The Lead-Lag Relation between VIX Futures and SPX Futures

Christine Bangsgaard*[a] and Thomas Kokholm†[a,b]

[a]Aarhus BSS, Aarhus University, Department of Economics and Business Economics, Denmark

[b]Danish Finance Institute

Abstract

We analyze the lead-lag relationship between VIX futures and SPX futures on a sample of high-frequency data. We find that the two futures markets are weakly connected when market volatility is low. In contrast, when volatility is high, their prices are highly negatively correlated and with the VIX futures leading the SPX futures. We study the determinants of the lead-lag relation, and find that an improvement in the relative liquidity of one market strengthens the lead of that market. In addition, we compute a measure of cross-market activity and find that days of high activity are associated with a strengthened VIX futures leadership. The results provide some indication that VIX futures hedging activities of dealers impact the lead-lag relation. We also document that, when dealers at an aggregate level are in a negative gamma position, an increase in SPX option dealers' rebalancing activities further strengthens an existing VIX futures leadership.

Keywords: Lead-lag relation; high-frequency data; cross-correlation; price discovery; VIX futures hedging; cross-market activity.

*chba@econ.au.dk
†thko@econ.au.dk

1 Introduction

Lead-lag relations generally reflect some degree of inefficiency of financial markets, and arise due to available information not being properly reflected in market prices across securities at a given point in time. As a consequence, past price changes in the leading security can be used to predict future price changes in the lagging security. As such, the identification of possible lead-lag patterns among securities has practical value for both speculation and hedging, which in the past couple of decades has lead to a growing body of academic literature on lead-lag relations.

We study the lead-lag relation between VIX futures and E-mini SPX futures (referred to as SPX futures hereafter) on a sample of transactions time-stamped down to the millisecond and collected over the period from September 2013 to September 2020. We find that the two futures markets exhibit little connection under low levels of volatility while their prices are characterized by a stronger negative correlation under high volatility. Moreover, when the volatility in the market is high, the VIX futures lead the SPX futures on average terms. Hence, the nature of the lead-lag relation is highly dependent on the volatility regime.

Traditionally, changes to the price of a given equity index and its volatility are considered to be connected via two channels (see Bekaert and Wu (2000); Bollerslev et al. (2006)): first, if volatility is priced, an increase in the level of volatility impacts the price of the index negatively as investors require a lower price in order to be compensated for the higher level of risk. This channel is traditionally labeled the *volatility feedback effect*. Second, a drop to the value of the index leads to higher volatility due to the so-called *leverage effect*. Hence, speculation in the level of volatility of the SPX index or the price of the SPX index can be performed both with VIX futures and SPX futures, respectively. This implies that the relative liquidity of the two markets potentially plays a role for which market first reflect new information. Consistent with a price discovery channel, we find that an increase in the liquidity of VIX futures relative to the liquidity of SPX futures results in a strengthening of the VIX futures lead and vice versa.

Moreover, we compute a measure of cross-market activity between the VIX futures and

SPX futures using the methodology introduced in Dobrev and Schaumburg (2018), and find that higher levels of cross-market trading is linked with a stronger VIX futures lead. Overall, we interpret cross-market activity as a combined measure of the activity of two types of agents: the first type consists of high-frequency traders executing orders in one market as reaction to price changes in the other market. Hence, in high volatility scenarios where VIX futures tend to lead SPX futures and the markets are more tightly connected, high-frequency traders can exploit this link and trade in SPX futures as a reaction to changes in the VIX futures price. As these high-frequency traders will trade in the same direction as the anticipated price change in SPX futures, they have the potential to increase the magnitude of the cross-correlations. With an existing VIX futures lead this will be reflected in a stronger VIX futures lead and vice versa. Alternatively, cross-market activity can be driven by VIX futures dealers setting up their hedge in response to a change in their VIX futures net position. In order to hedge their positions in volatility, dealers typically employ various options-based hedging strategies (Chang, 2017). Since these strategies entail trading the underlying in the form of the SPX futures, VIX futures trading would lead to subsequent hedging activities in the SPX futures. These transactions would influence the lead-lag relationship in the direction of a VIX futures leadership.

In addition, we consider how the aggregate net gamma position of SPX option dealers can impact the lead-lag relation. If new information is first incorporated in the VIX futures market, the SPX futures market will subsequently adjust to reflect the new information by moving in the opposite direction of the VIX futures price. The SPX futures price change has the consequence that option dealers with a negative gamma position rebalance their hedge positions by trading in the same direction as the price change (Ni et al., 2021). A negative aggregate gamma position of dealers could therefore result in an even stronger VIX futures leadership as the initial VIX futures price movement continues to have an impact on the SPX futures over a longer time frame. In line with this reasoning, and conditioned on the gamma position being negative, we find that a change in the gamma imbalance towards a more negative position is associated with a strengthening of the VIX futures lead. In contrast, we find no significant impact of a gamma position change on the lead-lag relation when the gamma position is in the positive domain.

The existence of a lead-lag relation between VIX futures and SPX futures may appear at odds with what we would expect for two large and liquid markets. However, other studies show that even for markets that are considered large and actively traded, one market can lead the price changes of another market (Chen et al., 2016; Dao et al., 2018). Furthermore, the evidence on the leadership of the VIX futures may be somewhat surprising in light of the larger SPX futures market. Nevertheless, the findings are consistent with other research showing that VIX futures plays a dominant role in relation to other SPX-related markets. In particular, several studies show that the VIX futures lead the VIX index (Shu and Zhang, 2012; Frijns et al., 2016; Bollen et al., 2017; Chen and Tsai, 2017), which by construction of the VIX index translates into VIX futures leading SPX options. In addition to leading SPX options, VIX futures also lead VIX options (Bollen et al., 2017).

To assess the lead-lag relation between volatility markets and equity markets, the VIX futures and the SPX futures markets are natural candidates. Among the different traded volatility products, the market for VIX futures is the biggest exchange-traded venue for volatility and allows for "pure" volatility speculation without having to delta-hedge, as opposed to volatility speculation via options. Moreover, and as mentioned above, other studies have found that VIX futures lead other volatility sensitive products such as SPX options and VIX options. The VIX futures market is also more liquid compared to VIX options (Hilal et al., 2011) and SPX options (Johnson, 2017). Since the VIX futures value is a measure of the expected future volatility of the SPX index, we should use a traded equity product closely connected to the SPX index. Hasbrouck (2003) shows that across the E-mini SPX futures, the floor-traded SPX futures, and ETFs tracking the SPX index, the E-mini SPX futures contribute the most to price discovery. Using more recent data, Buckle et al. (2018) show that the largest contribution to price discovery does not come from the SPX futures but from ETFs tracking the SPX index. Similarly, Chen et al. (2016) find that more price discovery occurs in SPY than in SPX futures but with the roles reversed under high volatility periods. In particular, we are interested in the lead-lag relation in high volatility scenarios as this is where the VIX futures and SPX futures prices are more tightly linked. Moreover, we connect the lead-lag relation to hedging activities of dealers, which are typically carried out using futures due to the unfunded nature of these. Hence, these results are in favor of relying on the SPX futures.

A large number of studies on lead-lag relations among SPX and its derivatives already exist. The findings of Frijns et al. (2016) reveal that VIX futures have some predictive power of the SPX index. While it has also been shown that SPX futures lead the SPX index (Hasbrouck, 2003), these findings leave it unclear to which extend VIX futures lead or lag SPX futures. As mentioned above, there is also evidence that VIX futures lead the VIX index which is equivalent to leading SPX options. Moreover, it has been shown by Chen et al. (2016) that SPX futures provide greater contribution to price discovery than SPX options. Thus, both VIX futures and SPX futures lead SPX options, but again the VIX futures and SPX futures lead-lag relation cannot be inferred from these studies. While Lee et al. (2017) show that the VIX futures basis (the difference between VIX futures and the VIX index) has some predictive power for the SPX futures returns, the information contained in the VIX futures price and VIX futures basis may be distinct. We therefore contribute to the literature by providing the first evidence on the lead-lag relation between the VIX futures and SPX futures market. Contrary to Lee et al. (2017) who use daily data, we analyze the lead-lag relation from high-frequency data, which allows for examining its time-variation.

