Forecasting stock market volatility and application to volatility timing portfolios

Dohyun Chun[1], Hoon Cho[2], and Doojin Ryu[3,*]

Highlights

- We forecast the U.S. stock market volatility and apply them to the asset allocation problem.
- Machine learning and model averaging methods are applied to integrate predictive information.
- High-dimensional models have higher accuracy compared to the standard volatility model.
- LASSO-based volatility timing portfolios show outstanding investment performance.

[*] Corresponding author. E-mail: sharpjin@skku.edu (D. Ryu)

[1] College of Government and Business, Yonsei University, Gangwon, Republic of Korea, ORCID: https://orcid.org/0000-0003-3031-4011

[2] College of Business, Korea Advanced Institute of Science and Technology, Seoul, Republic of Korea, ORCID: https://orcid.org/0000-0003-2322-320X

[3] Department of Economics, Sungkyunkwan University, Seoul, Republic of Korea, ORCID: https://orcid.org/0000-0002-0059-4887

Abstract

This study predicts stock market volatility and applies them to the standard problem in finance, namely, asset allocation. Based on machine learning and model averaging approaches, we integrate the drivers' predictive information to forecast market volatilities. Using various evaluation methods, we verify that those high-dimensional models have better predictive performance relative to the standard volatility models. Furthermore, we construct volatility timing portfolios and discover that portfolios based on high-dimensional models mostly yield higher Sharpe ratios compared with the market. Among others, the least absolute shrinkage and selection operator (LASSO) generates the most accurate forecasts, leading to outstanding investment performance, regardless of the forecasting horizon.

JEL classification: C52, C53, C55, G11, G17

Keywords: Asset allocation; Machine learning; Model averaging; Volatility forecasting; Volatility timing portfolio

1. Introduction

Volatility is an essential component that describes the properties of financial assets. As a result, better volatility specifications lead to more accurate asset valuations. As a measure of risk, volatility dynamics are related to investment and trading decisions, risk management, and portfolio allocations. Hence, accurate volatility forecasting is crucial to both researchers and market practitioners.

In this study, we forecast aggregate stock market volatility and utilize them to solve the asset allocation problem. In particular, we first collect a wide range of economic and financial indicators that are known to or are likely to affect financial volatility. Then, by integrating their predictive information through the high-dimensional models, we generate information-intensive forecasts for the short-, medium-, and long-term U.S. stock market volatilities (Christiansen, Schmeling, and Schrimpf, 2012; Dangl and Hailing, 2012; Nonejad, 2017; Vrontos, Galakis, and Vrontos, 2021). For the aggregate U.S. stock market volatility, daily, weekly, and monthly Standard & Poor (S&P) 500 realized volatilities (RVs) are estimated using intraday data. The evaluation of the prediction performance indicates that machine learning and model averaging methods perform significantly better than the benchmark heterogeneous autoregressive (HAR) model, during our sample period. Finally, we utilize the volatility forecasts to implement a dynamic asset allocation strategy, i.e., a volatility timing strategy (Fleming, Kirby, and Ostdiek, 2001, 2003). Because their performance relies on the accuracy of the volatility estimations, studies often assess the economic value of volatility forecasts using a volatility timing portfolio. In this study, we construct volatility targeting (Barroso and Santa-Clara, 2015; Moreira and Muir, 2017; Harvey, Hoyle, Korgaonkar, Rattray, Sargaison, Van Hemert, 2018; Cederburg, O'Doherty, Wang, and Yan, 2020; Mylnikov, 2021; Stamos and Zimmerer, 2021) and risk targeting (Bollerslev, Hood, Huss, and Pedersen, 2018) portfolios to evaluate the investment performance. We verify that volatility timing portfolios based on machine learning and model averaging methods enjoy significantly higher returns and Sharpe ratios than the market portfolio in general.

Each high-dimensional model has distinct statistical properties. Therefore, their prediction performance depends on the characteristics of the target and explanatory variables. To find an efficient volatility prediction method, we compare the out-of-sample performance of high-dimensional models. When only a small number of variables contain predictive information and they have a linear relationship with the target variable, an L_1-penalized regression has an advantage, because it identifies an efficient subset of the explanatory variables and simplifies the model. We present evidence supporting this hypothesis that due to sparsity and linearity of the predictor variables, the least absolute shrinkage and selection operator (LASSO), a representative L_1-penalized regression, provides the most accurate forecasts, leading to large economic gains for investors in terms of asset allocations. The volatility timing strategies based on the LASSO forecasts have the highest returns, Sharpe ratios, and certainty equivalent returns in most cases, providing at least 24% and at most 42% higher Sharpe ratios than the market portfolio.

This study contributes to the literature by producing statistically and economically superior volatility forecasts and utilizing them to construct efficient volatility timing portfolios. Recent studies by Wang, Ma, Wei, and Wu (2016), Liu, Tang, and Zhou, (2019), Jayawardena, Todorova, Li, and Su (2020), and Qu and Zhang (2021) also apply the volatility timing strategy to assess the direct economic value of the volatility prediction. Our study differs in that we collect a variety of financial volatility predictors and integrate them through appropriate high-dimensional approaches. Moreover, we predict daily, weekly, and monthly volatilities and construct corresponding portfolios to investigate the predictability and the investment performance with respect to forecasting horizons. Finally, we revisit various evaluation approaches for volatility prediction. To assess the statistical significance of the improvement in prediction accuracy, we apply the model confidence set of Hansen, Lunde, and Nason (2011), the hypothesis-testing frameworks of Clark and West (2007) and Giacomini and White (2006), and the out-of-sample R^2 following Campbell and Thompson (2007) and Welch and Goyal (2008). These approaches allow us to identify the efficient volatility prediction method.

This paper proceeds as follows. Section 2 introduces target and predictor variables. Section 3 describes the high-dimensional approaches used in this study. Section 4 evaluates the out-of-sample predictive performance of high-dimensional models. Section 5 shows the performance of volatility timing portfolios. Section 6 concludes the paper.

2. Target and predictor variables

2.1. Realized volatility estimation

In this study, we consider daily, weekly, and monthly S&P 500 RVs for the target variable, from January 2, 2004, to December 31, 2020 (4,311 days). The daily (i.e., open-to-open) RV can be decomposed into open-to-close and close-to-open RVs. First, for the open-to-close RV, we apply a pre-averaging RV estimator (PRV) of Jacod, Li, Mykland, and Podolskij (2009). Second, for the close-to-open RV, we use the square of the overnight return following Martens (2002) and Koopman, Jungbacker, and Hol (2005). In consequence, the daily RV is defined as

$$\widehat{RV}_{oo,n} = \widehat{RV}_{pre,n} + r_{co,n}^2, \tag{1}$$

where $\widehat{RV}_{pre,n}$ is the open-to-close PRV estimate, and $r_{co,n}$ is the close-to-open return on day n. Finally, daily (RV_d), weekly (RV_w), and monthly (RV_m) target RVs can be calculated as

$$RV_{d,n+1} = \widehat{RV}_{oo,n+1},\ RV_{w,n+1} = \frac{\sum_{i=n+1}^{n+5} \widehat{RV}_{oo,i}}{5}, \text{ and } RV_{m,n+1} = \frac{\sum_{i=n+1}^{n+20} \widehat{RV}_{oo,i}}{20}. \tag{2}$$

We consider the logarithm of volatilities following Andersen, Bollerslev, Diebold, and Ebens (2001), Christiansen, Schmeling, and Schrimpf (2012), and Paye (2012). Fig. 1 shows the daily S&P 500 returns and RV estimates during our sample period.

[Fig. 1 here]

2.2. Predictor variables

The predictor variables we used in this study can be categorized into six groups, described as follows.

(a) The U.S. stock market variables: Existing studies uncovered various stock market indicators to predict market volatility. The predictor variables related to stock characteristics are the open-to-close S&P 500 return (*SPX*), total daily dollar volume of the New York Stock Exchange (NYSE, *Totdvol*), S&P 500 ETF (SPX, VOO, IVV) trading volume (*SPXETFvol*), market aggregate bid-ask spread-based illiquidity (*BAS*), Amihud's (2002) price impact measure (*Amihud*), Nagel's (2012) short-term reversal factor (*STR*), an aggregate measure of earnings news (*Earnnews*), and the realized skewness (*Rskew*) and kurtosis (*Rkurt*) of equity returns.

(b) Index option-related variables: Informed traders often realize their information through options and futures markets. Therefore, the options and futures market indicators are widely used to predict stock market behaviors. We include an implied volatility index (*VIX*), the implied volatilities of at-the-money calls and puts (*ATMIV*), the put/call option volume ratios on U.S. equities (*PCR_EQT*) and the S&P 500 index (*PCR_SPX*), and the slope of the index option-implied volatility curve, calculated as the difference between the out-of-the-money (OTM) put and call option implied volatilities (*IVslope1*, *IVslope2*).

(c) Investor attention, sentiment, and economic uncertainty variables: Investors' irrational behavior can affect the stock market. We use variables that measure investor attention and sentiment, such as daily aggregate abnormal trading volumes (*Abtv*), a market-wide Google search volume index (*GSV*) using the keywords "stock market," "S&P 500," and "Nasdaq", the daily news sentiment index (*News*), the cross-sectional dispersion of firm returns (*Stdret*), the economic policy uncertainty (*EPU*) index, and an economic surprise index for the United States (*SURP*).

(d) Interest rate and financial condition variables: Many studies describe the countercyclicality of financial volatility. Regarding macroeconomic conditions, we consider various interest rate spreads, including the default spread (*DEF*), term spread (*TERM*), TED spread (*TED*), and high-yield bond spread (*HYBSpread*). Summarizing representative macro variables, the Goldman Sachs Financial Condition Index (*FINCON*) is used as a proxy for overall macroeconomic conditions.

(e) Global market variables: Global market indicators are essential predictors of U.S. stock volatilities, owing to financial globalization and overnight information. To capture the overseas market movements, we include the Hang Seng (*HSX*), Shanghai Composite (*SCX*), Nikkei 225 (*NIKKEI*), EURO STOXX 50 (*STOXX*), and FTSE100 (*UKX*) returns, the All Country World Index of Morgan Stanley Capital International (MSCI ACWI index, *ACWI*), as well as PRV estimators for the Hang Seng (*HSXvol*), NIKKEI 225 (*NIKKEIvol*), and EURO STOXX 50 (*STOXXvol*). In addition, we consider various global economic indicators, including the U.S. dollar index (*Dollarind*), Emerging Market Bond Index Spread of J.P. Morgan (*EMBISpread*), and Citi Economic Surprise Indices for China (*SURP_CN*) and the European Union (EU, *SUPR_EU*) as proxies for global economic conditions.

