The Long-run Relationship of Gold and Silver and the Influence of Bubbles and Financial Crises

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Abstract

This paper analyzes the long-run relationship between gold and silver prices. We closely follow Escribano and Granger (1998) and extend their study. We use a longer sample period from 1970-2011 and study the role of bubbles and financial crises for the relationship between gold and silver prices. We find clear evidence for a co-integration relationship between gold and silver with gold prices driving the relationship. The analysis also indicates that the results are influenced by bubble-like episodes and financial crises.

Keywords: co-integration; nonlinear error-correction; Granger causality; gold; silver; bubbles; financial crisis

JEL classification: C22; G1; G110

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Introduction

Gold and Silver have been used as a store of value and currency for thousands of years suggesting that there is a long-run relationship between the two precious metals. However, there are several factors that may drive their prices away from each other. Such factors are industrial demand for silver and jewelery, dental demand and central bank demand for gold.[1] The recent popularity of commodities as an investment and a hedge[2] against adverse financial or economic events may constitute an additional force that either creates an otherwise non-existent long-run relationship or strengthens a pre-existing long-run relationship.[3] If investment demand indeed creates links, periods which can be primarily attributed to investor behavior such as bubbles or financial crises may provide additional insights as to what factors drive long-run relationships.

This study is motivated by the large price changes in commodities, especially by changes in the precious metals gold and silver in recent years, in particular between 2004 and 2011, and the research questions raised by Escribano and Granger (1998). Escribano and Granger (1998) analyzed the relationship between gold and silver prices for the period between 1971 and 1994 and found that gold and silver are co-integrated mainly due to a specific bubble and post-bubble period. They also suggest that the two markets may be in the process of becoming separated. Since we are not aware of any study that has comprehensively answered the question whether the events after the publication of the Escribano and Granger study supported their conjecture regarding the separation of the two markets, this study aims to close this gap in the literature. In addition, the other question regarding the role of bubbles for co-integration has to the best of our knowledge also not been conclusively answered.

There are other studies that investigate the relationship between gold and silver prices but most of these studies use data that makes a comparison with the Escribano and

[1] See World Gold Council (www.gold.org) for more details.

[2] Baur and Lucey (2010) and Baur and McDermott (2010) analyze the role of gold as a hedge and a safe haven against adverse financial events.

[3] Batten et al. (2012) find evidence for fractional dependency between gold and silver prices that may be exploited for trading profit.

Granger study either impossible or the samples analyzed are too short to answer the questions raised by the authors. For example, Ciner (2001) analyzes the period from 1994 to 1998 and finds no evidence for a long-run co-integrating relationship. Figuerola-Ferretti and Gonzalo (2010) use the period from 1990 to 2009 and find that gold and silver are co-integrated only under weak US dollar and high volatility conditions. Lucey and Tully (2003) use the period from 1978 to 2002 and find that a stable long-run relationship exists. Liu and Chou (2003) use daily data from 1983 to 1995[4] and Adrangi, Chatrath and David (2000) use intra-day data between 1993 and 1995 with a focus on price discovery.

The above list of papers illustrates that there is no paper that replicates and extends the analysis by Escribano and Granger (1998) with the aim to answer the following questions: (i) Are gold and silver prices co-integrated? (ii) Is the relationship stable? and (iii) Do bubbles or financial crises establish a link and thus render gold and silver prices co-integrated?

We are interested in the existence of the long-run relationship between gold and silver but also aim to investigate the causes of such a long-run relationship, that is, do investors cause such relationships or are fundamentals responsible for the connection between the two prices?[5] There is a link to the literature on financialization which analyzes whether the use of commodities as an investment makes their prices co-move more with other assets, especially stock indices (e.g. see Tang and Xiong, 2012).

We contribute to the literature by using a relatively long sample period of gold and silver spot prices. The sample period which spans more than 40 years from 1970 to 2011 includes the sample period used by Escribano and Granger (1998) and allows us to replicate and thus revisit their estimations and examine how a longer sample period affects the findings and conclusions. The sample period used in this study is the longest economically reasonable time frame as gold and silver prices were regulated prior to 1970. Another interesting feature of the longer sample period is that the data contains more

[4]Liu and Chou (2003) argue that spread trading may be one of the major reasons for a close relationship (see also Wahab et al., 1994).

[5]We assume that fundamental factors dominate in the long-run and that investor behavior can influence the relationship merely in the short-run.

periods of extreme price behaviour similar to the “bubble” period in the late 1970s and thus enriches an analysis of the role of such periods relative to the original study.[6]

We show that periods of extreme price behaviour indeed positively influence the statistical evidence for a long-run co-integration relationship. In “normal” periods, gold and silver prices are not co-integrated. The Vector Error Correction Model and derived Granger causality tests further show that gold drives the long-run relationship despite its lower volatility relative to silver.

The paper is structured as follows. Section 1 presents descriptive statistics of the data, section 2 contains the estimation results with a detailed analysis of the role of bubbles and financial crises. Section 3 presents the results of the Error-Correction Model and Granger causality tests. A sub-sample analysis which enables a direct comparison with the Escribano and Granger (1998) study is performed in section 4 and section 5 summarizes the findings and concludes.