The paper is also related to the literature on the relation between lagged equity market returns and volatility: Carr and Wu (2006) study the cross-correlation function for SPX index returns and VIX index changes at a daily frequency and find marginal evidence of SPX returns having some predictive power for VIX index changes. Similarly, Bollerslev et al. (2006) find a significant negative correlation between the absolute value of SPX returns (volatility proxy) and lagged SPX returns both sampled at a five-minute frequency. At the same time, correlations between returns and lagged absolute returns are close to zero. The same pattern is found by Bollerslev et al. (2012) using the squared VIX index as the volatility measure. Dufour et al. (2012) use the same data frequency but for the floor-traded SPX futures and estimate the volatility by computing realized volatility, bipower variation and implied volatility, respectively. Results based on the first two volatility measures, show that the volatility feedback effect is almost absent while it becomes more important once the implied volatility is taken into account. In our study, using data at the millisecond frequency and a maximum lag up to one minute, the volatility feedback effect appear to be more important than the leverage effect. Though the correlation between lagged SPX futures returns and contemporaneous VIX futures returns is non-zero for the smaller lags, the cross-correlation function's decay towards zero is even slower for the part showing the relation between lagged VIX futures returns and contemporaneous SPX futures returns.

The remainder of the paper is structured as follows: First, we introduce the methodology to quantify the lead-lag relation in Section 2. Section 3 presents the data and the results of the empirical analysis. Finally, Section 4 concludes.

2 Lead-lag methodology

Many studies on lead-lag relations are concerned with securities that are closely linked such that a cointegrating relation of the prices can be assumed. In those settings, the information share (Hasbrouck, 1995) or the common factor component weight approach (Gonzalo and Granger, 1995) are often applied. However, it is inappropriate to impose the assumption of cointegration when describing the relation between VIX futures and SPX futures. Instead, we analyze the lead-lag relationship through the cross-correlation function. In Section 2.1, we describe how to obtain the cross-correlation function using the techniques of Hayashi and Yoshida (2005); Hoffmann et al. (2013), and Section 2.2 presents three different quantifications of the lead-lag relation based on the cross-correlation function.

2.1 Estimation of the cross-correlation function

Based on Hayashi and Yoshida (2005) and Hoffmann et al. (2013), we estimate the cross-correlation for two securities, A and B, as

$$\hat{\rho}_{HY}(\vartheta) = \frac{\sum_{i=1}^{n_A}\sum_{j=1}^{n_B}\Delta_{t_i^A}X^A\Delta_{t_j^B}X^B 1_{\left\{(t_{i-1}^A,t_i^A]\cap(t_{j-1}^B-\vartheta,t_j^B-\vartheta]\neq\emptyset\right\}}}{\sqrt{\sum_{i=1}^{n_A}\left(\Delta_{t_i^A}X^A\right)^2}\sqrt{\sum_{j=1}^{n_B}\left(\Delta_{t_j^B}X^B\right)^2}}, \tag{1}$$

where $\Delta_{t_i^k}X^k = X_{t_i^k}^k - X_{t_{i-1}^k}^k$ is the log-return of asset $k = A,B$, and $i = 1,\dots,n_k$ meaning that the returns entering in equation (1) are computed between every single tick, t_i^k. The product of two returns is included in the sum whenever the time intervals, over which the returns are realized, are overlapping. Inspired by Dao et al. (2018), we use Figure 1 to illustrate this. By focusing on the first two line segments, we ignore the possibility of shifting the time-stamps so $\vartheta = 0$. As an example, consider the three intervals of asset B, J_1, J_2 and J_3. J_1 intersects with I_2, J_2 intersects with I_2, while J_3 intersects with I_2, I_3 and I_4. Thus, the contribution to the sum based on each of the three intervals is $\Delta_{t_2^A}X^A\Delta_{t_1^B}X^B$, $\Delta_{t_2^A}X^A\Delta_{t_2^B}X^B$, and $(\Delta_{t_2^A}X^A+\Delta_{t_3^A}X^A+\Delta_{t_4^A}X^A)\Delta_{t_3^B}X^B$, respectively. Hence, it is possible that the same return can contribute to the sum more than once as it will enter every time the interval intersects with one of the intervals of the other asset. Note that when implementing equation (1), the return over I_1 will not influence the correlation as the indicator function equals zero for intervals that do not intersect with any of the intervals of the other asset.

Repeatedly adjusting the time-stamps of the second asset by different values of ϑ allows us to compute the cross-correlation function. The shift of the time-stamps is illustrated for asset B in the lower part of Figure 1. Note that only the time-stamps of one of the two assets are shifted while the time-stamps of the other remain fixed. The returns, $\Delta_{t_i^k}X^k$, are invariant to the shift of the time-stamps, meaning that only the indicator function changes as we vary the value of ϑ. Thus, it is the same returns that enter equation (1) for each ϑ, but they are multiplied and summed in different ways.

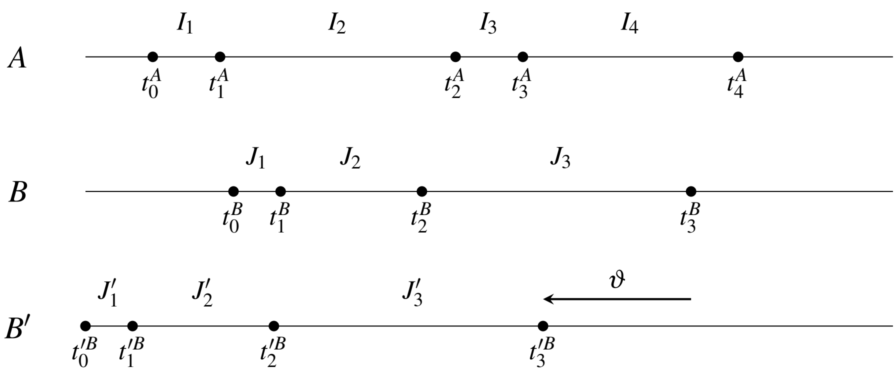

Figure 1: Illustration of the time-stamp adjustment for the cross-correlation functions.

2.2 Lead-lag time, lead-lag correlation, and lead-lag ratio

To measure the lead-lag relation between two assets, Hoffmann et al. (2013) define the lead-lag time (LLT) as the value of ϑ that maximizes the absolute value of the cross-correlation function, $|\hat{\rho}_{HY}(\vartheta)|$, across all ϑ on some grid. If the absolute correlation is maximized at a point $\vartheta \neq 0$, then one asset is leading the other. Under certain assumptions, the point is a consistent estimator of the true LLT (Hoffmann et al., 2013). While LLT measures the amount of time by which an asset leads the other, knowledge of the value of the cross-correlation function at the point corresponding to LLT is also informative about the nature of the lead-lag relation. The value of the cross-correlation at this point is referred to as the lead-lag correlation (LLC).

Both LLT and LLC focus on a single point of the cross-correlation function. However, the rest of the cross-correlation function also contains relevant information about the direction and strength of the lead-lag relation. The lead-lag ratio (LLR) of Huth and Abergel (2014) accounts exactly for this by compressing the entire cross-correlation function into a single measure of the lead-lag relation. Considering all the positive time-stamp adjustments, $\vartheta_1, \ldots, \vartheta_p$, LLR is defined as

$$LLR = \frac{\sum_{i=1}^{p} \hat{\rho}_{HY}^2(\vartheta_i)}{\sum_{j=1}^{p} \hat{\rho}_{HY}^2(-\vartheta_j)}. \tag{2}$$

The ratio captures the relative forecasting ability of one asset over the other. When LLR > 1

it means that the correlations at positive lags are overall larger than the correlations at negative lags. Thus, the asset for which the time-stamps are kept fixed will lead the asset for which the time-stamps are adjusted (asset A will lead asset B in Figure 1). The conclusion of the leadership is the opposite if LLR < 1 where the asset with fixed time-stamps lags the other (asset B leads asset A). Compared with LLT, LLR takes into account the overall predictive power of the returns of one asset on the returns of the other asset by summing the squared correlations. The conclusion on the lead-lag relation drawn from LLT is generally more sensitive to the shape of the cross-correlation function as small variations in its shape could shift LLT from a positive to a negative value and vice versa. LLT may also have limited ability to capture differences in the strength of the leadership that may exist even when two LLTs are close to each other. For instance, consider the two cases illustrated in Figure 2 where LLT and LLC are identical but the behavior of the cross-correlation function to the right of LLT is very different. In the plot to the left, the cross-correlation function goes to zero very quickly, while in the plot to the right, the cross-correlation slowly decays to zero for values of ϑ higher than LLT. Hence, the lead-lag relation depicted in the plot to the left is much stronger than the one to the right. Still, LLT and LLC will be identical, and only LLR will capture this difference in the strength of the lead-lag relation. In fact, LLR could lead to a different conclusion on the lead-lag relation than LLT.