(f) Commodity variables: A large strand of the literature shows that commodities have predictive power over stock markets. The last set of predictor variables is related to the price of commodities. This study considers the Commodity Research Bureau (CRB) commodity index (*CRBCMDT*), gold futures price (*Gold*), WTI futures price (*WTI*), and oil ETF implied volatility index (*OilVol*) as predictor variables.

Table 1 summarizes the descriptions and sources of the variables.

[Table 1 here]

3. High-dimensional models

3.1. Machine learning models

Machine learning has demonstrated its potential to improve the measurement of risk premium, which is basically a prediction problem in finance (Gu, Kelly, and Xiu, 2020). Machine learning simply identifies the best functional relationships between target and predictor variables based on statistical properties. Collaborated with economists' intuition, recently, machine learning is widely adopted for equilibrium asset pricing and other prediction studies in the field of financial economics (see Chen, Pegler, and Zhu (2019), Bryzgalova, Pegler, and Zhu (2020), Feng, Giglio, and Xiu (2020), Gu, Kelly, and Xiu (2021), and Kim, Cho, and Ryu (2021a, 2021b, 2022), for example). Machine learning models also play an important role in this study because they are used to integrate the drivers' predictive information and predict aggregate stock market volatility.

In this subsection, we describe three machine learning models, the first of which is the LASSO (Tibshirani, 1996). The LASSO is a representative penalized regression that uses L_1-penalization for coefficient shrinkage and variable selection. The LASSO minimizes estimation error by considering the penalty parameter v:

$$a^s, \boldsymbol{b}^s = \underset{\alpha,\boldsymbol{\beta}}{\operatorname{argmin}} \frac{1}{N} \sum_{n=1}^{N} \{(y_{n+1} - \alpha - \boldsymbol{X_n\beta})^2 + \nu|\boldsymbol{\beta}|\}, \tag{3}$$

where y is a target variable, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_R)^{\mathrm{T}}$ and $\boldsymbol{X}_n = (X_{1,n}, X_{2,n}, \ldots, X_{R,n})$ are vectors of parameters and explanatory variables, respectively, the superscript T denotes the transpose, and R is the number of predictors. The penalty parameter v determines the intensity of coefficient shrinkage and variable selection. Moreover, we consider a ridge regression (Hoerl and Kennard, 1970), which is a penalized regression that uses an L_2-penalization. In contrast to the LASSO, the ridge regression mitigates multicollinearity with parameter shrinkage. The ridge regression minimizes the following equation:

$$a^s, \boldsymbol{b}^s = \underset{\alpha,\boldsymbol{\beta}}{\operatorname{argmin}} \frac{1}{N} \sum_{n=1}^{N} \{(y_{n+1} - \alpha - \boldsymbol{X_n}\boldsymbol{\beta})^2 + \nu |\boldsymbol{\beta}|^2\}. \quad (4)$$

Again, v determines the regularization strength in the ridge regression. For the LASSO and ridge regression, the optimal regulation intensity v is a hyper-parameter to determine.

However, penalized linear regression models cannot account for nonlinear and interactive relationships between predictor variables. Here, regression trees are representative machine learning approaches that can deal with these characteristics. A tree is designed along a recursive partitioning to pick and split the predictor variables to be as homogeneous as possible in relation to the target variable. This sequential branching approximates the functional relationship within each partition. To avoid overfitting, trees are often regularized using boosting. One of the most popular supervised machine learning algorithms, the gradient boosting regression tree (GBRT), applies the gradient boosting algorithm to a regression tree (Friedman, 2001). The GBRT generates optimal forecasts by combining forecasts from a variety of trees. Based on variance reduction, it provides feature importance scores for each predictor variable. For the GBRT, we should determine the maximum depth and the number of boosting stages through the validation procedure.

3.2. Model averaging methods

The usual model selection approaches often suffer from parameter instability and model uncertainty. To use the information contained in all candidate models, researchers take an average over all possible models, applying the weight vector that maximizes the information efficiency (Steel, 2020). In the Bayesian model averaging (BMA) framework, the optimal weight vector is derived based on Bayesian inference (Hoeting, Madigan, Raftery, and Volinsky, 1999; Koop, 2003). Specifically, models for all possible combinations of the explanatory variables are estimated. Then, their weighted average is calculated, where the optimal weights correspond to the posterior probability. For instance, the optimal expectation of the target variable y is calculated as

$$E(y|\boldsymbol{X}) = \sum_{j=1}^{J} p\left(M^j \middle| y, \boldsymbol{X}\right) \times y|M^j, \boldsymbol{X} \text{ where } p\left(M^j \middle| y, \boldsymbol{X}\right) = \frac{p\left(y \middle| M^j, \boldsymbol{X}\right) p(M^j)}{\sum_{i=1}^{J} p\left(y \middle| M^i, \boldsymbol{X}\right) p(M^i)}. \tag{5}$$

Here, M^j denotes a forecast model j, J is the total number of candidate models, and $p\left(M^j \middle| y, \boldsymbol{X}\right)$ denotes the model's posterior probability. Christiansen, Schmeling, and Schrimpf (2012), Dangl and Hailing (2012), Wang, Ma, Wei, and Wu (2016), and Nonejad (2017) apply the BMA to forecast stock market volatility, and find that integrating information using the BMA improves the out-of-sample performance. Instead of obtaining a weighted average, one can simply select the forecast combination with the highest posterior probability, called Bayesian model selection (BMS). Considering the large dimensional set of variables, we employ the birth-death Markov chain Monte Carlo (MCMC) sampler. As we do not have enough prior knowledge, we assume a uniform model prior, and assign unit information prior to Zellner's g-prior. We confirm that the empirical results are robust to the MCMC sampler and prior settings. The posterior inclusion probability of each predictor represents the sum of the probability that the predictor is included in each candidate model, which can be used as a proxy for the variable importance in a BMA inference.

In contrast to Bayesian averaging frameworks, frequentist methods do not rely on the priors of the parameter values, and focus mainly on the asymptotic behavior of the estimators. From a linear regression perspective, frequentist model averaging (FMA) estimators can be expressed as

$$\boldsymbol{b}^{FMA} = \sum_{j=1}^{J} \omega^j \, \boldsymbol{b}^j, \tag{6}$$

where $\boldsymbol{b}^j$ and ω^j are an estimator and a weight, respectively, corresponding to the candidate model j. Based on information criteria, Buckland, Burnham, and Augustin (1997) suggest optimal weights as

$$\omega^j = \frac{exp\left(-\frac{\mathcal{F}^j}{2}\right)}{\sum_{i=1}^{J} exp\left(-\frac{\mathcal{F}^i}{2}\right)}, \tag{7}$$

where $\mathcal{F}^j$ is an information criterion for model j. Hansen (2007) introduces a least-squares model averaging approach that minimizes the Mallows criterion, namely, Mallows model averaging (MMA). Consider the homoscedastic regression model given by

$$y_n = \sum_{j=1}^{\infty} \beta^j X_{j,n} + \epsilon_{0,n}, \tag{8}$$

where $E(\epsilon_{0,n}|\boldsymbol{X}_n) = 0$, $E(\epsilon_{0,n}^2|\boldsymbol{X}_n) = \sigma^2$, and $\boldsymbol{X}_n = (X_{1,n}, X_{2,n}, \ldots)$. Assume that the first κ^j variables are used in the model j and $0 < \kappa^1 < \kappa^2 < \cdots$. Then, the model j can be written as

$$y_n = \sum_{j=1}^{\kappa^j} \beta^j X_{j,n} + \epsilon_n^j \text{ or } \boldsymbol{Y} = \boldsymbol{X}^j \boldsymbol{\beta}^j + \boldsymbol{\epsilon}^j, \tag{9}$$

where $\boldsymbol{Y}$ is a $N \times 1$ target vector, $\boldsymbol{X}^j$ is a $N \times \kappa^j$ matrix, and $\boldsymbol{\beta}^j$ is a $\kappa^j \times 1$ parameter vector. Then, the optimal weight of the MMA can be obtained as

$$\widehat{\boldsymbol{\omega}}^{\text{MMA}} = \underset{\boldsymbol{\omega}}{argmin}\{(\boldsymbol{Y} - \boldsymbol{Xb})^{\text{T}}(\boldsymbol{Y} - \boldsymbol{Xb}) + 2\sigma^2\Phi(\boldsymbol{\omega})\}, \tag{10}$$

where $\boldsymbol{b} = \sum_{j=1}^{J} \omega^j \begin{pmatrix} \boldsymbol{b}^j \\ 0 \end{pmatrix}$, $\Phi(\boldsymbol{\omega}) = \text{tr}(\Psi(\boldsymbol{\omega}))$, $\text{tr}(\cdot)$ is the trace of a matrix, $\Psi(\boldsymbol{\omega}) = \sum_{j=1}^{R} \omega^j \Psi^j$, and $\Psi^j = \boldsymbol{X}^j(\boldsymbol{X}^{j,\text{T}}\boldsymbol{X}^j)^{-1}\boldsymbol{X}^{j,\text{T}}$. Generalizing the MMA, Hansen and Racine (2012) propose jackknife model averaging (JMA) for non-nested and heteroscedastic models. Under the JMA framework, optimal weights can be obtained as

$$\widehat{\boldsymbol{\omega}}^{\text{JMA}} = \underset{\boldsymbol{\omega}}{argmin}\left\{(\boldsymbol{Y} - \widetilde{\boldsymbol{\mu}}(\boldsymbol{\omega}))^{\text{T}}(\boldsymbol{Y} - \widetilde{\boldsymbol{\mu}}(\boldsymbol{\omega}))\right\}, \tag{11}$$

where $\widetilde{\boldsymbol{\mu}}(\boldsymbol{\omega}) = \sum_{j=1}^{J} \omega^j \widetilde{\boldsymbol{\mu}}^j$, $\widetilde{\boldsymbol{\mu}}^j = (\tilde{\mu}_1^j, \tilde{\mu}_2^j, \ldots, \tilde{\mu}_N^j)^{\text{T}}$, $\tilde{\mu}_n^j = \boldsymbol{X}_n^j(\boldsymbol{X}_{-n}^{j,\text{T}}\boldsymbol{X}_{-n}^j)^{-1}\boldsymbol{X}_{-n}^{j,\text{T}}\boldsymbol{Y}_n$, and $\boldsymbol{X}_{-n}$ represents the matrix without the n^{th} row. Using $\widehat{\boldsymbol{\omega}}^{\text{MMA}}$ and $\widehat{\boldsymbol{\omega}}^{\text{JMA}}$, we obtain optimal estimators and forecasts across all candidate models.