1 Data

We use $T = 499$ monthly observations from January 1970 to July 2011. Figure 1 presents the prices of gold and silver in US dollars.[7] The two panels at the top are plots of gold and silver prices and their log versions. The plots also highlight two specific episodes, i.e. the bubble period in 1979-1980 and the financial crisis in 2008. Panels three and four at the bottom contain the scatter plots (cross plots) of the gold and silver prices and in logs, respectively. The time-series plots demonstrate that the prices are connected and that there are two distinct periods in which silver seems to change its relationship with the price of gold. These periods are the “bubble” in 1979 and 1980 as defined and analyzed by Escribano and Granger (1998) and the 2010-2011 episode. The scatter plots indicate a positive relationship between the two price series and their logarithms.

[6]Escribano and Granger (1998) call the period in which the silver market was cornered, roughly between June 1979 until March 1980, by the Hunt brothers a “bubble” but note that the period does not completely correspond to the concept of a bubble found in the financial literature (page 82).

[7]The data is obtained from DataStream. The mnemonics are GOLDBLN and SLVCASH. The gold price is Gold Bullion US dollar per troy ounce and the silver price is Silver Fix cash US dollar cents per troy ounce.

*** Insert Figure 1 about here ***

Many papers refer to the gold-silver ratio as an indication of the stability of the relationship between the two precious metals. Figure 2 plots the gold-silver ratio against time and illustrates that there is an upward trend until the year 1991 and a downward trend from 1991 until 2011. The average ratio is around 55 ranging from around 30 in the 1970s to around 100 in the early 1990s. The evolution of the gold-silver ratio suggests that the relationship between gold and silver is not stable. This result also implies that spread trading, i.e. simultaneously taking a long position in one asset and a short position in the other asset, may work for some shorter periods but is a rather risky strategy in the long-run.

*** Insert Figure 2 about here ***

Table 1 displays descriptive statistics for the prices and returns of gold and silver. The table shows that there are substantial differences in the statistical properties of both gold and silver prices and gold and silver returns.

*** Insert Table 1 about here ***

2 Long-Run Relationship of Gold and Silver

This section examines whether there is a long-run co-integrating relationship between gold and silver prices. We follow Escribano and Granger (1998) and define dummy variables to identify the influence of specific periods on the equilibrium relationship. We use a dummy variable to capture the "bubble" or, alternatively, period of market manipulation in the silver market from September 1979 until March 1980.[8] We refer to this dummy

[8] We follow the terminology used by Escribano and Granger (1998) and call this period a "bubble".

as DB. We also use a dummy (labelled $D04$) that captures the recent bull period in commodities also associated with an episode of financialization as discussed inter alia by Tang and Xiong (2012). To analyze the role of a financial crises and market turmoil we further define a dummy variable (DC) that represents the period of the global financial and economic crisis from August 2008 to February 2009 (e.g. see Bank for International Settlements report (BIS, 2009) inter alia). Finally, we define another bubble period in the silver market supported by a graphical analysis of the gold and silver price series in figure 2. The figure suggests that there is a similar phenomenon in 2010 and 2011 as observed during the period from September 1979 until March 1980. We label the dummy for this period $D10$.

The first step of our analysis tests if gold and silver prices are I(1). All the tests show that gold and silver prices and the logarithms of the prices are integrated of order 1. As the results are easy to reproduce, we do not present the details of the results here. Given that gold and silver prices are I(1) we estimate the following equilibrium model

$$PG_t = c + \beta PS_t + \boldsymbol{\gamma} \boldsymbol{X}_t + Z_t \tag{1}$$

where PG denotes the price of gold, PS denotes the price of silver and $\boldsymbol{X}$ is a matrix of dummy variables and interaction terms explained in more detail below. The error term is represented by Z_t. Note that Equation (1) is an equilibrium equation and expresses the association between the variables without assuming exogeneity of any variable.[9]

Table 2 presents the estimation results for a regression of the price of gold on a constant and the price of silver without any dummy variables or interaction terms labeled LR1; a model with three dummies as defined above is labeled LR2; and a specification that contains the three dummies and interaction terms of the dummies with the price of silver is labeled LR3.

[9]An alternative specification with silver prices on the left hand side of the equation yields similar results for Z_t.

*** Insert Table 2 about here ***

In the table, DF1 stands for Dickey-Fuller test with no constant and no trend, DF2 is such a test with a constant and no trend and DF3 is a test with a constant and a trend. Our cointegration regressions are not standard because each equation contains one regular I(1) variable, three dummies and dummies multiplied with the I(1) variable. Following the practice by Escribano and Granger (1998), the equivalent number of explanatory variables will be either 1, 4, or 7 for our regressions. We compute the critical values for the null that the residuals are non-stationary ("H_0: the residual is I(1)") using the tables by MacKinnon (2010); for example, the 95% critical value for 4 explanatory variables in the case of a Dickey-Fuller regression with a constant and no trend is $-4.0960 - 11.2349/498 - 11.1750/498^2 = -4.1186$. In the table, numbers under column $CV5\%, Nmax$ are critical values of the Dickey-Fuller statistics such that $Nmax$ is the number of explanatory variables in the Dickey-Fuller regressions. In our models LR1, LR2 and LR3 the number of explanatory variables is 1, 4, and 7, respectively.