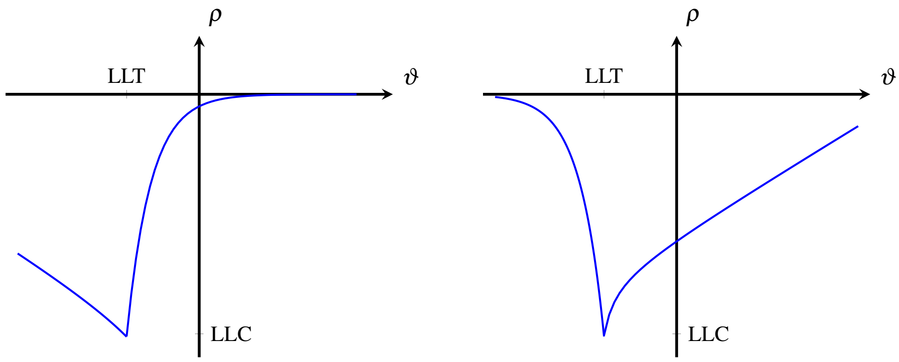

Figure 2: Illustration of cross-correlation functions.

3 Empirical analysis

In this section, we present the empirical analysis. First, Section 3.1 describes the data used for the analysis. Next, Section 3.2 presents the results on the overall lead-lag relationship between VIX futures and SPX futures. In Section 3.3, we analyze potential drivers of the lead-lag relation.

3.1 Data

For the analysis, we collect data over the period from September 2013 to September 2020. Tick-by-tick trade data on SPX futures (ES) and VIX futures (VX) are obtained from the Tick Data database. The time-stamps of the trades are available with millisecond precision. For each sample date, the VIX futures contract used for the analysis is the one closest to an expiry of 30 days, and the SPX futures contract used is the one closest to expiry except when the time to expiry is less than six days where we shift to the next contract. We focus only on trades during the regular trading hours of VIX futures, 9:30-16:15 EST. Any date where the exchanges closed earlier is removed from the sample. Trades with a negative price are removed. For the purpose of computing the cross-correlation function of Section 2, trades sharing the same time-stamp are replaced by a single trade with a price equal to the median price of the trades. Table 1 shows trading and dollar volume for VIX futures and SPX futures. Clearly, SPX futures are more heavily traded than the VIX futures, both when measured in terms of trading and dollar volume.

Time series of the SPX index, VIX index, and VIX futures closing prices are collected from the CBOE homepage. Daily data on open interest and the Black-Scholes gamma of SPX options is provided by OptionMetrics. Days of scheduled release of information on the U.S. Consumer Price Index, Producer Price Index, Employment Situation, or Gross Domestic Product are obtained from Archival Federal Reserve Economic Data (ALFRED).

Table 1: Descriptive statistics of the VIX futures and the SPX futures markets. The statistics are computed from daily observations over the sample period. For each sample date, the VIX futures contract used is the one with an expiry closest to 30 days. The SPX futures contract is the one closest to expiration except when this is less than six days. The variables are measured over 9:30-16:15 EST.

		VIX futures	SPX futures
Trading volume	Mean	80,571	1,277,935
	Median	70,311	1,182,843
	Min	18,432	286,808
	Max	386,637	3,983,301
	Std. dev.	43,909	466,961
Dollar volume (in mm)	Mean	1,403	152,660
	Median	1,143	140,128
	Min	280	29,911
	Max	10,365	567,136
	Std. dev.	962	60,909

3.2 The lead-lag relationship

In this section, we present the overall results on the lead-lag relation based on the methodology detailed in Section 2. For the computation of cross-correlations, we keep the time-stamp of the SPX futures trades fixed and shift the time-stamps of the VIX futures trades. To estimate the cross-correlation function, the grid of ϑ is chosen so that it is finer around zero and less dense as we move away from zero. We do this, since we expect the lead-lag time to be small, so we want to be able to capture variations in the correlation at a higher detail around zero. Hence, the grid is chosen as

$$-60,-59.9,...,-1.1,-1,-0.99,...,-0.11,-0.1,-0.099,...,-0.001,0,0.001,...,0.099,0.1,0.11,...,0.99,1,1.1,...,59.9,60,$$

where the numbers are in seconds and where the largest value (± 60 seconds) reflect the maximum allowed lead-lag. When the grid is most narrow, the length between two grid points is one millisecond, which corresponds to the precision at which the trades are measured.

Figure 3 depicts the median of the cross-correlation. From Panel (a), we see that the peak of the function is around a lag of zero, and the cross-correlation function is close to zero for lags greater than approximately 20 seconds in absolute value. Zooming in on the timeshifts up to one second in Panel (b), we observe a skewed shape of the cross-correlation function with more weight on the left part of the curve. The asymmetry translates into an LLR measure less than one. Hence, on average, VIX futures lead the SPX futures.

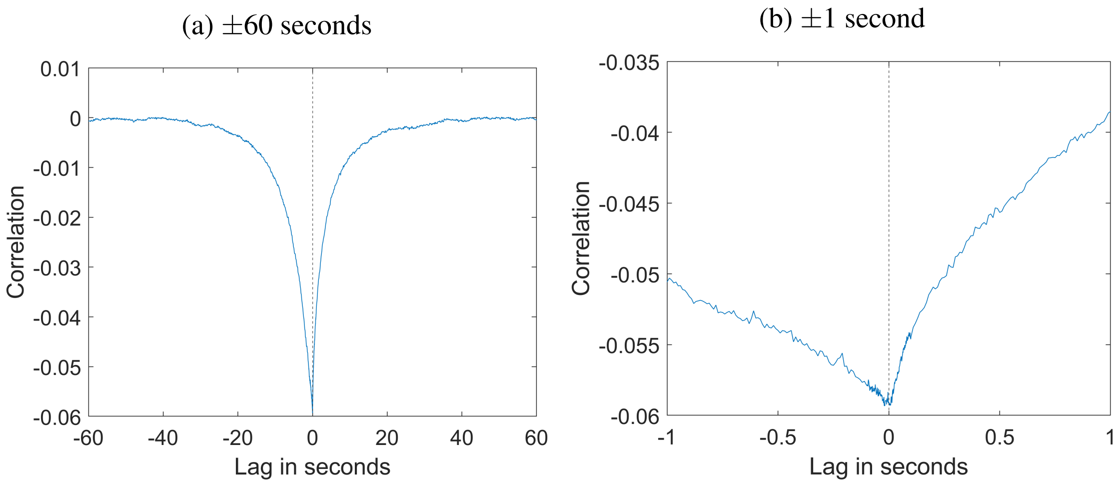

Figure 3: Median cross-correlation function. If the absolute cross-correlation is maximized at a positive (negative) lag it means that the SPX futures (VIX futures) lead.

Figure 4 shows the time series of the three lead-lag measures of Section 2.2 together with the time series of the VIX index and the SPX index. The shaded areas of the chart represent the days with a VIX level belonging to the 60% upper quantile corresponding to values above 15.35. Inspection of the upper panel of Figure 4 confirms what Figure 3 indicates, namely that on most days, the VIX futures lead the SPX futures (LLR less than one). We also notice some interesting features of the lead-lag measures on days of high volatility: First, LLR seems to be more stable and around a level of approximately 0.8. Second, LLT is erratic when volatility is low while close to zero in high volatility regimes. Third, LLC gets more pronounced when volatility is high.

In order to understand the very erratic nature of the LLT measure, Figure 5 depicts stylized cross-correlation curves in a state with a low and high level of market volatility, respectively. The figure illustrates that in scenarios where the level of volatility is low, the overall level of the cross-correlation curve is close to zero and the position of LLT suffers from instability. There is a risk that the markets are not close enough connected during low volatility regimes for LLT to properly reflect the amount of time it takes before the lagging return series has incorporated the same information as the leading series. This means that under low volatility one should be careful to read too much meaning into LLT and LLR about the direction of the lead-lag relation. On the other hand, when the volatility is at a high level the curve gets more peaked and with

