The Argument for Bonds in Strategic Asset Allocation

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Abstract

A number of well-known practitioners such as Warren Buffett and Jeremy Siegel have long advocated a strategic asset allocation in which investors hold a majority of their assets in equities. However, in this simple straightforward study we find that in order to maximize the well-known Sharpe ratio, most investors over the period 1995-2019 would have been better off holding a majority of their assets in bonds rather than stocks. While interest rates have decreased during this twenty-five year period and thus enhanced bonds relative returns, our results are quite convincing. Even though our results are obviously ex post, investors very often make decisions on future investment based on past performance and thus our results could provide guidance to investors for future investing decisions.

Key words: Strategic Asset Allocation, Asset Classes, Investing, Optimization, Sharpe Ratio

1. Introduction

What is the optimal strategic allocation of assets for the average person? That is, how much should the average person allocate to stocks, bonds, developed markets, emerging markets, real estate, or commodities? In the literature there is wide array of opinions on this question. For example, Warren Buffet recommends that the average person should hold 90 percent of their assets in the Standard and Poor's 500 index and the remaining 10 percent in short-term government bonds.[1] Jeremy Siegel, the well-known Wharton school finance professor and author of the book Stocks for the Long Run, recommends 75 percent in the U.S. stock market and 25 percent in bonds.[2]

In the endowment space, David Swensen, the well-known CIO of the Yale endowment, recommends a more complicated portfolio of 30 percent US large cap stocks, 15 percent foreign developed stocks, 5 percent emerging market stocks, 15 percent in long-term U.S. government bonds, 15 percent in Treasury Protected Investment Securities (TIPS), and 20 percent in real estate investment trusts.[3] Similarly, Mohamad El-Erian, former Harvard endowment manager and former head of Pimco, recommends 18 percent in U.S. large cap stocks, 18 percent in foreign developed stocks, 15 percent in emerging market stocks, 6 percent in long-term U.S government bonds, 11 percent in foreign bonds, 6 percent in TIPS, 13 percent in commodities and 13 percent in real estate investment trusts.[4] Finally Fidelity, one of the largest mutual fund companies in the world, recommends that the average person hold a portfolio of 49 percent U.S. large cap stocks, 21 percent

[1] See page 20 of letter to Berkshire Hathaway's investors in 2013. See https://www.berkshirehathaway.com/letters/2013ltr.pdf.

[2] See https://www.cnbc.com/2020/02/08/whartons-jeremy-siegel-60-40-portfolio-doesnt-cut-it-anymore.html.

[3] See Swensen (2005), p. 34.

[4] El-Erian (2008), p. 198.

foreign developed stocks, 5 percent short-term U.S. government bonds and 25 percent U.S. government long-term bonds.[5]

With all these differing opinions what is the average investor to do when determining their strategic asset allocation? In this paper we investigate optimal strategic asset allocation through examining what portfolio weights would have maximized the well-known Sharpe ratio, which is the ratio of portfolio return in excess of the risk-free return to portfolio standard deviation. It provides a simple, easy-to-understand measure of risk-adjusted return that investors have seen for over 50 years.

In this study we look at 16 overlapping 10-year periods (over the period 1995-2019) to determine what weights in a portfolio would have maximized the Sharpe ratio for a range of differing levels of risk. By doing this we show how the optimal portfolio would differ depending upon the risk-level chosen by the investor.

Using our approach, we find that at most levels of risk, domestic and emerging market bonds should have been a significantly larger portion of one's strategic asset allocation than what most of the experts (as profiled above) indicate. Specifically, we find that with a portfolio with a risk level equal to a four percent annualized standard deviation, the investor should have held, on average, a portfolio of approximately 79 percent in a U.S. aggregate bond index in order to maximize the Sharpe ratio. But as we allowed the risk level to increase, the percentage held in U.S. bonds decreases substantially in favor of emerging market bonds. Indeed, for a portfolio constrained to be equal to a 10 percent annualized standard deviation, we find the percentage that should have been held in emerging market bonds is a remarkable allocation of 62.87 percent on

[5] See Fidelity Investments (2018), p. 5.

average. Hence, in sum, while most expert opinion suggests that investors would be better off with strategic allocation of 50 percent or more in equities, our research indicates that domestic and emerging market bonds should have been a much larger allocation in order to maximize the Sharpe ratio at most risk levels. Only at the highest risk levels do we find that the investor should hold more than 50 percent in equities and most of these should be in emerging markets.

What do these results mean for investors? Of course, our results are ex post and based on 10-year horizons during a twenty-five year period (1995-2019) when interest rates have generally declined and thus improved the relative performance of bonds. And there is no way to know if such low interest rates will hold in the near future. But that being said, our results are quite convincing. Bonds whether they be domestic or emerging markets should have been a much larger proportion of one's portfolio than is often touted. Moreover, investors very often make future investment decisions based on past performance and thus our results could provide guidance to investors for future investing decisions. Also, there is relatively little recent evidence that bond yields are predictive of future 10-year stocks returns. For example, in a Wall Street Journal article (June 7, 2020) Rob Arnott states that the R-squared between bond yields and future 10-year stock returns is a mere three percent, indicating that bond yields are not very predictive of future long-term stock returns.[6] Hence, the low bond yields that exist today do not necessarily imply that stock returns will be better over the next 10-years and thus give further credence to our results suggesting that bonds should have a higher weight in most investor's strategic asset allocations.

[6] Mark Hulbert, "Will Stocks Trail Bonds Over the Next Decade?" *Wall Street Journal*, June 7, 2020. As a comparison consider that the cyclically adjusted price/earnings ratio (or CAPE) developed by John Campbell and Robert Shiller has an R-squared over 50% when predicting long-term stock returns.

The rest of this paper is organized as follows. Section 2 describes the data. Section 3 describes our methodology. Section 4 explains the results. Section 5 provides a robustness test and we conclude with Section 6.

2. Data

The data for the paper are all obtained from Morningstar. We use index data from all the major asset classes including domestic equities, domestic bonds, commodities, emerging market equities, emerging market bonds, real estate, developed market equities, and developed market bonds. Since the data for some of the bond indexes only go back to the 1995, the monthly returns are extracted between January 1995 and December 2019.