The results for model LR1 show that the residuals of this regression are I(0) as the DF1, DF2 and DF3 indicate a rejection of the null hypothesis of non-stationarity. The relationship between gold and silver prices is represented by the coefficient β. It is estimated at 0.392 and statistically significant. The second specification, LR2, includes the dummy variables for the bubble, the crisis period and the period of financialization (or, alternatively, the bull phase in commodities) and shows that the test statistics reject the null hypothesis of non-stationarity in residuals and thus support the conclusion that gold and silver prices are co-integrated. The estimate of the coefficient β is now 0.358, and does not change noticeably from the first specification, still depicting the positive long run relationship. The dummy variables, DC and $D04$ are both positive and significant which implies that these periods lead to divergence of gold and silver. The third specification, labeled LR3, includes the interaction terms and leads to substantial changes in the test statistics and conclusions. The long-run relationship represented by β is estimated at

0.342 and is, again, comparable with those estimated in the other specifications. In contrast, the Dickey-Fuller test statistics do not support a co-integration relationship. This is due to the inclusion of the interaction term of the silver bubble (DB) with the silver price. The fact that the inclusion of the dummies leads to "no co-integration" suggests that the periods represented by the dummies cause the long-run relationship and confirms a similar result reported by Escribano and Granger (1998).

We also analyze whether there is a cointegrating relationship between the logarithm of gold and silver prices. Similar to the above three specifications, we estimate a model without any dummies, which we label LR1 in Table 3; a model with the first three dummies, which we label LR2; and a model that contains the three dummies and interaction terms of the dummies with the logarithm of silver price, which we label LR3. The obtained DF statistics are now much lower than those in Table 2. Comparing the Dickey-Fuller statistics with the critical values $CV5\%, Nmax$, we fail to reject the null that there is no co-integrating relationship specified by the LR2 model between the log prices of gold and silver. The results thus indicate that there is no long-run co-integration relationship between log prices. Granger and Escribano (1998) also found differences between price levels and logs but mainly in the role of the dummy variables, i.e. "fewer dummies are needed for the logs of prices than for the prices" (page 87).

In order to analyze the influence of the last two years of the sample period which appears to be as volatile as in the bubble period in the late 1970s and early 1980s, we re-estimate the model and swap the dummy for the bull phase ($D04$) with the dummy $D10$ which is equal to one if the observations belong to the years 2010 and 2011. The results are presented in table 4 and confirm the hypothesis derived from a graphical inspection of the data. The coefficient estimates for the bubble dummy DB and the potential alternative bubble period $D10$ are statistically significant. The estimate of $D10$ is highly significant and estimated at 0.244. The inclusion of an interaction term with this dummy further leads to the non-rejection of the null hypothesis of "no co-integration". In other words, if the interaction term is included and thus a change of the gold-silver relationship is allowed, statistical evidence for a long-run association disappears. This finding indicates that specific episodes are responsible for the long-run co-integration relationship.

*** Insert Table 4 about here ***

Given our results above, we use the long-run relationship established by the models with the dummy variables but without the interaction terms to analyze the dynamics of this equilibrium within an error-correction model framework in the next section.

3 Error-Correction Models

We have established in the previous section that gold and silver prices follow a dynamic equilibrium relationship:

$$GP_t = const + \alpha_1 DB_t + \alpha_2 DC_t + \alpha_3 D04_t + \beta SP_t + Z_t \tag{2}$$

The residuals obtained from the above equilibrium regression, Z_t, is a measure of the degree of disequilibrium. To capture the idea that the relationship between gold and silver prices is in disequilibrium at any given time, but has a tendency to adjust itself towards the equilibrium, we use the ECM characterized by the following two equations:

$$\Delta GP_t = \sum_{i=1}^{12} \beta_{1,i} \Delta GP_{t-i} + \sum_{i=1}^{12} \beta_{1,12+i} \Delta SP_{t-i} + f(Z_t) \tag{3}$$

$$\Delta SP_t = \sum_{i=1}^{12} \beta_{2,i} \Delta GP_{t-i} + \sum_{i=1}^{12} \beta_{2,12+i} \Delta SP_{t-i} + f(Z_t) \tag{4}$$

We use alternative functional forms for $f(Z_t)$ to model the possible nonlinear effect of

Z_t on the gold and silver prices:

$$f(Z_t) = \begin{cases} c_1 Z_{t-1} \\ c_2 Z_{t-1}^2 \\ c_3 Z_{t-1}^2 + c_4 Z_{t-1}^3 \\ c_5 Z_{t-1} + c_6 Z_{t-1} I_{Z_{t-1}>0} \\ c_7 Z_{t-1} + c_8 Z_{t-1} \Delta GP_{t-1} + c_9 Z_{t-1} \Delta SP_{t-1} \end{cases} \tag{5}$$

In the standard Error-Correction Model (ECM), the coefficients c_i represent the strength of the disequilibrium correction. The models represented by M1 to M5 below correspond to five alternative functional forms for $f(Z_t)$ as described above.

Nonlinear error-correction models try to describe the nonlinear adjustment to the long-run equilibrium relationships. There is a large variety of nonlinear error-correction models. These include threshold autoregressive models as introduced by Granger and Lee (1989), the smooth transition ECM of Granger and Terasvirta (1993), the threshold model of Balke and Fomby (1997), cubic polynomials of Escribano (2004) and the Markov switching ECM by Krolzig et al. (2002) and Psaradakis et al. (2004). Recently, Seo (2011) developed asymptotic theory for a class of nonlinear regime switching ECMs. Escribano and Mira (2002) apply the concept of near epoch dependence to extend the Granger Representation Theorem to the nonlinear ECMs and Kristensen and Rahbek (2007) study Gaussian likelihood-based estimators for a broad class of nonlinear vector ECMs including the smooth transition models.

We include twelve lags in the error-correction model and estimate the model using the efficient method outlined by Engle and Yoo (1991) and Escribano and Granger (1998). Engle and Yoo (1991) show that the method is fully efficient; when they compare their method with the Johansen ML procedure, they find that their three-step estimator achieves the same limiting distribution as the Johansen approach. Table 6 reports estimation results of the long run relationship of $GP_t = const + \alpha_1 DB_t + \alpha_2 DC_t + \alpha_3 D04_t + \beta SP_t$ and Table

7 contains the estimation results of the ECM model, described by equations (3) and (4). We report the estimates for the first lag of prices, ΔGP_{t-1} and ΔSP_{t-1}, in the second and third columns, whereas the remaining estimates are associated with alternative linear and nonlinear functional forms of $f(Z_t)$ as described by equation (5).

The results in Table 6 show that the estimates of the equilibrium equation are almost invariant across the five models and highly significant, implying that the long run relationship is stable across models. Moreover, the coefficient estimates are comparable with those of the long run regression in the previous section in Table 2.

*** Insert Table 6 about here ***

*** Insert Table 7 about here ***

In Table 7, all estimates of the coefficients for the first lag of the gold price in both equations are negative and seven out of ten are statistically significant. In contrast, all coefficients for the first lag of the silver price are positive and eight estimates are statistically significant with the remaining two being marginally insignificant. That is, all models predict that a current increase of the silver price will result in an increase in both silver and gold prices in the next month, whereas an increase in the price of gold will lead to a future decrease in both silver and gold prices.

The standard ECM, $M1$, is given by $f(Z_{t_1}) = c_1 Z_{t-1}$. The estimates for coefficient c_1 in the gold and silver equations are 0.024 and 0.279, respectively, meaning that when there is disequilibrium, the speed of correction toward the equilibrium is higher for the silver price. The corresponding t-stats are 1.452 and 1.866, indicating that the estimates are marginally insignificant and significant for gold and silver, respectively.

The estimates in models $M2$ and $M3$ indicate that the quadratic term $Z_{t_1}^2$ is insignificant, whereas the cubic term $Z_{t_1}^3$ is important in the model. The estimates of R^2 in these two models confirm the important role of the cubic relationship, justifying our usage of nonlinear relationship between the degree of disequilibrium and current prices.

In model $M4$, the effect of the degree of disequilibrium on the current gold and silver prices is asymmetric according to: $f(Z_t) = Z_{t-1} I_{Z_{t-1}>0}$. The estimated results for this model suggest that this structural form is important in the gold price equation but not supported by the data in the silver equation as the estimated t-stats for c_6 in the two equations are 1.951 and -0.542, respectively. The model demonstrates that only gold exhibits an asymmetric effect.

The last four rows are estimated values of the ECM, where $f(Z_{t_1}) = c_7 Z_{t-1} + c_8 Z_{t-1} \Delta GP_{t-1} + c_9 Z_{t-1} \Delta SP_{t-1}$; that is, we test whether the relationship between changes in gold and silver prices and the degree of disequilibrium is dependent on previous gold and silver price levels. The estimates of c_8 in the gold and silver equations are 1.256 and 15.34 with corresponding t-stats of 2.765 and 2.702, respectively. The results show that the interaction term between the gold price and the disequilibrium is important. On the other hand, the estimates of c_9 in the gold and silver equations are -0.074 and 2.723 with corresponding t-stats of -0.088 and 0.563 indicating that the relationship between changes in gold and silver prices and the degree of disequilibrium is independent on previous silver prices. These findings suggest that the gold price plays a dominating role in the long-run relationship.

In Figure 3, we plot the predicted values of $f(Z_t)$ that are entailed by models $M1$ to $M5$ in equation 5 against time to see how well the models perform at different periods. Consistent with the higher values of R^2, the graphs of $f(Z_t)$ from models M3 and M5 are more responsive during the bubble periods, particularly the bubble period from September 1979 to March 1980 and the bull period from January 2004 until 2011. The results confirm that these nonlinear structures are important in describing the long term relationship between gold and silver prices, especially during the volatile bubble and pull periods.

*** Insert Figure 3 about here ***

3.1 Granger Causality

The error correction model estimates demonstrate that the equilibrium error Z significantly impacts the price of silver but not the price of gold. This result indicates that gold is the driving force in the gold-silver relationship and that silver adjusts to any deviations from the equilibrium. If the price of gold is the driving force in this relationship it also causes the price of silver. Granger causality tests confirm this result. The test statistics show that the gold price does Granger-cause the silver price and that the silver price does not Granger-cause the gold price.[10]

4 Sub-sample Analysis

In this section, we divide the 40-year data set into two approximately equal 20 year samples. The sub-sample spans the period from Jan 1970 to July 1990, which is similar to that used by Escribano and Granger (1998). The second sub-sample covers the remaining 20 years. Table 8 reports the Dickey-Fuller statistics for the residuals of the regressions (three sets of regressions with (i) no dummies, (ii) dummies and no interaction terms and (iii) dummies and interaction terms) similar to those estimated in section 2 but for the two sub-sample periods. The table contains results for the prices of gold and silver.[11]

*** Insert Table 8 about here ***

Table 8 confirms the finding by Escribano and Granger (1998) that there is a co-integrating relationship between the gold and silver prices. However, the statistics for the second sub-sample indicate that there is weaker evidence for a long run relationship between gold and silver prices. This result is fully consistent with the prediction by Escribano and Granger (1998) that gold and silver prices separate in the 1990s.

[10] The results are not reported due to space considerations.

[11] The estimation results based on log-prices yield similar qualitative results and can be obtained from the authors upon request.

5 Summary and Concluding Remarks

This paper analyzed the long-run relationship of gold and silver prices for a 40-year period from 1970 until 2011. We closely followed and thus revisited the study by Escribano and Granger (1998) and examined the role of episodes of extreme price behaviour on the existence and stability of an equilibrium relationship. We found that such episodes, in particular periods that resemble typical features of a bubble, can influence such relationships. An error-correction model and Granger causality tests further reveal that the price of gold drives the price of silver and thus the long-run relationship despite the greater volatility of the silver price. Finally, a sub-sample analysis confirm a prediction by Escribano and Granger (1998). There is evidence that the prices of gold and silver separated and decoupled in the 1990s. Our findings are a step towards a better understanding of the forces that create or destroy long-run equilibrium relationships. The empirical findings also show that the gold and silver price relationship is not stable with possibly strong implications for the efficient-market hypothesis and arbitrageurs.

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Table 1: **Descriptive statistics.**

	Min	Mean	Max	Std	Skewness	Kurtosis	AC(1)
Gold Prices	35	392	1579	268	1.86	7.43	0.97
Gold Returns	-0.29	0.01	0.39	0.06	0.49	9.09	0.02
Silver Prices	132	740	4372	587	2.86	13.78	0.93
Silver Returns	-0.47	0.01	0.74	0.10	0.74	12.48	0.03

The price of gold is denominated in US dollars per troy ounce and the price of silver is denominated in US dollar cents per troy ounce.

Table 2: **Test of long-run relationships, sample period: Jan 1970 to July 2011.**

	Const	SP	DB	DC	D04	DB*SP	DC*SP	D04*SP	No. Obs	R^2
LR1										
Est.	0.101	0.392							499	0.74
Tstat	(2.667)	(6.247)								
LR2										
Est.	0.099	0.358	-0.476	0.143	0.175				499	0.85
Tstat	(3.870)	(9.990)	(-5.743)	(3.171)	(3.125)					
LR3										
Est.	0.108	0.342	0.099	0.267	0.101	-0.224	-0.097	0.060	499	0.87
Tstat	(3.040)	(6.928)	(2.775)	(2.160)	(1.303)	(-4.547)	(-0.916)	(0.888)		

		Dickey-Fuller Test		
	Statistic	CV5%, N=1	CV5%, Nmax	DW
LR1				
DF1	-4.403	-1.941	-1.941	1.974
DF2	-4.399	-2.868	-2.868	
DF3	-5.916	-3.419	-3.419	
LR2				
DF1	-4.655	-1.941	-4.119	2.040
DF2	-4.651	-2.868	-4.119	
DF3	-5.102	-3.419	-4.458	
LR3				
DF1	-3.074	-1.941	-4.741	2.012
DF2	-3.070	-2.868	-4.741	
DF3	-3.027	-3.419	-5.023	

The top panel presents the coefficient estimates for three different models denoted LR1, LR2 and LR3. The bottom panel displays the test statistics and critical values based on the residuals Z_t of the cointegration equations. DF1, DF2 and DF3 denote a Dickey-Fuller test specification with no constant and no trend, with a constant and no trend and with a constant and a trend, respectively. The second column displays the test statistic and the third and fourth column present the 5% critical values (CV) based on Dickey-Fuller and McKinnon, respectively.

Table 3: **Test of long-run relationships, sample period: Jan 1970 to July 2011, log prices.**

	Const	logSP	DB	DC	D04	DB*logSP	DC*logSP	D04*logSP	Obs	R^2
LR1										
Est.	-1.076	1.064							499	0.76
Tstat	(-1.473)	(9.575)								
LR2										
Est.	-1.327	1.105	-1.022	0.279	0.004				499	0.78
Tstat	(-1.545)	(8.191)	(-5.213)	(5.164)	(0.032)					
LR3										
Est.	-1.739	1.171	3.245	3.185	2.316	-0.567	-0.420	-0.332	499	0.79
Tstat	(-1.778)	(7.538)	(3.300)	(3.545)	(2.234)	(-3.629)	(-3.248)	(-2.029)		

		Dickey-Fuller Test		
	Statistic	CV5%, N=1	CV5%, Nmax	DW
LR1				
DF1	-2.947	-1.941	-1.941	1.997
DF2	-2.947	-2.868	-2.868	
DF3	-2.707	-3.419	-3.419	
LR2				
DF1	-2.958	-1.941	-4.119	2.037
DF2	-2.957	-2.868	-4.119	
DF3	-2.615	-3.419	-4.458	
LR3				
DF1	-2.994	-1.941	-4.741	2.016
DF2	-2.993	-2.868	-4.741	
DF3	-2.615	-3.419	-5.023	

The top panel presents the coefficient estimates for three different models denoted LR1, LR2 and LR3. The bottom panel displays the test statistics and critical values based on the residuals Z_t of the cointegration equations. DF1, DF2 and DF3 denote a Dickey-Fuller test specification with no constant and no trend, with a constant and no trend and with a constant and a trend, respectively. The second column displays the test statistic and the third and fourth column present the 5% critical values (CV) based on Dickey-Fuller and McKinnon, respectively.

Table 4: **Test of long-run relationships, sample period: Jan 1970 to July 2011, D04 dummy substituted for D10 (dummy for 2010 and 2011).**

	Const	SP	DB	DC	D10	DB*SP	DC*SP	D10*SP	Obs	R^2
LR1										
Est.	0.101	0.392							499	0.74
Tstat	(2.667)	(6.247)								
LR2										
Est.	0.109	0.374	-0.524	0.291	0.244				499	0.83
Tstat	(3.015)	(6.718)	(-4.636)	(8.529)	(2.050)					
LR3										
Est.	0.060	0.451	0.147	0.416	0.835	-0.333	-0.145	-0.287	499	0.87
Tstat	(1.737)	(8.211)	(4.323)	(4.284)	(14.968)	(-6.080)	(-1.423)	(-5.356)		

	Statistic	Dickey-Fuller Test CV5%, N=1	CV5%, Nmax	DW
LR1				
DF1	-4.4029	-1.941	-1.941	1.974
DF2	-4.3986	-2.868	-2.868	
DF3	-5.9161	-3.419	-3.419	
LR2				
DF1	-4.7169	-1.941	-4.119	2.024
DF2	-4.7118	-2.868	-4.119	
DF3	-5.8567	-3.419	-4.458	
LR3				
DF1	-3.6598	-1.941	-4.741	2.010
DF2	-3.6562	-2.868	-4.741	
DF3	-4.6908	-3.419	-5.023	

The top panel presents the coefficient estimates for three different models denoted LR1, LR2 and LR3. The bottom panel displays the test statistics and critical values based on the residuals Z_t of the cointegration equations. DF1, DF2 and DF3 denote a Dickey-Fuller test specification with no constant and no trend, with a constant and no trend and with a constant and a trend, respectively. The second column displays the test statistic and the third and fourth column present the 5% critical values (CV) based on Dickey-Fuller and McKinnon, respectively.

Table 5: **Test of long-run relationships, sample period: Jan 1970 to July 2011, D04 dummy substituted for D10 (dummy for 2010 and 2011), log prices.**

	Const	logSP	DB	DC	D10	DB*logSP	DC*logSP	D10*logSP	Obs	R^2
LR1										
Est.	-1.076	1.064							499	0.76
Tstat	(-1.473)	(9.575)								
LR2										
Est.	-1.492	1.132	-1.066	0.259	-0.167				499	0.78
Tstat	(-1.956)	(9.674)	(-6.863)	(3.215)	(-0.829)					
LR3										
Est.	-1.592	1.148	3.099	5.355	6.170	-0.543	-0.729	-0.815	499	0.79
Tstat	(-2.092)	(9.816)	(4.042)	(4.798)	(7.787)	(-4.614)	(-4.401)	(-6.785)		

		Dickey-Fuller Test		
	Statistic	CV5%, N=1	CV5%, Nmax	DW
LR1				
DF1	-2.947	-1.941	-1.941	1.997
DF2	-2.947	-2.868	-2.868	
DF3	-2.707	-3.419	-3.419	
LR2				
DF1	-3.073	-1.941	-4.119	2.033
DF2	-3.072	-2.868	-4.119	
DF3	-2.894	-3.419	-4.458	
LR3				
DF1	-3.009	-1.941	-4.741	2.023
DF2	-3.009	-2.868	-4.741	
DF3	-3.012	-3.419	-5.023	

The top panel presents the coefficient estimates for three different models denoted LR1, LR2 and LR3. The bottom panel displays the test statistics and critical values based on the residuals Z_t of the cointegration equations. DF1, DF2 and DF3 denote a Dickey-Fuller test specification with no constant and no trend, with a constant and no trend and with a constant and a trend, respectively. The second column displays the test statistic and the third and fourth column present the 5% critical values (CV) based on Dickey-Fuller and McKinnon, respectively.

Table 6: **Estimation of the long run relationship, sample period: Jan 1970 - July 2011.**

	Const	SP	DB	DC	D04	R^2
Model M1						
Est.	0.100	0.358	-0.480	0.139	0.180	0.854
Tstat	(33.54)	(88.27)	(-29.27)	(9.01)	(30.81)	
Model M2						
Est.	0.097	0.359	-0.487	0.139	0.181	0.854
Tstat	(31.20)	(84.93)	(-28.47)	(8.67)	(29.73)	
Model M3						
Est.	0.097	0.358	-0.489	0.136	0.183	0.854
Tstat	(31.69)	(86.18)	(-29.07)	(8.56)	(30.50)	
Model M4						
Est.	0.097	0.359	-0.482	0.140	0.181	0.854
Tstat	(32.28)	(88.07)	(-29.23)	(9.02)	(30.65)	
Model M5						
Est.	0.101	0.357	-0.479	0.135	0.182	0.854
Tstat	(33.50)	(87.64)	(-29.10)	(8.68)	(30.91)	

The table displays the ECM estimates based on the long-run relationship given by: $GP_t = const + \alpha_1 DB_t + \alpha_2 DC_t + \alpha_3 D04_t + \beta SP_t + Z_t$.

Table 7: **Estimation of the ECMs, sample period: Jan 1970 to July 2011.**

	ΔGP_{t-1}	ΔSP_{t-1}	Z_{t-1}	Z^2_{t-1}	Z^3_{t-1}	$Z_{t-1}I_{Z_{t-1}>0}$	$Z_{t-1}\Delta GP_{t-1}$	$Z_{t-1}\Delta SP_{t-1}$	R^2
Model M1									
ΔGP_t	-0.251	0.053	0.024						0.18
Tstat	(-1.755)	(3.168)	(1.452)						
ΔSP_t	-0.469	0.176	0.279						0.17
Tstat	(-0.925)	(1.561)	(1.866)						
Model M2									
ΔGP_t	-0.230	0.044		0.083					0.18
Tstat	(-1.642)	(2.420)		(0.492)					
ΔSP_t	-0.365	0.261		-2.098					0.17
Tstat	(-0.706)	(1.729)		(-1.198)					
Model M3									
ΔGP_t	-0.314	0.080		0.121	1.195				0.21
Tstat	(-2.074)	(2.515)		(1.086)	(2.744)				
ΔSP_t	-1.430	0.726		-1.648	15.29				0.34
Tstat	(-2.340)	(2.537)		(-1.864)	(4.188)				
Model M4									
ΔGP_t	-0.256	0.050	-0.003			0.052			0.19
Tstat	(-1.794)	(3.102)	(-0.125)			(1.951)			
ΔSP_t	-0.462	0.182	0.336			-0.105			0.17
Tstat	(-0.898)	(1.631)	(1.465)			(-0.542)			
Model M5									
ΔGP_t	-0.319	0.088	0.010				1.256	-0.074	0.21
Tstat	(-2.233)	(2.893)	(0.818)				(2.765)	(-0.088)	
ΔSP_t	-1.259	0.588	0.087				15.34	2.723	0.30
Tstat	(-2.099)	(2.076)	(1.978)				(2.702)	(0.563)	

The table displays estimates of the ECMs based on the long-run relationship given by: $GP_t = const + \alpha_1 DB_t + \alpha_2 DC_t + \alpha_3 D04_t + \beta SP_t + Z_t$.

Table 8: **Test of long-run relationships, sub-sample analysis.**

Dickey-Fuller Test for long-run relationship, **sample period: Jan 1970 - July1990**					
	Statistic	CV5%, N=1	CV5%, Nmax	DW	Obs
LR1					
DF1	-2.830	-1.942	-1.942	2.018	246
DF2	-2.823	-2.874	-2.874		
DF3	-5.434	-3.431	-3.431		
LR2					
DF1	-6.209	-1.942	-4.142	2.005	246
DF2	-6.197	-2.874	-4.142		
DF3	-6.178	-3.431	-4.489		
LR3					
DF1	-5.648	-1.942	-4.473	2.000	246
DF2	-5.637	-2.874	-4.473		
DF3	-5.608	-3.431	-4.787		
Dickey-Fuller Test for long-run relationship, **sample period: Aug 1990 - July 2011**					
	Statistic	CV5%, N=1	CV5%, Nmax	DW	Obs
LR1					
DF1	-2.518	-1.942	-1.942	1.991	252
DF2	-2.508	-2.873	-2.874		
DF3	-2.438	-3.430	-3.431		
LR2					
DF1	-2.856	-1.942	-4.142	2.006	252
DF2	-2.848	-2.873	-4.142		
DF3	-2.861	-3.430	-4.489		
LR3					
DF1	-2.900	-1.942	-4.473	2.005	252
DF2	-2.891	-2.873	-4.473		
DF3	-2.892	-3.430	-4.787		

The table displays the test statistics and critical values based on the residuals Z_t of the cointegration equations. DF1, DF2 and DF3 denote a Dickey-Fuller test specification with no constant and no trend, with a constant and no trend and with a constant and a trend, respectively. The second column displays the test statistic and the third and fourth column present the 5% critical values (CV) based on Dickey-Fuller and McKinnon, respectively.

Figure 1: **Gold and Silver spot prices** The figure presents the prices of gold and silver in US dollars. The two panels at the top are plots of gold and silver prices and their log versions. The plots also highlight two specific episodes, i.e. the bubble period in 1979-1980 and the financial crisis in 2008. Panels three and four at the bottom contain the scatter plots (cross plots) of the gold and silver prices and their log version.

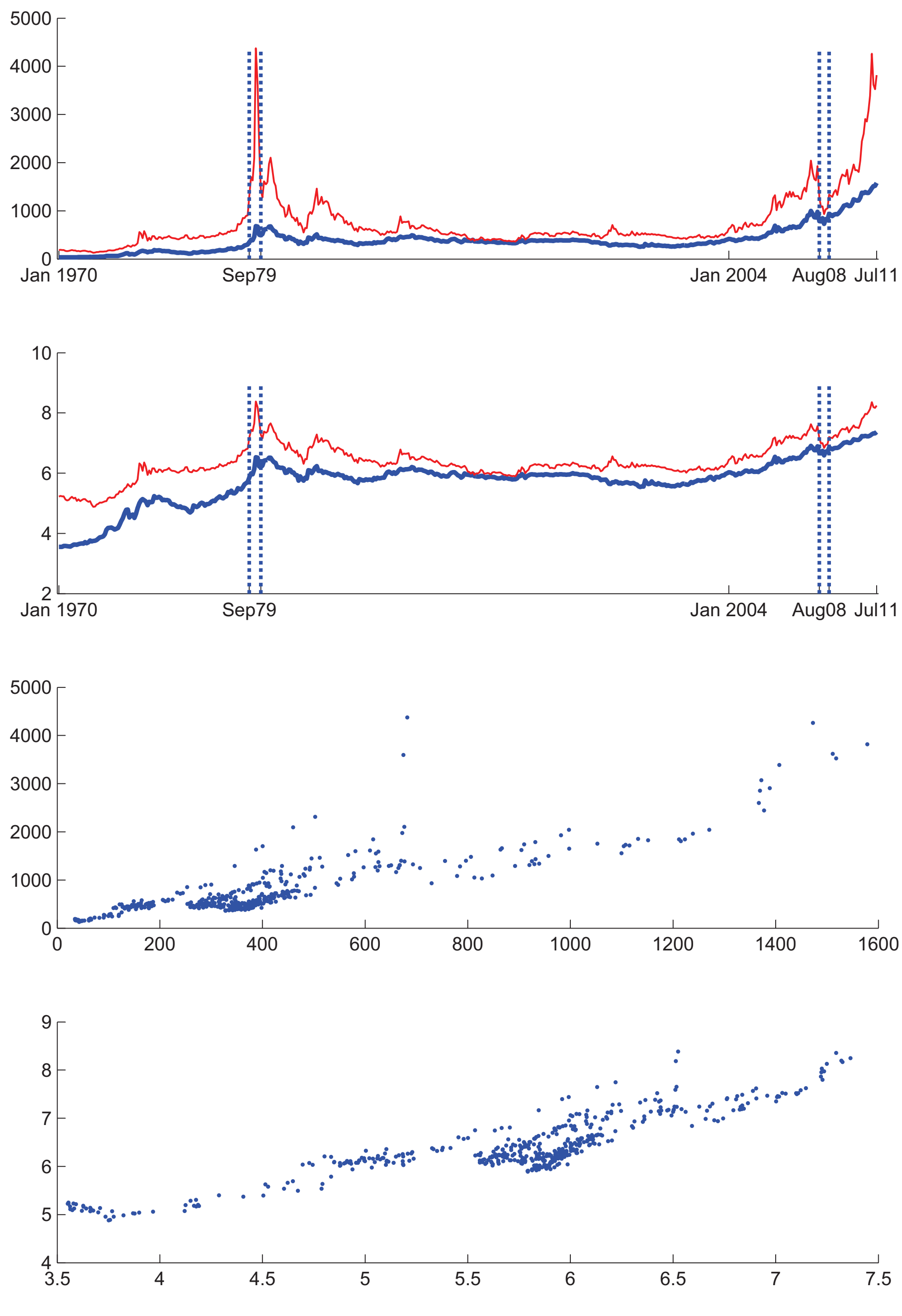

Figure 2: **Gold-silver ratio**

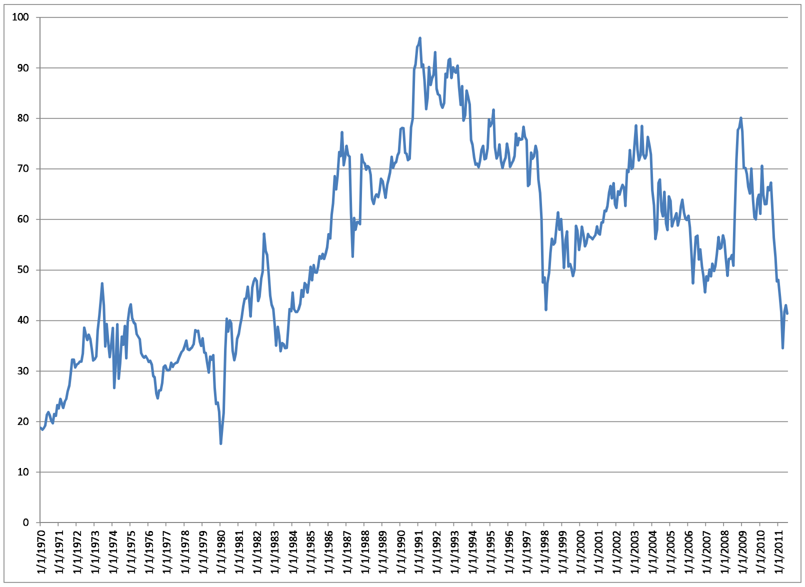

Figure 3: **Predicted values of** $f(Z_t)$ The figure shows the predicted values of $f(Z_t)$ obtained from models M1 (top) to M5 bottom).