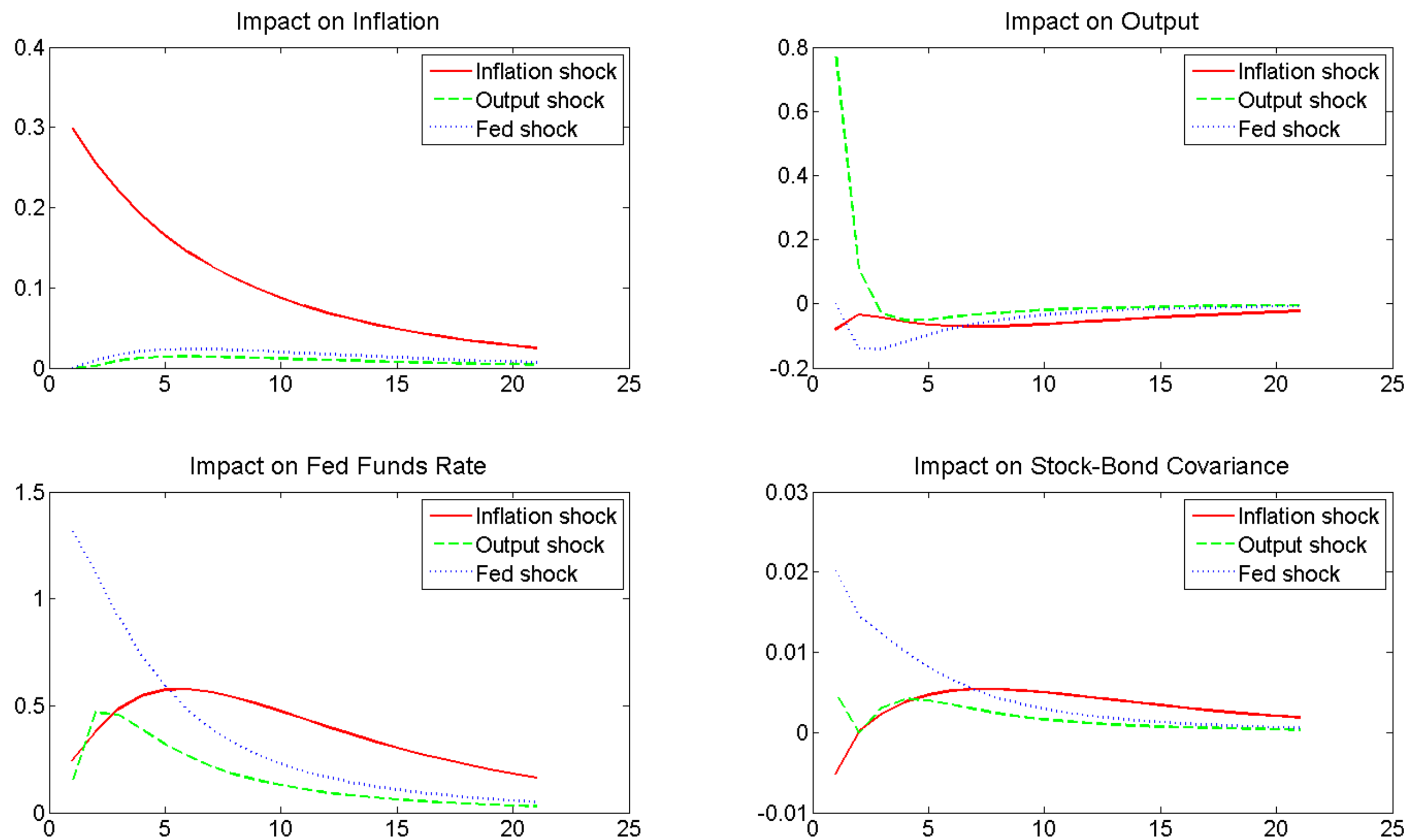


Figure 11: Impulse response functions for the countercyclical period 1965:1 - 1999:4. The shocks are retrieved from a VAR(1) with the ordering $X_t = [inflation, output, Fed\,funds\,rate, covariance]$ and where an orthogononalization is done using the Cholesky factor.