For domestic equities we use the Standard and Poor's 500 index. For domestic bonds we use the Bloomberg Barclays US Aggregate Bond Index. For commodities we use the S&P GSCI Index. For emerging market equities we use the MSCI Emerging Market Index. For emerging market bonds we use the Bloomberg Barclays Emerging Market USD Aggregate Bond Index. For real estate we use the Dow Jones US Real Estate Index. For developed country equities we use the Standard and Poor's Developed Ex-US Broad Market Value Index and for developed country bonds we use the Bloomberg Barclays Global Aggregate Ex USD Bond Index.[7] For the risk-free rate we use the 1-month T-Bill return from Ibbotson and Associates, Inc. extracted from Kenneth French's website.[8]

[7] Note that this index is not purely developed markets. While 96 percent of the index is composed of developed market countries it does contain about four percent in emerging markets.

[8] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html

3. Methodology

Mathematical optimization is performed to retrospectively identify the weights of the asset classes described above that a portfolio should have allocated so as to maximize the Sharpe ratio. The optimization is performed separately for 16 overlapping 10-year time periods and across a range of fixed levels of risk.

For the fixed levels of risk we begin by finding the optimal portfolio weights for a portfolio with a standard deviation equal to 4 percent. We then find the optimal portfolio weights for a portfolio with a standard deviation equal to 5 percent. We continue this process for each successive one unit increase in standard deviation unit we reach 17 percent. Hence, for each of the 16 10-year time periods, we have the portfolio weights to maximize the Sharpe ratio for fixed levels of risk ranging from 4 percent to 17 percent standard deviation.

We only use the fixed levels of risk between 4 percent and 17 percent standard deviation because these represent the levels of portfolio standard deviation that are possible for all of the 16 10-year investment periods. Other levels of standard deviation are excluded as they are not possible in all 16 10-year periods. Indeed, there are some levels of standard deviation (really high or low) that are only possible during a few of the 16 10-year periods.

Hence, the decision variables in the optimization problem are the weights in the various asset classes; the objective is to maximize the Sharpe Ratio; and the optimization problem is subject to constraints regarding the weights, portfolio return, and portfolio standard deviation. Specifically, the weights must sum to one; the annualized portfolio average return is the weighted average of the annualized average return of the indexes in the portfolio; and the annualized portfolio standard deviation must be equal to the specified fixed level. Appendix 1 details the

equations used to calculate performance measures and the optimization problem used to implement the optimization.

4. Results

As stated above we use data from 1995-2019. With these data we then examine the 16 10-year overlapping samples that span 1995-2019. We use 10-year samples as this captures a period long enough to implement strategic assets allocation and is a common time horizon used by investors. The first 10-year sample is 1995-2004, the next is 1996-2005, and subsequent samples are formed until the last one which is 2010-2019. We use the 16 overlapping periods so as to ensure our results are not just reflective of any particular time period.

In Table 1 we provide, for each asset class, the average annual returns, average standard deviation and the average Sharpe ratio across the 16 10-year periods. Hence, the table shows the average of these measures across the 16 different 10-year time periods. So for example the Sharpe ratio in Table 1 represents the average of the 16 Sharpe ratios of the 16 different 10-year periods.

We find that the domestic bonds had the highest average Sharpe ratio at 0.95 followed by emerging market bonds with a ratio of 0.76. The asset class with the lowest average Sharpe ratio was commodities with a ratio of -0.06. Table 1 clearly shows that in terms of the Sharpe ratio, domestic and emerging market bonds outperform all other asset classes.

Figure 1 then plots the 16 10-year Sharpe ratios that were used to calculate the averages used in for the Sharpe ratio in Table 1. The graph clearly shows that the Sharpe ratios of domestic bonds and emerging market bonds consistently attain the highest 10-year Sharpe ratios throughout 1995-2019 time period. Hence, no single 10-year time period skews the results. Figure 1 also shows how the Sharpe ratio of commodities dramatically declines in the later 10-year periods.

In Table 2 we provide the monthly correlations over the entire period 1995-2019 between the various asset types. Compared to other asset types, both the domestic bonds and commodities have substantially lower correlations to other assets.

In Figures 2-3 and Tables 3-4 we present the main results of the paper. In this analysis we show, for different levels of risk, the strategic asset allocation weightings that would maximize the average 10-year Sharpe ratio. As stated in Section 3, to deal with different levels of risk preferences we fix the annualized standard deviation to be equal to at a certain level and then examine what weighting of assets would maximize the Sharpe ratio. We examine fixed levels of risk starting at 4 percent standard deviation and then repeatedly increase the standard deviation constraint by one standard deviation unit until reaching a standard deviation constraint of 17 percent.

So if an investor wants a low risk portfolio we identify the strategic asset allocation weightings that would optimize the Sharpe ratio using a low standard deviation constraint. Conversely, for an investor who is willing to take on higher risk, we identify the weightings of assets that would maximize the Sharpe ratio at these higher levels of risk.

We then complete the above described optimization for each of the 16 ten-year samples. So for the 1995-2004 sample we calculate the asset allocation weightings that maximize the Sharpe ratio at each level of risk from 4 percent through 17 percent standard deviation. We then repeat this for 1996-2005 and every ten-year period until 2010-2019. Then, rather than present the results for each of the ten-year time periods, we present, for each level of risk, the averages of the 16 ten-year samples. So for example, in Table 3, the 6.96 percent weighting for domestic equity in the four percent standard deviation case (the first row of Table 3), is the average weight for domestic equity from the 16 10-year periods.

From left to right, the first column of Table 3 shows the level of standard deviation at which the portfolio is constrained. As explained above, this ranges from 4 percent to 17 percent. The second column provides the average annual return generated by the portfolio and the third column shows the average Sharpe ratio. The next columns show the average weights that would be assigned to each of the asset classes.