4. Predictive performance of forecasting models

4.1. **Out-of-sample prediction and evaluation**

From each high-dimensional model s, we can obtain the daily, weekly, and monthly RV forecasts as follows:

$$\text{High-dimensional model } s \text{ prediction: } \widehat{RV}_{s,n+1} = a^s + \boldsymbol{X}_n \boldsymbol{b}^s. \tag{12}$$

Model coefficients are recursively estimated from 500-day moving windows, which are split into 400 days of the training set and 100 days of the validation set. We conduct out-of-sample prediction from January 3, 2007, to December 31, 2020 (N = 3,498).

For the statistical evaluation of the out-of-sample predictive performance, we consider four statistics based on mean squared prediction errors (MSPEs). The first is the model confidence set (MCS) procedure proposed by Hansen, Lunde, and Nason (2011). Given a specific loss function, the MCS demonstrates a statistically superior subset of competing forecasts. The MCS procedure allows us to test whether a single forecasting model dominates its competitors based on their out-of-sample performance. For example, Wang, Ma, Wei, and Wu (2016) compare various volatility forecasting models using the MCS procedure, and show that dynamic model averaging generates better forecasts. Samuels and Sekkel (2017), Amendola, Braione, Candila, and Storti (2020), and Chiang, Liao, and Zhou (2021) identify the best subset of the forecasts and produce superior forecasts using the MCS procedure. Following these studies, we perform a block bootstrap procedure of 5,000 resamples.

To test the Granger causality and finite-sample prediction performance, we apply the frameworks of Clark and West (2007; CW) and Giacomini and White (2006; GW), respectively. The CW and GW statistics test different aspects of the equal predictive accuracy of the model: the CW statistic focuses on the Granger causality, whereas the GW statistic deals with the finite-sample performance. The detailed evaluation procedures are as follows. Let $MSPE_s$ be the MSPE of the model s forecasts and $\widehat{RV}_{BM,n}$ and $MSPE_{BM}$ be the fitted RV and MSPE, respectively, from the benchmark model. Then, the difference between $MSPE_s$ and $MSPE_{BM}$ is the finite-sample prediction accuracy of the HARX model s relative to the benchmark. Thus, the GW statistic tests the null hypothesis H_0: $E[MSPE_{BM} - MSPE_s] = 0$, as follows:[4]

$$GW_s = \frac{\bar{\xi}_s}{\sqrt{Var(\xi_{s,n})/N}},\ \ \bar{\xi}_s = \frac{1}{N}\sum_{n=1}^{N}\xi_{s,n},\ \ \xi_{s,n} = \left(RV_n - \widehat{RV}_{BM,n}\right)^2 - \left(RV_n - \widehat{RV}_{s,n}\right)^2. \tag{13}$$

Intuitively, the GW statistic evaluates the finite-sample improvement in terms of the prediction error. It has a negative value when the cost (i.e., prediction error) of incorporating additional predictors outweighs the benefit (i.e., prediction accuracy). In contrast, the CW statistic tests the large-sample prediction performance by adjusting the parameter estimation error. The CW statistic is calculated as follows:

$$\begin{aligned} CW_s &= \frac{\bar{\xi}_{adj,s}}{\sqrt{\mathrm{Var}(\xi_{adj,s,n})/N}}, \\ \bar{\xi}_{adj,s} &= \frac{1}{N}\sum_{n=1}^{N}\xi_{adj,s,n} = MSPE_{BM} - MSPE_s + \frac{1}{N}\sum_{n=1}^{N}\left(\widehat{RV}_{BM,n} - \widehat{RV}_{s,n}\right)^2, \\ \xi_{adj,s,n} &= \left(RV_n - \widehat{RV}_{BM,n}\right)^2 - \left[\left(RV_n - \widehat{RV}_{s,n}\right)^2 - \left(\widehat{RV}_{BM,n} - \widehat{RV}_{s,n}\right)^2\right]. \end{aligned} \tag{14}$$

Note that the term $\frac{1}{N}\sum_{n=1}^{N}\left(\widehat{RV}_{BM,n} - \widehat{RV}_{s,n}\right)^2$ in $\bar{\xi}_{adj,s}$ adjusts the incremental variance from the parameter estimation. Note that if the explanatory variable Granger causes the target variable, but does not improve the prediction accuracy, the CW test will be rejected, whereas

[4] We apply a Newey–West heteroscedasticity and autocorrelation-adjusted estimator for the variance of the statistics.

the GW test will not.

Finally, we consider the out-of-sample R^2, following Welch and Goyal (2008) and Campbell and Thompson (2007). The out-of-sample R^2 of the model (R^2_{oos}) and the increase in R^2_{oos} compared with the benchmark model (ΔR^2_{oos}) are calculated as

$$R^2_{oos} = 1 - \frac{MSPE_S}{MSPE_{HM}},\ \Delta R^2_{oos} = \left(1 - \frac{MSPE_S}{MSPE_{HM}}\right) - \left(1 - \frac{MSPE_{BM}}{MSPE_{HM}}\right) = \frac{MSPE_{BM} - MSPE_S}{MSPE_{HM}}, \tag{15}$$

where $MSPE_{HM}$ denotes the MSPE of the historical mean model. The ΔR^2_{oos} value captures the relative prediction error, scaled by the MSPE of the historical mean model.

4.2. Prediction results

In this subsection, we assess the out-of-sample predictive performance of the high-dimensional models relative to the benchmark forecasting. As a benchmark, we consider a HARSJ model, which incorporates exogenous regressors, the signed jump variation, and the negative semi-variance in the HAR model of Corsi (2009). Patton and Sheppard (2015) define a signed jump variation as

$$Jump_n = SV_{pos,n} - SV_{neg,n}, \tag{16}$$

where $SV_{pos,n} = \sum_{k=1}^{K} ir_{n,k}^2 1_{[ir_{n,k} \geq 0]}$ and $SV_{neg,n} = \sum_{k=1}^{K} ir_{n,k}^2 1_{[ir_{n,k} < 0]}$ denote positive and negative semi-variances, respectively, while $ir_{n,k}$ is the k^{th} five-minute return on day n, K stands for the total number of five-minute returns on day n, and $1_{[\cdot]}$ is an indicator function. Then, the HARSJ model is represented as

$$RV_{n+1} = \alpha + \vartheta_d RV_{d,n} + \vartheta_w RV_{w,n} + \vartheta_m RV_{m,n} + \vartheta_{nj} SV_{neg,n} + \vartheta_j Jump_n + \epsilon_{n+1}, \tag{17}$$

where $RV_{d,n}$, $RV_{w,n}$, and $RV_{m,n}$ are daily, weekly, and monthly historical RVs, respectively.[5]

Additionally, we calculate a simple combination of individual forecasts based on the HARSJ model. The HARSJ-X model, which incorporates the exogenous regressor $X_{r,n}$ in the HARSJ model, can be described as follows:

$$RV_{r,n+1} = \alpha + \vartheta_d RV_{d,n} + \vartheta_w RV_{w,n} + \vartheta_m RV_{m,n} + \vartheta_{nj} SV_{neg,n} + \vartheta_j Jump_n + \beta_r X_{r,n} + \epsilon_{r,n+1}. \tag{18}$$

Given the individual HARSJ-X forecasts $\widehat{RV}_{r,n+1}$, the simple forecast combinations are calculated as

$$\widehat{RV}_{c,n+1} = \sum_{r=1}^{R} \delta_{r,n+1} \widehat{RV}_{r,n+1}, \tag{19}$$

where $\widehat{RV}_{r,n+1}$ denotes the forecast from HARX model *r*. A mean forecast combination (*MFC*) is an equal-weighted average of forecasts, obtained by setting $\delta_r = \frac{1}{R}$. The discounted mean squared prediction error (*DMSPE*) uses the inverse of the historical MSPE for the weights, given by

$$\delta_{r,n+1} = \frac{\left(\widehat{RV}_{r,n} - RV_n\right)^{-2}}{\sum_{i=1}^{R} \left(\widehat{RV}_{i,n} - RV_n\right)^{-2}}. \tag{20}$$

5 We consider various extensions of the HAR model as benchmarks, and find that our HARSJ model has higher predictive power for the U.S. stock market realized volatilities than other models do. Nevertheless, the main findings of this study are robust to the benchmarks.

Table 2 reports the MSPE and mean absolute percentage error (MAPE), as well as GW and CW test results and the ΔR^2_{oos} value, relative to the benchmark HARSJ model. The results demonstrate the higher predictability and accuracy of models that are based on machine learning and model averaging methods, compared to the benchmark HARSJ model. The improvement in prediction accuracy, captured by CW, GW, and ΔR^2_{oos}, are statistically significant in most cases. Specifically, the LASSO generates the most accurate forecasts among others, regardless of the forecasting horizon. The LASSO forecasts have the largest ΔR^2_{oos} and the smallest MSPE and MAPE, and belong only to the 1% MCS for all forecasting horizons.

[Table 2 here]

Each model has distinct properties in dealing with the high-dimensional set of variables. Therefore, their prediction performance depends on the characteristics of the target and explanatory variables. We suppose that the LASSO outperforms the others in terms of out-of-sample forecasting because of the sparsity and linearity of the predictor variables. In terms of sparsity, Chun, Cho, and Ryu (2022) demonstrate that only a few volatility drivers have significant out-of-sample predictability beyond the historical volatilities. Moreover, to investigate the linearity between target and predictor variables, we identify significant predictors of each high-dimensional model. Table 3 displays key predictor variables for out-of-sample prediction of high-dimensional models, sorted by corresponding importance measures.[6] We discover that common key predictor variables such as the Hang Seng return (*HSX*) and volatility (*HSXvol*), bid-ask spread (*BAS*), implied volatility indices (*VIX*, *ATMIV*), high-yield bond spread (*HYBSpread*), and Google search volume index (*GSV*) have a theoretical linear

[6] Please see Appendix A for variance importnace measures in detail.

relationship with stock volatility. This is in line with the results of Chun, Cho, and Ryu (2022), which are based on the out-of-sample LASSO prediction. Fig. 2 shows scatter plots between these variables and the one-day-ahead daily S&P 500 RV. Here, we find that not only the volatility measures but also the other variables have a linear relationship with the target variable.