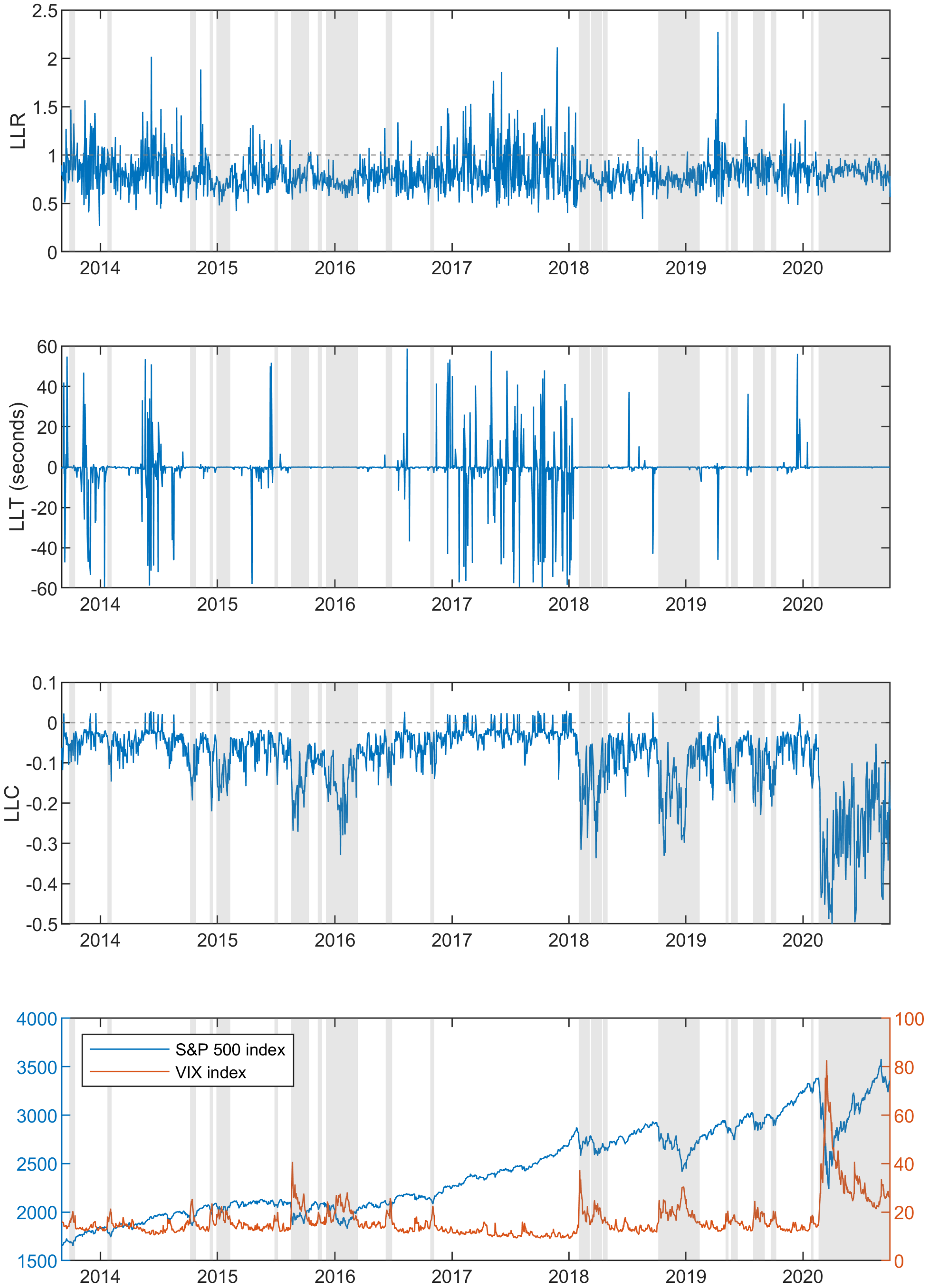

Figure 4: The lead-lag measures and VIX and SPX index over the sample period. The shaded areas represent the days with a level of the VIX index belonging to the 60% upper quantile.

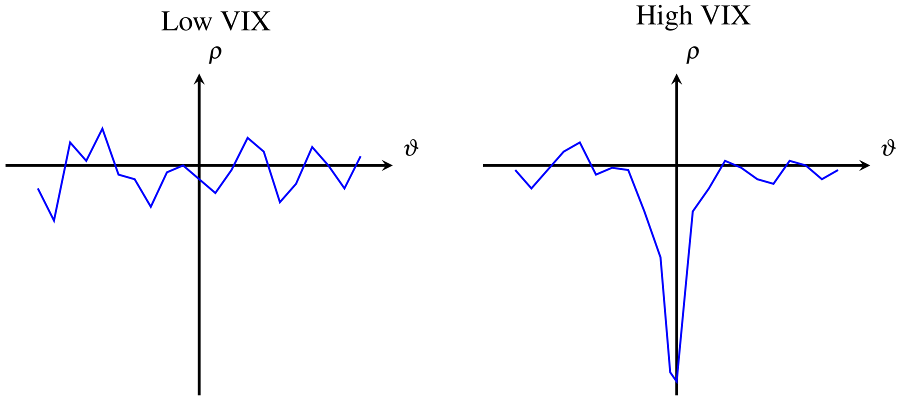

Figure 5: Stylized cross-correlation functions in states with low volatility and high volatility, respectively.

LLT positioned close to zero. For this reason, it is more reasonable to rely on the values of LLT and LLR as describing the lead-lag relation in high volatility regimes.

Table 2 reports descriptive statistics on the three lead-lag measures. In addition to using the full sample, we also report the statistics after splitting the sample into the period before and after the beginning of the covid-19 crisis using February 20, 2020, as the cut-off date. Based on the observations connected to Figure 4, we also compute the statistics conditioned on the volatility being in the upper quartile while excluding the period after the covid-19 outbreak which is characterized by a high level of VIX. Panel A of the table confirms our observations of the relation between LLC and volatility. In the two subsamples defined by the VIX upper quartile excluding the covid-19 period and the covid-19 period itself, LLC is negative for all days and generally its values are larger in magnitude. In terms of the mean and median values, the LLR measure is relatively stable across all four samples, and the values below one indicate that VIX futures lead. However, the variability in the LLR measure is much lower, conditioned on the volatility being high. Considering LLT, the picture is more extreme. On the full sample, LLT roughly varies in the interval $[-60 \text{ seconds}, 60 \text{ seconds}]$. In comparison, for the high volatility sample LLT is much more concentrated around zero with a minimum value of -2.900 seconds and a maximum value equal to 0.660 seconds. In this subsample, the distribution of LLT is not only more narrow around zero but also has more weight on the negative values with a mean and median of -0.027 and -0.004, respectively. Since a negative value of LLT indicates a VIX

Table 2: Descriptive statistics of the lead-lag measures. The covid-19 period covers all sample days after February 20, 2020. The second subsample first excludes all days after the covid-19 cutoff date and then includes only the days where the level of the VIX index is above its upper quartile. LLT is measured in seconds.

	Full sample			VIX upper quartile (before covid-19)			Before covid-19			Covid-19 period		
Panel A												
	LLR	LLT	LLC	LLR	LLT	LLC	LLR	LLT	LLC	LLR	LLT	LLC
Mean	0.818	-0.695	-0.091	0.768	-0.027	-0.143	0.818	-0.762	-0.074	0.820	0.008	-0.276
Minimum	0.266	-59.700	-0.501	0.480	-2.900	-0.337	0.266	-59.700	-0.337	0.567	-0.590	-0.501
Lower quartile	0.710	-0.290	-0.118	0.701	-0.020	-0.180	0.704	-0.390	-0.100	0.771	0.005	-0.345
Median	0.790	-0.012	-0.063	0.752	-0.004	-0.134	0.785	-0.018	-0.057	0.826	0.016	-0.274
Upper quartile	0.892	0.014	-0.035	0.829	0.007	-0.099	0.896	0.009	-0.033	0.878	0.026	-0.195
Maximum	2.274	58.700	0.029	1.471	0.660	-0.023	2.274	58.700	0.029	0.974	0.038	-0.052
Std. dev.	0.186	10.561	0.085	0.106	0.193	0.061	0.193	11.055	0.058	0.076	0.054	0.101
Panel B												
Prop. with LLR<1		89			97			88			100	
Prop. with LLT<0		61			59			66			18	
Prop. with LLT<0 & LLR<1		58			58			62			18	
Prop. with LLT>0 & LLR>1		8			2			9			0	
Prop. with LLT>0 & LLR<1		29			35			24			82	

futures leadership, the high volatility subsample is characterized by a VIX futures lead both in terms of LLR and LLT.

Focusing on the two subperiods defined by the onset of the covid-19 crisis, it seems that the covid-19 period is associated with a much tighter LLT and with much lower variability in all the lead-lag measures. Based on the high level of VIX characterizing this subperiod, this is consistent with the more stable behavior of the lead-lag measures in high volatility regimes. However, we note that with respect to the LLT measure, the covid-19 period differs from the sample conditioned on a high volatility level, where the measure based on the former sample is, on average, slightly positive while slightly negative in the latter.