The results in Table 3 provide some interesting results. First, in a simple analysis of the standard deviation and returns (columns 1 and 2) we see there are benefits to increasing risk in terms of increased return until the standard deviation reaches about 13 percent. After this point, the increases in risk do not cause commensurate increases in the average annual return. We illustrate this issue with Figure 2 which plots the first two columns of Table 3.

Second, at low the levels of risk, Table 3 shows that the portfolio that maximizes the Sharpe ratio consists mainly of domestic bonds. Indeed, at the 4 percent standard deviation risk level, the portfolio consists of 79.02 percent of domestic bonds. As the risk level is increased, the weightings shift from domestic bonds to emerging market bonds. Indeed, at the 10 percent standard deviation level, emerging market bonds represent 62.87 of the weight of the portfolio. Even as we increase the risk levels to 12 percent standard deviation we still find that more than 50 percent of the portfolio would be in bonds, whether they are domestic, emerging markets or developed markets. This result is further showcased in Table 4 which shows the optimal weightings of equities versus bonds (whether they be domestic, emerging market or developed markets). The table simply aggregates the percentage of assets in domestic, emerging and developed equities into equities and does the same for bonds. These results indicate that at low to moderate levels of risk, the Sharpe ratio is maximized by having higher weightings of bonds than many experts have recommended.

Second, contrary to Warren Buffett and Jeremy Siegel who argue that domestic equities should be the vast majority of one's holdings, we find that in most situations the weight of domestic equities is relatively low. Indeed, the highest weight for domestic equities is 29.76 percent at the 15 percent standard deviation risk level. In every other risk level, the weighting of domestic equities is significantly lower than this.

Third, we find that weightings for developed international equity and bonds are very small at all risk levels. In fact, it is never the case that combined weights of developed market equity and developed market bonds rise above 6 percent. These results are contrary to Solnik (1974) and Odier and Solnik (1993) who argue that developed international equities and bonds should consists of 20-30 percent of the average person's portfolio.

Fourth, we find that the weights to commodities is abysmally low at all levels of risk. The proportion to commodities is never more than 1.61 percent and in many cases the amounts are below 1%. These results are quite different than much of the literature which has advocated a relatively high proportion of assets to commodities. For example, Erb and Harvey (2006) advocate that for the investor who is comfortable with the volatility of a 60%/40% stocks/bonds portfolio, their portfolio should consist of 18 percent in commodity futures.

Fifth, at the highest levels of risk, the types of assets that should be held more are emerging market equities. Once the risk level hits a standard deviation of 12 percent, the percentage of assets to emerging markets stocks starts to increase dramatically. Indeed, at the highest level of risk, the 17 percent standard deviation, the allocation is 42.50 percent to emerging market stocks. However, as we mentioned before, the risk premium for bearing this increased risk is very small if at all. Above the 13 percent standard deviation level of risk, the average annual returns decline. Higher risk does not translate into higher return after this point.

The results in Table 3 are visually presented in Figure 3 which shows for each level of risk, the optimal portfolio weights by color. One can see that at low risk levels the percentage of assets to domestic bonds is high but as we increase the risk level we move out of domestic bonds and into emerging market bonds. At a risk level of about 12 percent standard deviation and above, the situation changes and investors should hold more emerging market stocks and possibly real estate. Throughout the various risk levels we find that domestic equities, developed market equities, developed market bonds, and commodities are not held at very high levels compared to what is often talked about in the literature or by investing experts.

5. Robustness Check of the Results

One possible problem with our results in Table 3-4 and Figure 3 is that the averaging of the 16 10-year periods is putting too much weight on the middle part of the 1995-2019 sample. Years in the middle of 1995-2019 are unduly represented in the sample and hence may be influencing the results. To deal with this issue, we provide Tables 5-7. These tables report the portfolio weightings for each of the 10-year periods at a specified level of risk. For example, Table 5 shows the portfolio weighing for each of the 16 10-year periods in the low risk condition (4 percent annualized standard deviation). Tables 6 and 7 show the same as Table 5 but at a medium level of risk (11 percent annualized standard deviation) and at the highest level of risk (17 percent annualized standard deviation). Hence, the reader can now see how the portfolio weightings change across the different 10-year time periods for low-risk, medium-risk, and high-risk conditions.

The results of Table 5 (the low-risk case of 4 percent annualized standard deviation) show that across all the time periods, the percentage of assets to domestic bonds is consistently at a high

level. The percentage to this index never drops below 54.90 percent (this in the 2009-2018 sample) and is often in the 70 or 80 percent range.

Table 6 (the medium risk case of 11 percent annualized standard deviation) shows that the percentage of assets to emerging market bonds is, for the most part, quite high but variable. There is a period in the early 2000s when the percentage to emerging market bonds is extremely high; for the 10-year samples that range between 1999-2008 and 2005-2014 (representing seven different 10-year periods), the percentage to emerging market bonds is usually 80 to 90 percent of the entire portfolio. Indeed, there is no 10-year period which includes the year 2008 that has a percentage weighting of less than 50 percent in emerging market bonds. So it is possibly that the inclusion of this year is driving our average results. Conversely, there are 10-year periods where the percentage to emerging market bonds is much lower. In 2009-2018 the percentage to emerging market bonds is only 25.81 percent and in 2010-2019 it is only 13.70 percent. Indeed, for 10-year periods that do not include the year 2008, the percentage weighting to emerging markets bonds averages only 28.36 percent.

While there does seem to be some variability in the asset allocation to emerging markets bonds across the 10-year samples, there is even greater variability of the weightings in other asset classes for Table 6. For example, for domestic equities, the results indicate that investor should hold little or no domestic equities for every 10-year period up until 2006-2015. After this time the weightings become quite high with domestic equities reaching 81.04 percent in the 2010-2019 time period. Similarly, the real estate index indicates that one should hold about 50 percent of their assets in this asset class for the first three 10-year periods (1995-2004, 1996-2005, 1997-2006) and then hold little or no assets in this class for every other 10-year period. So given the variability in other asset classes, our average results in Table 3 showing the investor (at the 11 percent

annualized standard deviation) should hold a high percentage in emerging market bonds seems a reasonable approximation.

Finally, in Table 7 (the high risk case of 17 percent annualized standard deviation) we again see the results are quite dependent upon the 10-year period chosen. For every 10-year period up until 2014, the percentage in emerging market stocks is quite high but then it completely drops off to zero in the 2006-2015 period and then remains there until the 2010-2019 period when it increases to 98.03 percent. For domestic equities the percentage is zero for many of the 10-year periods and then rises dramatically to 71.23 percent in the 2007-2016 period.

In sum, the robustness check does show that in some cases that the weightings are dependent on the time period. By choosing one 10-year period one can get very different results than another 10-year period. Indeed, investors should see the results over many different time periods as the period chosen can impact the results greatly. Given this we believe our averages shown in Table 3 are the reasonable way to capture the general tendencies of the data.

6. Conclusions

As we stated in the introduction, if an investor was to ask famous investors on what should be their strategic asset allocation, they would usually receive advice that they should hold the vast majority of their assets in equities. Indeed, Warren Buffett has recommend a strategic asset allocation of 90 percent in domestic equities. In this paper we try to answer this question for investors by determining what weights in a portfolio would maximize the Sharpe ratio for a range of differing levels of risk and many different 10-year time horizons (from 1995-2019). We believe that the Sharpe ratio method is useful to investors as it is grounded in the Modern Portfolio Theory and is

commonly used and reported by mutual funds and other financial service firms. Moreover, investors are familiar with it as it has been in use for over 50 years.

Using the averages of 16 overlapping 10-year time horizons, we find that at most levels of risk, bonds, whether they be U.S. or emerging market, should be a significantly larger portion of one's strategic asset allocation than what most of the experts have previously stated. Only at the highest risk levels do we find that the investor should hold more than 50 percent in equities and most of these should be in emerging markets. We also find that the gains to increased risk are very little to none after a certain point. Indeed, above a level of risk of 13 percent standard deviation, we find the average annual return actually declines.

In addition, we find that the strategic asset allocation weightings are quite sensitive to the time period chosen for evaluation. Using 10-year time periods, we find that strategic allocations to various asset classes can vary widely depending upon the time period chosen. For example, over the period 1999-2008 (for moderate risk of 11 percent standard deviation) we find the optimal allocation is 99.09 percent to emerging market bonds. However, over the period 2010-2019 we find that this percentage to emerging market bonds falls off to only 13.70 percent. Similarly, we find the same sort of results with domestic equities, emerging equities and real estate. Namely the 10-year time period chosen greatly impact the strategic allocation to the portfolio. Given this large variability in the results, we believe our use of the average of the 16 10-year periods is a reasonable approximation for investors to consider.

Our conclusions that bonds should have been a bigger proportion of the holdings of most investors is not unique. There are some recent papers that have touted the idea that bonds outperform stocks. For example McQuarrie (2019) has found that domestic bonds outperformed domestic equities in 38.7% of all 10-year periods since 1793. Moreover, McQuarrie finds that in

many of the 10-year periods where equities outperformed, bonds lagged behind equities by less than an annualized percentage point. Indeed, if one counts all the 10-year periods in which bonds either beat stocks or lagged behind by less than an annualized percentage point, bonds held their own 51% of the time since 1793. Given that bonds have significantly less volatility than bonds such a record gives some credence to the idea that bonds may outperform stocks on a risk-adjusted basis. Similarly, Arnott (2009) finds results that bonds outperform stocks. Using data that ends of as of February 2009 (which of course was when the stock market was still in midst of the 2008 financial crisis), he finds that for 10 years, 20 years, even 40 years, ordinary long-term Treasury bonds have outpaced the broad stock market.

As we stated in the introduction our results have implications for investors. While our results are based on a twenty-five year period (1995-2019) when interest rates have largely declined and thus improved the relative performance of bonds, our results, we believe, are quite helpful to investors going forward. Since investors rely on the past performance to guide their future investing decisions and since there is relatively little evidence that bond yields predict long-term equity returns, our results could provide guidance to investors for future investing decisions. Based on the past 25 years, bonds whether they be domestic or emerging markets should have been a much larger proportion of one's portfolio than is often stated by finance professionals and commentators. If the past is prologue investors should consider holding greater proportions of their assets in bonds.

Appendix 1

Performance measures are calculated as follows. Let:

$\bar{r}_i$ = annualized average return for index *i*

σ_i = annualized standard deviation for index *i*

$\bar{r}_f$ = annualized average return for the risk-free rate of return

r_{ij} = monthly return for index *i* in month *j*

r_{fj} = monthly return for the risk-free rate in month *j*

n = number of months in the given time period

$Sharpe\ Ratio_p$ = Portfolio Sharpe Ratio

$\bar{r}_p$ = annualized portfolio average return

σ_p = annualized portfolio standard deviation

w_i= weight of index *i* in the portfolio being maximized

ρ_{ij} = correlation between the returns of indexes *i* and *j*

m = number of indexes in the given universe

σ_K = A specified fixed level of standard deviation

$\bar{r}_i$, $\bar{r}_f$, and σ_i are calculated are as follows:

$$\bar{r}_i = \prod_{j=1}^{n}\left(1 + r_{ij}\right)^{\frac{12}{n}} - 1 \quad (1)$$

$$\bar{r}_f = \prod_{j=1}^{n}\left(1 + r_{fj}\right)^{\frac{12}{n}} - 1 \quad (2)$$

$$\sigma_i = \left(12 \cdot \frac{\sum_{j=1}^{n}\left(r_{ij} - \bar{r}_i\right)^2}{n-1}\right)^{\frac{1}{2}} \quad (3)$$

The optimization problem that maximizes the Sharpe Ratio solves for the weights in the given universe of indexes that maximizes the following equation,

$$Sharpe\ Ratio_p = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p}, \quad (4)$$

subject to the following constraints:

$$\sum_{i=1}^{m} w_i = 1 \quad (5)$$

$$w_i \geq 0,\ i = 1 \ldots m \quad (6)$$

$$\bar{r}_p = \sum_{i=1}^{m} w_i \cdot \bar{r}_i \quad (7)$$

$$\sigma_p = \left(\sum_{i=1}^{m}\sum_{j=1}^{m} w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{ij}\right)^{\frac{1}{2}} \quad (8)$$

$$\sigma_p = \sigma_K \quad (9)$$

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Table 1: Summary Statistics of the 16 10-year Investment Periods. In this table we present the average annual return, standard deviation, and Sharpe ratio for the sixteen overlapping ten-year investment periods over 1995-2019. Hence, for each of the 16 10-year investment periods, the annualized return, standard deviation and Sharpe Ratio is calculated. We then take the average of the 16 periods and provide this information below.

Asset Class	Average Annualized return	Average Annualized Standard Deviation	Average Sharpe Ratio
Domestic Equities	6.82%	15.03%	0.34
Domestic Bonds	5.26%	3.46%	0.95
Commodities	0.69%	23.00%	(0.06)
Emerging Market Equities	8.86%	23.21%	0.29
Emerging Market Bonds	9.52%	10.37%	0.76
Real Estate	9.41%	20.98%	0.39
Developed Market Equities	6.85%	17.22%	0.29
Developed Markets Bonds	4.64%	8.26%	0.31

Table 2: Correlations of monthly returns across all the asset classes over January 1995-December 2019. Here we simply examine the entire 1995-2019 period and provide the monthly correlations between the various asset classes.

Asset Class	Domestic Equities	Domestic Bonds	Commodities	Emerging Market Equities	Emerging Market Bonds	Real Estate	Developed Market Equities
Domestic Bonds	-0.02						
Commodities	0.28	-0.03					
Emerging Market Equities	0.73	-0.02	0.38				
Emerging Market Bonds	0.54	0.32	0.24	0.68			
Real Estate	0.58	0.18	0.17	0.51	0.47		
Developed Market Equities	0.81	-0.02	0.41	0.82	0.57	0.59	
Developed Markets Bonds	0.15	0.49	0.25	0.24	0.23	0.26	0.38

Table 3: Average Portfolio Weights to Maximize the Sharpe Ratio across all of the 16 10-year Investment Periods from 1995-2019. For each level of portfolio risk from 4 percent to 17 percent standard deviation we present average portfolio weights that optimize the Sharpe Ratio across all the 16 10-year periods from 1995-2019. So for example the 4 percent portfolio standard deviation represents a portfolio in which the standard deviation is constrained to be equal to 4 percent per year.

Portfolio Standard Deviation is constrained to be equal to:	**Average Portfolio Return**	**Average Sharpe Ratio**	**Average Portfolio Weight in *Domestic Equities***	**Average Portfolio Weight in *Domestic Bonds***	**Average Portfolio Weight in *Commodities***	**Average Portfolio Weight in *Emerging Market Equities***	**Average Portfolio Weight in *Emerging Market Bonds***	**Average Portfolio Weight in *Real Estate***	**Average Portfolio Weight in *Developed Markets Equities***	**Average Portfolio Weight in *Developed Markets Bonds***
4%	6.41%	1.10	6.96%	79.02%	0.72%	1.74%	7.26%	2.52%	1.77%	0.00%
5%	7.16%	1.03	7.40%	63.07%	0.95%	1.43%	20.72%	3.99%	2.00%	0.45%
6%	7.80%	0.97	7.88%	49.23%	1.09%	1.35%	32.08%	5.16%	2.29%	0.92%
7%	8.39%	0.91	8.58%	36.45%	1.23%	1.28%	42.16%	6.23%	2.56%	1.52%
8%	8.94%	0.87	10.00%	25.07%	1.36%	1.30%	50.17%	7.25%	2.82%	2.04%
9%	9.47%	0.83	11.19%	14.10%	1.49%	1.32%	58.18%	8.30%	3.07%	2.35%
10%	9.97%	0.80	13.81%	5.44%	1.61%	2.04%	62.87%	9.39%	3.32%	1.51%
11%	10.39%	0.76	18.94%	1.24%	1.44%	4.43%	58.81%	10.63%	3.57%	0.94%
12%	10.69%	0.72	22.65%	0.00%	0.69%	9.01%	50.35%	12.89%	3.75%	0.66%
13%	10.74%	0.67	24.48%	0.00%	0.62%	14.89%	40.45%	15.94%	3.43%	0.19%
14%	10.73%	0.62	24.41%	0.00%	0.56%	20.10%	38.21%	13.19%	3.53%	0.00%
15%	10.43%	0.56	29.76%	0.00%	0.50%	29.10%	26.31%	10.75%	3.57%	0.00%
16%	10.20%	0.51	24.41%	0.00%	0.44%	35.94%	22.11%	13.51%	3.59%	0.00%
17%	9.99%	0.47	18.95%	0.00%	0.26%	42.50%	19.22%	15.82%	3.25%	0.00%

Table 4: Average Portfolio Weights in Equities or Bonds. For each level of portfolio risk from 4 percent to 17 percent standard deviation we present average portfolio weights in total equities or in total bonds (including domestic, emerging market and developed) that optimize the Sharpe Ratio across all the 16 10-year periods from 1995-2019. So for example the 4 percent portfolio standard deviation represents a portfolio in which the standard deviation is constrained to be equal to 4 percent per year.

Portfolio Standard Deviation is constrained to be equal to:	**Average Portfolio Return**	**Average Sharpe Ratio**	**Average Portfolio Weight in *Equities***	**Average Portfolio Weight in *Bonds***
4%	6.41%	1.10	10.47%	86.28%
5%	7.16%	1.03	10.83%	84.24%
6%	7.80%	0.97	11.52%	82.23%
7%	8.39%	0.91	12.42%	80.13%
8%	8.94%	0.87	14.12%	77.28%
9%	9.47%	0.83	15.58%	74.63%
10%	9.97%	0.80	19.17%	69.82%
11%	10.39%	0.76	26.94%	60.99%
12%	10.69%	0.72	35.41%	51.01%
13%	10.74%	0.67	42.80%	40.64%
14%	10.73%	0.62	48.04%	38.21%
15%	10.43%	0.56	62.43%	26.31%
16%	10.20%	0.51	63.94%	22.11%
17%	9.99%	0.47	64.70%	19.22%

Table 5: Portfolio weights that optimize the Sharpe Ratio for portfolio standard deviation level equal to 4% for each overlapping ten-year investment periods over 1995-2019. Here we show the weightings that maximize the Sharpe ratio (with the risk-level fixed at a 4 percent standard deviation) for each of the 16 10-year investment periods.

