U.S. Treasury Bond Betas: 1961–2019

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Abstract

This study plumbs the limits of U.S. Treasuries (USTs) as a "safe asset" through lens neglected in the literature on the correlation between bond and equity returns. An asymmetric M-GARCH model confirms a shift from positive to negative correlations in recent decades. However, the variance around bond-stock covariance has increased, consistent with greater not lower covariance premiums. Spectral analysis shows that, like the contribution to overall variance in returns, high-frequency cycles of no longer than a week account for most of the covariance between the yardstick risk-free and risky assets, increasingly so over the years. But there is no consistent evidence that USTs are better hedges against shorter-lived shocks. Quantile regressions suggest that USTs are not particularly convex hedges, either. Even amid very low yields in recent years, the distribution of 10-year UST returns is wider as well as more negatively skewed conditioned on stock market swoons.

Introduction

The importance to investors of the correlation between returns on nominal U.S. Treasuries (USTs) and the S&P 500 equity index is hard to overstate. The covariance between the yardstick credit-risk-free asset and the risky asset figures profoundly in portfolio optimization and may define the ultimate limits to diversification. Also, the prevailing covariance informs expected returns on the risk-free asset. Investors' perceptions of the hedging value of USTs may significantly affect term premiums, synonymous with required excess returns on nominal risk-free bonds.

This empirical study addresses the bond beta along dimensions not found in the literature. To fix bearings, an unlike more common sampling at lower frequencies, an asymmetric M-GARCH of daily returns confirms the well-known shift over the last couple decades in the bond-stock correlation from positive to negative, on net and across UST maturities given data from June 30, 1961 through September 19, 2019. But, in contrast to longer-run decline in the so-called "vol of vol" of UST returns, this methodology also highlights that the variance of covariance has increased during the same period, notably for both the average-expected-short-rate and term-premium components of returns based on the ACM term structure model (Adrian et al., 2013). Therefore, on balance the quantity, but not necessarily the price, of UST covariance risk has declined over the years.

Also, this study uses spectral analysis to parse bond-stock correlations and betas along the frequency domain to help gauge whether longer-run phenomena such as the procyclicality of inflation, which draws on consumption-based asset pricing, or shorter-run dynamics, including flight-to-quality (FTQ) flows, primarily impact the correlation. Similar to the decomposition of variance, also unreported in the literature, higher frequency cycles of no longer than a week's length largely dominate the covariance between USTs and the S&P 500. Moreover, the importance of shorter time scales has increased, on net, over the sample. This result indirectly suggests that the bond-stock correlation appears to have owed more to investors' near-term perceptions or attitudes toward risk and liquidity, or arguably exceedingly frequent revisions in their beliefs about macroeconomic regimes, notably as the overall correlation has turned negative during the last 20 years. However, besides the greater power of high-frequency cycles, spectral analysis also suggests that the direction and magnitude of the correlation is similar across different time scales. Somewhat problematic for emphasis on FTQs as a primary driver of correlations, USTs have not been a

consistently more effective hedge against risky assets during short-run shocks than over the longerrun.

Another key neglected issue in empirical studies is more direct assessment of the convexity of USTs as a hedge against risky assets, or in consumption-based parlance, the worst states of the world. This study takes two approaches. First, simple OLS regressions given subsamples of S&P 500 returns nearer the tails of the distribution hardly imply that USTs are consistently more effective hedges on the downside, even during more recent decades. Second, more rigorous quantile regressions help assess whether the conditional distributions of UST returns are not only narrower but also more positively skewed when stock prices swoon, compared to when they rally. If anything, the results suggest the opposite for longer-dated USTs, the primary tenor of focus in the literature. Longer-dated USTs may correlate more negatively with S&P 500 returns over the last several years, again on net, but the densities of default-risk-free asset returns tend to be wider with more leftward skew, conditioned on very low equity returns. Thus, however robust the correlation between hedging demand for USTs and valuations based on regressions through the mean, positive convexity cannot explain the exceptionally low level of yields, as well as estimated term premiums, toward the end of the sample.

The next section briefly reviews the academic and practitioner literature and motivates further analyses with simple estimates of the impact of dynamic correlation on the level of UST yields, all else equal. The next section reviews the M-GARCH results from the early 1960s to fix bearings, followed by the spectral analysis. The final empirical section examines the conditional asymmetry of UST returns at different maturities conditioned on equity returns, before a brief discussion of caveats and suggestions for additional research.

Literature review and motivation

Academics and practitioners have long observed that the bond-stock correlation is dynamic. Yet debate regards not only the nature of which underlying factors affect the relation, but also their timing, from structural shifts to transitory shocks. Regarding the former, the bond-stock correlation lies at the intersection between macroeconomics and finance. Drawing on consumption-based asset pricing, the payoffs of nominal USTs are significantly tied to inflation. When "supply-side" or "stagflation" shocks dominate, as roughly from the late 1970s through the 1980s in the U.S., inflation and consumption are negatively correlated. Thus, when inflation is "countercyclical,"

nominal bonds tend to pay off (sell off) in more (less) favorable states. Along with a positive premium on risky assets, such as equities that correlate positively with consumption growth, investors also require a positive (inflation) risk premium on USTs, which in turn produces a positive nominal bond-stock correlation.

The converse holds when "demand-side" shocks prevail, approximately from the late 1990s through the most recent decade. If inflation positively correlates with output and consumption, nominal bonds conversely pay off during unwelcome states and thereby offset stock market and broader consumption risks. Under this state, or rather strictly when investors learn or expect demand-side disturbances to dominate (David and Veronesi, 2013), the bond-stock correlation is negative, and USTs comprise a "safe" asset, presumably in short supply (Caballero et al., 2017). Some studies incorporate an intervening role for the aggressiveness of monetary policy under supply and demand shocks, namely whether the central bank responses one-for-one to inflationary shocks (e.g., Song, 2017). But the takeaway from this macro-finance lens is that investors' perceptions of inflation cyclicality largely account for the shift of the stock-bond correlation from broadly positive to negative before and after the late 1990s.

However, the literature produces no consensus. For example, Baele et al. (2010), rather than shifting macroeconomic factors, find a more prominent role for liquidity factors, amid puzzlingly substantial time variation in correlations.² Indeed, and particularly at higher frequencies, flights-to-(and-from)-quality appear to buffet the bond-stock correlation.³ During stress investors may flee less liquid equities for the relative liquidity and safety of (nominal) UST bonds. To wit, based on an event-study methodology given data through 2000, Gulko (2002) finds that when equity prices drop more than 5 percent, the correlation between bond and stocks switches from positive to negative. Also, Connelly et al. (2005) find that sharp increases in stock market uncertainty and increased equity market turnover tend to be negatively correlated with the bond-stock correlation, even during

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¹ Song (2017) delineates three regimes from 1963 to 2014, including countercyclical inflation with accommodative monetary policy (with respect to the inflation component of the dual mandate, roughly before the Volker era), countercyclical inflation with aggressive policy (with respect to the inflation goal), and procyclical inflation with aggressive policy. Notably, the regime-switching model produces transition probabilities, which allow for upward-sloping yield curves, even under the a procyclical-aggressive policy regime. Loosely speaking, high-frequency variation in the bond-stock correlation may of course reflect investors' perceptions of the varying states, along the lines of David and Veronesi (2013).

² Also, although Campbell et al. (2017) largely attribute the secular decline in nominal UST bond term premiums to the change in covariance between inflation and the real economy, they also find that excess bond returns vary excessively relative to their model and suggest that other factors might help determine risk premia, perhaps including UST supply. ³ The term FTQ may conflate or subsume flights to quality and flights to liquidity, possibly distinct episodes.

periods of longer-run unconditional positive correlation. And, Bansal et al. (2014) report that a strong negative relation between the covariance of equity risk and equity returns and the bond-stock correlation, notably over 1997 to 2011. Therefore, these results largely suggest that USTs consistently offer diversification precisely when needed most. Then again, unless FTQs have become more frequent if not persistent, the net decline in the UST-S&P correlation likely also reflects longer-run factors, too.⁴

Before presenting analyses that bear on these arguments, quantification of the effect of UST hedging properties on valuation, notably relative to myriad other factors such as expectations for macroeconomic variables as well as supply factors, strongly motivates for practitioners more careful analysis of the dynamic bond-stock correlation.⁵ Without digressing fully into government bond "value," consider the following time-series regression,⁷

$$y_t^{UST} = \alpha + \beta_{\hat{\rho}} \hat{\rho}_t + \beta_X^T X_t + \varepsilon_t \tag{1.1}$$

where y_t^{UST} refers to the yield on USTs at t for some maturity; $\hat{\rho}_t$ a measure of the dynamic correlation (or beta) between UST (at the same maturity as y_t^{UST}) and S&P 500 returns, described more fully below; X_t is a matrix of control variables factors, from forecasts of macroeconomic factors and government budget deficits, as well as proxies of uncertainty around those variables; and ε_t is a standard error term that captures UST mis-valuation at t. Moreover, with roots in extreme bound analysis (Leamer, 1983), allow for multiple alternative specifications of X_t to acknowledge

⁴ Indeed, Campbell et al. (2018) argue that when agents have consumption-based habit formation preferences, time-varying macroeconomic dynamics and FTQ are not mutually exclusive explanations for the apparent structural change in the bond-stock correlation from positive to negative. Instead, recessions increase the risk-aversion of investors, which amplifies the correlation in the corresponding direction when inflation is procyclical and countercyclical.