As a supplement to the statistics in Panel A of Table 2, Panel B shows the proportion of the sample dates where LLR and LLT indicates either a VIX futures or SPX futures leadership. We observe that on most sample dates, LLR is below one meaning VIX futures lead. Especially, we notice that the proportion of days where VIX futures lead in terms of LLR is equal to or close to 100% for the two subsamples characterized by high volatility. For the same two subsamples, LLT behaves quite different. While 59% of the days in the VIX upper quartile subsample reveals a VIX futures lead, this is only 18% during the covid-19 period. Hence, in the covid-19 period there appears to be more disagreement across LLR and LLT on whether VIX futures lead or lag SPX futures. Here the construction of the lead-lag measures could play a role. As the location of the maximum of the absolute cross-correlation is more sensitive compared to the LLR measure, LLR would be the more reliable measure of the direction of the lead-lag relation.

Overall, we conclude that the behaviour of the lead-lag measures strongly depend on the volatility regime. In periods of high volatility, LLC is characterized by larger negative values, the lead-lag relation is short-lived, as indicated by LLT, but with a tendency for VIX futures to lead both when measured in terms of LLR and LLT. A similar dependence on volatility is found in Buccheri et al. (2021), where the lead-lag correlation among stocks strengthens when volatilities are high, while being more erratic in low volatility regimes. Other studies find that correlations at a daily frequency tend to increase between the VIX index and the SPX index when market movements are big (Cont and Kokholm, 2013; Todorov and Tauchen, 2011). The connection between the lead-lag relationship and the level of volatility can possibly be prescribed to different types of trading. First, in the context of high-frequency observations, Buccheri et al. (2021); Zhang (2010); Dobrev and Schaumburg (2018) argue that the relation between high-volatility and lead-lag relationships can be ascribed to high-frequency traders exploiting statistical dependencies across markets appearing when the volatility is high. In relation to VIX futures and SPX futures, this means that a stronger negative correlation in periods of high volatility is possibly exploited by high-frequency traders. Their trading activities could impact the markets by reducing LLT to almost zero and further amplifying the negative correlation which means reducing LLC. Alternatively, in high volatility regimes, the demand for long positions in volatility tends to be high, with a resulting high level of VIX futures hedging activities performed by market makers. As dealers hedge their exposure in VIX futures using SPX futures, VIX futures trading mechanically triggers SPX futures trading. These hedging activities could strengthen the VIX futures lead but also the link between the two markets as they create an additional channel connecting the VIX futures and SPX futures prices. Hence, hedging activities would pull LLR below one and LLT below zero albeit with the increased connectedness pushing LLT towards zero from below. As part of the analysis of the drivers of the lead-lag relation covered in Section 3.3, we explain in more detail and analyze these two types of activities and their impact on the lead-lag measures in Section 3.3.2.

3.3 Determinants of the lead-lag relation

In the following we analyze the potential drivers of the lead-lag relation by running a set of regressions with each of the three measures of the lead-lag relation, LLR, LLT, and LLC, as the dependent variable. In Section 3.3.1 we analyze the impact of the relative liquidity across the two markets while in Section 3.3.2 and 3.3.3 we consider the influence of dealers' hedging activities in relation to VIX futures hedging and SPX option rebalancing, respectively. In each of the sections, the main explanatory variables are described in further detail.

Along with our main explanatory variables, a set of control variables are included in the regressions. In order to control more broadly for market conditions, we include the level of the VIX index, VIX_t, and the SPX index return, SPX_t. In relation to other lead-lag studies, it has been shown that the level of volatility and SPX returns could influence the lead-lag relation. Chen et al. (2016) show a dependence on volatility as the relative informativeness of SPX futures and SPY is reversed under high volatility. Ren et al. (2019) suggest that the lead-lag relation between equity index options and the index is reversed when the index is not stable or up-trending. Finally, Lee et al. (2017) show how the predictability of the VIX futures basis on SPX futures returns changes across the SPX return distribution. We also control for the time to expiry of the specific VIX futures and SPX futures contracts used to compute the lead-lag measures. These are denoted by $Expiry_t^{VX}$ and $Expiry_t^{ES}$, respectively. Moreover, we include a macroeconomic news announcement dummy, d_t^{news}, equal to one for the days of the announcements since other studies have shown that lead-lag relations might differ on those days. For instance, Frino et al. (2000) show that the leadership of equity index futures relative to the equity index is strengthened around the time of macroeconomic announcements, while Chen and Tsai (2017) find that VIX futures lead the VIX index more on the days of the release.

The announcement days used for constructing the dummy variable are the days of the scheduled release of information on the U.S. Consumer Price Index, Producer Price Index, Employment Situation, or Gross Domestic Product.

The results of the regression for LLR, LLT and LLC are shown in Tables 3, 4, and 5, respectively. The regression results for LLC only use one of our three main explanatory variables since only this one variable is potentially influencing the strength of the correlation at the peak of the cross-correlation curve. Compared to the other regressions, the results for LLT in Table 4 are less appealing to interpret. A quick glance at Figure 4 reveals that LLT is fluctuating wildly around zero during low volatility periods while being close to zero when volatility is high. This can explain why the R-squared is close to zero across all the LLT regressions. Hence, the results confirm the statements of Section 2.2. Namely, that LLT is highly unstable and LLR is a more robust measure of the lead-lag relation. In the following sections, we therefore continue the analysis of how our main explanatory variables relate to the lead-lag relation relying on LLR.

3.3.1 Relative liquidity

Though the VIX futures and SPX futures prices are not linked by a no-arbitrage relation, the two futures contracts can still be used to achieve similar objectives. This could be the case for informed investors expressing their view on future market movements or investors buying protection against downside risk. There is mixed evidence on whether informed trading occurs at the index level. Pan and Poteshman (2006) do not find evidence that index option trading is informative about future changes in the index, while Li et al. (2017) find that informed SPX option trading takes place during the financial crisis. In terms of the latter motive, a short SPX futures position provides a hedge against stock market crashes, and with a negative correlation between the SPX and VIX index, so does a long VIX futures position (Szado, 2009; Hilal et al., 2011). Hence, the relative liquidity potentially plays a role for which market investors prefer to trade in and can therefore influence which market first reflect new information. Traditionally, price discovery tends to occur in the market with the highest level of liquidity. This is supported theoretically (Kyle, 1985) as well as empirically (Green et al., 2010; Chen et al., 2016;

Table 3: *The determinants of LLR*. The table shows regression results from regressing daily values of LLR on the relative liquidity of VIX futures and SPX futures, LQR_t, the peak cross-market trading between VIX futures and SPX futures, $PCMA_t$, and the aggregate SPX option net gamma position of dealers for days where it is positive, NGP_t^+, and for days where it is negative, NGP_t^-. VIX_t is the level of the VIX index, and SPX_t is the return of the SPX index on day t. d_t^{news} is a dummy variable equal to one on days with U.S. macroeconomic news announcements, and $Expiry_t^{VX}$ and $Expiry_t^{ES}$ denotes the number of days to expiry of the VIX futures and SPX futures contract, respectively. Newey-West t-statistics are in parentheses. ***, **, * indicates 1%, 5%, and 10% significance, respectively.

LLR_t	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	0.846***	0.859***	0.851***	0.885***	0.899***	0.915***	0.828***	0.857***	0.842***
	(64.623)	(36.775)	(40.535)	(76.486)	(42.737)	(42.294)	(103.740)	(36.886)	(41.867)
LQR_t	-22.257**	-18.469*	-22.400**						
	(-2.321)	(-1.675)	(-2.391)						
$PCMA_t$				-2.447***	-3.217***	-2.524***			
				(-7.602)	(-8.914)	(-7.689)			
NGP_t^+							0.052	0.045	0.055
							(1.326)	(1.136)	(1.399)
NGP_t^-							0.077***	0.066***	0.085***
							(5.533)	(4.498)	(5.715)
VIX_t		-0.001			0.002***			-0.001	
		(-0.711)			(4.253)			(-1.467)	
SPX_t			0.216			-0.365			-0.668**
			(0.724)			(-1.519)			(-2.192)
d_t^{news}		-0.000	-0.000		0.001	0.000		-0.001	-0.001
		(-0.038)	(-0.031)		(0.066)	(0.005)		(-0.110)	(-0.092)
$Expiry_t^{VX}$		-0.000	-0.000		-0.001*	-0.001		-0.000	-0.000
		(-0.435)	(-0.414)		(-1.665)	(-1.421)		(-0.465)	(-0.447)
$Expiry_t^{ES}$		0.000	0.000		-0.000	-0.000		-0.000	-0.000
		(0.138)	(0.173)		(-0.868)	(-0.578)		(-0.353)	(-0.420)
Adj. R^2 (%)	1.38	1.20	1.19	5.17	5.64	5.15	2.06	2.03	2.01
No. of Obs.	1761	1761	1761	1767	1767	1767	1763	1763	1763

Table 4: *The determinants of LLT*. The table shows regression results from regressing daily values of LLT on the relative liquidity of VIX futures and SPX futures, LQR_t, the peak cross-market trading between VIX futures and SPX futures, $PCMA_t$, and the aggregate SPX option net gamma position of dealers for days where it is positive, NGP_t^+, and for days where it is negative, NGP_t^-. VIX_t is the level of the VIX index, and SPX_t is the return of the SPX index on day t. d_t^{news} is a dummy variable equal to one on days with U.S. macroeconomic news announcements, and $Expiry_t^{VX}$ and $Expiry_t^{ES}$ denotes the number of days to expiry of the VIX futures and SPX futures contract, respectively. Newey-West t-statistics are in parentheses. ***, **, * indicates 1%, 5%, and 10% significance, respectively.

