4×4 Asset Allocation

Kenneth Cron
Private Investor

Maxim Golts[1]
4x4invest

We propose the following structured approach to asset allocation: all assets and liabilities in any portfolio should be thought of as means contributing to the following four ends:

- *Liquidity* maintenance: nominally safe and quickly accessible "cash-like" pool of assets,
- *Income* generation: relatively regular, certain and near-term cash payments,
- *Preservation* of (real) capital: assets expected to retain their value over time,
- *Growth*: more volatile assets and strategies expected to generate future cash payments.

We believe that all 4 areas should be "powered," giving our approach its 4×4 name. Further, we suggest that investors should start their asset allocation process by explicitly setting a strategic investment horizon over which they seek to achieve their goals, and building strategic 4×4 portfolios. Investment portfolios should then be rebalanced with some regular tactical frequency in order to re-align with the strategic investment horizon goals, while also managing tactical risk, return, and cash flows.

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[1] Golts: maxgolts@gmail.com. We appreciate the comments of Greg Jones, Mark Kritzman, Anton Malkin, Michael Roitman, and Seth Weingram.

Introduction: Focus on Four Investment Goals Over an Investment Horizon

Risk and reward of investing is often defined in terms of the nominal dollar value of the portfolio: dollar gains, dollar losses, dollar volatility and dollar value at risk, etc. However, the actual goals of individual and institutional investors are only indirectly related to the nominal dollar value and volatility of their portfolios. Investors may be better off explicitly focusing on their investment *goals* over an investment *horizon*, and manage their portfolios accordingly.

We suggest here that the assets and liabilities in any portfolio should be thought of as means contributing to (or detracting from) the following four ends:

- *Liquidity* maintenance with a pool of relatively safe and quickly accessible assets, providing a cushion in a crisis and dry powder for investment opportunities.
- *Income* generation helped by assets producing relatively regular, relatively certain, and relatively near-term cash payments.
- *Preservation* of real capital aided by relatively illiquid assets locked up for a longer period and expected to retain their real value over time.
- *Growth* of capital via potentially more volatile assets and alpha strategies expected to generate future cash flows, if allowed sufficient time to germinate and perform.

The four goals are schematically depicted in the **Figure 1** diagram. We believe that all four areas should be "powered" – which gives our asset allocation approach its 4×4 name – and that the natural diversification arising from paying attention to all four goals will allow the portfolio to perform in all four "terrains" or market environments, which can be stylistically described as recession, recovery, mid-cycle and late cycle.

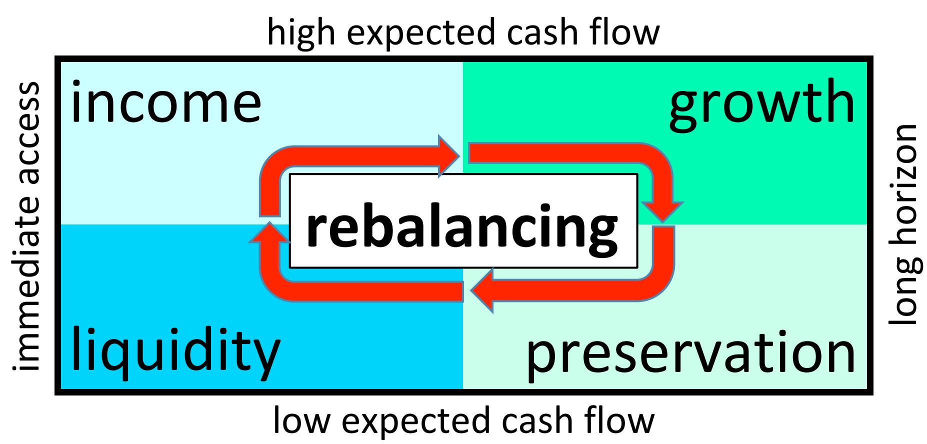

Figure 1. Four investment goals and some time horizon and cash flow characteristics of the investments likely associated with these goals.

We suggest that investors should start their asset allocation process by explicitly setting a *strategic investment horizon T* over which they would seek to achieve their goals: for example, one, three, or ten years, or even "forever" which we may proxy by thirty years. We believe that the relevant strategic performance and risk measures should be *investor-specific:* the rate of success of achieving the liquidity, income, preservation and growth goals, and the risk of not achieving these goals over the chosen investment horizon. Hence, investors should build *strategic portfolios* expected to be most likely to achieve the investment goals. No asset would be *a priori* "risk-free" in this paradigm: for example, too much cash in the strategic portfolio can increase the risk of not achieving the growth goal, and too much volatile equities can increase the risk of missing the preservation goal. We suggest that, *ceteris paribus*, strategic liquidity, preservation, income and growth allocations (of the surplus capital left after expected liability coverage) should all be roughly equal to each other. Should the investor have views, for example on the long-term economic environment, the strategic portfolios can be actively tilted accordingly.

Next step we would suggest in the asset allocation process is *tactical rebalancing*: a portfolio should be reviewed and rebalanced, if needed, with some regular frequency τ: for example, daily, weekly, monthly or quarterly. Each tactical rebalancing should seek to re-align the portfolio with the strategic investment horizon goals, while also managing tactical risk and, if the investor has any active tactical views, maximizing return (after transaction costs and, if relevant, taxes) on the rebalancing horizon.

We will address the details of asset allocation in the body of the paper, but we would like to mention explicitly in the introduction one, in our experience often neglected, asset: cash. Cash is a very important contributor to the liquidity goal, and "cash flow" is often understood to be synonymous with income, portfolios should be explicitly optimized with cash as an asset. We suggest that the tactical rebalancing portfolio construction should have explicit constraints on cash, from below, in order to ensure that the asset allocation process generates a regular desired cash flow, optimized on a tactical frequency, and contributing to the income and liquidity goals.

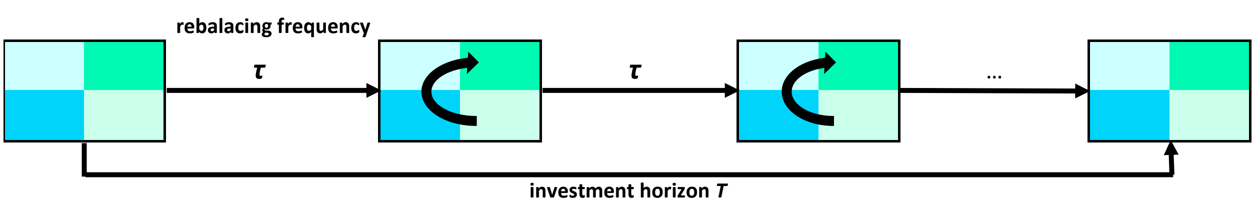

Figure 2. Strategic investment horizon T (for example, one, three, or ten years, or thirty years), and tactical rebalancing frequency τ (for example, daily, weekly, monthly or quarterly). We suggest that investors should first build strategic portfolios, and then rebalance allocations on a tactical frequency, re-aligning portfolios with the strategic goals and managing tactical risk and return (after transaction costs and, if relevant, taxes). We suggest that the tactical rebalancing portfolio construction should have explicit constraints on cash, from below, in order to ensure that the asset allocation process generates a regular desired cash flow.

Related Literature and Approaches

The study of investment preferences and goals has a long and storied history. Tobin [1958] investigated risk aversion and liquidity preferences. Bridgewater Associates have for a long time been thinking about inflation and real growth environments and implementing "All Weather" strategies inspiring the risk parity approach to asset allocation. Other well-known asset allocation approaches include equal weighting, minimum variance, mean-variance optimization, 60/40 equity/bond portfolio. For a comprehensive review of Modern Portfolio Theory, see Elton et al. [2014]. Chaves, Hsu, Li, and Shakernia [2011] conduct a horse race between a few of the popular strategies. Kinlaw, Kritzman and Turkington [2013] incorporate liquidity as a shadow asset into portfolio construction. Hayes, Primbs and Chiquoine [2015] modify the familiar mean–variance optimization framework by incorporating an illiquidity-related marginal penalty function that varies with each investor's liquidity needs. The goal-based investment approach has been explored by academics and practitioners alike and reflected in the growing goal-based wealth management literature. Merton [2014] argues for a liability-driven investment strategy whose aim is to improve the probability of achieving a desired retirement income rather than to maximize the capital value of the savings. Merton and Muralidhar [2020] provide an extensive literature review, and discuss income, investment preferences, and goal-based approaches in the context of a looming retirement crisis.

Asset Characteristics and Parameters

Our framework requires estimating expected (net) present values (abbreviated here as EPV) for all assets and liabilities. Estimation of future asset return distributions in general, and EPVs in particular, could present serious challenges and requires certain views about the asset returns, economic and regulatory environment, and career prospects (for individual investors). Such challenges are present in any asset allocation methodology and are usually met with investment research, modeling, and sophisticated computer simulations. We believe however that even simple "back of the envelope" EPVs with very conservative assumptions may lead to useful asset allocation insights in our framework.

We think of every asset and liability as a stochastic stream of future cash flows[2] C_t. Any final disposal or maturity of the asset/liability is treated as the last cash flow. Generally speaking,

[2] Cash flows could be positive (asset) or negative (liability) or mixed (asset/liability package). Hereinafter we will refer to an asset or liability simply as asset, so the EPV can be either positive or negative dollar amount. We treat an asset/liability package (e.g., mortgaged rental property) as 2 asset entities: the asset side and the liability side.

expected present value is computed as the expected dollar value of discounted[3] stream of our cash flows C_t.

$$EPV = E\left(\sum_t \frac{C_t}{(1+r_t)^t}\right)$$

The cash flows C_t can be considered in nominal terms or real terms, after expected inflation.

We will divide the parameters used in our asset allocation framework into three groups below. While we think that time-varying parameters in each of these groups can and do interact and impact each other, we believe that it is helpful to indicate the primary origin from which a parameter is arising.

Market parameters:

- r_0: fully liquid nominally safe asset return rate. This is the so-called "risk free" or "reference" rate usually set by the central bank, at least in the developed non-crisis economies.

Asset-specific parameters:

- α: expected return.
- σ: expected volatility.
- EM: expected weighted average maturity of the cash flow stream C_t. This concept is related to bond duration, but since we deal with expected values, it is not necessarily an indicator of the sensitivity to shifts of the current interest rate curve.

$$EM = E\left(\sum_t \frac{C_t}{(1+r_t)^t} t\right) / EPV$$

Investor-specific parameters:

- T: strategic investment horizon T over which investors seek to achieve their goals: for example, three, five, or ten years, or even "forever" which we may proxy by thirty years.
- τ: tactical rebalancing/trading frequency. This frequency could range from fractions of a second (ultra high frequency trading) to a day, a month, or a quarter.
- η: risk aversion parameter in the power utility function

$$u(c) = \frac{c^{(1-\eta)} - 1}{1-\eta},$$

[3] We take into account the probability distribution of future interest rates rather than the current interest rate curve. The general discounting notation is meant to indicate compounding appropriate for the nature and frequency of the expected cash flows.

where c is capital (usually assumed to be related to consumption). The higher the parameter η, the less capital it takes to lose the same amount of utility, see Exhibit [Risk Aversion] for illustrative examples.

4×4 Asset Decomposition

We treat all assets and liabilities as tools which help or hinder the investor to achieve their liquidity, preservation, income and growth goals. In this section we suggest a methodology to estimate an asset/liability impact on–or contribution to–the four goals.

Return *of* capital and return *on* capital

For every asset/liability C_t we will distinguish here between "return *of* capital" cash flows F_t (final sale/disposal/maturity of the asset) and the "return *on* capital" cash flow N_t (coupons, dividends, real estate rent, futures "roll return," FX "carry," royalties, systematic tax-managed sales of appreciated assets, labor-related income, etc.). While this distinction can arguably be somewhat artificial and ambiguous, we think that the implications for liquidity, transaction costs, taxes, accounting and ultimately re-allocation decisions are important enough to warrant separate consideration of "return *of* capital" and "return *on* capital" flow returns. Since we intend to (have to) hold this asset (liability) on our books for T years, we have:

$$EPV(C_t, T) = EPV(F_t, T) + EPV(N_t, T) = E\left(\sum_{t \le T} \frac{F_t}{(1 + r_t)^t}\right) + E\left(\sum_{t \le T} \frac{N_t}{(1 + r_t)^t}\right).$$

We will estimate separately the volatility of the "return *of* capital" cash flows F_t which we denote σ_F, the volatility of the "return *on* capital" cash flows N_t which we denote σ_N.

Estimating EPVs and expected returns

We will consider separately the (annualized) expected returns α_F arising from "return *of* capital" cash flows and (annualized) expected returns α_N arising from "return *on* capital" cash flows. These expected returns could be considered in nominal terms or real terms, after (model-estimated) expected inflation. The forecasting of such expected returns is the subject of much of the finance literature and practice, and is beyond the scope of this paper.

The EPVs, initial capital invested W into the asset C_t, and expected returns are related as follows (T is measured in years):

$$EPVF/W = EPV(F_t, T)/W = 1 + \alpha_F T, \quad \text{and} \quad EPVN/W = EPV(N_t, T)/W = \alpha_N T.$$

Note that we are using the *arithmetic* expected return setup here, which we believe is more appropriate for dynamic asset allocation problems, however there are valid arguments for using *geometric* expected returns as well.

Return of capital cash flows: liquidity and preservation

We will separate the "return of capital" cash flows F_t into liquidity and preservation parts. Heuristically, *liquidity* is quickly and easily accessible, less volatile part of the cash flows, while *preservation*—in particular, inflation protection—is helped by potentially more volatile investments, expected to retain their real value if held for a longer period of time. We use option pricing theory to quantify this intuition.

More specifically, we define the *preservation* part $EPVP$ of $EPVF = EPV(F_t, T)$ as

$$EPVP = \Pi_{\text{liquidity option}}(\tau, T, \sigma_F, \$1)EPVF,$$

where $\Pi_{\text{liquidity option}}$ is the price of the liquidity option as defined in Golts-Kritzman [2010, formula 5], τ, T, σ are the parameters discussed above, and the option is written on the $EPVF$ notional amount. We consider the remaining part $EPVL$ of $EPVF$ as *liquidity* contribution:

$$EPVL = EPVF - EPVP.$$

Intuitively, by buying the asset F_t we are locking up liquid capital in a volatile asset for a period of time *T,* hoping to achieve some capital gain. This "lock-up" gain should arise from the illiquidity discount built into the initial purchase price, and thus the gain is proxied by the price of the liquidity option. The longer the holding period T is relative to the rebalancing frequency τ, and the higher the volatility σ, the less liquid our asset is, and thus we are holding this asset for capital preservation. Conversely, for cash $\tau = T$ and $\sigma = 0$ (or very close to zero), and the price of the liquidity option is zero. Thus cash is 100% liquid, and money market instruments, such as discounted Treasury bills, are nearly 100% liquid.

Return on capital cash flows: income and growth

Similarly, we will separate the return *on* capital cash flows into income and growth parts. Heuristically, *income* for us is nearer and surer part of the return *on* capital flows, and *growth* is more distant and volatile part of the return *on* capital flows. Again, we use option pricing theory to quantify this intuition.

More specifically, we define the *growth* part $EPVG$ of the $EPVN = EPV(N_t, T)$ as

$$EPVG = \Pi_{\text{first-passage}}(T, \sigma_N, B, \$1)EPVN,$$

where $\Pi_{\text{first-passage}}$ is the price of the first-passage option as defined in Golts-Kritzman [2010, formula 3], T, σ are the parameters discussed above, and the option is written on the $EPV(N_t, T)$ notional amount, and the barrier B is obtained from the investor's risk aversion parameter η as follows:

$$B = c(\eta)EPVN, \text{ where } c(\eta) \text{ is such that the power utility function } u\big(c(\eta)\big) = -1.$$

According to Exhibit [Risk Aversion], the barrier ratio $c(\eta) = B/EPVN$ could be as low as 63% for $\eta = 1$ to 97% for $\eta = 200$. Intuitively, an investment into more distant and potentially volatile cash flows bears the risk of a total loss of this source of income (debt or equity issuer default, total loss of rental property, loss of employment, etc.). The more volatile the expected cash flows are, and the more risk-averse the investor is, the less she's willing to accept the risk of total loss of this source of income. Thus, this part of the asset cash flow is treated as "risky" and contributing to the growth goal.

We consider the remaining "surer and nearer" part $EPVI$ of $EPVN$ as the contribution to *income*:

$$EPVI = EPVN - EPVG.$$

For example, assuming very low probability of default on a short-term government bond, its coupons will be treated as nearly 100% income and nearly no growth.

Asset Risk Discussion

The risks associated with an asset are most commonly quantified by the volatility of its returns. However, *risk* is the possibility of significant adverse events occurring in the future and, if the definition of "significant" and "adverse" is *investor-specific*, then risk should be analyzed as such.

Building on the intuition above, we consider two types of "significant adverse events:" not having enough liquidity when needed, and a total loss of an income source. These risks should be quantified by the cost of "insurance" one can buy to protect against these events. Since both liquidity and first-passage options have positive vega, both risks rise when volatility of an asset rises, but the pricing of these risks is investor-specific.

Negative EPVs

EPV could be negative in several cases: for example a scheduled liability (say a bullet loan). In this case we treat the principal amount of the loan as a negative value, $EPVF$ is a negative value as well, and $EPVL, EPVP, EPVI, EPVG$ are all negative dollar amounts too.

When we deal with an asset/liability package (say a mortgaged commercial property) we separate it into the asset side (the rent-paying property) with a positive EPV and the liability side (the mortgage and the property costs). The asset and liability sides may or may not share characteristics such as volatility and expected weighted average maturity.

Asset Allocation Setup

We will consider an investor's portfolio as the collection of all assets and liabilities. Some of the features in the portfolio are fixed (such as existing liabilities and commitments) and some may be more flexible (such as the allocation of the liquid part of the capital).

Strategic portfolio process

We believe that investors should start their asset allocation process by building strategic portfolios seeking to achieve their goals over a strategic investment horizon *T*. Assume that the expected (net) present value $EPV(T)$ of the entire portfolio is positive. The key idea of the 4×4 process is to target the liquidity, preservation, income and growth allocations in the portfolio. An investor can set their own 4×4 allocation targets, or, in the absence of views, seek roughly equal liquidity, preservation, income and growth allocations.

More formally, let W be the total dollar amount of investor's portfolio containing *n* assets/liabilities. We will denote the dollars invested[4] as $W_1, W_2, \dots, W_n$ and the (vector of) asset weights as $w = (w_1, w_2, \dots, w_n)$, that is $w_i = W_i/W$, for $i = 1, \dots, n$. Similarly, let $EPV = EPV(T)$ be the total dollar expected value of all *n* assets/liabilities. We will denote the dollars expected value of each asset as $EPV_1, EPV_2, \dots, EPV_n$ and the expected value weights as $epv_1, epv_2, \dots, epv_n$, that is, $epv_i = EPV_i/EPV$, for $i = 1, \dots, n$[5]. Let

$$EPVL(T) = \sum_{i=1}^{n} \frac{EPVL_i}{EPV_i} W_i, \quad \text{and} \quad epvl = EPVL(T)/W,$$

and similar formulas for $epvp, epvi, epvg$. Further, let $\alpha^T = (\alpha_1^T, \alpha_2^T, \dots, \alpha_n^T)$ be the (vector of) expected returns over the strategic horizon *T* for $i = 1, \dots, n$, and let $r^T = \sum_{i=1}^{n} \alpha_i^T w_i^T$ be the expected return of the portfolio over the strategic horizon *T*.

Finally, let s_l, s_p, s_i, s_g, where $s_l + s_p + s_i + s_g = 1$ be the liquidity, preservation, income and growth allocation targets in the strategic portfolio.

We set up the strategic optimization as a Linear Programming problem:

$$\max_{w} r^T \qquad \underline{\text{subject to}} \qquad epvl = s_l, epvp = s_p, epvi = s_i, epvg = s_g,$$

and subject to various other investor-specific constraints. Denote the weights of the resulting strategic allocation portfolio $w^T = (w_1^T, w_2^T, \dots, w_n^T)$.

Tactical rebalancing process

We believe a portfolio should be rebalanced with some regular frequency τ: for example, daily, weekly, monthly or quarterly. Each tactical rebalancing should seek to re-align the portfolio with the strategic investment horizon goals, while managing tactical risks and return in a transaction-cost- and tax-aware manner[6].

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[4] The estimation of "dollars invested" in the human/labor capital presents some special challenges, see for example Blundell et al [1999].

[5] Both w_i and epv_i can be positive (asset) or negative (liability) or zero.

[6] In particular, capital gain tax issues could be quite complicated and unwieldy, see Chacko et al [2005] for a discussion and a possible approach using forward-solving simulation techniques.

More formally, denote the (vector of) asset weights as $w^\tau = (w_1^\tau, w_2^\tau, \ldots, w_n^\tau)$, and let $\alpha^\tau = (\alpha_1^\tau, \alpha_2^\tau, \ldots, \alpha_n^\tau)$ be the (vector of) expected returns over the rebalancing horizon τ for $i = 1, \ldots, n$. (We could have $\alpha^\tau = 0$ if the investor does not have tactical alphas). Let $r^\tau = \sum_{i=1}^{n} \alpha_i^\tau w_i^\tau$ be the expected portfolio return at the tactical horizon τ.

We set up the tactical rebalancing optimization problem as follows[7]:

$$\max_w \sum_{i=1}^{n} \alpha_i^\tau w_i^\tau - \nu \sum_{i=1}^{n} (w_i^\tau - w_i^T)^2 - \frac{1}{2\lambda} (w^\tau)' \Sigma^{\text{robust}} w^\tau - [\text{transaction cost/tax penalty}],$$

subject to various investor-specific constraints.

One, often overlooked, asset to be highlighted is cash. We believe cash should be explicitly included in the set of assets, so that the tactical portfolio construction process should produce an allocation to cash. Including a constraint $w_{\text{cash}}^\tau \geq c_{\text{cash}}^\tau$ would allow a generation of a cash flow at the tactical frequency τ (if such a cash flow is required), and would also help achieve the income and liquidity goals strategically.

Asset Allocation: Illustration of Investor-Specific 4x4 Decompositions

Individual Investors

Let us consider an asset allocation which may be relevant for a higher net worth individual with 10 years left in her career, and thus has relatively little of her portfolio EPV coming from human capital, with a relatively large portion invested into public and private securities and alternative vehicles. We set strategic horizon at 10 years. The 4×4 decompositions for a notional asset allocation would depend on the investor-specific needs and preferences: rebalancing frequency and risk aversion, as well as our asset-specific alpha and risk assumptions.

Let us consider two examples: the first one might be relevant for a more risk-seeking and tolerant individual (tactical rebalancing frequency $\tau = 1$ year and the risk aversion parameter $\eta = 20$). Our second example is for more risk-averse person (tactical rebalancing frequency $\tau = 1/52$ years, or one week, and the risk aversion parameter $\eta = 200$). For the risk-seeker, the loss of 15% of her net worth would result in the same loss of power utility as the loss of 3% for the risk-averse person.

We illustrate the assets' contributions to the Liquidity, Income, Preservation, and Growth goals in **Exhibit [Risk-Seeking]** and **Exhibit [Risk-Averse]**. Note that since the two investors have different preferences, the same assets contribute differently in terms of the four goals. In particular, the risk-averse individual may have too much in Growth and not enough Income and Liquidity, while the risk-seeker may consider allocating a bit more to Growth.

[7] For an example of a robust mean-variance optimization problem setup, see Golts-Jones [2009].

Institutional Investors

Our asset allocation framework can be equally relevant in the institutional environment. The assets and liabilities will be different and depending on the institution in question, but any assets and liabilities will still have an impact on the same four goal categories and should be managed accordingly.

Conclusion

Investors focusing solely on the nominal asset dollar prices often neglect one or more of our four goal categories. Some individuals and institutions suffer from cash flow or liquidity problems even when they are asset-rich, such problems are more likely to manifest themselves in turbulent market conditions and result in asset fire sales at depressed prices. Others may be too risk-averse and miss good opportunities to grow their assets or protect against inflation. Further, in our experience investors could sometimes be myopic, not properly balancing the strategic and tactical goals and risks in a disciplined fashion. We believe that our 4×4 framework with explicit strategic portfolios, rebalanced at tactical frequency to re-align with strategic goals and take advantage of short-term opportunities, is a framework well suited for achieving a truly balanced and diversified portfolio.

Appendix: Exhibits

Exhibit [Risk Aversion]

η	c	loss relative to c= 1	u[c]
1	0.37	63%	-1
2	0.50	50%	-1
3	0.58	42%	-1
4	0.63	37%	-1
5	0.67	33%	-1
6	0.70	30%	-1
7	0.72	28%	-1
8	0.74	26%	-1
9	0.76	24%	-1
10	0.77	23%	-1
20	0.85	15%	-1
30	0.89	11%	-1
50	0.92	8%	-1
100	0.95	5%	-1
200	0.97	3%	-1

Notes: η is the parameter is the power utility function, c is the amount of capital resulting in the 1.00 loss of utility relative to c=1. The higher the parameter η, the less capital it takes to lose 1.00 in terms of power utility.

Exhibit [Risk-Seeking]

Assets/Liabilities	Decomposition of the Asset Contributions to the Four Goals				Portfolio Weight
	Liquidity	Preservation	Income	Growth	
Money Market Instruments	91%	0%	9%	0%	15%
5 year US Treasuries	50%	0%	50%	0%	5%
10 year US Treasuries	33%	0%	50%	16%	5%
30 year US Treasuries	33%	0%	47%	20%	5%
Investment Grade Bonds	14%	0%	84%	1%	5%
High Yield Bonds	9%	0%	55%	36%	5%
Inflation-linked Bonds	47%	53%	0%	0%	5%
Rent-generating Real Estate	7%	60%	11%	23%	5%
Non-rent Real Estate	10%	90%	0%	0%	5%
High Dividend Public Equities	27%	6%	47%	20%	5%
Growth Public Equities	8%	9%	22%	61%	5%
Private Equity	0%	50%	10%	40%	5%
Commodity-related Instruments	7%	60%	11%	23%	5%
Crypto Assets	0%	50%	7%	43%	5%
Alternatitive and Absolute Return	0%	4%	20%	75%	5%
Life Insurance Payoff	0%	100%	0%	0%	5%
Human/Labor Capital	0%	0%	71%	29%	10%
TOTAL	**26%**	**24%**	**29%**	**21%**	**100%**

Notes: strategic horizon T =10 years, tactical rebalancing frequency τ =1 year, risk aversion parameter $\eta = 20$. *The 4×4 decomposition also depends on our asset-specific alpha and risk assumptions.*

Exhibit [Risk-Averse]

Assets/Liabilities	Decomposition of the Asset Contributions to the Four Goals				Portfolio Weight
	Liquidity	Preservation	Income	Growth	
Money Market Instruments	91%	0%	9%	0%	15%
5 year US Treasuries	50%	0%	39%	11%	5%
10 year US Treasuries	33%	0%	11%	56%	5%
30 year US Treasuries	15%	18%	10%	57%	5%
Investment Grade Bonds	14%	0%	27%	58%	5%
High Yield Bonds	9%	0%	11%	80%	5%
Inflation-linked Bonds	0%	100%	0%	0%	5%
Rent-generating Real Estate	0%	67%	2%	31%	5%
Non-rent Real Estate	0%	100%	0%	0%	5%
High Dividend Public Equities	0%	33%	10%	57%	5%
Growth Public Equities	0%	17%	4%	79%	5%
Private Equity	0%	50%	2%	48%	5%
Commodity-related Instruments	0%	67%	2%	31%	5%
Crypto Assets	0%	50%	1%	49%	5%
Alternatitive and Absolute Return	0%	5%	4%	92%	5%
Life Insurance Payoff	0%	100%	0%	0%	5%
Human/Labor Capital	0%	0%	15%	85%	10%
TOTAL	**20%**	**30%**	**9%**	**41%**	**100%**

Notes: strategic horizon T =10 years, tactical rebalancing frequency τ =1/52 years (one week), risk aversion parameter $\eta = 200$. *The 4×4 decomposition also depends on our asset-specific alpha and risk assumptions.*

References

Blundell, R., L. Dearden, C. Meghir, and B. Sianesi “Human Capital Investment: The Returns from Education and Training to the Individual, the Firm and the Economy” *Fiscal Studies* (1999) vol. 20, no. 1, pp. 1–23.

Chaves, D., J. Hsu, F. Li, and O. Shakernia “Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios” *Journal of Investing* Spring 2011, Vol. 20, No. 1: pp. 108-118.

Chacko, G., M. Desai, M. Golts, and V. Novakovsky “A Forward-Solving Numerical Technique for Dynamic Consumption and Portfolio Allocation Problems” (March 15, 2005). Available at SSRN: https://ssrn.com/abstract=687622 or http://dx.doi.org/10.2139/ssrn.687622

Elton, E., M. Gruber, S. Brown, and W. Goetzmann, Modern Portfolio Theory and Investment Analysis, 9th Edition, Wiley, 2014.

Golts, M., and G. C. Jones “A Sharper Angle on Optimization,” October 5, 2009. Available at SSRN: https://ssrn.com/abstract=1483412 or http://dx.doi.org/10.2139/ssrn.1483412

Golts, M., and M. Kritzman “Liquidity Options” *Journal of Derivatives* Fall 2010, Vol. 18, No. 1: pp. 80-89.

Hayes, M., J. A. Primbs, and B. Chiquoine “A Penalty Cost Approach to Strategic Asset Allocation with Illiquid Asset Classes” Journal of Portfolio Management, Winter 2015, Vol. 41, No. 2: pp. 33-41.

Kinlaw, W., M. Kritzman, and D. Turkington “Liquidity and Portfolio Choice: A Unified Approach” *Journal of Portfolio Management*, Winter 2013, Vol. 39, No. 2: pp. 19-27.

Merton, Robert C. "The Crisis in Retirement Planning." *Harvard Business Review* 92, nos. 7/8 (July–August 2014): 43–50.

Merton, Robert C., and A. Muralidhar "A Six-Component Integrated Approach to Addressing the Retirement Funding Challenge." *Journal of Investment Management*, Vol. 18, No. 4 (2020): pp. 1-27.

Tobin, J. “Liquidity Preference as Behavior Towards Risk” *The Review of Economic Studies*, Vol. 25, No. 2 (Feb., 1958), pp. 65-86.