False Safe Haven Assets: Evidence From the Target Volatility Strategy Based on Recurrent Neural Network [1]

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Abstract

This paper examines which safe haven assets should be used when improving out-of-sample portfolio performance. We define a market state with recurrent neural network (RNN) volatility predictions and construct an investment strategy that dynamically combines equity, cash, and safe havens. The equity is allocated by targeting the volatility, and investing in safe havens depends on the predicted market state. We consider the S&P500 index with 13 safe haven assets, such as long-term government bonds, commodities, gold, and other precious metals. Other indices, NIKKEI225, NIFTY50, and STOXX50, are examined for robustness. With analysis conducted over a 20-year sample period, we find that RNN delivers sound predictions to construct the volatility targeting strategy. Among considered assets, only long-term Treasury bonds act as a safe haven and improve the strategy performance. Other considered assets have no such potential. Our findings are relevant to portfolio managers and investors actively managing portfolio risk.

Keywords: Asset allocation strategy, Target volatility, Safe haven, Recurrent neural networks, Machine Learning

JEL Codes: G11 (portfolio construction), C45 (neural networks and related topics), C32 (time series analysis), G15 (international financial markets), C58 (financial econometrics)

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1. Introduction

Volatility becomes crucial, in times of elevated risk, when investors fly to safety (Baele et al., 2020; Beber et al., 2009; Longstaff, 2004). It is well documented that this safety is obtained by investing in government bonds (Hsu et al., 2020); however, other asset classes, e.g., gold (Cai et al., 2001; Koutsoyiannis, 1983; Triki & Ben Maatoug, 2021) or precious and industrial metals (Ali et al., 2020), are also a good choice. Adrian et al. (2019) suggest, however, that when market volatility is high, the effect of the flight-to-safety is weak. The uncertainty about the level of market volatility influences portfolio efficiency, but it can be reduced by applying an appropriate strategy. This strategy should be based on nonlinear market volatility forecasts, as the linear models fail to predict them (Adrian et al., 2019, Bekaert & Hoerova, 2014; Bollerslev et al., 2013).

In this study, we assess the efficiency of volatility-targeting portfolio, which assumes the allocation between stocks, assets of potential safe haven characteristics, and cash. We enhance the strategy's ability with market volatility forecasts, generated by recurrent neural networks, and test its performance within the out-of-the-sample approach. That will allow this strategy to be investable.

The strategy of target volatility has its origins within the global financial crisis. Insurance companies had to reconsider risk management techniques and find a way to control the volatility of their long-term investments (Hocquard et al., 2013). The main goal of the target volatility strategy is to maintain desired portfolio volatility level. The critical challenge of this strategy lies in the accurate prediction of market volatility. The better the predictions, the minor differences between both required and realized portfolio volatility are. The prediction accuracy becomes especially important when markets are not stable. There are different approaches to enhance the prediction accuracy during times of highly elevated market volatility, but none of them are ideal (D. H. Kim et al., 2009; H. Y. Kim & Won, 2018; Y. Kim & Enke, 2016, 2018). Generally, the forecasts' mean square error increases as market volatility increases; therefore, the challenge of finding the most accurate model for market volatility prediction – which can be used for portfolio volatility forecasts – still seems to be opened.

There are a few approaches that are most broadly used for nonlinear volatility modelling and forecasting. The autoregressive conditional heteroskedasticity (ARCH) models are based on daily returns, and are aimed to estimate latent market volatility. Another approach is based on the measures employing intraday returns and, thus, calculate unobserved integrated variance (Hansen & Lunde, 2005). In this case, linear *ARFIMA* models are applied to non-parametric volatility estimates, such as realized variance or bipower variation. The third approach uses implied volatility, which relies on option pricing. Here, the historical returns or volatility are not needed; however, the choice of the option-pricing model becomes crucial. Each of these approaches requires different assumptions about the distribution of the given data (Poon & Granger, 2005). At the same time, an increasing number of studies offer a machine learning approach that allows for the overcoming of these difficulties (Christensen et al., 2021; H. Y. Kim & Won, 2018; Y. Kim & Enke, 2018). In this article, we continue the successful path of utilizing machine learning techniques to forecasting volatility and use a recurrent neural network, represented by Gated recurrent units (*GRU*), to predict realized bipower variation (*RBV*). We compare the results with *ARFIMA* and *GARCH* or *GJR-GARCH*, which aim to predict the latent market volatility.

The novelty of this study is to enrich the target volatility strategy with potential safe haven assets[2], and thus increase the efficiency of the portfolio. First, we generate volatility forecasts from several competing models. Second, we apply these forecasts in the target volatility strategy and check the performance of the portfolios that are measured by Sharpe and Calmar ratios. Third, if the target volatility strategy (based on our forecasts) gets better results than an index, we examine the safe haven asset. In this way, we verify whether an investment – expected to have a safe haven characteristic – increases portfolio efficiency; otherwise, we may conclude that this is not a safe-have asset. It is essential to highlight that our approach provides entirely out-of-sample conclusions—unlike most previous literature. We test safe haven assets' impact on the target volatility strategy on the 90th, 95th, and 99th percentile of predicted one-day-ahead volatility and 10%, 20%, and 30% maximum weight allocations.

In theory, the target volatility strategy, as well as an approach with safe haven assets, should act in the same way. The target volatility strategy should reduce portfolio risk by changing equity allocation, depending on the expected market volatility. Adding safe haven assets to a portfolio should reduce its standard deviation and drawdowns; this is due to the negative correlation between equity and safe haven assets when markets are sinking. In other words, if an investor can find periods when reducing the equity weights in a portfolio increases the portfolio's efficiency, they could be interested, at the same time, in investing in a safe haven asset. Adding such an asset to a portfolio should be a natural investment decision, since low equity allocation in a portfolio leaves plenty of space for other investments.

[2] We define a safe haven asset after Baur (2010). A safe haven is an asset that is uncorrelated or negatively correlated with a given asset or portfolio in times of market stress or turmoil. This assumption allows us to focus on the property of asset classes to reduce losses, particularly in extreme adverse market conditions; this is unlike in the case of a hedge or diversifier assets, with regard to which zero or negative correlation property is only required to hold on average.

We consider the 13 assets that are the most discussed in the literature as potential safe havens. We include four types of long-term government bonds; precious metals, namely gold; silver and platinum; industrial metals, namely oil and copper; and agricultural raw commodities such as cocoa, wheat, soybeans, and corn. We exclude from testing foreign exchange currencies, since the evidence on their property of safe haven in relation to the stock market is mixed.[3] We also exclude cryptocurrencies because of their unclear diversification benefits for stock portfolios in stressful market conditions.[4] Besides, the general trend of cryptocurrency appreciation persistence over the studied period can bias the conclusion on their short-term ability to act as a safe haven for stock prices.

To this end, we conduct a considerably large-scale empirical analysis for the S&P500 index with 21 years of daily history spanning from 2000 to 2020. We test the strategy results with an 11-year-window (2010-2020) that covers a few significant market sale-off periods, including the turbulences initiated by the COVID-19 pandemic. We find that the pure strategy targeting volatility of 10% and *GRU*-MULTIVAR daily volatility predictions overperform the index by 21.3% and 83%, in terms of the Sharpe and Calmar ratios. The pure strategy results demonstrate that our *GRU*-MULTIVAR predicts one-day-ahead market volatility. Therefore, we move to the next step and include each of the safe haven assets as a strategy potential enrichment for periods of expected elevated volatility. In this setting, we can verify the role of additional investments in the fully out-of-sample approach.

Our findings are the following. First, none of the tested assets improve the strategy performance in periods of extremely high expected volatility. This finding contradicts the primary goal of a safe haven asset: protecting a portfolio against market turmoil. Second, U.S. long-term Treasuries enhance the strategy performance in times of moderately elevated expected volatility. They may add 5% to the portfolio performance that is measured with the Sharpe ratio and even more than 10% in the case of the Calmar ratio. These findings are confirmed with other volatility prediction methods. Moreover, we verify the robustness with Japanese, Indian, and European stock markets and find similar results for NIKKEI225 and NIFTY50. The volatility seems to be more challenging to predict for STOXX50, where our pure strategy fails to overperform the index. Thus, in this case, we resign from testing the impact of adding the safe haven assets. Finally, we define the five most significant drawdowns in our

[3] see Campbell et al. (2010), Grisse and Nitschka (2013), Kopyl and Lee (2016), Ranaldo and Söderlind (2010)—among others.

[4] The studies of (Conlon et al., 2020; Conlon & McGee, 2020; Hussain Shahzad et al., 2020; Klein et al., 2018; Smales, 2019) provide contrary results on that issue to the studies of (Ali et al., 2020; Będowska-Sójka & Kliber, 2021; Bouri et al., 2017, 2020).

testing sample (2010-2020), and visualize the strategy performance in the market stress period. The clean target volatility strategy overperforms the S&P500 index in all reported drawdowns. We observe the safe haven properties only in long-term bonds, especially in Treasuries. It is worth highlighting that the largest of the drawdowns occurred during the COVID-19 pandemic. Our findings contrast with the emerging literature about safe havens during the pandemic, which reports gold's safe haven properties.

Our study aims to contribute to two strands of research. First, we develop a new methodology to test defensive properties of assets, thus contributing to research on safe havens. We propose a machine learning out-of-sample approach to define the market state in terms of its volatility level. This framework enables finding periods of market turmoil when safe haven assets should provide a shelter to the portfolio. Second, by utilizing forecasts of realized volatility via recurrent neural networks for volatility targeting, we contribute to studies on target volatility strategy.

The remainder of the article is organized as follows. Section 2 reviews the literature concerning safe haven assets and volatility targeting strategies. It also conducts a more detailed discussion of our contributions. Section 3 explains the stages of our research and presents the models used to forecast market volatility, the framework of target volatility strategy, and the procedure of testing safe haven assets. Section 4 provides the details and results of our empirical study. Section 5 provides robustness and additional tests. Finally, Section 6 concludes. Additional results are available in the Online Appendix.

2. Literature review

Our study considers 13 different assets from two groups: commodities and bonds. We test their safe haven properties by adding them to the target volatility strategy. This section reviews the main studies on safe haven assets and the target volatility strategy, elaborates on how they relate to our research, and presents our contribution.

Baur and Lucey (2010) shows that gold is a safe haven for stocks within a short period of about three weeks. Baur and McDermott (2010) extend this research and examine all major world markets. They find that gold is a safe haven for the United States and Europe, but not for many other regions, including emerging markets. Li and Lucey (2017; 2015) prove that the same safe asset features are shared by other precious metals as silver, platinum, and palladium, but they may change in time. Będowska-Sójka and Kliber (2021) examine gold, Bitcoin, and Ether as potential safe havens and find that from January 2015 to March 2021 only gold was a strong safe-haven against the stock market indices. This feature evaporated at the beginning of the COVID-19 pandemic crisis. Ali et al. (2020) propose a new methodology based on a cross-quantilogram, and investigate the role of four commodity groups (energy, agricultural, industrial, and precious metals) on international stock markets. They find safe haven features for precious and industrial metals. The majority of other researchers studying potential safe haven characteristics for commodities underline the sought-after qualities in gold. The results for other commodities are mixed and depend on the timeframe, market, and method used to analyze data. In terms of methodology, the researchers typically use quantile regressions, GARCH, or cross-quantilogram. Some also use alternative approaches, e.g., Markov-switching (He et al., 2018) or wavelet analysis (Bouri et al., 2020).[5] However, to the best of our knowledge, none of them uses machine learning to define periods of market turmoil. Our approach is to determine the market state with recurrent neural networks (RNN). We predict realized variance with RNN, and our forecasts define the market state where safe haven assets should be uncorrelated or negatively correlated with the equity market. With realized volatility forecasts based on RNN, our approach is out-of-sample and fully investable.

Furthermore, our study aims to contribute to the fast-growing body of research on the role of safe haven assets during the COVID-19 pandemic. Adekoya et al. (2021) analyze a period of 91 days from January 2, 2020 to May 15, 2020, and find that stock market risk could be effectively hedged by gold during the crisis. These findings are supported by Ji et al., (2020), who define the COVID-19 period from December 1, 2019 to March 31, 2020, and find safe haven assets properties not only in gold, but also in soybean. In contrast, Akhtaruzzaman et al. (2021) split the analyzed period to Phase I (December 31, 2019 to March 16, 2020) and phase II (March 17 to April 24, 2020). They demonstrate that gold served as safe haven asset for the stock market during Phase I and lost its defensive properties during Phase II. From the international market perspective, Yousaf et al. (2021) investigate the role of gold against 13 Asian stock markets. They define the COVID-19 subperiod from January 1, 2020 to May 5, 2020, and indicate that gold was a strong safe haven only in China, Indonesia, Singapore, and Vietnam. Our findings question whether gold and other commodities exhibit safe haven asset characteristics during the COVID-19 sell-off. We define the COVID-19 period as when the S&P500 index experienced its maximum drawdown in our testing sample. The starting date is February 19, 2020, and the end date reaches March 23, 2020. We find that all commodities, including gold, deteriorate performance of our strategy. Among 13 tested assets, only U.S. Treasuries turn out to be a safe haven.

[5] Ali et al. (2020) demonstrate a list of 50 studies related to commodities as safe haven assets for stock market indices.

Another subject of research are bonds that act as a safe haven for equity markets. The diversifying role of bonds in the investment portfolio is related to assets decoupling (Gulko, 2002) and flight-to-quality (Baur & Lucey, 2009).[6] The essence of flight-to-quality is associated with the research on safe haven assets. Both phenomena are supposed to increase diversification benefits to the portfolio.[7] Likewise, for commodity research, the effect of flight-to-quality is not constant, and changes depending on the crisis period, market, and methodology used (Baur & Lucey, 2009; Flavin et al., 2014; Gulko, 2002; Habib & Stracca, 2015; Hartmann et al., 2004). From the methodological perspective, many earlier studies investigate the relationship between stock returns and bond market variance (Fleming et al., 1998; Gulko, 2002; Scruggs & Glabadanidis, 2003). Hartmann et al. (2004) use extremal dependence measure, while Baur & Lucey (2009) draw conclusions based on regressions. Flavin et al. (2014) employ a regime-switching framework within which both common and idiosyncratic shocks are separated. Finally, with regressions and the wavelet approach, Papadamou et al. (2021) demonstrate that flights-to-quality took place during the COVID-19 pandemic and occurred simultaneously across 10 countries. However, analogous to the research on commodities, none uses machine learning to define the market turmoil periods when safe haven assets should add value to the portfolio. Our study uses a different approach to evaluate the role of government bonds as a safe haven. We determine the periods of market stress with predictions for realized variance made with RNN. To the best of our knowledge, this is the first study to define commodities and bonds as safe haven characteristics with RNN.

Our paper also contributes to the research on targeting the volatility in the investment strategy. Hocquard et al. (2013) present empirical results of a target volatility strategy. They estimate volatility predictions with the GARCH(1,1) model and demonstrate a 42% increase in the Sharpe ratio of target volatility strategy to a globally diversified portfolio. Perchet et al. (2015) use five different GARCH models to predict the next day's volatility, and compare the target volatility strategy results with a buy-and-hold investment in the S&P500 index. They demonstrate that strategy built with a clean GARCH overperforms the buy-and-hold by 24.4%. Kim and Enke (2016, 2018) predict the implied market volatility with neural networks, and report a significant overperformance of the target volatility strategy based on neural networks to the buy-and-hold and other forecasting methods. Finally, Zakamulin (2019) and Bongaerts

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[6] Baur & Lucey (2009) define flight-to-quality from stocks to bonds as a significant decrease in the correlation coefficient in a crisis period compared to a benchmark period resulting in a negative correlation level.

[7] To unify the terminology, our study relates only to safe haven assets, and investigates the diversification benefits of bonds along with commodities. Our approach is a common procedure (Baur & McDermott, 2016; Flavin et al., 2014; Gulko, 2002).

(2020) propose simple modifications in the target volatility strategy to reduce transaction costs. All these studies use the same basic concept of the target volatility strategy as we do, but we are the first to construct a target volatility strategy with predictions for realized variance based on RNN, and to utilize recent innovations in predicting the realized volatility in the target volatility strategy.[8]

3. Methods

In this section, we first describe the general setup for daily volatility prediction models. Then, we present five prediction methods; these include three financial time-series models (*GARCH*, *GJR-GARCH*, and *ARFIMA*), as well as two types of recurrent neural networks that are applied to predict market volatility (univariate and multivariate, *RNN*s with *GRU* layers). In the next step, we present the target volatility strategy's implementation to assess the portfolio volatility based on earlier predictions. Finally, we explain how we verify the safe haven characteristics of assets that are covered in the study.

3.1 The general model of market volatility

We apply both univariate and multivariate approaches. In the former, there are *GARCH* models that use historical returns, as well as *ARFIMA* and *GRU-UNIVAR* models that utilize historical volatility estimates. In the latter, *GRU-MULTIVAR* extends the data input scope of daily observations for the realized daily volatility (*BPVSPX500*), gold (*GOLD*), or crude prices (*CRUDE*)—as well as yields of government (*USGOV3YI*) and corporate bonds (*USCORPAA3YI*) (H. Y. Kim & Won, 2018)[9].

3.2 Financial time-series models

When obtaining forecasts of market volatility, we rely on two alternative approaches. First, returns of the indices are modelled with two basic *GARCH* specifications; second, we apply ARFIMA specification to realized bivariate variation (*RBV*). Both classes of models are estimated using G@RCH and *ARFIMA* packages (Doornik & Hendry, 2005). When forecasting, we use a rolling sample of size T (T is equal to the number of observations in the primary 2006.01-2009.12 window) and either set one-step ahead conditional variance forecasts (within the underlying *GARCH* specifications) or one-step ahead realized bivariate variation

[8] See H. Y. Kim & Won (2018).

[9] Realized daily volatility (*BPVSPX500*) is represented as the monthly index volatility but *ARFIMA*, *GRU-UNIVAR*, and *GRU-MULTIVAR* predicts the realized daily volatility before the transformation, which is presented in equation 24.

(within the *ARFIMA* specification) Henceforth, the one-step-ahead variance forecast from a model is denoted as v_{t+1}.

3.2.1 *GARCH* model

The *GARCH* specification was proposed by Bollerslev (1986). A univariate time series, returns r_t is represented as follows:

$$r_t = E(r_t|F_{t-1}) + \varepsilon_t, \tag{1}$$

where $E(.|.)$ is the expectation operator, ε_t is an error term (also called an innovation process) such as $E(\varepsilon_t) = 0, E(\varepsilon_t, \varepsilon_s) = 0$ if $t \neq s$. In the *GARCH*, the specification innovation process is presented as:

$$\varepsilon_t = \sigma_t z_t, \tag{2}$$

where z_t is IID process, with $E(z_t) = 0$ and $var(z_t) = 1$.

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \tag{3}$$

We apply plain-vanilla *GARCH*(1,1) specification, as it often serves as a benchmark model.

3.2.2 *GJR-GARCH* model

We also consider the asymmetric model that accounts for the so-called leverage effect, which is a commonly observed feature in the financial markets: volatility reacts differently to big negative returns causes than to big positive returns. The specification of the *GJR-GARCH* model is the following (Glosten et al., 1993):

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2 + \gamma_i S_{t-i} \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{4}$$

where S_{t-1} is a dummy variable with the value of 1 for $\varepsilon_t < 0$—and 0 otherwise. The γ parameter measures a potential leverage effect.

3.2.3 *ARFIMA* model

We also apply *ARFIMA* specification to volatility measures, obtained on the basis of intraday market returns. This concept was introduced by Andersen and Bollerslev (1998), who proposed an estimate of unknown integrated variance called realized variance (*RV*). Although RV is a very popular measure (Będowska-Sójka, 2018), it is quite sensitive to jumps within the variance. Therefore, Barndorff–Nielsen and Shephard (2004) introduced another measure called the realized bipower variation (*RBV*) that is also based on the intraday returns but is robust to jumps. The formula is the following:

$$RBV_t(\Delta) = \frac{\pi}{2}\sum_{n=1}^{1/\Delta}|r_{t,n}||r_{t,n+1}|, \tag{5}$$

where $r_{t,n}$ is *n*-th intraday return on day *t* and Δ denotes the frequency of intradaily returns. The specification of *ARFIMA* model is as follows:

$$\Phi(L)(1-L)^{\delta}(RBV_t) = \Theta(L)e_t, \ \ e_t \sim N\,\mathrm{ID}(0,\sigma_e^2), \tag{6}$$

where the operator $(1-L)^{\delta}$ accounts for long-memory. In our study, we use *ARFIMA*(0,*δ*,0) specification. We apply realized bipower variations that are calculated on the basis of five-minute returns. We also examined the forecasts of realized variance, but finally chose *RBV* due to its higher forecasting accuracy.

3.3 Recurrent neural networks (*RNN*)

3.3.1 Development of *RNN*s

Recurrent neural networks (*RNN*s) are a powerful type of ANN model that can analyze time dynamics. Introduced by Elman (1990), *RNN* is a rich family of models that capture temporal dependencies between inputs and outputs that consist of sequences of non-independant points (Lipton et al., 2015). The ability to process input data that are structured in sequences makes it a practical method for learning patterns in time-series data (Salehinejad et al., 2017).

A basic *RNN* looks quite similar to a traditional feedforward network. The difference lies in connections pointing backward. Figure 1 (left) presents the simplest possible *RNN* structure, consisting of three neurons (nodes). The first one receives input x_t, the second one produces output y_t, and the third one is called hidden node h_t. The hidden notes are latent variables that are not observed directly but are the key points of *RNN*; with the ability to receive input, produce output, and send that output back to itself. They are responsible for memorizing patterns in a sequence (Cong et al., 2020). Figure 1 (right) visualizes the unrolled network when the same recurrent network is presented once per time step. In each time t the recurrent neuron receives the input x_t, as well as its output from the previous step— y_{t-1}. Recurrent neurons have two sets of weights: one for the input x_t and the other for the outputs from the previous time step, $y_{(t-1)}$ (Géron, 2019). The hidden state output h_t and output y_t at step t can be represented as:

$$h_{(t)} = \sigma\left(W_{hx}^T x_{(t)} + W_{hh}^T h_{(t-1)} + b_h\right) \tag{7}$$

$$y_{(t)} = f_0\left(W_{yh}^T h_{(t)} + b_y\right), \tag{8}$$

where W_{hx} and W_{hh} are two matrices of weights in the hidden state, consisting of recurrent neurons, W_{yh} is the matrix of weights in the output layer; b_h and b_y are the bias vectors in the hidden and output layer, respectively; and σ is the activation function. Finally, f_0 states for functions that transfer from h_t to the output values.[10] Depending on a task's nature, *RNN* can be traditionally configured in three different methods: sequence-to-sequence network, sequence-to-vector network, and a vector-to-sequence network.[11] For volatility prediction, it is suitable to use the sequence-to-vector network. We can feed the *RNN* with a sequence of past observations and predict the daily volatility only for step T. Therefore, the equation (2) can be used only once for each daily prediction.