LLT_t	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	-1.314***	-2.709***	-2.180**	-1.692**	-3.067***	-2.966***	-0.284	-1.842*	-1.264
	(-2.717)	(-2.727)	(-2.225)	(-2.531)	(-2.835)	(-2.643)	(-0.741)	(-1.692)	(-1.201)
LQR_t	501.237**	219.882	493.440**						
	(2.172)	(0.888)	(2.187)						
$PCMA_t$				36.877**	35.908**	37.259**			
				(2.372)	(2.084)	(2.403)			
NGP_t^+							-5.179*	-4.912*	-5.120*
							(-1.762)	(-1.666)	(-1.733)
NGP_t^-							-0.305	0.057	-0.224
							(-0.571)	(0.123)	(-0.397)
VIX_t		0.049			0.009			0.038*	
		(1.524)			(0.458)			(1.795)	
SPX_t			-26.928**			-13.434			-7.535
			(-2.428)			(-1.466)			(-0.857)
d_t^{news}		-0.084	-0.087		-0.103	-0.104		-0.009	-0.015
		(-0.149)	(-0.154)		(-0.183)	(-0.184)		(-0.017)	(-0.027)
$Expiry_t^{VX}$		0.024	0.023		0.031	0.031		0.023	0.022
		(1.099)	(1.056)		(1.397)	(1.413)		(1.019)	(1.004)
$Expiry_t^{ES}$		0.004	0.004		0.007	0.007		0.005	0.006
		(0.406)	(0.372)		(0.617)	(0.655)		(0.514)	(0.581)
Adj. R^2 (%)	0.17	0.03	0.05	0.31	0.18	0.20	0.51	0.40	0.35
No. of Obs.	1761	1761	1761	1767	1767	1767	1763	1763	1763

Table 5: *The determinants of LLC*. The table shows regression results from regressing daily values of LLC on the peak cross-market trading between VIX futures and SPX futures, $PCMA_t$. VIX_t is the level of the VIX index, and SPX_t is the return of the SPX index on day t. d_t^{news} is a dummy variable equal to one on days with U.S. macroeconomic news announcements, and $Expiry_t^{VX}$ and $Expiry_t^{ES}$ denotes the number of days to expiry of the VIX futures and SPX futures contract, respectively. Newey-West t-statistics are in parentheses. ***, **, * indicates 1%, 5%, and 10% significance, respectively.

LLC_t	(1)	(2)	(3)
Constant	0.021***	0.074***	0.039***
	(5.521)	(6.876)	(4.042)
$PCMA_t$	-4.143***	-2.462***	-4.162***
	(-21.728)	(-12.267)	(-20.997)
VIX_t		-0.006***	
		(-6.237)	
SPX_t			0.028
			(0.170)
d_t^{news}		-0.001	0.001
		(-0.503)	(0.243)
$Expiry_t^{VX}$		-0.000	-0.000
		(-0.301)	(-1.584)
$Expiry_t^{ES}$		-0.000	-0.000
		(-0.685)	(-1.622)
Adj. R^2 (%)	71.23	85.51	71.50
No. of Obs.	1767	1767	1767

Frijns et al., 2018). Higher liquidity results in limited price concession leaving more room for informed traders to generate profit (Kyle, 1985; Fleming et al., 1996).

To test this hypothesis, we define the relative liquidity of the two markets as the Amihud illiquidity measure (Amihud, 2002) computed for the SPX futures market relative to the same measure computed for the VIX futures market, $LQR_t = AMH_t^{ES}/AMH_t^{VX}$. For each of the two markets, the Amihud measure is obtained as

$$AHM_t = \frac{1}{N}\sum_{i=1}^{N}\frac{|r_{i,t}|}{Vol_{i,t}^{\$}}, \tag{3}$$

with N being the number of 5-minute intervals during the trading day, and $r_{i,t}$ and $Vol_{i,t}^{\$}$ is the return and dollar volume, respectively, over the ith interval on day t. Figure 6 shows the relative liquidity variable, LQR_t, across the sample period. Since the Amihud measure is inversely related to the level of liquidity, we expect that an increase (decrease) in LQR_t leads to a stronger (weaker) VIX futures lead.

The coefficient on the relative liquidity ratio in columns (1)-(3) of Table 3 is negative and

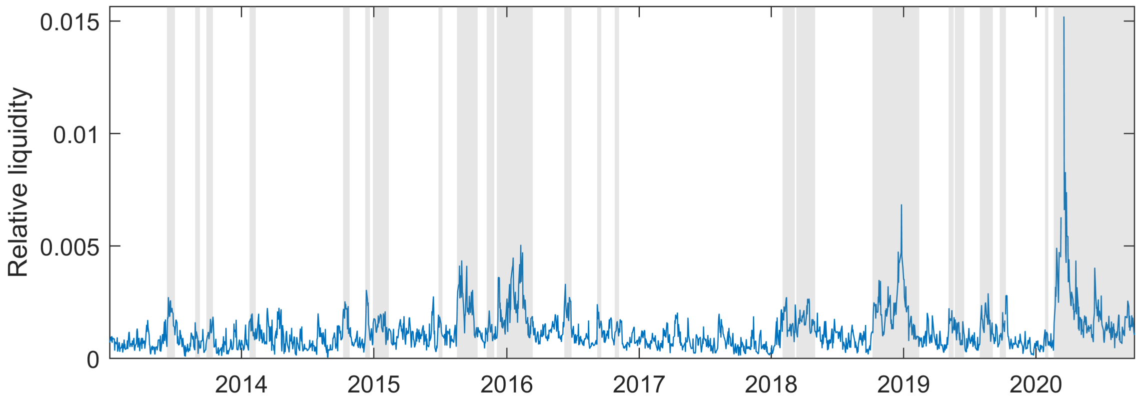

Figure 6: Relative liquidity of VIX futures and SPX futures. The shaded areas represent the days with a level of the VIX index belonging to the 60% upper quantile.

significantly different from zero. Hence, when the relative liquidity improves in favor of the VIX futures market, the VIX futures lead is strengthened while the opposite holds for the reverse scenario. This relation also exists after controlling for the level of VIX and SPX returns, and the pattern would be consistent with investors preferring to trade in the most liquid market.

3.3.2 VIX futures hedging

While Section 3.3.1 relates to the price discovery process, this section and Section 3.3.3 are not concerned with factors determining where new information is first incorporated into prices. Instead, the focus is on hedging transactions that can change the way that VIX futures and SPX futures are connected. In this section, we look at how VIX futures hedging by dealers create a potential link between the two futures markets as VIX futures dealers can hedge using SPX futures. The observation that hedging activities can impact the lead-lag relation between two markets if market makers in one market trade in the other market to hedge is not new. Option dealers setting up delta hedges is an important example of this (Easley et al., 1998; Chan et al., 2002; Schlag and Stoll, 2005; Hu, 2014).

When market participants want to trade VIX futures, typically major dealers in financial markets take the other side of the trade. Market makers and dealers are subject to strict risk requirements and profit from their flow of transactions and not from risk-taking. In order to hedge their positions in volatility, dealers typically employ various options-based hedging strategies

(Chang, 2017). Following a change in the net position of the dealer, delta hedging the option position requires the dealer to trade the underlying index. For instance, take a dealer with a short position in volatility. The dealer hedges the short volatility exposure by buying options on the SPX index while delta hedging using SPX futures. In principle, the dealer can enter long positions in either put or call options. However, in practice, the dealer has to buy call options since this matches the short position of the end-user demand (Garleanu et al., 2008; Goyenko and Zhang, 2019). In addition, the positive delta of the call option position has to be hedged by a short SPX futures position. Alternatively, dealers could turn to a more approximate hedge utilizing the negative correlation between VIX futures and SPX futures prices. Here the dealer would not use options to hedge but only take a position in SPX futures. In either case, a short (long) VIX futures position of the dealer is hedged by selling (buying) SPX futures.

The hedging activity implies that once VIX futures dealers experience a change in their net position, they hereafter trade in the SPX futures market to set up the hedge. Hence, the hedging mechanisms have the potential to impact the lead-lag relation between VIX futures and SPX futures. Since the hedging strategy implies that a VIX futures trade is followed by SPX futures trading, it works in favor of a VIX futures leadership. Therefore, we expect that greater levels of hedging activity would strengthen the lead of the VIX futures. In addition, the price impact from the trades will contribute to the negative correlation between VIX futures and SPX futures prices. This means that greater hedging activity could eventually cause a larger negative correlation between the two futures prices pushing down the value of LLC.

In order to analyze how VIX futures hedging activities influence the lead-lag relation we would like to measure the extend of such activities. Generally, cross-market trading between two markets emerges from situations where trading in one market would generate trading in the other market. This makes the VIX futures hedging activity a potential source for cross-market trading between VIX futures and SPX futures. To proxy the part of the trading activity related to cross-market trading, we use the cross-market activity measure introduced by Dobrev and Schaumburg (2018). Based on high-frequency data, the idea behind the measure is to identify all so-called active time-stamps. A time-stamp is active if the specified activity takes place at that time-stamp. In our case, the relevant activity is trading. The total number of time-stamps with simultaneous activity is then summed over the trading day and shows how often both markets are active at the same time. Assuming that $\vartheta = 0$, this number can be obtained as

$$X_{\vartheta}^{raw} = \sum_{i=|\vartheta|+1}^{N-|\vartheta|} 1_{\{\text{market } A \text{ active in period } i\} \cap \{\text{market } B \text{ active in period } i-\vartheta\}}, \tag{4}$$

where N is the total number of time-stamps. With data at millisecond frequency, N is the total number of milliseconds over the trading day. The number of cross-active time-stamps can be scaled by the total number of active time-stamps in the least active market to measure cross-market activity as a proportion of the total activity

$$X_{\vartheta}^{rel} = \frac{X_{\vartheta}^{raw}}{\min\left\{\sum_{i=|\vartheta|+1}^{N-|\vartheta|} 1_{\{\text{market } A \text{ active in period } i\}}, \sum_{i=|\vartheta|+1}^{N-|\vartheta|} 1_{\{\text{market } B \text{ active in period } i-\vartheta\}}\right\}}. \tag{5}$$

In order to only capture the activity with a cross-market dimension, a further adjustment is implemented to account for the simultaneous activity which would occur simply by randomness. This gives the cross-market activity in excess of what would be expected by coincidence, given that activity in the two markets is independent of each other. It is defined as

$$X_{\vartheta} = X_{\vartheta}^{rel} - X_{\infty}^{rel}, \tag{6}$$

where the adjustment term is given by $X_{\infty}^{rel} = 1/\left(2\left(T_2 - T_1\right)\right)\sum_{|\vartheta|=T_1+1}^{T_2} X_{\vartheta}^{rel}$ for sufficiently large $T_2 > T_1$. In addition to simultaneous activity where $\vartheta = 0$, time-stamps are shifted forward and backward in time when considering non-zero values of ϑ. For a set of different values of ϑ, a full curve for the proportion of cross-market activity can be obtained. We illustrate this in Figure 7, and denote the value of the time-stamp shift, ϑ, corresponding to the maximum of the curve by the cross-market activity time (CMAT), while the peak cross-market activity (PCMA) is the value of the cross-market activity, X_{ϑ}, at the point.

For each sample date, our measure of cross-market trading is PCMA. For the implementation of the measure, we use a time-shift, ϑ, within $[-1000, 1000]$ milliseconds where incre-

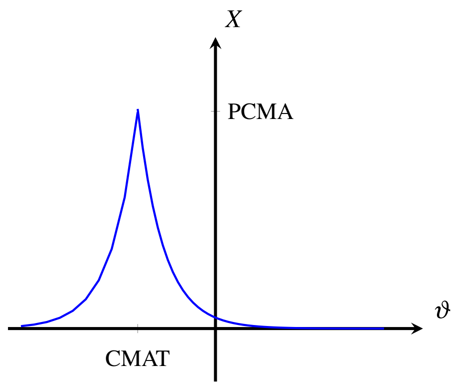

Figure 7: Illustration of cross-market activity function.

ments are of one millisecond and with $T_1 = 500$ and $T_2 = 1000$ as in Dobrev and Schaumburg (2018). Figure 8 plots the peak cross-market trading variable, $PCMA_t$, over the sample period. We see a clear connection between the proportion of trading related to cross-market activity and the level of the VIX index as PCMA increases during periods of high VIX indicated by the shaded areas. PCMA reaches its highest level at the beginning of the covid-19 pandemic.

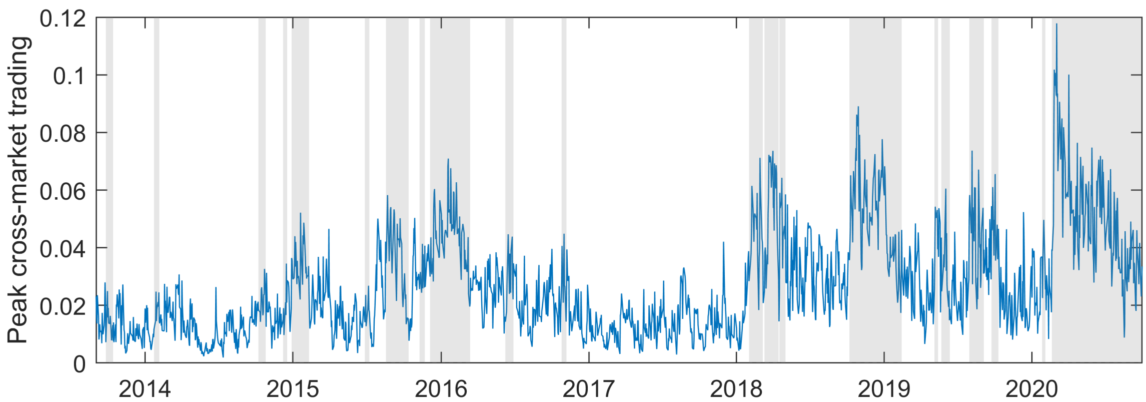

Figure 8: Peak cross-market trading for VIX futures and SPX futures. The shaded areas represent the days with a level of the VIX index belonging to the 60% upper quantile.

Table 3 shows the regression results from regressing LLR on PCMA. Since cross-market trading can arise from hedging by VIX futures dealers, the strong significance of the cross-market trading variable in columns (4)-(6) can indicate that VIX futures hedging influences the lead-lag relationship and pushes the markets in the direction of a VIX futures leadership.[1]

[1]In principle, VIX futures hedging activity is only consistent with the part of the cross-market trading where the VIX futures trades lead the SPX futures trades. However, as noted by Dobrev and Schaumburg (2018), the estimation of the location of the peak cross-market trading is often unreliable while the proportion of cross-market

The relation between cross-market trading and LLC is shown in Table 5. We see that when cross-market trading increases, the negative correlation between VIX futures and SPX futures returns gets more pronounced. With investors buying (selling) VIX futures and VIX futures dealers hedging in response to this by selling (buying) SPX futures, the hedging mechanisms could create a greater negative correlation. To the extend that hedging activities are reflected in cross-market trading, this can explain why a greater proportion of cross-market trading pushes down the value of LLC.

It is uncertain to which extend cross-market activity is driven by VIX futures hedging activities. Alternatively, it could arise from high-frequency traders exploiting the relation between equity markets and volatility. Once the SPX futures price moves, these traders may have strategies in place for how to trade in the VIX futures market and vice versa. Particularly, this type of strategies could be used in the high VIX periods where the two futures markets are more negatively correlated as documented in Section 3.2. Besides the referenced papers of Section 3.2, this is related with empirical studies showing that profitable trading opportunities tend to occur more often under high volatility. For instance, Marshall et al. (2013); Budish et al. (2015) show that arbitrage opportunities between the two ETFs, IVV and SPY, and between SPX futures and SPY, respectively, are more frequent under high volatility. Though the SPX and VIX futures are not characterized by the same no-arbitrage relation, mislocations of prices and short-lived correlations, which create profitable trading opportunities for high-frequency traders, would still be expected to be more likely as volatility increases. Based on this reasoning, the proportion of cross-market trading under high VIX periods would be higher due to an increased presence of high-frequency traders exploiting the stronger connection between the two futures prices. This is also evident from Figure 8 as days with the VIX index belonging to the upper 60% quantile seem to be characterized by a high level of PCMA. Empirically, the correlation between PCMA and VIX equals 0.67. Recall that under high VIX periods we have documented a VIX futures leadership. This means that the negative correlation between LLR and cross-market trading observed in Table 3 could arise from the more active high-frequency

trading is much easier to reliably estimate. Thus, we do not make any distinction in whether the cross-market trading on a given day occurs at an offset where VIX futures trades lead SPX futures trades or the reverse.

traders in high VIX periods.