⁵ Whenever nominal yields breach historical lows or break out of prevailing trading ranges, Wall Street analysts and academic bloggers alike invariably proffer explanations, usually univariate espousals of a single pet factor amid a zoo of other plausible considerations. But as in the case of the cross-section of equity returns (Gu et al., 2018; Durham, 2000; Harvey et al., 2016), or even long-run macroeconomic growth (Levine and Renelt, 1992), the safer stories for yields are likely multivariate.

⁶ Common measures of "value" for government bonds only consider inflation (Asness et al., 2013; Brooks et al., 2018). However, considering a policy-rule framework, say, other factors might similarly anchor "fundamental" value, without breaching other factors such as momentum, carry, or betting-against-beta.

⁷ For similar simple regression-based approaches to anchor yields as well estimated term premiums, see (Li and Wei, 2013) or Durham (2008).

⁸ See Durham (2013) for a similar framework on the context of equity index level valuation. Of course, machine learning methods usefully address a multitude of correlates or features, no less for bond returns (Bianchi et al., 2019). But rather than produce the best prediction, the simple motivation here to isolate the effect of a specific independent variable, for which OLS seems best suited.

likely bias in the selection of proxies for underlying factors. Table 1 reports estimates across 288 alternative specifications of (1.1) for 10-year ZC UST yields, given monthly data from February 1996 through June 2018.⁹

Just to motivate further analyses, the magnitudes of the effect of $\hat{\rho}_t$ on yields rivals if not exceeds most other factors. The estimated coefficient(s) are statistically significant in each of the 288 regressions, and the coefficients suggest that a 1-standard-error increase in the correlation (beta) corresponds to a 38-basis-point (33-basis-point) increase in yields, all else equal and averaging over 288 models. The magnitude of the effect compares to, say, 29- and 29-basis point (8- and 14-basis-point) responses to 1-SE positive shocks to, say, the mean and the standard deviation of survey-based expected GDP growth (inflation), respectively. Moreover, unlike the bond-stock correlation proxies, other factors are hardly robust across alternative specifications, including the inflation measures as well as proxies for expected budget deficits and foreign demand for USTs. In short, these results suggest that dynamic bond-stock correlations matter as much for valuation as other factors related to the macroeconomic outlook or bond supply, which arguably receive notably far greater attention among practitioners.

An asymmetric multivariate-GARCH lens

Turning to direct analyses of the relation, previous studies nearly ubiquitously rest on unconditional correlation measures, commonly sampled at monthly or quarterly frequencies. Instead, an M-GARCH model also provides useful bearings on time variation in the bond-stock correlation. Conditional models may afford not only another lens on the trajectory of dynamic correlations but also a finer read on the higher moments around those trends, such as the so-called "vol of vol" and notably the (unconditional) variance around conditional covariance.

Among many models in the literature, the analyses follow the asymmetric approach from Cappiello et al. (2006), applied in a bi-variate setting. ¹⁰ The underlying data comprise nominal (ZC)

$$\Sigma_t = D_t P_t D_t$$

⁹ Data are monthly observations timed to an estimate of when respondents complete the Consensus Economics surveys. Further details on the regressions are available on request.

¹⁰ That is, the estimates refer not to a comprehensive variance-covariance matrix of all asset under review but separate bi-variate estimates between returns on the UST maturity and component in question and the S&P 500. See Cappiello et al. (2006) for complete details, but underlying dynamic variance-covariance, Σ_t follows from

zero-coupon yields based on Gürkaynak, Sack, and Swanson (2006) (GSW), which at the time of writing covers June 30, 1961 through September 19, 2019. However, the estimates for fitted UST yields, as well as average expected short rates and ZC term premiums, follow an update of Adrian et al. (2013), which spans the same period. Log differences in the daily S&P 500 index values comprise returns, and (synthetic) constant-maturity returns for yields as well as the expected rate and term premium components, *y*, follow

$$r_{t+1}^{\tau-year} = -D\Delta y_{t+1} + \frac{1}{2}D^2(\Delta y_{t+1})^2$$
(1.2)

where D is the duration or maturity in the case of ZC instruments. Also, to evaluate correlations on a notionally equivalent basis, 2- and 5-year returns are levered up to the 10-year duration point. ¹²

Chart 1 displays the estimated asymmetric, dynamic conditional correlation (A-DCC) coefficients between S&P 500 returns and 2-, 5-, and 10-year nominal USTs over the full sample. Notwithstanding a mean-reversion parameter anchored by the unconditional sample correlation(s) in Cappiello et al. (2006), the A-DCCs suggest an orthogonal relation between the credit-risk-free and yardstick risky asset class through the 1960s, a positive association beginning in the 1970s and prevailing roughly through the 1990s, and a notable net decline thereafter with mostly, but not exclusively, negative readings. As Table 2 indicates, the trajectories of the A-DCCs are broadly

where D is the diagonal matrix of conditional volatilities, which follow TGARCH/AVGARCH; P is the correlation matrix, Q^* is a scaling matrix, with

$$P_{t} = Q_{t}^{*-1} Q_{t} Q_{t}^{*}$$

$$\overline{P} = E \left\{ \varepsilon_{t} \varepsilon_{t}^{T} \right\}$$

$$\varepsilon_{x} = r_{x} \sigma_{x}^{-1}$$

The key dynamics, in the symmetric case, follow

$$Q_{t} = (1 - a - b)\overline{P} + a\varepsilon_{t-1}\varepsilon_{t-1}^{T} + Q_{t}$$

where parameters a and b are estimated using maximum likelihood methods.

¹¹ Ten-year zero-coupon GSW estimates start from in August 16, 1971, but Adrian et al. (2013) backfill these estimates to 1961 using the contemporaneous GSW coefficients. Also, the GSW curve culls on- as well as first- and second-off-the-run U.S. Treasury securities. Given that these securities at times of stress command considerable liquidity premiums, these estimates may understate the negative correlation between UST and equity returns. However, the results are similar using other fitted curves, such as US, which do embed some information from the most liquid issues but do not cover as lengthy a sample as the GWS estimates.

¹² The leverage adjustment simply comprises $D = \frac{10}{\tau}$ for all maturities.

similar across maturities. Before and after 1998, the average A-DCC across 2-, 5-, and 10-year securities declined from 0.19 to -0.21, from 0.21 to -0.23, and from 0.22 to -0.23, respectively. The corresponding asymmetric dynamic conditional (A-DC) betas, which of course incorporate the time-varying ratios of volatility between USTs and the S&P 500, similarly imply a long-run increase in the hedging value of USTs.

None of these observations necessarily break new ground. However, the daily-return-based asymmetric M-GARCH highlights that amid the apparent structural decline in correlations, the shift does not seem to accompany a diminution of the volatility around correlations. To the contrary, visual inspection of the series after 1998 in Chart 1 suggests comparatively meaningful variation in A-DCC coefficients. To be more precise, the simple standard deviations of the A-DCCs before and after the breakpoint increase from 0.04 to 0.18, from 0.07 to 0.21, and from 0.04 to 0.23 for 2-, 5-, and 10-year USTs, respectively. This result contrasts with the corresponding estimates of conditional volatility for each UST maturity, based on an asymmetric GARCH, shown in Chart 2 and listed in the bottom panel of Table 2. "Vol of vol," captured by the standard deviation of the conditional volatility, declines notably along with the diminished level of return volatility after the break, from 4.36 to 2.97, from 6.04 to 2.63, and from 5.19 to 2.45 percent (annualized and levered to the 10-year maturity), respectively.

A possible inference from the M-GARCH across maturities is that variance risk premiums declined along with the level of variance. But the increased variance around covariance over the last couple decades is by contrast consistent with greater covariance premiums, even as on average the value of USTs as a risky asset hedge improved. In other words, the quantity but not necessarily the price of covariance risk has decreased. Another plausible interpretation is that, although investors may have "learned" about the procyclicality of inflation over the last two decades, on net, the level of uncertainty around those "beliefs" has increased the past 20 years (David and Veronesi, 2013). Or, put another way, perhaps they increasingly attached greater uncertainty around the odds on shifts given the prevailing regime in cyclicality (Song, 2017), not dissimilar in spirit to the "excess sensitivity" of distant-horizon forward rates to macroeconomic news (Gürkaynak et al., 2005), say.

¹³ Campbell et al. (2017), Burkhardt and Hasseltoft (2012), and Baele (2010) report similar inferences using unconditional correlations using 3-year and 5-year rolling windows of daily and quarterly returns, as well as non-overlapping quarterly estimates using daily returns, respectively.

Another noteworthy result regards the decomposition of UST returns into expected rates and term premiums. At first blush, equity return correlations with both components largely follow the long-run trends as previously discussed with respect to aggregate returns. However, the distinction may be meaningful at times. Focusing on the post-1998 sample, Chart 3 shows that the average 10-year expected rate (synthetic) return A-DCC with the S&P 500 is persistently positive from the middle of 2012 toward the end of 2014, not only after the global financial crisis but also safely following the date after which studies document inflation procyclicality. Meanwhile, the corresponding correlation for the 10-year term premium component is safely negative. Chart 4 conveys the same substance for anticipated rates through the 2-year horizon, obviously more relevant to near-term monetary policy expectations, and indeed the shorter-dated expected-rate return A-DCC exceeded even the measure for gold.

Strong caveats about false precision with term structure models aside, the results for this episode, spanning the so-called "taper tantrum," are more consistent with perceptions of a "forecast signal" rather than a so-called "Fed put." Assuming causation runs from discount rates to risky asset prices, and noting that the simultaneously negative covariance between returns and the term premium component remains consistent with an effective hedge, a positive dynamic correlation connotes that upward (downward) revisions in the near-term policy path convey good (bad) news about consumption, which boosts (lowers) stock prices. Thus, when investors perceive the central bank has superior information about the either inflation or the real economy, or colloquially whenever markets think the Fed knows more than they do, the expected-rate component detracts somewhat from the hedging value of USTs.

Yet another interpretation of this configuration of positive and negative expected-rate and term premium correlations with the S&P 500, respectively, is that the central bank takes the long- as opposed to the short-run view on the tradeoff between broader financial conditions and future GDP growth (Adrian et al., 2019). That is, policymakers lean against accommodative conditions to ameliorate the downside skew of further-dated growth projections that owe to burgeoning vulnerabilities, at the expense of the nearer-term outlook. In any event, the meaningful variation of the covariance, with respect to either component of USTs, may imply abundant uncertainty around

¹⁴ Somewhat dissimilar to Song (2017), Burkhardt and Hasseltoft (2012), and Campbell et al. (2018), who consider a break around the turn of the century, Chen et al. (2016) date the regime shift toward a positive correlation between (10-year forward) consumption growth and inflation in the late 2000s. Even so, their measure does appear to increase steady from around the mid-1980s.

investors' beliefs about prevailing inflation cyclicality and central bank reaction functions, even in recent decades.

Spectral Analysis

Yet no M-GARCH can capture relevant subtleties. For example, daily sampling identifies meaningful, and persistent to the eye, conditional variation in the correlations. But spectral decomposition seems better suited to address precisely the power of high- versus low-frequency cycles that in turn might reflect more transitory FTQs, on the one hand, or the shifts in the cyclicality of inflation, on the other. Economists have long used spectral analysis to decompose stationary time series into sums of periodic functions and thereby visualize data in the frequency domain. Briefly, spectral and co-spectral power isolate the variability or the correlation, respectively, of time series that owes to fluctuations at specific frequencies or horizons. Methods such as the discrete Fourier transformation (DFT) of data sampled at fixed intervals produce a lens on the high-frequency (HF), mid-frequency (MF), and low-frequency (LF) components over time that is often concealed to the eye in raw time series.

Applications to empirical finance are rarer. But as a welcome exception, Chaudhuri and Lo (2015, 2016) (CL) accordingly decompose U.S. equity return volatilities and correlations. They report that these key statistics change not only over time, as is well known, but also across frequencies over time. The share of equity return volatility that traces to HF as opposed to LF cycles has increased

$$r_{t} = \mu + \int_{0}^{\pi} \alpha(\omega) \cos(\omega t) d\omega + \int_{0}^{\pi} \delta(\omega) \sin(\omega t) d\omega$$

where ω refers to a frequency. Given a subsample of two return series, say, r_i^{UST} and $r_i^{S&P}$ from 1,2,...,t,t-1,...T-1, the covariance, ωv , of the interval can be written as

$$\operatorname{cov}_{r^{S\&P},r_{i}^{UST}} = \frac{1}{T} \sum_{t=1}^{T-1} \left(r_{i}^{UST} - E\left\{r^{UST}\right\} \right) \left(r_{i}^{S\&P} - E\left\{r^{S\&P}\right\} \right) = \frac{1}{T} \sum_{t=1}^{T-1} \frac{1}{T} \operatorname{Re} \left[R_{\omega}^{UST} R_{\omega}^{S\&P} \right]$$

where is the expectations operator; R_{ω}^{UST} and $R_{\omega}^{S\&P}$ are the T-point DFT coefficients of the subsample of r_{t}^{UST} and $r_{t}^{S\&P}$ at frequency ω ; and the sum over $\frac{1}{T} \operatorname{Re} \left[R_{\omega}^{UST} R_{\omega}^{S\&P} \right]$, or the co-spectrum, is proportional to the sample covariance of r_{t}^{UST} and $r_{t}^{S\&P}$. The sum of over a band of frequencies is proportional to the contribution of that band to the overall covariance, and to obtain the variance decomposition of, say, UST returns, substitute r_{t}^{UST} for $r_{t}^{S\&P}$ and r_{ω}^{UST} for $r_{t}^{S\&P}$ is the previous expressions.

¹⁵ With using spectral analysis, Campbell et al. (2017) reference considerable high-frequency variation in bond-stock correlations and betas, amid substantial low-frequency movements.

¹⁶ Examples include, say, applications to seasonal adjustments of macroeconomic data. Also, see Engle (1974) for an early application to the permanent income hypothesis, and Hamilton (1994) for an overview.

¹⁷ A stationary return series, r, can be expressed as a weighted sum of periodic functions and its mean, μ , as in

over the past two decades. "Fast-moving" strategies increasingly on net account for more overall market variability than "slower-moving" trading rules. 18

Spectral analysis of the nominal UST term structure seems overdue in general and relevant to the hedging properties of bonds. For instance, the more salient the consumption-based mechanism from inflation procyclicality, the more likely LF cycles account for the variation in association. Conversely, to the degree that FTQs buffet the correlation, or perhaps "excess sensitivity" to perceptions or uncertainty around shifting macroeconomic regimes, the greater power for HF phenomena. To address the issue, the following accepts some basic methodological choices from Chaudhuri and Lo (2015) and delineates the frequency domain into HF, MF, and LF cycles that span 2 to 5 days (i.e., at least 1 cycle per week), 5 to 21 days (between 1 cycle per week and 1 cycle per month), and beyond 21 days (less than 1 cycle per month), respectively. Also, 3-year rolling windows capture changes in the frequency domain over time.¹⁹

Before turning to the covariance, consider first the decomposition of UST variance, undocumented in the literature. To fix bearings, Chart 5 shows the full-sample decomposition of S&P 500 returns. Very similar to the results in Chaudhuri and Lo (2016), HF trends on average account for about 56 percent of stock return variance, and the proportion ranges from 38 to 76 percent, with a steady net increase (drop) in the HF (LF) share over the sample. The corresponding estimates for 2-year USTs, shown in Chart 6, largely tells the same story, with a mean HF share over the period of about 53 percent and a net upward trajectory that spans minimum and maximum shares of 26 to 71 percent. With the increased relative importance of HF cycles for UST returns, especially toward the front of the term structure, the same broad conclusions about equity markets seem germane to USTs, too. Besides technology and increased competition, another factor especially relevant to USTs is the steady increase in Federal Reserve transparency, ²⁰ particularly including far

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¹⁸ More broadly, they show that the key ingredients of modern portfolio theory—mean returns and higher moments as well as correlations, betas, and alphas—have clear counterparts in the frequency domain.

¹⁹ Strictly speaking, the 3-year as opposed to longer-dated rolling DFTs represent shorter-time Fourier transforms that address the potential issue of non-stationarity in return time series. Chaudhuri and Lo (2016) cite a tradeoff in that longer windows produce better frequency resolution but are less equipped to capture time-variation in the statistical properties of signals than shorter windows.

²⁰ This increased (decreased) share of HF (LF) cycles may have some implications for systematic trading strategies in the UST market. Again, in the case of equities, CL explain that the waning degree of LF power is consistent with diminished longer-run serial correlation in stock returns. The literature on time series (Moskowitz et al., 2012) and cross-sectional government bond momentum (Durham, 2015) is far smaller and arguably less compelling. Nonetheless, LF momentum strategies may have lost further luster, and other factors such as "value," very likely operate at much lower frequencies.

more detailed policy announcements from the mid-1990s and other innovations such as modern press conferences.²¹ Transparency likely affects not only the average expected rate but also uncertainty around the policy path. Indeed, Table 3 documents similar shares and trajectories for the expected-rate and term-premium components of USTs at the 2-, 5-, and 10-year maturities, and HF phenomena contribute approximately equivalently to expected rates and required returns.

Turning to the relation between bond and stock returns, Table 4 reports the results from the spectral decomposition of the covariance of each UST maturity-component returns with the S&P 500, and Chart 7 illustrates the relative sizes and paths for the frequency shares over the full sample for 2-year UST-S&P covariance. A few observations are noteworthy. First, to date on net, the share at the end of the sample of HF cycles is comparable to the HF share for 2-year UST return variance (61 and 62 percent respectively). Chart 8, which illustrates the corresponding calculations for 10-year USTs, as well as Table 4 that covers all maturities and components, produce very similar results. True, the mean HF shares over the full sample are very modestly lower for covariance than for covariance, for example, about 50 (49) versus 56 (53) percent for 10-year (2-year) USTs. Yet the results still suggest that the underlying forces that shift correlations are also primarily HF phenomena, perhaps as opposed to slower developments in the cyclicality of inflation or other longer-run macroeconomic fundamentals.

However, a second observation is that the HF share of covariance has notably increased from 1961 to 2019, on net. As Table 4 reports and Chart 8 (Chart 7) shows for the 10-year (2-year) UST, the HF proportion rose from about 44 (23) during the first 3-year rolling estimate ending in July 1964 to again around 60 (61) percent at the end of the sample. This net increase in the HF share might suggest decreased influence of (perceived) macroeconomic regimes, perhaps. However, a third possibly comprises a caveat, insofar as the shares across time scales are themselves highly variable over the sample, especially compared to the relatively steady increase in the relative importance of HF cycles to total variances. Even so, the results if anything lend more credence to the notion that the stock-bond correlation owes more to HF phenomena. Perhaps most obviously these include transitory swings in investors' attitudes toward and perceptions of risky assets following, say, Connelly et al. (2005) or Bansal et al. (2014). But at the same time, even though

²¹ At the same time, the results plausibly connote increasingly considerable noise in UST markets, akin to equities. However, further analysis is required to assess if, say, the excess sensitivity puzzle has worsened over the years. A decomposition of the covariance between the front and the back ends of the term structure in the frequency domain is outside the scope of this study.

inflation cyclicality unlikely changes overnight, investors may revise their uncertainty or beliefs (David and Veronesi, 2013) around perceptions of prevailing regimes, too, arguably in an excessively frequent manner.