[Table 3 here]

[Fig. 2 here]

If only a small number of predictors improve the predictive ability, and the predictors are linearly correlated with the target variable, the parsimonious linear model is likely to perform well in out-of-sample forecasting, owing to the bias-variance tradeoff. In particular, the prediction bias decreases with the number of explanatory variables as the model's explanatory power increases. However, in finite-sample predictions, additional predictors increase the parameter estimation error. Accordingly, adding extra predictors has both benefits (i.e., bias reduction) and costs (i.e., an increase in the prediction variance), leading to the bias-variance tradeoff for the finite-sample prediction. The LASSO identifies an efficient subset of the explanatory variables and simplifies the model. This reduced model has statistical advantages, owing to its stability and addressable estimates. Accordingly, the LASSO has an advantage in using a sparse set of variables that are linearly correlated with the target variable.

5. Volatility timing strategies

Considering that only pure statistical rules are applied in high-dimensional approaches, their economic meanings and implications remain debatable. For financial applications, we apply the volatility timing strategy and assess the direct economic value of the volatility prediction.

In particular, we solve the asset allocation problem of an investor who determines the optimal weighting of a market portfolio and a risk-free asset based on market volatility forecasts. Let w_t^p be the portion of an investment in the market portfolio on day *t*. Then, the portion of the investment in the risk-free asset is given by $1 - w_t^p$, and the portfolio return on day *t* ($r_{p,t}$) is expressed as

$$r_{p,t} = w_t^p r_{m,t} + \left(1 - w_t^p\right) r_{f,t}, \tag{21}$$

where $r_{m,t}$ and $r_{f,t}$ are the market portfolio return and the risk-free rate, respectively. In this study, we regard the open-to-open S&P 500 return and the three-month T-bill rate as the market portfolio return and the risk-free rate, respectively.[7]

The optimal weights of the volatility targeting portfolios are calculated as follows:

$$w_t^{vt} = \frac{1}{\gamma} \frac{V_t}{\hat{\sigma}_{m,t}}, \tag{22}$$

where V_t is the target volatility, and $\hat{\sigma}_{m,t}$ is the fitted volatility of the stock market, conditional on the past information set. For the target volatility, we apply both the fixed target and the moving variance of the stock market return. In accordance with the prediction horizons, we construct daily, weekly, and monthly portfolios, as follows. An investor rebalances a daily portfolio based on the daily volatility forecasts. For the weekly portfolios, the investor predicts the weekly volatility and forms the portfolio on day *t*, holds the position for a week, and

[7] To assess the real investment performance, we skip 15 seconds after the market opens. Moreover, we restrict the range of w_t^p from -3.0 to 3.0, considering the investment in the index portfolio.

rebalances the portfolio again on day *t*+4. The monthly portfolios are constructed in the same way, with a 20-day holding period.

For portfolios based on the high-dimensional model, we use daily, weekly, and monthly volatility forecasts from the out-of-sample prediction in Section 4.2. In addition to the high-dimensional models, we consider the MFC and DMSPE introduced in Section 4.2. Moreover, two standard volatility models are adopted, the first of which is the HARSJ model, presented in Eq. (17). The second model is the GARCH (1,1) model of Bollerslev (1986):

$$r_t = \mu + \varepsilon_t^g,\ \varepsilon_t^g | I_{t-1} \sim N(0, h_t),\ h_t = \alpha^g + \beta^g h_{t-1} + \tau^g \left(\varepsilon_{t-1}^g\right)^2, \tag{23}$$

where h_t is the conditional volatility of the return, α^g, β^g, and τ^g are model parameters, and I_t denotes the information set. We estimate the daily, weekly, and monthly conditional volatilities using the HARSJ and GARCH models. As the benchmark, we consider a buy-and-hold market portfolio in which an investor buys a market portfolio at the beginning of the investment period, holds it without adjustment, and sells it at the end of the investment period. This can be obtained by setting $w_t^p = 1$ in Eq. (21).

For the performance measures, we employ the Sharpe ratio and the certainty equivalent returns (CERs). The Sharpe ratio captures the risk-return tradeoff by calculating the expected excess return in units of standard deviations. The CER is the risk-free return that is equivalent to holding a risky portfolio. A higher CER portfolio provides higher utility to an investor, considering their risk aversion. The Sharpe ratio and CER are computed as

$$Sharpe\ ratio = \frac{E\left[r_{p,t} - r_{f,t}\right]}{\sigma_{p,t}} = \frac{\frac{\sum_{t=1}^{T}\left(r_{p,t} - r_{f,t}\right)}{T}}{\sqrt{\frac{\sum_{t=1}^{T}\left(r_{p,t} - r_{f,t}\right)^2}{T-1}}}, \tag{24}$$

$$CER = E[r_{p,t}] - \frac{1}{2}\gamma\sigma_{p,t}^2 = \frac{1}{T}\sum_{t=1}^{T} r_{p,t} - \frac{1}{2}\gamma\frac{\sum_{t=1}^{T} r_{p,t}{}^2}{T-1},$$

respectively, where γ is the investor's risk aversion coefficient. We report the annualized Sharpe ratio and CER for γ=3.

Tables 4, 5, and 6 show the performance of the daily, weekly, and monthly volatility targeting portfolios, respectively. The first three columns represent the volatility targeting portfolios with fixed target volatility corresponding to an annualized volatility of 15%. The last three columns stand for those with the target volatilities corresponding to the one-year rolling standard deviations of the S&P 500 returns.[8] Overall, the volatility timing strategies tend to outperform the static market portfolio. Compared with the market portfolio, most of the volatility timing portfolios have higher average returns over the evaluation period. Their higher Sharpe ratios and CERs imply that they also generate higher risk-adjusted returns. In particular, among the volatility timing portfolios, those that time with the LASSO forecasts perform best, regardless of the strategies and forecasting horizons. They enjoy the highest average returns, Sharpe ratios, and CERs in most cases: at least 24%, and at most 42% higher Sharpe ratios than the market portfolio. Fig. 3 summarizes the findings, showing the cumulative log-returns of the monthly volatility targeting portfolios, ordered by the final cumulative returns in descending order. Here, we find that the volatility timing portfolios outperform the market portfolio in general, and the LASSO portfolios consistently have the highest cumulative returns over the 14-year testing sample.

[Table 4 here]

[8] We verify that the main results are robust to the target volatility. The results for other target volatilities are available upon request.

[Table 5 here]

[Table 6 here]

[Fig. 3 here]

The final strategy is the risk-targeting strategy (Bollerslev, Hood, Huss, and Pedersen, 2018). The utility maximization problem of a mean-variance investor is expressed as

$$max_{w_t^p} U\{E(r_{p,t+1}|I_t), \sigma_{p,t+1}\} = E(r_{p,t+1}|I_t) - \frac{1}{2}\gamma\sigma_{p,t+1}^2. \tag{25}$$

Solving the maximization problem, we obtain the optimal portfolio weight for the investor as follows:

$$w_t^{mv} = argmax_{w_t^p} \cdot U(r_{p,t+1}) = \frac{1}{\gamma}\frac{E(r_{m,t+1}|I_t) - r_{f,t+1}}{\sigma_{m,t+1}^2}. \tag{26}$$

Assuming a constant Sharpe ratio, we can express Eqs. (25) and (26) as follows (by omitting the constant):

$$U(r_{p,t+1}) = w_t^p \cdot \varrho \cdot \sigma_{m,t+1} - \frac{\gamma}{2}\left(w_t^p\right)^2 \sigma_{m,t+1}^2 \text{ and } w_t^{rt} = \frac{\varrho}{\gamma}\frac{1}{\hat{\sigma}_{m,t+1}^2}, \tag{27}$$

where $\varrho = \frac{E[r_{m,t+1} - r_{f,t+1}]}{\sigma_{m,t+1}}$. Then, the realized utility of the investor can be written as

$$Realized\ utility = \frac{\varrho^2}{\gamma}\frac{\sigma_{m,t+1}}{\hat{\sigma}_{m,t+1}} - \frac{\varrho^2}{2\gamma}\frac{\sigma^2_{m,t+1}}{\hat{\sigma}^2_{m,t+1}}. \tag{28}$$

In this study, we set ϱ=0.3 and γ=3. Note that the realized utility depends on the ratio of the true and the estimated volatilities, that is, the volatility prediction accuracy. Table 7 reports the realized utility of the risk-targeting strategies and the *t*-statistics for testing the significance of the realized utility difference, relative to the model with the highest utility. The results suggest that the LASSO delivers the highest utility among the studied models. In particular, the negative and significant *t*-statistics imply that the LASSO forecasts provide significantly higher utility than that of the other models.

[Table 7 here]

6. Conclusion

In this study, we forecast aggregate stock market volatility using high-dimensional models and apply them to the standard problem of asset allocation in finance. For the forecasting model, we adopt representative machine learning and model averaging methods to integrate predictive information contained in the wide range of predictors. The high-dimensional models play an important role in this study because they provide information-intensive forecasts corresponding to the time-varying market state. Using various performance measures, we find that the LASSO provides the most accurate forecasts within our framework. Furthermore, we construct daily, weekly, and monthly volatility timing portfolios based on volatility forecasts from each high-dimensional model. The asset allocation results suggest that a LASSO-based portfolio outperforms all others, regardless of the strategies and forecasting horizons. Notably, economic gain from the volatility timing portfolio can be regarded as the economic value of the volatility prediction. This economic evaluation procedure provides economic justification for the results of out-of-sample forecasting, as well as useful financial applications for accurate volatility prediction.

Compliance with Ethical Standards

Funding:

Conflict of Interests: Author Dohyun Chun declares that he has no conflict of interest. Author Hoon Cho declares that he has no conflict of interest.

There is no conflict of interest regarding the publication of this paper.

Reference

Amendola, A., Braione, M., Candila, V., Storti, G., 2020. A model confidence set approach to the combination of multivariate volatility forecasts. International Journal of Forecasting 36, 873-891.

Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. Journal of Financial Markets 5 (1), 31-56.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Ebens, H., 2001. The distribution of realized stock return volatility. Journal of Financial Economics 61 (1), 43-76.

Barroso, P., Santa-Clara, P., 2015. Momentum has its moments. Journal of Financial Economics 116 (1), 111-120.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31 (3), 307-327.

Bollerslev, T., Hood, B., Huss, J., Pedersen, L.H., 2018. Risk everywhere: Modeling and managing volatility. Review of Financial Studies 31 (7), 2729-2773.

Bryzgalova, S., Pelger, M., Zhu, J., 2020. Forest through the trees: Building cross-sections of stock returns. Available at SSRN 3493458.

Buckland, S.T., Burnham, K.P., Augustin, N.H., 1997. Model selection: an integral part of inference. Biometrics 53 (2), 603-618.

Campbell, J.Y., Thompson, S.B., 2007. Predicting excess stock returns out of sample: Can anything beat the historical average? Review of Financial Studies 21 (4), 1509-1531.

Cederburg, S., O'Doherty, M.S., Wang, F., Yan, X.S., 2020. On the performance of volatility-managed portfolios. Journal of Financial Economics 138 (1), 95-117.

Chen, L., Pelger, M., Zhu, J., 2019. Deep learning in asset pricing. arXiv preprint arXiv:1904.00745.

Chiang, I.-H.E., Liao, Y., Zhou, Q., 2021. Modeling the cross-section of stock returns using sensible models in a model pool. Journal of Empirical Finance 60, 56-73.

Christiansen, C., Schmeling, M., Schrimpf, A., 2012. A comprehensive look at financial volatility prediction by economic variables. Journal of Applied Econometrics 27 (6), 956-977.

Cho, H., Chun, D., and Ryu, D., 2022. Discovering the drivers of stock market volatility in a data-rich world. Working Paper.

Clark, T.E., West, K.D., 2007. Approximately normal tests for equal predictive accuracy in nested models. Journal of Econometrics 138 (1), 291-311.

Corsi, F., 2009. A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics 7 (2), 174-196.

Dangl, T., Halling, M., 2012. Predictive regressions with time-varying coefficients. Journal of Financial Economics 106 (1), 157-181.

Feng, G., Giglio, S., Xiu, D., 2020. Taming the factor zoo: A test of new factors. The Journal of Finance, 75 (3), 1327-1370.

Fleming, J., Kirby, C., Ostdiek, B., 2001. The economic value of volatility timing. Journal of Finance 56 (1), 329-352.

Fleming, J., Kirby, C., Ostdiek, B., 2003. The economic value of volatility timing using "realized" volatility. Journal of Financial Economics 67 (3), 473-509.

Friedman, J. H., 2001. Greedy function approximation: a gradient boosting machine. Annals of Statistics 29(5) 1189-1232.

Giacomini, R., White, H., 2006. Tests of conditional predictive ability. Econometrica 74 (6), 1545-1578.

Gu, S., Kelly, B., Xiu, D., 2020. Empirical asset pricing via machine learning. Review of Financial Studies 33 (5), 2223-2273.

Gu, S., Kelly, B., Xiu, D., 2021. Autoencoder asset pricing models. Journal of Econometrics 222 (1), 429-450.

Hansen, B.E., 2007. Least squares model averaging. Econometrica 75 (4), 1175-1189.

Hansen, P.R., Lunde, A., Nason, J.M., 2011. The model confidence set. Econometrica 79 (2), 453-497.

Hansen, B.E., Racine, J.S., 2012. Jackknife model averaging. Journal of Econometrics 167 (1), 38-46.

Harvey, C.R., Hoyle, E., Korgaonkar, R., Rattray, S., Sargaison, M., Van Hemert, O., 2018. The impact of volatility targeting. Journal of Portfolio Management 45 (1), 14-33.

Hoerl, A.E., Kennard, R.W., 1970. Ridge regression: Biased estimation for nonorthogonal problems. Technometrics 12 (1), 55-67.

Hoeting, J.A., Madigan, D., Raftery, A.E., Volinsky, C.T., 1999. Bayesian model averaging: a tutorial. Statistical Science 14 (4), 382-401.

Jacod, J., Li, Y., Mykland, P.A., Podolskij, M., Vetter, M., 2009. Microstructure noise in the continuous case: the pre-averaging approach. Stochastic Processes and Their Applications 119 (7), 2249-2276.

Jayawardena, N.I., Todorova, N., Li, B., Su, J.-J., 2016. Forecasting stock volatility using after-hour information: Evidence from the Australian Stock Exchange. Economic Modelling 52, 592-608.

Kim, H., Cho, H., Ryu, D., 2021a. Forecasting consumer credit recovery failure: Classification approaches. Journal of Credit Risk 17 (3), 117-140.

Kim, H., Cho, H., Ryu, D., 2021b. Predicting corporate defaults using machine learning with geometric-lag variables. Investment Analysts Journal 50 (3), 161-175.

Kim, H., Cho, H., Ryu, D., 2022. Corporate bankruptcy prediction using machine learning methodologies with a focus on sequential data. Computational Economic 59, 1231-1249

Koop, G.M., 2003. Bayesian econometrics. John Wiley & Sons Inc.

Koopman, S.J., Jungbacker, B., Hol, E., 2005. Forecasting daily variability of the S&P 100 stock index using historical, realised and implied volatility measurements. Journal of Empirical Finance 12 (3), 445-475.

Liu, F., Tang, X., Zhou, G., 2019. Volatility-managed portfolio: does it really work?. Journal of Portfolio Management, 46 (1), 38-51.

Martens, M., 2002. Measuring and forecasting S&P 500 index-futures volatility using high-frequency data. Journal of Futures Markets 22 (6), 497-518.

Moreira, A., Muir, T., 2017. Volatility-managed portfolios. Journal of Finance 72 (4), 1611-1644.

Mylnikov, G., 2021. Volatility Targeting: It's Complicated!. Journal of Portfolio Management, 47 (8), 57-74.

Nagel, S., 2012. Evaporating liquidity. Review of Financial Studies 25 (7), 2005-2039.

Nonejad, N., 2017. Forecasting aggregate stock market volatility using financial and macroeconomic predictors: Which models forecast best, when and why? Journal of Empirical Finance 42, 131-154.

Patton, A.J., Sheppard, K., 2015. Good volatility, bad volatility: Signed jumps and the persistence of volatility. Review of Economics and Statistics 97 (3), 683-697.

Paye, B.S., 2012. 'Déjà vol': Predictive regressions for aggregate stock market volatility using macroeconomic variables. Journal of Financial Economics 106 (3), 527-546.

Qu, H., Zhang, Y., 2021. Asymmetric multivariate HAR models for realized covariance matrix: A study based on volatility timing strategies. Economic Modelling, 105699.

Samuels, J.D., Sekkel, R.M., 2017. Model confidence sets and forecast combination. International Journal of Forecasting 33 (1), 48-60.

Stamos, M., Zimmerer, T., 2021. Managing portfolio volatility. Journal of Portfolio Management 47 (4), 99-109.

Steel, M. F., 2020. Model averaging and its use in economics. Journal of Economic Literature 58 (3), 644-719.

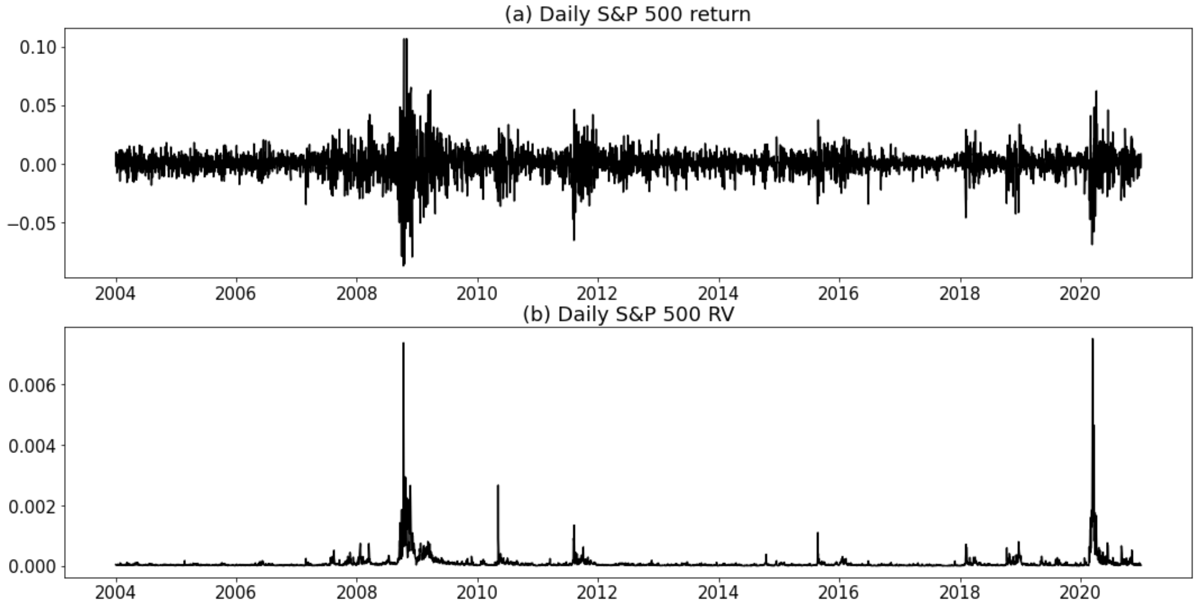
Tibshirani, R., 1996. Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society: Series B (Methodological) 58 (1), 267-288.

Vrontos, S. D., Galakis, J., Vrontos, I. D., 2021. Implied volatility directional forecasting: a machine learning approach. Quantitative Finance, 21 (10), 1687-1706.

Wang, Y., Ma, F., Wei, Y., Wu, C., 2016. Forecasting realized volatility in a changing world: A dynamic model averaging approach. Journal of Banking & Finance 64, 136-149.

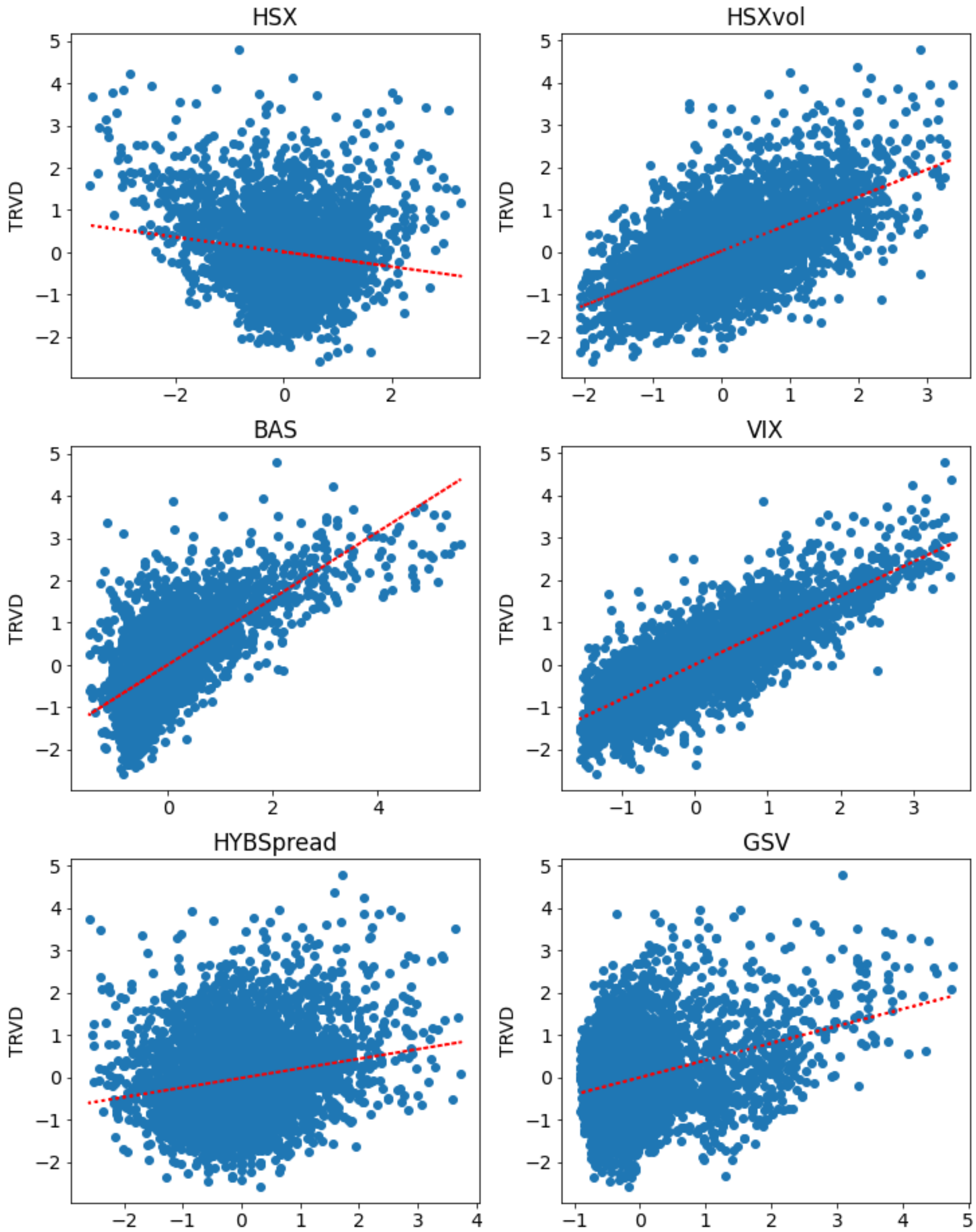
Welch, I., Goyal, A., 2008. A comprehensive look at the empirical performance of equity premium prediction. Review of Financial Studies 21 (4), 1455-1508.

Fig. 1. Daily S&P 500 index returns and realized volatility estimates.



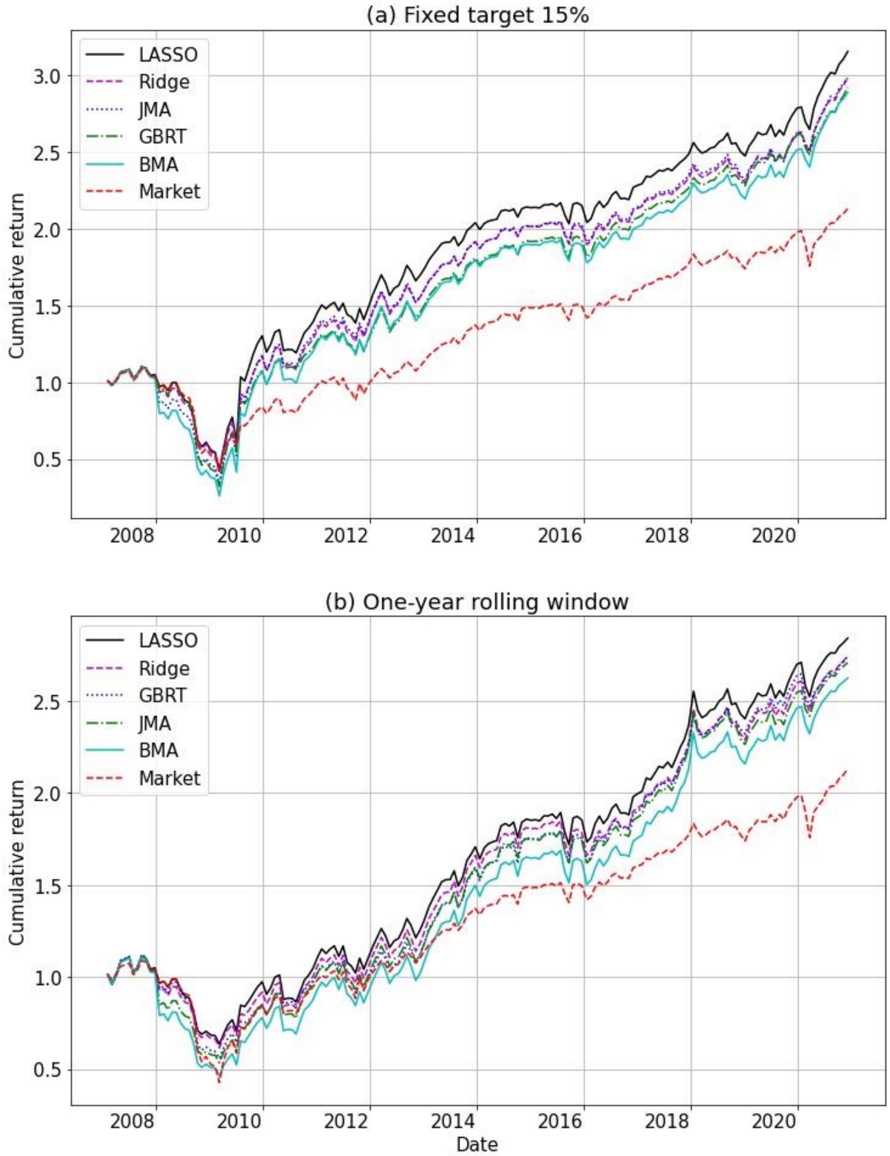
Notes. This figure illustrates daily open-to-open S&P 500 returns (upper panel) and the PRV (lower panel) for the period January 2, 2004, to December 31, 2020. The year is shown on the x-axis.

Fig. 2. Scatter plots between one-day-ahead daily S&P 500 RV and key predictor variables.



Notes. This figure illustrates scatter plots between the one-day-ahead daily S&P 500 RV and Hang Seng return (*HSX*), Hang Seng return volatility (*HSXvol*), bid-ask spread (*BAS*), VIX, high-yield bond spread (*HYBSpread*), and Google search volume index (*GSV*). The red dotted line illustrates the regression line.

Fig. 3. Cumulative return of monthly volatility targeting portfolios.



Notes. This figure presents the cumulative log-returns of monthly volatility targeting portfolios (a) with an annualized target volatility of 15% and (b) with the target volatility of the one-year rolling standard deviation of the market returns. The legend is ordered by the final cumulative returns in descending order. The evaluation period is from January 3, 2007, to December 31, 2020 (3,498 days).

Table 1. Summary of predictor variables.

Predictor	Description	Source
U.S. stock market variables		
SPX	S&P 500 index return	Bloomberg
Totdvol	NYSE trading volume	Bloomberg
SPXETFtdv	S&P 500 ETF trading volume	Bloomberg
BAS	Market aggregate bid-ask spread measure	CRSP
Amihud	Market aggregate Amihud measure	CRSP
STR	Short-term reversal factor	French's website
Earnnews	Total number of quarterly earnings news	CRSP
Rskew	Realized skewness	Intraday data from CBOE
Rkurt	Realized kurtosis	Intraday data from CBOE
Index option-related variables		
VIX	CBOE volatility index: VIX	Bloomberg
ATMIV	ATM call and put option IVs	Bloomberg
IVslope1	Difference between OTM put and call option IVs	Bloomberg
IVslope2	Difference between deep OTM put and call option IVs	Bloomberg
PCR_EQT	CBOE equity put/call volume ratios	CBOE
PCR_SPX	S&P500 index put/call volume ratios	CBOE
Investor attention, sentiment, and economic uncertainty variables		
Abtv	Abnormal NYSE dollar volume	CRSP
GSV	Google search volume keyword: "stock market"&"S&P 500"&"Nasdaq"	Google
News	News sentiment index of Shapiro, Sudhof, and Wilson (2020)	FRBSF
Stdret	Cross-sectional return standard deviation	CRSP
EPU	Economic policy uncertainty	World News Bank Service
SURP	U.S. Citi economic surprise index	Bloomberg
Interest rate and financial condition variables		
DEF	Default spread (BAA - AAA)	Moody's
TERM	Term spread (10Y-3M)	Moody's
TED	TED spread (3M LIBOR – 3M T-bill)	FRB
HYBSpread	Spread between low-grade bond yield and spot treasury curve	ICE
FINCON	Goldman Sachs U.S. financial conditions index return	Bloomberg
Global market variables		
HSX	Hang Seng index return	Hong Kong Exchanges
SCX	Shanghai composite index return	Shanghai Stock Exchange
NIKKEI	Nikkei225 index return	Tokyo Stock Exchange
STOXX	EURO STOXX index return	EURO STOXX
UKX	FTSE 100 index return	Bloomberg
ACWI	MSCI ACWI index return	Bloomberg
HSXvol	PRV of HSX	FirstRate Data
NIKKEIvol	PRV of NIKKEI	FirstRate Data
STOXXvol	PRV of STOXX	FirstRate Data
Dollarind	Dollar index return	Bloomberg
EMBISpread	J.P. Morgan EMBI global spread	Bloomberg
SURP_CN	China Citi economic surprise index	Bloomberg
SURP_EU	EU Citi economic surprise index	Bloomberg
Commodity variables		
CRBCMDT	CRB commodity index return	CRB
Gold	Gold futures return	Bloomberg
WTI	WTI futures return	Bloomberg
OilVol	Oil ETF implied volatility	CBOE

Notes. This table summarizes the predictor variables by reporting their symbols, descriptions, and sources.

Table 2. Out-of-sample prediction accuracy of high-dimensional models.

Model	MSPE	MAPE	CW	GW	ΔR^2_{oos}
Panel A. Daily					
LASSO	0.304***	0.464***	12.708***	7.281***	3.109
Ridge	0.337	0.516	11.507***	-0.361	-0.311
GBRT	0.342	0.493	11.140***	-1.414	-0.912
BMA	0.310	0.473	14.362***	4.186***	2.539
BMS	0.316	0.479	14.113***	2.903***	1.901
JMA	0.308	0.477	14.518***	3.972***	2.663
MMA	0.308	0.477	14.349***	3.972***	2.681
DMSPE	0.330	0.478	5.832***	3.589***	0.345
MFC	0.330	0.479	6.376***	5.068***	0.358
Panel B. Weekly					
LASSO	0.271***	0.400***	8.503***	4.470***	2.729
Ridge	0.290	0.419	9.058***	0.319	0.275
GBRT	0.307	0.437	10.254***	-1.970	-1.996
BMA	0.279	0.421	10.990***	1.782**	1.661
BMS	0.284	0.426	10.844***	1.028	1.005
JMA	0.274	0.459	11.173***	1.762**	2.271
MMA	0.276	0.459	11.510***	1.601*	2.108
DMSPE	0.287	0.409	5.257***	4.150***	0.633
MFC	0.287	0.410	4.898***	3.980***	0.593
Panel C. Monthly					
LASSO	0.360***	0.432***	10.211***	4.503***	5.765
Ridge	0.361	0.436	11.708***	4.068***	5.577
GBRT	0.376	0.481	11.203***	1.603*	3.140
BMA	0.391	0.487	12.145***	0.387	0.786
BMS	0.396	0.488	12.108***	-0.041	-0.084
JMA	0.369	0.567	12.024***	2.058**	4.376
MMA	0.374	0.512	11.898***	1.498*	3.536
DMSPE	0.391	0.445	4.269***	2.026**	0.752
MFC	0.389	0.446	5.494***	3.676***	1.177

Notes. This table reports the prediction accuracy of the high-dimensional models. The table reports the MSPE and MAPE; *** means that the model belongs to the 1% MCS. The table displays the CW and GW statistics; ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. The table also shows an increase in the out-of-sample R^2 relative to the benchmark HARSJ model (ΔR^2_{oos}). The variances of the statistics are calculated considering heteroscedasticity and autocorrelation.

Table 3. Significant variables for out-of-sample prediction of the high-dimensional model.

Rank	LASSO	Ridge	GBRT	JMA	MMA	BMA
Panel A. Daily						
1	HSX	BAS	SPX	ATMIV	ATMIV	BAS
2	BAS	ATMIV	VIX	BAS	BAS	HSX
3	ATMIV	HSXvol	NIKKEI	VIX	VIX	ATMIV
4	HSXvol	VIX	UKX	HSX	HSX	HSXvol
5	VIX	GSV	BAS	SPX	SPX	VIX
6	NIKKEI	HSX	Earnnews	HSXvol	FINCON	NIKKEI
7	HYBSpread	OilVol	HYBSpread	FINCON	HSXvol	HYBSpread
8	Earnnews	Stdret	TERM	ACWI	OilVol	Earnnews
9	STR	NIKKEI	SPXETFtdv	OilVol	ACWI	EMBISpread
10	UKX	HYBSpread	Totdvol	HYBSpread	HYBSpread	SPX
Panel B. Weekly						
1	HSX	HSXvol	SPX	ACWI	ACWI	HSX
2	HSXvol	Stdret	STR	FINCON	HSX	HSXvol
3	VIX	GSV	VIX	HSX	FINCON	STR
4	ATMIV	ATMIV	STOXX	VIX	ATMIV	GSV
5	SURP_EU	BAS	HYBSpread	ATMIV	VIX	NIKKEI
6	STR	VIX	Totdvol	SPX	SPX	VIX
7	NIKKEI	HSX	SPXETFtdv	HSXvol	HSXvol	SURP_EU
8	GSV	NIKKEI	DEF	SURP_EU	SURP_EU	IVslope1
9	HYBSpread	HYBSpread	Abtv	OilVol	GSV	BAS
10	IVslope1	OilVol	SURP	GSV	OilVol	ATMIV
Panel C. Monthly						
1	OilVol	ATMIV	SPX	VIX	VIX	VIX
2	News	VIX	DEF	ATMIV	ATMIV	News
3	VIX	OilVol	VIX	OilVol	OilVol	OilVol
4	PCR_EQT	HSXvol	Totdvol	News	News	PCR_EQT
5	HSXvol	News	ATMIV	PCR_EQT	PCR_EQT	ATMIV
6	GSV	BAS	SPXETFtdv	SURP_CN	SURP_CN	SURP_EU
7	ATMIV	GSV	STOXX	SPX	SPX	HSXvol
8	Earnnews	PCR_EQT	HYBSpread	SURP_EU	SURP_EU	SURP_CN
9	SURP_EU	Earnnews	UKX	HSXvol	GSV	Earnnews
10	SURP_CN	NIKKEI	IVslope2	FINCON	HSXvol	SURP

Notes. This table reports the significant variables for the out-of-sample prediction of each model and forecasting horizon. The measures of variable importance are presented in Appendix A. Full results are provided in Appendix B.

Table 4. Performance of daily volatility targeting portfolios.

Model	Avg.ret (%)	Sharpe	CER	Avg.ret (%)	Sharpe	CER
	Fixed target 15%			One-year rolling window		
LASSO	11.55	0.560	6.083	14.06	0.593	6.631
Ridge	11.18	0.532	5.542	13.38	0.552	5.668
GBRT	11.26	0.537	5.646	13.90	0.578	6.262
BMA	10.70	0.518	5.282	12.88	0.549	5.694
BMS	10.81	0.523	5.381	13.00	0.554	5.804
JMA	11.06	0.537	5.648	13.17	0.563	5.994
MMA	11.09	0.539	5.684	13.21	0.564	6.022
DMSPE	11.28	0.538	5.665	13.82	0.577	6.255
MFC	11.35	0.543	5.748	13.89	0.581	6.348
HARSJ	11.32	0.541	5.712	13.87	0.580	6.324
GARCH	8.87	0.481	4.705	10.93	0.531	5.533
Market	8.55	0.418	3.480	8.55	0.418	3.480

Notes. This table shows the performance of the daily volatility targeting portfolios based on the daily volatility forecasts from each model, reporting the annualized average return (*Avg.ret*), Sharpe ratio (*Sharpe*), and CER. The first three columns represent volatility targeting portfolios with fixed target volatility corresponding to an annualized volatility of 15%. The last three columns stand for the target volatility corresponding to the one-year rolling standard deviation of the market returns. The rows labeled "*Market*" present the results for the buy-and-hold market portfolio.

Table 5. Performance of weekly volatility targeting portfolios.

Model	Avg.ret (%)	Sharpe	CER	Avg.ret (%)	Sharpe	CER
	Fixed target 15%			One-year rolling window		
LASSO	12.06	0.600	6.176	14.49	0.629	6.709
Ridge	11.33	0.555	5.264	13.53	0.582	5.641
GBRT	12.17	0.589	5.954	14.94	0.638	6.909
BMA	11.17	0.561	5.397	13.58	0.604	6.194
BMS	11.14	0.558	5.335	13.52	0.600	6.096
JMA	11.62	0.591	5.989	13.76	0.619	6.540
MMA	11.66	0.593	6.033	13.81	0.622	6.591
DMSPE	11.97	0.587	5.917	14.30	0.614	6.369
MFC	11.79	0.579	5.748	14.17	0.609	6.245
HARSJ	11.80	0.579	5.751	14.17	0.609	6.251
GARCH	8.90	0.448	3.212	11.71	0.514	4.149
Market	8.42	0.484	4.072	8.42	0.484	4.072

Notes. This table shows the performance of the weekly volatility targeting portfolios based on the weekly volatility forecasts from each model, reporting the annualized average return (*Avg.ret*), Sharpe ratio (*Sharpe*), and CER. The first three columns represent volatility targeting portfolios with fixed target volatility corresponding to an annualized volatility of 15%. The last three columns stand for the target volatility corresponding to the one-year rolling standard deviation of the market returns. The rows labeled "*Market*" present the results for the buy-and-hold market portfolio.

Table 6. Performance of monthly volatility targeting portfolios.

Model	Avg.ret (%)	Sharpe	CER	Avg.ret (%)	Sharpe	CER
	Fixed target 15%			One-year rolling window		
LASSO	12.71	0.695	7.729	14.86	0.688	7.897
Ridge	12.04	0.665	7.162	13.70	0.677	7.596
GBRT	12.04	0.672	7.263	13.26	0.656	7.175
BMA	11.22	0.584	5.730	13.02	0.599	5.983
BMS	11.18	0.583	5.705	12.91	0.594	5.876
JMA	11.80	0.632	6.605	13.67	0.651	7.086
MMA	11.75	0.629	6.547	13.63	0.648	7.037
DMSPE	11.81	0.652	6.920	13.45	0.660	7.268
MFC	11.30	0.634	6.575	13.19	0.651	7.061
HARSJ	11.21	0.629	6.477	13.09	0.646	6.969
GARCH	6.81	0.326	0.358	8.40	0.349	-0.218
Market	7.80	0.500	4.186	7.80	0.500	4.186

Notes. This table shows the performance of the monthly volatility targeting portfolios based on the monthly volatility forecasts from each model, reporting the annualized average return (*Avg.ret*), Sharpe ratio (*Sharpe*), and CER. The first three columns represent volatility targeting portfolios with fixed target volatility corresponding to an annualized volatility of 15%. The last three columns stand for the target volatility corresponding to the one-year rolling standard deviation of the market returns. The rows labeled "*Market*" present the results for the buy-and-hold market portfolio.

Table 7. Realized utility of risk-targeting portfolios.

	Daily		Weekly		Monthly	
Model	Realized utility	*t*-stat	Realized utility	*t*-stat	Realized utility	*t*-stat
LASSO	1.377	Best	1.405	Best	1.406	Best
Ridge	1.332	-1.389	1.403	-0.439	1.280	-1.043
GBRT	1.365	-3.932***	1.386	-2.469**	1.375	-1.985**
BMA	1.372	-2.623***	1.390	-2.285**	1.387	-2.064**
BMS	1.369	-3.789***	1.386	-2.236**	1.385	-2.349**
JMA	1.369	-2.162**	1.396	-2.063**	1.393	-1.356
MMA	1.369	-2.185**	1.395	-2.227**	1.394	-1.288
DMSPE	1.372	-2.583***	1.401	-1.020	1.398	-1.037
MFC	1.372	-2.431**	1.399	-1.277	1.390	-1.922*
HAR	1.371	-2.868***	1.398	-1.556	1.384	-2.389**
GARCH	0.914	-19.310***	0.898	-10.370***	0.862	-5.563***

Notes. This table reports the realized utility of risk-targeting portfolios, based on volatility forecasts from each model. *t*-stat is used to test the significance of the realized utility relative to the model with the highest utility. "*Best*" represents the model with the highest utility, and a negative statistic indicates that the best model outperforms another in terms of the realized utility gain; ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Appendix A. Variable importance measures

In this section, we introduce variable importance measures corresponding to each high dimensional model, following Chun, Cho, and Ryu (2022). For example, for the LASSO, we consider three evaluation indicators: the percentage included (PI), average absolute coefficient (AAC), and an increase in adjusted R^2 values of the encompassing regressions (ΔR_q^2). The PI gauges the number of out-of-sample predictions containing the predictor, calculated as

$$\mathrm{PI}_r = \frac{\text{The number of out}-\text{of}-\text{sample predictions that select the predictor } r}{\text{Total number of out}-\text{of}-\text{sample predictions}} \times 100(\%). \tag{A.1}$$

Second, the average absolute coefficient (AAC) of a predictor is obtained by averaging the absolute coefficients over out-of-sample predictions. Third, we calculate the difference between the adjusted R^2 of the following regressions:

$$\begin{aligned} &(\mathrm{R1})\ \ RV_n = \alpha_1 + \zeta_r \widehat{RV}_{r,n} + \omega_{r,n}, \\ &(\mathrm{R2})\ \ RV_n = \alpha_2 + \varphi_r \widehat{RV}_{r,n} + \varphi_s \widehat{RV}_{s,n} + \upsilon_{s,r,n}, \end{aligned} \tag{A.2}$$

where $\widehat{RV}_{r,n}$ and $\widehat{RV}_{s,n}$ are the RV forecasts from HARSJ-X model r and the high-dimensional model s, respectively. We define ΔR_q^2 as an increase in the adjusted R^2 of regression (R2) relative to that of regression (R1). Intuitively, smaller ΔR_q^2 signifies that the predictor r plays an important role in the high-dimensional model s forecast. In accordance with the model properties, we calculate the average rank of the PI, AAC (in descending order), and ΔR_q^2 (in ascending order) for LASSO, AAC and ΔR_q^2 for the ridge regression, that of the PI, feature importance, and ΔR_q^2 for the GBRT, that of the PI, AAC, and ΔR_q^2 for the JMA and MMA, and that of the posterior inclusion probabilities and ΔR_q^2 for the BMA.

Appendix B. Supplementary tables

Table B.1. Variable importance based on the out-of-sample ridge prediction: Full results.

Daily		Weekly		Monthly	
Predictor	avg.coef.	Predictor	avg.coef.	Predictor	avg.coef.
BAS	0.053	HSXvol	0.040	ATMIV	0.118
ATMIV	0.224	Stdret	0.026	VIX	0.203
HSXvol	0.050	GSV	0.027	OilVol	0.059
VIX	0.290	ATMIV	0.189	HSXvol	0.034
GSV	0.029	BAS	0.037	News	-0.005
HSX	-0.023	VIX	0.256	BAS	0.025
OilVol	0.113	HSX	-0.020	GSV	0.032
Stdret	0.029	NIKKEI	-0.015	PCR_EQT	-0.011
NIKKEI	-0.018	HYBSpread	0.016	Earnnews	0.008
HYBSpread	0.018	OilVol	0.096	NIKKEI	-0.012
PCR_EQT	0.030	IVslope1	0.017	Amihud	0.007
NIKKEIvol	0.022	PCR_EQT	0.022	HSX	-0.009
IVslope1	0.016	SURP_EU	-0.024	IVslope2	0.007
SPX	-0.012	ACWI	-0.007	IVslope1	0.017
UKX	-0.015	STOXXvol	0.041	SURP_EU	-0.036
EPU	0.019	STR	-0.011	SURP_CN	0.025
FINCON	0.011	FINCON	0.007	PCR_SPX	0.006
ACWI	-0.010	Amihud	0.014	FINCON	0.005
Earnnews	0.016	EMBISpread	0.003	SURP	0.009
STR	-0.009	IVslope2	-0.005	HYBSpread	0.015
Amihud	0.015	SURP	0.001	Gold	0.011
Abtv	0.019	SPX	-0.006	Stdret	0.024
STOXXvol	0.046	Rkurt	0.003	SPX	-0.004
SURP_EU	-0.017	Abtv	0.011	STOXXvol	0.023
EMBISpread	0.002	STOXX	0.000	DEF	0.007
SCX	-0.011	UKX	-0.014	CRBCMDT	0.005
PCR_SPX	0.006	SCX	-0.005	Abtv	0.006
TERM	0.007	News	-0.009	NIKKEIvol	0.020
News	-0.010	NIKKEIvol	0.018	Dollarind	-0.004
SURP	-0.002	Earnnews	0.008	ACWI	-0.003
CRBCMDT	-0.003	SPXETFtdv	-0.006	EPU	0.006
SURP_CN	0.003	PCR_SPX	0.004	TED	0.003
STOXX	-0.001	EPU	0.016	EMBISpread	0.001
IVslope2	0.002	CRBCMDT	0.001	UKX	-0.008
TED	0.004	DEF	0.006	Rskew	0.004
WTI	0.004	SURP_CN	0.011	SPXETFtdv	-0.006
Dollarind	-0.007	WTI	0.002	Rkurt	0.001
SPXETFtdv	-0.006	Totdvol	-0.002	STR	-0.014
DEF	0.000	Rskew	0.002	STOXX	0.001
Rkurt	0.004	Dollarind	-0.003	SCX	0.003
Gold	0.001	Gold	0.003	Totdvol	-0.003
Totdvol	0.000	TED	0.002	TERM	-0.002
Rskew	0.000	TERM	0.000	WTI	0.001

Notes. This table reports the average coefficient based on the out-of-sample ridge prediction. The table is sorted by the average rank of AAC and ΔR_q^2.

Table B.2. Variable importance based on the out-of-sample GBRT prediction: Full results.

Daily			Weekly			Monthly		
Predictor	FI	avg.coef.	Predictor	FI	avg.coef.	Predictor	FI	avg.coef.
SPX	99.06	0.161	SPX	98.03	0.144	SPX	95.57	0.080
VIX	98.03	0.034	STR	89.59	0.041	DEF	93.45	0.076
NIKKEI	93.37	0.014	VIX	91.05	0.027	VIX	80.05	0.027
UKX	88.25	0.011	STOXX	91.68	0.019	Totdvol	94.08	0.063
BAS	83.36	0.010	HYBSpread	84.22	0.028	ATMIV	75.81	0.016
Earnnews	85.42	0.012	Totdvol	98.31	0.135	SPXETFtdv	85.88	0.020
HYBSpread	78.50	0.010	SPXETFtdv	89.28	0.019	STOXX	92.65	0.050
TERM	85.91	0.014	DEF	94.74	0.035	HYBSpread	90.62	0.055
SPXETFtdv	90.19	0.015	Abtv	82.08	0.020	UKX	92.60	0.063
Totdvol	99.66	0.182	SURP	88.56	0.036	IVslope2	79.67	0.012
STR	75.96	0.027	ACWI	72.36	0.010	GSV	78.10	0.012
SCX	78.62	0.009	NIKKEI	76.39	0.007	Abtv	83.22	0.018
SURP	85.11	0.027	GSV	76.73	0.011	PCR_SPX	76.70	0.024
Abtv	79.47	0.012	Amihud	80.10	0.025	Amihud	83.13	0.026
PCR_SPX	76.84	0.019	ATMIV	67.70	0.013	SURP	90.71	0.061
GSV	75.76	0.007	IVslope1	75.73	0.008	STR	90.28	0.069
Amihud	71.81	0.018	UKX	88.39	0.033	TED	80.67	0.015
IVslope1	79.65	0.006	PCR_SPX	72.21	0.023	BAS	75.39	0.018
STOXXvol	86.71	0.009	SCX	72.24	0.007	Dollarind	67.67	0.012
ATMIV	64.29	0.010	TERM	78.53	0.022	TERM	83.99	0.027
ACWI	70.21	0.006	BAS	67.21	0.013	HSX	70.78	0.007
NIKKEIvol	75.59	0.006	FINCON	62.78	0.005	IVslope1	75.64	0.012
DEF	80.47	0.011	Earnnews	67.12	0.012	ACWI	66.27	0.012
HSX	68.15	0.004	STOXXvol	72.01	0.006	FINCON	68.70	0.008
EPU	82.73	0.008	IVslope2	70.13	0.006	HSXvol	66.58	0.008
HSXvol	67.90	0.004	Stdret	60.32	0.006	Earnnews	65.07	0.012
STOXX	68.27	0.008	HSXvol	58.98	0.005	News	77.47	0.013
FINCON	69.75	0.004	HSX	55.12	0.004	PCR_EQT	61.21	0.009
TED	71.93	0.006	TED	73.96	0.008	NIKKEI	64.64	0.007
Stdret	66.90	0.004	News	69.95	0.013	SCX	74.76	0.012
News	73.38	0.009	CRBCMDT	65.98	0.006	Gold	65.15	0.006
IVslope2	72.33	0.005	SURP_CN	63.41	0.008	SURP_CN	68.38	0.010
Dollarind	64.47	0.008	EPU	71.84	0.009	EPU	79.33	0.011
OilVol	66.90	0.004	Dollarind	59.49	0.011	OilVol	63.04	0.008
Gold	71.81	0.004	NIKKEIvol	62.72	0.007	CRBCMDT	68.84	0.008
CRBCMDT	71.50	0.004	WTI	49.63	0.004	Stdret	67.01	0.009
PCR_EQT	51.72	0.006	Rkurt	57.32	0.003	EMBISpread	64.21	0.005
EMBISpread	60.15	0.003	OilVol	53.54	0.005	Rskew	62.35	0.004
SURP_CN	65.95	0.006	EMBISpread	50.83	0.003	WTI	68.50	0.006
SURP_EU	52.66	0.006	SURP_EU	55.55	0.009	SURP_EU	63.55	0.010
WTI	66.30	0.004	PCR_EQT	49.14	0.007	NIKKEIvol	64.58	0.006
Rkurt	67.24	0.003	Gold	59.26	0.004	STOXXvol	64.21	0.008
Rskew	57.00	0.003	Rskew	51.89	0.003	Rkurt	63.09	0.004

Notes. This table reports the feature importance (FI) and average coefficient based on the out-of-sample GBRT prediction. The table is sorted by the average rank of PI, feature importance, and ΔR_q^2.

Table B.3. