Risk Parity Is Not Short Volatility

(Not That There's Anything Wrong with Short Volatility)*

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There have been increasingly frequent claims that risk parity strategies are hiding an implicit short volatility exposure or behave as though they are short volatility. In order to test the veracity of these claims, we simulate stylized versions of three-asset-class (equity, fixed income, and commodities) risk parity and short volatility strategies, and we compare the trading behavior and returns of each. We conclude that the two strategies' similarities are overstated, and we find no empirical evidence to support the claimed hidden exposure. Even with conservative assumptions designed to heighten the similarity of the two strategies, their trades are uncorrelated (or even slightly negative correlated) at almost any horizon. Though their returns are moderately correlated, the correlation is explained by common exposure to equities and bonds, not by common exposure to gamma or other forms of convexity. Controlling for these static underlying exposures, we find that the returns of the two strategies are almost orthogonal, with short volatility explaining less than one percent of the total variance of risk parity returns. We extend our analysis to consider equity and fixed income asset classes in isolation, where we observe very similar results.

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Introduction

Some recent additions to the literature on alternative strategies (e.g., Bhansali and Harris (2018) and Cole (2017)), along with financial newspapers[1], have expressed concerns about risk parity portfolios' relationship to market volatility. Many risk parity portfolios target constant risk, reducing positions when asset price volatility has risen. This trading behavior has generated two related concerns. First, by selling after volatility rises, risk parity may exacerbate market losses and possibly even pose a systemic risk, particularly when considered in tandem with the hedging behavior of volatility-selling options strategies, where the pair of strategies' "uncoordinated but correlated behavior could trigger a significant volatility event."[2] The second is that risk parity is implicitly short volatility because the two strategies exhibit similar behavior during market down environments.

While such a connection between risk parity and short volatility seems intuitive, we are unaware of any serious attempt to empirically test this linkage. This paper will remedy that by constructing replicable simulations of the two strategies, seeking empirical answers to these two questions:

1) Do risk parity and short volatility trades coincide in a way that constitutes a systemic risk to financial markets?

2) Do risk parity strategy returns exhibit meaningful exposure to short volatility?

Our strategy simulations indicate the answer to both of these questions is "no." Although both strategies are contingent on volatility in some sense and there are circumstances in which the two strategies behave similarly, the empirical relationship between them in practice is weak. Despite sharing common exposure to equities, the ways in which these two strategies trade and earn returns are distinct.

Risk parity's returns come from long exposure to multiple asset classes, and in many cases it adjusts its exposures to these assets based on perceived changes in their risk. A typical short volatility strategy, on the other hand, derives its return by selling options to harvest the volatility risk premium, and in many cases it subsequently trades to neutralize the options' exposures to changes in underlying asset prices. In short, when prices change, risk parity earns (positive or negative) PL while short volatility adjusts its equity holdings by delta-hedging[3]; when volatility changes, risk parity adjusts its holdings while short volatility earns PL.

[1] For example, the *Wall Street Journal* article "Is this Obscure Wall Street Invention Responsible for the Market Selloff?" (https://www.wsj.com/articles/is-this-obscure-wall-street-invention-responsible-for-the-market-selloff-1518085802)

[2] Bhansali and Harris (2018).

[3] Many short volatility strategies, such as covered calls or put-writing strategies do not delta-hedge. So in many cases there may not even be a "destabilizing" trade. Further, every short volatility options contract must have a long option counterpart. Some of these long option positions are also delta-hedged, which may be viewed as "stabilizing" trades.

Our simulations demonstrate that these two strategies are empirically distinct. Even under conservative assumptions designed to heighten the two strategies' trading behavior similarity, we find that their trades are uncorrelated (or even slightly negative correlated). This is also true even when we ignore the idiosyncrasies of short volatilities' delta-hedging, such as clean-up trades that liquidate hedges whenever cash-settled option contracts expire. Instead, there appears to be a relationship between the *direction* of the risk parity trade and the *magnitude* of the short volatility trade. The implication is that when risk parity has large liquidations, short volatility will likely have large trades, but the direction of these trades is unpredictable. In fact, within equities (which has been the focus of the recent concern), the largest 20% of risk parity sells were twice as likely to be associated with short volatility *buys* than sells. Of course, nearly any two strategies on the same assets are going to sell at the same time sometimes, but the worrying claim about the destabilizing nature of risk parity and short volatility as a pair is undermined by the fact that short volatility appears to sell less frequently and consistently in concert with risk parity than does the exact opposite of short volatility, long volatility.

On the second question, we find modestly positive return correlations between the strategies[4]; however, this is explained by their common exposures to the stock or bond market, not by common exposure to gamma or other forms of convexity. Once we control for these static underlying market exposures, returns were close to uncorrelated, with short volatility returns explaining less than 1% of risk parity's return variance. Risk parity's average exposure to its underlying assets, on the other hand, accounted for over 85% of its return variance. Based on these results, we conclude that once we adjust for their common passive exposures to equity and fixed income, risk parity and short volatility's returns were effectively orthogonal to one another.[5]

This paper is laid out as follows: Section 2 covers the basic concepts of both risk parity and short volatility strategies. Section 3 describes the data and explains the simulation examples we use in our analysis. Section 4 discusses the trading behavior of the two strategies, and provides an answer to first main question above. Section 5 examines the return aspect of the relationship, decomposing the returns of our risk parity simulation into exposures to both the underlying assets and short volatility, answering the second question. Section 6 concludes.

[4] Specifically, there's a correlation of 0.3 between the equity component of risk parity and our equity short volatility simulation. The three-asset-class risk parity returns have correlations of 0.2 and 0.1 with equity short volatility and fixed income short volatility, respectively.

[5] A similar result holds for the equity and fixed-income portions of a risk parity portfolio when considered in isolation. For the former, 80% of the variance is explained by general equity exposure and less than 1% by an equity short volatility strategy. For the latter, approximately 93% of the variance in returns is explained by general fixed-income exposure and approximately 0% is explained by a fixed-income short volatility strategy.

Section 2 – Understanding Risk Parity and Short Volatility

Both risk parity and short volatility are volatility-contingent, but they differ significantly in the *ways* in which they are related to market volatility. Risk parity's position sizes depend on levels of volatility, while short volatility's returns are contingent on changes in volatility.

Risk Parity

Risk parity's investment approach, described in works such as Maillard et al (2010), Qian (2011), and Asness et al (2012), seeks to balance risk levels across asset classes such as equities, government bonds, commodities, currencies, and other instruments. Methodologies differ among managers, but typically involve relatively larger notional positions in asset classes with lower risk levels (like government bonds) and smaller notional positions in asset classes with higher risk levels (like equities) than traditional portfolios.[6] With larger notional weights on low-volatility assets, risk parity strategies often incorporate leverage to provide a portfolio risk level closer to that of traditional allocations. Further, some, but not all, risk parity providers dynamically adjust positions over time to maintain both risk balance across asset classes and consistent portfolio volatility as market risk levels change.

These dynamic risk parity funds are typically the focus of pieces written about systemic risk. For example, consider a volatility-controlled S&P 500 strategy. The strategy has larger notional exposure when volatility is expected to be low and has smaller positions when volatility is expected to be higher in order to reduce the chance of over-realizing risk.[7][8]

This is the type of volatility-contingency that has raised concerns, since the mechanism described above can cause the strategy to sell in response to an increase in forecasted volatility. However, as we describe in Section 4, we believe that these concerns are generally unfounded. Additionally, the total assets invested in risk parity strategies (likely around $150 billion) is small relative to the size of the underlying markets (global equity index futures have a daily volume traded of around $500 billion, for example).[9] Risk parity strategies generally invest in

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[6] An example of a traditional portfolio here is "60-40" which holds 60% of its notional value in equities and 40% of its notional value in bonds.

[7] Specifically, given any risk model or prediction of future volatility $\hat{\sigma}$ and a desired level of volatility V, the portfolio will invest $V/_{\hat{\sigma}}$ in the risky asset. As an example, consider an investor that desires a 10% volatility investment in the S&P 500 and expects the S&P 500 to realize 12.5% volatility over the target period. Then this investor would invest $10\%/_{12.5\%} = 80\%$ of his portfolio in the underlying asset (the S&P 500 in this case) and the remainder in cash. If the investor is correct about future equity volatility, then this 80% notional investment in the S&P 500 should realize a volatility level of $.8 * 12.5\% = 10\%$ volatility over that target period. Maintaining this volatility target will require rebalancing as the predicted level of volatility changes, and estimated risk levels below the desired target V will require the use of leverage.

[8] It is important to note that this strategy, though sized based on estimates of market volatility, always maintains a long exposure to the S&P 500, earning positive returns when the S&P 500 gains and negative returns when the S&P 500 loses. This is in contrast to strategies that sometimes get compared to risk parity like trend-following, which takes positive or negative exposures to assets depending on the sign of the return of the asset over various historical windows.

[9] These claims are elaborated in Hurst et al (2015) and Mendelson (2018)

extremely liquid assets like equity index, government bond, and commodity futures with large market size and daily trading volume; this is particularly true for the S&P 500, where critics are most focused.

Short Volatility

As the name would suggest, short volatility strategies are also "volatility-contingent", though in a very different manner than risk parity. Short volatility strategies are short instruments (options) whose fundamental values are tied to the underlying asset's volatility. Due to their convex payoff profiles, both put and call[10] options' expected payouts are positively related to their underlying asset's realized volatility. Thus, their prices are also positively related to their underlying assets' anticipated volatility.

Short volatility strategies seek to earn the volatility risk premium (VRP).[11] Some portfolio managers then delta-hedge[12] their short option positions, though covered call, put underwriting, and iron condor funds are three examples of non-delta-hedged short volatility strategies with significant assets under management.[13]

One common directionally-neutral short volatility strategy sells both call and put options with strike prices near the current underlying asset's price.[14] This portfolio is initially indifferent to the direction of the underlying asset's price changes because the call option's sensitivity to those price changes is of equal magnitude and opposite sign of the put option's sensitivity. Thus, at inception, the short options may require no hedge at all. However, because the sensitivities depend on the underlying asset price, a "delta"-hedge is required to maintain neutral exposure as the underlying price changes. This delta hedge continues to adjust as prices change until the options expire, at which point the hedge position is closed.

This delta-hedging behavior for short option positions is a source of concern for those worried about large-scale, simultaneous underlying asset unwinds. Delta hedge trades for both short call and put options point in the same direction, selling as underlying prices fall and buying as prices rise.[15]

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[10] Call (put) options are the right, but not the obligation, to buy (sell) the underlying reference asset at a fixed strike price K on a specified date. If the call (put) option's strike price is below (above) the current market price of the underlying asset, it is said to be "in the money". If the call (put) option's strike price is above (below) the current market price of the underlying asset, it is said to be "out of the money". An option on an asset with a current price $S_t \approx K$ is said to be "at the money".

[11] Bakshi and Kapadia (2003) analyzed delta-hedged index option returns and found evidence in favor of a volatility risk premium. Hill et al. (2006) and Israelov and Nielsen (2015) observed that covered call returns are higher because of the spread between implied and realized volatility. Israelov, Nielsen, and Villalon (2016) showed that exposure to downside risk, through selling put options, captured both the equity risk premium and the volatility risk premium.

[12] This "delta hedging" arises because options are jointly exposed to changes in the price and realized volatility of the underlying asset, and the hedge required to neutralize the option to market movements is dependent on the relationship between the current underlying asset price S_t and the strike price K, among other things. A call option that is very far in the money ($S_t \gg K$) should change in value close to one-to-one with the underlying asset because it is unlikely to expire out of the money, while a call option that is very far out of the money ($S_t \ll K$) should not change in value much at all with the underlying asset because it is unlikely to expire in the money.

[13] See Israelov and Nielsen (2015) for discussion of such strategies.

[14] These are generally referred to as straddles for at-the-money put and call options with the same strike and maturity and strangles for out-of-the-money put and call options with the same maturity.

[15] Delta hedging a short call option requires a long position in the underlying asset, and that long position falls (selling the underlying asset) as the underlying price falls. Delta hedging a short put option requires a short position in the underlying asset, and that short position increases (again, selling the underlying asset) as the underlying price falls. The opposite is true for both types of options as underlying asset prices rise.

Section 3 – Data and Simulation Methodology

To examine the relationship between risk parity and short volatility strategies, we construct historical simulations of stylized versions for each strategy using daily data between January 1996 and April 2018.

The OptionMetrics IVY database provides daily closing bid and ask quotes, implied volatilities, dividends, and option deltas for the S&P 500 options analyzed in this paper. Bloomberg provides underlying equity index values and USD LIBOR. Equity index and 10-year bond futures prices and returns, which are used to calculate position sizing and returns for both short volatility and risk parity, are also derived from Bloomberg data. Commodity futures prices and returns are derived from Bloomberg and Commodity Systems Inc (CSI).

The equity short volatility strategy sells a front-month 50-delta S&P 500 straddle and a front-month 25-delta S&P 500 strangle every month.[16] The total option notional sold is 100% of net asset value (NAV), with an equal number of options at each strike. Options are held to expiration and fully delta-hedged each day at the close. Daily short option returns are calculated as the negated option price change, plus the appreciation of the delta hedge and interest from the option premium:

$$R_{i,t} = w_{i,t-1} * \left(\left(P_{opt,i,t-1} - P_{opt,i,t} \right) + \Delta_{opt,i,t-1} \left(P_{fut,t} - P_{fut,t-1} \right) + R_{f,t}\, P_{opt,i,t-1} \right)$$

where $R_{i,t}$, $P_{opt,i,t}$, $\Delta_{opt,i,t}$, and $w_{i,t}$ are the return, price, delta exposure, and portfolio weight of option i, $P_{fut,t}$ is the price of the underlying futures contract, $R_{f,t}$ is the risk free rate (assumed to be US 3-Month LIBOR), all at date t. The fixed income short volatility strategy is constructed similarly, except using options on US 10-year bond futures and hedged using those same futures.

The risk parity simulation targets equal-realized-volatility (5% annualized) in three assets: S&P 500 futures, US 10-year bond futures, and the Goldman Sachs GSCI Commodity Index, implemented using futures contracts.[17] We must forecast volatility in order to dynamically size positions to target constant volatility. We use an exponentially-weighted moving volatility of daily returns with a 35-day half-life.[18] We (conservatively) assume no trade delay, with risk parity trades happening instantaneously at each business day's close based on the updated volatility estimates including that day's return.[19] In practice, many real world implementations hold static positions over time, so their trade sizes may be unrelated to market movements. For the ones that do have a

[16] A 50-delta straddle consists of a 50-delta (i.e., at-the-money) call option and a 50-delta put option, where both have the same expiration date. A 25-delta strangle consists of a 25-delta (i.e., out-of-the-money) call option and 25-delta put option, again both with the same expiration date.

[17] Due to diversification, this strategy is expected to realized close to 10% volatility overall, rather than 15%, the sum of the individual asset class volatility targets.

[18] Our results are robust to different specifications. We do find that trade correlation between the strategies is negatively related to the half-life of the volatility estimate used to size risk parity positions. Slower volatility forecast adjustments are associated with less correlated trading behavior between risk parity and short volatility.

[19] As we show in the Appendix sub-section "Effect of Trade Delay on Risk Parity vs. Short Volatility Trade Relationship", our results are robust to this assumption, and if anything are even stronger if we instead assume a more realistic delay of one or two days in the ex-ante volatility estimate.

responsive risk model, they often experience a trade delay of at least one day (perhaps even two or three days) and additionally smooth trades in order to reduce price impact and transaction costs. Our assumptions should therefore lead to a conservative estimate of the strength of the trading relationship between risk parity and short volatility.

Risk parity positions for each asset k on date t are a function of the asset's prior day's estimated volatility

$$w_{k,t-1} = \frac{5\%}{\sigma_{k,t-1}}$$

Asset class returns are thus

$$RP_{k,t} = w_{k,t-1} * r_{k,t}$$

where $r_{k,t}$ is the return on date t for asset k. Risk parity fund returns are

$$RP_{fund,t} = RP_{spx,t} + RP_{us10y,t} + RP_{gsci,t}$$

The simulation's NAV responds to the returns of the different components, and the relative returns of the different assets can cause position drift over the course of a day that either increases or decreases trade sizes. For instance, if the GSCI portion of the portfolio falls one day while the others gain, the resulting portfolio will be underweight GSCI and overweight bonds and equities unless risk estimates change. This drift alone induces trading. Therefore, when calculating the risk parity strategy's trade sizes, we consider the combined effect of changes in forecasted volatility and the position drift relative to the floating NAV. A thorough discussion of this calculation is provided in the Appendix.

Section 4 – Trade Perspective

It has been argued that both risk parity and short volatility are inherently destabilizing due to the assumption that equity market crashes cause both strategies to sell significantly into the crash (the former via increased risk in equity markets and the latter via delta-hedging). Occasionally, a stronger assertion is made: that these strategies may actually *create* a "vicious cycle" by selling, exacerbating crashes, selling further as a result, and so on.[20] Our simulations demonstrate the trading behaviors of these strategies are not as strongly linked as they may first appear, and that the "vicious cycle" fears are grossly overstated.

Trade Perspective: Short Volatility Hedge Trades

A case study illustrates the short volatility strategy's trading behavior. The two months between mid-July and mid-September of 2011 were volatile, with the S&P 500 falling significantly the first month before partially recovering in the second. The top panel in **Exhibit 1** shows the level of the S&P 500 over this period, along with

[20] For example, Cole (2017) and Lehmann (2013).

the dates at which each set of options was sold and their expiration dates. The bottom panel shows the delta hedge position in the S&P 500 over the same period.

This two-month period demonstrates that the relationship between spot return and hedge delta strongly depends on the time period being examined. First, the period from July 15 (the date of the initial option sale) through August 18 (the day before the option expiration date) supports the typical story of falling prices causing the strategy to sell the underlying asset. As the S&P 500 fell severely over the course of the month, the put option approached a delta of -1 and the call approached a delta of 0, so the delta hedge position was strongly negative.

However, after the option expires on August 19, the strategy must engage in a large positive "clean-up" trade to zero out the old hedge positions. Because of this, the relationship between the S&P 500 return and the hedge traded completely breaks down if you consider the period from July 15 to August 19. Over this period, the S&P 500 is still down significantly but the total net hedge trade is approximately *zero*.

Over the next month (from August 19 to September 15), the relationship breaks down even further. As the S&P 500 partially recovers from its previous lows, by September 15 the delta hedge has become strongly positive in the exact same magnitude as it was negative in the previous month, since the calls have a delta close to 1 and the puts are deeply out-of-the-money with a delta approaching 0. Over the two-month period from July 15 to September 15, the S&P 500 is still down significantly but the short volatility strategy has been a strong net *buyer* of the S&P 500. This contradicts the typical story that hedge trades and spot returns should be in the same direction during this kind of market downturn.

An even subtler point emerges when we look more closely at the period between August 5th and the expiration of the first option. This period is particularly volatile, with prices whip-sawing across a span of nearly 100 points. We might expect that this would induce considerable trading for hedging purposes, but in fact it hardly results in any trading at all. This is because the put and call options had moved so far in and out-of-the-money, respectively, and there was so little time to expiration remaining, that their deltas became inelastic to price movements in the underlying asset.[21] In this case (and also more generally), the very fact that the strategy has already sold the underlying asset reduced the potential need for future sales, an often overlooked hole in the "vicious cycle" theory. Moneyness and time to expiry (along with asset price volatility) jointly determine the hedge positions; changes in these variables routinely dampen hedge position sensitivity to changes in underlying price.

[21] For those familiar with Black-Scholes greek terminology, this is another way of saying that the portfolio had a small gamma exposure.

Stepping back from this case study, **Exhibit 2**[22] demonstrates that these issues interfere more generally with the narrative of rampant equity selling by short volatility strategies during market sell-offs. When we examine the specific period of option position inception to one day before those options' expiration date, the S&P 500's return is negatively related to the final hedge position. But this relationship is completely severed once we add the expiration date to the period, due to the clean-up trade. Finally, if we look ahead to one day prior to the *following* expiration, we find a positive but much weaker relationship overall. In particular, there are several points in the upper left quadrant, indicating periods (our case study being one of them) in which the S&P 500's return was negative and yet the short volatility strategy was a net buyer of equities.

While the charts in **Exhibit 2** focus on time periods from one expiration date to another (with or without a one-day offset), we also expect these kinds of nuances to show up more generally. **Exhibit 3** plots the hedge position versus the S&P 500's return for four different horizons, confirming three stylized facts about the relationship between the spot return and the hedge position:

1) The hedge delta is not one-to-one with price changes, but instead can be either more or less sensitive to price changes at very short horizons depending on moneyness, time to expiration, and spot price volatility.

2) The clean-up trade that occurs at expiration dramatically weakens the relationship even at very short horizons.

3) The relationship also weakens as horizon increases, particularly as the horizon includes one or more expiration dates.

Trade Perspective: Risk Parity Trades

Risk parity trades are often similarly misunderstood. Unlike short volatility strategies, risk parity trading behavior is not a function of prices, but rather of perceived levels of market risk. To the extent that directional changes in price are (negatively) correlated to directional changes in volatility, we might expect trades across the two strategies to coincide. However, the reality is more complicated.

Our stylized risk parity strategy adjusts its equity exposure based on forecasted market volatility. Whereas positive (negative) returns generally led to buying (selling) for short volatility, market volatility estimates are

[22] In the top chart, one can observe that the hedge deltas traded during months with very negative spot returns (less than -10%, for example) are slightly less negative than months with moderately negatively spot returns (such as -5%). The explanation is that our short volatility simulation is being run assuming a fixed NAV. When we initially sell the options, the positions are sized such that the option notional exposure equals the NAV (i.e., our portfolio is sized to have a leverage of 1.0). But on subsequent days, the NAV will remain fixed while the option quantities are unchanged, but notional exposure will vary since the spot is changing. Specifically, the notional exposure (as a percent of NAV) will be lower than 1 if there has been an S&P crash since the last option sale.

This causes the observed effect where very negative spot returns have less negative deltas than months where the spot was down only 5%. In both of those scenarios, the puts will have deltas of approximately 1 and the calls will have deltas of approximately 0, so the portfolio delta *as a percent of the option notional exposure* would be very close to 0.5 in both cases. However, the notional will be lower in the months when there was a very negative spot return, so the delta exposure *as a percent of NAV* will be lower in those months as well based on the square of returns, and so are agnostic to the return's sign: large returns in either direction will tend to increase volatility estimates, and small returns will do the opposite. We see this in **Exhibit 4**, which shows the changes in forecasted S&P 500 volatility against its daily returns.

This demonstrates a major difference between the trades of risk parity and short volatility strategies. Unlike the latter, risk parity trades are primarily driven by a quantity (changes in volatility estimates) which varies with the absolute value of the spot return. Additionally, other factors further dilute the relationship. A large magnitude return need not change the volatility estimate (and thus need not induce any trading at all) if the volatility level was already high.[23] Further, the risk parity position is a function of the *inverse* of the volatility estimate, adding another non-linear layer to the relationship.[24] Add to all of this the fact that positions relative to the strategy's NAV change themselves based on returns,[25] and the end result is that the relationship between underlying asset returns and risk parity trades is much less direct than it is sometimes described.

Exhibit 5 confirms the noisy relationship between risk parity trades and equity returns. There's clear evidence of an upside-down-'U'-shaped profile because large magnitude returns often lead to increased volatility estimates and de-leveraging for the strategy. However, large magnitude returns are also often associated with small sells (or even buys) because despite being large in absolute terms, they may be small relative to existing volatility forecasts. Indeed, as volatility estimates adjust following large market moves, ever larger returns are required to generate further sells.

Consider, for example, an extreme scenario where a very large move causes the volatility estimate to double, and risk parity cuts its position in half. Because volatility is already elevated, a significantly larger move is required to cause the volatility estimate to again double, and the effect on the position size would only be half as large. Specifically, suppose that the risk parity strategy holds 50% of its NAV in equities with an equity volatility estimate of 10%. It would take a one-day absolute move of approximately 7.5% to cause the volatility estimate based on a 35-day half-life to double to 20%, resulting in the strategy selling half of its holdings (25% of NAV). Based on this new position, it would require return *twice as large in magnitude* (an approximately 15% one-day absolute return) for volatility to again double to 40%, and yet would only result in half the trade size, selling 12.5% of NAV. It is clear from this example that the potential for a risk parity portfolio to actually generate a cycle of heavy continued selling over many days is made nearly impossible by mathematical fact alone.[26]

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[23] This fact explains why the observations in **Exhibit 4** do not perfectly fall along a line, but instead fan out around a trend.

[24] We estimate a -0.7 correlation between changes in ex-ante S&P 500 volatility and the corresponding change in desired S&P 500 position for our risk parity strategy, which is large in magnitude, but far from perfect.

[25] We estimate a 0.95 correlation between the change in desired risk parity equity position and the actual trade due to this fact.

[26] The specific numbers provided in this scenario do not account for trading due to changing relative asset-class NAVs, which further reduces the required trade sizes (see appendix for a thorough decomposition of risk parity trades); this only considers the effect of changing volatility

Perhaps surprising, there's also evidence of a slightly negative one-day relationship between equity returns and risk parity equity trades, which may be attributable to the "leverage" within a portfolio that seeks less than 100% exposure to equities. While the upside-down-'U'-shape is the primary takeaway from **Exhibit 5**, the lack of significantly positive correlation between risk parity's trades and equity's returns is important because it is this correlation that is the primary transmission mechanism for trading directionally in line with short volatility. This slight negative overall correlation is exhibited over short horizons of one to three days. However, over longer horizons, the relationship between equity returns and the cumulative risk parity equity trade becomes increasingly positive, as shown in **Exhibit 6**. Nevertheless, **Exhibit 6** makes clear that this relationship is quite distinct from the relationship between equity returns and short volatility hedge trades, which is also included for reference.

Exhibits 3, 5, and **6** demonstrate that the connections between equity returns and the trades of both risk parity and short volatility strategies are complex. But while equity returns are the main channel through which the trades of strategies are connected, we are ultimately concerned with the connection itself, regardless of the mechanism. **Exhibit 7** shows the relationship between equity trades of risk parity and short volatility, with an overall one-day correlation that is very close to zero[27]. This is true whether we include option expiration dates (which generate clean-up trades) or exclude them. Roughly as often as these strategies line up to generate large trades in the same direction, the opposite occurs, creating large trades in opposite directions. There are instances where trades will line up and be large in magnitude, for instance when there is a large return over a period when no options expire *and* the options are not very far in- or out-of-the-money *and* the return is negative *and* ex-ante equity volatility was low to begin with. This does happen at times in our sample, but it is far from the only possible trading outcome. Furthermore, following such a scenario, the potential for a self-reinforcing cycle of continued significant and coincident selling is eliminated as the required initial conditions such as low ex-ante volatility and near-the-moneyness of options trades no longer hold.

Even over horizons longer than one day, it remains the case that the two strategies' trades have approximately no full-sample correlation. In **Exhibit 8**, we vary the horizon from one to sixty-five business days, and for each of these we record the correlation between the short volatility strategy's cumulative delta traded with the risk

estimates on positions.

[27] A risk parity strategy that targets volatility using VIX instead of recent historical volatility would see equity trades that are materially correlated with short volatility's delta-hedges. We do not believe that this realistically captures the approach taken by the risk parity community for the simple reason that it is a less effective way of targeting risk. This is because implied volatility includes the additional noise associated with time varying volatility risk premium, reducing its forecasting power. Even still, the residualized returns of a risk parity strategy constructed using VIX remain lowly correlated to short volatility.

parity strategy's cumulative equity trade. Regardless of horizon, the correlation remains flat or slightly negative, and is generally not statistically different than zero based on bootstrapped confidence intervals.

Of course, there's an important distinction between uncorrelated and unrelated. We find that risk parity's trades and short volatility's trades are uncorrelated, meaning we cannot ascertain whether one strategy is buying or selling by knowing the direction of the other strategy's trades. However, the upside-down 'U'-shaped pattern in **Exhibit 7** shows that their trades are not unrelated. The absolute nature of the relationship means that the direction of the risk parity trade is related to the *magnitude* of the short volatility trade. If we know the magnitude of the short volatility trade (for instance, if short volatility has a large-sized trade, either buying or selling), then we can infer something about the direction of the risk parity trade (risk parity is a likely seller because short volatility's large-sized trade indicates a large equity move, which would mean an increase in estimated volatility).[28] In the same vein, knowing the direction of the risk parity trade (say, that risk parity has a significant liquidation trade) only suggests that short volatility has a large magnitude trade, but does not inform whether short volatility is a buyer or seller because the increased volatility could be due to either a large positive or negative equity return.

This relationship is not straightforward, and can make it difficult to draw clear, concise conclusions about the level of concern that investors and regulators should have about these strategies' joint behavior. There are dates in our simulation when the two both experience significant negative trades on the same day, which would not be surprising for any arbitrary pair of strategies on the same assets.[29] So it is natural that we examine those instances where these strategies sell at the same time to determine if the frequency, magnitude, or the aftermath of these events is enough to warrant particular concern.

Exhibit 9 tallies the number of observations that jointly fall within particular quintiles for each strategy excluding option expiration dates, and the joint distribution follows the same upside-down-'U' pattern we discussed with **Exhibit 7**. If risk parity is buying (80th percentile or higher, the top row), then the short volatility trade is likely to fall in the middle three quintiles (20th – 80th percentile), while if risk parity is strongly selling (20th percentile or lower, the bottom row), then short volatility is likely to have a large trade, though in an unknown direction (less than 20th percentile or greater than 80th percentile). While that is to be expected, two unexpected results appear as well. First, conditioning on a large sell for risk parity, short volatility was actually somewhat more likely to *buy* than sell, with more than double the observations falling in the lower right corner than the lower left corner. Second, the distribution of risk parity trades conditional on a large sell for short volatility (the left-most column)

[28] This is, of course, an oversimplification. A large delta-hedge trade could occur due to small-in-magnitude price changes around the strike price near option expiration, for instance, and that would not necessarily cause an increase in estimated volatility.

[29] This can be seen by our finding that risk parity and *long volatility* also have days in which they both experience significant negative trades.

is strikingly spread out: there is hardly any difference between four of the five quintile buckets, meaning we can't even be confident that risk parity would be selling in that instance at all.[30]

While counting observations indicates the frequency of these joint events, **Exhibit 9** does not necessarily mean that the tails of these distributions are completely benign. **Exhibit 10** reports statistics on each strategy's trades conditional on the direction and magnitude of the other strategy's trade, excluding option expiration dates for clarity. These tables provide a number of interesting observations. As expected, if short volatility has large trades (buys or sells), the average risk parity trade is also negative. For instance, in the 1st percentile largest short volatility selling days, risk parity's average trade is a sell of -3.3%, and the average percentile of its trade is 10%.[31] But none of the bucket t-statistics are statistically significant at a 5% confidence level, suggesting that there is enough dispersion in trades between the strategies that we cannot draw definitive conclusions about the direction of risk parity trades even in short volatility's most extreme tail events.

Another anticipated result is that the direction of short volatility's trade is not related to risk parity trade's percentile, but the magnitude of those trades certainly is: there is no discernable pattern for any of the measures of short volatility trade level (average trade, t-statistic, average percentile, percent of days selling), but the standard deviation of the short volatility trade increases monotonically from 2.6% when risk parity is buying its largest quantities to 19.7% when risk parity is selling. This further confirms the connection between risk parity trade direction and short volatility trade magnitude.

Exhibit 11 repeats the exercise after including option expiration dates. The same results broadly hold, though the clean-up trades add some noise to the conclusions, especially for the largest magnitude short volatility trade buckets, in which those clean-up trades are highly represented. Still, it is remarkable that adding those expiration dates back into the analysis in **Exhibit 11** leads to risk parity actually *buying* on average when short volatility is committing its largest sells (1st percentile).[32] If the actual average risk parity trade is a buy even in the 1st percentile of short volatility trades, then it would seem that much of the worry about the joint behavior of these strategies is misplaced.

Perhaps the most interesting result in **Exhibit 10** (and **11**) has to do not with instances of extreme selling, but rather of extreme buying. Risk parity was more likely selling when short volatility was *buying* (it sold 100% of the time when short volatility was in its 99th percentile) than when short volatility was *selling* (only 91% of the time when short volatility was in its 1st percentile). The average percentile of risk parity trades was lower when short

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[30] Recall that **Exhibit 9** does not include option expiration dates, and therefore leaves out clean-up trades that are likely in practice to further obscure this relationship.

[31] Note that this is also true when short volatility is buying – in the largest 1% of buying days for short volatility (99th percentile), risk parity's average trade is a sell of -1.6%, slightly smaller than that associated with short volatility's largest sells, and with an average trade percentile of 5%, slightly more consistently negative than that associated with short volatility's largest sells.

[32] These clean-up trades necessarily exist for any cash-settled short volatility strategy that delta-hedges (that is, exactly the set of strategies we are concerned with).

volatility bought (5%) than when it sold (10%). And despite the fact that the average risk parity trade was slightly less negative, the t-statistic of that average is larger in magnitude (-1.66 vs -0.68) when short volatility bought versus when it sold. These results hold true in less extreme percentiles (95^{th}-99^{th} versus 5^{th}-1^{st}, 90^{th}-95^{th} versus 10^{th}-5^{th}). They also hold true in **Exhibit 11** when we include option expiration dates.

All of this points to the fact that short volatility was as likely, if not even slightly more likely, to mitigate risk parity's selling than it was to align with that selling in our simulation. To be clear, risk parity and short volatility do experience periods of mutual selling; but as we've noted before, this is true of many pairs of strategies on the same assets. And **Exhibits 9**, **10**, and **11** provide an excellent example of this truism: instances in which risk parity sells and short volatility buys are by definition also instances of joint selling between risk parity and *long volatility*, the exact opposite of short volatility. The consensus of the recent articles that motivated our research is that risk parity and short volatility are so intertwined as to constitute a danger to financial markets; but this seems hard to reconcile with the fact that risk parity reduced its positions more frequently and consistently in line with the *opposite* strategy of shorting volatility.

Adding additional robustness to these findings, this section is replicated in the Appendix for fixed-income-specific short volatility and risk parity simulations. The results are quite similar, with short volatility trade magnitude being clearly linked to risk parity trade direction, but no evidence that risk parity selling is any more likely to occur with short volatility selling than it is to occur with short volatility buying. See the Appendix for further details.

Risk Parity Trade Attribution

To better explain why these two strategies' trades are uncorrelated, we decompose risk parity's equity trades into various components based on the *reasons* for those trades.

Specifically, we attribute risk parity trades to the following three sources:

1) changes in the S&P 500 volatility forecast
2) effect of the PL of risk parity's equity exposures
3) effect of the PL of risk parity's other two asset class exposures

See the Appendix for more detail on precisely how these are calculated.

The first of these components breaks out the hypothetical trade deriving *solely* from the change in S&P 500 volatility forecasts. This component turns out to be the most important by far for determining the overall trade size, explaining 95% of the one-day trade variance. This is the component that is speculated to provide risk parity with short volatility characteristics. However, its correlation (0.09) to short volatility delta trades is rather small.

The second component in our attribution highlights the effect of the PL of risk parity's equity bucket. For the intuition behind this, suppose that on day *t-1* the risk parity weight on the S&P 500 ($p_{SPX,t-1}$) is 0.5, and then on day t, the S&P 500 falls 5%. To isolate the effect of the equity bucket's PL, we can estimate what our trade would be if (hypothetically) the volatility forecast remained constant and if the other buckets had zero PL. The desired post-trade risk parity weight ($p_{SPX,t}$) remains 0.5 because forecasted volatility is unchanged. But due to the equity PL, the new NAV is 97.5% of the old NAV, while the pre-trade S&P 500 position would 0.5*(1 - 5%) = 47.5% of the old NAV, which is equivalent to 47.5%/97.5% ≈ 48.7% of the new NAV. Therefore, a trade of 50% - 48.7% ≈ +1.3% is needed to reach the desired weight.

This second component is a "leverage effect",[33] which disappears if the risk parity weight on the S&P 500 were exactly one. This effect is consistent with similar leverage effects observed in leveraged ETF rebalance processes, such as Ivanov and Lenkey (2014) and Cheng and Madhavan (2009). In our risk parity simulation, the component is fairly minor (much smaller than the first component), explaining only about 1.5% of the one-day trade variance. However, it has a much larger impact on overall correlation to short volatility trades because it is *strongly negatively* correlated to them (-0.58 at a one-day horizon). This negative correlation makes sense because, if the average weight is consistently less than one (which is the case in our simulation given a 5% risk target and is likely to be true for almost all risk parity implementations given the relatively high volatility of equities as an asset class), then the component will be highly negatively correlated to the S&P 500 return (buying when the market has fallen), which is itself positively correlated to the delta hedge trade (selling when the market has fallen).[34]

The final component is what remains after subtracting the other two, capturing the equity trade induced by the PL from the two non-equity risk parity exposures (fixed income and commodities). Similar to the previous example, this effect exists because the PL from those exposures affects the risk parity NAV, thereby causing a

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[33] The "leverage effect" generally refers to the relationship between a change in an asset's price and its expected volatility. Applied to equities, under the Merton (1974) framework which views equity as a call option on a firm's value, stock has a levered exposure to its underlying firm. Therefore, its leverage is inversely related to firm value, leading to an increase in its volatility as the firm's value declines. This is the principle at play when the S&P 500's volatility is expected to increase as equity markets fall.

A separate leverage effect occurs for any portfolio that is not fully invested in a single asset. Because our stylized risk parity's exposure to equity is less-than-one relative to the total portfolio's NAV, its exposure to equity is *positively* related to firm values. Thus, the portfolio's allocation to equities tend to fall as equity value declines, inducing the strategy to *buy* in order to maintain its risk target.

So there are two leverage effects at play, and they go in opposite directions. The first is that the equity itself tends to become more volatile as equities fall. The second is that the portfolio's allocation to equity tends to decline as equities fall. Throughout the paper, we refer to the first effect as "changes in ex-ante volatility" and the second effect as the "leverage effect."

To make matters more confusing, risk parity's allocation to fixed income tends to be greater-than-one due to fixed income's low volatility. Thus, the fixed income leverage effect in risk parity is in the more traditional direction, where exposure to fixed income tends to increase as bonds decline in value.

[34] These relationships are further discussed in Israelov and Tummala (2018), which proposes a hedging overlay strategy designed to capitalize on the idea that "leverage effect" rebalance trades are typically in the opposite direction of short volatility delta-hedge trades.

deviation between the current equity weight and the desired equity weight. Empirically, this effect was slightly more important than the previous component, explaining 4% of the one-day variance in trades. But since fixed income and commodities PL are not highly correlated with short volatility trade sizes, it was similarly uncorrelated (-0.01).

To summarize, the above attribution analysis sheds some light on our original observation of a -0.06 correlation between risk parity and short volatility equity trades. Great attention is paid to the potential for joint trading due to the modest inverse relationship between equity returns and equity volatility. Although the trades deriving solely from changes in forecasted volatility were slightly positively correlated to short volatility trades, this correlation was more than canceled out by the negative correlation induced by the leverage effect, which is often ignored. **Exhibit 12** shows that this story also holds for longer horizons.

The Appendix applies this attribution to the fixed-income-specific versions of the strategies, with similar results, though the effects of the last two components differ due to the fact that the fixed income long-term volatility level is closer to the risk target of 5%.

Section 5 – Return Perspective

More important for investors than the explicit timing of trades, however, is the question of return sensitivities. Recently, market commentators claim that risk parity is implicitly short volatility.[35] In this section, we argue that these fears are unjustified: once we control for risk parity's general long exposures to its underlying assets, the returns of a volatility-targeted risk parity portfolio are almost orthogonal to the returns of short volatility strategies.[36]

Comparing Risk Parity and Short Volatility

To isolate a potential short volatility exposure within risk parity, our first step constructs (for both equity and fixed income) beta-neutral short volatility return series by removing the full-sample exposures to their underlying assets. Removing the average long underlying market exposure should help suss-out the "volatility-contingent" portions of each in their pure sense. If risk parity's action of changing exposures in response to market volatility quietly embedded short volatility exposure, it should show up as a loading on these.

We regress the three-asset-class risk parity strategy returns on its three underlying assets (S&P 500, US 10-year bond returns, and the Goldman Sachs GSCI Commodity Index) as well as on the full-sample beta-neutral equity

[35] For example, Cole (2017) classifies risk parity as an "implicit short volatility" strategy that "use[s] financial engineering to generate excess returns by exposure to the same risk factors as a short option portfolio." Bhansali and Harris (2018) also describes risk parity funds as "implicit volatility sellers".

[36] Both have a common exposure to long equities, but that commonality does not lend credence to the claim that risk parity is hiding short volatility exposure. For example, a Dow 30 ETF does not contain hidden short volatility exposure just because both it and short volatility are positively exposed to the S&P 500 index.

and fixed income short volatility series. **Exhibit 13** reports the regression output, along with a risk decomposition based on the fitted values of the regression.[37]

The results are striking: risk parity gets almost none of its risk from short volatility.[38] As **Panel 2** of **Exhibit 13** shows, less than 1% of risk parity's risk comes from the two beta-neutral short volatility strategies (equity and fixed income short volatility), with the remaining 99% coming from sources other than short volatility. In fact, most of risk parity's risk comes simply from its average exposures to its three underlying assets – 28% from equities, 24% from fixed income, and 34% from commodities. The remaining 14% of the risk came from the unexplained regression model residuals, which are likely driven by the time-varying exposures to the underlying asset classes.

Comparing the Risk Parity Equity Component and Equity Short Volatility

To provide a clearer picture, we simplify our analysis by restricting it to a comparison between the *equity component* of the risk parity simulation and our *equity* short volatility series.

The left panel of **Exhibit 14** scatter plots S&P 500 daily returns (*x*-axis) against the daily return to the equity component of risk parity (*y*-axis). Unsurprisingly, the risk parity equity component is positively exposed to the S&P 500 through its time-varying but always-long exposure to the asset; positive (negative) return for the S&P 500 necessarily means positive (negative) return for that sleeve of risk parity. The slope of the trend-line (0.24) is roughly the average exposure to equities.[39] But risk parity equity returns are not perfectly correlated to S&P 500 returns, because this strategy does not simply hold a constant position all of the time. Instead, as risk forecasts change, the volatility-targeted equity component changes its exposure dynamically. The end effect of volatility-targeting is an hourglass pattern due to the rotation of the line expressing the sensitivity of the risk parity equity component to the underlying equity return.[40]

The second panel in **Exhibit 14** plots S&P 500 returns against short volatility returns. This relationship is strikingly different from risk parity's, precisely because the short volatility strategy earns its returns based on a more complex function of underlying asset returns. Recall that short volatility sells S&P 500 options (both calls and puts), and that those options become more valuable (and therefore provide negative returns to the strategy)

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[37] The "Risk Contribution" row is defined as the covariance of the component returns (regression coefficient times the explanatory variable returns) with risk parity returns, divided by the variance of the risk parity returns. The component "From Other" is defined as the regression residuals of the model fit in Panel 1 of **Exhibit 13**.

[38] The regression coefficient on equity short volatility is statistically insignificant, while the coefficient on fixed income short volatility is positive and statistically significant, but economically small (a 37% exposure to the fixed income short volatility strategy used here takes only 0.2% volatility given the low overall risk level of short volatility)

[39] This is materially lower than the average notional exposure to equities (0.35) simply because the act of holding smaller notional positions during periods of high volatility (and thus higher notional positions during periods of low volatility) mechanically reduces the risk taken by the strategy relative to holding a constant notional position equal to the average position of the strategy. See Asness-Hood-Huss (FAJ, 2015) for further explanation of this phenomenon.

[40] It is worth noting that this 'rotation' of the exposure line never turns negative – there are exactly zero data points in the 2nd and 4th quadrants, because risk parity never takes short positions in the underlying equities. It is only ever takes long positions in the S&P 500.

when volatility rises. As such, large magnitude S&P 500 returns *of either sign* can result in negative short volatility returns.[41]

It may be tempting to view **Exhibit 14** as confirmation of the claims about risk parity's hidden short volatility exposure, since it demonstrates that both strategies tend to lose money when the S&P 500 declines. However, while it is true that both strategies are positively exposed to equities, this is also the case with many other investments like single name stocks, global equity indices, credit indices, real estate, etc. Using just their common positive equity exposure to justify this claim would therefore be misleading. In order to single out short volatility as a hidden component of risk parity, you would need to find that the *equity residuals* of risk parity equity returns (the hourglass shape) are related to short volatility. But this requires a more careful analysis; it cannot be deduced simply from the plots in **Exhibit 14**.

To further the point, we observe the effect of removing the full-sample betas to S&P 500 returns of both the risk parity equity component and the short volatility strategy. We compare those two beta-neutralized returns in **Exhibit 15**. These beta-neutral risk parity returns are nearly unrelated to beta-neutral short volatility returns, with an estimated correlation coefficient of only 0.09. Further, data points corresponding to negative outliers for both return series (3rd quadrant outliers) do not appear particularly prominent. In other words, any claim that risk parity equity returns experience downside risk at the same time as short volatility returns is explained by the simple fact that risk parity is long equities overall.[42] There is nothing special about risk parity that makes it connected to short volatility; it is simply exposed to equities.

We continue our examination of this relationship by regressing risk parity's equity component returns on S&P 500 returns and beta-neutral short volatility returns. **Exhibit 16** reports the regression output, along with a risk decomposition based on the fitted values of the regression.

The results confirm what we'd seen previously for three-asset-class risk parity: almost none of the equity component's risk comes from short volatility.[43] As **Panel 2** of **Exhibit 16** shows, less than 1% of risk parity equity component risk comes from short volatility, with the remaining 99% coming from sources other than short volatility. Approximately 80% of the risk comes from simple equity exposure, with the remaining 20% coming

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[41] Realized volatility affects option returns over the entire time to expiration of the options. This means that when delta-hedged options experience realized volatility higher than their implied volatilities over the full period between inception and expiration, they will tend to lose money over that entire period. For shorter horizons, however, they may still experience short term fluctuations in returns. The short term (ie, 1-day) returns of sold options are determined by implied volatility, which is a function of both expectations of future realized volatility and the size of the volatility risk premium. To the extent that investors demand increased compensation for taking short options positions as underlying prices fall, the implied volatility does not need to respond to large positive and negative returns as symmetrically as realized volatility typically does, rising more in sharply falling markets than in sharply rising markets. Indeed, our short volatility returns do maintain some exposure to S&P 500 returns (the returns are 0.31 correlated) despite their quadratic nature, reflecting this empirical observation.

[42] And we would expect the same of a basic equity portfolio, a basic credit index, and so on. These all have long exposure to equity risk, and therefore experience negative returns at the same time that short volatility realizes losses. That does not mean that these strategies are actually short volatility strategies.

[43] The regression coefficient on short volatility is positive and statistically significant, but is economically small (a 12% exposure to the short volatility strategy used here takes only 0.2% volatility given the low overall risk level of short volatility).

from the unexplained regression model residuals. A non-volatility-targeted risk parity equity exposure would get 100% of its risk from the S&P 500, so it is clear that our simulation is getting some risk from some additional source; it is also clear that that additional source is not short volatility.

Finally, we note that any risk parity return exposure to short volatility, while already immaterial at a one day horizon, is even weaker over longer periods. **Exhibit 17** shows both the estimated regression coefficient and the risk contribution from short volatility as a function of return horizon. Both fall from already low levels to roughly zero at a one month horizon, even becoming slightly negative over longer periods.

The Appendix replicates this section using fixed-income-specific versions of the simulations for robustness and find nearly identical results.

Section 6 – Conclusion

Our contribution in this paper is to rigorously examine the relationship between the risk parity and short volatility strategies in an empirical sense. We construct replicable simulations of the two strategies and demonstrate that, far from being the same strategy, they show remarkably little direct connection in terms of either trading behavior or portfolio returns.

Trades for short volatility and the equity portion of the risk parity strategy are roughly uncorrelated at any horizon. Their relationship is better described as quadratic, with the direction of risk parity trades connected to the magnitude, but agnostic to the direction, of short volatility trades. Conditional on risk parity selling on a given day, short volatility is as likely (if not more likely) to buy on that day than it is to sell.

Additionally, once we control for risk parity's positive average betas to its underlying assets, we find that the returns of these two strategies are almost orthogonal, with short volatility explaining less than one percent of the total variance of risk parity returns. The same is true for both the equity and fixed income[44] components even when considered in isolation of the other asset classes, making absurd the notion of hidden short volatility exposure for risk parity strategies.

We also reiterate that these simulations were built using assumptions selected to heighten the similarities between the strategies. They therefore likely *overestimate* commonality in trading. For example, our simulations assume that risk parity immediately executes its trades in response to a market event changing its volatility estimate; in reality, practitioners delay trading by at least one day. We chose a very responsive risk model which increases the magnitude of trades in response to market moves; in practice many portfolios instead hold static positions over time. We assume that both the risk parity and short volatility strategies trade only S&P 500 and

[44] As shown in the Appendix.

its derivatives for their equity components; it is common practice to hold more diversified, global equity positions in both strategies. We assume that our short volatility strategy will always trade daily to be completely delta-neutral; some option sellers do not delta hedge at all, hedge infrequently, or only hedge a fraction of their exposures. We ignore the necessary existence of offsetting long option positions; some option buyers also delta-hedge in a way that would net against the trades of option sellers. In light of these assumptions, that we still fail to find a compelling connection beyond simple equity beta is a strong statement against claims the two strategies are the same.

Based on the results herein, we reject the idea that risk parity (or a volatility-targeted strategy in general) is taking a bet on falling volatility. We also reject the idea that the trading behavior of these two strategies is so connected as to constitute a systemic risk to the financial system. Risk parity takes long positions in equity markets; short volatility takes short positions in options markets. These are materially different things, even while they both interact with common equity markets.

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Appendix

Comparing Fixed Income Short Volatility and Risk Parity: Trade Perspective

Section 4 of the paper used an equity short volatility simulation and the equity component of a three-asset-class risk-parity simulation to examine the relationship between the trading behavior of these two strategies. However, our conclusions are not equity-specific, and can in fact be applied to other asset classes. Specifically, in this Appendix section we find very similar results when we consider the trades of a *fixed income* short volatility strategy and the *fixed income* component of our risk parity simulation.

To examine this relationship, we again use stylized historical simulations using daily data between January 1996 and April 2018. For US 10-year bond options, price data comes from the Chicago Mercantile Exchange (CME). 10-year bond futures prices and returns, which are used to calculate position sizing and returns for both short volatility and risk parity, are also derived from Bloomberg data. The risk parity simulation is identical to the one used in the previous sub-sections, except instead of just examining the equity exposure, we now take a look at the component consisting of US 10-year bond futures. For the fixed income short volatility strategy, we also use the same portfolio construction as the previously-described equity short volatility strategy, except applied to options on US 10-year bond futures and hedged using those same futures.

One notable difference between the fixed income short volatility strategy and the S&P 500 short volatility strategy is that options on US 10-year bond futures are physically settled, rather than cash settled. This means that for our fixed income comparison, there are none of the "clean-up" trades that we'd seen in the equity short volatility simulation.[45] In the equity comparison, these clean-up trades added a significant amount of noise to the relationship between short volatility and risk parity trades.

Nevertheless, as we see in the next few charts, by and large the equity results do extend to fixed income even in the absence of clean-up trades. **Exhibit A1**, for example, plots the relationship between the fixed income risk parity trades and the underlying asset return, again finding the two to be nearly uncorrelated overall (correlation of 0.07), but with a clear (negative) absolute relationship.

Additionally, it is still the case that risk parity trades and short volatility hedge trades are uncorrelated but non-linearly related for fixed income. In **Exhibit A2**, we observe a negligible full-sample correlation of 0.07 between

[45] Derivatives contracts like options can be "cash settled", meaning that the losing party delivers cash equal to the value of the final contract loss to the winning party, or "physically settled" where the underlying asset itself, and not just its cash value, must be delivered. Cash settled options require a clean-up trade for hedgers because they hold positions in the underlying asset at expiration that must be liquidated, whereas with physically settled options the hedger can simply deliver the underlying position. For instance, if a call option seller is hedging their position and the option expires in the money, the delta hedge will consist of exactly one long position in the underlying for each option traded. If this option is physically settled (as is the case for the fixed income options used in our simulation), then the option seller can simply deliver the long position in the underlying in exchange for the strike price. However, if it is cash settled, the seller must deliver the difference between the market price of the underlying and the strike price, and then recoup the strike by selling the underlying asset in the underlying market (the clean-up trade).

the two. And this correlation remains negligible over longer horizons up to 65 days (as seen in **Exhibit A3**), never exceeding 0.14. Similar to the equity case study, the direction of the risk parity trade tells us something about the magnitude of the short volatility trade, and vice versa. Large-sized delta-hedge trades (buys or sells) are often, but not always, associated with risk parity deleveraging, and significantly-sized risk parity trades tell us little about short volatility's trading direction.

Exhibits A4 and **A5** replicate **Exhibits 9** and **10**, respectively, for fixed income. **Exhibit A4** shows a remarkably clear upside-down-'U' relationship. Unlike with equities, this relationship is quite symmetric, with short volatility generally being equally likely to buy or sell given a large risk parity sell. **Exhibit A5** also repeats the findings of **Exhibit 10**, again with slightly more symmetry when conditioning on extreme short volatility trades. Risk parity has similar percentages of days selling when short volatility either sells or buys its most extreme amounts (86% of the time when buying, 89% when selling), for instance. The t-statistics are of a similar magnitude (-1.02 when short volatility is buying, -1.20 when it is selling) as well. This additional symmetry likely appears because fixed income generally exhibits less correlation between prices and volatility, and because the risk parity allocation to fixed income is closer to full leverage, meaning that there are fewer opportunities for leverage-based rebalancing to interfere with the underlying relationship.

Last, we can also apply the trade attribution methodology introduced in the equity-only section in order to gain further insight on the reasons for the lack of relationship between the two. Just as with equity risk parity, the "Change in Volatility Estimate" component is by far the most important driver for fixed income risk parity trades (explaining 91% of the trade variance) and it has a 1-day correlation of 0.06 to short volatility delta traded. Compared to this, the other two components have much weaker effects on the correlation, and essentially cancel each other out. The "FI-Bucket PL" component (which measures the "leverage effect") explains only 0.5% of the variance in trade size, though it has a large positive (+0.66) correlation to short volatility delta traded. This is less impactful than we'd seen for equity risk parity, because the 5% volatility target is much closer to the underlying fixed income volatility (3.7%) than it is to the underlying equity volatility (17.3%). Additionally, the correlation to short volatility delta traded is strongly positive, whereas for equity we'd seen a strongly negative relationship. This is because the fixed income leverage in the risk parity simulation is typically greater than one, while the equity leverage is typically less than one. Finally, the "Other-Bucket PL" component accounts for 9% of the trade variance, and partially cancels out the previous component by having a negative correlation (-0.15) with short volatility delta traded. All of these relationships hold at longer trade horizons, as seen in **Exhibit A6**. Overall, despite the lack of clean-up trades, the conclusions reached for equities hold up well when extended to fixed income.

Comparing Fixed Income Short Volatility and Risk Parity: Return Perspective

For further validation, we can also extend the return analysis applied to equity-specific versions of the strategies in **Section 5** to the fixed income short volatility and fixed income risk parity series that we introduced in the previous Appendix section.

To start off, **Exhibit A7** reproduces **Exhibit 14** from the sub-section on equities. On the left, we show a scatterplot of the relationship between daily returns of the risk parity fixed income component (*y*-axis) and US 10-year futures returns (*x*-axis). Unsurprisingly, this is similar to equities in showing a very strong correlation between the two (0.96). On the right, we show the analogous chart for short volatility, scatter plotting fixed income short volatility (*y*-axis) against the underlying US 10-year treasury futures (*x*-axis). Just as we report for equities, this latter relationship is again highly distinct from the one on the left, since large underlying asset returns of either direction tend to cause short volatility losses. One notable difference compared to equities is that the right panel shows a correlation of 0.00, whereas for equities, short volatility returns and underlying asset returns had a moderately positive correlation (0.31). This can be explained by the lack of correlation between bond returns and bond volatility. Unlike in equities, fixed income implied volatility spikes are approximately equally likely to be accompanied by down-moves in the underlying asset as up-moves.

In **Exhibit A8**, we remove the risk parity fixed income component's full-sample beta to US 10-year bond futures, and we compare those beta-neutral returns to beta-neutral fixed income short volatility. As with equities, if short volatility is really a hidden component of risk parity returns, then this should become even more apparent once the risk parity returns are "purified" by factoring out risk parity's passive long-term bond exposure. However, as with **Exhibit 15** for equities, this is not the case empirically. The two series (beta-neutral risk parity and beta-neutral fixed income short volatility) turn out to have correlation of only 0.01. While there are of course some points in the lower-left quadrant of the scatter[46], the other quadrants are heavily represented as well and overall the relationship looks very noisy.

To look at this more rigorously, we can replicate the main-body sub-section by regressing risk parity's fixed income daily returns on both the underlying asset returns (US-10 year bond futures) and beta-neutral fixed income short volatility returns. **Exhibit A9** provides the output of this regression along with a risk decomposition based on the fitted values.

The results, once again, mirror what we report for equities. The short volatility sub-component ("From Short Volatility" in the decomposition) has an extremely small economic impact, contributing essentially nothing to either the return or risk of the risk parity fixed income component. Moreover (and unlike in equities), the

[46] In particular, the most prominent lower-left outlier occurred on March 18, 2009, when beta-neutral fixed income short volatility was down 0.7% and risk parity's beta-neutral fixed income component had a return of -1.1%. These large moves were caused by the Federal Reserve announcing it would buy $300 billion in long-term Treasuries.

regression coefficient on short volatility is not statistically significant, with a t-stat of 1.1. Changing the horizon does not alter this story, as can be observed in **Exhibit A10**.

As with equities, the evidence does not support a notion of fixed income risk parity having a hidden fixed income short volatility exposure. In fact, from a returns perspective the case in fixed income seems even weaker than the (already weak) case for equities.

Calculation of Risk Parity Trades

As mentioned in the paper, for each asset class *k* (equities, fixed income, or commodities), we calculate the return of that asset class's risk parity bucket as:

$$RP_{k,t} = \frac{0.05}{\sigma_{k,t-1}} * r_{k,t}$$

where $r_{k,t}$ is the return on date t for asset k.

Let NAV_t be the NAV of the entire risk parity strategy at the close of day *t*. Since our simulation allows this NAV to float over time along with the PL of the strategy, we calculate the NAV each day as:

$$NAV_t = NAV_{t-1} * (1 + RP_{fund,t})$$

In order to calculate risk parity trade sizes, we assume that trades occur instantaneously at the close of each day. Therefore, in dollar terms, the post-trade position size for asset *k* at the close of day *t* will need to be:

$$PostTradePos_{k,t} = \frac{0.05}{\sigma_{k,t}} * NAV_t$$

Based on this, we can then grow the previous day's post-trade position to derive that the pre-trade position size for asset *k* at the close of day *t*:

$$PreTradePos_{k,t} = \frac{0.05}{\sigma_{k,t-1}} * NAV_{t-1} * (1 + r_{k,t})$$

And since we now know what both our pre-trade and post-trade positions will be, we can calculate each day's trade (as a percent of NAV) simply by taking the difference between these two and dividing by the NAV:

$$\begin{aligned} Trade_{k,t} &= \frac{1}{NAV_t} * \left(PostTradePos_{k,t} - PreTradePos_{k,t}\right) \\ &= \frac{1}{NAV_t} * \left(\frac{0.05 * NAV_t}{\sigma_{k,t}} - \frac{0.05 * NAV_{t-1}}{\sigma_{k,t-1}} * \left(1 + r_{k,t}\right)\right) \\ &= 0.05 * \left(\frac{1}{\sigma_{k,t}} - \frac{1}{\sigma_{k,t-1}} * \left(\frac{1 + r_{k,t}}{1 + RP_{fund,t}}\right)\right) \end{aligned}$$

Decomposition of Risk Parity Trades

In the previous subsection of the appendix, we calculated the trade size each day for each asset (as a percent of NAV) as:

$$Trade_{k,t} = 0.05 * \left(\frac{1}{\sigma_{k,t}} - \frac{1}{\sigma_{k,t-1}} * \left(\frac{1 + r_{k,t}}{1 + RP_{fund,t}}\right)\right)$$

In order to decompose this, we can first break out the hypothetical trade deriving solely from the change in the volatility estimate. This is equivalent to what the trade size would be in a hypothetical fixed-NAV version of a risk parity simulation:

$$VolChangeTrade_{k,t} = 0.05 * \left(\frac{1}{\sigma_{k,t}} - \frac{1}{\sigma_{k,t-1}}\right)$$

From this, we can rearrange the first equation to derive an alternate expression of our trades in terms of this new component:

$$Trade_{k,t} = VolChangeTrade_{k,t} + 0.05 * \left(\frac{RP_{fund,t} - r_{k,t}}{\sigma_{k,t-1} * (1 + RP_{fund,t})}\right)$$

Next, we break out the trades induced by the PL specifically coming from risk parity's *k*-asset bucket (as opposed to the effect of the two buckets' PL). To do this, we consider what the trade would be if the other two assets had zero PL on day *t* (implying that $RP_{fund,t}$ would be equal to $RP_{k,t}$) *and* if our volatility estimate did not change (implying that $VolChangeTrade_{k,t} = 0$). Based on the previous equation, this is given by:

$$AssetPLTrade_{k,t} = 0.05 * \left(\frac{RP_{k,t} - r_{k,t}}{\sigma_{k,t-1} * (1 + RP_{k,t})}\right)$$

Lastly, the remaining part of the risk parity trade for asset *k* is attributable to the PL coming from risk parity's other two buckets (the non-*k* buckets). After some rearranging, this remainder can be expressed as:

$$OtherPLTrade_{k,t} = 0.05 * \left(\frac{\left(RP_{fund,t} - RP_{k,t}\right) * (1 + r_{k,t})}{\sigma_{k,t-1} * \left(1 + RP_{fund,t}\right) * (1 + RP_{k,t})}\right)$$

These three components provide a full decomposition of each day's risk parity trades:

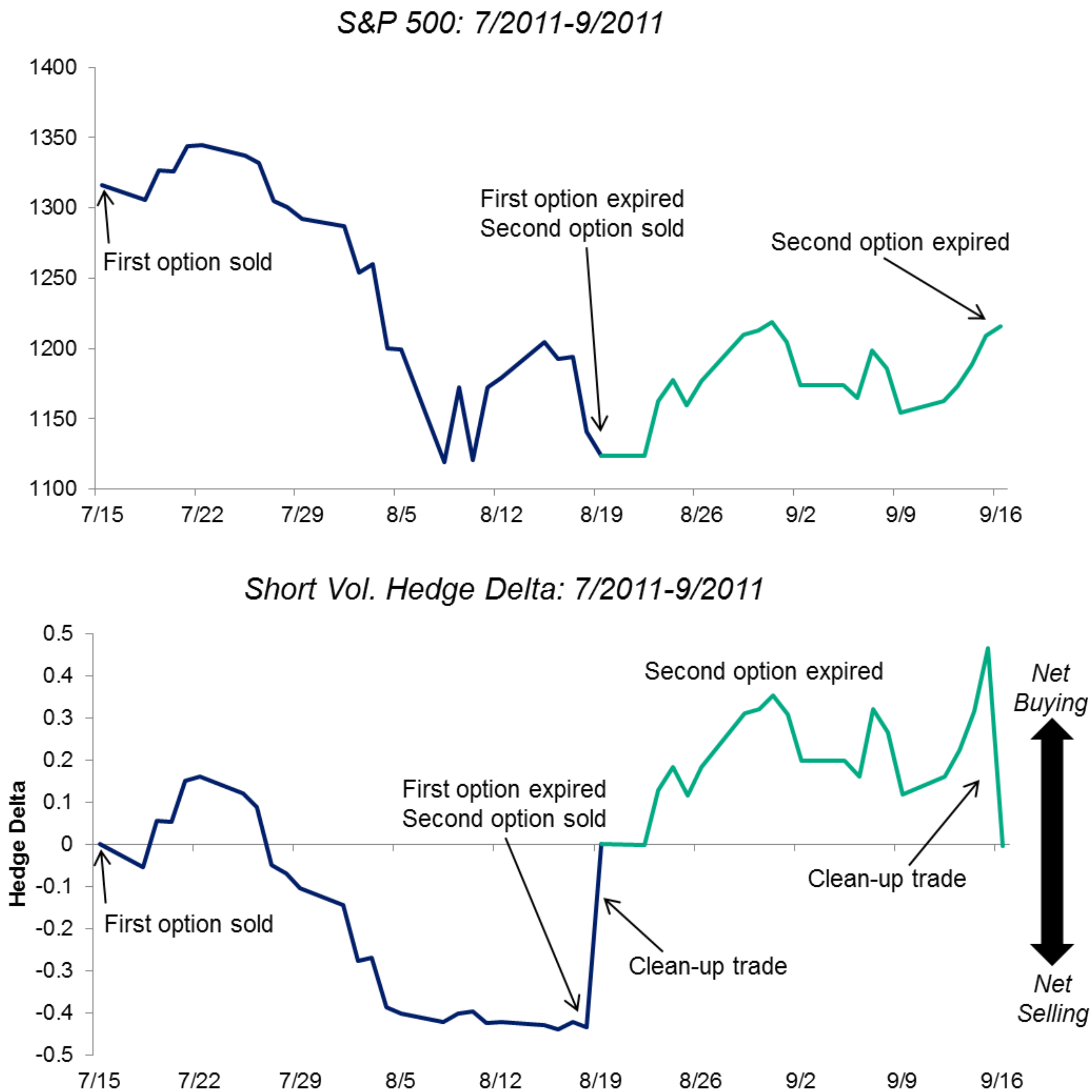
$$Trade_{k,t} = VolChangeTrade_{k,t} + AssetPLTrade_{k,t} + OtherPLTrade_{k,t}$$

Effect of Trade Delay on Risk Parity vs. Short Volatility Trade Relationship

The risk parity simulation being considered in this paper makes the conservative assumption that there is no trade delay, meaning that trades happen instantaneously at the close based on an updated ex-ante volatility estimate which includes that day's return. Under that assumption, we estimated a -0.06 one-day correlation between short volatility equity trades and risk parity equity trades. However, if you instead assume a more realistic delay of one or two days in the ex-ante volatility estimate, then we find that the correlation becomes *even more negative*, falling to -0.17. The trade-delayed correlation to short volatility trades remains below the original one over long horizons as well, as seen in **Exhibit A11**.

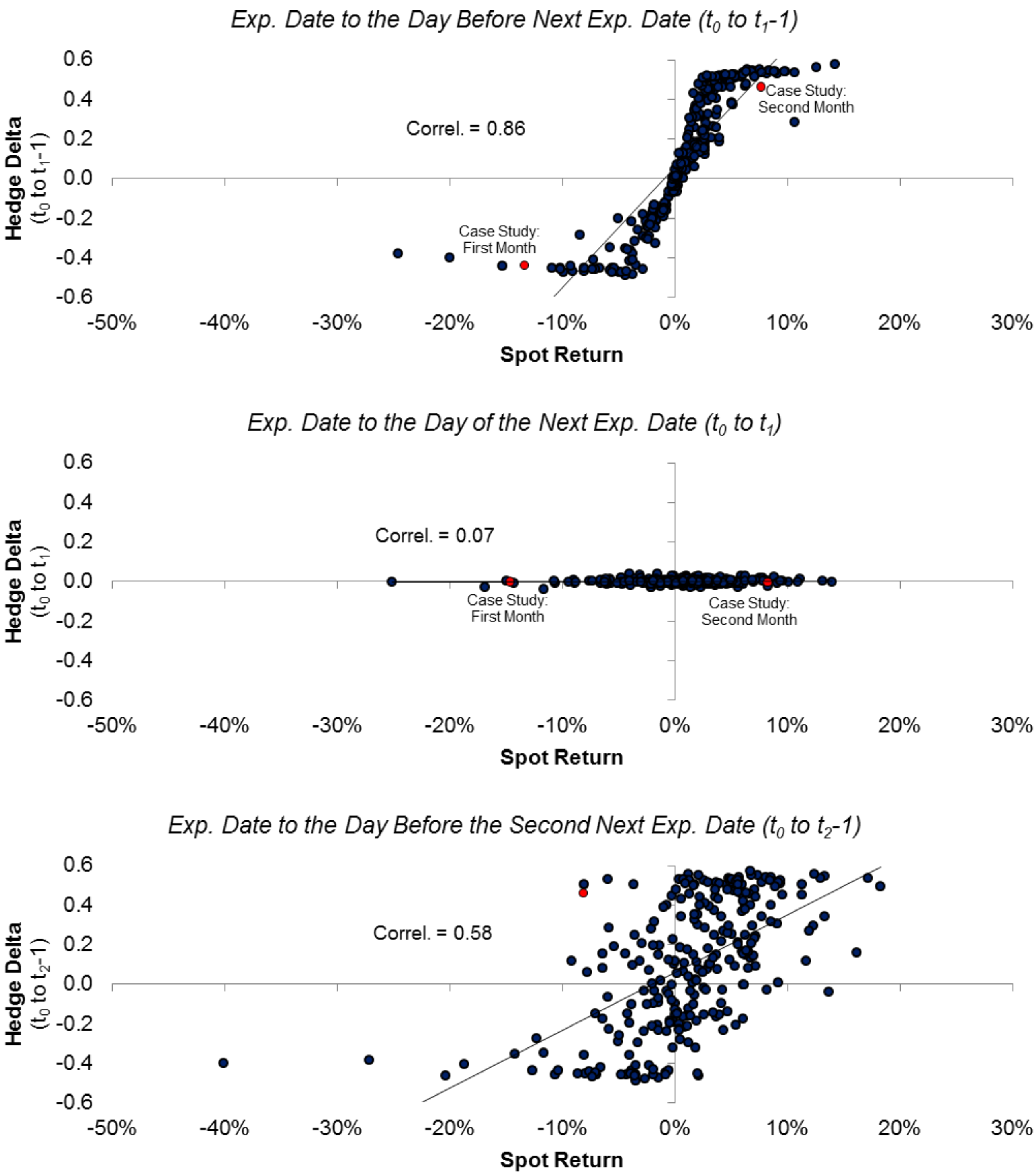
The decomposition methodology introduced in the "Risk Parity Trade Attribution" sub-section can help us explain why the one-day correlation is lower for the trade-delayed versions of the risk parity simulation. As we observed with the zero-delay risk parity simulation, the positive correlation from the "Change in Volatility Estimate" component had been approximately cancelled out by the negative correlation from the "Equity-Bucket PL" component (as a result of the "leverage effect"). However, this is no longer the case when we introduce a trade delay. With the delay, the "Change in Volatility Estimate" component is now based on the *lagged* change in ex-ante volatility, which is uncorrelated (at a one-day horizon) with the delta hedge trade. But the "Equity-Bucket PL" component is nearly identical regardless of the lag, since it is driven by the S&P 500's return and the leverage, not by changes in the ex-ante volatility estimate. Therefore, the effect of introducing the trade delay is that the negative correlation from the "Equity-Bucket PL" component drags down the overall correlation even more strongly than before.

Exhibit 1: Short Volatility Strategy Case Study



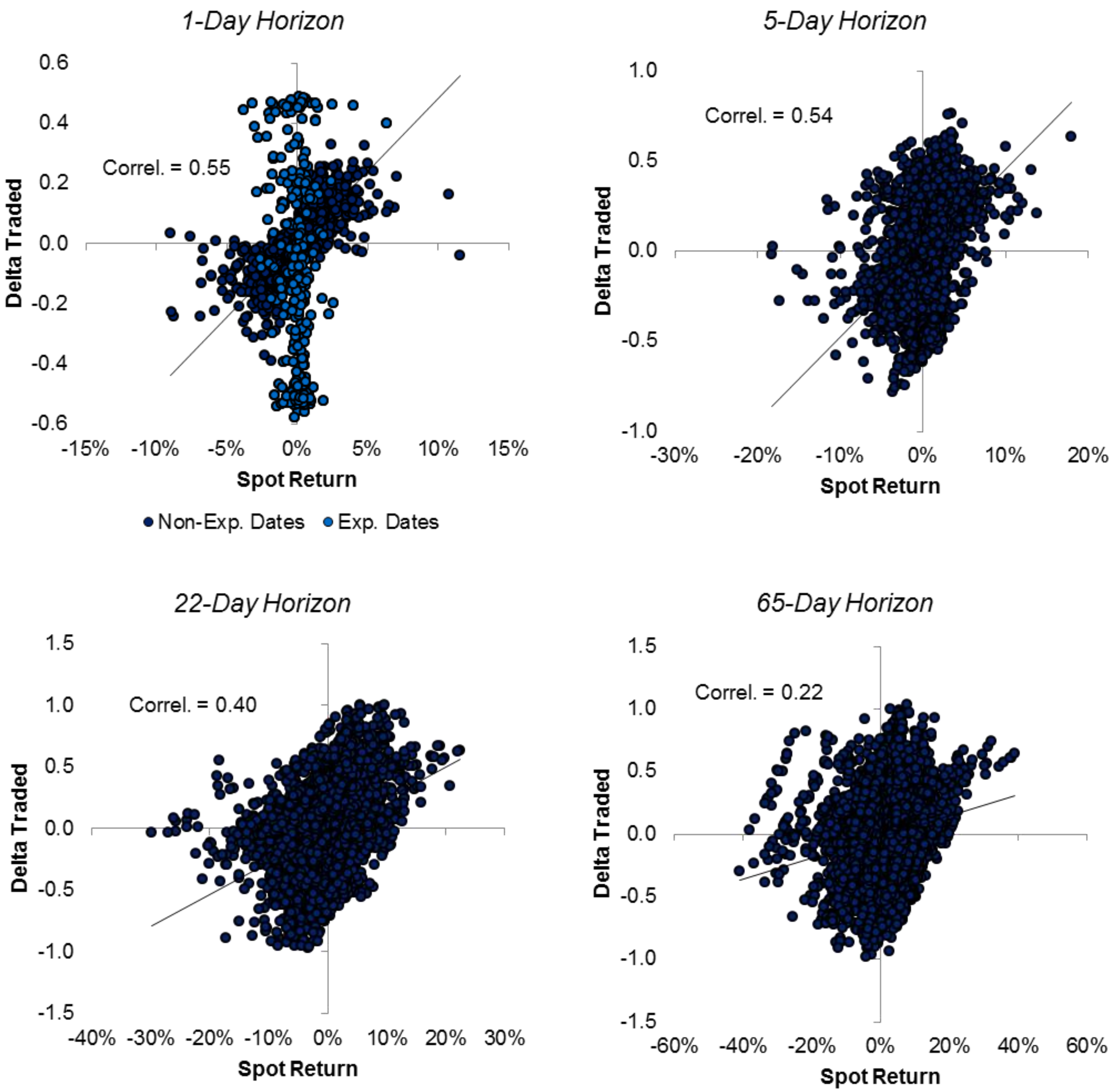
Source: AQR, Bloomberg, OptionMetrics. The top panel shows the S&P 500 level over the course of two monthly option expiration cycles. Over the same period, the bottom panel shows the total delta-hedge position in the underlying asset taken by our stylized short volatility strategy. The date range is from July 15, 2011 through September 16, 2011.

Exhibit 2: Short Volatility Hedge Delta vs. Spot Return



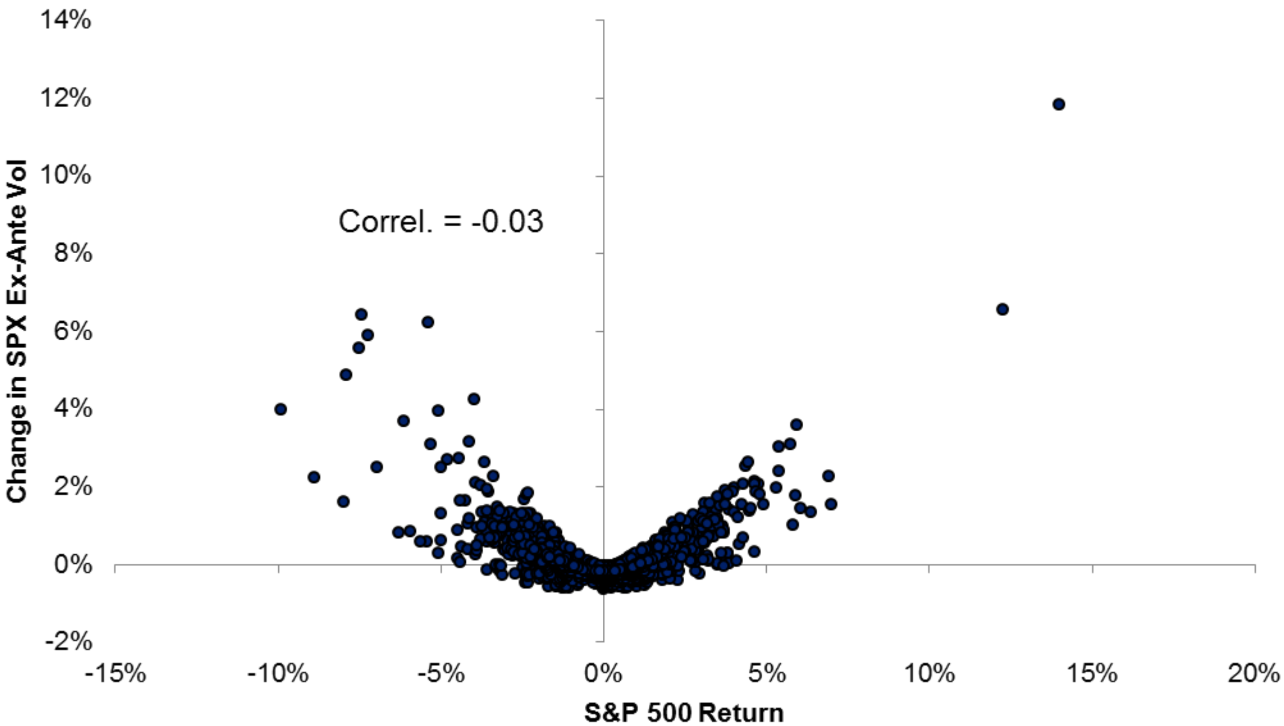
Source: AQR, Bloomberg, OptionMetrics. Each point in the top scatter chart corresponds to a time period from one monthly S&P 500 option expiration date until the day before the next monthly S&P 500 option expiration date. The x-values of those points are the S&P 500 returns over those periods, and the y-values are the cumulative delta-hedge trade in the underlying asset by our stylized short volatility strategy over those periods. The middle chart is similar except the time periods are from one monthly expiration date until the day *of* the next expiration date (inclusive), and the bottom chart's time periods are from one monthly expiration date until the day before the *second* next expiration date. In all charts, the months from the case study (from July 15, 2011 to September 16, 2011) are highlighted in red. The date range is from January 1996 through April 2018.

Exhibit 3: Short Volatility Delta Traded vs. Spot Return



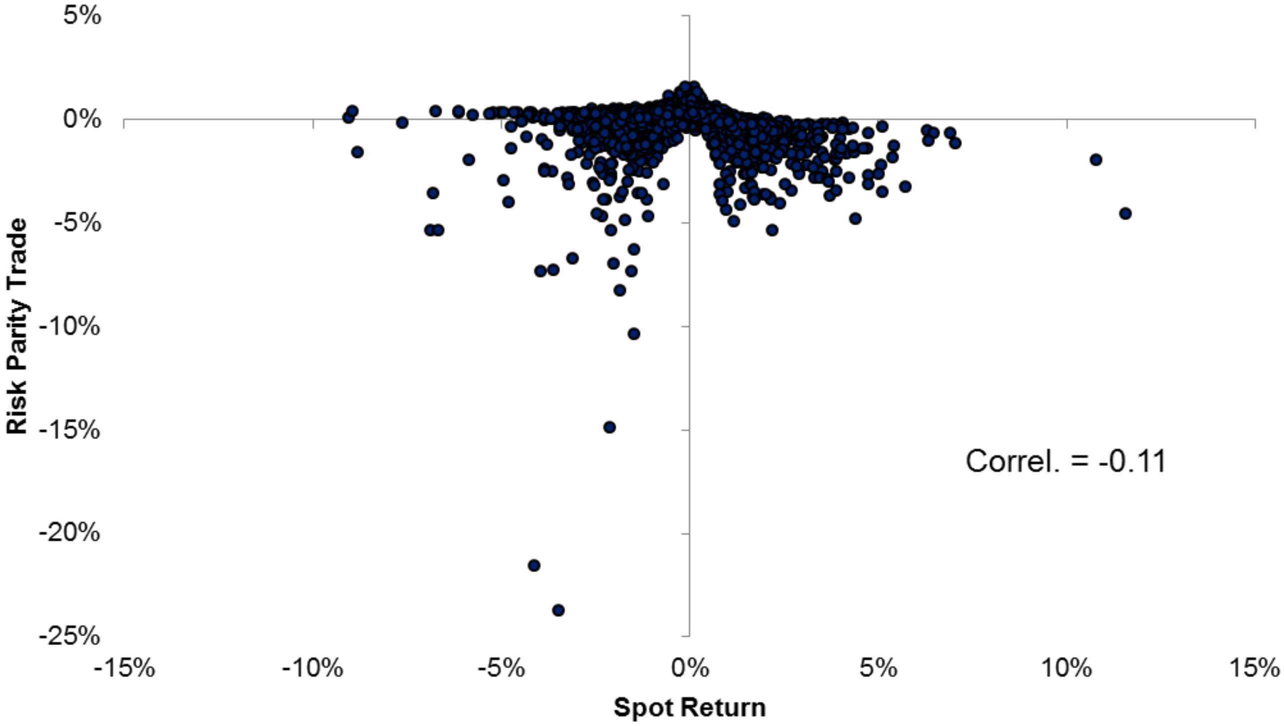
Source: AQR, Bloomberg, OptionMetrics. The scatter charts plot the S&P 500 return (x-axis) vs the cumulative delta-hedge traded of our stylized short volatility simulation (y-axis) over various horizons. The top left uses a 1-day horizon (i.e., daily returns), while the top right uses a 5-business-day horizon (i.e., weekly returns), the bottom left uses 22-business-day returns (i.e., monthly returns), and the bottom right uses 65-business-day returns (i.e., quarterly returns). The date range is from January 1996 through April 2018.

Exhibit 4: S&P 500 Return vs. Change in SPX Ex-Ante Volatility (1-Day Horizon)



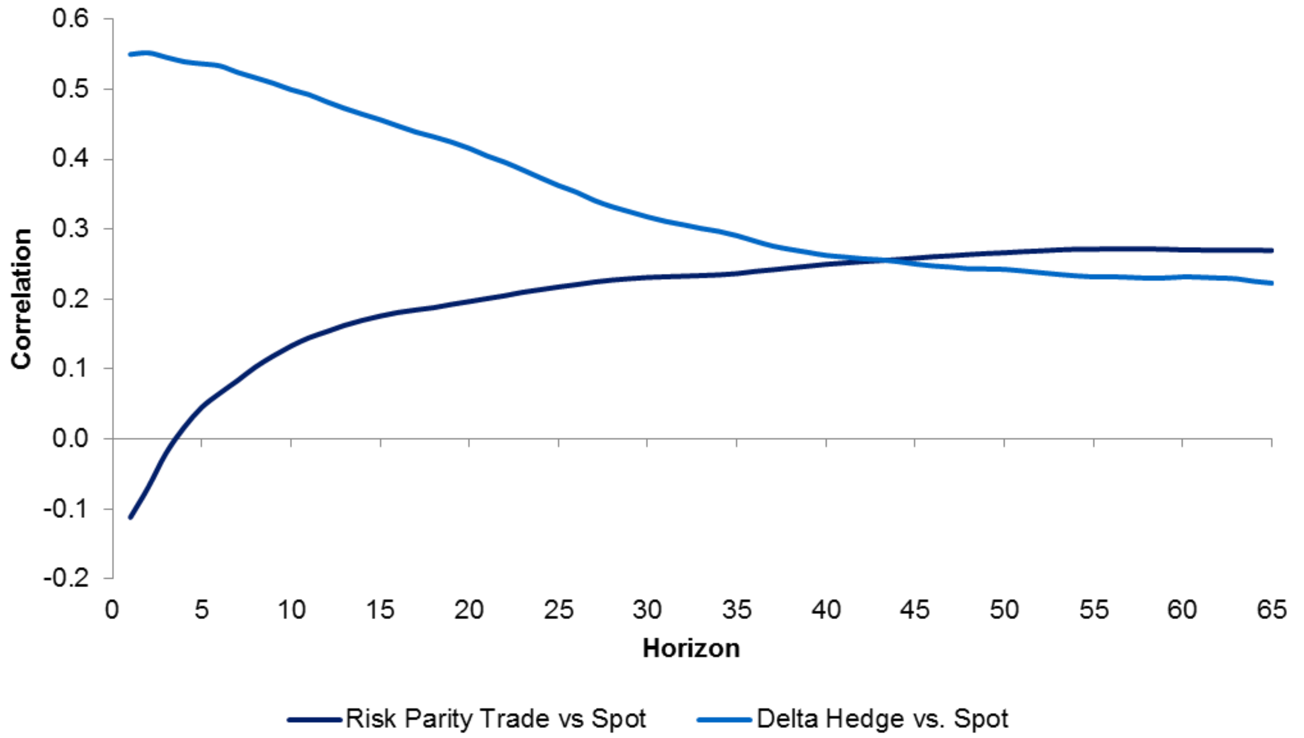
Source: AQR, Bloomberg. The scatter plot shows the daily S&P 500 return (x-axis) vs the corresponding daily change in our risk parity strategy's ex-ante S&P 500 volatility estimate. The date range is from January 1990 through April 2018.

Exhibit 5: Spot Return vs. Risk Parity Trade (1-Day Horizon)



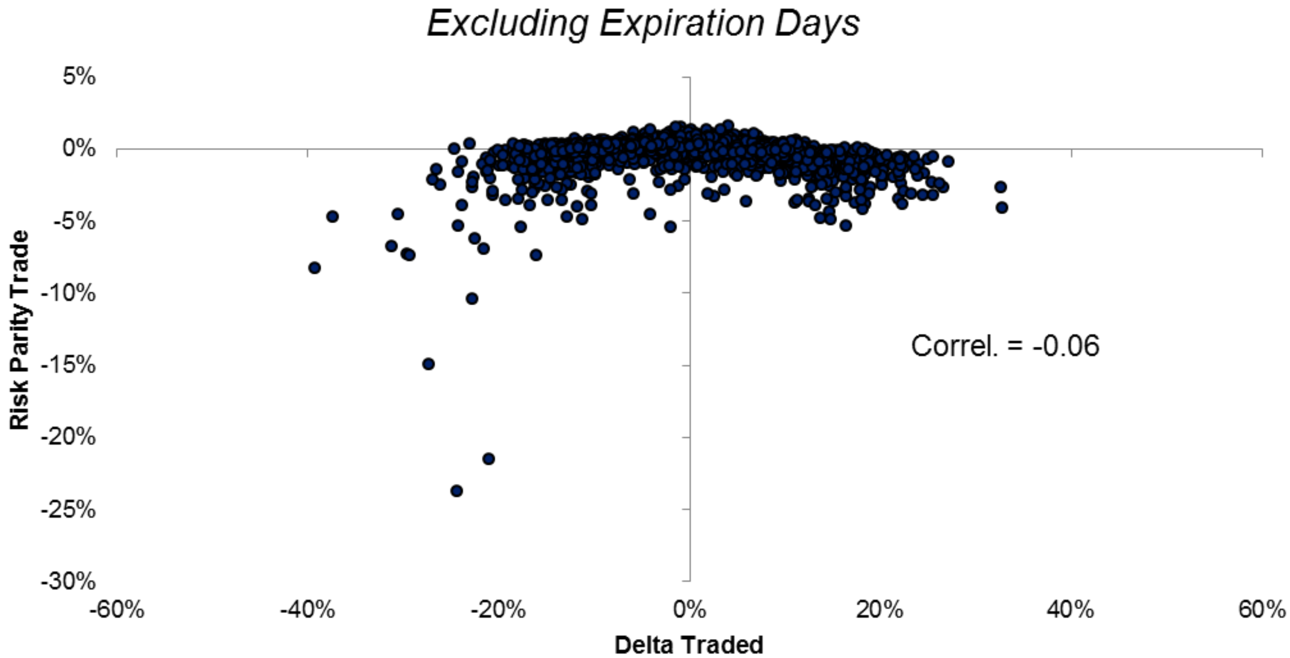
Source: AQR, Bloomberg. The scatter plot shows the daily S&P 500 return (x-axis) vs the corresponding daily trade in the risk parity strategy's S&P 500 component (as a percent of NAV). The date range is from January 1996 through April 2018.

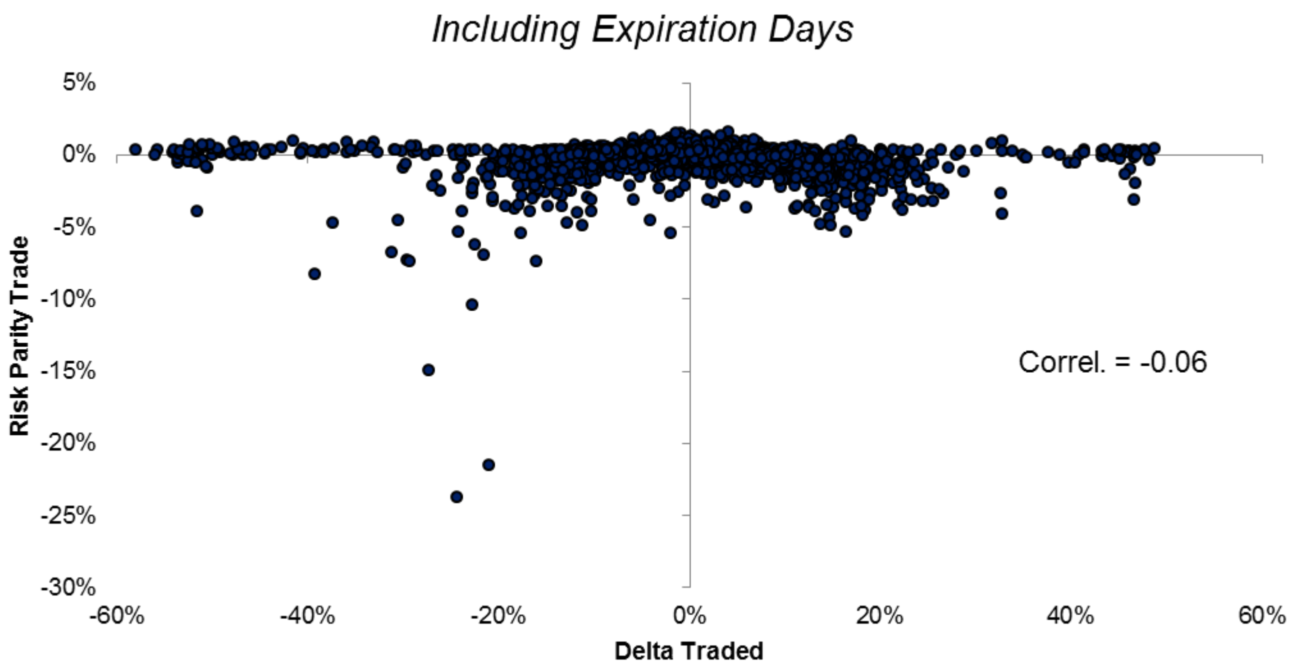
Exhibit 6: Risk Parity Trade and Delta Hedge, Correlation to Spot Return by Horizon



Source: AQR, Bloomberg, OptionMetrics. The solid line in the plot shows the correlation over various horizons between the S&P 500 return and the corresponding cumulative trade in our risk parity strategy's S&P 500 component (as a percent of NAV). The dotted line shows the correlation over various horizons between the S&P 500 return and corresponding cumulative delta-hedge traded in our stylized short volatility strategy. The horizon ranges from 1 day through 65 business days. The date range is from January 1996 through April 2018.

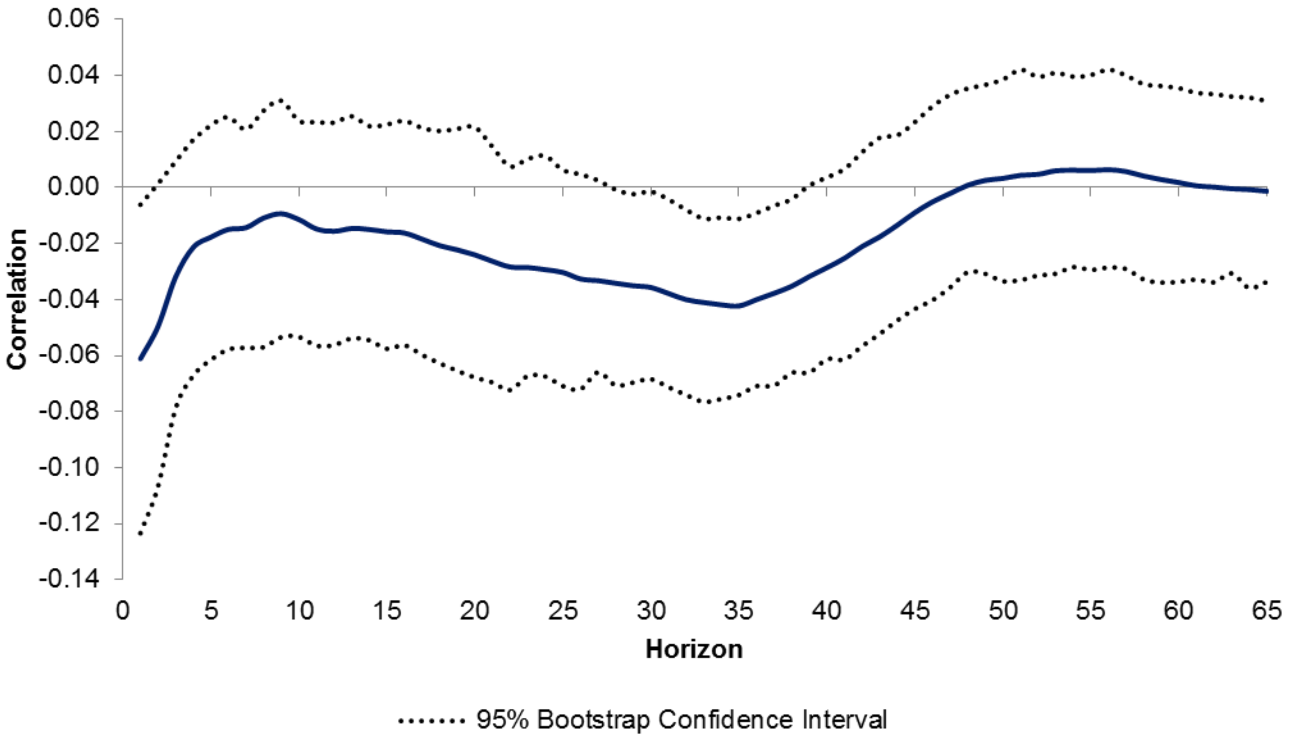
Exhibit 7: Delta Traded vs. Risk Parity Trade (1-Day Horizon)





Source: AQR, Bloomberg, OptionMetrics. The scatters above plot the relationship between the daily delta-hedge traded in our stylized short volatility simulation (x-axis) vs the daily trade in our risk parity strategy's S&P 500 component (y-axis). The bottom chart includes all days within our time period, while the top excludes option expiration dates. The date range is from January 1996 through April 2018.

Exhibit 8: Correlation between Delta Traded and Risk Parity Equity Trade, by Horizon



Source: AQR, Bloomberg, OptionMetrics. The chart shows the correlation over various horizons between the cumulative trade in our risk parity strategy's S&P 500 component (as a percent of NAV) and the corresponding cumulative delta-hedge traded in our stylized short volatility strategy. To construct the "95% Bootstrap Confidence Interval" for each horizon, we first constructed 1000 randomly-sampled (with replacement) series of dates, each of which was equal in length to our original series (which ranged from January 1996 through April 2018). Then, we selected the corresponding pairs of returns for each of those dates, and using those we generated 1000 simulated correlations. The 2.5th and 97.5th percentiles of these correlations for each horizon are plotted in the dotted lines of the above chart. The horizon ranges from 1 day through 65 business days. The date range is from January 1996 through April 2018.

Exhibit 9: Joint Trade Quintile Day Counts, Excluding Expiration Days

	(<-- selling)		Short Vol Trade Percentile		(buying -->)
Risk Parity Trade Percentile	**<20th**	**20th – 40th**	**40th – 60th**	**60th – 80th**	**>80th**
>80th	103	392	374	212	30
60th – 80th	260	346	291	172	42
40th – 60th	238	256	284	264	69
20th – 40th	217	85	124	376	309
<20th	292	33	37	87	661

Source: AQR, Bloomberg, OptionMetrics. The figure reports the number of days in the full sample where strategies jointly trade in particular quintiles. The date range is from January 1996 through April 2018.

Exhibit 10: Average Trades Conditional On Large Moves, Excluding Expiration Days

Risk Parity Trades Conditional On Short Volatility Trades

			(<-- buying)	Short Vol Trade Percentile			(selling -->)	
IF	Short Vol Trades Are In This Percentile…	99th	95th-99th	90th-95th	Middle 80th	10th-5th	5th-1st	1st
THEN	**Average Risk Parity Trades**	-1.6%	-1.1%	-0.6%	0.2%	-0.1%	-0.6%	-3.3%
	Standard Deviation of Risk Parity Trades	1.0%	1.0%	0.7%	0.4%	0.6%	1.1%	4.8%
	T-Statistics for Average Risk Parity Trades	-1.66	-1.13	-0.87	0.47	-0.18	-0.58	-0.68
	Average Percentile of Risk Parity Trades	5%	9%	18%	57%	40%	26%	10%
	Percentage of Days Risk Parity Sells	100%	98%	87%	19%	40%	65%	91%

Short Volatility Trades Conditional On Risk Parity Trades

			(<-- buying)	Risk Parity Trade Percentile			(selling -->)	
IF	Risk Parity Trades Are In This Percentile…	99th	95th-99th	90th-95th	Middle 80th	10th-5th	5th-1st	1st
THEN	**Average Short Vol Trades**	0.2%	-0.2%	-0.2%	0.1%	5.5%	3.7%	-3.2%
	Standard Deviation of Short Vol Trades	2.6%	3.0%	3.1%	6.3%	11.5%	14.7%	19.7%
	T-Statistics for Average Short Vol Trades	0.08	-0.06	-0.08	0.02	0.48	0.25	-0.16
	Average Percentile of Short Vol Trades	48%	46%	45%	49%	68%	60%	46%
	Percentage of Days Short Vol Sells	47%	49%	52%	46%	27%	38%	53%

Source: AQR, Bloomberg, OptionMetrics. The tables report the average trade size for each strategy conditional on percentile trade by the other strategy, where percentiles are based on the full distribution of the conditioning strategy's trade sizes, excluding option expiration dates. We also report t-statistics for this conditional average. We report the conditional average of the percentile of the trade size to put the trade sizes into perspective. Finally, we report the percentage of days in each bucket that the strategy sells. For example, if consider only days for which a risk parity trade is in the bottom 1st percentile, then the average short volatility trade on those days is -3.2%, and the average of the short volatility trade percentiles on those days relative to the entire (unconditional) set of short volatility trades is 46%, very close to the median overall. The t-statistic in this bucket is -0.68, which indicates that we would not be able to reject the hypothesis that the true average short vol trade in this bucket is zero with any confidence, though that finding is tempered by the fact that short vol sells on 91% of those days and buys on the others. The date range is from January 1996 through April 2018.

Exhibit 11: Average Trades Conditional On Large Moves, Including Expiration Days

Risk Parity Trades Conditional On Short Volatility Trades

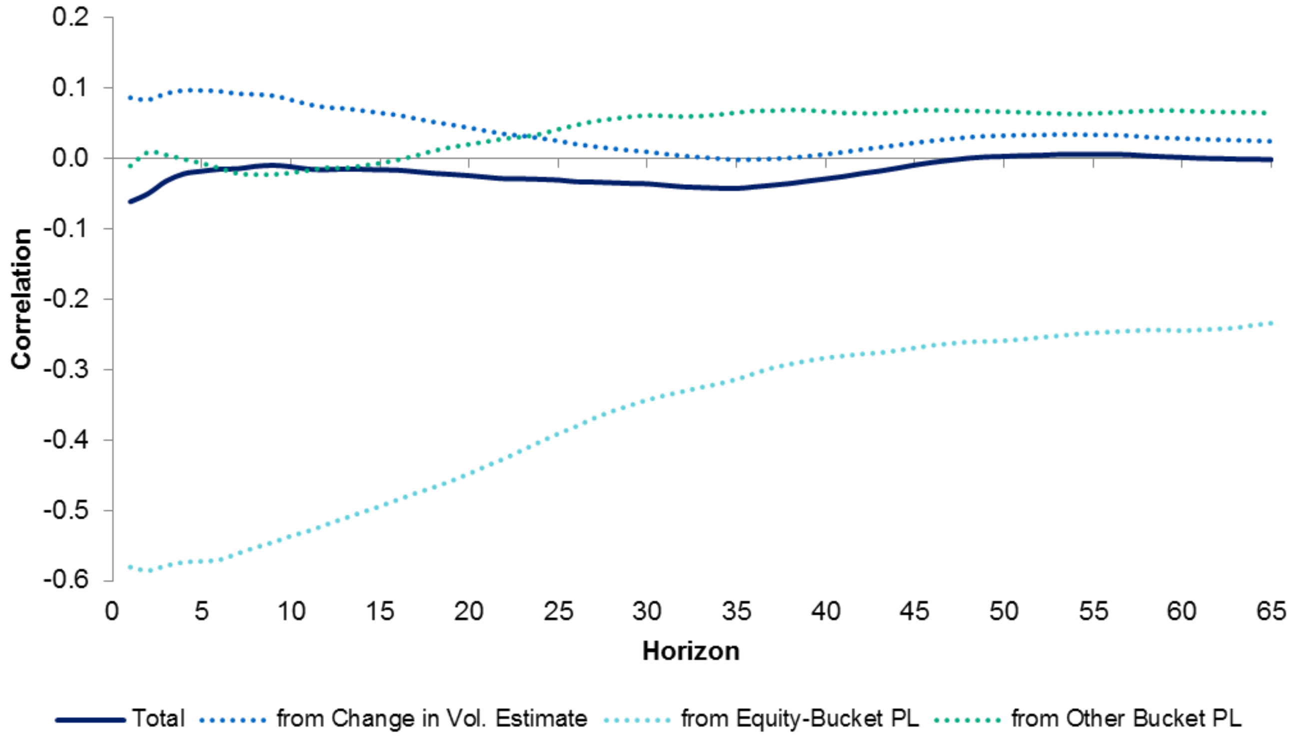
			(<-- buying)	Short Vol Trade Percentile			(selling -->)	
IF	Short Vol Trades Are In This Percentile…	99th	95th-99th	90th-95th	Middle 80th	10th-5th	5th-1st	1st
THEN	Average Risk Parity Trades	-0.5%	-1.1%	-0.7%	0.2%	-0.3%	-1.1%	0.1%
	Standard Deviation of Risk Parity Trades	1.2%	1.0%	0.7%	0.4%	0.8%	2.8%	0.6%
	T-Statistics for Average Risk Parity Trades	-0.41	-1.01	-0.92	0.43	-0.37	-0.39	0.15
	Average Percentile of Risk Parity Trades	37%	14%	16%	56%	34%	31%	55%
	Percentage of Days Risk Parity Sells	50%	91%	89%	20%	51%	57%	21%

Short Volatility Trades Conditional On Risk Parity Trades

			(<-- buying)	Risk Parity Trade Percentile			(selling -->)	
IF	Risk Parity Trades Are In This Percentile…	99th	95th-99th	90th-95th	Middle 80th	10th-5th	5th-1st	1st
THEN	Average Short Vol Trades	-1.2%	-1.3%	-1.3%	-0.3%	4.8%	4.1%	-3.5%
	Standard Deviation of Short Vol Trades	11.0%	8.4%	7.4%	9.7%	13.1%	14.9%	21.3%
	T-Statistics for Average Short Vol Trades	-0.10	-0.15	-0.18	-0.03	0.37	0.27	-0.16
	Average Percentile of Short Vol Trades	48%	46%	45%	49%	67%	61%	46%
	Percentage of Days Short Vol Sells	50%	50%	53%	47%	29%	38%	53%

Source: AQR, Bloomberg, OptionMetrics. The tables report the average trade size for each strategy conditional on percentile trade by the other strategy, where percentiles are based on the full distribution of the conditioning strategy's trade sizes, including option expiration dates. We also report t-statistics for this conditional average. We report the conditional average of the percentile of the trade size to put the trade sizes into perspective. Finally, we report the percentage of days in each bucket that the strategy sells. For example, if consider only days for which a risk parity trade is in the bottom 1st percentile, then the average short volatility trade on those days is -3.5%, and the average of the short volatility trade percentiles on those days relative to the entire (unconditional) set of short volatility trades is 46%, very close to the median overall. The t-statistic in this bucket is -0.16, which indicates that we would not be able to reject the hypothesis that the true average short vol trade in this bucket is zero with any confidence, and that is strengthened by the fact that short vol sells on approximately half of those days (53%) and buys on the others. The date range is from January 1996 through April 2018.

Exhibit 12: Decomposition of Correlation between Delta Traded and Risk Parity Trade, by Horizon



Source: AQR, Bloomberg, OptionMetrics. The solid line shows the correlation over various horizons between the cumulative trade in our risk parity strategy's S&P 500 component (as a percent of NAV) and the corresponding cumulative delta-hedge traded in our stylized short volatility strategy. Using the methodology described in the "Decomposition of Risk Parity Trades" section of the appendix, we then decomposed the risk parity trades into three components: trades deriving from changes in volatility estimates, trades induced by the equity-bucket PL, and trades induced by the PL of the two non-equity buckets. The three dotted lines show the correlations over various horizons between the cumulative trades for each of these components with the corresponding cumulative delta-hedge traded in our stylized short volatility strategy. The horizon ranges from 1 day through 65 business days. The date range is from January 1996 through April 2018.

Exhibit 13: Results of 3-Asset-Class Risk Parity Regression on Underlying Assets and Beta-Neutral Short Volatility

Panel 1: Regression output

	Coefficient	T-Stat	P-Value
S&P 500	0.23	101.6	0.00
US 10-Year	1.31	117.3	0.00
GSCI	0.23	112.3	0.00
Beta-Neutral Equity Short Volatility	0.04	1.4	0.17
Beta-Neutral FI Short Volatility	0.37	4.5	0.00
Intercept	0.00	0.8	0.44

Panel 2: Risk decomposition

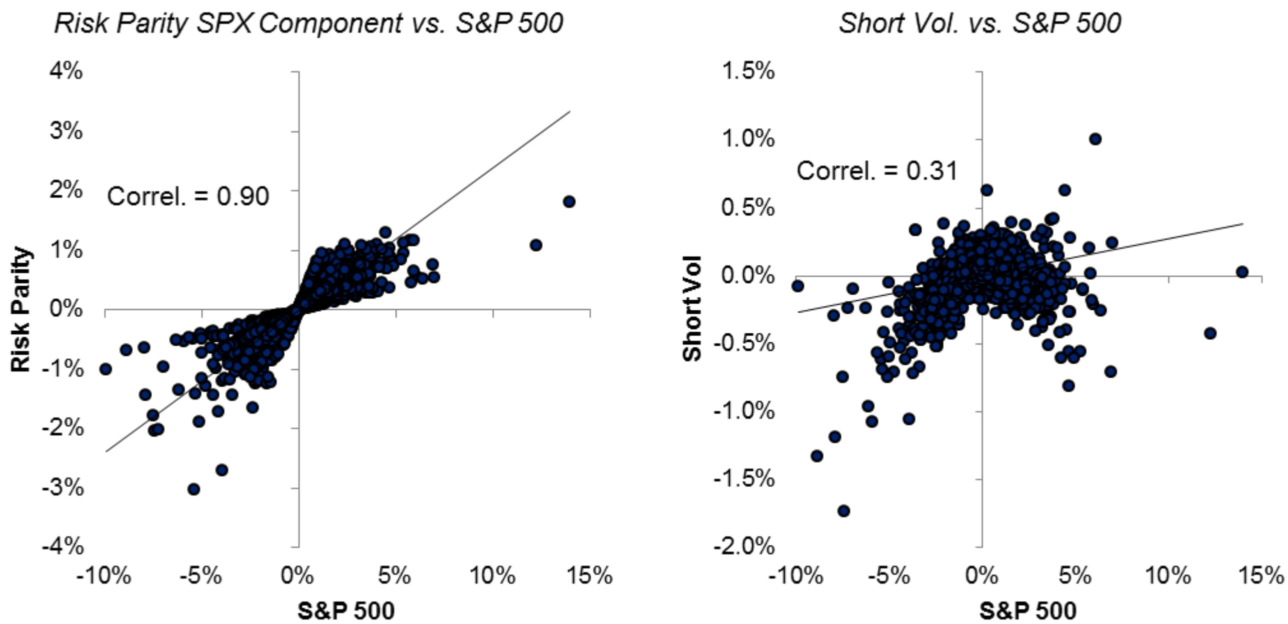
	3-Bucket Risk Parity	From S&P 500	From US 10-Year	From GSCI	From Beta-Neutral EQ Short Vol	From Beta-Neutral FI Short Vol	From Other
Return	6.0%	1.7%	2.6%	0.9%	0.1%	0.2%	0.5%
Volatility	8.2%	4.4%	4.9%	4.7%	0.1%	0.2%	3.1%
Sharpe	0.72	0.39	0.53	0.19	0.89	1.30	0.16
Risk Contribution	-	28%	24%	34%	0%	0%	14%

Source: AQR, Bloomberg, OptionMetrics. The top panel shows the output of a multivariate regression in which we regressed the daily returns of our three-bucket risk parity simulation against its three underlying assets (S&P 500, US 10-year bond returns, and the Goldman Sachs GSCI Commodity Index), a beta-neutral equity short volatility series and a beta-neutral fixed income short volatility series. To construct beta-neutral equity short volatility returns, we ran a full-sample regression of our stylized short volatility simulation against the S&P 500 in order to estimate its long-term equity beta. The beta-neutral returns are the residuals of that regression. Similarly, for the beta-neutral fixed income short volatility series, we ran a full-sample regression of our stylized fixed income short volatility simulation against the US 10-year bond returns, and used the residuals of that regression.

For the bottom panel, we decompose the risk parity equity bucket daily returns into six components (“from S&P 500”, “from US 10-Year”, “from GSCI”, “from beta-neutral equity short volatility”, “from beta-neutral fixed income short volatility” and “from other”) using the betas calculated in the multivariate regression in the top panel. The panel then shows summary statistics for the three-bucket risk parity returns and for each of the components. Risk contribution is defined as the covariance of the component with the full strategy, divided by the variance of the full strategy.

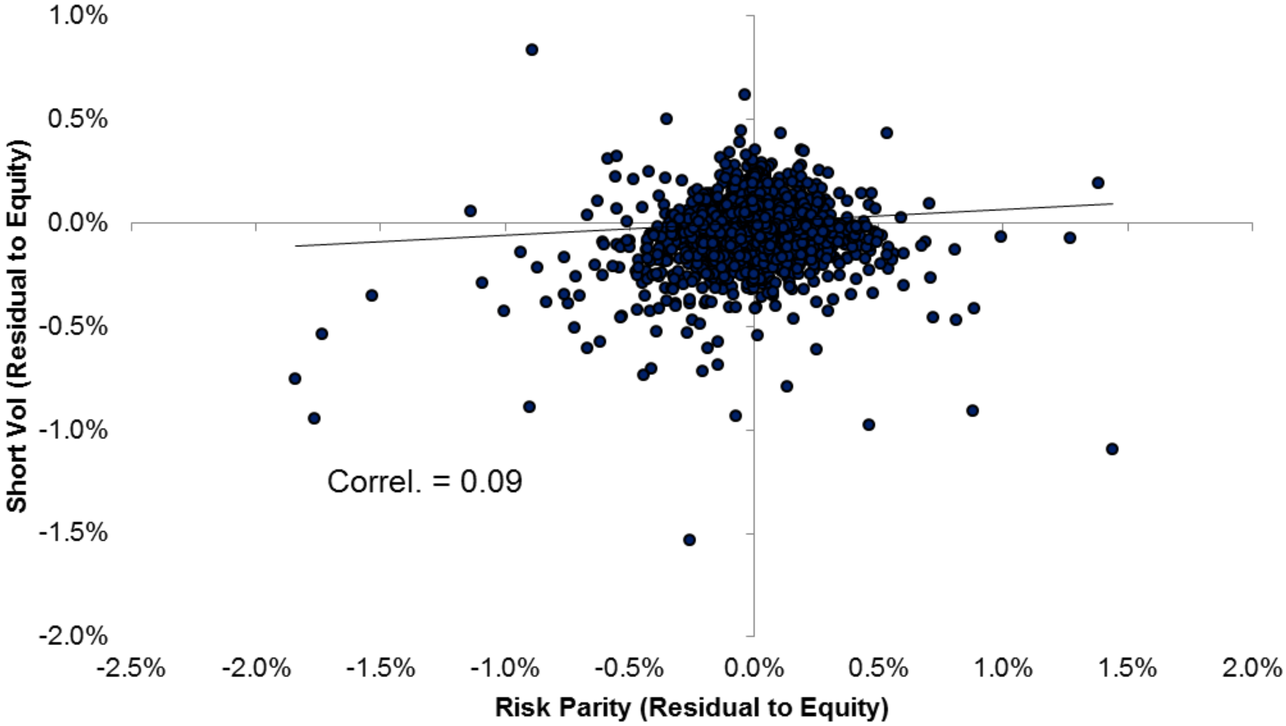
The date range is from January 1996 through April 2018.

Exhibit 14: Return Correlations of S&P 500, Risk Parity SPX Component, and Short Volatility



Source: AQR, Bloomberg, OptionMetrics. The left panel scatter plots S&P 500 daily returns (x-axis) against the daily return to the equity component of risk parity (y-axis). The right panel plots S&P 500 daily returns (x-axis) against the daily returns of our stylized short volatility simulation (y-axis). The date range is from January 1996 through April 2018.

Exhibit 15: Short Volatility vs. Risk Parity SPX Component (Residuals to Equity)



Source: AQR, Bloomberg, OptionMetrics. The plot scatters the beta-neutral returns of the equity component of risk parity (x-axis) against the daily beta-neutral returns of the stylized short volatility simulation (y-axis). To construct beta-neutral returns, for each series we ran a full-sample regression against the S&P 500 in order to estimate its long-term equity beta. The beta-neutral returns are the residuals to those two regressions. The date range is from January 1996 through April 2018.

Exhibit 16: Results of Risk Parity Regression on S&P 500 and Beta-Neutral Short Volatility

Panel 1: Regression output

	Coefficient	T-Stat	P-Value
Beta-Neutral Short Volatility	0.12	6.7	0.00
S&P 500	0.24	154.3	0.00
Intercept	0.00	0.9	0.38

Panel 2: Risk decomposition

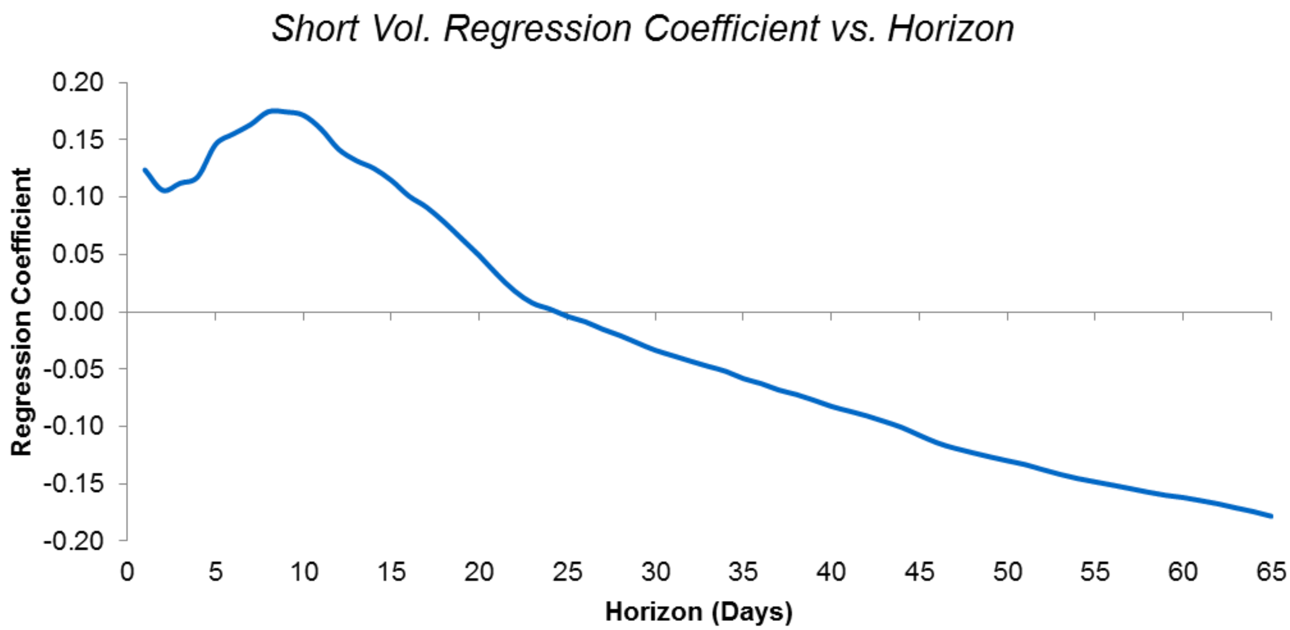
	Risk Parity S&P 500	From S&P 500	From Beta-Neutral Short Vol	From Other
Return	2.3%	1.8%	0.2%	0.4%
Volatility	5.1%	4.5%	0.2%	2.2%
Sharpe	0.46	0.39	0.89	0.18
Risk Contribution	-	80%	0%	20%

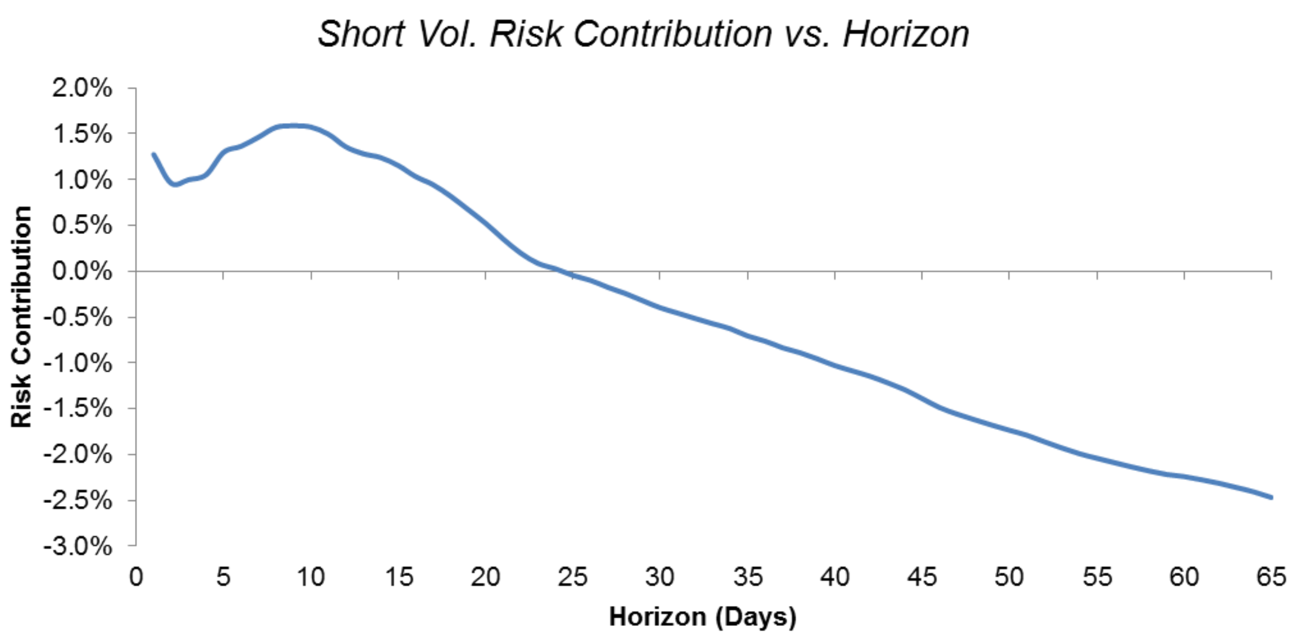
Source: AQR, Bloomberg, OptionMetrics. The top panel shows the output of a multivariate regression in which we regressed the daily returns of our risk parity simulation's equity component against both a beta-neutral short volatility series and the S&P 500. To construct beta-neutral short volatility returns, we ran a full-sample regression of our stylized short volatility simulation against the S&P 500 in order to estimate its long-term equity beta. The beta-neutral returns are the residuals of that regression.

For the bottom panel, we decompose the risk parity equity bucket daily returns into three components ("from S&P 500", "from beta-neutral short volatility", and "from other") using the betas calculated in the multivariate regression in the top panel. The panel then shows summary statistics for the risk parity equity bucket returns and for each of the three components. Risk contribution is defined as the covariance of the component with the full strategy, divided by the variance of the full strategy.

The date range is from January 1996 through April 2018.

Exhibit 17: Results by Horizon for Regressions of Risk Parity on S&P 500 and Beta-Neutral Short Volatility



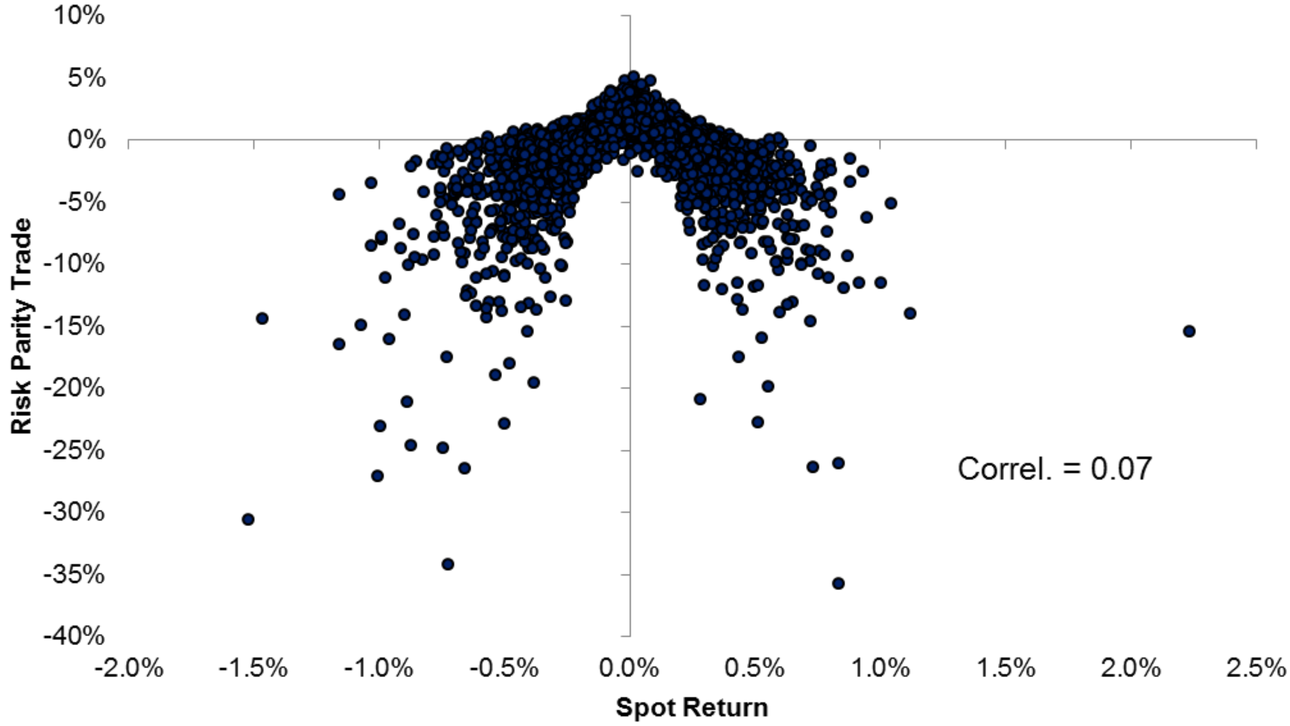


Source: AQR, Bloomberg, OptionMetrics. The two charts are derived from multivariate regressions in which (over various horizons) we regressed the returns of our risk parity simulation's equity component against both a beta-neutral short volatility series and the S&P 500. To construct beta-neutral short volatility returns, for each horizon we ran a full-sample regression of our stylized short volatility simulation against the S&P 500 in order to estimate its long-term equity beta at that horizon. The beta-neutral returns are the residuals of those regressions.

The top chart shows the coefficient on beta-neutral short volatility as a function of the horizon. The bottom chart shows the risk contribution of the short volatility component of risk-parity derived using these coefficients.

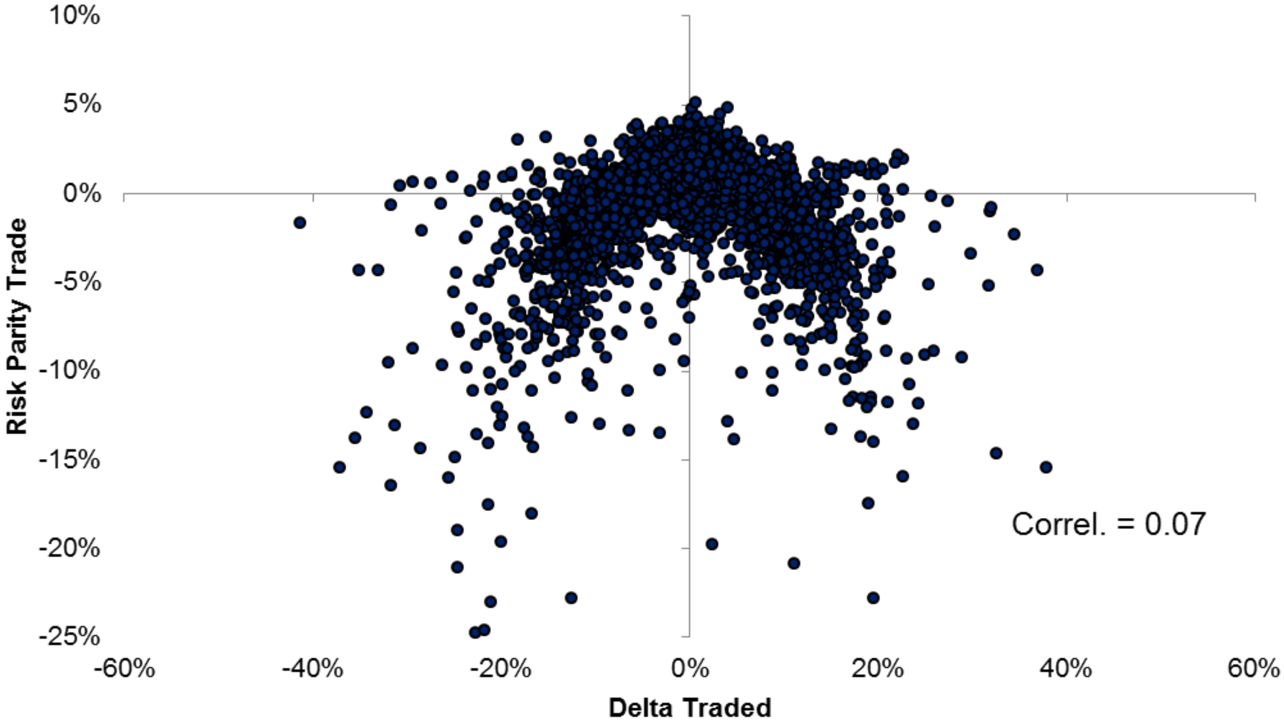
Risk contribution is defined as the covariance of the component with the full strategy, divided by the variance of the full strategy. The horizon ranges from 1 day through 65 business days. The date range is from January 1996 through April 2018.

Exhibit A1: Fixed Income: Spot Return vs. Risk Parity Trade (1-Day Horizon)



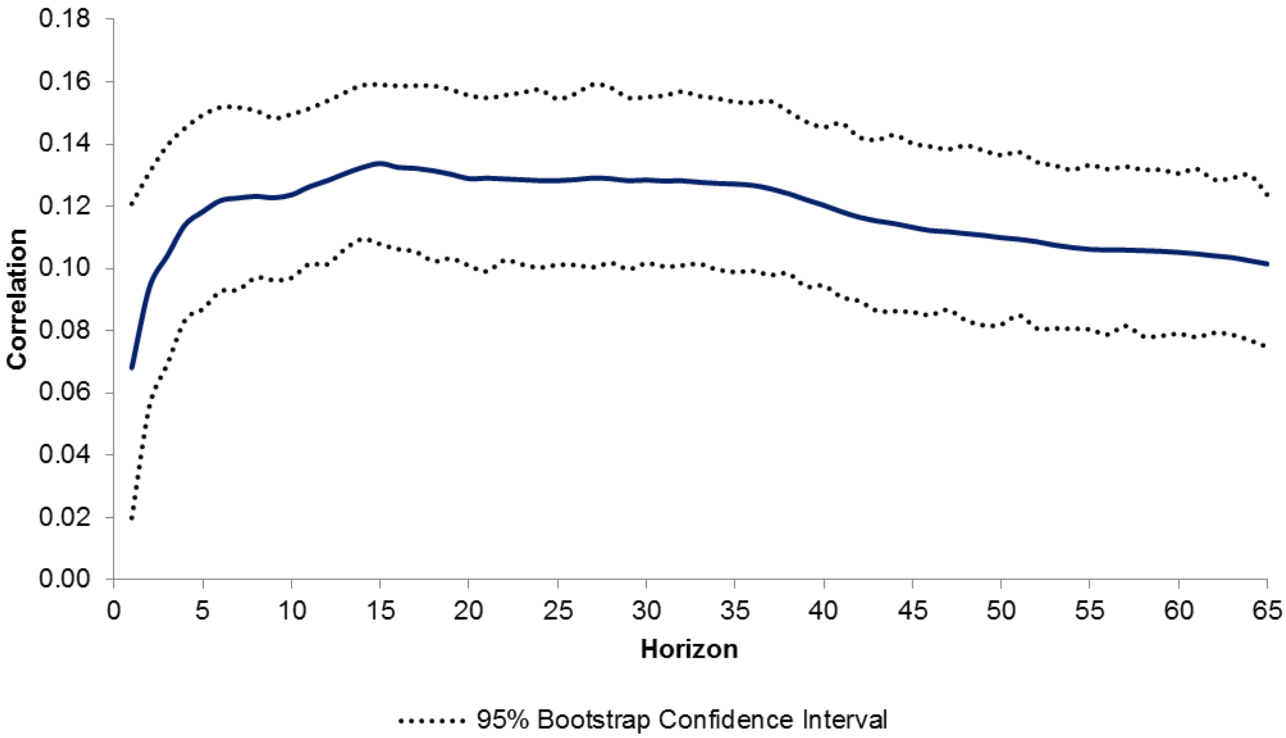
Source: AQR, Bloomberg. The scatter plot shows the daily US 10-year bond return (x-axis) vs the corresponding daily trade in our risk parity strategy's fixed income component (as a percent of NAV). The date range is from February 1993 through April 2018.

Exhibit A2: Fixed Income: Delta Traded vs. Risk Parity Trade (1-Day Horizon)



Source: AQR, Bloomberg. The scatter plots the relationship between the daily delta-hedge traded in our stylized US 10-year bond short volatility simulation (x-axis) vs the daily trade in our risk parity strategy's fixed income component (y-axis). The bottom chart includes all days within our time period, while the top excludes option expiration dates. The date range is from February 1993 through April 2018.

Exhibit A3: Fixed Income: Correlation between Delta Traded and Risk Parity Trade, by Horizon



Source: AQR, Bloomberg. The chart shows the correlation over various horizons between the cumulative trade in our risk parity strategy's fixed income component (as a percent of NAV) and the corresponding cumulative delta-hedge traded in our stylized US 10-year bond short volatility strategy. To construct the "95% Bootstrap Confidence Interval" for each horizon, we first constructed 1000 randomly-sampled (with replacement) series of dates, each of which was equal in length to our original series (which ranged from February 1993 through April 2018). Then, we selected the corresponding pairs of returns for each of those dates, and using those we generated 1000 simulated correlations. The 2.5th and 97.5th percentiles of these correlations for each horizon are plotted in the dotted lines of the above chart. The horizon ranges from 1 day through 65 business days. The date range is from February 1993 through April 2018.

Exhibit A4: Joint Trade Quintile Day Counts, Fixed Income

	(<-- selling)		Short Vol Trade Percentile		(buying -->)
Risk Parity Trade Percentile	**<20th**	**20th – 40th**	**40th – 60th**	**60th – 80th**	**>80th**
>80th	60	352	501	348	53
60th – 80th	92	364	405	370	83
40th – 60th	175	354	281	336	169
20th – 40th	392	196	91	202	433
<20th	595	48	37	58	576

Source: AQR, Bloomberg. The figure reports the number of days in the full sample where the strategies jointly trade in particular quintiles. The date range is from February 1993 through April 2018.

Exhibit A5: Average Trades Conditional On Large Moves, Fixed Income

Risk Parity Trades Conditional On Short Volatility Trades

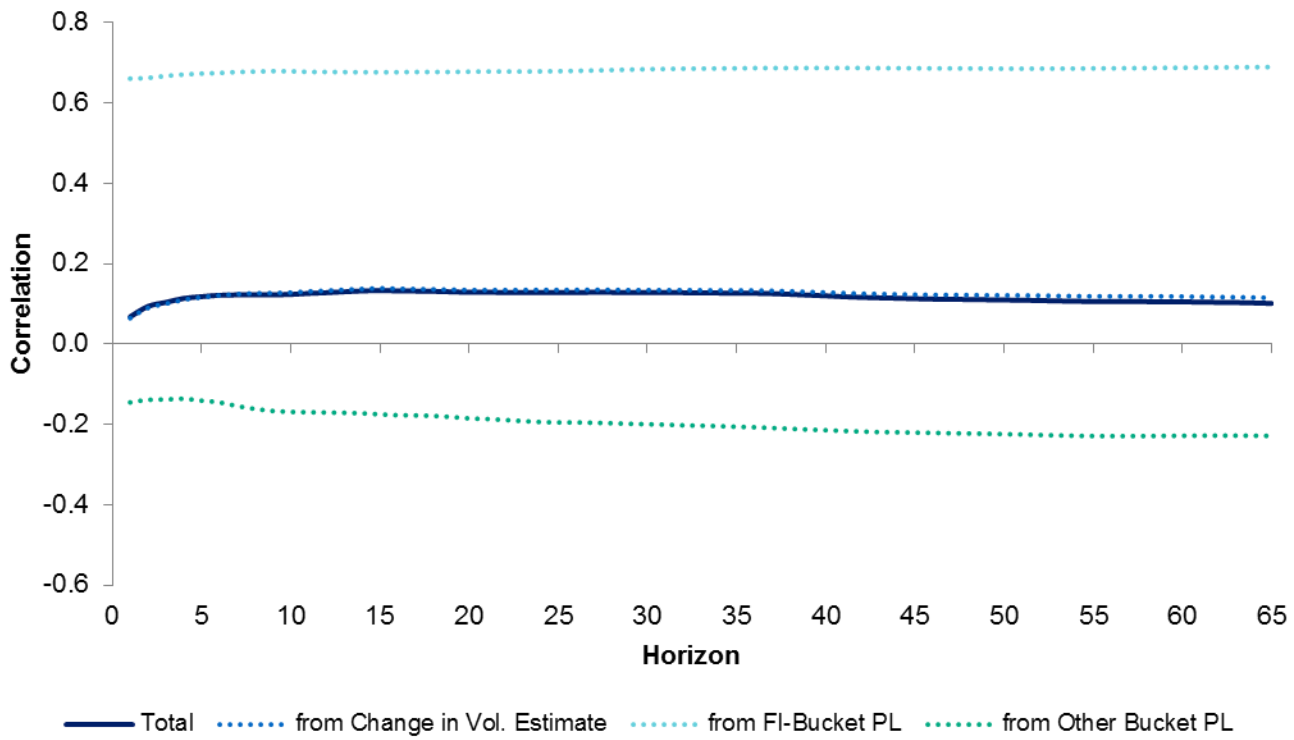
			(<-- buying)		Short Vol Trade Percentile		(selling -->)	
IF	Short Vol Trades Are In This Percentile…	99th	95th-99th	90th-95th	Middle 80th	10th-5th	5th-1st	1st
THEN	Average Risk Parity Trades	-6.6%	-3.2%	-1.3%	0.8%	-1.6%	-3.8%	-9.2%
	Standard Deviation of Risk Parity Trades	6.5%	2.7%	2.2%	1.4%	2.6%	3.6%	7.7%
	T-Statistics for Average Risk Parity Trades	-1.02	-1.21	-0.59	0.55	-0.59	-1.06	-1.20
	Average Percentile of Risk Parity Trades	15%	13%	24%	58%	22%	13%	9%
	Percentage of Days Risk Parity Sells	86%	92%	79%	17%	79%	92%	89%

Short Volatility Trades Conditional On Risk Parity Trades

			(<-- buying)		Risk Parity Trade Percentile		(selling -->)	
IF	Risk Parity Trades Are In This Percentile…	99th	95th-99th	90th-95th	Middle 80th	10th-5th	5th-1st	1st
THEN	Average Short Vol Trades	-0.8%	0.4%	0.4%	0.4%	0.3%	-0.6%	-5.0%
	Standard Deviation of Short Vol Trades	3.9%	2.8%	3.5%	6.0%	12.3%	15.5%	20.3%
	T-Statistics for Average Short Vol Trades	-0.20	0.14	0.11	0.06	0.02	-0.04	-0.25
	Average Percentile of Short Vol Trades	45%	50%	50%	50%	50%	48%	39%
	Percentage of Days Short Vol Sells	49%	43%	43%	45%	50%	52%	60%

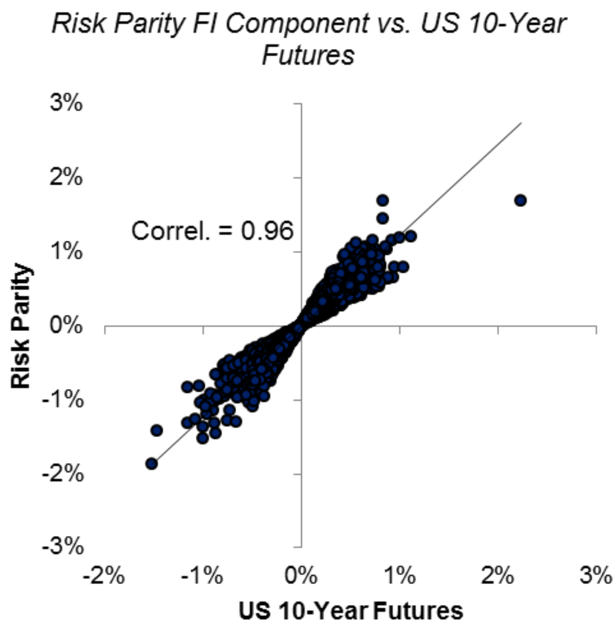
Source: AQR, Bloomberg, The tables report the average trade size for each strategy conditional on percentile trade by the other strategy, where percentiles are based on the full distribution of the conditioning strategy's trade sizes. We also report t-statistics for this conditional average. We report the conditional average of the percentile of the trade size to put the trade sizes into perspective. Finally, we report the percentage of days in each bucket that the strategy sells. For example, if consider only days for which a risk parity trade is in the bottom 1st percentile, then the average short volatility trade on those days is -5.0%, and the average of the short volatility trade percentiles on those days relative to the entire (unconditional) set of short volatility trades is 39%, somewhat close to the median overall. The t-statistic in this bucket is -0.25, which indicates that we would not be able to reject the hypothesis that the true average short vol trade in this bucket is zero with any confidence, and that is strengthened by the fact that short vol sells on only slightly more than half of those days (60%) and buys on the others. The date range is from February 1993 through April 2018.

Exhibit A6: Fixed Income: Decomposition of Correlation between Delta Traded and Risk Parity Trade, by Horizon



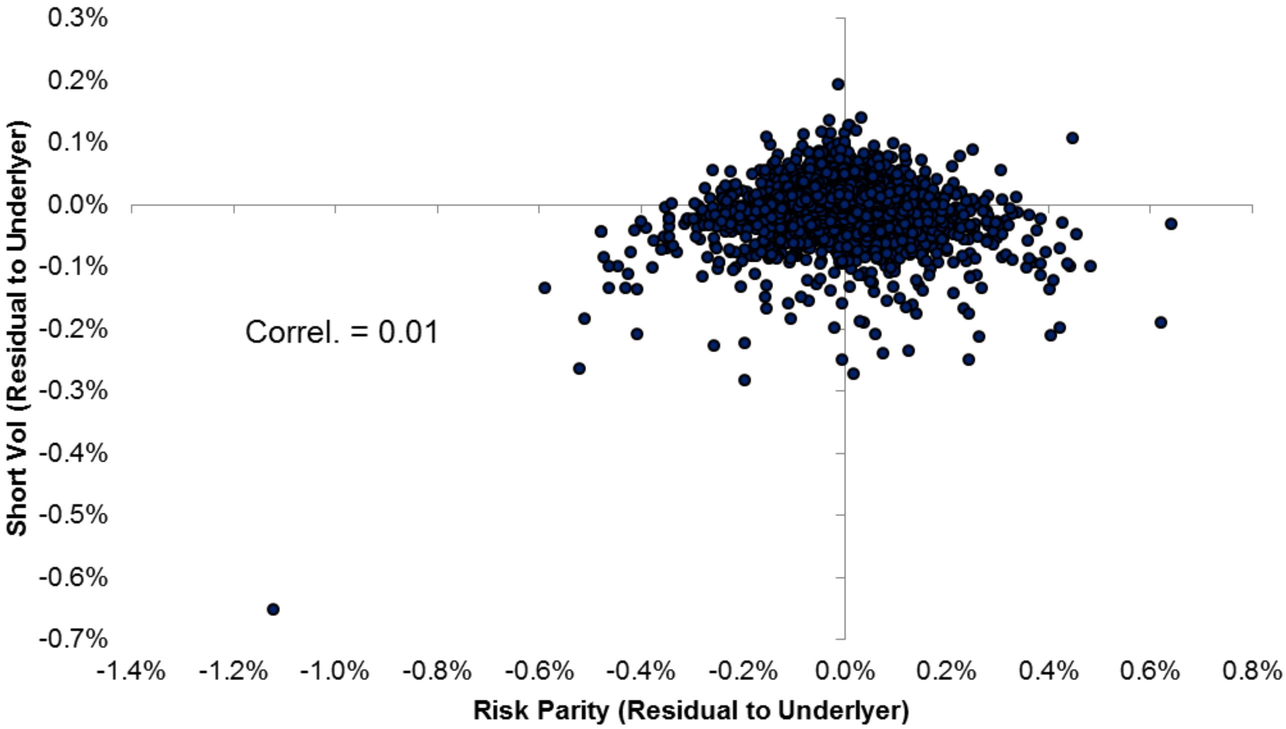
Source: AQR, Bloomberg. The solid line shows the correlation over various horizons between the cumulative trade in our risk parity strategy's fixed income component (as a percent of NAV) and the corresponding cumulative delta-hedge traded in our stylized US 10-year bond short volatility strategy. Using the methodology described in the "Decomposition of Risk Parity Trades" section of the appendix, we then decomposed the risk parity trades into three components: trades deriving from changes in volatility estimates, trades induced by the fixed-income-bucket PL, and trades induced by the PL of the two non-FI buckets. The three dotted lines show the correlations over various horizons between the cumulative trades for each of these components with the corresponding cumulative delta-hedge traded in our stylized short volatility strategy. The horizon ranges from 1 day through 65 business days. The date range is from February 1993 through April 2018.

Exhibit A7: Fixed Income: Return Correlations of US 10-Year, Risk Parity FI Component, and FI Short Volatility



Source: AQR, Bloomberg. The left panel scatter plots US 10-year bond daily returns (x-axis) against the daily return to the fixed income component of risk parity (y-axis). The right panel plots US 10-year daily returns (x-axis) against the daily returns of our stylized fixed income short volatility simulation (y-axis). The date range is from February 1993 through April 2018.

Exhibit A8: Fixed Income Short Volatility vs. Risk Parity FI Component (Residuals to Underlying Asset Returns)



Source: AQR, Bloomberg. The plot scatters the beta-neutral returns of the fixed income component of risk parity (x-axis) against the daily beta-neutral returns of the stylized fixed income short volatility simulation (y-axis). To construct beta-neutral returns, for each series we ran a full-sample regression against the US 10-year bond returns in order to estimate its long-term equity beta. The beta-neutral returns are the residuals to those two regressions. The date range is from February 1993 through April 2018.

Exhibit A9: Fixed Income: Results of FI Risk Parity Regression on US 10-Year and Beta-Neutral FI Short Volatility.

Panel 1: Regression output

	Coefficient	T-Stat	P-Value
Beta-Neutral Short Volatility	0.03	1.1	0.29
US 10-Year	1.26	295.8	0.00
Intercept	0.00	0.3	0.77

Panel 2: Risk Decomposition

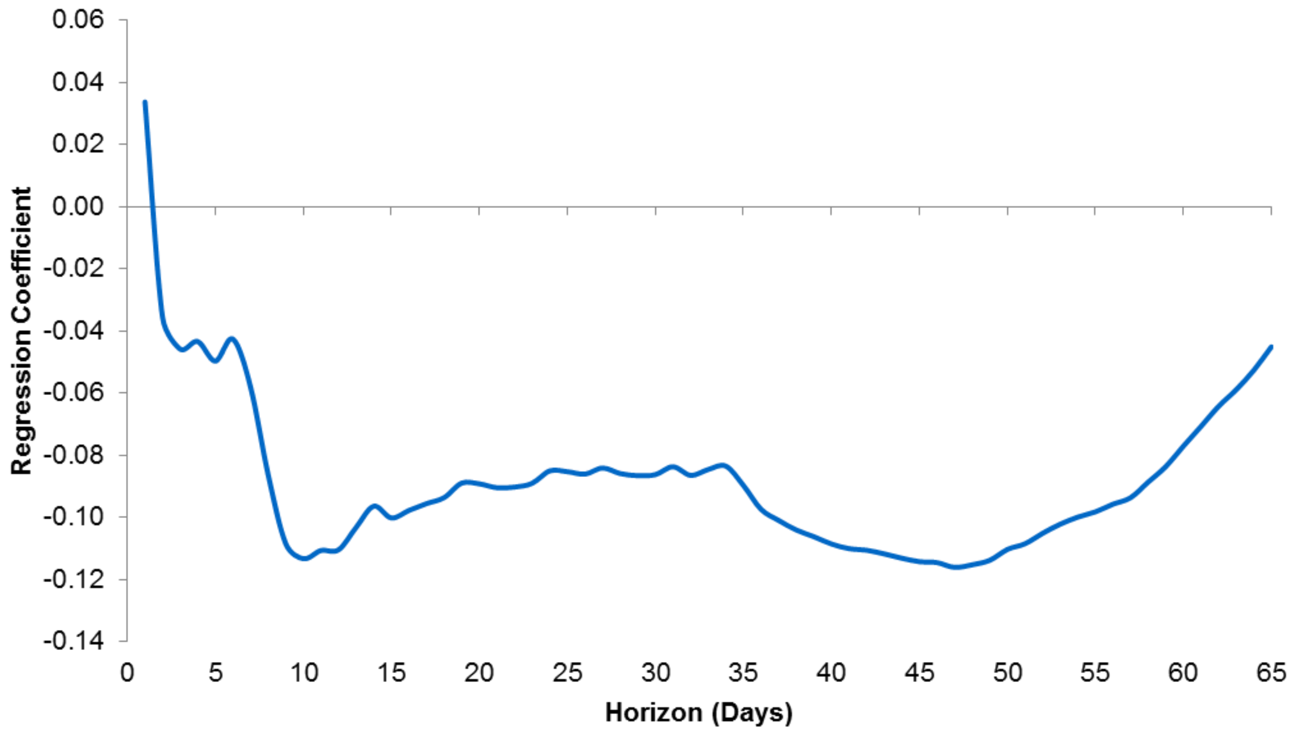
	Risk Parity FI	From US 10-Yr	From Beta-Neutral Short Vol	From Other
Return	2.7%	2.6%	0.0%	0.1%
Volatility	5.0%	4.8%	0.0%	1.3%
Sharpe	0.54	0.54	1.36	0.06
Risk Contribution	-	93%	0%	7%

Source: AQR, Bloomberg. The top panel shows the output of a multivariate regression in which we regressed the daily returns of our risk parity simulation's fixed income component against both a beta-neutral fixed income short volatility series and the US 10-year bond returns. To construct beta-neutral fixed income short volatility returns, we ran a full-sample regression of our stylized fixed income short volatility simulation against the US 10-year bond returns in order to estimate its long-term beta. The beta-neutral returns are the residuals of that regression.

For the bottom panel, we decompose the risk parity equity bucket daily returns into three components ("from US 10-Year", "from beta-neutral short volatility", and "from other") using the betas calculated in the multivariate regression in the top panel. The panel then shows summary statistics for the risk parity fixed income bucket returns and for each of the three components. Risk contribution is defined as the covariance of the component with the full strategy, divided by the variance of the full strategy.

The date range is from February 1993 through April 2018.

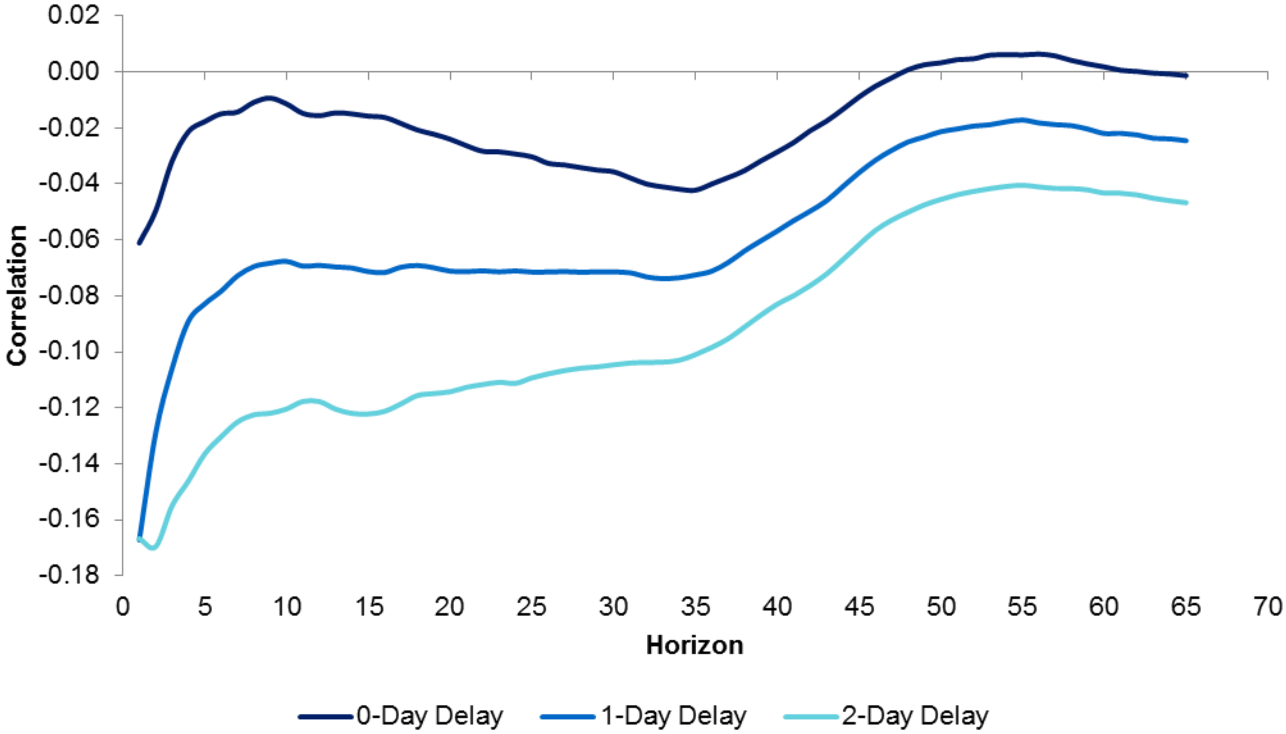
Exhibit A10: Fixed Income: Short Vol. Coefficient by Horizon for Regressions of FI Risk Parity vs. US 10-Year and Beta-Neutral FI Short Volatility



Source: AQR, Bloomberg. The chart above is derived from multivariate regressions in which (over various horizons) we regressed the returns of our risk parity simulation's fixed income component against both a beta-neutral fixed income short volatility series and the US 10-year bond returns. To construct beta-neutral fixed income short volatility returns, for each horizon we ran a full-sample regression of our stylized fixed income short volatility simulation against the US 10-year bond returns in order to estimate its long-term equity beta at that horizon. The beta-neutral returns are the residuals of those regressions.

The chart shows the coefficient on beta-neutral fixed income short volatility as a function of the horizon. The horizon ranges from 1 day through 65 business days. The date range is from February 1993 through April 2018.

Exhibit A11: Delta Traded and Risk Parity Trade Correlation by Horizon and Risk Model Vol. Delay Length



Source: AQR, Bloomberg, OptionMetrics. The chart shows the correlation over various horizons between the cumulative trade in three different versions of our risk parity strategy's S&P 500 component (as a percent of NAV) and the corresponding cumulative delta-hedge traded in our stylized short volatility strategy. The three versions of our risk parity simulation differ in their assumptions about how long the lag should be for the last return used in constructing the ex-ante volatility estimate. Our original simulation (the "0-Day Delay" version) assumes no lag (i.e., trades happen instantaneously at the close based on an updated ex-ante volatility estimate which includes that day's return). The "1-Day Delay" and "2-Day Delay" simulations assume a one and two day lag, respectively, in the ex-ante volatility estimate. The horizon ranges from 1 day through 65 business days. The date range is from January 1996 through April 2018.