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Portfolio Rebalancing: Tradeoffs and Decisions[†]

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[†] We would like to thank Wes Crill, Wei Dai, Savina Rizova, and Bhanu Singh for their helpful comments. Special thanks to Marlena Lee for thoughtful discussions and suggestions.
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Abstract

This paper identifies a clear tradeoff between tracking error—performance differences relative to a targeted asset allocation—and turnover—a proxy for rebalancing costs—that can help guide investors' rebalancing choices. We find that calendar-based approaches, while convenient, tend to lead to less efficient rebalancing tradeoffs than rebalancing with tolerance bands. Further improvements can be gained with tiered approaches that apply different tolerance bands across and within asset classes. We do not find evidence that rebalancing choices can reliably increase expected returns. Finally, our study evaluates how rebalancing choices relate to asset allocation and how they may impact a portfolio's maximum drawdowns and shorter-term return differences to the target allocation.

1. Introduction

Rebalancing can help investors maintain an asset allocation that aligns with their needs, goals, and risk tolerance. As the performance of portfolio components varies over time, component weights may deviate from their target allocations, exposing investors to a different risk-return profile than that of the intended allocation. Rebalancing is a tool to manage such deviations.

Deciding when and how to rebalance a portfolio requires weighing benefits and costs. The costs of rebalancing include explicit costs, such as brokerage commissions, custody and exchange fees, or taxes, and implicit costs, such as bid-ask spreads and potential market impact. All else equal, more frequent rebalancing reduces deviations from the target allocation but also drives up the associated costs.

When it comes to return differences, we do not find evidence that rebalancing choices can reliably increase expected returns. Instead, realized return differences across rebalancing approaches are typically due to style drift. For example, more frequent rebalancing tends to reduce returns because rebalancing tends to reduce the allocation to asset classes with higher expected returns. However, if a greater allocation to higher-expected-return asset classes was appropriate, it should be reflected in the target allocation. Noise in returns can also impact the realized return outcomes of different rebalancing approaches, suggesting that investors should avoid choosing a rebalancing method based solely on historical returns.

With returns out of the running, how should investors decide between different rebalancing approaches? We identify a clear tradeoff between tracking error—performance differences relative to the target asset allocation—and turnover—a proxy for rebalancing costs. This tradeoff can help guide investor choices when deciding between rebalancing approaches. Rigid calendar-based approaches can lead to less efficient rebalancing tradeoffs—lower turnover for a given level of tracking error and vice versa—compared to deviation-based approaches. Further efficiencies can be gained with tiered approaches that apply different tolerance bands for a portfolio's split between

equities and fixed income relative to components within equities and fixed income. In addition, this paper evaluates how rebalancing choices relate to asset allocation and how they may impact a portfolio's maximum drawdowns and shorter-term return differences to the target asset allocation.

2. Testing Different Rebalancing Methods with Historical Data

To help illustrate and quantify the tradeoffs involved in portfolio rebalancing, we test two categories of rebalancing methods. The first set are calendar-based, in which rebalancing happens at certain calendar dates. We also evaluate rebalancing triggered by how much portfolio weights have deviated from the target allocation. This approach, rebalancing with tolerance bands, rebalances a portfolio when the deviation of the portfolio's asset allocation from the target exceeds a percentage tolerance band: 1%, 2.5%, 5%, 7.5%, or 10%.

Using historical data to form hypothetical portfolios, we first examine the impact of rebalancing methods on portfolio returns. We then evaluate portfolio turnover, one proxy for potential rebalancing costs, and tracking error relative to those of a monthly rebalanced portfolio with the same asset allocation. While different investors may have different preferences on how much tracking error to incur relative to the target asset allocation, investors should prefer lower turnover for a given level of tracking error. Conversely, lower tracking error is preferred for a given level of turnover.

3. Case Study 1: Calendar-based vs. Tolerance Band Rebalancing

In the first case study, we analyze the impact of the calendar-based and tolerance band rebalancing approaches on a hypothetical US 60/40 portfolio composed of 60% in the Russell 3000 Index and 40% in the Bloomberg Barclays US Aggregate Bond Index from 1979 to 2019. Within tolerance band rebalancing, we further consider rebalancing fully to target and rebalancing to bound, where the bound is the nearest edge of the tolerance band. Under both approaches, if the deviation is only within the bound, no rebalancing will be triggered. To illustrate those two approaches, **Exhibit 1** shows a hypothetical rebalancing scenario for the 60/40 portfolio under a 5% tolerance band rebalancing approach. When market movements lead to an asset allocation of 67% equities before rebalancing, the rebalancing-to-target approach sets the allocation to 60% equities (the target), while the rebalancing-to-bound approach sets the allocation to 65% equities (the nearest bound). Similarly, a mirroring mechanism will take place when the equity allocation drops below the lower bound of the band—55% in this case. Compared to rebalancing to bound, the rebalancing-to-target approach makes a larger adjustment for each rebalancing and hence leaves more room for deviations before the next rebalancing is triggered. As a result, the rebalance-to-target approach is likely to trigger fewer rebalancing events—an implication that we will discuss further in the next section.

We test a total of 14 different rebalancing approaches. Four are calendar-based methods ranging from monthly to annual rebalancing, and 10 are deviation-based approaches with five different

thresholds for both rebalancing to target and rebalancing to bound. As **Exhibit 2** demonstrates, returns before rebalancing costs among those 14 rebalancing approaches are similar. For example, the portfolio rebalanced to bound with a 10% tolerance band delivered the highest return (0.89%), whereas the portfolio rebalanced monthly had the lowest return (0.87%), yielding the largest return difference between any two rebalancing approaches of 2 basis points (bps) monthly. This is intuitive, as equities have higher expected returns than fixed income and would generally gain a larger share of the portfolio over time with less frequent rebalancing. In addition, while volatility tends to increase slightly with wider bands within the tolerance band/rebalance-to-bound approach, we find that there are no meaningful differences in volatility within both the calendar-based and tolerance band/rebalance-to-target approaches.

As return and volatility differences across rebalancing approaches are mostly small to nonexistent, deciding which approach to choose comes down to costs and tracking error. **Exhibit 3** illustrates the tradeoff between turnover and tracking error under the 14 different rebalancing methods for the same US 60/40 portfolio. It is evident that more frequent rebalancing tends to come with less tracking error and more turnover. Furthermore, for a given level of tracking error, calendar-based rebalancing approaches tend to have higher portfolio turnover than approaches rebalancing with tolerance bands. Rebalancing to bound instead of rebalancing to target can further improve rebalancing tradeoffs.

The above results represent one realized outcome over the full sample period. We also study the range of outcomes over different time periods. To this end, we examine rebalancing results over the 253 20-year rolling periods with a one-month step size starting from January 1979. **Exhibit 4** shows the resulting distributions for both tracking error in **Panel A** and turnover in **Panel B** and for each rebalancing approach. The results indicate a similar pattern as in the full sample analysis: approaches with tolerance bands generally yield better turnover-tracking error tradeoffs than calendar-based approaches, with rebalance to bound allowing for further improvement over rebalance to target.

Unlike rebalancing with tolerance bands, the calendar-based approach can potentially leave large deviations from the target unchecked until the next rebalancing date. For example, **Exhibit 5** shows the weights in equities under three different rebalancing approaches. Large deviations that exist under annual rebalancing shown in **Panel A** tend to be corrected more quickly under the tolerance band approaches shown in **Panel B** and **Panel C**. A further comparison between Panel B and C suggests that more rebalancing events are likely to occur when rebalancing to bound instead of rebalancing to target, but each with smaller adjustments in constituent weights than under approaches rebalancing to target. Since the Russell 3000 Index had higher average returns than the Bloomberg Barclays US Aggregate Bond Index from 1979 to 2019, we also observe in Exhibit 5 that weights in equities are more often located asymmetrically near the upper bound than near the lower bound of the tolerance band.

4. Case Study 2: Tiered Tolerance Band Rebalancing

For asset allocations with multiple components within the equity and fixed income parts of a portfolio, we can apply tiered tolerance bands. For example, we may want to apply a different tolerance band to the split between equities and fixed income relative to components within equities and fixed income. We again consider both rebalancing to target and rebalancing to bound in the case of a rebalancing event.

Different asset classes can have a different magnitude of impact on tracking error. More volatile asset classes, such as equities, tend to contribute more to overall tracking error than less volatile asset classes, such as fixed income. Relative to fully rebalancing every component within a portfolio, tiered rebalancing mitigates turnover by rebalancing only within asset classes whose components deviate by more than the specified tolerance band, as long as the overall split between asset classes is not out of balance. **Exhibit 6** shows two hypothetical rebalancing scenarios for a 60/40 portfolio with multiple equity and fixed income components under a tiered rebalancing approach with 5%/5% tolerance bands. In **Panel A**, market movements lead to a total equity weight of 67% before rebalancing. This exceeds the first equity/fixed income split tier of the 5%/5% band, so the portfolio is fully rebalanced. In the second scenario, as shown in **Panel B**, no rebalancing occurs between equities and fixed income because the split between the two stays within the 5% tolerance band (64%/36% for equities/fixed income). However, within equities, the weight of component A exceeds its target by more than 5%. As a result, rebalancing occurs within the equity part of the portfolio.[1]

Exhibit 7 shows the performance of a multicomponent global 60/40 portfolio under tiered tolerance band rebalancing as well as calendar-based and simple tolerance band approaches over the sample period from January 1988 to December 2019.[2] Returns before rebalancing costs are similar, and observed differences are generally mirrored by differences in average equity allocations. Noise in realized returns can also impact results across rebalancing approaches.[3]

As before, we believe these results suggest that rebalancing decisions should not be based on the premise of improving expected returns, but instead on considerations of the tradeoff between tracking error and turnover. **Exhibit 8** illustrates this tradeoff for the global 60/40 portfolio. Applying tiered rebalancing can improve rebalancing tradeoffs—lower turnover for a given level of tracking error and vice versa—compared to rebalancing with tolerance bands and calendar-based rebalancing, for both rebalancing to target shown in **Panel A** and rebalancing to bound shown in **Panel B**. These findings are consistent with the premise that introducing two tiers helps generate better rebalancing tradeoffs because it focuses the rebalancing activity on the appropriate level of the portfolio by distinguishing between rebalancing across asset categories and rebalancing within

[1] Similarly, if any component within the fixed income part of the portfolio deviates from the target weight by more than 5%, a rebalancing within the fixed income will be triggered. Note that the tolerance band in the second tier applies to each component's *relative* weight within their respective asset category.

[2] Specifically, the global 60/40 portfolio consists of 60% global equities (20% Russell 1000 Index, 10% Russell 2000 Index, 20% MSCI World ex USA Index (net div.), 10% MSCI Emerging Markets Index (gross div.)) and 40% fixed income (20% Bloomberg Barclays US Government Bond Index, and 20% Bloomberg Barclays US Credit Bond Index).

[3] For example, in unreported results, we compare the returns of 12 annual rebalancing methods for the global 60/40 portfolio, each rebalanced in a different month. Expected returns across these strategies should be similar. Empirically, however, we find the largest average monthly return difference between any two of these annual methods to be 0.03% and reliably different from zero (t=2.99), likely the result of random chance.

asset categories. In addition, as shown in Exhibit 7, most of the within-asset-class rebalancing occurred in equities, which is consistent with the intuition that components of more volatile asset categories have a greater tendency to deviate from target weights than components of less volatile asset categories.

Similar to the first case study, we also evaluate 20-year rolling periods observed each month starting from January 1988—a total of 145 rolling sample periods. **Exhibit 9** shows the range of outcomes for tracking error and turnover under each rebalancing approach, with results similar to those reported for the full sample period.

5. Tracking Error and Its Implications

Tracking error may depend on a variety of factors, such as a portfolio's asset allocation and market conditions. As highlighted in the previous sections, less frequent rebalancing is generally associated with higher tracking error and higher average allocations in higher-expected-return asset classes, such as equities, through time. **Exhibit 10** shows that investors who prefer rebalancing methods associated with higher tracking error may not necessarily experience higher maximum drawdowns. However, the results indicate that rebalancing less frequently can lead to meaningful short-term return differences relative to a monthly rebalanced portfolio. As would be expected, such discrepancies tend to grow as tolerance bands widen. For example, in the US 60/40 allocation, the largest instances of under- and outperformance relative to a monthly rebalanced portfolio over a 12-month period are –0.2% and 0.2%, respectively, for the 1%-to-target approach, compared to –2.9% and 3.7%, respectively, for the 10%-to-target approach.

Tracking error expectations should also reflect a portfolio's asset mix. **Exhibit 11** shows that, for a given rebalancing approach, higher equity allocations tend to be associated with higher overall tracking error relative to a monthly rebalanced portfolio with the same asset allocation. In addition, the range of tracking error across rebalancing approaches is wider for portfolios with larger equity allocations. For example, the difference in tracking errors between 10%- and 1%-to-target rebalancing is 1.47% (1.77% vs. 0.30%) for an allocation with 80% equities, more than double the 0.64% difference between those two approaches for an allocation with 20% equities (0.75% vs. 0.11%). This may suggest that investors sensitive to tracking error should be particularly mindful when choosing rebalancing approaches if their portfolios have high allocations to equities.

6. Rebalancing Costs: Integrated Portfolios vs. Portfolios with Many Components

Portfolios with fewer components can be more efficiently rebalanced than portfolios with many components. To illustrate this point, we compare a portfolio composed of the stocks in a single global equity index, the MSCI All Country World IMI Index (gross div.), with another globally diversified equity portfolio consisting of eight components, with each component's weight roughly

representing the respective asset category's weight in the global equity market today.[4] The average returns of this allocation have been similar to those of the overall market during this period. The same is not true for turnover, however. **Exhibit 12** shows the annualized turnover for the portfolio with multiple components, grouped by different rebalancing methods. Annualized turnover ranges from 0.6% to 8.8%, with a larger magnitude when rebalancing occurs more frequently. Note that the holistic portfolio in this comparison has zero turnover, by definition.[5] Our analysis therefore suggests that, where appropriate, an integrated investment solution can be more economical in terms of rebalancing costs than an otherwise similar portfolio with many components. In addition, recent research by Dai et al (2021) shows that, in the pursuit of multiple premiums, an integrated approach—when compared to a combination approach with multiple components—can lead to not only lower costs but also greater reliability of outperformance and better risk control.

7. Conclusion

Tracking error vs. turnover is a key tradeoff investors should consider when developing their approach to decisions around rebalancing. Calendar-based approaches, while convenient, tend to lead to less efficient rebalancing tradeoffs and allow for substantial deviations from the target allocation, especially in the case of significant market moves far from the scheduled rebalancing date. By contrast, rebalancing with tolerance bands can help reduce large portfolio deviations from the target in a timely manner but requires continuous monitoring. For asset allocations with multiple components in each asset class, rebalancing with tiered bands may further improve rebalancing tradeoffs by reducing unnecessary turnover. Rebalancing costs should also be taken into consideration when investors choose between investing in an integrated portfolio solution and a portfolio with many components. This is because a multicomponent portfolio, when compared to an integrated portfolio with otherwise similar characteristics, can incur meaningfully higher costs due to rebalancing but with little expected difference in returns.

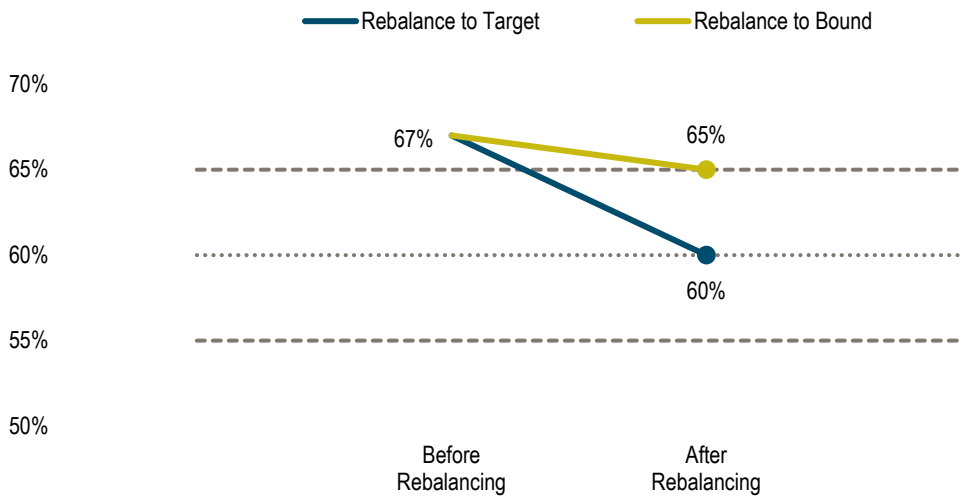
[4] Specifically, the allocation is 22.5% Russell 1000 Value Index, 22.5% Russell 1000 Growth Index, 2.5% Russell 2000 Growth Index and 2.5% Russell 2000 Value Index, 36% MSCI World ex USA Index (gross div.), 4% S&P Developed ex US SmallCap Index (gross div.), 9% MSCI Emerging Markets Index (gross div.), and 1% MSCI Emerging Markets Small Cap Index (gross div.). As a robustness check, we also chose the beginning of the sample period as the reference point to set component weights and repeated the same exercise. We found similar results.

[5] Turnover within each component index is excluded from this comparison.

Exhibits

EXHIBIT 1

Rebalance to Target vs. Rebalance to Bound with a 5% Tolerance Band



For illustrative purposes only.

EXHIBIT 2

Comparing Rebalancing Results for a US 60/40 Portfolio Under Calendar-based and Tolerance Band Rebalancing Approaches, January 1979–December 2019

Rebalancing Method	Number of Rebalancing Events	Average Monthly Return	Annualized Compound Return	Annualized Standard Deviation	Average Equity Allocation
Calendar-Based					
Monthly	492	0.87%	10.39%	9.67%	60.08%
Quarterly	164	0.87%	10.45%	9.64%	60.10%
Annual (December)	41	0.87%	10.44%	9.68%	60.52%
Annual (June)	41	0.87%	10.40%	9.56%	60.22%
Tolerance Band					
Rebalance to Target					
1%	169	0.87%	10.39%	9.67%	60.08%
2.5%	61	0.87%	10.45%	9.69%	60.41%
5%	22	0.88%	10.50%	9.73%	61.22%
7.5%	11	0.88%	10.49%	9.81%	61.91%
10%	7	0.89%	10.64%	9.76%	61.33%
Rebalance to Bound					
1%	248	0.87%	10.44%	9.68%	60.39%
2.5%	168	0.87%	10.49%	9.72%	61.20%
5%	103	0.88%	10.55%	9.85%	62.43%
7.5%	79	0.88%	10.59%	10.04%	63.93%
10%	63	0.89%	10.60%	10.20%	65.20%

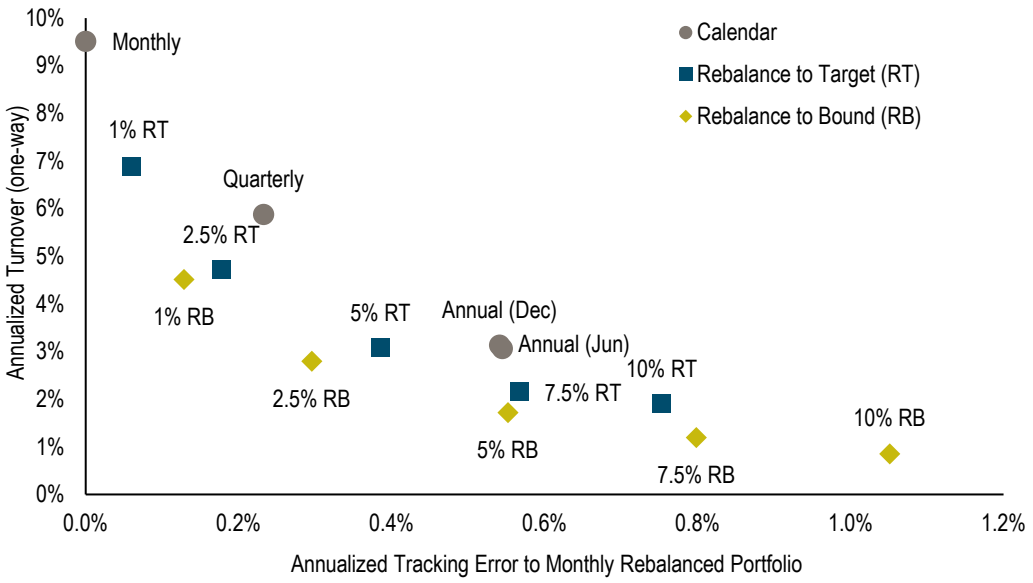
Past performance, including hypothetical performance, is no guarantee of future results.

Filters were applied to data retroactively and with the benefit of hindsight.

The hypothetical US 60/40 portfolio consists of 60% Russell 3000 Index and 40% Bloomberg Barclays US Aggregate Bond Index. Bounds are checked monthly. Rebalancing to target resets the portfolio's asset allocation to its target in case of a rebalancing event. Rebalancing to bound resets the portfolio's asset allocation to the nearest edge of a tolerance band in case of a rebalancing event. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. Indices are not available for direct investment.

EXHIBIT 3

Tracking Error vs. Turnover Tradeoffs Under Calendar-based and Tolerance Band Rebalancing Methods for a US 60/40 Portfolio, January 1979–December 2019

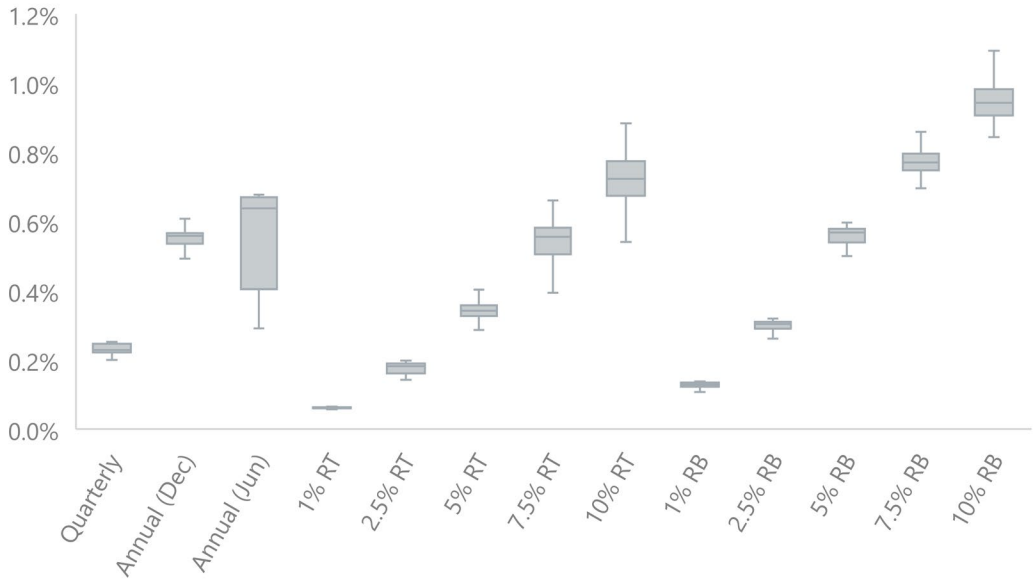


We assume a hypothetical portfolio composed of 60% Russell 3000 Index and 40% Bloomberg Barclays US Aggregate Bond Index. Bounds are checked monthly. Reported turnover is one-way turnover. Annualized tracking error is with respect to a monthly rebalanced hypothetical portfolio. In the chart, "RT" abbreviates "Rebalance to Target" and "RB" abbreviates "Rebalance to Bound". Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. Indices are not available for direct investment.

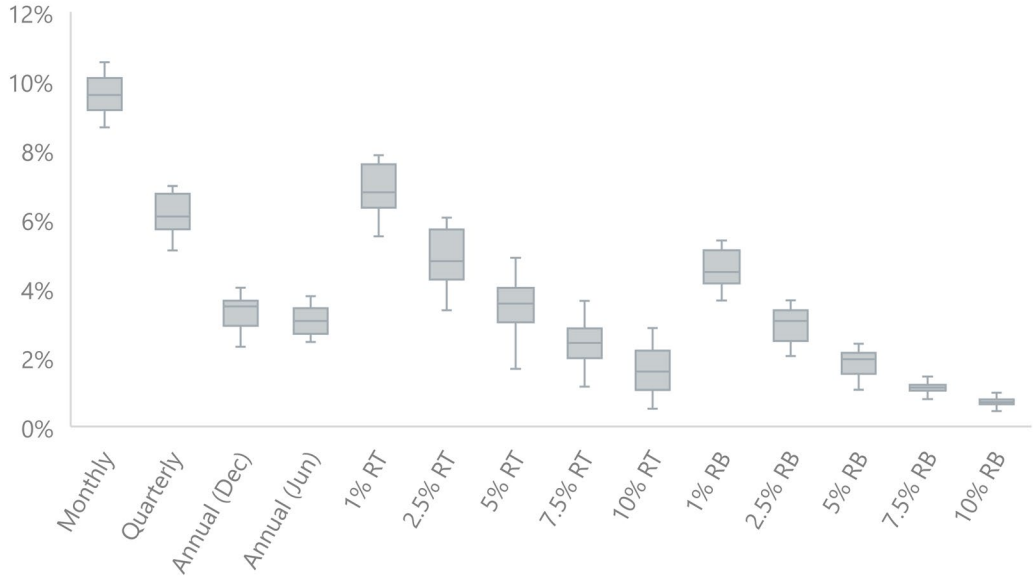
EXHIBIT 4

Distribution of Tracking Error and Turnover Across 20-Year Rolling Periods, US 60/40 Portfolio, January 1979–December 2019

PANEL A: TRACKING ERROR VS. MONTHLY REBALANCED PORTFOLIO



PANEL B: ANNUALIZED ONE-WAY TURNOVER

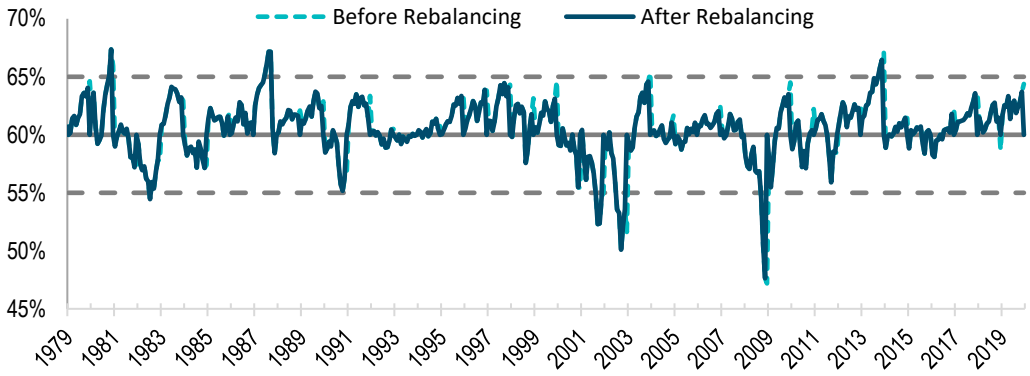


We assume a hypothetical portfolio composed of 60% Russell 3000 Index and 40% Bloomberg Barclays US Aggregate Bond Index. Bounds are checked monthly. Annualized tracking error is with respect to a monthly rebalanced hypothetical portfolio. In the chart, "RT" abbreviates "Rebalance to Target" and "RB" abbreviates "Rebalance to Bound." The box and whisker plot shows the distribution of data into quartiles, with the box representing the range from the 1st to the 3rd quartile. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. Indices are not available for direct investment.

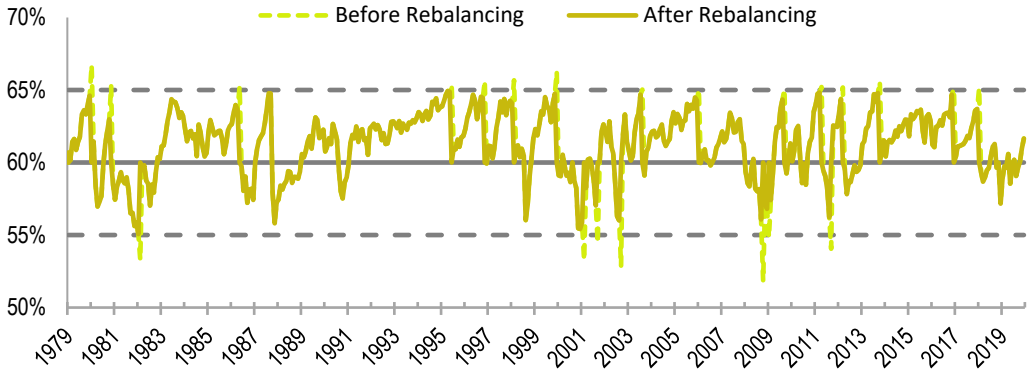
EXHIBIT 5

Weight in Equities Under Different Rebalancing Methods for a US 60/40 Portfolio, January 1979–December 2019

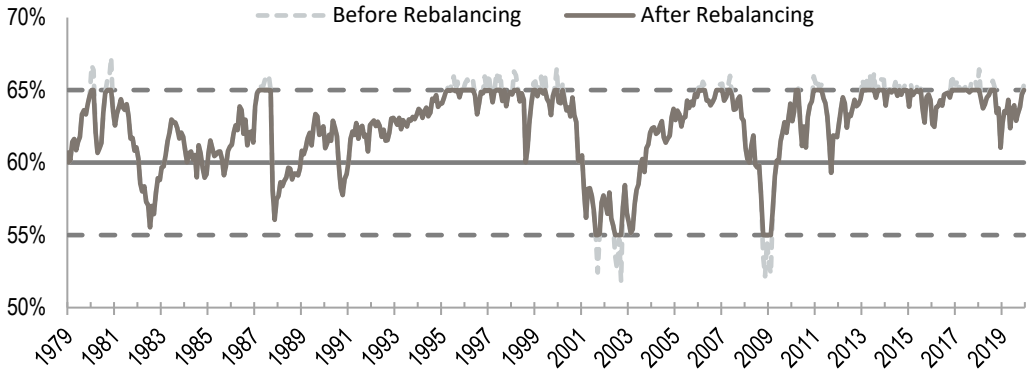
PANEL A: ANNUAL REBALANCING (DECEMBER)



PANEL B: 5% TOLERANCE BAND (REBALANCE TO TARGET)



PANEL C: 5% TOLERANCE BAND (REBALANCE TO BOUND)

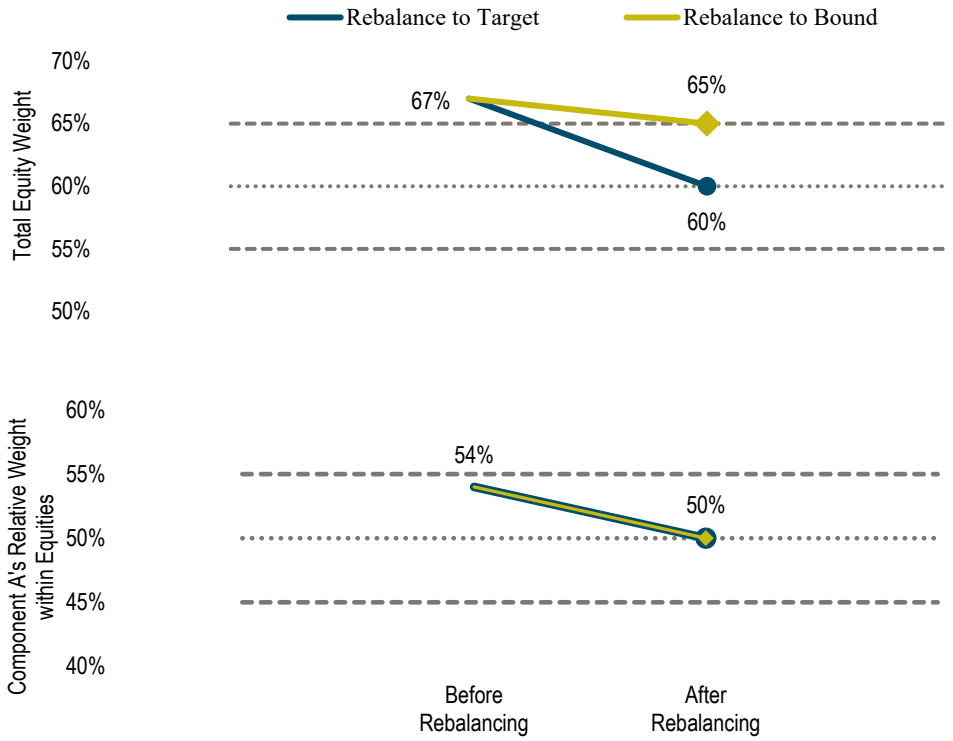


Bounds are checked monthly. Weights are month-end weights before and after rebalancing of the Russell 3000 Index in a hypothetical allocation targeting 60% weight in the Russell 3000 Index and 40% weight in the Bloomberg Barclays US Aggregate Bond Index. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. Indices are not available for direct investment.

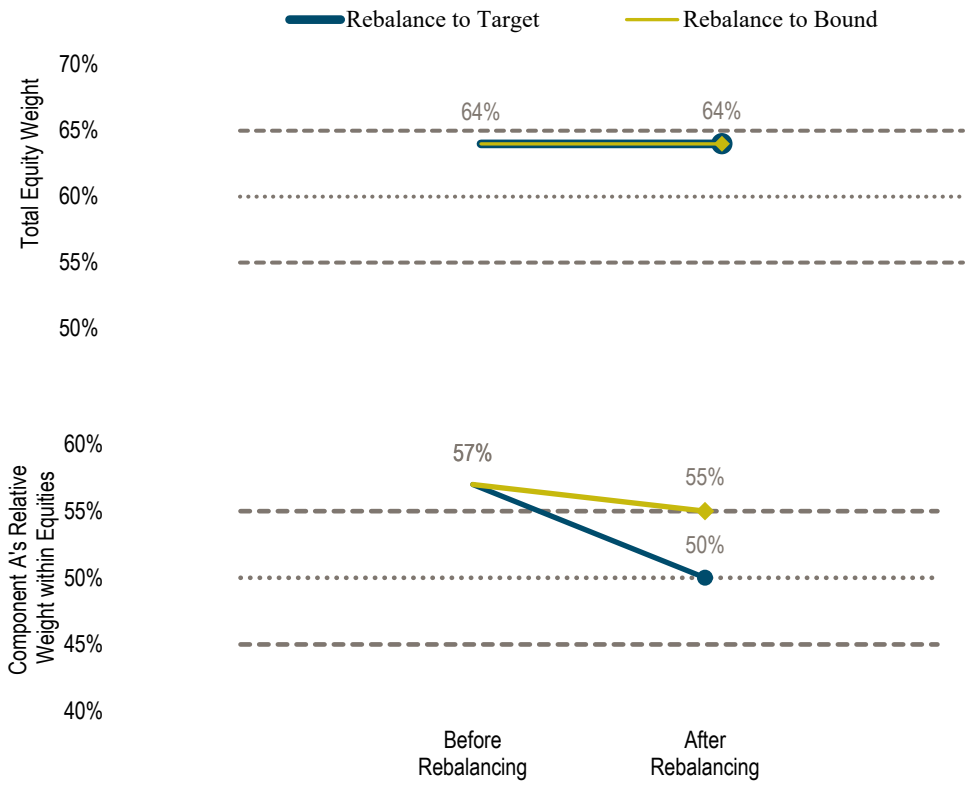
EXHIBIT 6

Tiered Rebalancing with 5%/5% Tolerance Bands

PANEL A: FULLY REBALANCING ALL COMPONENTS



PANEL B: REBALANCING ONLY EQUITY COMPONENTS



For illustrative purposes only.

EXHIBIT 7

Comparing Rebalancing Results for a Global 60/40 Portfolio Under Calendar-based, Tolerance Band, and Tiered Band Rebalancing Approaches, January 1988–December 2019

	Number of Rebalancing Events						
Rebalancing Method	Full Reb.	Equity Only	Fixed Only	Average Monthly Return	Annualized Compound Return	Annualized Standard Deviation	Average Equity Allocations
Calendar-Based							
Monthly	384	—	—	0.72%	8.45%	9.38%	60.05%
Quarterly	128	—	—	0.72%	8.56%	9.34%	60.12%
Annual (December)	32	—	—	0.73%	8.69%	9.24%	60.29%
Annual (June)	32	—	—	0.71%	8.44%	9.12%	60.01%
Rebalance to Target							
Tolerance Band							
1%	140	—	—	0.72%	8.47%	9.36%	60.10%
2.5%	46	—	—	0.73%	8.66%	9.36%	60.33%
5%	18	—	—	0.73%	8.70%	9.42%	61.68%
7.5%	9	—	—	0.77%	9.12%	9.56%	62.32%
10%	6	—	—	0.77%	9.13%	9.57%	62.58%
Tiered Band							
Tiered 1%/1%	141	73	1	0.72%	8.49%	9.38%	60.10%
Tiered 2.5%/2.5%	48	23	0	0.73%	8.58%	9.38%	60.25%
Tiered 5%/5%	20	4	0	0.74%	8.74%	9.46%	61.67%
Tiered 7.5%/7.5%	8	6	0	0.74%	8.69%	9.55%	62.17%
Tiered 10%/10%	6	4	0	0.75%	8.81%	9.65%	63.12%
Rebalance to Bound							
Tolerance Band							
1%	183	—	—	0.72%	8.50%	9.35%	60.32%
2.5%	116	—	—	0.73%	8.60%	9.37%	61.09%
5%	80	—	—	0.74%	8.78%	9.51%	62.75%
7.5%	59	—	—	0.75%	8.88%	9.72%	64.10%
10%	28	—	—	0.74%	8.75%	9.84%	64.69%
Tiered Band							
Tiered 1%/1%	183	71	0	0.72%	8.52%	9.36%	60.32%
Tiered 2.5%/2.5%	115	64	0	0.73%	8.60%	9.38%	61.09%
Tiered 5%/5%	80	21	0	0.74%	8.73%	9.51%	62.74%
Tiered 7.5%/7.5%	59	17	0	0.74%	8.76%	9.69%	64.08%
Tiered 10%/10%	23	24	0	0.73%	8.62%	9.81%	64.65%

Past performance, including hypothetical performance, is no guarantee of future results.

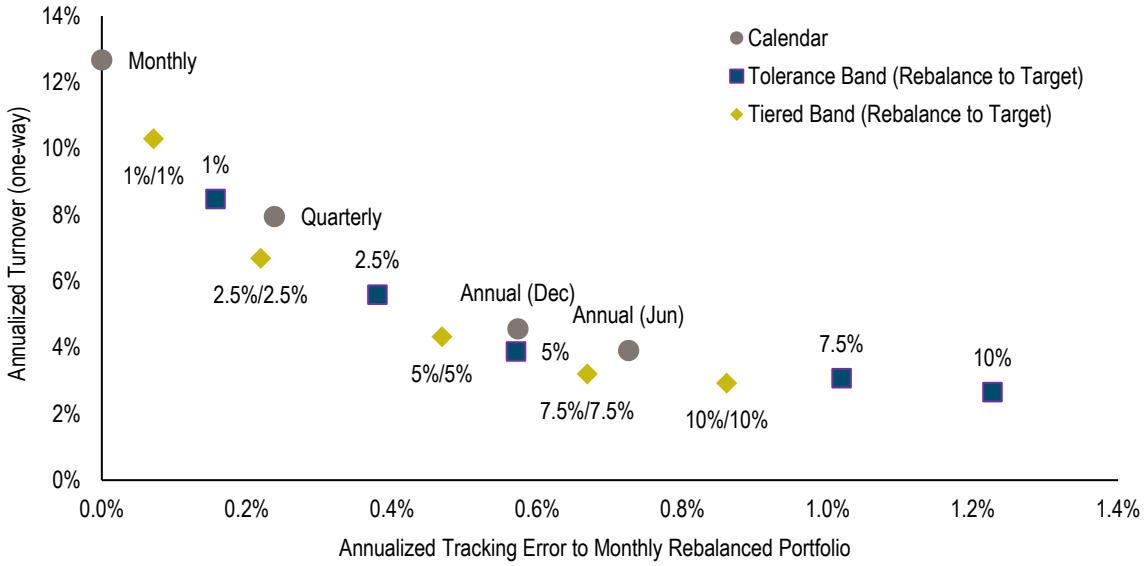
Filters were applied to data retroactively and with the benefit of hindsight.

We assume a hypothetical portfolio composed of 20% Russell 1000 Index, 10% Russell 2000 Index, 20% MSCI World ex USA Index (net div.), 10% MSCI Emerging Markets Index (gross div.), 20% Bloomberg Barclays US Government Bond Index, and 20% Bloomberg Barclays US Credit Bond Index. Bounds are checked monthly. Reported turnover is one-way turnover. Annualized tracking error is with respect to a monthly rebalanced hypothetical portfolio. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. Indices are not available for direct investment.

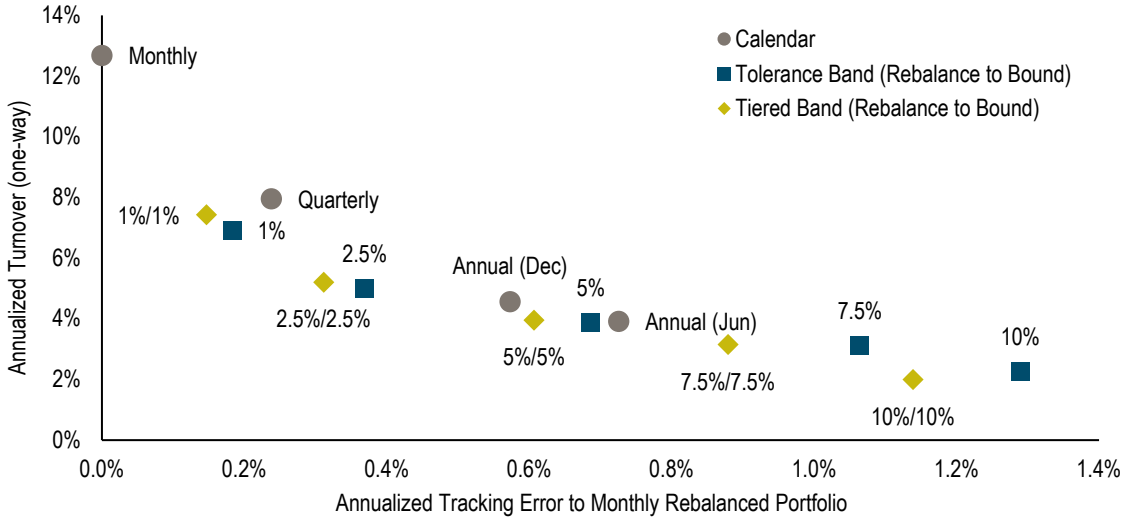
EXHIBIT 8

Tracking Error vs. Turnover Tradeoffs for a Global 60/40 Portfolio Under Calendar-based, Tolerance Band, and Tiered Band Rebalancing Approaches, January 1988–December 2019

PANEL A: REBALANCING TO TARGET



PANEL B: REBALANCING TO BOUND

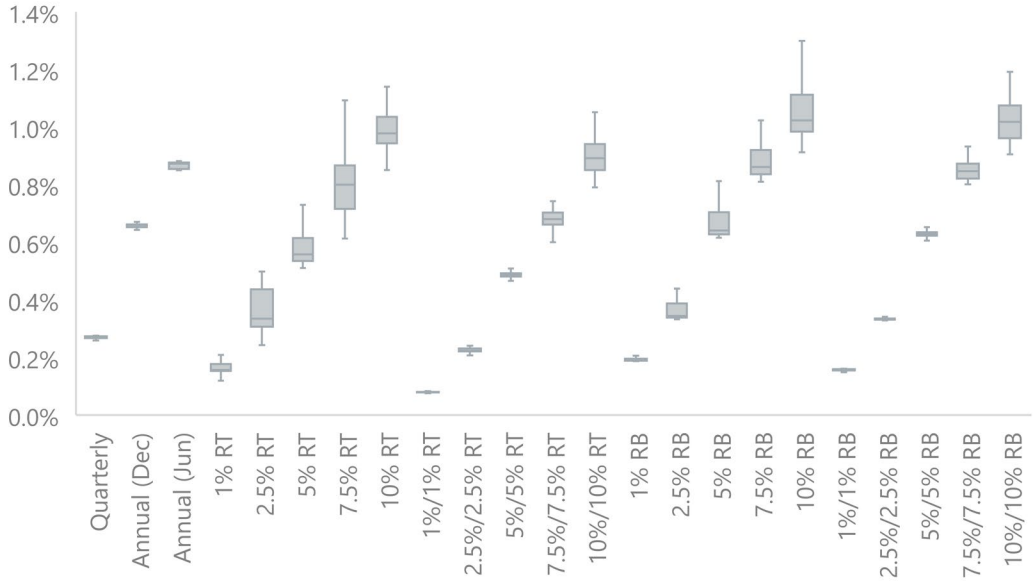


We assume a hypothetical portfolio composed of 20% Russell 1000 Index, 10% Russell 2000 Index, 20% MSCI World ex USA Index (net div.), 10% MSCI Emerging Markets Index (gross div.), 20% Bloomberg Barclays US Government Bond Index, and 20% Bloomberg Barclays US Credit Bond Index. Bounds are checked monthly. Reported turnover is one-way turnover. Annualized tracking error is with respect to a monthly rebalanced hypothetical portfolio. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. MSCI data © MSCI 2021, all rights reserved. Indices are not available for direct investment.

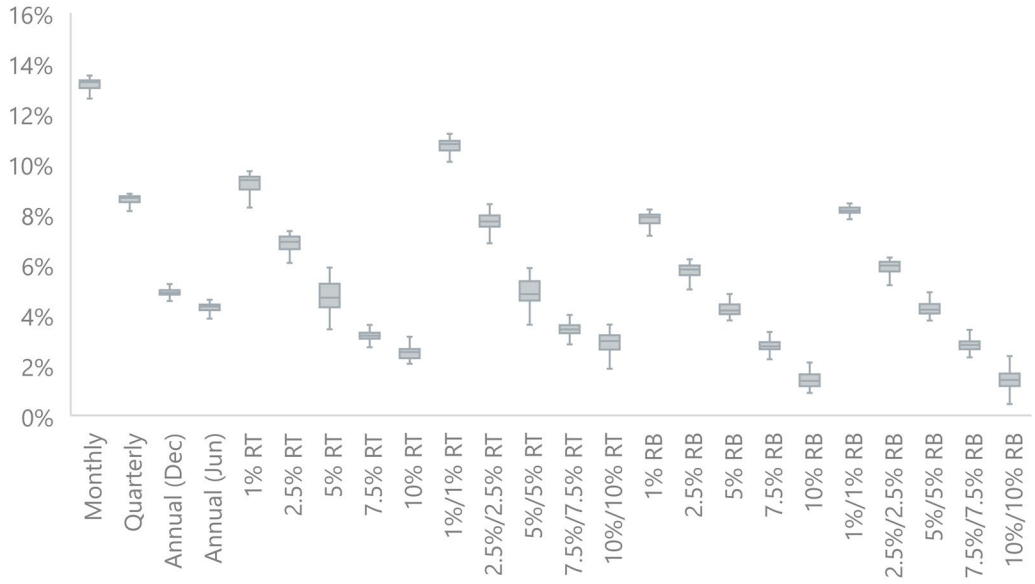
EXHIBIT 9

Distribution of Tracking Error and Turnover Across 20-Year Rolling Periods, Global 60/40 Portfolio, January 1988–December 2019

PANEL A: TRACKING ERROR VS. MONTHLY REBALANCED PORTFOLIO



PANEL B: ANNUALIZED ONE-WAY TURNOVER



We assume a hypothetical portfolio composed of 20% Russell 1000 Index, 10% Russell 2000 Index, 20% MSCI World ex USA Index (net div.), 10% MSCI Emerging Markets Index (gross div.), 20% Bloomberg Barclays US Government Bond Index, and 20% Bloomberg Barclays US Credit Bond Index. Bounds are checked monthly. Annualized tracking error is with respect to a monthly rebalanced hypothetical portfolio. In the chart, "RT" abbreviates "Rebalance to Target" and "RB" abbreviates "Rebalance to Bound." The box and whisker plot shows the distribution of data into quartiles, with the box representing the range from the 1st to the 3rd quartile. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. MSCI data © MSCI 2021, all rights reserved. Indices are not available for direct investment.

EXHIBIT 10

Comparing Return Differences and Maximum Drawdowns for Different Levels of Tracking Error Under Various Rebalancing Approaches

PANEL A: US 60/40 PORTFOLIO, JANUARY 1979–DECEMBER 2019

Rebalancing Method	Annualized Tracking Error	Average Equity Allocations	Maximum Drawdown	Min of 12-Month Rolling Return Difference vs. Monthly Reb.	Max of 12-Month Rolling Return Difference vs. Monthly Reb.
Calendar-Based					
Monthly	—	60.1%	32.7%	—	—
Quarterly	0.23%	60.1%	32.2%	–0.6%	0.8%
Annual (December)	0.54%	60.5%	30.9%	–2.2%	2.0%
Annual (June)	0.54%	60.2%	30.3%	–3.4%	2.4%
Tolerance Band					
Rebalance to Target					
1%	0.06%	60.1%	32.7%	–0.2%	0.2%
2.5%	0.18%	60.4%	32.3%	–0.5%	1.0%
5%	0.39%	61.2%	32.2%	–1.6%	1.2%
7.5%	0.57%	61.9%	32.5%	–2.3%	1.6%
10%	0.75%	61.3%	31.8%	–2.9%	3.7%
Rebalance to Bound					
1%	0.13%	60.4%	32.3%	–0.2%	0.7%
2.5%	0.30%	61.2%	31.9%	–0.7%	1.3%
5%	0.55%	62.4%	31.8%	–1.7%	1.8%
7.5%	0.80%	63.9%	31.9%	–2.3%	2.8%
10%	1.05%	65.2%	30.9%	–3.1%	3.7%

PANEL B: GLOBAL 60/40 PORTFOLIO, JANUARY 1988–DECEMBER 2019

Rebalancing Method	Annualized Tracking Error	Average Weight in Equities	Maximum Drawdown	Min of 12-Month Rolling Return Difference vs. Monthly Reb.	Max of 12-Month Rolling Return Difference vs. Monthly Reb.
Calendar-Based					
Monthly	—	60.0%	36.0%	—	—
Quarterly	0.24%	60.1%	35.5%	–0.3%	0.9%
Annual (December)	0.57%	60.3%	33.9%	–0.7%	3.2%
Annual (June)	0.73%	60.0%	33.5%	–5.3%	2.6%
Rebalance to Target					
Tolerance Band					
1%	0.16%	60.1%	35.8%	–0.6%	0.8%
2.5%	0.38%	60.3%	35.7%	–1.1%	1.9%
5%	0.57%	61.7%	35.5%	–2.2%	2.6%
7.5%	1.02%	62.3%	34.8%	–2.6%	6.1%
10%	1.23%	62.6%	34.3%	–2.6%	5.2%
Tiered Band					
Tiered 1%/1%	0.07%	60.1%	35.9%	–0.1%	0.4%
Tiered 2.5%/2.5%	0.22%	60.2%	35.7%	–0.6%	0.8%
Tiered 5%/5%	0.47%	61.7%	35.5%	–0.8%	2.6%
Tiered 7.5%/7.5%	0.67%	62.2%	36.0%	–1.5%	2.0%
Tiered 10%/10%	0.86%	63.1%	37.4%	–3.1%	4.4%
Rebalance to Bound					
Tolerance Band					
1%	0.18%	60.3%	35.6%	–0.7%	0.8%
2.5%	0.37%	61.1%	35.2%	–1.1%	1.6%
5%	0.69%	62.7%	34.9%	–1.7%	3.4%
7.5%	1.06%	64.1%	35.6%	–2.6%	6.3%
10%	1.29%	64.7%	37.1%	–3.9%	5.2%
Tiered Band					
Tiered 1%/1%	0.15%	60.3%	35.6%	–0.3%	0.8%
Tiered 2.5%/2.5%	0.31%	61.1%	35.2%	–0.7%	1.6%
Tiered 5%/5%	0.61%	62.7%	34.9%	–1.3%	2.1%
Tiered 7.5%/7.5%	0.88%	64.1%	35.6%	–2.0%	3.1%
Tiered 10%/10%	1.14%	64.7%	37.1%	–3.9%	3.2%

Past performance, including hypothetical performance, is no guarantee of future results.

Filters were applied to data retroactively and with the benefit of hindsight.

The hypothetical US 60/40 portfolio composed of 60% Russell 3000 Index and 40% Bloomberg Barclays U.S. Aggregate Bond Index. The global 60/40 portfolio composed of 20% Russell 1000 Index, 10% Russell 2000 Index, 20% MSCI World ex USA Index (net div.), 10% MSCI Emerging Markets Index (gross div.), 20% Bloomberg Barclays US Government Bond Index, and 20% Bloomberg Barclays US Credit Bond Index. Bounds are checked monthly. Reported turnover is one-way turnover. Annualized tracking error is with respect to a monthly rebalanced hypothetical portfolio. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. Indices are not available for direct investment.

EXHIBIT 11

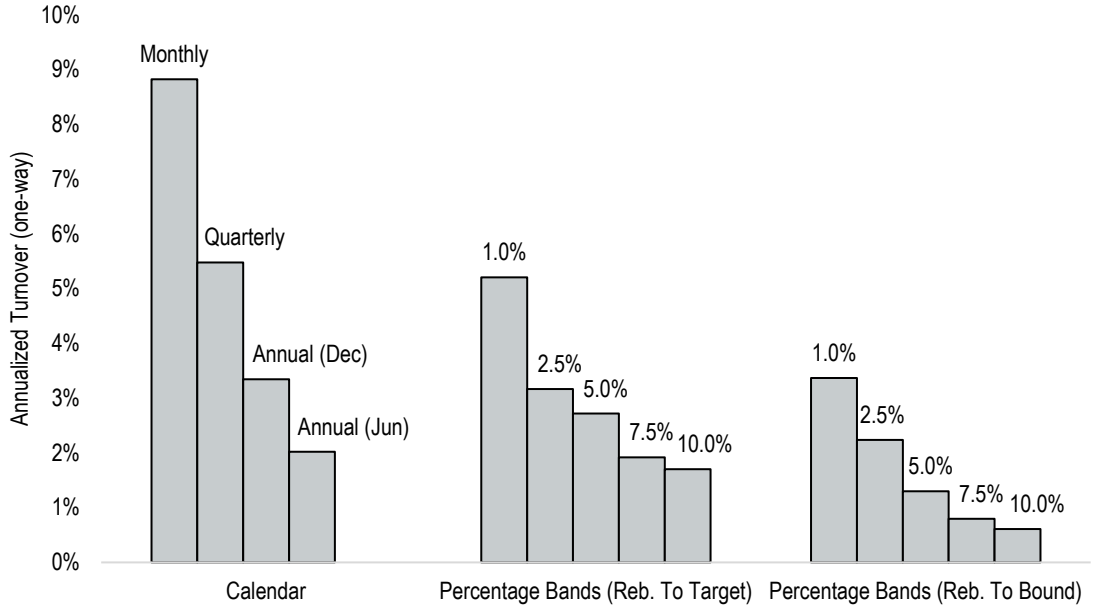
Annualized One-Way Tracking Error vs. Monthly Rebalanced Portfolio with Different Equity Allocations for Global Portfolios, January 1988–December 2019

Rebalancing Method	80% Equity	60% Equity	40% Equity	20% Equity
Calendar-Based				
Monthly	—	—	—	—
Quarterly	0.20%	0.24%	0.22%	0.15%
Annual (December)	0.48%	0.57%	0.53%	0.34%
Annual (June)	0.59%	0.73%	0.66%	0.42%
Rebalance to Target				
Tolerance Band				
1%	0.30%	0.16%	0.12%	0.11%
2.5%	0.88%	0.38%	0.28%	0.29%
5%	1.13%	0.57%	0.57%	0.46%
7.5%	1.43%	1.02%	0.81%	0.61%
10%	1.77%	1.23%	1.02%	0.75%
Tiered Band				
Tiered 1%/1%	0.07%	0.07%	0.07%	0.06%
Tiered 2.5%/2.5%	0.25%	0.22%	0.20%	0.19%
Tiered 5%/5%	0.48%	0.47%	0.42%	0.40%
Tiered 7.5%/7.5%	0.74%	0.67%	0.63%	0.52%
Tiered 10%/10%	0.99%	0.86%	0.78%	0.63%
Rebalance to Bound				
Tolerance Band				
1%	0.24%	0.18%	0.16%	0.13%
2.5%	0.60%	0.37%	0.32%	0.31%
5%	1.06%	0.69%	0.61%	0.56%
7.5%	1.47%	1.06%	0.93%	0.74%
10%	1.81%	1.29%	1.12%	0.93%
Tiered Band				
Tiered 1%/1%	0.15%	0.15%	0.14%	0.12%
Tiered 2.5%/2.5%	0.35%	0.31%	0.29%	0.28%
Tiered 5%/5%	0.66%	0.61%	0.57%	0.51%
Tiered 7.5%/7.5%	1.00%	0.88%	0.82%	0.68%
Tiered 10%/10%	1.33%	1.14%	1.02%	0.88%

The global portfolio composed of X% equity and (1-X%) fixed income, where X=20, 40, 60, and 80. Within equity, X%/3 Russell 1000 Index, X%/6 Russell 2000 Index, X%/3 MSCI World ex USA Index (net div.), X%/6 MSCI Emerging Markets Index (gross div.); within fixed income, (1-X%)/2 Bloomberg Barclays US Government Bond Index, and (1-X%)/2 Bloomberg Barclays US Credit Bond Index. Bounds are checked monthly. Reported turnover is one-way turnover. Annualized tracking error is with respect to a monthly rebalanced hypothetical portfolio. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. MSCI data © MSCI 2021, all rights reserved. Indices are not available for direct investment.

EXHIBIT 12

Turnover Under Different Rebalancing Approaches for a Global Equity Portfolio with Multiple Components, June 1994–December 2019



We assume a hypothetical portfolio composed of 22.5% Russell 1000 Value Index, 22.5% Russell 1000 Growth Index, 2.5% Russell 2000 Growth Index and 2.5% Russell 2000 Value Index, 36% MSCI World ex USA Index (gross div.), 4% S&P Developed ex US SmallCap Index (gross div.), 9% MSCI Emerging Markets Index (gross div.), and 1% MSCI Emerging Markets Small Cap Index (gross div.). Bounds are checked monthly. Reported turnover is one-way turnover. Source: Dimensional. Bloomberg Barclays data provided by Bloomberg. Copyright 2021 S&P Dow Jones Indices LLC, a division of S&P Global. Frank Russell Company is the source and owner of the trademarks, service marks, and copyrights related to the Russell Indexes. MSCI data © MSCI 2021, all rights reserved. Indices are not available for direct investment.

Reference

Dai, Wei, Namiko Saito, and Stephen Watson. 2021. "Pursuing Multiple Premiums: Combination vs. Integration." Available at SSRN: https://ssrn.com/abstract=3793594.

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