Finally, the relative power of different frequency bands does not speak to the direction of covariance and ultimately the hedging value of USTs per se across different time scales. For example, Gulko (2002) and Connelly et al. (2005) suggest the correlation between stocks and bonds turns negative or "decouples" amid FTQs, but notably only temporarily and amid an unconditionally positive correlation largely prior to 2000. These arguments therefore imply that bonds are better hedges over shorter as opposed to longer horizons. Also, in general and considering the modern era of inflation procyclicality, investors conceivably may still accept lower returns on USTs, insofar as they especially insure portfolios against shorter-lived shocks.²²

Spectral analysis helps assess this issue, too. So-called band-spectrum betas (Engle, 1974) directly determine whether HF UST betas are less than, or more negative, compared to LF UST betas. Charts 9 and 10 show the results for 2- and 10-year USTs, respectively. In contrast to this hypothesis, the band-spectrum betas are largely consistent across time scales for both maturities, apart perhaps from the early 1970s as well as the early to mid-1980s. During these episodes of some band-spectrum beta divergence, HF are strictly less than estimated LF betas. But notably, the betas are positive rather than negative across time scales during this era, inconsistent with any hedge. In sum, and however indirectly, the relative share of HF cycles points toward more transitory phenomena, and the band spectrum betas are somewhat problematic for the view that FTQs primarily determine correlations, insofar as USTs have not been especially better hedges in the short run.²³

Convexity and quantile regressions

A key strand of both consumption- and FTQ-based arguments about USTs as a hedge is that bonds pay off not only on average but also especially, or perhaps nearly exclusively, in bad

²² In a similar application, Durham (2019) finds that bitcoin returns are largely orthogonal or negatively correlated with the S&P 500 over HFs but positively correlated over longer cycles, which implies that any hedging value of cryptocurrencies wanes as the investment extends.

²⁵ These findings might cast doubt on whether the net secular decline in term premiums over the sample particularly owes particularly to shorter-run hedging properties of USTs. Then again, trivially, longer-dated term premiums assume investment horizons of equivalent length. Investors may plausibly value consistency in hedging value across different time scales, which this spectral analysis largely conveys.

states of the world. Consider both components of yields. In the contexts of contemporary low nominal term premiums globally, negative required returns in the direst states plausibly drag down the weight-average bond return across all scenarios. With respect to expected short rates, a more fulsome assessment of the "Fed put" beyond "regressing through the mean" seems essential, given that hedging value associated with central banks' response to worsening financial conditions ultimately trace most to the left tail, not to the center of the distribution of risky asset returns. Besides the existing literature on the bond-stock correlation, neither M-GARCHs of time-varying correlations and betas nor the spectral analysis address this asymmetry in investors' calculation of expected returns or their assessment of policy responses to deteriorating financial conditions.

Therefore, this section takes two tacks at asymmetries in correlations. The first is informal yet illustrative. Table 5 reports simple OLS regressions of daily UST returns on S&P 500 returns, with limited samples near the tails of the S&P sample distribution over the full period, including sub-samples when equity returns are less-than-or-equal-to the 0.5 and 5.0 percentiles of the distribution, on the downside, and greater-than-or-equal-to the 95 and 99.5 percentiles, on the upside. An archetypal "convex" hedge has a negative beta for the sub-sample near the left of the equity return distribution, but a positive beta to the right. Or, perhaps more realistically and specific to possibly negative term premiums, UST betas should be increasingly negative toward the left tail, indicative of an effective hedge against the worst of risky asset returns.

But results are mixed at best. The full-sample for 2-year USTs does produce a negative beta (-0.17) at 0.5 percentile of S&P 500 returns (i.e., days with -3.34 percent or lower returns on stocks), across 72 observations, as listed in Table 5. However, as Chart 11 suggests, although the betas modestly increase to the right of the distribution (the green bars), the coefficient at the 99.5 percentile of returns is strictly negative (-0.10), although the estimate at the extreme right is not statistically significant. Also, strictly speaking the results for the 10-year (the red bars) imply a negatively not a positively convex hedge at the extremes, with a perversely positive (negative) beta at the 0.5 (99.5) percentiles of the distribution, 0.02 (-0.18). Chart 12 shows that this result owes largely to the estimated term premium component of yields, with perverse betas across the extreme tails of 0.13 and -0.12, respectively, although only the former is statistically significant.

These full-sample results arguably should be insensitive to correlation regimes. Indeed, the arguments in Gulko (2002) or Connelly et al. (2005) imply that more extreme returns drive transitory covariance changes, even when a positive correlation between USTs and stocks largely prevails. But

to test whether these findings are robust to the more recent two decades, Chart 13 shows the corresponding results for the limited sample after 1998. Briefly, the results are substantively quite similar and indicate no robust positive schedule of betas across the distribution of S&P 500 returns, consistent with a consistently positively convex hedge, or an asset that primarily, not to mention exclusively, pays off in the worst states.

The second approach at asymmetries comprises quantile regressions to trace the full distributions of USTs conditioned on risky asset returns. This analysis transcends the first moment and addresses whether the conditional densities of UST returns are, first, narrower and, second, more positively skewed to the upside when equity markets slide, as opposed to when risky assets rally. Presumably, and appealing to the logic of a higher momentum CAPM (e.g., Kraus and Litzenberger, 1976; Harvey and Siddique, 2000), required returns or term premiums on USTs should decline, insofar as the conditional distributions imply less uncertainty (the second moment) and positive skew, compared to upside scenarios of similar magnitude (the third moment). More colloquially, the question is whether UST returns are more positively skewed when the S&P 500 swoons than negatively skewed when stocks rally by an equivalent magnitude, apart from the sign of the hedge.

There are several references on quantile regression (e.g., Koenker and Hallock, 2001). In this application to UST returns, r^{UST} , the θ^{th} regression quantile is any β that solves

$$\min_{\beta} \frac{1}{T} \sum_{t=1}^{T} \left[\theta - I \left(r_{t}^{UST} < X_{t} \beta_{\theta} \right) \right] \left[r_{t}^{UST} < X_{t} \beta_{\theta} \right]$$
(1.3)

where X_t includes an intercept and returns on the risky asset, $r_t^{S\&P}$, T is the time-series sample size, and $I(\cdot)$ is the indicator function. ²⁴ Following the general approach from Ghysels et al. (2011) and Ghysels (2014), (1.3) produces the quantile function (QF), including the 5th, 25th, 50th, 75th, and 95th percentiles of the UST distribution, in turn conditioned on a given S&P 500 return. The full conditional distribution follows a skewed t distribution interpolated from the QF, following Azzalini

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²⁴ Quantile regression minimizes the sum of absolute rather than squared errors and puts differential weights on the errors based on whether an error is above or below the quantile.

and Capitanio (2003).²⁵ With respect to higher moments, which are independent of the skewed t parameters, the conditional width of the distribution at a given point, $\hat{W}_{\theta} \mid r_i^{S\&P}$, follows

$$\hat{W}_{\theta} \mid r_t^{S \& P} = \hat{Q}_{\theta} \mid r_t^{S \& P} - \hat{Q}_{100-\theta} \mid r_t^{S \& P}$$

$$\tag{1.4}$$

Regarding the skew, the following refers to conditional Bowley (1920) skew of a given (fitted) quantile of UST returns, $S_{\theta} \mid r_t^{S \& P}$, as in

$$\hat{S}_{\theta} \mid r_{t}^{S \& P} = \frac{\left(\hat{Q}_{\theta} \mid r_{t}^{S \& P} - \hat{Q}_{50} \mid r_{t}^{S \& P}\right) - \left(\hat{Q}_{50} \mid r_{t}^{S \& P} - \hat{Q}_{100 - \theta} \mid r_{t}^{S \& P}\right)}{\hat{Q}_{\theta} \mid r_{t}^{S \& P} - \hat{Q}_{100 - \theta} \mid r_{t}^{S \& P}}$$

$$(1.5)$$

where $\hat{Q}_{\theta} \mid r_{t}^{S\&P}$ is the fitted value from the quantile regressions for a given quantile, or $X_{t}\hat{\beta}_{\theta}$, and the results focus on the distribution at $\theta = 95$ and $\theta = 75$, the interquartile range.

Before turning to the conditional distributions and higher moment measures, consider the schedule of quantile betas with respect to increasing values of θ , as shown in Chart 14 for 2-, 5-, and 10-year USTs over the full sample of daily returns. True, the magnitudes of the full sample betas are small. Nonetheless, for each maturity the betas are positive at θ = 5 but negative for θ = 95. As such, conditioned on positive (negative) S&P 500 returns, the left tail increases (decreases) and the right tail decreases (increases), and therefore the width of the conditional distribution decreases (increases), all else equal. Also, with respect to skew, and more generally speaking, when the quantile betas are increasingly greater toward the right of the asset return distribution, asset returns are positively (negatively) skewed, conditioned on positive (negative) market returns. Conversely, if the quantile betas decline toward the right of the distribution, asset returns are negatively (positively) skewed, conditioned on positive (negative) market returns. Briefly, insofar as the distribution widens and skews leftward conditioned on negative S&P 500 returns, the schedule of quantile betas implies a negatively rather than positively convex hedge, all else equal.

To go beyond intuition and to assess empirically the comparative symmetry of conditional UST return distributions, which includes the schedule of quantile alphas, Chart 15 shows two densities for the 2-year UST based on the estimated quantile regression coefficients. The first traces out the distribution, condition on "downside" 0.5 percentile S&P 500 returns (red), and the second

²⁵ See Adrian et al. (2019) for a similar application.

²⁶ This brief illustration abstracts from relative size of the quantile regression intercepts.

shows the corresponding density conditioned on an "upside" or positive return of equal magnitude (green). Clearly, the central tendencies of both distributions nearly overlap and are very close to zero, indicative of orthogonality with the S&P 500 rather than an outright hedge over the period. But turning to higher moments, and largely consistent with the configuration of betas in Chart 14, the width of the "downside" density from $\theta = 5$ to $\theta = 95$, about 209 basis points, clear exceeds that for the "upside" density, 160 basis points, which is consistent with greater uncertainty about conditional UST returns in unwelcome as opposed to favorable risky asset return scenarios. Also, based on the Bowley skew readings, the downside distribution is skewed negatively, $\hat{S}_{\theta95} \mid r_i^{S&P(-3.34\%)} = -0.104$, whereas the upside density is tilted to the upside, $\hat{S}_{\theta95} \mid r_i^{S&P(+3.34\%)} = 0.132$. In short, at least over the full sample, the conditional distribution of 2-year USTs does not suggest a particularly convex hedge with respect to higher moments.

The top third panel of Table 6 largely suggests the same inference across maturities and the components of yields. In each case for the full sample beginning in 1961, the downside densities are more negatively skewed as well as wider than the corresponding upside distributions (Columns 3 and 4, respectively), from $\theta = 5$ to $\theta = 95$ along the distributions. Also, the interquartile ranges are similarly wider for every downside distribution compared to the corresponding upside conditional density (Column 8), although the comparative skew measures nearer the center of the distributions are somewhat mixed.

Again, it seems worthwhile to isolate the results since 1998, when the overall bond-stock correlation was largely negative and nominal USTs hedged (mean) S&P 500 returns on average. Chart 16 shows the upside and downside densities for the 2-year UST, given only information after 1998. Consistent with the prevailing negative correlation, the distribution conditioned on a 0.5 percentile decline of S&P 500 returns lies to the right of the origin, and the corresponding density for outlying positive equity returns (4.32%) is to the left. Turning to telling higher moments, the widths are very similar (153 basis points versus 148 basis points). But, the skew metrics for this period do suggest more favorable hedging properties for the 2-year maturity. Consistent with a convex hedge, or rather an asset that provides comparatively superior protection on the downside, $\hat{S}_{\theta95} \mid r_i^{S\&P(-4.32\%)} = 0.171$, whereas $\hat{S}_{\theta95} \mid r_i^{S\&P(4.32\%)} = -0.131$. And, as the middle third of Table 6 indicates, the same comparative configuration for the second and third moments similarly holds for the interquartile range (Columns 7 and 8).

However, this result for the post-1998 sample is not consistent across the term structure. As Chart 17 indicates, the downside density for 10-year USTs is about 21 basis points wider, and the downside distribution skew is notably negative, $\hat{S}_{\theta 95} \mid r_t^{S\&P(-4.32\%)} = -0.342$, as the corresponding upside density tilts leftwards, $\hat{S}_{\theta95} \mid r_t^{S\&P(4.32\%)} = 0.235$. Also, perusing the middle third panel of Table 6 and considering each maturity and component of yields, the downside distributions on the whole are more negatively skewed and are ubiquitously wider between the 5th and 95th percentiles, and the aggregate results for the interquartile range are mixed. Sample divisions are ultimately arbitrary, but as a cursory but relevant robustness check, the lower third panel reports the results for the most recent 3-year sample, notably with yields and estimated term premiums near sample lows. The safest inference during this period is that, again, USTs do not appear to be consistently more positively skewed, with narrower distributions, conditioned on very negative equity returns. Chart 18 for the 10-year yield is illustrative. The downside density is both by visual inspection and by measurement notably wider, by about 64 basis points, and it is clearly more negatively skewed than the corresponding distribution conditioned on very positive returns ($\hat{S}_{\theta95} \mid r_t^{S\&P(-3.25\%)} = -0.377$ versus $\hat{S}_{\theta 95} \mid r_t^{S\&P(+3.25\%)} = 0.271$). Therefore, in general, explicit assessment of conditional distributions casts some doubt about USTs as an effectively convex hedge with respect to the fuller distribution of risky asset returns. This inference holds even for the most recent period and during an era of historically low yields and term premiums and especially for longer-dated maturities, the primary focus of the literature.

Discussion

The bond-stock correlation matters not only for the limits of portfolio diversification but also for required returns on the risk-free asset class. The preceding analysis reconfirms the familiar net shift from positive to negative covariance between the yardstick risk-free and risky assets over the full sample for the U.S. But an asymmetric M-GARCH based on daily returns also suggests meaningfully greater (unconditional) variation around the correlation over the last 20 years, in both the estimated term premium and expected path components of USTs, which strictly speaking is consistent with higher rather than lower covariance premiums. Spectral analysis further indicates that HF cycles account for nearly as much of the variation in covariance with the S&P 500 as overall variance in individual asset returns, increasingly so even as the correlation declined during the period. Yet there is no evidence that the hedging value of USTs is any greater in the nearer term as

opposed to the longer run. Also, even in more recent times of very low government bond yields and estimated negative term premiums, simple OLS as well as quantile regressions connote that USTs do not seem to hedge the very worst states of the world more effectively. Again, Table 1 suggests that time-varying correlations contribute meaningfully to valuations. Nonetheless, it seems possible to oversell the hedging value of UST as an explanation for extremely low government longer-dated bond yields toward the end of the sample, particularly regarding insurance against short-lived shocks or positive convexity with respect to stock market risks. Collateral value and supply issues aside, from investors' perspective the so-called "safe asset shortage" seems acute indeed (Caballero et al., 2017).

Although the existing literature seems to neglect higher frequency sampling and models of conditional correlation, spectral analysis, and quantile regression methods, there is no dearth of caveats with the preceding analyses or further lines of inquiry to explore. For example, besides obvious applications to other government bond markets beyond the U.S., this study focuses on nominal as opposed to real yields, like most but not all studies, with an emphasis on inflation cyclicality as well as FTQs to nominal USTs. However, a substantial portion of variation in the overall bond-stock correlation could owe to the real component of yields, too, even though consumption-based theory suggests that pure exposure to real borrowing costs should carry a negative premium. For example, Howard (2016) argues that real bond and stock returns necessarily become negatively correlated near the nominal lower bound. In any case, despite more limited timeseries data, correlations between TIPs, as well as further decompositions of real and inflation risk premiums (D'Amico et al., 2018), and S&P 500 returns might produce a finer read on the what drives the correlation, liquidity premiums in TIPs aside. Also, the analyses consider beta and correlations with respect to the risky asset class directly, not measures of consumption, in this empirical study for practitioners that bears on diversification and hedging.²⁷ The issue is not necessarily that the consumption-based asset pricing model is mis-specified. Rather, it may be worthwhile to use similar techniques, namely examining the frequency domain as well as conditional asymmetry, sampled at lower frequencies of the underlying macroeconomic series. Finally, the correlation metrics in this study, as well as several others, are directionless, distinct from Grangercausality or "interconnectedness" (Diebold and Yilmaz, 2015). Insofar as FTQs drive the

²⁷ Moreover, over long samples the industry composition of equity indices likely changes, and the relative concentration of inherently "cyclical" stocks may affect the correlation dynamics.

correlation, or more generally if "cross-market rebalancing" transmits shocks from one asset class to another (Kodres and Pritsker, 2002), a reasonable prior is that the association runs from the risky to the risk-free asset. By contrast, with pure discount-factor or monetary policy shocks, the relation likely runs in the reverse direction. Even thorough consideration of dynamic correlations, differences across time scales, and conditional asymmetries miss this potential subtlety.

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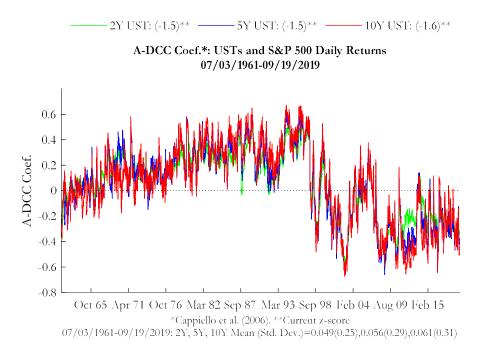


Chart 2

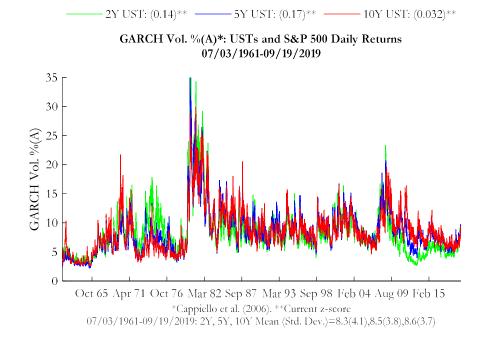


Chart 3

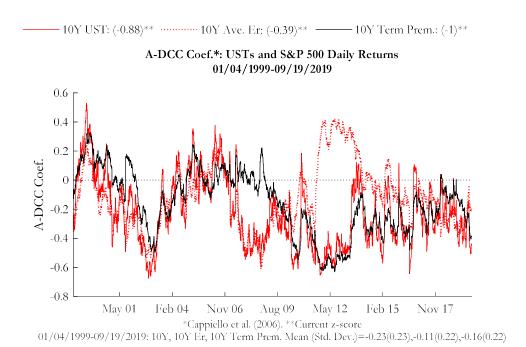
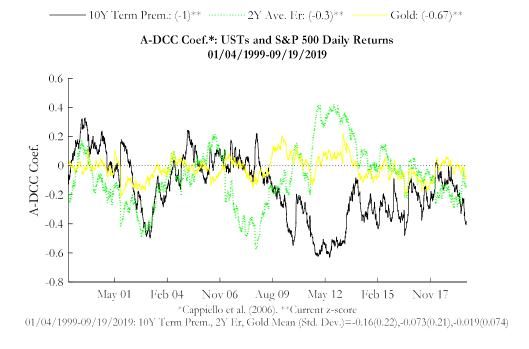


Chart 4



S&P 500 Spectral Decomposition (Variance) 07/14/1964-09/19/2019 80 60 40

% of Total

20

Oct 65 Apr 71 Oct 76 Mar 82 Sep 87 Mar 93 Sep 98 Feb 04 Aug 09 Feb 15

Based on 3-year rolling daily returns

High (medium) [low] frequency defined as 1 cycle every <=5(5<=21)[>21] days.

High, medium, and low current values=62°,30°,8°, of total.

Chart 6

PEGL Oct 65 Apr 71 Oct 76 Mar 82 Sep 87 Mar 93 Sep 98 Feb 04 Aug 09 Feb 15 Based on 3-year rolling daily returns High (medium) |low| frequency defined as 1 cycle every <=5(5<=21)|>21| days. High, medium, and low current values=62%,29%,99% of total.

——HIGH ——MEDIUM ——LOW

2Y UST Spectral Decomposition (Covariance with S&P 500) 07/14/1964-09/19/2019

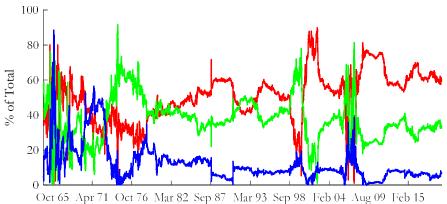


Based on 3-year rolling daily returns High (medium) [low] frequency defined as 1 cycle every <=5(5<=21)[>21] days. High, medium, and low current values=61%,32%,7% of total.

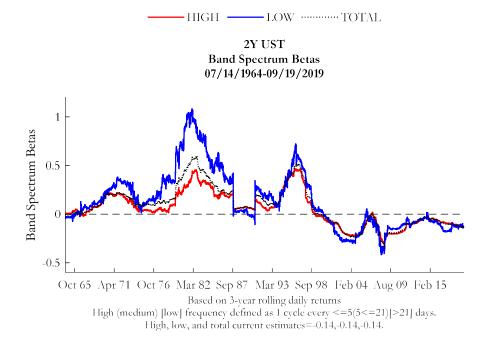
Chart 8

——HIGH ——MEDIUM ——LOW

10Y UST Spectral Decomposition (Covariance with S&P 500) 07/14/1964-09/19/2019



Based on 3-year rolling daily returns
High (medium) [low] frequency defined as 1 cycle every <=5(5<=21)[>21] days.
High, medium, and low current values=59%,33%,7% of total.



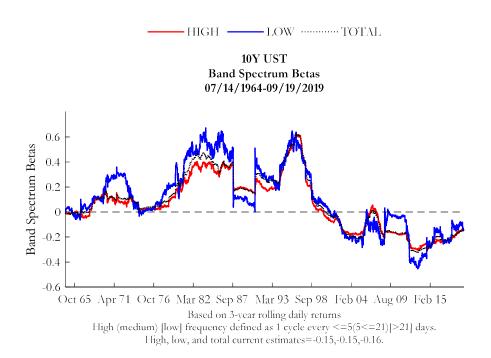


Chart 11

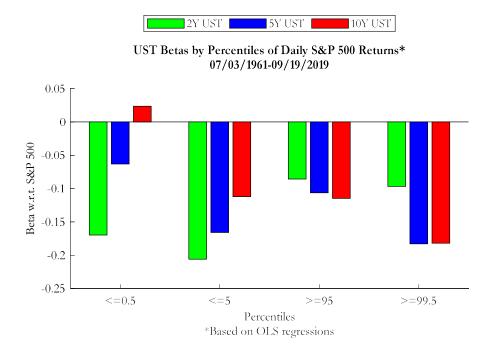


Chart 12

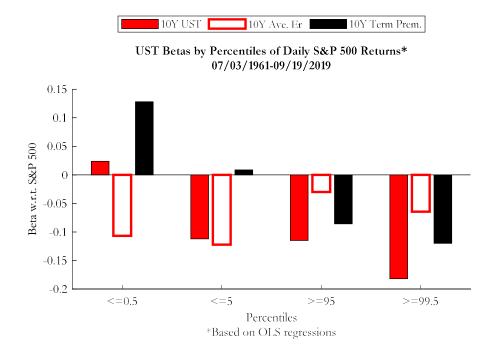


Chart 13

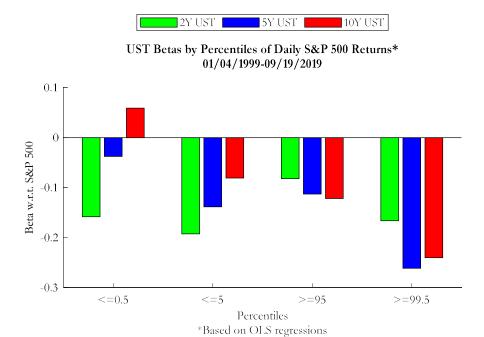
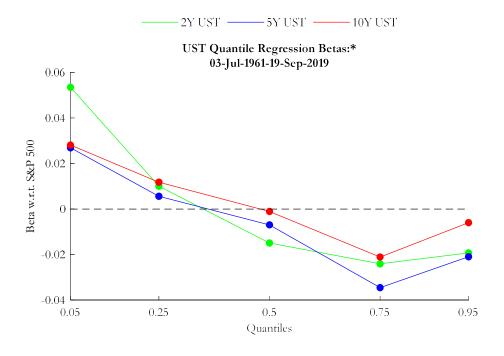
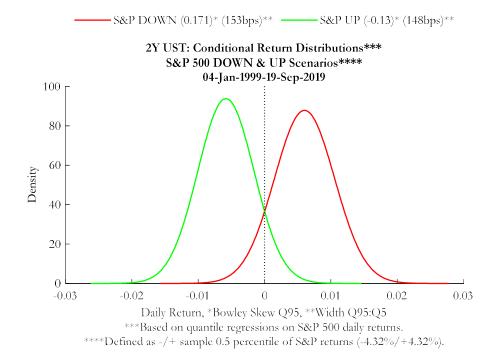
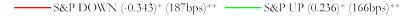


Chart 14



2Y UST: Conditional Return Distributions*** S&P 500 DOWN & UP Scenarios**** 03-Jul-1961-19-Sep-2019 80 60 Density 40 20 -0.03 -0.02 -0.01 0.02 0.03 0.01 Daily Return, *Bowley Skew Q95, **Width Q95:Q5 ***Based on quantile regressions on S&P 500 daily returns. ****Defined as -/+ sample 0.5 percentile of S&P returns (-3.34%/+3.34%).





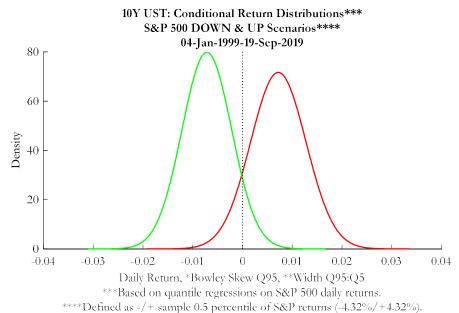


Chart 18

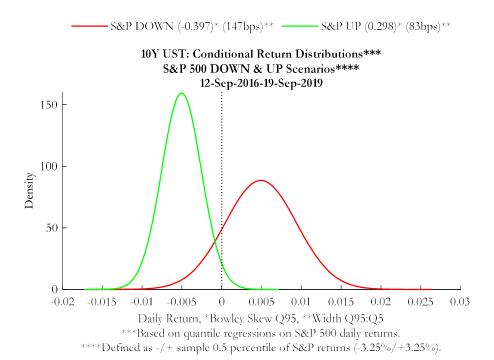


Table 1

UST: Coefficients: 10-year Zero-Coupon Yields
Level OLS Regressions:
Feb 12 1996-Jun 11 2018

Independent Variable	Average Beta	Min Beta	Max Beta	Ave. p value	% Significant	Ave. R^2	# Regs.
Short Rate (3M)	1.02	0.79	1.24	0.00	1.00	0.86	288
Exp. Inflation (CE)	0.08	-0.12	0.20	0.18	0.44	0.86	96
Oil Spot Price	-0.20	-0.32	-0.06	0.01	0.94	0.86	96
% Odds on Deflation (CE)	-0.06	-0.18	0.15	0.24	0.43	0.86	96
Exp. GDP Growth (CE)	0.29	0.19	0.40	0.00	1.00	0.87	96
Exp. Unemployment (CE)	0.37	0.16	0.58	0.00	1.00	0.86	96
% Odds on < 0 GDP Growth (CE)	-0.10	-0.25	0.13	0.25	0.40	0.85	96
Exp. Budget Surplus/GDP (CE)	0.04	-0.42	0.49	0.25	0.56	0.86	144
Foreign Custody Holdings	-0.07	-0.29	0.22	0.20	0.51	0.86	144
Std. Dev. GDP Growth (CE)	0.29	0.00	0.46	0.03	0.92	0.87	144
Local Equity Market GARCH Return Vol.	0.04	-0.17	0.22	0.21	0.42	0.85	144
Std. Dev. Inflation (CE)	0.14	-0.06	0.32	0.14	0.60	0.84	72
Rate Survey Std. Dev. (CE)	0.14	-0.01	0.28	0.10	0.68	0.85	72
GARCH #-year Return Vol.	0.34	0.16	0.49	0.00	1.00	0.87	72
GARCH 1-year Return Vol.	0.35	0.16	0.54	0.00	1.00	0.87	72
M-GARCH A-DCC	0.38	0.25	0.45	0.00	1.00	0.86	144
M-GARCH A-DC Beta	0.33	0.22	0.43	0.00	1.00	0.85	144

Table 2

Bi-variate M-GARCH Summary Statistics (U.S Treasuries w.r.t. S&P 500)

07/03/1961-09/19/2019

Cappiello et al. (2006)

			Mean Before	Mean After	Sample Std.	Std. Dev.	Std. Dev. After	Estimate (03-	Estimate (19-
Asset	M-GARCH Stat.	Sample Mean	(1999)	(1999)	Dev.	Before (1999)	(1999)	Jul-1961)	Sep-2019)
2Y UST	A-DCC Coef.	0.05	0.19	-0.21	0.25	0.04	0.18	-0.36	-0.33
2Y Ave. E {r}	A-DCC Coef.	0.06	0.13	-0.07	0.18	0.18	0.21	0.06	-0.14
2Y Term Prem.	A-DCC Coef.	0.00	0.12	-0.20	0.23	0.12	0.19	-0.39	-0.43
5Y UST	A-DCC Coef.	0.06	0.21	-0.23	0.29	0.07	0.21	-0.26	-0.38
5Y Ave. E {r}	A-DCC Coef.	0.05	0.13	-0.10	0.20	0.18	0.22	0.04	-0.18
5Y Term Prem.	A-DCC Coef.	0.03	0.15	-0.19	0.26	0.01	0.22	-0.24	-0.45
10Y UST	A-DCC Coef.	0.06	0.22	-0.23	0.31	0.04	0.23	-0.29	-0.44
10Y Ave. E{r}	A-DCC Coef.	0.05	0.14	-0.11	0.21	0.19	0.22	0.04	-0.20
10Y Term Prem.	A-DCC Coef.	0.04	0.15	-0.16	0.24	0.08	0.22	-0.24	-0.38
Gold	A-DCC Coef.	-0.02	-0.02	<u>-0.02</u>	0.07	0.16	0.07	-0.22	<u>-0.07</u>
2Y UST	A-DC Beta	0.06	0.15	-0.10	0.18	0.03	0.10	-0.16	-0.27
2Y Ave. E {r}	A-DC Beta	0.06	0.11	-0.03	0.13	0.10	0.09	0.02	-0.06
2Y Term Prem.	A-DC Beta	0.01	0.06	-0.07	0.10	0.09	0.07	-0.17	-0.25
5Y UST	A-DC Beta	0.06	0.17	-0.13	0.21	0.13	0.13	-0.08	-0.32
5Y Ave. E {r}	A-DC Beta	0.05	0.10	-0.04	0.13	0.10	0.10	0.01	-0.10
5Y Term Prem.	A-DC Beta	0.02	0.08	-0.09	0.14	0.00	0.11	-0.08	-0.25
10Y UST	A-DC Beta	0.06	0.18	-0.13	0.23	0.10	0.15	-0.10	-0.35
10Y Ave. E{r}	A-DC Beta	0.04	0.08	-0.04	0.11	0.09	0.08	0.01	-0.09
10Y Term Prem.	A-DC Beta	0.03	0.10	-0.10	0.17	0.04	0.15	-0.11	-0.26
Gold	A-DC Beta	<u>-0.02</u>	-0.02	<u>-0.02</u>	0.10	0.03	0.09	<u>-0.04</u>	<u>-0.08</u>
2Y UST	GARCH Vol. %(A)	8.29	8.80	7.37	4.12	4.36	2.97	3.37	8.84
2Y Ave. E {r}	GARCH Vol. %(A)	8.03	8.94	6.39	4.49	4.18	2.64	3.11	5.15
2Y Term Prem.	GARCH Vol. %(A)	5.71	6.11	5.00	2.97	0.03	1.25	3.41	6.18
5Y UST	GARCH Vol. %(A)	8.54	8.48	8.66	3.85	6.04	2.62	2.44	9.21
5Y Ave. E{r}	GARCH Vol. %(A)	7.58	8.24	6.40	4.00	4.36	2.45	2.63	5.77
5Y Term Prem.	GARCH Vol. %(A)	6.15	6.18	6.09	2.72	1.91	1.82	2.72	6.09
10Y UST	GARCH Vol. %(A)	8.61	8.47	8.86	3.70	5.19	2.45	2.70	8.73
10Y Ave. E{r}	GARCH Vol. %(A)	6.17	6.65	5.30	3.22	3.63	1.99	2.09	4.92
10Y Term Prem.	GARCH Vol. %(A)	8.36	8.60	7.94	4.09	2.31	2.62	3.42	7.33
Gold	GARCH Vol. %(A)	14.12	13.65	14.96	8.40	5.23	5.18	1.46	12.80

Table 3

Spectral Decompositions (Variance)

Share of Frequency Cycles

HIGH (MED.) [LOW] frequency defined as 1 cycle every <=5(5<=21)[>21] days.

07/03/1961-09/19/2019

Frequency Maximum Start End Asset Band Mean Minimium S&P 500 HIGH 0.56 0.38 0.76 0.55 0.62 2Y UST HIGH 0.53 0.36 0.71 0.53 0.62 0.540.34 0.70 2Y Ave. $E\{r\}$ HIGH 0.620.64 0.49 0.74 2Y Term Prem. HIGH 0.63 0.66 0.66 5Y UST HIGH 0.54 0.41 0.67 0.47 0.61 5Y Ave. $E\{r\}$ HIGH 0.55 0.35 0.62 0.67 0.65 5Y Term Prem. HIGH 0.58 0.42 0.69 0.66 0.60 10Y UST 0.56 0.58 HIGH 0.46 0.67 0.63 10Y Ave. $E\{r\}$ 0.55 0.36 0.65 0.62 HIGH 0.67 10Y Term Prem. 0.580.41 0.71 0.57 HIGH 0.69 Gold HIGH 0.62 0.47 0.70 0.60 0.59 S&P 500 MEDIUM 0.34 0.18 0.49 0.35 0.30 2Y UST 0.23 0.29 **MEDIUM** 0.34 0.44 0.32 2Y Ave. $E\{r\}$ **MEDIUM** 0.33 0.22 0.46 0.28 0.29 2Y Term Prem. 0.27 MEDIUM 0.30 0.21 0.42 0.26 5Y UST **MEDIUM** 0.34 0.25 0.43 0.37 0.29 5Y Ave. $E\{r\}$ 0.23 0.27 **MEDIUM** 0.33 0.47 0.31 5Y Term Prem. **MEDIUM** 0.33 0.24 0.44 0.26 0.30 10Y UST **MEDIUM** 0.26 0.29 0.30 0.34 0.43 10Y Ave. $E\{r\}$ **MEDIUM** 0.33 0.24 0.47 0.27 0.31 10Y Term Prem. 0.23 0.25 **MEDIUM** 0.34 0.44 0.33 Gold **MEDIUM** 0.29 0.21 0.41 0.31 0.31 S&P 500 LOW 0.09 0.04 0.20 0.10 0.08 2Y UST LOW 0.13 0.05 0.30 0.15 0.09 2Y Ave. $E\{r\}$ LOW 0.13 0.05 0.28 0.10 0.08 2Y Term Prem. LOW 0.07 0.03 0.16 0.08 0.07 5Y UST LOW 0.12 0.06 0.21 0.16 0.10 5Y Ave. $E\{r\}$ LOW 0.12 0.06 0.23 0.08 0.07 5Y Term Prem. LOW 0.09 0.05 0.22 0.07 0.11 10Y UST LOW 0.10 0.06 0.19 0.08 0.12 10Y Ave. $E\{r\}$ LOW 0.11 0.06 0.22 0.08 0.07 10Y Term Prem. LOW 0.08 0.04 0.22 0.11 0.06

0.05

0.16

0.09

0.10

Gold

LOW

0.09

Table 4

Spectral Decompositions (Covariance with S&P 500)

Share of Frequency Cycles

HIGH (MED.) [LOW] frequency defined as 1 cycle every <=5(5<=21)[>21] days.