Basic *RNN* configuration has a limited capability of the sequence length that may be learnt. Therefore, it may be inadequate to train for tasks that involve long-term dependencies (Bengio et al., 1994). To tackle this problem, several types of cells with long-term memory have been introduced. The most popular model, Long Short-Term Memory (*LSTM*), is proposed by Hochreiter and Schmidhuber (1997). It consists of multiplicative gate units that learn to open and close access to the constant error flow. Figure 2 presents the anatomy of *LSTM* architecture. In comparison to basic *RNN*, there's an additional data flow between each recurrent neuron that consists of long term memory $c_{(t)}$. Besides, instead of one hidden layer – used in basic *RNN* structure – there are four layers. The main layer $g_{(t)}$ has the traditional role of analyzing the current inputs $x_{(t)}$ and the previous (short-term) state $h_{(t-1)}$. Three additional layers control the data flow between time steps. Each time step consists of three gates (Figure 2 shows them aggregated as "GATES" for clarity): *forget gate* (controlled by $f_{(t)}$), which controls what parts of the long-term memory should be added to the long-term state; *input gate* (controlled by $i_{(t)}$), which regulates which part of g_t should be added to the long-term memory; and *output gate* (controlled with $o_{(t)}$), which decides what parts of the long term memory should be read and output at this time step.[12] From a formal perspective, the data flow can be described with the following equations:

$$i_{(t)} = \sigma\left(W_{xi}^T x_{(t)} + W_{hi}^T h_{(t-1)} + b_i\right) \tag{9}$$

$$f_{(t)} = \sigma\left(W_{xf}^T x_{(t)} + W_{hf}^T h_{(t-1)} + b_f\right) \tag{10}$$

$$o_{(t)} = \sigma\left(W_{xo}^T x_{(t)} + W_{ho}^T h_{(t-1)} + b_o\right) \tag{11}$$

$$g_{(t)} = tanh\left(W_{xg}^T x_{(t)} + W_{hg}^T h_{(t-1)} + b_g\right) \tag{12}$$

$$c_{(t)} = f_{(t)} \otimes c_{(t-1)} + i_{(t)} \otimes g_{(t)} \tag{13}$$

[10] The functions used for final data transformation depends on the nature of the task. For classification, tasks typically softmax function is used. In case of predicting the volatility, it may be an identity function.
[11] Another type of *RNN* configuration is more complex when sequence-to-vector model is connected with vector-to-sequence. This type of configuration is called Encoder-Decoder.
[12] More details concerning *LSTM* architecture can be found in Géron (2019).

$$y_{(t)} = h_t = o_{(t)} \otimes tanh(c_{(t)}), \tag{14}$$

where W_{xi}, W_{xf}, W_{xo}, W_{xg} denote weight matrices between each of four layers, the input $x_{(t)}$; W_{hi}, W_{hf}, W_{ho}, W_{hg} states for a set of weights matrices between each of the four layers, and the short-term state $h_{(t-1)}$; b_i, b_f, b_o, b_g represents the bias terms for each of the four layers.

While *LSTM*s have shown the capability of learning patterns from long sequences, they have high memory requirements due to them consisting of multiple memory cells. Therefore, a simpler mechanism that significantly reduces the number of parameters is attractive for relatively less complicated tasks. Cho (2014) proposes Gated Recurrent Units (*GRU*) as a simplified version of *LSTM* that reduces the computation cost and performs just as well (Cong et al., 2020; Greff et al., 2015). *GRU* architecture merges two state vectors into a single vector $h_{(t)}$. Both an input gate and a forget gate are controlled with a single gate. Additionally, there is no output gate; the full state vector is output at every time step. Instead, there's a new gate controller $r_{(t)}$. Its role is to control which part of the previous state will be shown to the main layer. Equations 17-20 describe the formal computation details:

$$z_{(t)} = \sigma(W_{xz}^T x_{(t)} + W_{hz}^T h_{(t-1)} + b_z) \tag{15}$$

$$r_{(t)} = \sigma(W_{xr}^T x_{(t)} + W_{hr}^T h_{(t-1)} + b_r) \tag{16}$$

$$g_{(t)} = tanh(W_{xg}^T x_{(t)} + W_{hg}^T (r_{(t)} \otimes h_{(t-1)}) + b_g) \tag{17}$$

$$h_{(t)} = z_{(t)} \otimes h_{(t-1)} + (1 - z_{(t)}) \otimes g_{(t)} \tag{18}$$

3.3.2 Model training, validation, and testing

Our prediction task is relatively simple, in terms of *RNN* capacities related to process translating or image processing. In the univariate model, we predict the next day market volatility measured with realized bivariate variation (*RBV*) – described in equation 5 – with 22 days of daily historical observations.[13] In the multivariate model, we extend the data scope for four additional features that create an input matrix of 108 daily observations. These features are gold, oil, T-bonds, and commercial bonds; features that Kim and Won (2018) demonstrate to be informative in volatility prediction. We use a stacked *GRU* specification that consists of an input layer, two

[13] The history of 22 daily observations represents a typical approach to include a sequence of monthly rolling, daily data; it is commonly used in tasks concerning the prediction of stock price index volatility (H. Y. Kim & Won, 2018).

stacked hidden *GRU* layers, and the output layer that transforms the output from the hidden layer into a single volatility prediction number.[14]

We divide our dataset into two subsamples: training and testing. This split allows us to deliver entirely out-of-sample testing results. The training sample is used to estimate the model subject to specific tuning hyperparameters values. We use five-fold cross-validation and repeatedly search for hyperparameters that minimize the predictive mean squared error. With data from the training sample, we optimize the number of neurons in both hidden layers, a batch size, a level of l2 regularization on weights and bias, and a dropout ratio.[15]

We implement four regularization techniques in order to deal with the possible overfitting. First, we use l2 or ridge regularization on weights and bias. This standard machine learning technique controls overfitting where a penalty is set for the objective function (Gu et al., 2020a). Second, we use dropout as a technique that randomly ignores some of the neurons during the training (Cong et al., 2020). The parameters for both l2 regularization and dropout are set with the cross-validation. Third, we implement another machine learning regularization tool known as “early-stopping.” As the training process progresses and improves results of the training set, this technique monitors the error on the validation set and stops training on an optimal epoch —in terms of this error. Finally, as a fourth regularization technique, we employ ensamble approach. We use multiple random seeds to initialize neural network estimation and construct model predictions as an average of estimates from all networks. This method increases the prediction stability as with a stochastic nature of weights initialization, different training may find different function optima (Gu et al., 2020b).

We train *RNN* with an efficient version of a stochastic gradient descent method: an adaptive moment estimation algorithm (named “Adam”). It computes adaptive learning rates for parameters with estimates of the gradients’ first and second moments (Kingma & Ba, 2015). We process all *RNN* calculations in Python—with Keras and Tensorflow packages.

[14] We have tested *GRU* and *LSTM* specification. *GRU* provides identical MSE with much lower computation cost than *LSTM*.

[15] As an alternative for cross-validation, we can traditionally split training data into training and validation sets (Gu et al., 2020b; Tobek & Hronec, 2020). This approach is more common when a dataset covers many, full economic cycles. Although it has a significant drawback when a dataset is not extremely long and include one or two economic cycles. The split between training and validation may include unrepresentative periods. For instance, splitting the training data for the period of 01/01/2000 to 31/12/2010, in typical proportions (70% to 30%), may result in a relatively low volatility in the training data and rather high volatility in the validation sample. This type of split may be unrepresentative for a testing period and the tuned hyperparameters may not be universal. This observation is also confirmed with a different prediction task that was conducted by a different machine learning method (Kaczmarek & Perez, 2021).

3.4 Target volatility strategy

Once we prepare market volatility predictions, we can use them to construct a plain vanilla version of the target volatility strategy. The fundamental principle of the strategy, which is focused on the efficient asset allocation, is quite simple. It dynamically adjusts the tactical allocation between a risky asset (e.g., stocks) and a risk-free asset (e.g., T-bills) in order to maintain a constant level of target volatility for portfolio within changing market conditions (Hocquard et al., 2013; Y. Kim & Enke, 2016, 2018; Perchet et al., 2015). The asset allocation between risky and risk-free assets is calculated along with the concept of the modern portfolio theory. The risk-return profile for the strategy is defined with the proportion between risky assets ω_e, and risk-free assets ω_{rf}, where $\omega_{rf} = 1 - \omega_e$. Thus, the portfolio expected return $E(r_p)$ and risk σ_p can be described as:

$$E(r_p) = \omega_e E(r_e) + (1 - \omega_e) r_f, \tag{19}$$

$$\sigma_p^2 = \omega_e^2 \sigma_e^2 + (1 - \omega_e)^2 \sigma_{rf}^2 + 2\omega_e (1 - \omega_e) \sigma_{e,rf}, \tag{20}$$

where, $E(r_e)$ represents expected return of a risky asset, r_f is the risk-free asset return; σ_e, σ_{rf}, and $\sigma_{e,rf}$ represents a standard deviation of the risky asset, a risk-free asset, and a covariance between them. For calculation clarity, we assume that $r_f = 0$, and its risk is $\sigma_{rf} = 0$. This assumption answers a very present level of short-term interest rates and simplifies the equations (19) and (20) to:

$$E(r_p) = \omega_e E(r_e), \tag{21}$$

$$\sigma_p = \omega_e \sigma_e. \tag{22}$$

As we exclude the leverage, we can represent the proportion of the risky asset as:

$$\omega_e = min\left(\frac{\sigma_p}{\sigma_e}, 100\%\right). \tag{23}$$

This equation demonstrates the idea of the target volatility strategy, in which the weight of risky asset ω_e is equal to the proportion between required (target) monthly volatility; measured by a monthly standard deviation of risky asset returns and the predicted volatility.

As our forecasts are generated for daily volatility, $v_{(t+1)}$, a basic transformation into monthly estimates, is applied according to the formula:

$$\sigma_e = 22 * v_{(t+1)}. \tag{24}$$

Throughout all of our simulations, we assume that the required volatility σ_p is 10% and it represents the monthly standard deviation of equity index returns.

3.5 Safe haven assets

Applying a target volatility strategy allows for the control of portfolio volatility. In practice for every predicted day, we calculate weights of equity assets and cash allowing to control portfolio volatility and – therefore – optimizing portfolio efficiency. One of the possibly beneficial solutions in days of abnormal market volatility is replacing a portion of cash with safe haven investments. We choose 13 assets, which were analyzed in the literature, as potential safe haven assets in order to verify their impact on the efficiency of the portfolio based on target volatility strategy. We do this verification in three main steps.

First, we forecast the market volatility with five different methods and analyze which of them provides the smallest prediction errors. Second, we construct the target volatility strategy and observe the portfolios' efficiency—compared to the benchmark. Third, we examine each of these 13 assets as potential safe haven candidates by including them in our target volatility strategy in periods when the predicted volatility is high. We define the elevated periods of volatility as three quantiles: 0.9, 0.95, and 0.99.[16] These points mark the line of demarcation, where we either include the tested asset in the portfolio or not. Additionally, since the desired weight of a safe haven investment in the portfolio is not obvious and may depend on an investor risk appetite, we test portfolios that include three different safe haven assets maximum weights: 10%, 20%, 30%. We exclude the possibility of leverage (all weights are non-negative); furthermore – at any time – the entire portfolio weight of equity, safe haven asset, and cash sums up to 100%.

Next, we measure the portfolio performance with Sharpe and Calmar ratios. We calculate the mean of these different settings and seek for portfolios with the increased values of Sharpe and Calmar performance measures by more than 10%, in comparison to such values for portfolios with clean target volatility strategy. Once we find them, we call the asset that is included in them a safe haven.

We can formalize this methodology and define the expected return of a portfolio, based on target volatility strategy and includes safe haven asset, as:

$$E(r_{p+sa}) = \omega_e * E(r_e) + \left(\omega_{sa} \middle| \left(\sigma_e \geq Q_{(90,95,99)}\right)\right) * r_{sa} \tag{25}$$

$$\omega_{sa} = min\big((1-\omega_e), \langle 10\%, 20\%, 30\% \rangle\big), \tag{26}$$

where, ω_{sa} and r_{sa} denote unconditional safe haven asset weight and return, respectively, and $Q_{(90,95,99)}$ describe the 90, 95, and 99 quantiles of predicted volatilities σ_e in the testing period.

[16] This approach resembles the conditional volatility targeting, as described by Bongaerts et al. (2020).

4. Empirical study

4.1 Data

Our empirical case study is conducted on the U.S. market with the S&P500 index, which aggregates 500 of the most liquid companies from the world's biggest U.S. stock market. Findings from the S&P500 can be considered as a good starting point for a broader analysis that covers other equity markets. The data for indices; 13 assets of potential safe haven explanatory, as well as characteristics variables (besides *RBV*); are obtained from Refinitiv Datastream. The data for realized bipower variation (*RBV*) are from Realized Library.[17] The input data is described in panels A and B of Table A.1 in the Online Appendix.

The study period ranges from January 1, 2000 to December 31, 2020. We divide this period into the training and testing periods. Our training period data covers January 2000 up to December 2009. Our testing period runs from January 2010 and ends on December 2020, totalling 11 years and 2,869 daily observations. We update the prediction models' parameters with the extending window approach. In the case of financial time-series models, which was described in Section 3.2, the parameters are updated daily. Due to high computational costs, the parameters for recurrent neural networks (*RNN*) from Section 3.3 are updated annually. Finally, with limited historical data availability for *USCORPAA3YI,* we shorten the training period for *GRU-MULTIVAR* and start it from April 11, 2002.

Table 1 presents the descriptive statistics of prediction models' input data. Panel C of Table A.1 in the Online Appendix presents the detailed list of tested assets. Table A.2. in the Online Appendix demonstrates descriptive statistics of these assets' daily returns.

4.2 Market volatility predictions and target volatility strategy

Our first empirical task is to verify whether market volatility prediction models, presented in Sections 3.2 and 3.3, provide enough accurate market volatility forecasts to leverage efficiency of the portfolio based on the target volatility strategy. First, we look at the Diebold-Mariano (DM) equal forecast tests, which are visualized in Table 2. The tests suggest that our models deliver different accuracy. Therefore, it is worth testing target volatility strategies—based on their predictions—independently. Figure 3 visualizes that all of our prediction models keep quite close to realized volatility BV. To be more precise, Table 3 compares prediction errors and demonstrates that *GRU-MULTIVAR* provides the most accurate prediction in terms of both the mean squared error (MSE) and the mean average percentage error (MAPE). The difference

[17] Oxford-Man Institute of Quantitative Finance (Heber et al., 2009)

in accuracy between *GRU-MULTIVAR*, *GRU-UNIVAR*, and *ARFIMA* is not that high and reaches 9.7-38.7%, depending on the prediction method and the error measure; however, in the case of *GARCH* and *GJR-GARCH*, it is extended to 179.6-371.0%. This suggests that it is more accurate to predict realized volatility with RBV than with returns of the indices that are modelled by *GARCH*. However, it is not clear which of these two approaches dedicated to predicting market volatility delivers better portfolio performance in terms of target volatility strategy. We will investigate that in the next paragraph. For now, we can say that models that are oriented to predict RBV – directly – deliver more accurate predictions.

Figure 4 delivers the first look at the target volatility strategy results. The top subfigure demonstrates the difference in compounded return between *SPX500TR* and the strategy built with *GRU-MULTIVAR* predictions. The strategy delivers a lower compounded return with a lower volatility. These results are provided with frequent changes of equity weight ω_e, what we can read from the bottom subfigure.[18] Table 4 demonstrates that the trade-off between lower return and lower volatility favours the target volatility performance. Along with information from Panel A, the target volatility strategy – based on prediction from *GRU-MULTIVAR* – overperforms the benchmark in terms of Sharpe and Calmar ratios. The overperformance reaches 21.3% and 83.0%—respectively. These findings correspond well with other studies about target volatility strategy that report its relative overperformance to the benchmark (Bongaerts et al., 2020; Hocquard et al., 2013; Y. Kim & Enke, 2016, 2018; Perchet et al., 2015; Zakamulin, 2019).[19] The remaining panels present the results of strategies that are built with other prediction methods. The differences between the strategy performance, depending on the prediction method, are not very high. It is worth highlighting that the relative overperformance is the highest with *ARFIMA*. Besides, strategies built with *GARCH* and *GJR-GARCH* that generate considerably higher prediction errors perform almost equal to *GRU-MULTIVAR*, *GRU-UNIVAR*, or *ARFIMA*.

The results of the target volatility strategies confirm that our models provide quality forecasts. Our volatility predictions can indicate when to either increase or decrease the equity allocation, as well as reduce the negative impact of increased stock market instability on portfolio performance. Once we know when the market is to suffer from excessive volatility,

[18] Our model assumes that there are no transactions costs; however, it can be modified e.g., by adding a tunning parameter that allows for control of the aggressiveness of the risk exposure adjustments in response to volatility changes, and no-transaction region around the targeted risk exposure (Zakamulin, 2019).

[19] For the sake of brevity, we do not precisely compare our target volatility implementation with other studies. All are built on different time horizons and have different reference points. For example, Perchet et al. (2015) report results for 1990-2013, when their strategy overperforms the S&P500 index by 24.4% in terms of the Sharpe ratio. Our implementation covers the period of 2010-2021 and compares results with the S&P Total Return index.

we can verify whether there are instruments that could further increase the portfolio performance, when included to it with proper timing. If such assets do exist, we can call them safe havens.

4.3 Testing instruments with have safe haven characteristics

We verify if any of the 13 assets, that the literature indicates to have safe haven characteristics, can increase the portfolio performance during the market turmoil. Along with the methodology described in Section 3.5, we add our tested assets to the portfolio; depending on 90, 95, and 99 quantiles of predicted volatility. The black line on Figure 5 justifies such numbers. It visualizes how the standard deviation of *SPX500TR* daily returns increases with a rising quantile of expected volatility. The line curvature is the highest when the quantile exceeds 90. This phenomenon is observable for all our models. In theory, adding a safe haven asset to a portfolio in a period when the standard deviation of returns is high should increase that portfolio's performance.

Now, we direct our attention towards potential safe haven assets. The correlation heatmap from Figure 6 shows that the correlation between daily returns of SPX500TR, as well as all of the tested assets, are low. We observe similar dependence in the case of the correlations between assets from different asset classes. Such results justify investigating how they can impact the target volatility strategy's performance during times of high market volatility.

We examine the capabilities of each safe haven asset with five prediction methods, three quantiles of predicted volatility (90, 95, 99), and three caps for the maximum weight (10%, 20%, 30%). That makes 45 portfolios for each asset and two performance measures—Sharpe and Calmar ratios. To obtain the final precise results, we calculate the average performance of different quantiles and caps. Table 4 reports the mean values of Sharpe and Calmar ratios from all configurations for five forecasting models.

We start the analysis with the results for the strategies based on *GRU*-MULIVAR that present the lowest prediction errors (see Table A.3 in the Online Appendix for details). A total of 7 (Sharpe ratio) and 9 (Calmar ratio) out of 13 portfolios of equity, tested assets, and cash underperform pure target volatility strategy. *SILVER*, *PLATINUM*, *COPPER*, *COCOA,* and *CORN* reduce the pure strategy performance by more than 5%. *OIL* destroys more than half of it. *SWISSGOV10_PI* doesn't change it. Only *GOLD, USGOV10_PI,* and *BUNDGOV10_PI,* marginally overperform it.

Each of the four assets that do not underperform presents the same characteristic. They add value to the portfolio when predicted volatility is above the 90th percentile. *USGOV30_PI* demonstrates the same. Especially bonds (*USGOV30_PI*, *USGOV10_PI*, *BUNDGOV10_PI*, and *SWISSGOV10_PI)* increase the strategy overperformance with the rising maximum weight limit strategy (cap). In case of *USGOV30_PI* and *USGOV10_PI,* the portfolio overperformance is quite reliable as its Sharpe ratio increases by more than 5% and Calmar ratio by more than 10%. This value disappears, however, in both the 95th and 99th percentile. Overall, that means that – in case of the strategy based on *GRU-MULTIVAR* – none of the assets tested to have a safe haven characteristic that demonstrates a meaningful, positive impact on the portfolio's performance in the highest volatility quantiles. Two assets – *USGOV30_PI* and *USGOV10_PI* – however, have a positive and reliable effect on portfolio results with, not extreme, but elevated levels of predicted volatilities.

The results obtained for the strategies built with the rest of the volatility prediction methods confirm these observations. The negative impact on the portfolio's performance is noticeable once *OIL*, *SILVER*, *PLATINUM*, *COPPER*, *COCOA*, and *CORN* are tested. Moreover, there is no asset that reliably improves results with Sharpe and Calmar ratio means. We may see the same tendencies, as with *GRU-MULTIVAR* predictions, after digging deeper. Adding bonds provides some benefits to portfolios that are based on the 90th predicted volatility percentile. In particular, *USGOV30_PI* and *USGOV10_PI* demonstrate the best impact across different prediction methods. There are only two exceptions to this pattern. First, predictions based on *GRU-UNIVAR* cooperate better with *GOLD*, which is observable in the 90th and 95th quantile. Second, *GJR-GARCH* doesn't work with *USGOV30_PI*, especially when measured with the Calmar ratio. However, these observations are isolated only for those particular forecasting methods.

Our main finding is that none of the 13 assets we verify in out-of-sample testing configuration prove to be a safe haven. None of them provide benefits to the portfolio when predicted volatility is exceptionally high. This observation is in contradiction to numerous studies that indicate bonds (Flavin et al., 2014; Habib & Stracca, 2015), commodities (Gorton & Geert Rouwenhorst, 2006; Xia et al., 2019), gold (Baur & McDermott, 2016; Reboredo, 2013; Salisu et al., 2020; Salisu & Adediran, 2020), or other precious metals (Li & Lucey, 2017) as assets that present either strong or weak safe haven characteristics with the in-sample testing. Among the rare literature that verifies safe haven features with the out-of-sample approach, Daskalaki and Skiadopoulos (2011) prove that commodities are not safe havens. That is consistent with our findings. Despite the low correlation with equities that emerge with good rationality to look for diversification benefits, the growing presence of index funds in commodity markets, increases their co-movements with stocks when volatility is enormously elevated. Boubaker (2020) researches gold and silver defensive characteristics in the out-of-sample setting and finds that, since the end of the First World War, gold serves well as a safe haven—while silver does not. ETFs' rising role may also impact the gold features; additionally, once we search the gold characteristic in the period of 2010 to 2021, the past observations might not be longer valid. Finally, along with our conclusions, bonds seem to present some safe haven characteristics with the out-of-sample setting (Adrian et al., 2019; de Goeij & Marquering, 2009). It might seem reasonable to include the U.S. long-term Treasuries in the target volatility investment strategy for the periods of elevated, but not only extreme, expected market volatility.

5. Robustness and additional tests

This section verifies if the base results that we observe in the U.S. equity market are universal and can be used for other markets. We use Japanese stocks (represented with *NIKKEI225TR*), as well as the Indian market (*NIFTY50TR*) and European equity (*STOXX50TR*) as for robustness tests. Furthermore, we demonstrate how the strategy based on *GRU-MULTIVAR* works during five of the most severe drawdowns of *SPX500TR* in the testing sample.

5.1 Data

We acquire additional data of realized bipower variation (*BPVNIKKEI225, BPVNIFTY50,* and *BPVSTOXX50*) from the Oxford-Man Institute of Quantitative Finance, Realized Library (Heber et al., 2009). Our univariate prediction models are based on *NIKKEI225PI*, *NIFTY50PI,* and *STOXX50PI*. In all *GRU-MULTIVAR* specifications, we universally use *GOLDLOG* and *CRUDELOG* and extend it with bond indices specific for each country: *JPGOV3YI* and *JPCORPAA3YI* for Japan; *INDGOV3YI* for India; and *EUGOV3YI* and *EUCORPAA3YI*, in case of the European market. We reduce the number of inputs for India, as we do not have adequate corporate bond index data. The detailed information about explanatory variables is available in Table A.1 in the Online Appendix.[20]

5.2 Volatility predictions and the target volatility strategy by market

In all our specifications, *GRU-MULTIVAR* delivers the most accurate volatility predictions. Table B.3 in the Online Appendix confirms observations from the U.S. market. Two *RNN*s, along with *ARFIMA*, predict *RBV* with higher precision than both *GARCH* and *GJR-GARCH*. *GRU-MULTIVAR* delivers the best predictions across all specifications and error measures. DM

[20] The descriptive statistics for explanatory variables used for robustness checks is available upon request.

tests for differences in forecasts, presented in Table B.3, show that the distinctions in prediction accuracy for *GRU-MULTIVAR* are not that high. In terms of forecast accuracy for *BPVNIKKEI225* (measured with MSE), the difference between *ARFIMA* and *GRU-UNIVAR* is statistically insignificant. Furthermore, a similar situation occurs with predictions for *BPVNIFTY50,* where the difference between *ARFIMA* and *GRU-MULTIVAR* is statistically insignificant.

Our volatility predictions are accurate enough to create a successful target volatility strategy for two out of three markets that we test for robustness. Tables B.3-5 in the Online Appendix present results for strategies based on *NIKKEI225TR*, *NIFTY50TR,* and *STOXX50TR*—respectively. In terms of Sharpe and Calmar ratios, the target volatility strategy constructed with *GRU-MULTIVAR* on the Japanese market overperforms the index by 36.6% and 27.8%. Other prediction models beat the market, as well, but with a considerably lowest scale. The target volatility strategy runs very well with the Indian market, where the *GRU-MULTIVAR* approach reaches 35.5% and 78.0% relative overperformance with the Sharpe and Calmar ratio—respectively. Again, this is the best result among all models, although all of them demonstrate adequate precision to beat the broader market performance.

The situation is different in the European market. Predictions for *BPVSTOXX50* seem not to deliver proper input of the target volatility strategy. All of the specifications underperform the *STOXX50TR* index. It appears that the European market is more challenging to predict. Kaczmarek and Perez (2021) come to the same observation when forecasting stock excess returns with random forest. Low, long-term performance of large European caps may be the explanation for this phenomenon.[21]

To summarize, the target volatility strategies that are constructed on the Japanese and Indian markets deliver sound results. The volatility forecasts based on *GRU-MULTIVAR* are the most precise, both in prediction errors and strategy performance. We will use them to verify the tested assets' safe haven characteristics in the next Section. Contrary to this, the target volatility strategies that are settled with the *STOXX50TR*, representing European stocks, underperform the benchmark. Thus, the predictions for *BPVSTOXX50* are not reliable to test the safe haven features.

[21] The *GRU-MULTIVAR* prediction accuracy might be increased by adding other explanatory variables; this is, however, outside the scope of this study.

5.3 Testing assets characteristics

We first concentrate on the Japanese market that is reported in Table B.3 in the Online Appendix. The majority of tested assets deteriorate the results of the strategy. U.S. Treasuries fail to improve the performance, even in the 90th quantile of predicted volatility. Only one of the tested assets has a positive impact on the strategy results. It is *SOYABEANS* that improves the performance, especially in the 90th and 95th percentile. The positive effect that *SOYABEANS* has on the strategy is reported with all prediction models and reaches on average 5-10%. It is the most observable with *GRU-UNIVAR* and GRJ-*GARCH*, where results reported for the 90th quantile are improved by almost 50%. The positive impact is much smaller for higher percentiles.

The first look for panel A in Table B.4 from the Online Appendix, with results for the Indian market, shows that all assets fail to improve the strategy performance. It is, again, *OIL* that has the worst impact, but even U.S. Treasuries spoil the results. Other panels report a bit better outcomes for U.S. bonds; furthermore, in the case of *GARCH*, there is a positive impact of *WHEAT* with the 90th percentile. Because different models provide different results, and that *GRU-MULTIVAR* demonstrates that no asset presents the safe haven characteristic, we may say that there is no positive impact of adding tested assets to the target volatility strategy on the Indian market. Yet, we cannot exclude the notion that Treasuries may have some minor positive effects with particular strategy settings.

In summary, tests for Japanese and Indian markets demonstrate that most assets do not represent any safe haven characteristic. All assets fail to improve strategy results when the predicted volatility is exceptionally high. In the Japanese market, we should consider *SOYABEANS* as an exciting investment opportunity to stabilize the strategy results, but we should include it with either the 90th or 95th percentile of predicted volatility. Tests for the Indian market don’t present any assets that are beneficial for the strategy. The common observation for the Japanese and Indian markets is that U.S. Treasuries might enhance portfolio performance in some particular settings. We abstain from analyzing the impact of tested assets on the European market, as predictions don’t provide a reliable ground for building a target volatility strategy. Robustness checks compose well with the U.S. market's primary observation. No assets present the safe haven characteristic for strongly elevated volatility; furthermore, U.S. Treasuries show some positive impact on portfolio performance when the volatility is at the 90th and 95th percentile. We might extend these concussions to include *SOYABEANS* in order to present safe haven characteristics in the Japanese market, with the volatility levels analogical to the Treasuries.

5.4 Asset characteristics during stress conditions

In this section, we take a different approach to test the robustness of our main findings. We define five of the most severe drawdowns for *SPX500TR* in the testing sample, and verify the performance of the target volatility strategy and the characteristics of our tested assets in these extreme market conditions. We compare Sharpe ratios during periods where market index returns were negative, so we use a modified version of the Sharpe ratio (Israelsen, 2005).[22] Table 5 demonstrates that the deepest drawdown took place in 2020 and was related to the COVID-19 pandemic. Interestingly, it was both the most severe and the shortest drawdown in the testing period.

The target volatility strategy based on *GRU-MULTIVAR* overperforms index *SPX500TR* in terms of the modified Sharpe ratio in each of the five reported drawdowns. Table 6 demonstrates that the biggest relative overperformance occurred in the most significant drawdown during the COVID-19 pandemic. Figure 7 visualizes the differences in cumulative returns between the strategy and *SPX500TR* (left subfigures) and changes of equity ω_e and safe haven assets weights ω_{sa} (right subfigures) during the reported drawdowns. There is no universal pattern of how the strategy weights equity and safe haven depending on the drawdown. The most straightforward pattern occurs with the COVID-19 pandemic when boosting predicted volatility reduces equity share and increases safe haven asset weight a few days after the drawdown's start. This conservative asset allocation remains unchanged until the end of the drawdown. The assets weighing during other drawdowns are not constant, but reduce the risk compared to the clean equity strategy.

Assets tested as safe havens fail to substantially increase the Sharpe ratio (see Table 6). During the COVID-19 pandemic, only Treasuries increase the Sharpe ratio. The improvement reaches 10.9% in relation to the pure target volatility strategy. Adding *BUNDGOV10_PI* does not change the results, and all other assets, including gold, do not provide any safe haven features. These findings contrast to other studies that report safe haven characteristics in gold during the COVID-19 pandemic (Adekoya et al., 2021; Akhtaruzzaman et al., 2021; Ji et al., 2020). Looking at different drawdown periods, there is no asset to improve results in the second biggest drawdown; however, there are four in the third, seven in the fourth, and one in the fifth drawdown. Along with our previous findings, typically, only long-term government bonds positively impact the overall performance. This rule has some minor exceptions, where *GOLD*,

[22] We do not use the Calmar ratio to compare results during drawdowns as correction of maximum drawdown measure in analogy to standard deviation in the Sharpe ratio is not intuitive.

WHEAT, and *CORN* add some value in the fourth drawdown and *PLATINIUM* in the fifth one. However, we see no lasting rule in these exceptions.

6. Conclusions

This paper explores the quality of potential safe haven assets that are included in a volatility-targeting portfolio, with the intention of adding value to its performance. We utilize two RNN models, represented with *GRU-MULTIVAR* and *GRU-UNIVAR*; we also use three time series methods (*ARFIMA*, *GARCH*, and *GJR-GARCH*) in order to generate stock market volatility forecasts that are applied in the target volatility strategy realized in this portfolio. We compare the portfolio performance measured by Sharpe and Calmar ratios with the benchmark proxy of S&P500, as well as other indices, for robustness. When our strategy wins with the benchmarks, we examine the potential safe haven characteristics of 13 assets from different asset classes of long-term governmental bonds, precious and raw metals, as well as agricultural raw commodities. To this end, we test these assets in terms of their impact on the portfolio efficiency on the 90th, 95th and 99th percentile of predicted one-day-ahead volatility and 10%, 20%, and 30% maximum weight allocations. This way, we depart from most of the previous literature and provide entirely out-of-sample conclusions that make the strategy applicable in the real world. Depending on the direction of this impact and obtained values of Sharpe and Calmar ratios, in comparison to the pure version of our strategy, we conclude about the potential of these assets as safe haven investments.

Based on a considerably large-scale empirical analysis of stocks from the S&P500 with 21 years of history (ranging from 2000 to 2020) and an 11-year testing window (which includes the capital market turmoil initiated by the COVID-19 pandemic), we find that the pure strategy targeting volatility of 10% and *GRU*-MULTIVAR daily volatility predictions wins with the index by 21.3% and 83%—in terms of the Sharpe and Calmar ratios. This result justifies enriching the strategy with the assets of safe haven potential. As we introduce such assets into the portfolios, we observe that none of the tested assets improve the strategy performance in extremely high expected volatility times. U.S. long-term Treasuries enhance the strategy performance in times of moderate expected volatility. They add 5% to the portfolio's Sharpe ratio and more than 10% to the portfolio's Calmar ratio. These findings are confirmed with other volatility prediction methods and robustness tests for both NIKKEI225 and NIFTY50. The STOXX50 volatility seems to be more challenging to predict, where our pure strategy fails to win with the index. Therefore, in this case, we resign from testing the impact of adding the safe haven assets.

Overall, our results – based on utilizing advanced methods of prediction within the out-of-sample approach – show that a manager of a volatility-targeting portfolio is made better off by including within it only U.S. long-term Treasuries as safe haven assets. Other considered assets, including gold, proved to have no potential of being safe havens.

Our findings are relevant to portfolio managers and all investors using an active approach to manage portfolio risk. Volatility targeting with predictions from recurrent neural networks is an innovatory tool for dynamic risk management. It may be used both to increase the risk/reward relationship and to control the drawdown risk. A new perspective on the utility of safe havens indicates that investors should be cautious in their expectations of controlling the risk associated with market turmoil using commodities, including gold. On the other hand, our research confirms that Treasuries can be considered as safe havens. Finally, the market state-dependent method to place long-term government bonds in the portfolio illuminates an interesting approach to benefit from their safe haven characteristics for investors who are not interested in holding long-term bonds positions.

Future research should look into the benefits of other machine learning methods that can be utilized to predict market volatility, as well as the performance of volatility-targeting portfolios. Such a performance can be measured by ratios that take into account market risk factors that affect the dynamics of the portfolio. This is beyond the current paper but it deserves to become a topic for future research.

Figures and Tables

Figure 1. Basic *RNN* structure (left side) and its unrolled representation (right side)

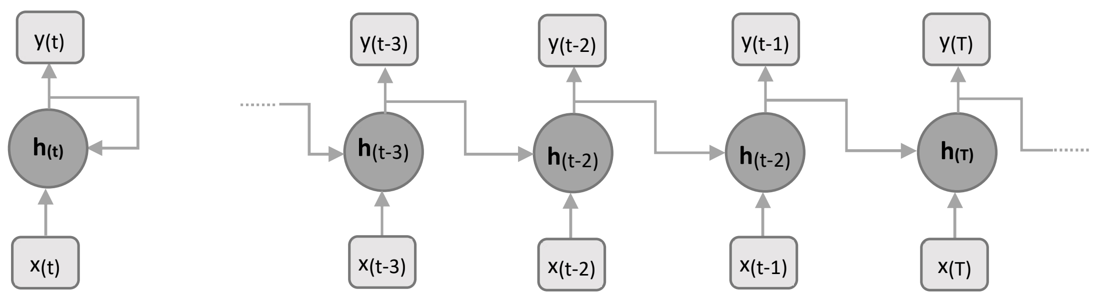


Figure 2. *LSTM* structure

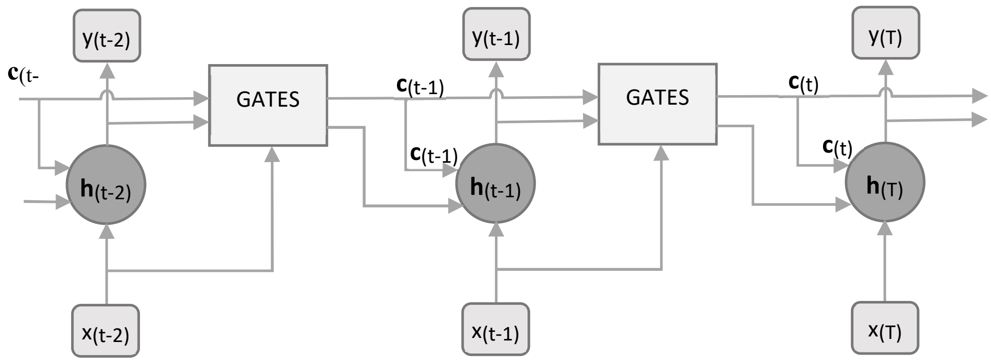


Figure 3. Predicted vs. realized volatility of SPX500PI daily returns

This figure demonstrates line plots between the daily volatility of *SPX500PI* returns and dates. The volatility of returns is represented as realized bi-power variation of intraday 5-minute returns and transformed to monthly volatility—along with equation 24. The light-grey lines state for realized volatility and dark-grey for the predicted volatility. Each of five subfigures represents a different method for predictions calculation: multivariate approach with gated recurrent units (*GRU-MULTIVAR*); univariate approach with gated recurrent units (*GRU-UNIVAR*); autoregressive fractionally integrated moving average (*ARFIMA*); generalized autoregressive conditional heteroskedasticity (*GARCH*); and Glosten, Jagannathan, and Runkle *GARCH* (*GJR-GARCH*). The training sample starts from 2000, and the testing sample period runs from 2010 to 2020.

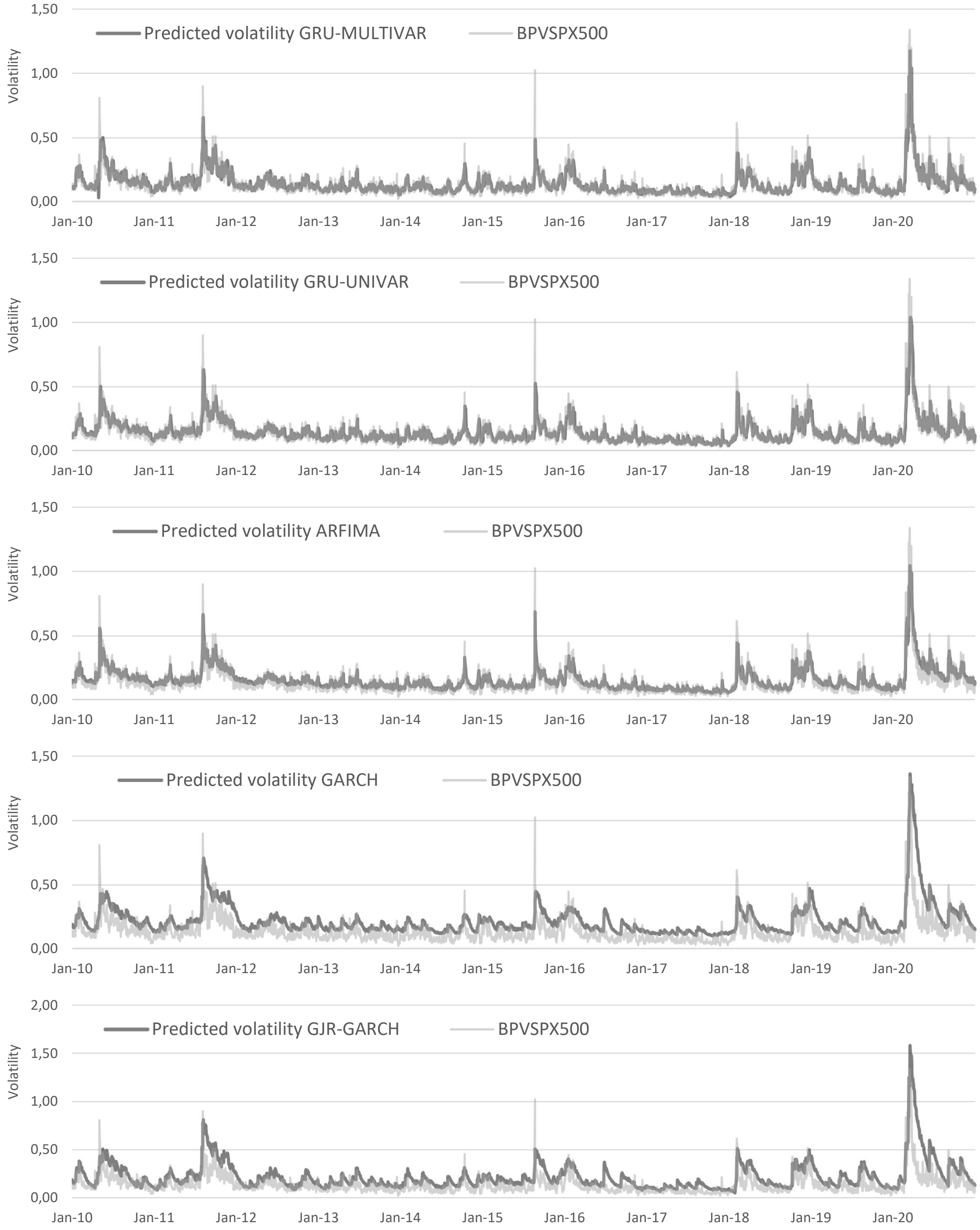


Figure 4. Target volatility strategy equity weights in time

This figure shows line plots between either compounded return or equity weight and dates. The top subfigure represents compounded returns of the *SPX500TR* Index, as well as the target volatility strategy that is based on a multivariate approach with gated recurrent units (*GRU-MULTIVAR*). The bottom subfigure visualizes the equity weight ω_e changes in the target volatility strategy (target volatility = 10%). The weights are calculated along with equation 23. The training sample starts from April 11, 2002, and the testing sample period runs from January 1, 2010 to December 31, 2020..

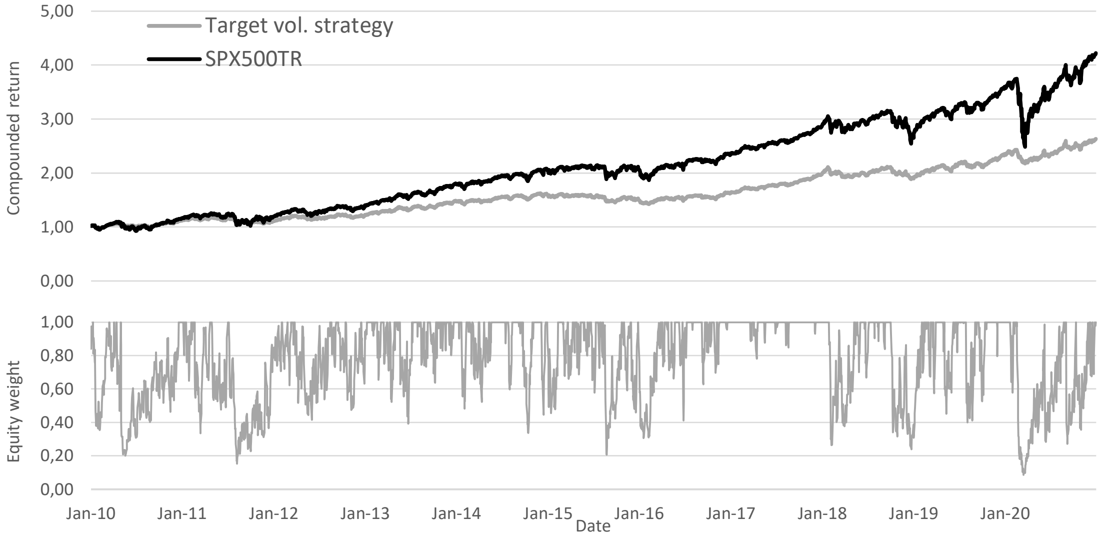


Figure 5. Index returns depending on predictions quantile

This figure presents line plots between daily returns measures of S&P 500 COMPOSITE - TOT RETURN IND and its daily predicted volatility quantiles. Each of the five subfigures represents a different method for predictions calculation: multivariate approach with gated recurrent units (*GRU-MULTIVAR*); univariate approach with gated recurrent units (*GRU-UNIVAR*); autoregressive fractionally integrated moving average (*ARFIMA*); generalized autoregressive conditional heteroskedasticity (*GARCH*); and Glosten, Jagannathan, and Runkle (*GJR-GARCH*). Subfigures present four lines: minimum daily return in predicted volatility quantile (min), maximum daily return in predicted volatility quantile (max), median of daily returns in predicted volatility quantile (median), and standard deviation of daily returns in predicted volatility quantile (std). The sample period runs from January 1, 2010 to December 31, 2020.

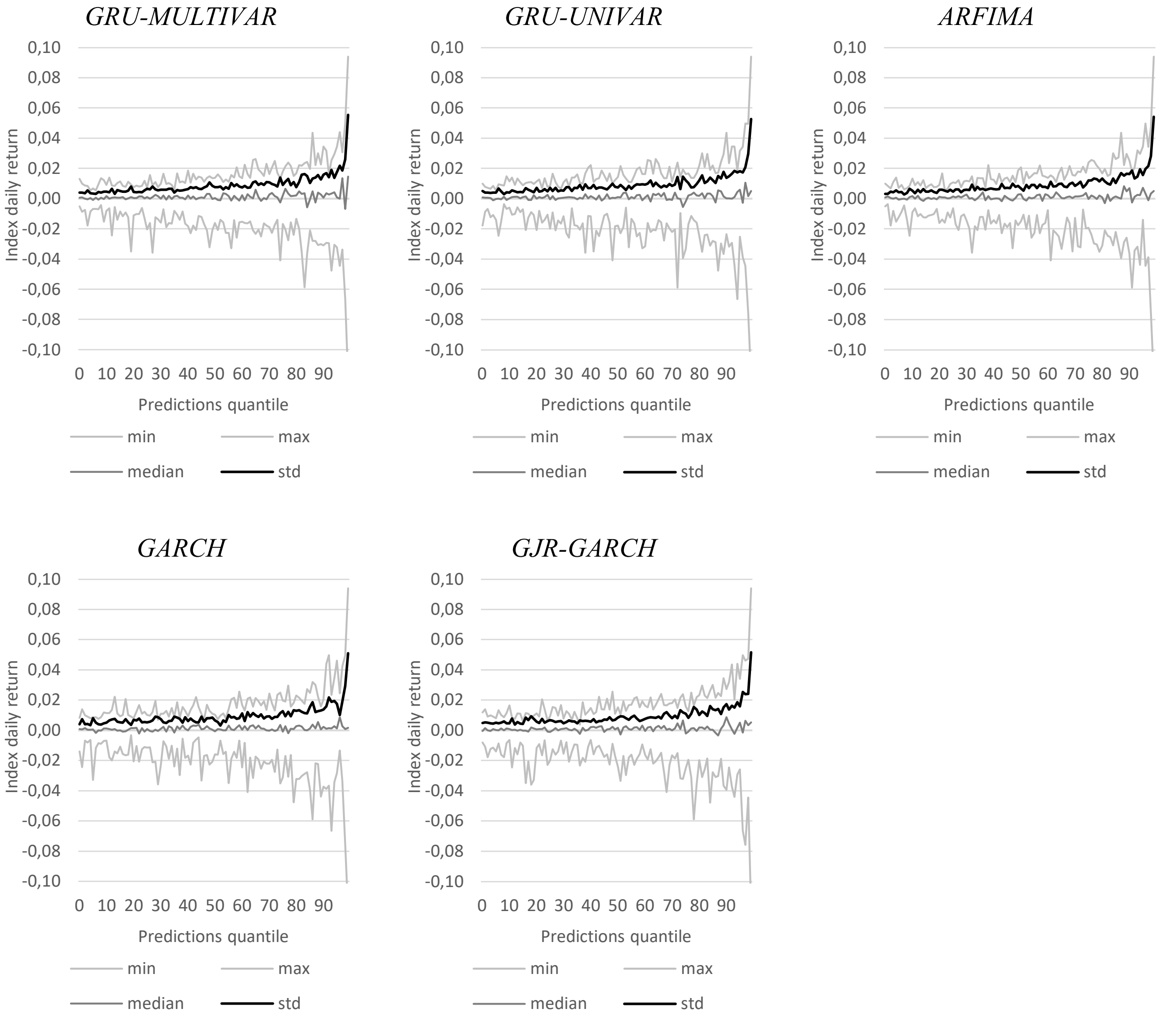


Figure 6. Correlation heatmap of daily returns between SPX500TR and assets tested as a safe haven

This figure presents the Pearson correlation heatmap of daily returns for SPX500TR and assets tested as save haven (the detailed description of assets is shown in table A1 in the Online Appendix). The sample period runs from January 1, 2010 to December 31, 2020.

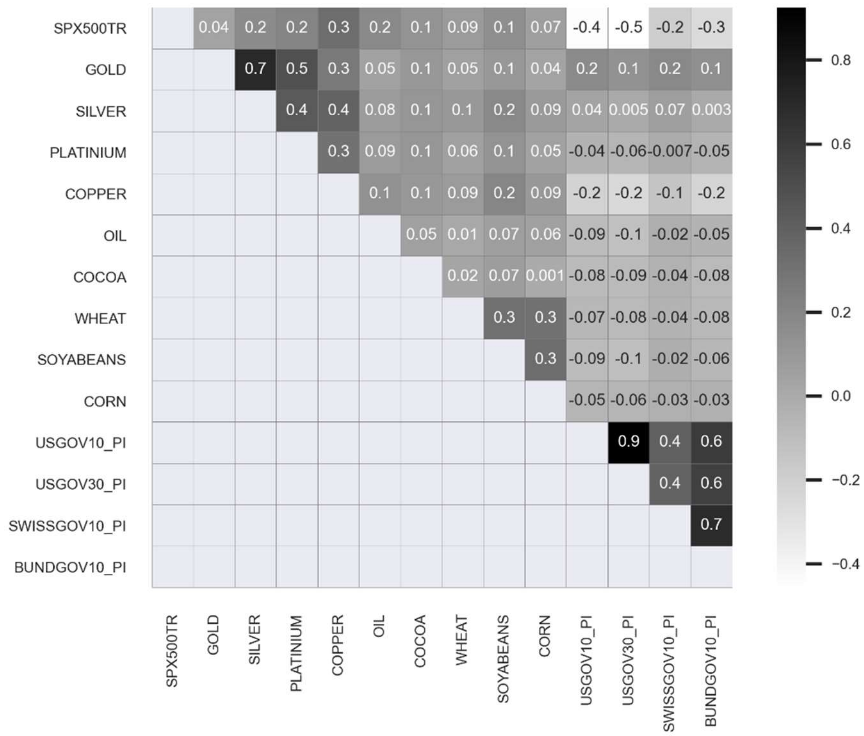
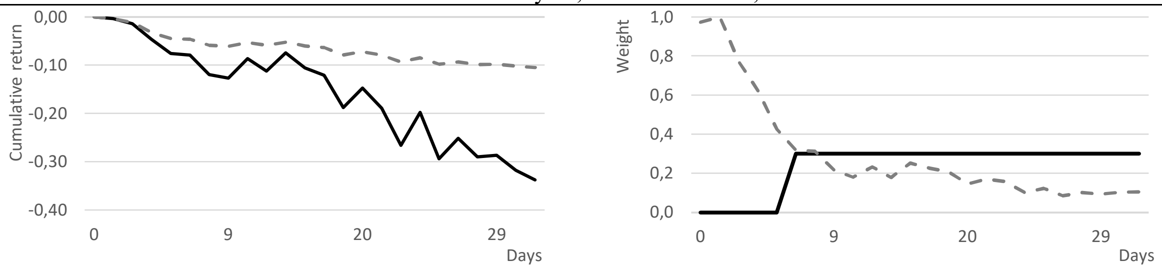


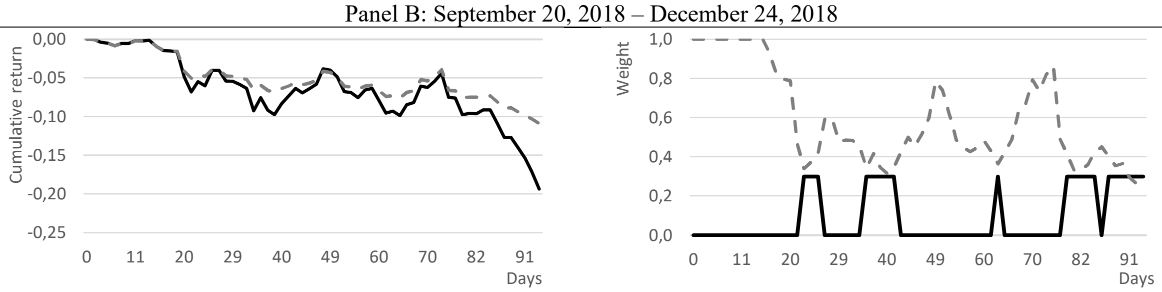
Figure 7. Strategy results during the top five drawdowns of the S&P500 Total Return index

This figure presents the target volatility strategy performance during the five most severe drawdowns in the testing sample (Table 5 shows the list of drawdowns). The figure is divided into five panels where each panel demonstrates a different drawdown period. The panels consist of two subfigures. The left subfigure presents cumulative returns for the S&P500 Total Return index (black continuous line) and target volatility strategy without safe haven asset based on predictions from *GRU-MULTIVAR* (grey dotted line). The right subfigure visualizes the weight of equity ω_e (grey dotted line) and weight of safe haven asset ω_{sa} (black continuous line) in target volatility strategy extended with conditional investments in a safe haven asset. The strategy is built according to Equations 25 and 26. For transparency, we show only results for a scenario with 90% volatility quantile and 30% maximum share of a safe haven asset. The testing sample period runs from January 1, 2010 to December 31, 2020.

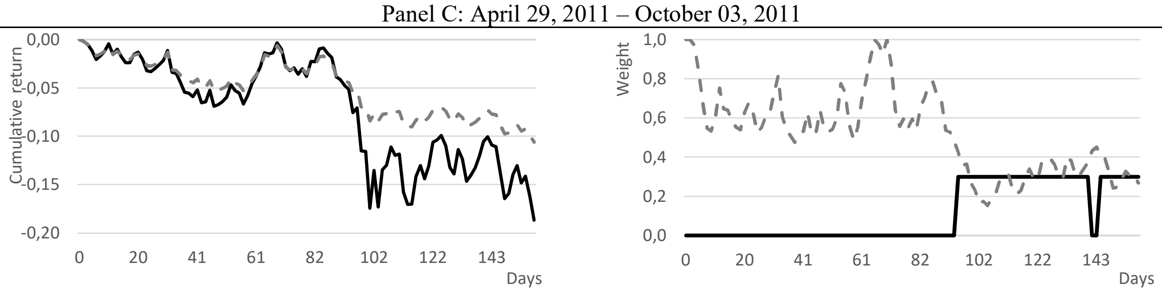
Panel A: February 19, 2020 – March 23, 2020



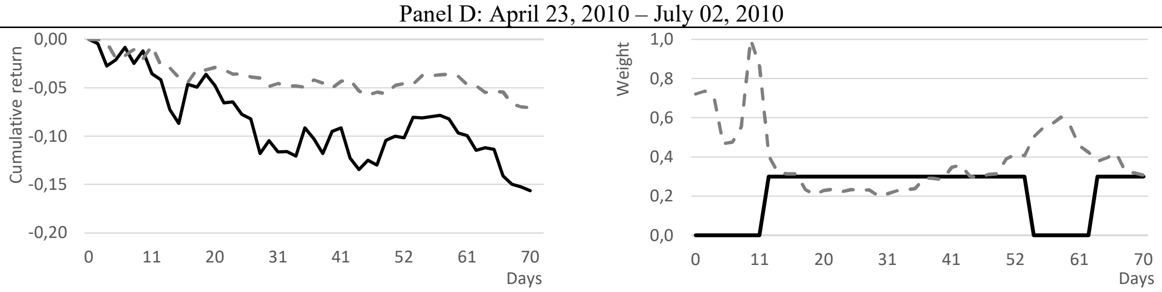
Panel B: September 20, 2018 – December 24, 2018



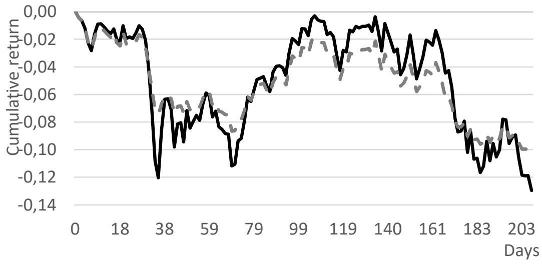
Panel C: April 29, 2011 – October 03, 2011



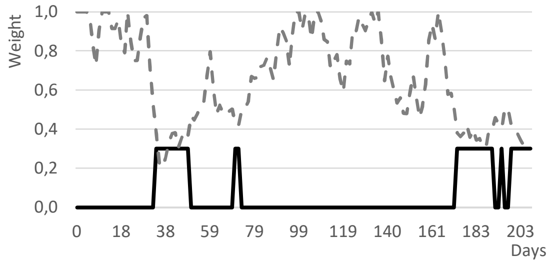
Panel D: April 23, 2010 – July 02, 2010



Panel E: July 20, 2015 – February 11, 2016



0,00
-0,02
-0,04
-0,06
-0,08
-0,10
-0,12
-0,14
Cumulative return
0 18 38 59 79 99 119 140 161 183 203
Days



1,0
0,8
0,6
0,4
0,2
0,0
Weight
0 18 38 59 79 99 119 140 161 183 203
Days

Table 1. Explanatory variables descriptive statistics

This table presents summary statistics for explanatory variables. *BPVSPX500* is the explanatory variable for four univariate prediction models and covers a period starting from January 1, 2000 to December 31, 2020. The remaining variables highlighted with * are the explanatory variables for the multivariate prediction model and, due to data availability limitation, cover the period from April 11, 2002 to December 21, 2020. *BPVSPX500 and BPVSPX500** are calculated along with equation 24. *SPX500PI*, *GOLDLOG,* and *CRUDELOG* are calculated as logarithms of daily differences. *USGOV3YI* and *USCORPAA3YI* are estimated with daily differences. The table columns present the mean (Mean); standard deviation (Std); skewness (Skew); excess kurtosis (Kurt); the number of daily observations, for which the given variable is available (Count); t-statistic of the Jarque-Bera test, for variable's normal distribution (Jarque-Bera); the Augmented Dickey-Fuller test, for the presence of a unit root in a sample (ADF), and the unit root test of Zivot and Andrews (1992), URZA, which allows a structural break in any point in time. $***$ and $**$ denote a rejection of the null hypothesis at the 1% and 5% significance level—respectively. Detailed description for each variate is presented in Table A1 in the Online Appendix.

	Mean	Std	Skew	Kurt	Count	Jarque-Bera	ADF	URZA
BPVSPX500	0.16951	0.1246	3.52	21.03	5265	107 694.2***	-7.8***	-22.1***
*BPVSPX500**	0.16331	0.1267	3.72	22.28	4701	107 829.3***	-7.2***	-19.5***
*SPX500PI**	0.00026	0.0125	-0.45	12.39	4701	30 181.5***	-16.5***	-78.4***
*GOLDLOG**	0.00039	0.0112	-0.48	5.33	4701	5 725.4***	-69.0***	-69.1***
*CRUDELOG**	0.00028	0.0271	0.56	18.97	4701	70 536.2***	-12.2***	-69.8***
*USGOV3YI**	-0.00068	0.0433	-0.01	7.56	4701	11 169.1***	-11.2***	-72.2***
*USCORPAA3YI**	-0.00046	0.0328	0.05	442.84	4701	38 331 121.0***	-47.6***	-94.3***

Table 2. Diebold-Mariano (DM) equal forecast tests

This table visualizes the results of Diebold-Mariano (DM) equal forecast accuracy tests for daily volatility of SPX500PI Index, measured with realized bi-powered variation (*BPVSPX500*). *BPVSPX500* is transformed to represent monthly standard deviation—according to equation 24. The table shows two prediction error measures: mean squared errors (MSE) and mean average percentage error (MAPE). Each column represents a different prediction method: autoregressive fractionally integrated moving average (*ARFIMA*); generalized autoregressive conditional heteroskedasticity (*GARCH*); Glosten, Jagannathan, and Runkle (*GJR-GARCH*); multivariate method with gated recurrent units (*GRU-MULTIVAR*); and univariate approach with gated recurrent units (*GRU-UNIVAR*). *GRU-MULTIVAR* uses the following input features: *BPV*, *GOLDLOG*, *CRUDELOG*, *GOVBOND*, and *CORPBONDAA*. $***$ and $**$ denote a rejection of the null hypothesis at the 1% and 5% significance level—respectively. The sample period runs from January 1, 2010 to December 31, 2020.

		GARCH	***GJR-GARCH***	***GRU-MULTIVAR***	***GRU-UNIVAR***
ARFIMA	MSE	14.5***	13.7***	-3.0***	4.9***
	MAPE	53.1***	46.2***	-17.7***	-4.2***
GARCH	MSE		6.7***	-14.8***	-12.9***
	MAPE		-10.9***	-50.5***	-47.6***
GJR-GARCH	MSE			-14.1***	-12.4***
	MAPE			-47.2***	-42.8***
GRU-MULTIVAR	MSE				6.5***
	MAPE				12.6***

Table 3. Prediction errors

This table demonstrates the results of out-of-sample prediction errors for daily volatility of *SPX500PI* Index, measured with realized bi-powered variation (*BPVSPX500*). *BPVSPX500* is transformed to represent monthly standard deviation—according to equation 24. The table shows two prediction error measures: mean squared errors (MSE) and mean average percentage error (MAPE). Each column represents a different prediction method: autoregressive fractionally integrated moving average (*ARFIMA*); generalized autoregressive conditional heteroskedasticity (*GARCH*); Glosten, Jagannathan, and Runkle *GARCH* (*GJR-GARCH*); univariate approach with gated recurrent units (*GRU-UNIVAR*); and multivariate method with gated recurrent units (*GRU-MULTIVAR*). *GRU-MULTIVAR* uses the following input features: *BPV*, *GOLDLOG*, *CRUDELOG*, *GOVBOND*, and *CORPBONDAA*. The sample period runs from January 1, 2010 to December 31, 2020.

	ARFIMA	***GARCH***	***GJR-GARCH***	***GRU-UNIVAR***	***GRU-MULTIVAR***
MSE	0.0034	0.0127	0.0146	0.0043	0.0031
MAPE	0.3197	0.7920	0.7320	0.3046	0.2618

Table 4. Safe haven assets in target volatility strategy based on *SPX500TR* Index

This table presents the mean Sharpe (SR) and Calmar ratios (CR) of target volatility strategies. Results cover the period from January 1, 2010 to December 31, 2020. The table has six columns: 1) presented strategy name (Strategy); 2) results of target volatility strategy, based on predictions from *GRU-MULTIVAR*; 3) results based on *GRU-UNIVAR*; 4) on *ARFIMA*; 5) on *GARCH*; and 6) on *GJR-GARCH*. The first two rows represent the target volatility strategy's results, without any safe haven asset (STRAT_CLEAN), and the S&P500 Total Return index (BENCHMARK). The remaining rows state the results of strategies that include additional assets. A full description of assets is presented in Table A1 in Online Appendix. For better transparency, only the first raw (STRAT_CLEAN) shows nominal values of SR and CR. All other rows present the difference between STRAT_CLEAN and the modified strategy results (or the benchmark). The SR and CR represent the mean performance of nine strategies with: three different quantile thresholds (90%, 95%, and 99%) and three different maximum exposures on safe haven weight allocation (cap 10%, cap 20%, and cap 30%). Detailed results for each of the strategies included in both the SR and CR are presented in Table A.3 in the Online Appendix.

Strategy	***GRU-MULTIVAR***		***GRU-UNIVAR***		***ARFIMA***		***GARCH***		***GJR-GARCH***	
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.97	0.75	0.95	0.70	1.00	0.80	0.95	0.71	0.88	0.71
BENCHMARK	-0.17	-0.34	-0.15	-0.29	-0.20	-0.38	-0.15	-0.29	-0.08	-0.30
GOLD	0.01	0.00	0.04	0.04	0.01	-0.05	0.00	-0.03	0.01	-0.06
SILVER	-0.05	-0.19	0.01	-0.09	-0.03	-0.24	-0.07	-0.23	-0.07	-0.28
PLATINIUM	-0.09	-0.24	-0.02	-0.11	-0.07	-0.28	-0.08	-0.26	-0.10	-0.31
COPPER	-0.04	-0.12	-0.03	-0.07	-0.03	-0.15	-0.05	-0.15	-0.05	-0.20
OIL	-0.47	-0.51	-0.71	-0.58	-0.75	-0.66	-1.09	-0.75	-0.75	-0.64
COCOA	-0.07	-0.12	-0.04	-0.05	-0.08	-0.19	-0.10	-0.16	-0.08	-0.19
WHEAT	0.01	-0.01	-0.01	-0.03	0.00	-0.01	-0.04	-0.02	-0.04	-0.06
SOYABEANS	-0.02	-0.05	0.01	-0.02	0.00	-0.03	-0.03	-0.04	-0.04	-0.10
CORN	-0.04	-0.09	0.00	-0.03	-0.06	-0.11	-0.09	-0.13	-0.08	-0.18
USGOV10_PI	0.01	0.01	0.02	0.01	0.01	0.00	0.03	0.02	0.02	0.01
USGOV30_PI	0.02	-0.01	0.03	0.01	0.01	-0.04	0.04	0.03	0.02	-0.05
SWISSGOV10_PI	0.00	0.00	-0.01	0.00	-0.01	-0.03	-0.01	-0.01	-0.01	-0.02
BUNDGOV10_PI	0.00	0.01	0.00	0.01	-0.01	-0.02	0.00	0.00	0.00	-0.01

Table 5. The most severe five drawdowns for the S&P500 Total Return index in the testing sample

This table shows the most severe drawdowns in the testing sample. Results cover the period from January 1, 2010 to December 31, 2020. The table has five columns: 1) the first five worst drawdowns are presented (Worst drawdown period); 2) the size of drawdown in % (Net drawdown); 3) the start date of the drawdown (Peak date); 4) the end date of the drawdown (Valley date); and 5) the drawdown duration in days (Duration in days).

Worst drawdown period	**Net drawdown**	**Peak date**	**Valley date**	**Duration in days**
1	33.8	2020-02-19	2020-03-23	33
2	19.4	2018-09-20	2018-12-24	95
3	18.6	2011-04-29	2011-10-03	157
4	15.6	2010-04-23	2010-07-02	70
5	13.0	2015-07-20	2016-02-11	206

Table 6. Safe haven assets in target volatility strategy based on the *SPX500TR* Index during the most severe five drawdowns

This table shows the mean modified Sharpe ratios (Israelsen, 2005) of target volatility strategy based on predictions from *GRU-MULTIVAR* during the five most severe drawdowns in the testing sample. Results cover the period from January 1, 2010 to December 31, 2020. The table has six columns: 1) presented strategy name (Strategy); 2) mean adjusted Sharpe ratios for the period February 19, 2020 – March 23, 2020 (1); 3) mean adjusted Sharpe ratios for the period September 20, 2018 – December 24, 2018 (2); 4) mean adjusted Sharpe ratios for the period April 29, 2011 – October 03, 2011 (3); 5) mean adjusted Sharpe ratios for the period April 23, 2010 – July 02, 2010 (4); 6) mean adjusted Sharpe ratios for the period July 20, 2015 – February 11, 2016 (5). For better transparency, only the first row (STRAT_CLEAN) shows nominal values of the adjusted Sharpe ratio. All other rows present the difference between STRAT_CLEAN and the modified strategy results (or the benchmark). The adjusted Sharpe ratio represents the mean performance of nine strategies with three different quantile thresholds (90%, 95%, and 99%), and three different maximum exposures on safe haven weight allocation (cap 10%, cap 20%, and cap 30%).

Strategy	**1**	**2**	**3**	**4**	**5**
STRAT_CLEAN	-0.092	-0.039	-0.023	-0.029	-0.018
BENCHMARK	-0.667	-0.074	-0.079	-0.128	-0.024
GOLD	-0.019	0.000	0.000	**0.001**	0.000
SILVER	-0.063	0.000	-0.009	0.000	0.000
PLATINIUM	-0.071	0.000	-0.003	-0.008	**0.001**
COPPER	-0.033	-0.001	-0.008	-0.006	0.000
OIL	-0.239	-0.004	-0.005	-0.012	-0.001
COCOA	-0.020	0.000	-0.002	-0.006	0.000
WHEAT	-0.011	0.000	-0.005	**0.002**	-0.001
SOYABEANS	-0.013	-0.001	-0.004	-0.002	0.000
CORN	-0.027	0.000	-0.005	**0.001**	0.000
USGOV10_PI	**0.010**	0.000	**0.001**	**0.002**	0.000
USGOV30_PI	**0.010**	0.000	**0.003**	**0.003**	-0.001
SWISSGOV10_PI	-0.003	0.000	**0.001**	**0.001**	0.000
BUNDGOV10_PI	0.000	0.000	**0.001**	**0.001**	0.000

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Appendices for " False Safe Haven Assets: Evidence From the Target Volatility Strategy Based on Recurrent Neural Network "

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Abstract

This appendix presents additional materials for the article "False safe haven assets: evidence from the target volatility strategy based on recurrent neural network." It consists of two sections: additional tables for strategy simulations on S&P500 and tables for robustness checks on NIKKEI225, NIFTY50, and STOXX50. Appendix A contains additional tables for the study. Table A1 presents the list of variables used in the study, along with their names, symbols, and description; it includes additional information and the data source. Table A2 provides descriptive statistics for the assets tested as safe havens. Finally, table A3 extends results presented in Table 4 from the main text with three different quantiles and three maximum safe asset weights that we use to calculate reported mean results. Appendix B presents five tables that visualize results for robustness tests. Table B1 shows prediction errors for NIKKEI225, NIFTY50, and STOXX50 volatility. Table B2 demonstrates Diebold-Mariano (DM) equal forecast tests for all three indices realized volatility. Finally, Tables B.3-B.5 provide detailed strategy results for NIKKEI225, NIFTY50, and STOXX50, respectively.

Appendix A. Additional Tables for the Study

Tables for Online Appendix

Table A.1. A detailed description of variables

Table A.1. explains the variables used in the research. Panels organize the rows. Panel A presents the volatility measures for four stock indices. Panel B presents the variates used to predict volatility. Panel C visualizes assets that are tested to have a safe haven characteristic. Finally, panel D shows the total return indices used to calculate target volatility strategies' performance. The table consists of four columns. The first column provides the name of each variable. The second column shows the symbol assigned to the variate. The third column describes how the variable is computed and measured. Finally, the fourth column indicates the source of the data used in this study—for the given variable.

Variate	Symbol	Description	Data source
		Panel A: Volatility measures	
S&P 500 bi-powered variation	*BPVSPX500*	Bi-powered variation is calculated as $RBV_t(\Delta) = \frac{\pi}{2}\sum_{n=1}^{\frac{1}{\Delta}}\lvert r_{t,n}\rvert\lvert r_{t,n+1}\rvert$, where $r_{t,n}$ is n-th intraday return on day t and Δ denotes the frequency of intradaily returns. For each of four indices (S&P500, STOXX50, NIKKEI225 and NIFTY50) the bi-powered daily variation taken from the source we transform into monthly standard deviation as $BPV = 22 * \sqrt{RBV_t(\Delta)}$.	Oxford-Man*
STOXX 50 bi-powered variation	*BPVSTOXX50*		Oxford-Man*
NIKKEI 225 bi-powered variation	*BPVNIKKEI225*		Oxford-Man*
NIFTY 50 bi-powered variation	*BPVNIFTY50*		Oxford-Man*
		Panel B: Explanatory variables	
S&P 500 COMPOSITE - PRICE INDEX	*SPX500PI*	One day log-return of (Standard and Poor's 500 Composite, price index), Datastream symbol: S&PCOMP	Datastream
STOXX EUROPE 50 - PRICE INDEX	*STOXX50PI*	One day log-return of (STOXX Europe 50, price index), Datastream symbol: DJSTO50	Datastream
NIKKEI 225 STOCK AVERAGE - PRICE INDEX	*NIKKEI225PI*	One day log-return of (Nikkei 225 Stock Average, price index), Datastream symbol: JAPDOWA	Datastream
CNX NIFTY (50) - PRICE INDEX	*NIFTY50PI*	One day log-return of (CNX Nifty (50) , price index), Datastream symbol: INNSE50	Datastream
Gold Bullion LBM $/t oz DELAY	*GOLDLOG*	One day log-return of (Gold Bullion London Bullion Market United States Dollar Per Metric Tonne Ounce Delay), Datastream symbol: GOLDBLN	Datastream
Crude Oil-WTI Spot Cushing U$/BBL	*CRUDELOG*	One day log-return of (Crude Oil-West Texas Intermediate Spot Cushing United States Dollar Per Barrel), Datastream symbol: CRUDOIL	Datastream
US GOVERNMENT BOND SERIES 3 YEAR - RED. YIELD	*USGOV3YI*	One day difference in yield of (United States Government Bond Series 3 Years), Datastream symbol: GBUS03Y	Datastream

RF US CORP BMK AA 3Y - RED. YIELD	*USCORPAA3YI*	One day difference in yield of (Refinitiv United States Corp Benchmark AA 3 Years), Datastream symbol: TRUCBYC	Datastream
GERMANY GOVERNMENT BOND 3 YEAR - RED. YIELD	*EUGOV3YI*	One day difference in yield of (Germany Government Bond 3 Years), Datastream symbol: GBBD03Y	Datastream
RF EURO CORP BMK AA 3Y - RED. YIELD	*EUCORPAA3YI*	One day difference in yield of (Refinitiv Euro Corp Benchmark AA 3 Years), Datastream symbol: TRECBYC	Datastream
JAPAN GOVERNMENT BOND SERIES 3 YR - RED. YIELD	*JPGOV3YI*	One day difference in yield of Japan Government Bond Series 3 Years, Datastream symbol: GBJP03Y	Datastream
RF JP CORP BMK AA 3Y - RED. YIELD	*JPCORPAA3YI*	One day difference in yield of Refinitiv Japanese Corp Benchmark AA 3 Years, Datastream symbol: TRJCBYC	Datastream
RF INDIA GVT BMK BID YLD 3Y - RED. YIELD	*INDGOV3YI*	One day difference in yield of Refinitiv India Government Benchmark Bid Yield 3 Years, Datastream symbol: TRIN3YT	Datastream
	Panel C: Assets tested as save haven		
Gold Bullion LBM $/t oz DELAY	*GOLD*	One day return of Gold Bullion London Bullion Market United States Dollar Per Metric Tonne Ounce Delay, Datastream symbol: GOLDBLN	Datastream
S&P GSCI Silver Total Return - RETURN IND. (OFCL)	*SILVER*	One day return of Standard and Poors Goldman Sachs Commodity Index(GSCI) Silver Total Return, Datastream symbol: GSSITOT	Datastream
London Platinum Free Market $/Troy oz	*PLATINIUM*	One day return of London Platinum Free Market United States Dollar Per Troy Ounce, Datastream symbol: PLATFRE	Datastream
LME-Copper Grade A Cash U$/MT	*COPPER*	One day return of London Metal Exchange (LME)-Copper Grade A Cash United States Dollar Per Metric Tonne, Datastream symbol: LCPCASH	Datastream
Crude Oil-WTI Spot Cushing U$/BBL	*OIL*	One day return of Crude Oil-West Texas Intermediate Spot Cushing United States Dollar Per Barrel, Datastream symbol: CRUDOIL	Datastream
Cocoa-ICCO Daily Price US$/MT	*COCOA*	One day return of Cocoa-International Cocoa Organization(ICCO) Daily Price USA United States Dollar Per Metric Tonne, Datastream symbol: COCINUS	Datastream
Wheat No.2,Soft Red U$/Bu	*WHEAT*	One day return of Wheat Number 2, Soft Red USD / Bushel, Datastream symbol: WHEATSF	Datastream
Soyabeans, No.1 Yellow $/Bushel	*SOYABEANS*	One day return of Soyabeans, Number 1 Yellow USD / Bushel, Datastream symbol: SOYBEAN	Datastream
Corn US No.2 South Central IL $/BSH	*CORN*	One day return of Corn USA Number 2 South Central IL$ / BSH, Datastream symbol: COTSCIL	Datastream
US BENCHMARK 10 YEAR DS GOVT. INDEX - CLEAN PRICE INDEX	*USGOV10PI*	One day return of United States Benchmark 10 Year Datastream Government Index, Datastream symbol: BMUS10Y	Datastream
US BENCHMARK 30 YEAR DS GOVT. INDEX - CLEAN PRICE INDEX	*USGOV30PI*	One day return of United States Benchmark 30 Year Datastream Government Index, Datastream symbol: BMUS30Y	Datastream
SW BENCHMARK 10 YEAR DS GOVT. INDEX - CLEAN PRICE INDEX	*SWISSGOV10PI*	One day return of SW Benchmark 10 Year Datastream Government Index, Datastream symbol: BMSW10Y	Datastream

BD BENCHMARK 10 YEAR DS GOVT. INDEX - CLEAN PRICE INDEX	*BUNDGOV10PI*	One day return of BD Benchmark 10 Year Datastream Government Index, Datastream symbol: BMBD10Y	Datastream
	Panel C: Total return indices		
S&P 500 COMPOSITE - TOT RETURN IND	*SPX500TR*	One day total return of Standard and Poor's 500 Composite, Datastream symbol: S&PCOMP, total return index	Datastream
STOXX EUROPE 50 - TOT RETURN IND	*STOXX50TR*	One day total return of STOXX Europe 50, Datastream symbol: DJSTO50	Datastream
NIKKEI 225 STOCK AVERAGE - TOT RETURN IND	*NIKKEI225TR*	One day total return of Nikkei 225 Stock Average, Datastream symbol: JAPDOWA	Datastream
CNX NIFTY (50) - TOT RETURN IND	*NIFTY50TR*	One day total return of CNX Nifty (50), Datastream symbol: INNSE50	Datastream

* Oxford-Man states for Oxford-Man Institute of Quantitative Finance, Realized Library (Heber et al., 2009).

Table A.2. Descriptive statistics for assets tested as save haven

This table presents descriptive statistics for assets assumed to possess safe haven characteristics. It covers a period ranging from January 1, 2010 to December 31, 2020. Each raw represents the variable name. All assets are represented as daily differences. The table columns present the mean (Mean); standard deviation (Std); skewness (Skew); excess kurtosis (Kurt); minimum (Min); 5th percentile (5%); 25th percentile (25%); median (50%); 75th percentile (75%); 95th percentile (95%); max (Max); and the number of daily observations, for which the given variable is available (Count). A detailed description for each variate is presented in Table A1 in the Online Appendix.

	mean	**std**	**skew**	**kurt**	**min**	**5%**	**25%**	**50%**	**75%**	**95%**	**max**	**count**
GOLD	0.0002	0.0098	-0.6009	6.7208	-0.0966	-0.0157	-0.0044	0.0002	0.0054	0.0155	0.0558	2869
SILVER	0.0003	0.0191	-0.7106	6.4891	-0.1771	-0.0303	-0.0077	0.0000	0.0090	0.0302	0.0801	2869
PLATINIUM	0.0000	0.0134	-0.3430	6.6703	-0.1343	-0.0203	-0.0073	0.0000	0.0074	0.0207	0.0979	2869
COPPER	0.0001	0.0134	-0.0716	2.8505	-0.0778	-0.0215	-0.0070	0.0000	0.0073	0.0219	0.0691	2869
OIL	-0.0011	0.0674	-34.5770	1517.0740	-3.0597	-0.0365	-0.0108	0.0000	0.0110	0.0330	0.3502	2869
COCOA	0.0000	0.0147	0.1050	2.3771	-0.0924	-0.0243	-0.0084	0.0000	0.0081	0.0234	0.0920	2869
WHEAT	0.0004	0.0225	0.7247	15.8701	-0.2186	-0.0321	-0.0107	0.0000	0.0108	0.0330	0.2702	2869
SOYABEANS	0.0002	0.0133	-0.3604	5.5978	-0.1034	-0.0205	-0.0062	0.0000	0.0069	0.0210	0.0787	2869
CORN	0.0004	0.0233	3.4194	107.3153	-0.3154	-0.0266	-0.0083	0.0000	0.0085	0.0281	0.4902	2869
USGOV10_PI	0.0001	0.0043	-0.1337	2.4809	-0.0244	-0.0066	-0.0024	0.0002	0.0026	0.0066	0.0211	2869
USGOV30_PI	0.0002	0.0097	0.0964	5.3172	-0.0667	-0.0156	-0.0052	0.0003	0.0058	0.0149	0.0855	2869
SWISSGOV10_PI	0.0001	0.0026	-0.2879	6.3624	-0.0245	-0.0041	-0.0013	0.0001	0.0014	0.0040	0.0194	2869
BUNDGOV10_PI	0.0002	0.0035	-0.2590	2.9074	-0.0198	-0.0056	-0.0017	0.0002	0.0022	0.0054	0.0227	2869

Table A.3. Safe haven assets in target volatility strategy based on SPX500TR Index – extended results

This table presents the Sharpe (SR) and Calmar ratios (CR) of target volatility strategies. Results cover the period from January 1, 2010 to December 31, 2020. The table is divided into five panels: panel A shows results of target volatility strategy, based on predictions from *GRU-MULTIVAR*; panel B shows results based on *GRU-UNIVAR*; panel C on *ARFIMA*; panel D on *GARCH*; and panel E on *GJR-GARCH*. In each panel, the two first rows represent the target volatility strategy's results, without any safe haven asset (STRAT_CLEAN), and the S&P500 Total Return index (BENCHMARK). The remaining rows state the results of strategies that include additional assets. A full description of assets is presented in Table A1 in Online Appendix. For better transparency, only the first raw in each panel (STRAT_CLEAN) shows nominal values of both SR and CR. All other rows present the difference between STRAT_CLEAN and the modified strategy results (or the benchmark). The strategies presented in the table are calculated with three different quantile thresholds: 90%, 95%, and 99%. Each quantile threshold represents the required predicted volatility to include the tested safe haven asset in the strategy. Furthermore, we demonstrate three different maximum exposures on safe haven weight allocation (cap 10%, cap 20%, and cap 30%) and the mean of these three ratios (mean_cap). Finally, the last two columns aggregate the different strategies and present the average result (mean) from all configurations.

Panel A: Target volatility strategy based on *GRU-MULTIVAR*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75	0.97	0.75
BENCHMARK	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34	-0.17	-0.34
GOLD	0.02	0.03	0.03	0.03	0.04	0.01	0.03	0.02	0.00	0.01	0.00	-0.01	-0.01	-0.03	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00	-0.04	0.00	-0.01	0.01	0.00
SILVER	0.00	-0.06	-0.02	-0.20	-0.06	-0.28	-0.03	-0.18	-0.03	-0.08	-0.07	-0.23	-0.12	-0.33	-0.07	-0.21	-0.02	-0.05	-0.04	-0.19	-0.06	-0.28	-0.04	-0.17	-0.05	-0.19
PLATINIUM	-0.06	-0.13	-0.13	-0.28	-0.20	-0.39	-0.13	-0.27	-0.04	-0.12	-0.09	-0.27	-0.14	-0.37	-0.09	-0.25	-0.02	-0.08	-0.05	-0.22	-0.08	-0.31	-0.05	-0.20	-0.09	-0.24
COPPER	-0.02	-0.04	-0.04	-0.15	-0.07	-0.23	-0.04	-0.14	-0.02	-0.02	-0.05	-0.13	-0.08	-0.22	-0.05	-0.12	-0.01	-0.02	-0.03	-0.08	-0.05	-0.16	-0.03	-0.09	-0.04	-0.12
OIL	-0.65	-0.65	-1.05	-0.78	-1.47	-0.92	-1.05	-0.78	-0.08	-0.26	-0.20	-0.43	-0.33	-0.51	-0.21	-0.40	-0.05	-0.21	-0.14	-0.38	-0.23	-0.47	-0.14	-0.35	-0.47	-0.51
COCOA	-0.04	-0.04	-0.09	-0.15	-0.15	-0.24	-0.10	-0.15	-0.04	-0.03	-0.07	-0.15	-0.12	-0.24	-0.08	-0.14	-0.02	-0.01	-0.03	-0.08	-0.05	-0.15	-0.03	-0.08	-0.07	-0.12
WHEAT	0.01	0.02	0.01	0.03	-0.01	-0.01	0.01	0.01	0.01	-0.02	0.00	-0.03	-0.01	-0.05	0.00	-0.03	0.01	0.00	0.02	0.00	0.03	0.01	0.02	0.00	0.01	-0.01
SOYABEANS	-0.01	-0.01	-0.02	-0.10	-0.03	-0.17	-0.02	-0.09	-0.01	0.00	-0.02	-0.03	-0.04	-0.09	-0.03	-0.04	0.00	-0.01	0.00	-0.02	-0.01	-0.03	0.00	-0.02	-0.02	-0.05
CORN	-0.02	-0.05	-0.05	-0.16	-0.09	-0.24	-0.06	-0.15	-0.01	-0.01	-0.03	-0.04	-0.06	-0.12	-0.03	-0.06	-0.01	-0.01	-0.02	-0.03	-0.04	-0.10	-0.02	-0.05	-0.04	-0.09
USGOV10_PI	0.02	0.03	0.03	0.05	0.05	0.08	0.03	0.05	0.00	-0.01	0.01	-0.01	0.01	-0.02	0.01	-0.01	0.00	0.00	0.00	-0.01	0.01	-0.01	0.00	-0.01	0.01	0.01
USGOV30_PI	0.04	0.03	0.06	0.06	0.08	0.08	0.06	0.06	0.01	-0.03	0.01	-0.05	0.01	-0.08	0.01	-0.05	0.00	-0.02	-0.01	-0.03	-0.01	-0.05	-0.01	-0.03	0.02	-0.01
SWISSGOV10_PI	0.01	0.01	0.01	0.02	0.02	0.03	0.01	0.02	0.00	-0.01	-0.01	-0.02	-0.01	-0.03	-0.01	-0.02	-0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	0.00	0.00
BUNDGOV10_PI	0.00	0.02	0.01	0.03	0.01	0.05	0.01	0.03	0.00	-0.01	-0.01	-0.02	-0.01	-0.03	-0.01	-0.02	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.01

Panel B: Target volatility strategy based on *GRU-UNIVAR*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70	0.95	0.70
BENCHMARK	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29
GOLD	0.03	0.04	0.06	0.06	0.08	0.08	0.06	0.06	0.03	0.04	0.06	0.07	0.08	0.10	0.05	0.07	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.04	0.04
SILVER	0.01	0.01	0.00	-0.11	-0.03	-0.20	-0.01	-0.10	0.03	0.04	0.05	-0.09	0.06	-0.17	0.04	-0.07	-0.01	-0.01	-0.02	-0.09	-0.03	-0.18	-0.02	-0.09	0.01	-0.09
PLATINIUM	-0.01	0.00	-0.02	-0.14	-0.05	-0.23	-0.02	-0.13	0.01	0.01	0.00	-0.13	-0.01	-0.22	0.00	-0.11	-0.01	-0.01	-0.03	-0.10	-0.05	-0.20	-0.03	-0.10	-0.02	-0.11
COPPER	-0.01	-0.05	-0.03	-0.13	-0.05	-0.20	-0.03	-0.13	-0.01	-0.01	-0.03	-0.02	-0.05	-0.09	-0.03	-0.04	-0.01	0.00	-0.03	-0.03	-0.04	-0.11	-0.03	-0.04	-0.03	-0.07
OIL	-0.64	-0.61	-1.04	-0.73	-1.46	-0.88	-1.04	-0.74	-0.55	-0.57	-0.96	-0.70	-1.41	-0.85	-0.97	-0.71	-0.04	-0.14	-0.11	-0.32	-0.20	-0.41	-0.12	-0.29	-0.71	-0.58
COCOA	-0.04	-0.02	-0.08	-0.09	-0.12	-0.16	-0.08	-0.09	-0.01	0.00	-0.02	-0.02	-0.04	-0.10	-0.02	-0.04	-0.01	-0.01	-0.02	-0.01	-0.04	-0.08	-0.02	-0.03	-0.04	-0.05
WHEAT	0.01	0.02	0.01	-0.01	0.00	-0.04	0.01	-0.01	-0.01	-0.03	-0.04	-0.05	-0.07	-0.08	-0.04	-0.05	0.00	-0.01	0.00	-0.02	0.00	-0.03	0.00	-0.02	-0.01	-0.03
SOYABEANS	0.01	0.00	0.02	-0.05	0.03	-0.09	0.02	-0.05	0.00	-0.01	0.00	-0.02	-0.01	-0.04	0.00	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	-0.02
CORN	0.03	0.00	0.04	-0.04	0.03	-0.08	0.03	-0.04	0.00	-0.01	0.00	-0.02	-0.01	-0.03	0.00	-0.02	-0.02	-0.02	-0.03	-0.03	-0.05	-0.06	-0.03	-0.04	0.00	-0.03
USGOV10_PI	0.01	0.02	0.03	0.03	0.04	0.04	0.03	0.03	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.02	0.01
USGOV30_PI	0.03	0.03	0.06	0.05	0.08	0.05	0.06	0.05	0.02	0.01	0.03	0.01	0.03	0.00	0.02	0.01	0.01	-0.01	0.01	-0.01	0.02	-0.02	0.01	-0.01	0.03	0.01
SWISSGOV10_PI	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.01	-0.01	-0.01	0.00
BUNDGOV10_PI	0.00	0.01	0.00	0.02	0.00	0.03	0.00	0.02	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.00	0.01

Panel C: Target volatility strategy based on *ARFIMA*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80	1.00	0.80
BENCHMARK	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38	-0.20	-0.38
GOLD	0.03	0.02	0.04	-0.02	0.05	-0.06	0.04	-0.02	0.00	-0.01	0.00	-0.06	-0.01	-0.11	-0.01	-0.06	-0.01	-0.02	-0.01	-0.07	-0.02	-0.12	-0.01	-0.07	0.01	-0.05
SILVER	0.01	-0.12	-0.01	-0.25	-0.05	-0.33	-0.02	-0.23	-0.01	-0.13	-0.04	-0.26	-0.08	-0.35	-0.04	-0.25	-0.01	-0.11	-0.03	-0.24	-0.05	-0.33	-0.03	-0.23	-0.03	-0.24
PLATINIUM	-0.03	-0.18	-0.07	-0.31	-0.12	-0.40	-0.07	-0.30	-0.03	-0.16	-0.07	-0.29	-0.12	-0.39	-0.08	-0.28	-0.02	-0.14	-0.04	-0.26	-0.07	-0.35	-0.05	-0.25	-0.07	-0.28
COPPER	-0.03	-0.08	-0.06	-0.20	-0.11	-0.29	-0.06	-0.19	0.01	-0.06	0.01	-0.13	0.00	-0.19	0.00	-0.13	-0.01	-0.06	-0.02	-0.14	-0.04	-0.21	-0.02	-0.14	-0.03	-0.15
OIL	-0.65	-0.69	-1.05	-0.81	-1.47	-0.96	-1.06	-0.82	-0.67	-0.70	-1.08	-0.82	-1.50	-0.96	-1.08	-0.83	-0.03	-0.18	-0.09	-0.34	-0.17	-0.44	-0.10	-0.32	-0.75	-0.66
COCOA	-0.07	-0.13	-0.14	-0.25	-0.21	-0.34	-0.14	-0.24	-0.02	-0.09	-0.05	-0.19	-0.08	-0.26	-0.05	-0.18	-0.02	-0.06	-0.04	-0.15	-0.07	-0.22	-0.04	-0.14	-0.08	-0.19
WHEAT	0.02	0.02	0.02	-0.01	0.00	-0.06	0.01	-0.02	0.01	0.00	0.01	-0.01	-0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	-0.01	-0.01	-0.03	0.00	-0.02	0.00	-0.01
SOYABEANS	0.00	0.01	0.00	-0.07	-0.02	-0.15	-0.01	-0.07	0.00	-0.01	-0.01	-0.02	-0.02	-0.06	-0.01	-0.03	0.00	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.00	-0.03
CORN	-0.03	-0.05	-0.08	-0.15	-0.14	-0.24	-0.09	-0.15	-0.03	-0.04	-0.07	-0.12	-0.12	-0.20	-0.08	-0.12	-0.01	-0.02	-0.02	-0.07	-0.04	-0.12	-0.02	-0.07	-0.06	-0.11
USGOV10_PI	0.02	0.03	0.04	0.06	0.06	0.07	0.04	0.05	0.00	-0.02	0.00	-0.03	0.00	-0.05	0.00	-0.03	0.00	0.00	0.00	-0.01	0.00	-0.02	0.00	-0.01	0.01	0.00
USGOV30_PI	0.04	0.04	0.07	0.07	0.10	0.06	0.07	0.06	-0.01	-0.04	-0.02	-0.08	-0.04	-0.12	-0.02	-0.08	-0.01	-0.02	-0.02	-0.11	-0.04	-0.18	-0.02	-0.10	0.01	-0.04
SWISSGOV10_PI	0.00	0.00	0.00	-0.01	0.01	-0.04	0.00	-0.02	0.00	-0.01	-0.01	-0.03	-0.02	-0.05	-0.01	-0.03	-0.01	-0.01	-0.02	-0.04	-0.02	-0.08	-0.02	-0.05	-0.01	-0.03
BUNDGOV10_PI	0.00	0.01	0.01	0.02	0.01	0.02	0.01	0.02	-0.01	-0.02	-0.01	-0.04	-0.02	-0.06	-0.01	-0.04	-0.01	-0.02	-0.01	-0.03	-0.02	-0.06	-0.01	-0.04	-0.01	-0.02

Panel D: Target volatility strategy based on *GARCH*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71	0.95	0.71
BENCHMARK	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29	-0.15	-0.29
GOLD	0.00	0.01	-0.01	-0.02	-0.03	-0.08	-0.01	-0.03	0.01	0.01	0.02	0.00	0.02	-0.05	0.02	-0.01	0.00	0.00	0.00	-0.02	-0.01	-0.08	0.00	-0.04	0.00	-0.03
SILVER	-0.04	-0.13	-0.11	-0.27	-0.19	-0.36	-0.11	-0.25	0.00	-0.10	-0.03	-0.24	-0.08	-0.32	-0.04	-0.22	-0.02	-0.10	-0.04	-0.24	-0.08	-0.33	-0.05	-0.22	-0.07	-0.23
PLATINIUM	-0.06	-0.17	-0.13	-0.32	-0.21	-0.41	-0.13	-0.30	-0.02	-0.13	-0.05	-0.27	-0.09	-0.35	-0.05	-0.25	-0.02	-0.11	-0.05	-0.25	-0.09	-0.34	-0.06	-0.24	-0.08	-0.26
COPPER	-0.04	-0.10	-0.10	-0.23	-0.17	-0.32	-0.11	-0.22	-0.01	-0.03	-0.02	-0.12	-0.05	-0.20	-0.03	-0.12	-0.01	-0.03	-0.03	-0.14	-0.05	-0.21	-0.03	-0.13	-0.05	-0.15
OIL	-0.72	-0.65	-1.08	-0.74	-1.46	-0.88	-1.08	-0.76	-0.70	-0.64	-1.06	-0.74	-1.44	-0.87	-1.07	-0.75	-0.73	-0.64	-1.10	-0.75	-1.51	-0.88	-1.11	-0.76	-1.09	-0.75
COCOA	-0.09	-0.10	-0.18	-0.25	-0.28	-0.37	-0.18	-0.24	-0.03	-0.05	-0.07	-0.17	-0.11	-0.25	-0.07	-0.16	-0.02	-0.01	-0.03	-0.10	-0.05	-0.17	-0.03	-0.10	-0.10	-0.16
WHEAT	-0.02	0.01	-0.07	-0.06	-0.13	-0.13	-0.07	-0.06	-0.01	0.00	-0.04	-0.01	-0.08	-0.03	-0.05	-0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	-0.04	-0.02
SOYABEANS	-0.03	-0.03	-0.07	-0.12	-0.11	-0.20	-0.07	-0.12	0.00	0.01	0.00	0.01	0.00	-0.02	0.00	0.00	-0.01	-0.01	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	-0.03	-0.04
CORN	-0.05	-0.06	-0.12	-0.18	-0.22	-0.26	-0.13	-0.17	-0.03	-0.01	-0.06	-0.08	-0.10	-0.18	-0.06	-0.09	-0.03	-0.02	-0.07	-0.12	-0.10	-0.23	-0.07	-0.12	-0.09	-0.13
USGOV10_PI	0.02	0.02	0.05	0.03	0.07	0.05	0.05	0.03	0.01	0.01	0.03	0.01	0.04	0.02	0.02	0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.02
USGOV30_PI	0.05	0.03	0.08	0.07	0.10	0.10	0.08	0.07	0.02	0.01	0.04	0.03	0.04	0.04	0.04	0.03	0.01	0.00	0.01	0.01	0.01	-0.03	0.01	-0.01	0.04	0.03
SWISSGOV10_PI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01
BUNDGOV10_PI	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	0.00	0.00

Panel E: Target volatility strategy based on *GJR-GARCH*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71	0.88	0.71
BENCHMARK	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30	-0.08	-0.30
GOLD	0.01	0.01	0.01	-0.05	0.00	-0.10	0.01	-0.05	0.01	0.01	0.01	-0.05	0.01	-0.10	0.01	-0.05	0.00	-0.01	0.00	-0.08	-0.01	-0.14	0.00	-0.07	0.01	-0.06
SILVER	-0.03	-0.17	-0.09	-0.31	-0.16	-0.39	-0.09	-0.29	-0.03	-0.16	-0.08	-0.30	-0.14	-0.38	-0.08	-0.28	-0.02	-0.15	-0.05	-0.28	-0.08	-0.37	-0.05	-0.27	-0.07	-0.28
PLATINIUM	-0.07	-0.22	-0.16	-0.37	-0.25	-0.46	-0.16	-0.35	-0.04	-0.18	-0.09	-0.32	-0.15	-0.41	-0.09	-0.31	-0.02	-0.15	-0.05	-0.29	-0.08	-0.37	-0.05	-0.27	-0.10	-0.31
COPPER	-0.03	-0.12	-0.07	-0.25	-0.13	-0.33	-0.08	-0.23	-0.01	-0.11	-0.03	-0.23	-0.06	-0.31	-0.03	-0.22	-0.01	-0.07	-0.03	-0.17	-0.04	-0.24	-0.03	-0.16	-0.05	-0.20
OIL	-0.67	-0.66	-1.01	-0.75	-1.39	-0.89	-1.03	-0.77	-0.67	-0.66	-1.02	-0.75	-1.39	-0.89	-1.03	-0.77	-0.08	-0.25	-0.21	-0.41	-0.33	-0.50	-0.21	-0.39	-0.75	-0.64
COCOA	-0.04	-0.11	-0.10	-0.23	-0.16	-0.32	-0.10	-0.22	-0.05	-0.11	-0.10	-0.22	-0.15	-0.31	-0.10	-0.21	-0.01	-0.05	-0.03	-0.13	-0.05	-0.20	-0.03	-0.13	-0.08	-0.19
WHEAT	0.01	0.02	0.00	-0.04	-0.04	-0.08	-0.01	-0.03	-0.05	-0.05	-0.11	-0.13	-0.17	-0.21	-0.11	-0.13	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	-0.04	-0.06
SOYABEANS	-0.02	-0.04	-0.05	-0.14	-0.09	-0.22	-0.05	-0.13	-0.03	-0.04	-0.06	-0.14	-0.09	-0.21	-0.06	-0.13	-0.01	0.00	-0.01	-0.01	-0.02	-0.05	-0.01	-0.02	-0.04	-0.10
CORN	-0.04	-0.09	-0.09	-0.20	-0.15	-0.29	-0.09	-0.19	-0.05	-0.09	-0.11	-0.22	-0.17	-0.31	-0.11	-0.21	-0.03	-0.05	-0.06	-0.16	-0.09	-0.24	-0.06	-0.15	-0.08	-0.18
USGOV10_PI	0.02	0.02	0.03	0.03	0.05	0.03	0.03	0.02	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	-0.04	0.00	-0.01	0.02	0.01
USGOV30_PI	0.04	0.01	0.06	-0.02	0.07	-0.05	0.05	-0.02	0.02	0.01	0.03	-0.01	0.02	-0.03	0.02	-0.01	0.00	-0.02	-0.01	-0.12	-0.02	-0.20	-0.01	-0.11	0.02	-0.05
SWISSGOV10_PI	0.00	0.00	0.00	0.00	0.00	-0.03	0.00	-0.01	0.00	0.00	-0.01	-0.01	-0.01	-0.05	-0.01	-0.02	-0.01	-0.01	-0.01	-0.03	-0.02	-0.08	-0.01	-0.04	-0.01	-0.02
BUNDGOV10_PI	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.02	-0.02	-0.06	-0.01	-0.03	0.00	-0.01

Appendix B. Tables for robustness tests

Table B.1. Prediction errors for *BPVNIKKEI225, BPVNIFTY50, BPVSTOXX50*

This table demonstrates the results of out-of-sample prediction errors for daily volatility of *NIKKEI225PI*, *NIFTY50PI*, and *STOXX50PI* Indices, measured with realized bi-powered variation (*BPV*). BPV is transformed to represent monthly standard deviation—according to equation 24. The table shows two prediction error measures: mean squared errors (MSE) and mean average percentage error (MAPE). Each column represents a different prediction method: autoregressive fractionally integrated moving average (*ARFIMA*); generalized autoregressive conditional heteroskedasticity (*GARCH*); Glosten, Jagannathan, and Runkle (*GJR-GARCH*); univariate approach with gated recurrent units (*GRU-UNIVAR*); and multivariate method with gated recurrent units (*GRU-MULTIVAR*). *GRU-MULTIVAR* uses the following input features: *BPV*, *GOLDLOG*, *CRUDELOG*, *GOVBOND*, and *CORPBONDAA*. The sample period runs from January 1, 2010 to December 31, 2020.

		ARFIMA	***GARCH***	***GJR-GARCH***	***GRU-UNIVAR***	***GRU-MULTIVAR***
NIKKEI225PI	MSE	0.0050	0.0266	0.0243	0.0053	0.0045
	MAPE	0.3372	1.1679	1.0467	0.3119	0.3205
NIFTY50TR	MSE	0.0037	0.0176	0.0216	0.0046	0.0035
	MAPE	0.2817	0.8927	0.9626	0.3641	0.2427
STOXX50TR	MSE	0.0043	0.0126	0.0134	0.0054	0.0039
	MAPE	0.3426	0.6634	0.5999	0.3722	0.3057

Table B.2. Diebold-Mariano (DM) equal forecast tests for *BPVNIKKEI225, BPVNIFTY50, BPVSTOXX50*

This table visualizes the results of Diebold-Mariano (DM) equal forecast accuracy tests for daily volatility of *NIKKEI225PI*, *NIFTY50PI*, and *STOXX50PI* Indices, measured with realized bi-powered variation (BPV). BPV is transformed to represent monthly standard deviation—according to equation 24. The table is divided into three panels: panel A presents the results for *BPVNIKKEI225*, panel B for *BPVNIFTY50*, and panel C for *BPVSTOXX50*. The table shows two prediction error measures: mean squared errors (MSE) and mean average percentage error (MAPE). Each column represents a different prediction method: autoregressive fractionally integrated moving average (*ARFIMA*); generalized autoregressive conditional heteroskedasticity (*GARCH*); Glosten, Jagannathan, and Runkle (*GJR-GARCH*); multivariate method with gated recurrent units (*GRU-MULTIVAR*); and univariate approach with gated recurrent units (*GRU-UNIVAR*). *GRU-MULTIVAR* uses the following input features: *BPV*, *GOLDLOG*, *CRUDELOG*, *GOVBOND*, and *CORPBONDAA*. $***$ and $**$ denote a rejection of the null hypothesis at the 1% and 5% significance level—respectively. The sample period runs from January 1, 2010 to December 31, 2020.

		GARCH	***GJR-GARCH***	***GRU-MULTIVAR***	***GRU-UNIVAR***
		Panel A: BPVNIKKEI225			
ARFIMA	MSE	35.3***	28.4***	-2.2**	0.8
	MAPE	81.6***	72.5***	-3.9***	-7.1***
GARCH	MSE		-6.2***	-33.1***	-29.7***
	MAPE		-19.8***	-78.0***	-82.9***
GJR-GARCH	MSE			-27.4***	-24.1***
	MAPE			-73.3***	-73.9***
GRU-MULTIVAR	MSE				3.2***
	MAPE				-2.1**
		Panel B: *BPVNIFTY50*			
ARFIMA	MSE	22.2***	20.1***	-0.7	2.2**
	MAPE	77.8***	80.1***	-13.1***	21.8***
GARCH	MSE		9.0***	-22.5***	-19.1***
	MAPE		19.9***	-75.9***	-83.1***
GJR-GARCH	MSE			-19.7***	-17.6***
	MAPE			-80.0***	-86.1***
GRU-MULTIVAR	MSE				6.4**
	MAPE				28.5***
		Panel C: *BPVSTOXX50*			
ARFIMA	MSE	18.9***	15.0***	-4.1***	7.3***
	MAPE	14.9***	18.0***	-7.2***	5.1***
GARCH	MSE		2.1**	-19.6***	-15.7***
	MAPE		6.0***	-13.7***	-16.5***
GJR-GARCH	MSE			-16.0***	-13.0***
	MAPE			-16.1***	-22.1***
GRU-MULTIVAR	MSE				7.8***
	MAPE				6.7***

Table B.3. Safe haven assets in target volatility strategy based on *NIKKEI225TR* Index

This table presents the Sharpe (SR) and Calmar ratios (CR) of target volatility strategies. Results cover the period from January 1, 2010 to December 31, 2020. The table is divided into five panels: panel A shows results of target volatility strategy, based on predictions from *GRU-MULTIVAR*; panel B shows results based on *GRU-UNIVAR*; panel C on *ARFIMA*; panel D on *GARCH*; and panel E on *GJR-GARCH*. In each panel, the two first rows represent the target volatility strategy's results without any safe haven asset (STRAT_CLEAN) and the NIKKEI 225 STOCK AVERAGE - TOT RETURN IND (*BENCHMARK*). The remaining rows state the results of strategies that include additional assets. A full description of assets is presented in Table A1 in the Online Appendix. For better transparency, only the first raw in each panel (STRAT_CLEAN) shows nominal values of both SR and CR. All other rows present the difference between STRAT_CLEAN and the modified strategy results (or the benchmark). The strategies that are presented in the table are calculated with three different quantile thresholds: 90%, 95%, and 99%. Each quantile threshold represents the required predicted volatility to include the tested safe haven asset in the strategy. Furthermore, we demonstrate three different maximum exposures on safe haven weight allocation (cap 10%, cap 20%, and cap 30%) and the mean of these three ratios (mean_cap). Finally, the last two columns aggregate the different strategies and present the average result (mean) from all configurations.

Panel A: Target volatility strategy based on *GRU-MULTIVAR*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46	0.71	0.46
BENCHMARK	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10	-0.19	-0.10
GOLD	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	0.01	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00
SILVER	-0.01	0.02	-0.03	-0.02	-0.05	-0.08	-0.03	-0.03	0.01	0.03	0.02	0.05	0.01	-0.01	0.01	0.02	-0.01	0.00	-0.02	-0.01	-0.03	-0.04	-0.02	-0.02	-0.01	-0.01
PLATINIUM	-0.01	-0.01	-0.04	-0.04	-0.06	-0.11	-0.04	-0.05	-0.01	0.02	-0.02	-0.02	-0.03	-0.08	-0.02	-0.03	-0.01	-0.01	-0.03	-0.02	-0.04	-0.05	-0.03	-0.02	-0.03	-0.04
COPPER	0.01	0.04	0.02	0.07	0.02	0.07	0.02	0.06	0.01	0.02	0.02	0.04	0.02	0.07	0.01	0.04	-0.01	0.01	-0.01	0.00	-0.02	0.00	-0.01	0.00	0.01	0.03
OIL	-0.45	-0.37	-0.80	-0.48	-1.20	-0.63	-0.81	-0.49	0.03	0.06	0.02	-0.06	-0.01	-0.13	0.01	-0.04	-0.01	0.01	-0.03	0.00	-0.07	-0.09	-0.03	-0.03	-0.28	-0.19
COCOA	-0.02	-0.04	-0.04	-0.07	-0.06	-0.10	-0.04	-0.07	-0.02	-0.03	-0.04	-0.05	-0.06	-0.08	-0.04	-0.05	-0.01	-0.01	-0.02	-0.02	-0.03	-0.03	-0.02	-0.02	-0.03	-0.05
WHEAT	0.02	0.01	0.03	0.00	0.02	-0.01	0.02	0.00	0.00	0.00	-0.01	-0.02	-0.02	-0.03	-0.01	-0.02	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.00
SOYABEANS	0.02	0.03	0.03	0.04	0.04	0.05	0.03	0.04	0.02	0.04	0.04	0.07	0.07	0.11	0.04	0.07	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.04
CORN	0.01	0.02	0.01	0.03	0.01	0.03	0.01	0.02	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.01	-0.01	0.00	-0.01	0.00	-0.01	0.00	0.00	0.01
USGOV10_PI	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
USGOV30_PI	0.00	-0.01	0.00	-0.02	0.00	-0.03	0.00	-0.02	-0.01	-0.01	-0.02	-0.02	-0.04	-0.04	-0.03	-0.02	-0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.01	-0.02
SWISSGOV10_PI	-0.01	-0.01	-0.02	-0.02	-0.03	-0.03	-0.02	-0.02	-0.01	-0.01	-0.02	-0.02	-0.03	-0.02	-0.02	-0.02	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	-0.02	-0.01
BUNDGOV10_PI	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01

Panel B: Target volatility strategy based on *GRU-UNIVAR*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33	0.57	0.33
BENCHMARK	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03	-0.05	0.03
GOLD	0.01	0.03	0.03	0.06	0.04	0.08	0.03	0.06	0.01	0.02	0.01	0.03	0.01	0.04	0.01	0.03	0.01	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.02	0.03
SILVER	0.03	0.06	0.05	0.08	0.07	0.05	0.05	0.06	0.01	0.02	0.01	0.05	0.01	0.01	0.01	0.03	0.01	0.01	0.03	0.02	0.04	0.02	0.02	0.02	0.03	0.04
PLATINIUM	0.02	0.04	0.04	0.07	0.05	0.04	0.04	0.05	-0.01	0.00	-0.03	0.01	-0.04	-0.04	-0.03	-0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
COPPER	0.01	0.02	0.02	0.05	0.03	0.08	0.02	0.05	0.01	0.01	0.01	0.03	0.01	0.05	0.01	0.03	0.00	0.01	0.00	0.01	0.00	0.02	0.00	0.01	0.01	0.03
OIL	-0.37	-0.26	-0.68	-0.36	-1.06	-0.50	-0.70	-0.37	0.02	0.04	0.01	-0.01	-0.02	-0.06	0.00	-0.01	-0.01	0.01	-0.02	0.01	-0.05	-0.05	-0.03	-0.01	-0.24	-0.13
COCOA	0.01	0.02	0.02	0.01	0.03	0.00	0.02	0.01	-0.01	0.00	-0.02	0.00	-0.03	-0.01	-0.02	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	0.00	0.00
WHEAT	0.02	0.02	0.03	0.04	0.03	0.06	0.02	0.04	-0.02	-0.03	-0.04	-0.06	-0.07	-0.09	-0.04	-0.06	0.01	0.01	0.03	0.02	0.04	0.03	0.03	0.02	0.00	0.00
SOYABEANS	0.04	0.04	0.07	0.09	0.10	0.14	0.07	0.09	0.00	-0.01	0.00	-0.02	0.00	-0.03	0.00	-0.02	0.02	0.01	0.03	0.03	0.04	0.04	0.03	0.03	0.03	0.03
CORN	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.08	-0.04	-0.05	0.00	0.01	0.01	0.01	0.01	0.02	0.01	0.01	-0.01	-0.02
USGOV10_PI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	-0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
USGOV30_PI	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	-0.01	0.00	-0.02	0.00	-0.03	0.00	-0.02	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00
SWISSGOV10_PI	-0.01	0.00	-0.02	-0.01	-0.03	-0.01	-0.02	-0.01	-0.01	0.00	-0.01	0.00	-0.02	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
BUNDGOV10_PI	0.00	0.00	0.00	0.01	-0.01	0.01	0.00	0.01	0.00	0.00	-0.01	0.01	-0.01	0.01	-0.01	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00

Panel C: Target volatility strategy based on *ARFIMA*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33	0.56	0.33
BENCHMARK	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03	-0.03	0.03
GOLD	0.01	0.03	0.02	0.07	0.03	0.08	0.02	0.06	0.01	0.02	0.02	0.05	0.02	0.07	0.01	0.05	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.02	0.04
SILVER	0.03	0.07	0.05	0.04	0.07	0.02	0.05	0.04	0.00	0.02	0.00	0.00	-0.01	-0.03	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.01	0.01
PLATINIUM	0.00	0.02	0.00	-0.02	-0.01	-0.06	0.00	-0.02	-0.01	0.01	-0.03	-0.04	-0.05	-0.09	-0.03	-0.04	-0.01	0.00	-0.02	-0.01	-0.03	-0.04	-0.02	-0.02	-0.02	-0.02
COPPER	0.01	0.03	0.01	0.05	0.01	0.04	0.01	0.04	0.01	0.02	0.01	0.04	0.01	0.03	0.01	0.03	0.00	0.00	-0.01	0.00	-0.02	0.00	-0.01	0.00	0.00	0.02
OIL	-0.38	-0.28	-0.68	-0.37	-1.05	-0.51	-0.71	-0.39	-0.01	0.01	-0.04	-0.04	-0.09	-0.09	-0.05	-0.04	-0.04	-0.03	-0.08	-0.08	-0.14	-0.14	-0.09	-0.08	-0.28	-0.17
COCOA	-0.02	-0.01	-0.04	-0.03	-0.07	-0.05	-0.05	-0.03	-0.02	-0.01	-0.05	-0.02	-0.07	-0.04	-0.05	-0.03	-0.01	0.00	-0.02	0.00	-0.03	-0.01	-0.02	0.00	-0.04	-0.02
WHEAT	0.00	0.00	-0.01	0.00	-0.02	0.00	-0.01	0.00	-0.01	-0.02	-0.02	-0.03	-0.03	-0.04	-0.02	-0.03	0.02	0.02	0.04	0.05	0.06	0.07	0.04	0.05	0.00	0.00
SOYABEANS	0.03	0.05	0.05	0.08	0.07	0.08	0.05	0.07	0.01	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.01	0.03	0.02	0.04	0.03	0.02	0.02	0.03	0.03
CORN	0.02	0.02	0.03	0.04	0.04	0.07	0.03	0.04	-0.02	-0.02	-0.04	-0.04	-0.07	-0.06	-0.04	-0.04	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.00
USGOV10_PI	0.00	0.00	-0.01	0.00	-0.01	0.01	-0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
USGOV30_PI	-0.01	0.00	-0.02	0.00	-0.03	0.00	-0.02	0.00	-0.01	0.00	-0.01	0.00	-0.02	0.00	-0.01	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	-0.01	0.00
SWISSGOV10_PI	-0.01	0.00	-0.02	-0.01	-0.03	-0.01	-0.02	-0.01	-0.01	-0.01	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.02	-0.01
BUNDGOV10_PI	-0.01	0.00	-0.02	-0.01	-0.02	-0.01	-0.02	-0.01	-0.01	0.00	-0.01	0.00	-0.02	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00

Panel D: Target volatility strategy based on *GARCH*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33	0.53	0.33
BENCHMARK	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03
GOLD	0.03	0.04	0.05	0.03	0.07	0.02	0.05	0.03	0.01	0.03	0.02	0.00	0.02	-0.02	0.01	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.03	0.02
SILVER	0.05	0.01	0.09	-0.02	0.10	-0.05	0.08	-0.02	-0.01	-0.03	-0.02	-0.09	-0.05	-0.13	-0.03	-0.08	0.00	0.00	-0.01	-0.03	-0.01	-0.05	-0.01	-0.03	0.02	-0.04
PLATINIUM	0.02	-0.03	0.02	-0.08	0.02	-0.12	0.02	-0.08	-0.01	-0.04	-0.03	-0.10	-0.06	-0.14	-0.03	-0.09	-0.01	-0.01	-0.03	-0.05	-0.05	-0.09	-0.03	-0.05	-0.02	-0.07
COPPER	0.04	0.04	0.08	0.02	0.10	0.01	0.07	0.02	0.00	0.01	0.00	-0.03	-0.01	-0.06	0.00	-0.03	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.00
OIL	-0.47	-0.31	-0.72	-0.38	-1.05	-0.51	-0.74	-0.40	-0.56	-0.34	-0.82	-0.41	-1.15	-0.54	-0.84	-0.43	0.03	0.03	0.04	-0.04	0.02	-0.08	0.03	-0.03	-0.52	-0.29
COCOA	-0.01	0.00	-0.03	-0.04	-0.06	-0.08	-0.03	-0.04	-0.04	-0.04	-0.08	-0.07	-0.12	-0.11	-0.08	-0.07	0.00	0.00	-0.01	0.01	-0.02	0.01	-0.01	0.01	-0.04	-0.03
WHEAT	0.00	0.01	-0.01	0.01	-0.03	-0.01	-0.02	0.01	0.01	0.02	0.01	0.04	0.00	0.05	0.01	0.04	0.01	0.02	0.03	0.03	0.04	0.05	0.03	0.03	0.01	0.03
SOYABEANS	0.06	0.06	0.11	0.11	0.15	0.14	0.10	0.10	0.01	-0.01	0.02	-0.01	0.03	-0.02	0.02	-0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.04	0.03
CORN	0.01	-0.02	0.00	-0.04	-0.01	-0.06	0.00	-0.04	-0.02	-0.04	-0.05	-0.07	-0.08	-0.09	-0.05	-0.07	0.00	0.01	-0.01	-0.01	-0.01	-0.04	-0.01	-0.01	-0.02	-0.04
USGOV10_PI	-0.01	0.00	-0.03	-0.01	-0.04	-0.02	-0.03	-0.01	0.00	0.00	-0.01	0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
USGOV30_PI	-0.03	-0.02	-0.07	-0.07	-0.11	-0.11	-0.07	-0.07	-0.01	0.00	-0.01	-0.04	-0.03	-0.07	-0.02	-0.04	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	-0.03	-0.04
SWISSGOV10_PI	-0.02	-0.01	-0.04	-0.02	-0.06	-0.04	-0.04	-0.03	-0.01	0.00	-0.02	-0.01	-0.04	-0.03	-0.03	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	-0.01
BUNDGOV10_PI	-0.01	0.00	-0.02	0.00	-0.04	-0.02	-0.02	-0.01	0.00	0.01	-0.01	0.00	-0.02	-0.02	-0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00

Panel E: Target volatility strategy based on *GJR-GARCH*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37	0.60	0.37
BENCHMARK	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02	-0.08	-0.02
GOLD	0.04	0.07	0.08	0.13	0.11	0.13	0.08	0.11	-0.01	-0.01	-0.02	-0.02	-0.03	-0.03	-0.02	-0.02	0.00	-0.01	-0.01	-0.02	-0.02	-0.03	-0.01	-0.02	0.02	0.02
SILVER	0.07	0.06	0.12	0.02	0.16	0.00	0.12	0.03	0.01	0.01	0.01	-0.06	-0.01	-0.11	0.00	-0.05	0.00	0.00	0.00	-0.02	-0.01	-0.06	-0.01	-0.03	0.04	-0.02
PLATINIUM	0.03	0.01	0.04	-0.05	0.04	-0.09	0.04	-0.04	0.00	-0.01	-0.01	-0.08	-0.03	-0.13	-0.01	-0.07	-0.03	-0.02	-0.06	-0.08	-0.10	-0.14	-0.06	-0.08	-0.01	-0.06
COPPER	0.04	0.07	0.07	0.03	0.08	0.00	0.06	0.03	0.02	0.06	0.03	0.01	0.04	-0.03	0.03	0.02	-0.01	0.00	-0.03	-0.01	-0.04	-0.05	-0.03	-0.02	0.02	0.01
OIL	-0.54	-0.36	-0.83	-0.44	-1.17	-0.57	-0.85	-0.45	0.01	0.01	-0.02	-0.09	-0.08	-0.15	-0.03	-0.07	-0.01	-0.02	-0.04	-0.10	-0.08	-0.15	-0.04	-0.09	-0.31	-0.21
COCOA	-0.03	0.00	-0.06	-0.04	-0.09	-0.09	-0.06	-0.04	-0.02	0.00	-0.04	-0.02	-0.07	-0.05	-0.04	-0.02	-0.03	-0.02	-0.05	-0.05	-0.08	-0.09	-0.05	-0.05	-0.05	-0.04
WHEAT	-0.03	-0.03	-0.06	-0.05	-0.11	-0.08	-0.07	-0.05	-0.01	-0.01	-0.02	-0.03	-0.04	-0.05	-0.03	-0.03	0.03	0.01	0.06	0.03	0.08	0.04	0.06	0.03	-0.01	-0.02
SOYABEANS	0.02	0.01	0.04	0.02	0.05	0.02	0.04	0.02	0.01	0.00	0.03	0.00	0.03	-0.01	0.02	0.00	0.03	0.02	0.05	0.03	0.07	0.05	0.05	0.03	0.04	0.01
CORN	-0.01	-0.03	-0.03	-0.06	-0.06	-0.09	-0.03	-0.06	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.03	-0.04	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	-0.02	-0.03
USGOV10_PI	0.00	0.00	-0.01	0.00	-0.02	-0.01	-0.01	0.00	-0.01	-0.02	-0.02	-0.03	-0.03	-0.05	-0.02	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01
USGOV30_PI	-0.01	-0.01	-0.03	-0.03	-0.05	-0.04	-0.03	-0.03	-0.03	-0.04	-0.05	-0.08	-0.08	-0.11	-0.05	-0.08	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	-0.02	-0.03
SWISSGOV10_PI	-0.02	-0.01	-0.03	-0.02	-0.05	-0.02	-0.03	-0.02	-0.02	-0.01	-0.03	-0.02	-0.05	-0.03	-0.03	-0.02	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	-0.02	-0.01
BUNDGOV10_PI	-0.01	0.00	-0.02	0.00	-0.02	0.01	-0.02	0.00	-0.01	-0.01	-0.03	-0.02	-0.04	-0.03	-0.03	-0.02	0.00	0.00	-0.01	0.00	-0.02	-0.01	-0.01	0.00	-0.02	-0.01

Table B.4. Safe haven assets in target volatility strategy based on the *NIFTY50TR* Index.

This table presents the Sharpe (SR) and Calmar ratios (CR) of target volatility strategies. The results cover the period from January 1, 2010 to December 31, 2020. The table is divided into five panels: panel A shows results of target volatility strategy, based on predictions from *GRU-MULTIVAR*; panel B shows results based on *GRU-UNIVAR*; panel C on *ARFIMA*; panel D on *GARCH*; and panel E on *GJR-GARCH*. In each panel, the first two rows represent the target volatility strategy's results without any safe haven asset (STRAT_CLEAN) and the CNX NIFTY (50) - TOT RETURN IND (BENCHMARK). The remaining rows state the results of strategies that include additional assets. A full description of assets is presented in Table A1 in Online Appendix. For better transparency, only the first raw in each panel (STRAT_CLEAN) shows nominal values of both SR and CR. All other rows present the difference between STRAT_CLEAN and the modified strategy results (or the benchmark). The strategies presented in the table are calculated with three different quantile thresholds: 90%, 95%, and 99%. Each quantile threshold represents the required predicted volatility to include the tested safe haven asset in the strategy. Furthermore, we demonstrate three different maximum exposures on safe haven weight allocation (cap 10%, cap 20%, and cap 30%) and the mean of these three ratios (mean_cap). Finally, the last two columns aggregate the different strategies and present the average result (mean) from all configurations.

Panel A: Target volatility strategy based on *GRU-MULTIVAR*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50	0.84	0.50
BENCHMARK	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22	-0.22
GOLD	0.00	-0.01	-0.01	-0.03	-0.02	-0.04	-0.01	-0.03	0.00	-0.01	0.00	-0.01	-0.01	-0.02	0.00	-0.01	0.01	0.01	0.02	0.01	0.03	0.02	0.02	0.01	0.00	-0.01
SILVER	-0.01	0.01	-0.05	-0.06	-0.09	-0.12	-0.05	-0.06	-0.02	-0.01	-0.06	-0.08	-0.11	-0.14	-0.07	-0.08	-0.01	0.00	-0.02	-0.05	-0.04	-0.11	-0.03	-0.05	-0.05	-0.06
PLATINIUM	-0.04	-0.03	-0.08	-0.11	-0.13	-0.17	-0.08	-0.10	-0.04	-0.02	-0.08	-0.10	-0.12	-0.17	-0.08	-0.10	-0.01	0.00	-0.03	-0.06	-0.04	-0.11	-0.03	-0.06	-0.06	-0.09
COPPER	0.00	0.03	-0.01	-0.01	-0.03	-0.05	-0.02	-0.01	-0.01	0.01	-0.02	-0.01	-0.03	-0.05	-0.02	-0.02	-0.01	0.00	-0.02	-0.01	-0.03	-0.05	-0.02	-0.02	-0.02	-0.02
OIL	-0.51	-0.41	-0.87	-0.51	-1.26	-0.65	-0.88	-0.52	-0.55	-0.42	-0.92	-0.53	-1.31	-0.67	-0.93	-0.54	-0.60	-0.43	-0.98	-0.55	-1.37	-0.68	-0.99	-0.55	-0.93	-0.54
COCOA	-0.01	0.02	-0.03	0.00	-0.05	-0.04	-0.03	-0.01	0.00	0.00	-0.01	0.01	-0.02	-0.02	-0.01	-0.01	-0.01	0.00	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	-0.02	-0.01
WHEAT	-0.01	-0.02	-0.04	-0.04	-0.08	-0.06	-0.05	-0.04	0.02	0.01	0.03	0.02	0.03	0.02	0.03	0.02	0.01	0.01	0.02	0.01	0.02	0.02	0.02	0.01	0.00	0.00
SOYABEANS	-0.01	-0.01	-0.03	-0.02	-0.05	-0.03	-0.03	-0.02	-0.01	0.00	-0.02	-0.01	-0.03	-0.01	-0.02	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	-0.02	-0.01
CORN	-0.05	-0.04	-0.11	-0.07	-0.17	-0.14	-0.11	-0.08	-0.02	-0.02	-0.05	-0.05	-0.08	-0.10	-0.05	-0.06	-0.02	-0.01	-0.04	-0.03	-0.06	-0.09	-0.04	-0.04	-0.07	-0.06
USGOV10_PI	0.00	-0.01	0.01	-0.01	0.01	-0.02	0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.00	-0.01
USGOV30_PI	0.00	-0.02	-0.01	-0.04	-0.02	-0.05	-0.01	-0.04	-0.02	-0.02	-0.03	-0.04	-0.06	-0.06	-0.03	-0.04	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	-0.01	-0.02
SWISSGOV10_PI	0.00	-0.01	-0.01	-0.01	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.01	-0.01	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	-0.01	-0.01
BUNDGOV10_PI	-0.01	-0.01	-0.01	-0.02	-0.02	-0.03	-0.01	-0.02	-0.01	-0.01	-0.03	-0.02	-0.04	-0.04	-0.03	-0.02	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	-0.02	-0.02

Panel B: Target volatility strategy based on *GRU-UNIVAR*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44	0.76	0.44
BENCHMARK	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13
GOLD	0.00	-0.02	-0.01	-0.04	-0.02	-0.06	-0.01	-0.04	-0.01	-0.01	-0.02	-0.04	-0.03	-0.06	-0.02	-0.04	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	-0.01	-0.02
SILVER	-0.04	-0.07	-0.10	-0.14	-0.16	-0.19	-0.10	-0.13	-0.05	-0.07	-0.10	-0.14	-0.16	-0.19	-0.10	-0.14	-0.01	-0.03	-0.03	-0.07	-0.05	-0.11	-0.03	-0.07	-0.08	-0.11
PLATINIUM	-0.05	-0.08	-0.11	-0.15	-0.17	-0.20	-0.11	-0.14	-0.03	-0.07	-0.07	-0.13	-0.11	-0.18	-0.07	-0.13	-0.02	-0.05	-0.04	-0.10	-0.07	-0.14	-0.05	-0.10	-0.08	-0.12
COPPER	-0.04	-0.06	-0.09	-0.11	-0.14	-0.15	-0.09	-0.11	-0.02	-0.05	-0.05	-0.09	-0.08	-0.12	-0.05	-0.09	-0.02	-0.04	-0.04	-0.08	-0.06	-0.11	-0.04	-0.08	-0.06	-0.09
OIL	-0.58	-0.39	-0.90	-0.48	-1.27	-0.62	-0.92	-0.50	-0.62	-0.40	-0.95	-0.50	-1.33	-0.63	-0.97	-0.51	-0.64	-0.41	-0.99	-0.51	-1.36	-0.64	-1.00	-0.52	-0.96	-0.51
COCOA	-0.03	-0.05	-0.07	-0.09	-0.11	-0.13	-0.07	-0.09	-0.03	-0.05	-0.06	-0.09	-0.09	-0.13	-0.06	-0.09	-0.01	-0.02	-0.02	-0.04	-0.03	-0.06	-0.02	-0.04	-0.05	-0.08
WHEAT	0.00	0.01	-0.01	0.00	-0.03	0.00	-0.01	0.00	0.00	0.01	-0.01	0.00	-0.02	-0.01	-0.01	0.00	0.00	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
SOYABEANS	0.02	0.00	0.03	-0.01	0.03	-0.02	0.02	-0.01	0.01	0.01	0.02	0.00	0.03	-0.01	0.02	0.00	0.00	0.01	0.00	0.00	-0.01	-0.01	0.00	0.00	0.01	0.00
CORN	-0.01	-0.03	-0.06	-0.06	-0.13	-0.09	-0.07	-0.06	-0.07	-0.07	-0.16	-0.13	-0.24	-0.18	-0.16	-0.13	-0.01	-0.03	-0.03	-0.07	-0.05	-0.10	-0.03	-0.07	-0.08	-0.08
USGOV10_PI	0.02	0.02	0.03	0.04	0.05	0.05	0.03	0.04	0.01	0.01	0.01	0.03	0.02	0.03	0.01	0.02	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.02	0.02
USGOV30_PI	0.04	0.05	0.07	0.09	0.10	0.13	0.07	0.09	0.01	0.04	0.02	0.06	0.03	0.07	0.02	0.05	0.02	0.02	0.03	0.03	0.04	0.04	0.03	0.03	0.04	0.06
SWISSGOV10_PI	0.00	-0.01	0.00	-0.03	-0.01	-0.04	0.00	-0.03	-0.01	-0.02	-0.02	-0.03	-0.03	-0.05	-0.02	-0.03	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.02
BUNDGOV10_PI	0.01	-0.01	0.01	-0.01	0.01	-0.02	0.01	-0.01	-0.01	-0.01	-0.02	-0.03	-0.03	-0.04	-0.02	-0.03	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.02

Panel C: Target volatility strategy based on *ARFIMA*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45	0.78	0.45
BENCHMARK	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15	-0.14	-0.15
GOLD	-0.01	-0.01	-0.03	-0.02	-0.05	-0.05	-0.03	-0.03	0.00	-0.01	0.00	-0.02	-0.01	-0.04	0.00	-0.03	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	-0.01	-0.01
SILVER	-0.01	-0.04	-0.04	-0.10	-0.09	-0.14	-0.05	-0.09	0.00	-0.04	-0.01	-0.10	-0.03	-0.14	-0.01	-0.09	-0.01	-0.02	-0.03	-0.06	-0.05	-0.10	-0.03	-0.06	-0.03	-0.08
PLATINIUM	-0.04	-0.06	-0.08	-0.13	-0.13	-0.18	-0.08	-0.12	-0.03	-0.07	-0.06	-0.13	-0.10	-0.18	-0.06	-0.12	-0.01	-0.04	-0.03	-0.09	-0.06	-0.13	-0.04	-0.09	-0.06	-0.11
COPPER	-0.02	-0.03	-0.04	-0.06	-0.07	-0.10	-0.04	-0.06	0.01	-0.02	0.01	-0.04	0.01	-0.07	0.01	-0.04	-0.01	-0.03	-0.03	-0.07	-0.05	-0.10	-0.03	-0.07	-0.02	-0.06
OIL	-0.53	-0.38	-0.87	-0.48	-1.25	-0.62	-0.89	-0.49	-0.51	-0.38	-0.85	-0.47	-1.23	-0.61	-0.86	-0.49	-0.60	-0.40	-0.96	-0.51	-1.33	-0.64	-0.96	-0.52	-0.90	-0.50
COCOA	-0.04	-0.04	-0.08	-0.08	-0.13	-0.12	-0.08	-0.08	0.00	-0.02	-0.01	-0.05	-0.02	-0.08	-0.01	-0.05	-0.01	-0.01	-0.02	-0.03	-0.02	-0.05	-0.02	-0.03	-0.04	-0.05
WHEAT	-0.01	0.00	-0.02	-0.01	-0.05	-0.02	-0.03	-0.01	0.02	0.01	0.04	0.02	0.04	0.03	0.03	0.02	0.01	0.01	0.02	0.01	0.02	0.02	0.02	0.01	0.01	0.01
SOYABEANS	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
CORN	-0.01	-0.01	-0.03	-0.04	-0.06	-0.07	-0.04	-0.04	-0.02	-0.02	-0.04	-0.07	-0.07	-0.10	-0.05	-0.06	-0.02	-0.02	-0.03	-0.06	-0.05	-0.09	-0.03	-0.05	-0.04	-0.05
USGOV10_PI	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00
USGOV30_PI	0.01	-0.01	0.01	-0.02	0.01	-0.03	0.01	-0.02	-0.02	-0.02	-0.04	-0.03	-0.06	-0.05	-0.04	-0.03	0.02	0.01	0.03	0.02	0.04	0.03	0.03	0.02	0.00	-0.01
SWISSGOV10_PI	-0.01	-0.01	-0.02	-0.02	-0.03	-0.04	-0.02	-0.02	-0.01	-0.01	-0.02	-0.01	-0.02	-0.03	-0.02	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01
BUNDGOV10_PI	-0.01	-0.01	-0.02	-0.02	-0.03	-0.03	-0.02	-0.02	-0.01	-0.01	-0.03	-0.02	-0.04	-0.04	-0.03	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	-0.01

Panel D: Target volatility strategy based on *GARCH*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45	0.77	0.45
BENCHMARK	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15	-0.13	-0.15
GOLD	-0.02	-0.02	-0.04	-0.05	-0.08	-0.07	-0.05	-0.04	-0.05	-0.05	-0.11	-0.11	-0.16	-0.15	-0.11	-0.10	0.01	0.00	0.01	-0.02	0.02	-0.03	0.01	-0.02	-0.05	-0.06
SILVER	-0.08	-0.11	-0.19	-0.19	-0.30	-0.25	-0.19	-0.18	-0.10	-0.13	-0.21	-0.22	-0.32	-0.28	-0.21	-0.21	-0.03	-0.09	-0.07	-0.16	-0.12	-0.22	-0.07	-0.16	-0.16	-0.18
PLATINIUM	-0.09	-0.14	-0.19	-0.23	-0.29	-0.29	-0.19	-0.22	-0.10	-0.14	-0.21	-0.24	-0.32	-0.30	-0.21	-0.23	-0.03	-0.09	-0.07	-0.16	-0.12	-0.21	-0.07	-0.16	-0.16	-0.20
COPPER	-0.06	-0.08	-0.12	-0.14	-0.20	-0.19	-0.12	-0.14	-0.03	-0.07	-0.08	-0.13	-0.13	-0.17	-0.08	-0.12	-0.02	-0.07	-0.05	-0.12	-0.07	-0.16	-0.05	-0.11	-0.08	-0.12
OIL	-0.71	-0.44	-0.99	-0.52	-1.32	-0.64	-1.01	-0.53	-0.68	-0.43	-0.96	-0.51	-1.31	-0.64	-0.98	-0.53	-0.76	-0.45	-1.04	-0.53	-1.38	-0.66	-1.06	-0.55	-1.02	-0.53
COCOA	-0.05	-0.08	-0.11	-0.14	-0.18	-0.19	-0.11	-0.14	-0.01	-0.05	-0.02	-0.10	-0.04	-0.14	-0.02	-0.10	-0.02	-0.05	-0.04	-0.09	-0.06	-0.14	-0.04	-0.09	-0.06	-0.11
WHEAT	0.05	0.05	0.07	0.07	0.06	0.09	0.06	0.07	0.01	0.01	0.00	0.00	-0.02	-0.02	0.00	0.00	0.01	0.03	0.02	0.03	0.02	0.03	0.02	0.03	0.02	0.03
SOYABEANS	0.02	0.01	0.02	-0.01	0.01	-0.02	0.02	-0.01	-0.01	0.00	-0.02	-0.02	-0.03	-0.04	-0.02	-0.02	0.00	0.00	-0.01	-0.02	-0.02	-0.04	-0.01	-0.02	0.00	-0.01
CORN	-0.03	-0.05	-0.08	-0.10	-0.14	-0.14	-0.08	-0.10	-0.04	-0.06	-0.09	-0.12	-0.14	-0.17	-0.09	-0.12	-0.02	-0.06	-0.04	-0.11	-0.07	-0.15	-0.05	-0.11	-0.07	-0.11
USGOV10_PI	0.01	0.02	0.02	0.04	0.03	0.04	0.02	0.03	0.00	0.01	0.01	0.00	0.01	-0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.02	-0.01	0.01	0.00	0.01	0.01
USGOV30_PI	0.02	0.03	0.02	0.03	0.01	0.03	0.01	0.03	0.00	-0.01	-0.01	-0.04	-0.02	-0.07	-0.01	-0.04	0.02	0.02	0.03	0.00	0.04	-0.01	0.03	0.00	0.01	0.00
SWISSGOV10_PI	-0.01	-0.02	-0.02	-0.04	-0.03	-0.06	-0.02	-0.04	-0.01	-0.03	-0.02	-0.05	-0.03	-0.07	-0.02	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.05	-0.01	-0.04	-0.02	-0.04
BUNDGOV10_PI	0.00	-0.01	0.00	-0.02	0.00	-0.03	0.00	-0.02	-0.01	-0.02	-0.02	-0.04	-0.03	-0.06	-0.02	-0.04	-0.01	-0.02	-0.01	-0.03	-0.02	-0.05	-0.01	-0.03	-0.01	-0.03

Panel E: Target volatility strategy based on *GJR-GARCH*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51	0.79	0.51
BENCHMARK	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21	-0.15	-0.21
GOLD	-0.03	-0.04	-0.07	-0.08	-0.12	-0.12	-0.07	-0.08	-0.02	-0.04	-0.05	-0.08	-0.09	-0.12	-0.06	-0.08	0.00	-0.02	-0.01	-0.05	-0.02	-0.07	-0.01	-0.05	-0.05	-0.07
SILVER	-0.09	-0.14	-0.20	-0.24	-0.32	-0.31	-0.20	-0.23	-0.07	-0.14	-0.16	-0.24	-0.26	-0.30	-0.16	-0.22	-0.04	-0.13	-0.10	-0.22	-0.16	-0.28	-0.10	-0.21	-0.16	-0.22
PLATINIUM	-0.06	-0.15	-0.14	-0.24	-0.23	-0.31	-0.14	-0.23	-0.06	-0.16	-0.14	-0.26	-0.23	-0.32	-0.15	-0.24	-0.03	-0.12	-0.07	-0.20	-0.12	-0.26	-0.07	-0.19	-0.12	-0.22
COPPER	-0.03	-0.09	-0.08	-0.15	-0.13	-0.20	-0.08	-0.15	-0.03	-0.10	-0.08	-0.16	-0.12	-0.21	-0.08	-0.16	-0.02	-0.09	-0.04	-0.15	-0.07	-0.20	-0.04	-0.14	-0.07	-0.15
OIL	-0.73	-0.50	-1.01	-0.58	-1.35	-0.70	-1.03	-0.59	-0.72	-0.50	-1.00	-0.57	-1.34	-0.70	-1.02	-0.59	-0.09	-0.22	-0.23	-0.32	-0.36	-0.37	-0.23	-0.30	-0.76	-0.49
COCOA	-0.01	-0.08	-0.04	-0.15	-0.08	-0.20	-0.05	-0.14	0.02	-0.06	0.02	-0.12	0.02	-0.16	0.02	-0.11	-0.02	-0.06	-0.04	-0.12	-0.06	-0.17	-0.04	-0.11	-0.02	-0.12
WHEAT	0.10	0.09	0.16	0.13	0.18	0.17	0.15	0.13	0.04	0.03	0.06	0.03	0.06	0.04	0.05	0.03	-0.01	0.01	-0.01	-0.01	-0.02	-0.03	-0.01	-0.01	0.06	0.05
SOYABEANS	0.03	0.00	0.04	-0.01	0.04	-0.03	0.03	-0.01	0.00	-0.01	-0.01	-0.04	-0.03	-0.07	-0.02	-0.04	-0.01	-0.01	-0.02	-0.04	-0.04	-0.07	-0.02	-0.04	0.00	-0.03
CORN	-0.09	-0.10	-0.20	-0.19	-0.32	-0.25	-0.20	-0.18	-0.01	-0.06	-0.03	-0.12	-0.07	-0.16	-0.04	-0.11	-0.03	-0.07	-0.06	-0.13	-0.09	-0.19	-0.06	-0.13	-0.10	-0.14
USGOV10_PI	0.01	0.04	0.03	0.04	0.03	0.04	0.02	0.04	0.00	0.02	0.00	-0.01	-0.01	-0.04	0.00	-0.01	0.00	0.00	0.00	-0.02	0.00	-0.04	0.00	-0.02	0.01	0.00
USGOV30_PI	0.02	0.05	0.01	0.05	0.00	0.04	0.01	0.04	-0.01	-0.02	-0.03	-0.07	-0.06	-0.12	-0.03	-0.07	0.00	-0.01	0.00	-0.04	-0.01	-0.08	0.00	-0.04	-0.01	-0.02
SWISSGOV10_PI	0.00	-0.02	-0.01	-0.05	-0.01	-0.07	-0.01	-0.05	-0.02	-0.03	-0.03	-0.06	-0.05	-0.09	-0.03	-0.06	-0.01	-0.03	-0.02	-0.06	-0.03	-0.08	-0.02	-0.06	-0.02	-0.05
BUNDGOV10_PI	-0.01	-0.02	-0.01	-0.03	-0.02	-0.05	-0.01	-0.03	-0.02	-0.03	-0.05	-0.06	-0.07	-0.11	-0.05	-0.07	-0.01	-0.03	-0.02	-0.05	-0.03	-0.08	-0.02	-0.05	-0.03	-0.05

Table B.5. Safe haven assets in target volatility strategy based on *STOXX50TR* Index

This table presents the Sharpe (SR) and Calmar ratios (CR) of target volatility strategies. The results cover the period from January 1, 2010 to December 31, 2020. The table is divided into five panels: panel A shows results of target volatility strategy, based on predictions from *GRU-MULTIVAR*; panel B shows results based on *GRU-UNIVAR*; panel C on *ARFIMA*; panel D on *GARCH*; and panel E on *GJR-GARCH*. In each panel, the two first rows represent the target volatility strategy's results without any safe haven asset (STRAT_CLEAN) and the STOXX EUROPE 50 - TOT RETURN IND (BENCHMARK). The remaining rows states for results of strategies that include additional assets. A full description of assets is presented in Table A1 in Online Appendix. For better transparency, only the first raw in each panel (STRAT_CLEAN) shows nominal values of both SR and CR. All other rows present the difference between STRAT_CLEAN and the modified strategy results (or the benchmark). The strategies that are presented in the table are calculated with three different quantile thresholds: 90%, 95%, and 99%. Each quantile threshold represents the required predicted volatility to include the tested safe haven asset in the strategy. Furthermore, we demonstrate three different maximum exposures on safe haven weight allocation (cap 10%, cap 20%, and cap 30%) and the mean of these three ratios (mean_cap). Finally, the last two columns aggregate the different strategies and present the average result (mean) from all configurations.

Panel A: Target volatility strategy based on *GRU-MULTIVAR*

	quantile 90%								**quantile 95%**								**quantile 99%**								**mean**	
	cap 10%		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**		**cap 10%**		**cap 20%**		**cap 30%**		**mean_cap**			
	SR	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**	**SR**	**CR**
STRAT_CLEAN	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14	0.28	0.14
BENCHMARK	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.04
GOLD	0.02	0.01	0.04	0.02	0.05	0.03	0.03	0.02	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
SILVER	0.00	0.00	-0.02	-0.02	-0.04	-0.04	-0.02	-0.02	-0.03	-0.01	-0.07	-0.04	-0.10	-0.07	-0.07	-0.04	-0.04	-0.02	-0.08	-0.05	-0.12	-0.08	-0.08	-0.05	-0.05	-0.03
PLATINIUM	-0.04	-0.02	-0.09	-0.05	-0.13	-0.08	-0.09	-0.05	-0.05	-0.02	-0.10	-0.06	-0.15	-0.09	-0.10	-0.06	-0.01	0.00	-0.03	-0.02	-0.04	-0.05	-0.03	-0.02	-0.07	-0.04
COPPER	0.00	0.01	0.00	0.02	-0.01	0.00	0.00	0.01	-0.02	0.00	-0.04	0.00	-0.06	-0.03	-0.04	-0.01	-0.01	0.00	-0.03	-0.01	-0.05	-0.03	-0.03	-0.02	-0.02	-0.01
OIL	-0.35	-0.16	-0.56	-0.23	-0.88	-0.33	-0.60	-0.24	-0.05	-0.03	-0.11	-0.08	-0.17	-0.10	-0.11	-0.07	-0.03	-0.01	-0.07	-0.05	-0.11	-0.08	-0.07	-0.05	-0.26	-0.12
COCOA	-0.07	-0.04	-0.15	-0.08	-0.22	-0.12	-0.15	-0.08	-0.06	-0.02	-0.11	-0.06	-0.17	-0.09	-0.11	-0.06	-0.02	-0.01	-0.04	-0.02	-0.06	-0.03	-0.04	-0.02	-0.10	-0.05
WHEAT	0.08	0.05	0.13	0.09	0.18	0.14	0.13	0.09	-0.01	-0.02	-0.03	-0.03	-0.06	-0.04	-0.04	-0.03	0.02	0.01	0.04	0.02	0.06	0.03	0.04	0.02	0.04	0.03
SOYABEANS	-0.01	0.00	-0.03	-0.01	-0.05	-0.01	-0.03	-0.01	-0.02	-0.01	-0.05	-0.02	-0.08	-0.03	-0.05	-0.02	0.01	0.00	0.02	0.01	0.02	0.01	0.02	0.01	-0.02	-0.01
CORN	0.00	0.01	0.00	0.00	-0.01	-0.02	0.00	0.00	-0.07	-0.04	-0.13	-0.07	-0.20	-0.11	-0.13	-0.07	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.05	-0.03
USGOV10_PI	0.01	0.01	0.02	0.01	0.03	0.02	0.02	0.01	0.01	0.01	0.02	0.01	0.03	0.02	0.02	0.01	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.01	0.01
USGOV30_PI	0.03	0.01	0.06	0.02	0.08	0.03	0.05	0.02	0.02	0.01	0.05	0.02	0.06	0.03	0.04	0.02	-0.02	-0.01	-0.03	-0.02	-0.05	-0.03	-0.03	-0.02	0.02	0.01
SWISSGOV10_PI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	-0.01	0.00
BUNDGOV10_PI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.02	-0.01	-0.02	-0.02	-0.02	-0.01	-0.01	0.00

Panel B: Target volatility strategy based on *GRU-UNIVAR*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14	0.25	0.14
BENCHMARK	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04
GOLD	0.03	0.02	0.05	0.04	0.07	0.05	0.05	0.04	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.01	0.01
SILVER	0.00	-0.01	-0.02	-0.03	-0.04	-0.04	-0.02	-0.03	-0.03	-0.03	-0.07	-0.06	-0.11	-0.08	-0.07	-0.05	-0.02	-0.02	-0.04	-0.05	-0.07	-0.07	-0.04	-0.05	-0.04	-0.04
PLATINIUM	-0.02	-0.02	-0.04	-0.04	-0.06	-0.06	-0.04	-0.04	-0.02	-0.02	-0.04	-0.04	-0.07	-0.06	-0.05	-0.04	-0.02	-0.02	-0.04	-0.05	-0.06	-0.07	-0.04	-0.04	-0.04	-0.04
COPPER	-0.02	0.00	-0.04	-0.02	-0.06	-0.04	-0.04	-0.02	-0.04	-0.02	-0.09	-0.05	-0.13	-0.08	-0.09	-0.05	-0.02	-0.01	-0.04	-0.03	-0.07	-0.05	-0.04	-0.03	-0.06	-0.04
OIL	-0.42	-0.18	-0.62	-0.24	-0.91	-0.36	-0.65	-0.26	-0.04	-0.04	-0.09	-0.08	-0.14	-0.10	-0.09	-0.07	-0.03	-0.04	-0.07	-0.07	-0.11	-0.09	-0.07	-0.07	-0.27	-0.13
COCOA	-0.05	-0.03	-0.11	-0.07	-0.16	-0.10	-0.11	-0.06	-0.04	-0.02	-0.09	-0.05	-0.13	-0.08	-0.09	-0.05	-0.01	-0.01	-0.03	-0.02	-0.04	-0.03	-0.03	-0.02	-0.07	-0.04
WHEAT	-0.01	-0.01	-0.02	-0.02	-0.04	-0.02	-0.02	-0.02	-0.01	-0.01	-0.03	-0.02	-0.05	-0.02	-0.03	-0.02	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.02	-0.01
SOYABEANS	-0.04	-0.03	-0.08	-0.05	-0.12	-0.07	-0.08	-0.05	-0.02	-0.01	-0.05	-0.02	-0.08	-0.04	-0.05	-0.03	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.05	-0.03
CORN	-0.01	-0.01	-0.02	-0.02	-0.04	-0.03	-0.02	-0.02	-0.05	-0.03	-0.11	-0.06	-0.16	-0.10	-0.11	-0.06	-0.03	-0.02	-0.06	-0.04	-0.09	-0.06	-0.06	-0.04	-0.06	-0.04
USGOV10_PI	0.02	0.01	0.04	0.02	0.06	0.04	0.04	0.02	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.02	0.01
USGOV30_PI	0.05	0.03	0.10	0.06	0.14	0.08	0.09	0.06	0.02	0.00	0.03	0.01	0.04	0.01	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.02
SWISSGOV10_PI	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
BUNDGOV10_PI	0.01	0.01	0.03	0.02	0.04	0.03	0.03	0.02	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00

Panel C: Target volatility strategy based on *ARFIMA*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12	0.25	0.12
BENCHMARK	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06	0.09	0.06
GOLD	0.03	0.02	0.05	0.03	0.07	0.05	0.05	0.03	0.02	0.01	0.04	0.02	0.05	0.03	0.04	0.02	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.03	0.02
SILVER	0.02	0.02	0.03	0.01	0.03	0.00	0.03	0.01	0.01	0.01	0.01	0.00	0.00	-0.01	0.01	0.00	-0.02	-0.01	-0.04	-0.03	-0.07	-0.05	-0.05	-0.03	0.00	-0.01
PLATINIUM	-0.02	0.00	-0.04	-0.03	-0.07	-0.05	-0.05	-0.03	-0.02	-0.01	-0.05	-0.03	-0.08	-0.05	-0.05	-0.03	-0.02	-0.01	-0.03	-0.03	-0.05	-0.05	-0.03	-0.03	-0.04	-0.03
COPPER	0.00	0.01	-0.01	0.01	-0.03	-0.01	-0.02	0.00	0.00	0.00	-0.01	0.01	-0.01	-0.01	-0.01	0.00	-0.01	0.00	-0.02	0.00	-0.02	-0.01	-0.02	0.00	-0.01	0.00
OIL	-0.40	-0.16	-0.60	-0.22	-0.89	-0.33	-0.63	-0.24	-0.37	-0.16	-0.57	-0.22	-0.89	-0.32	-0.61	-0.23	-0.02	-0.01	-0.06	-0.04	-0.09	-0.07	-0.06	-0.04	-0.43	-0.17
COCOA	-0.10	-0.05	-0.19	-0.09	-0.29	-0.14	-0.19	-0.09	-0.05	-0.02	-0.10	-0.05	-0.14	-0.07	-0.10	-0.05	-0.03	-0.01	-0.05	-0.02	-0.08	-0.04	-0.05	-0.02	-0.11	-0.05
WHEAT	0.05	0.03	0.09	0.05	0.11	0.08	0.08	0.05	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.02	0.01	0.05	0.02	0.07	0.03	0.05	0.02	0.04	0.02
SOYABEANS	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01	-0.02	0.00	-0.03	-0.01	-0.05	-0.02	-0.03	-0.01	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	-0.01	0.00
CORN	-0.03	-0.01	-0.07	-0.04	-0.11	-0.07	-0.07	-0.04	-0.04	-0.02	-0.09	-0.04	-0.13	-0.06	-0.09	-0.04	-0.01	-0.01	-0.02	-0.01	-0.04	-0.02	-0.02	-0.01	-0.06	-0.03
USGOV10_PI	0.02	0.01	0.04	0.01	0.05	0.02	0.04	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
USGOV30_PI	0.04	0.02	0.08	0.03	0.11	0.04	0.08	0.03	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	0.02	0.01
SWISSGOV10_PI	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	-0.01	0.00
BUNDGOV10_PI	0.01	0.00	0.01	0.00	0.02	0.01	0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	-0.01	-0.01	-0.02	-0.01	-0.03	-0.02	-0.02	-0.01	-0.01	-0.01

Panel D: Target volatility strategy based on *GARCH*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17	0.32	0.17
BENCHMARK	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00	0.02	0.00
GOLD	0.02	0.01	0.03	0.01	0.04	0.02	0.03	0.01	0.00	0.00	-0.01	-0.02	-0.02	-0.04	-0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
SILVER	0.01	-0.01	0.00	-0.04	-0.02	-0.06	0.00	-0.03	-0.03	-0.03	-0.08	-0.07	-0.12	-0.09	-0.08	-0.06	-0.01	-0.02	-0.03	-0.06	-0.05	-0.08	-0.03	-0.05	-0.04	-0.05
PLATINIUM	-0.03	-0.03	-0.06	-0.07	-0.10	-0.10	-0.06	-0.06	-0.04	-0.03	-0.08	-0.08	-0.12	-0.11	-0.08	-0.07	-0.03	-0.02	-0.06	-0.07	-0.09	-0.10	-0.06	-0.06	-0.07	-0.07
COPPER	-0.02	-0.02	-0.04	-0.05	-0.07	-0.07	-0.04	-0.05	-0.03	-0.02	-0.07	-0.05	-0.10	-0.08	-0.07	-0.05	-0.02	-0.01	-0.04	-0.03	-0.06	-0.05	-0.04	-0.03	-0.05	-0.04
OIL	-0.43	-0.20	-0.62	-0.26	-0.92	-0.37	-0.65	-0.28	-0.51	-0.22	-0.70	-0.28	-1.00	-0.40	-0.74	-0.30	-0.03	-0.04	-0.08	-0.08	-0.14	-0.11	-0.08	-0.08	-0.49	-0.22
COCOA	-0.07	-0.04	-0.15	-0.10	-0.22	-0.14	-0.15	-0.09	-0.06	-0.03	-0.12	-0.08	-0.17	-0.11	-0.11	-0.07	-0.02	-0.01	-0.04	-0.02	-0.06	-0.05	-0.04	-0.03	-0.10	-0.06
WHEAT	-0.04	-0.03	-0.09	-0.05	-0.14	-0.08	-0.09	-0.05	-0.01	-0.02	-0.03	-0.04	-0.06	-0.05	-0.04	-0.04	0.01	0.00	0.01	0.00	0.02	0.00	0.01	0.00	-0.04	-0.03
SOYABEANS	-0.03	-0.01	-0.05	-0.05	-0.09	-0.07	-0.06	-0.04	-0.03	-0.02	-0.07	-0.05	-0.11	-0.07	-0.07	-0.05	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	-0.04	-0.03
CORN	-0.02	-0.01	-0.05	-0.04	-0.08	-0.07	-0.05	-0.04	-0.07	-0.05	-0.13	-0.09	-0.20	-0.12	-0.13	-0.08	-0.03	-0.02	-0.06	-0.05	-0.10	-0.08	-0.06	-0.05	-0.08	-0.06
USGOV10_PI	0.02	0.01	0.05	0.01	0.07	0.02	0.04	0.01	0.01	0.01	0.03	0.01	0.04	0.02	0.03	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.02	0.01
USGOV30_PI	0.06	0.02	0.10	0.04	0.14	0.05	0.10	0.04	0.04	0.02	0.07	0.03	0.09	0.04	0.06	0.03	0.00	0.00	-0.01	-0.01	-0.01	-0.03	-0.01	-0.01	0.05	0.02
SWISSGOV10_PI	0.01	0.01	0.02	0.01	0.03	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.02	-0.01	-0.03	-0.01	-0.02	-0.01	0.00	0.00
BUNDGOV10_PI	0.01	0.01	0.03	0.01	0.04	0.02	0.03	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	-0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	0.01	0.00

Panel E: Target volatility strategy based on *GJR-GARCH*

	quantile 90%								quantile 95%								quantile 99%								mean	
	cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap		cap 10%		cap 20%		cap 30%		mean_cap			
	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR	SR	CR
STRAT_CLEAN	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15
BENCHMARK	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
GOLD	0.02	0.01	0.03	0.03	0.04	0.04	0.03	0.03	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.01
SILVER	-0.01	0.02	-0.04	-0.03	-0.07	-0.05	-0.04	-0.02	0.00	0.03	-0.01	0.00	-0.02	-0.02	-0.01	0.00	0.00	0.00	-0.01	-0.01	-0.02	-0.04	-0.01	-0.01	-0.02	-0.01
PLATINIUM	-0.08	-0.03	-0.17	-0.08	-0.24	-0.12	-0.16	-0.08	-0.05	-0.01	-0.10	-0.05	-0.15	-0.08	-0.10	-0.05	-0.02	-0.01	-0.04	-0.02	-0.06	-0.05	-0.04	-0.03	-0.10	-0.05
COPPER	-0.04	0.00	-0.08	-0.04	-0.12	-0.07	-0.08	-0.04	-0.03	0.01	-0.06	-0.02	-0.09	-0.05	-0.06	-0.02	-0.01	0.00	-0.02	0.00	-0.03	0.00	-0.02	0.00	-0.05	-0.02
OIL	-0.45	-0.18	-0.64	-0.24	-0.93	-0.35	-0.67	-0.26	-0.45	-0.18	-0.65	-0.24	-0.94	-0.35	-0.68	-0.26	-0.01	0.01	-0.05	-0.04	-0.09	-0.07	-0.05	-0.03	-0.47	-0.18
COCOA	-0.05	-0.02	-0.11	-0.05	-0.17	-0.09	-0.11	-0.05	-0.03	-0.01	-0.05	-0.01	-0.08	-0.03	-0.06	-0.02	-0.02	-0.01	-0.04	-0.02	-0.06	-0.03	-0.04	-0.02	-0.07	-0.03
WHEAT	0.00	-0.01	-0.01	-0.01	-0.03	-0.02	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.05	-0.03	-0.03	-0.02	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	-0.01	-0.01
SOYABEANS	-0.03	-0.02	-0.07	-0.04	-0.11	-0.05	-0.07	-0.04	-0.02	-0.01	-0.05	-0.01	-0.08	-0.03	-0.05	-0.02	0.01	0.01	0.03	0.03	0.04	0.05	0.03	0.03	-0.03	-0.01
CORN	-0.06	-0.02	-0.13	-0.06	-0.19	-0.10	-0.13	-0.06	-0.06	-0.03	-0.12	-0.05	-0.17	-0.08	-0.12	-0.06	-0.02	-0.02	-0.05	-0.03	-0.08	-0.05	-0.05	-0.03	-0.10	-0.05
USGOV10_PI	0.02	0.01	0.05	0.02	0.07	0.03	0.05	0.02	0.01	0.00	0.02	0.01	0.03	0.01	0.02	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.02	0.01
USGOV30_PI	0.05	0.02	0.09	0.03	0.12	0.05	0.09	0.03	0.03	0.01	0.05	0.02	0.07	0.03	0.05	0.02	-0.01	0.00	-0.03	-0.01	-0.04	-0.01	-0.03	-0.01	0.04	0.02
SWISSGOV10_PI	0.01	0.00	0.02	0.01	0.02	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.02	-0.01	-0.03	-0.01	-0.02	-0.01	0.00	0.00
BUNDGOV10_PI	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	0.00	0.00

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