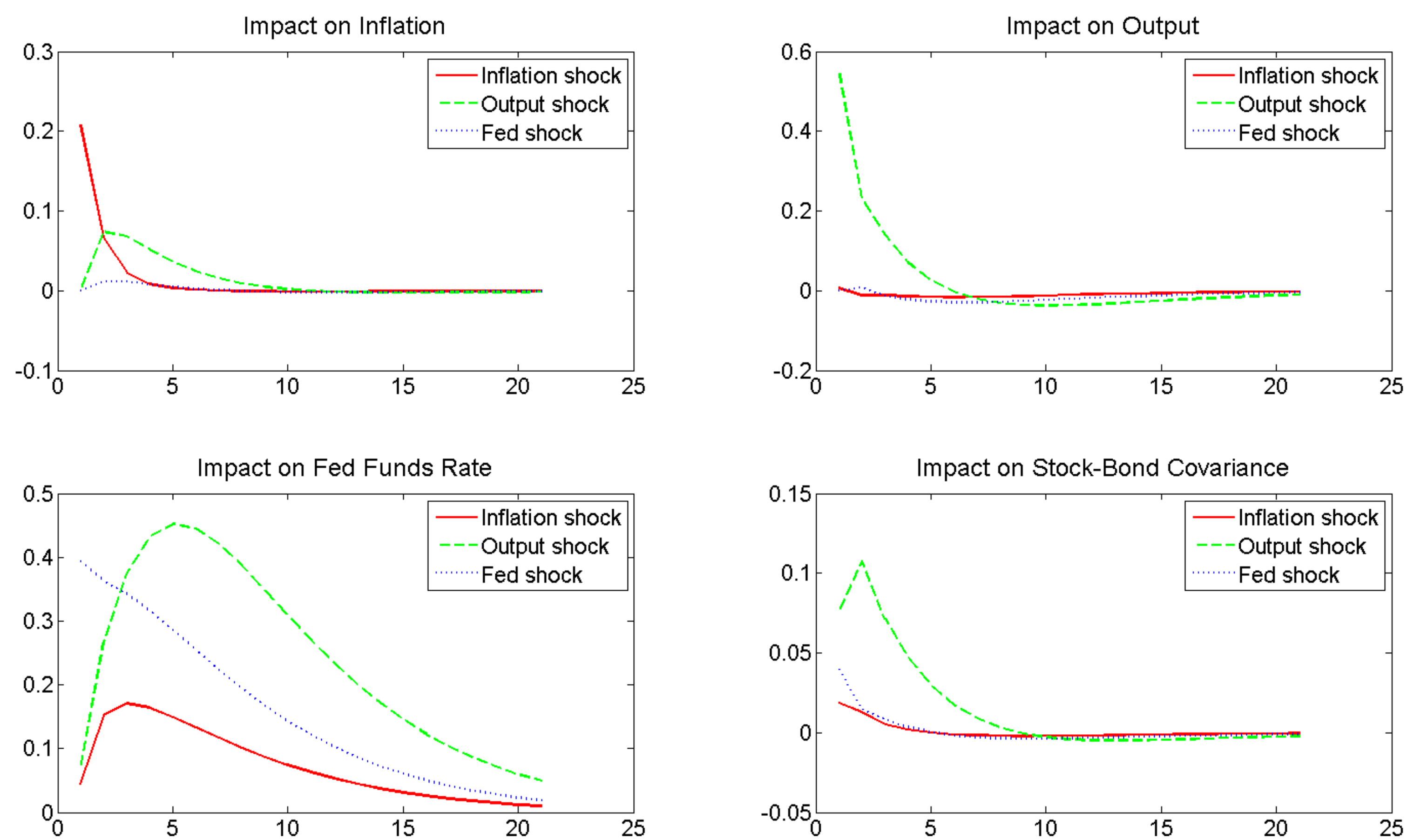


Figure 12: Impulse response functions for the procyclical period 2000:1 - 2011:4. The shocks are retrieved from a VAR(1) with the ordering $X_t = [inflation, output, Fed\, funds\, rate, covariance]$ and where an orthogononalization is done using the Cholesky factor.

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