07/03/1961-09/19/2019

Frequency Band Maximum Start End Asset Mean Minimium S&P 500 HIGH 0.56 0.38 0.76 0.55 0.62 0.00 0.97 2Y UST HIGH 0.49 0.23 0.61 2Y Ave. $E\{r\}$ HIGH 0.47 0.00 0.22 0.68 1.00 2Y Term Prem. 0.52 0.00 0.99 0.21 0.55 HIGH 5Y UST HIGH 0.50 0.00 0.96 0.20 0.61 5Y Ave. $E\{r\}$ HIGH 0.47 0.00 0.15 0.65 1.00 5Y Term Prem. HIGH 0.51 0.000.95 0.63 0.57 10Y UST 0.50 0.59 HIGH 0.000.90 0.44 10Y Ave. $E\{r\}$ 0.47 0.00 0.99 0.12 0.65 HIGH 10Y Term Prem. 0.50 0.00 0.99 0.50 0.55 HIGH Gold 0.00 0.82 HIGH 0.42 0.93 0.83 S&P 500 **MEDIUM** 0.34 0.18 0.49 0.35 0.30 2Y UST 0.97 0.32 **MEDIUM** 0.37 0.000.48 2Y Ave. $E\{r\}$ **MEDIUM** 0.38 0.000.96 0.43 0.28 2Y Term Prem. **MEDIUM** 0.34 0.000.97 0.35 0.35 5Y UST **MEDIUM** 0.38 0.000.86 0.57 0.32 5Y Ave. $E\{r\}$ 0.39 0.92 0.29 **MEDIUM** 0.000.47 5Y Term Prem. **MEDIUM** 0.36 0.000.96 0.02 0.35 10Y UST 0.38 0.92 0.52 0.33 MEDIUM 0.0010Y Ave. $E\{r\}$ **MEDIUM** 0.39 0.000.89 0.49 0.29 10Y Term Prem. 0.07 0.37 **MEDIUM** 0.38 0.000.95 Gold **MEDIUM** 0.42 0.00 1.00 0.13 0.03 S&P 500 LOW 0.09 0.04 0.20 0.10 0.08 2Y UST LOW 0.14 0.000.53 0.29 0.07 2Y Ave. $E\{r\}$ LOW 0.15 0.000.76 0.34 0.04 2Y Term Prem. LOW 0.14 0.00 0.99 0.44 0.10 5Y UST LOW 0.11 0.00 0.52 0.23 0.07 5Y Ave. $E\{r\}$ LOW 0.14 0.00 0.86 0.38 0.06 5Y Term Prem. LOW 0.13 0.00 0.98 0.36 0.08 10Y UST LOW 0.12 0.00 0.89 0.04 0.07 10Y Ave. $E\{r\}$ LOW 0.14 0.000.78 0.39 0.06 10Y Term Prem. LOW 0.12 0.00 0.93 0.43 0.08 Gold LOW 0.16 0.00 0.88 0.04 0.14

Table 5

OLS Betas by S&P 500 Return Percentiles
07/03/1961-09/19/2019

	Percentile	0 o D 500			ъ.	Fitted		
	(<=/>=	S&P 500	41 1	ъ.	Beta p	Return	DA0	01
Asset:	Pct.)	Return	Alpha	Beta	value	(@Pct.)	R^2	Obs.
2Y UST	0.5	-3.34%	-0.002	-0.169	0.008	0.37%	0.095	72
2Y UST	5	-1.52%	-0.004	-0.206	0.000	-0.10%	0.062	724
2Y UST	95	1.50%	0.003	-0.086	0.018	0.15%	0.008	724
<u>2Y UST</u>	<u>99.5</u>	<u>3.44%</u>	<u>0.004</u>	<u>-0.097</u>	<u>0.485</u>	0.06%	0.007	<u>72</u>
2Y Ave. $E\{r\}$	0.5	-3.34%	-0.002	-0.136	0.048	0.23%	0.055	72
2Y Ave. $E\{r\}$	5	-1.52%	-0.003	-0.149	0.000	-0.07%	0.040	724
2Y Ave. $E\{r\}$	95	1.50%	0.001	-0.026	0.422	0.11%	0.001	724
$2Y$ Ave. $E\{r\}$	<u>99.5</u>	<u>3.44%</u>	0.002	<u>-0.044</u>	0.723	<u>0.10%</u>	0.002	<u>72</u>
2Y Term Prem.	0.5	-3.34%	0.000	-0.033	0.132	0.14%	0.032	72
2Y Term Prem.	5	-1.52%	-0.001	-0.057	0.000	-0.02%	0.017	724
2Y Term Prem.	95	1.50%	0.001	-0.060	0.003	0.04%	0.012	724
2Y Term Prem.	<u>99.5</u>	<u>3.44%</u>	0.001	<u>-0.053</u>	0.342	<u>-0.04%</u>	0.013	<u>72</u>
10Y UST	0.5	-3.34%	0.007	0.024	0.677	0.63%	0.002	72
10Y UST	5	-1.52%	-0.002	-0.112	0.000	-0.01%	0.023	724
10Y UST	95	1.50%	0.003	-0.115	0.003	0.17%	0.012	724
<u>10Y UST</u>	<u>99.5</u>	<u>3.44%</u>	0.008	<u>-0.182</u>	0.207	<u>0.15%</u>	0.023	<u>72</u>
10Y Ave. $E\{r\}$	0.5	-3.34%	-0.001	-0.107	0.020	0.24%	0.075	72
10Y Ave. $E\{r\}$	5	-1.52%	-0.002	-0.122	0.000	-0.04%	0.050	724
10Y Ave. $E\{r\}$	95	1.50%	0.001	-0.030	0.206	0.07%	0.002	724
10 Y Ave. $E\{r\}$	<u>99.5</u>	<u>3.44%</u>	0.003	<u>-0.064</u>	0.473	0.09%	0.007	<u>72</u>
10Y Term Prem.	0.5	-3.34%	0.008	0.128	0.015	0.40%	0.082	72
10Y Term Prem.	5	-1.52%	0.000	0.009	0.717	0.03%	0.000	724
10Y Term Prem.	95	1.50%	0.002	-0.086	0.007	0.09%	0.010	724
10Y Term Prem.	<u>99.5</u>	3.44%	0.005	<u>-0.120</u>	0.272	0.06%	<u>0.017</u>	<u>72</u>
Gold	0.5	-3.34%	0.002	-0.041	0.686	0.29%	0.002	72
Gold	5	-1.52%	-0.001	-0.076	0.092	0.03%	0.004	724
Gold	95	1.50%	0.003	-0.129	0.044	0.08%	0.006	724
Gold	99.5	3.44%	0.012	-0.298	0.186	0.19%	0.025	72

Table 6

UST Conditional Quantile Regression-Based Return Distributions
Skew and Width Statistics: S&P 500 Return Scenarios
Sample End Date (09/16/2019)

Sample Start	Asset	(1): Bowley Skew Q95 S&P DOWN	(2): Bowley Skew Q95 UP	(3): (1)-(2)	(4): Q95:Q5 Width DOWN minus UP (bps)	(5): Bowley Skew Q75 S&P DOWN	(6): Bowley Skew Q75 UP	(7): (5)-(6)	(8): Q75:Q5 Width DOWN minus UP (bps)
7/3/1961	2Y UST	-0.10	0.13	-0.24	49	-0.07	0.16	-0.23	23
7/3/1961	2Y Ave. $E\{r\}$	-0.07	0.10	-0.17	46	0.00	0.03	-0.04	10
7/3/1961	2Y Term Prem.	-0.02	0.04	-0.07	18	-0.03	0.01	-0.04	6
7/3/1961	5Y UST	-0.04	0.03	-0.07	32	0.09	-0.08	0.17	27
7/3/1961	5Y Ave. E{r}	-0.04	0.07	-0.10	45	0.01	0.03	-0.02	11
7/3/1961	5Y Term Prem.	-0.12	0.06	-0.18	25	-0.03	-0.03	0.00	9
7/3/1961	10Y UST	-0.06	0.02	-0.08	23	0.04	-0.05	0.08	22
7/3/1961	10Y Ave. E{r}	-0.03	0.06	-0.09	36	0.00	0.06	-0.07	8
7/3/1961	10Y Term Prem.	-0.08	0.04	-0.12	20	0.00	-0.03	0.02	8
7/3/1961	Gold	<u>0.03</u>	<u>-0.01</u>	<u>0.04</u>	<u>79</u>	<u>0.32</u>	<u>-0.28</u>	<u>0.61</u>	<u>27</u>
1/4/1999	2Y UST	0.17	-0.13	0.30	4	0.10	-0.20	0.30	17
1/4/1999	2Y Ave. $E\{r\}$	0.31	-0.25	0.57	20	0.03	-0.03	0.06	6
1/4/1999	2Y Term Prem.	-0.08	0.06	-0.14	14	-0.11	0.02	-0.13	-3
1/4/1999	5Y UST	-0.22	0.10	-0.31	3	-0.29	0.27	-0.55	5
1/4/1999	5Y Ave. $E\{r\}$	0.20	-0.15	0.35	14	0.04	-0.01	0.06	9
1/4/1999	5Y Term Prem.	-0.24	0.10	-0.35	20	-0.10	0.00	-0.11	11
1/4/1999	10Y UST	-0.34	0.24	-0.58	21	-0.21	0.14	-0.35	4
1/4/1999	10Y Ave. E{r}	0.20	-0.14	0.35	9	0.01	0.04	-0.03	10
1/4/1999	10Y Term Prem.	-0.15	0.04	-0.19	29	-0.11	0.03	-0.14	7
1/4/1999	Gold	<u>-0.08</u>	<u>0.06</u>	<u>-0.14</u>	<u>65</u>	<u>0.05</u>	<u>-0.03</u>	<u>0.08</u>	<u>39</u>
9/7/2016	2Y UST	0.09	-0.07	0.16	43	-0.05	0.01	-0.07	15
9/7/2016	2Y Ave. $E\{r\}$	-0.06	0.10	-0.16	-21	-0.06	0.11	-0.17	6
9/7/2016	2Y Term Prem.	-0.05	-0.22	0.17	27	-0.27	0.11	-0.38	15
9/7/2016	5Y UST	-0.07	-0.01	-0.05	28	-0.24	0.29	-0.53	16
9/7/2016	5Y Ave. $E\{r\}$	-0.18	0.26	-0.44	-5	-0.33	0.98	-1.30	23
9/7/2016	5Y Term Prem.	-0.47	0.33	-0.80	56	-0.50	0.74	-1.24	26
9/7/2016	10Y UST	-0.38	0.27	-0.65	65	-0.51	0.91	-1.42	35
9/7/2016	10Y Ave. $E\{r\}$	-0.11	0.22	-0.33	-4	-0.17	0.50	-0.68	14
9/7/2016	10Y Term Prem.	-0.26	-0.01	-0.25	72	-0.07	-0.01	-0.05	22
9/7/2016	Gold	0.12	-0.11	0.23	80	0.10	-0.23	0.33	27