10-year Investment Period	Portfolio Standard Deviation is constrained to be equal to:	Portfolio Return	Sharpe Ratio	Portfolio Weight in *Domestic Equities*	Portfolio Weight in *Domestic Bonds*	Portfolio Weight in *Commodities*	Portfolio Weight in *Emerging Market Equities*	Portfolio Weight in *Emerging Market Bonds*	Portfolio Weight in *Real Estate*	Portfolio Weight in *Developed Market Equities*	Portfolio Weight in *Developed Market Bonds*
1995-2004	4%	8.99%	1.27	6.02%	75.44%	2.93%	0.00%	3.35%	12.25%	0.00%	0.00%
1996-2005	4%	7.69%	1.01	1.31%	73.81%	4.05%	0.00%	6.94%	12.24%	1.66%	0.00%
1997-2006	4%	7.70%	1.02	0.00%	74.85%	0.00%	0.00%	2.50%	13.41%	9.23%	0.00%
1998-2007	4%	7.48%	0.99	0.00%	73.05%	3.06%	3.64%	0.29%	2.49%	17.48%	0.00%
1999-2008	4%	6.03%	0.70	0.00%	88.85%	1.53%	3.15%	6.47%	0.00%	0.00%	0.00%
2000-2009	4%	6.67%	0.98	0.00%	91.79%	0.00%	2.36%	5.85%	0.00%	0.00%	0.00%
2001-2010	4%	6.60%	1.11	0.00%	92.63%	0.00%	7.37%	0.00%	0.00%	0.00%	0.00%
2002-2011	4%	6.49%	1.17	0.00%	89.91%	0.00%	5.73%	4.36%	0.00%	0.00%	0.00%
2003-2012	4%	6.21%	1.14	0.00%	88.63%	0.00%	5.59%	5.78%	0.00%	0.00%	0.00%
2004-2013	4%	5.27%	0.93	7.39%	79.72%	0.00%	0.00%	12.89%	0.00%	0.00%	0.00%
2005-2014	4%	5.41%	1.00	9.19%	77.18%	0.00%	0.00%	13.63%	0.00%	0.00%	0.00%
2006-2015	4%	5.08%	0.99	13.57%	77.80%	0.00%	0.00%	8.64%	0.00%	0.00%	0.00%
2007-2016	4%	4.89%	1.06	11.24%	78.07%	0.00%	0.00%	10.69%	0.00%	0.00%	0.00%
2008-2017	4%	4.89%	1.15	17.33%	79.31%	0.00%	0.00%	3.37%	0.00%	0.00%	0.00%
2009-2018	4%	6.37%	1.51	13.71%	54.90%	0.00%	0.00%	31.39%	0.00%	0.00%	0.00%
2010-2019	4%	6.85%	1.58	31.61%	68.39%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Average	4%	6.41%	1.10	6.96%	79.02%	0.72%	1.74%	7.26%	2.52%	1.77%	0.00%

Table 6: Portfolio weights that optimize the Sharpe Ratio for portfolio standard deviation level equal to 11% for each overlapping ten-year investment periods over 1995-2019. Here we show the weightings that maximize the Sharpe ratio (with the risk-level fixed at a 11 percent standard deviation) for each of the 16 10-year investment periods.

10-year Investment Period	Portfolio Standard Deviation is constrained to be equal to:	Portfolio Return	Sharpe Ratio	Portfolio Weight in *Domestic Equities*	Portfolio Weight in *Domestic Bonds*	Portfolio Weight in *Commodities*	Portfolio Weight in *Emerging Market Equities*	Portfolio Weight in *Emerging Market Bonds*	Portfolio Weight in *Real Estate*	Portfolio Weight in *Developed Market Equities*	Portfolio Weight in *Developed Market Bonds*
1995-2004	11%	13.42%	0.87	5.05%	2.10%	4.25%	0.00%	36.69%	51.91%	0.00%	0.00%
1996-2005	11%	12.63%	0.82	0.00%	0.00%	7.81%	0.00%	43.39%	48.80%	0.00%	0.00%
1997-2006	11%	11.85%	0.75	0.00%	8.30%	0.00%	0.00%	25.95%	58.28%	7.47%	0.00%
1998-2007	11%	10.74%	0.65	0.00%	4.67%	11.02%	4.28%	24.61%	5.78%	49.65%	0.00%
1999-2008	11%	9.58%	0.58	0.00%	0.91%	0.00%	0.00%	99.09%	0.00%	0.00%	0.00%
2000-2009	11%	10.57%	0.71	0.00%	0.00%	0.00%	0.38%	99.62%	0.00%	0.00%	0.00%
2001-2010	11%	10.91%	0.79	0.00%	3.79%	0.00%	18.45%	62.75%	0.00%	0.00%	15.01%
2002-2011	11%	11.26%	0.86	0.00%	0.00%	0.00%	6.19%	93.81%	0.00%	0.00%	0.00%
2003-2012	11%	12.34%	0.97	0.00%	0.00%	0.00%	13.54%	86.46%	0.00%	0.00%	0.00%
2004-2013	11%	8.93%	0.67	0.00%	0.00%	0.00%	13.32%	86.68%	0.00%	0.00%	0.00%
2005-2014	11%	7.97%	0.60	0.00%	0.00%	0.00%	14.72%	85.28%	0.00%	0.00%	0.00%
2006-2015	11%	7.00%	0.53	47.58%	0.00%	0.00%	0.00%	52.42%	0.00%	0.00%	0.00%
2007-2016	11%	6.82%	0.56	46.56%	0.00%	0.00%	0.00%	53.44%	0.00%	0.00%	0.00%
2008-2017	11%	7.74%	0.68	48.64%	0.00%	0.00%	0.00%	51.36%	0.00%	0.00%	0.00%
2009-2018	11%	11.92%	1.06	74.19%	0.00%	0.00%	0.00%	25.81%	0.00%	0.00%	0.00%
2010-2019	11%	12.53%	1.09	81.04%	0.00%	0.00%	0.00%	13.70%	5.27%	0.00%	0.00%
Average	11%	10.39%	0.76	18.94%	1.24%	1.44%	4.43%	58.81%	10.63%	3.57%	0.94%

Table 7: Portfolio weights that optimize the Sharpe Ratio for portfolio standard deviation level equal to 17% for each overlapping ten-year investment periods over 1995-2019. Here we show the weightings that maximize the Sharpe ratio (with the risk-level fixed at a 17 percent standard deviation) for each of the 16 10-year investment periods.

10-year Investment Period	Portfolio Standard Deviation is constrained to be equal to:	Portfolio Return	Sharpe Ratio	Portfolio Weight in *Domestic Equities*	Portfolio Weight in *Domestic Bonds*	Portfolio Weight in *Commodities*	Portfolio Weight in *Emerging Market Equities*	Portfolio Weight in *Emerging Market Bonds*	Portfolio Weight in *Real Estate*	Portfolio Weight in *Developed Market Equities*	Portfolio Weight in *Developed Market Bonds*
1995-2004	17%	8.88%	0.29	63.58%	0.00%	0.00%	36.42%	0.00%	0.00%	0.00%	0.00%
1996-2005	17%	9.71%	0.36	0.00%	0.00%	0.00%	56.12%	0.01%	43.87%	0.00%	0.00%
1997-2006	17%	11.22%	0.45	0.00%	0.00%	0.00%	53.05%	0.00%	46.94%	0.00%	0.00%
1998-2007	17%	12.85%	0.55	0.00%	0.00%	4.11%	43.92%	0.00%	0.00%	51.97%	0.00%
1999-2008	17%	9.44%	0.37	0.00%	0.00%	0.00%	55.56%	44.44%	0.00%	0.00%	0.00%
2000-2009	17%	10.33%	0.45	0.00%	0.00%	0.00%	53.29%	46.71%	0.00%	0.00%	0.00%
2001-2010	17%	13.58%	0.67	0.00%	0.00%	0.00%	53.87%	46.13%	0.00%	0.00%	0.00%
2002-2011	17%	12.80%	0.65	0.00%	0.00%	0.00%	55.56%	44.44%	0.00%	0.00%	0.00%
2003-2012	17%	14.65%	0.77	0.00%	0.00%	0.00%	57.61%	42.39%	0.00%	0.00%	0.00%
2004-2013	17%	10.27%	0.51	0.00%	0.00%	0.00%	58.00%	42.00%	0.00%	0.00%	0.00%
2005-2014	17%	8.39%	0.41	0.00%	0.00%	0.00%	58.65%	41.35%	0.00%	0.00%	0.00%
2006-2015	17%	6.99%	0.35	68.92%	0.00%	0.00%	0.00%	0.00%	31.08%	0.00%	0.00%
2007-2016	17%	6.06%	0.32	71.23%	0.00%	0.00%	0.00%	0.00%	28.77%	0.00%	0.00%
2008-2017	17%	7.99%	0.45	66.76%	0.00%	0.00%	0.00%	0.00%	33.24%	0.00%	0.00%
2009-2018	17%	12.43%	0.71	30.73%	0.00%	0.00%	0.00%	0.00%	69.27%	0.00%	0.00%
2010-2019	17%	4.23%	0.22	1.97%	0.00%	0.00%	98.03%	0.00%	0.00%	0.00%	0.00%
Average	17%	9.99%	0.47	18.95%	0.00%	0.26%	42.50%	19.22%	15.82%	3.25%	0.00%

Figure 1: Sharpe Ratio for each of the 16 overlapping ten-year investment periods over 1995-2019.

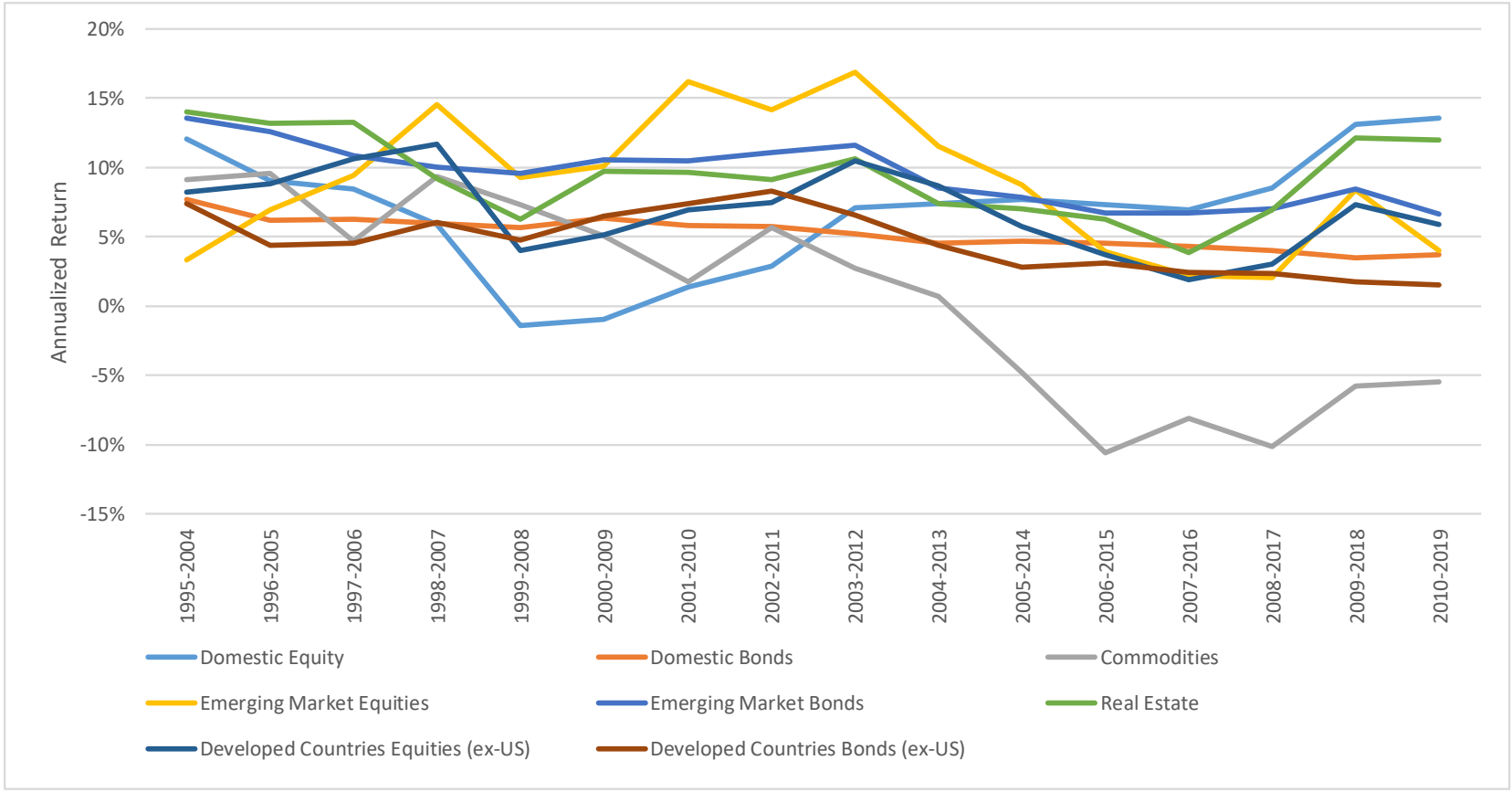

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Figure 2: Portfolio return for portfolios that optimize the Sharpe Ratio, presented across portfolio standard deviation levels ranging between 4% through 17%. Portfolio returns for each standard deviation are averaged over the sixteen overlapping ten-year investment periods over 1995-2019.

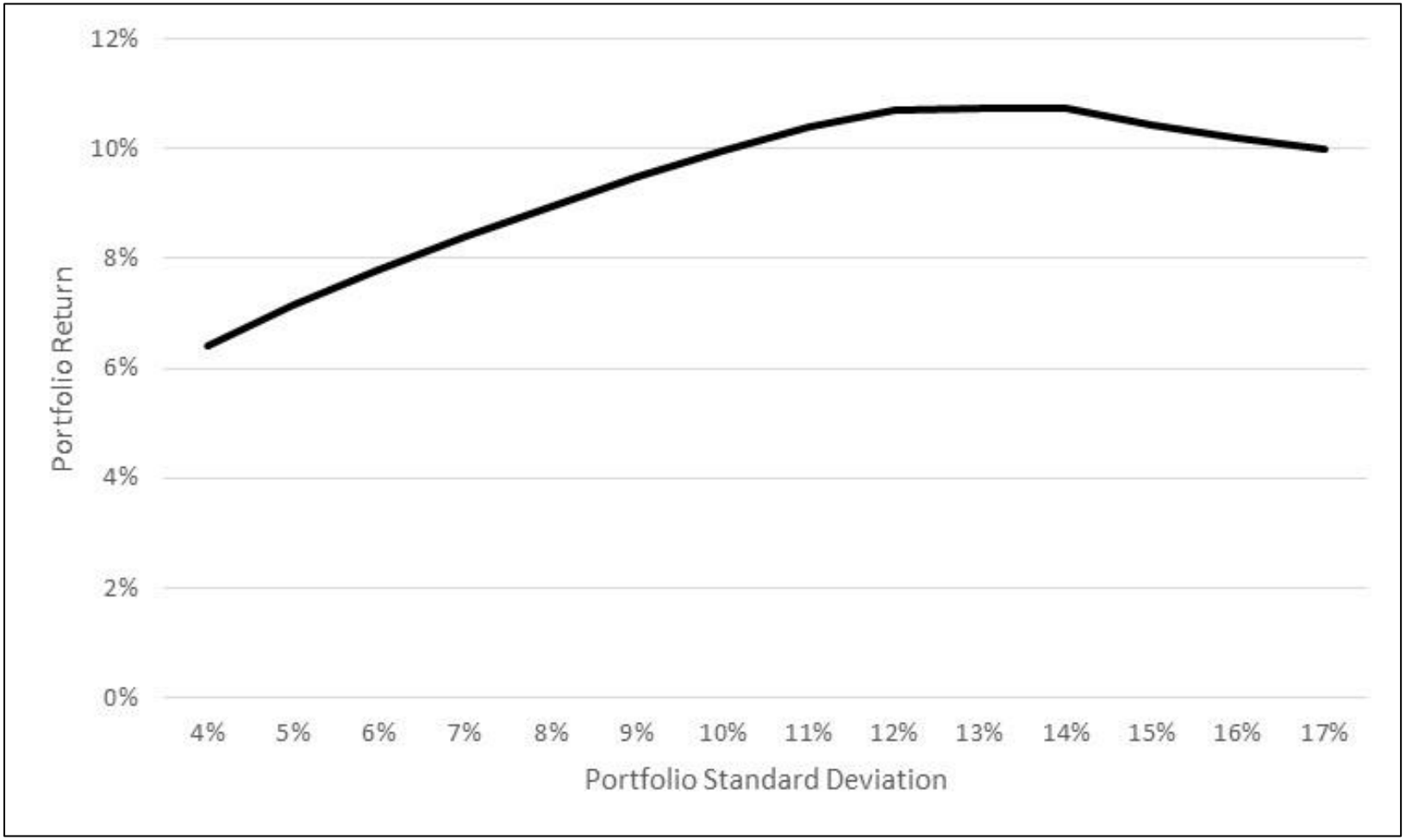

Figure 3: Portfolio weights that optimize the Sharpe Ratio presented across portfolio standard deviation levels ranging between 4% through 17%. Weights for each index for each standard deviation are averaged over the sixteen overlapping ten-year investment periods over 1995-2019.