Even if cross-market activity captures high-frequency trading activity rather than VIX futures hedging activity, it can still help explain the negative relation between LLC and PCMA shown in Table 5. This is the case under a feedback effect from the high-frequency trading. Besides attracting high-frequency traders, the negative correlation between the VIX futures and SPX futures prices could be further strengthened by the increased activity of high-frequency traders. Specifically, if their trading strategies are build to exploit the negative correlation between the two futures prices, their trading could push down the correlation even more. Based on this reasoning, the presence of high-frequency trading could amplify the magnitude of LLC and thereby influence LLR resulting in a strengthening of an existing leadership.

3.3.3 Option rebalancing

In general, option dealers with a negative gamma position rebalance their delta hedge in the direction of the market movements of the underlying while a positive net gamma position involves trading against the market movements (Ni et al., 2021). This means that when the aggregate net position of dealers is negative there is a risk of amplifying market movements in the underlying. Such amplification has been documented empirically for various markets on which options are written including the SPX futures (Baltussen et al., 2021; Barbon and Buraschi, 2021; Barbon et al., 2021).

We now want to understand how SPX option dealers' updating of delta hedges can impact the lead-lag relation between VIX futures and SPX futures. As is the case for the average sample date, assume that new information is first incorporated in the VIX futures market. Further, assume that the information results in a positive VIX futures return. This will later result in negative SPX futures returns possibly assisted by the presence of high-frequency traders or dealers' VIX futures hedging as explained in Section 3.3.2. Option dealers with a negative gamma position then rebalance in response to the negative SPX futures price movement by selling SPX futures resulting in further downward pressure on SPX futures. As long as the net gamma position is negative, the same holds but with opposite signs if the initial VIX futures return is negative. Thus, the negative net gamma position is an amplifier of the SPX futures price movement. Days where the aggregate net gamma position among dealers is negative could therefore result in an even stronger VIX futures leadership as the initial VIX futures price movement continues to have an impact on the SPX futures over longer time due to the option rebalancing. In terms of the cross-correlation function, this would produce a stronger asymmetry as we would have a slower decay of the curve for the negative timeshifts. We therefore expect that the value of LLR would be reduced as a result of dealers updating their delta hedges under a negative gamma position. The more negative the net gamma position is, the stronger this effect should be as a more negative net gamma position requires dealers to trade more in the SPX futures market for a given change in the SPX futures price. While we expect that the hedging activities impact LLR, the value of LLT should not be influenced as the location of the peak of the cross-correlation function should be unaffected. On days where dealers hold a positive net gamma position, the option rebalancing should not induce the same long-lasting impact of a given VIX futures price change as dealers trade in the opposite direction of the SPX futures price movement. Overall, we note that since the average/median sample date is characterized by a VIX futures leadership in terms of LLT, we expect that a negative net gamma position could amplify the VIX futures leadership as measured by LLR.

In order to analyze the impact of the net gamma position, we estimate the daily aggregate net gamma position of dealers. This is done using SPX options data from OptionMetrics and making some assumptions: As in Section 3.3.2, we first assume that end-user demand is long in put options and short in call options. This claim is empirically justified by Garleanu et al. (2008); Goyenko and Zhang (2019). Second, we assume that the dealer is the counterparty in all option trades, and finally that the dealers delta hedge their entire exposure. Similar to Baltussen et al. (2021); Barbon and Buraschi (2021), we use these assumptions to construct our proxy for the net gamma position at time t as

$$NGP_t = \sum_{i=1}^{N_t^C} \Gamma_t^{BS}(C^i) OI_t(C^i) - \sum_{i=1}^{N_t^P} \Gamma_t^{BS}(P^i) OI_t(P^i), \tag{7}$$

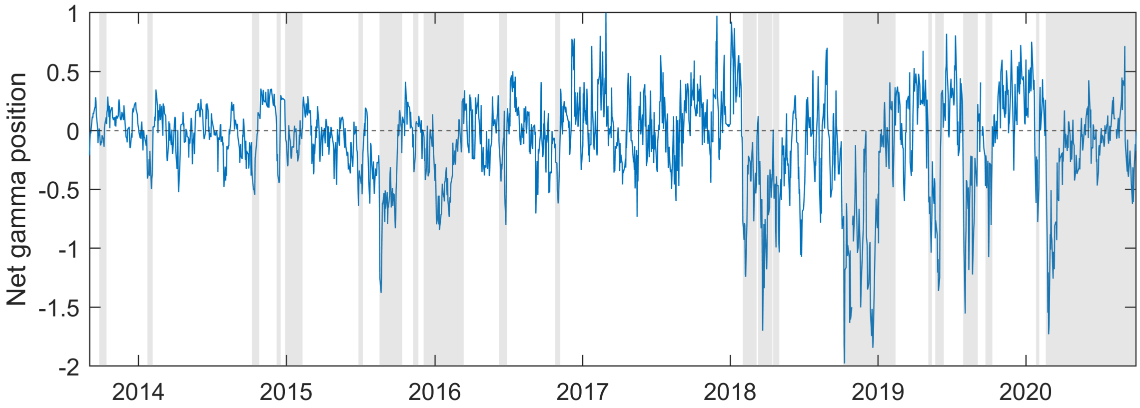

Figure 9: SPX option dealer net gamma position. The shaded areas represent the days with a level of the VIX index belonging to the 60% upper quantile.

where N_t^C is the number of call options traded at time t with an open interest greater than zero. $\Gamma_t^{BS}(C^i)$ denotes the Black-Scholes gamma of call option i, and $OI_t(C^i)$ is the option's open interest. Similar notation holds for put options.

Figure 9 depicts the estimated net gamma position throughout the sample period. The gamma of the option position fluctuates around zero. However, when markets are in turmoil, as measured by a high level of the VIX index, the position is mostly negative and can move rapidly from positive to negative when volatility increases.

Due to the different expectations about the impact of a positive and negative net gamma position, we decompose the series into two variables: NGP_t^+ for all the days where the position is positive, and NGP_t^- composed of all the days where the net gamma position is negative. The relation between LLR and these two variables is shown in columns (7)-(9) of Table 3. As expected, we see that changes in the gamma position when it is positive does not significantly influence LLR. On the other hand, a decrease in the gamma position when it is already negative is associated with a decrease in LLR. This is equivalent to a strengthening of the VIX futures lead. The more negative the gamma position, the more heavily option dealers will need to trade in the direction of a given SPX futures price change. This could create a prolonged impact of any SPX futures price change originating from VIX futures trading. Hence, the regression results are consistent with option rebalancing generating a slower decay of the left tail of the cross-correlation function which will work to reduce LLR. When the VIX futures already lead, our results indicate that the leadership could be further strengthened by the delta hedging of option dealers if their net gamma position is negative.

4 Conclusion

We study the lead-lag relationship between VIX futures and SPX futures on a high-frequency sample of transactions over the period from September 2013 to September 2020. To analyze the lead-lag relation, we estimate the cross-correlation function. The leadership strength is computed on a daily basis using various measures of lead-lag strength. The analysis reveals a large time variation in the lead-lag relation. Under low volatility, the lead-lag measures should be interpreted with caution as the two markets are not strongly connected. Under high volatility, returns exhibit a stronger negative correlation and a short-lived lead-lag with a tendency for VIX futures to lead SPX futures. The VIX futures leadership confirms the important role played by the VIX futures across the various SPX-related markets.

Our results show that an improvement in the relative liquidity of one market strengthens the lead of that market. Since investors can express their view on future market movements or buy protection against downside risk using both VIX futures and SPX futures, we interpret the results as evidence that relative liquidity influences the price discovery process.

Moreover, we find that an increase in the proportion of cross-market trading is associated with a strengthened VIX futures lead. As VIX futures dealers can hedge their VIX futures position through delta-hedged SPX option positions, their hedging strategy would involve trading SPX futures after providing liquidity in the VIX futures market. Thereby, the hedging activities would contribute to the level of cross-market activity. Hence, it is possible that hedging activities are driving a part of the VIX futures lead over SPX futures. Another possibility is that the relation between cross-market trading and a stronger VIX futures lead appears due to a greater presence of high-frequency traders under high volatility.

Finally, our analysis reveals that when market makers in SPX options are in an aggregate negative gamma position, an additional increase in the imbalance of the gamma position leads to a stronger VIX futures lead. In contrast, when the gamma position is positive, a change to the position does not impact the lead-lag relation between VIX futures and SPX futures.

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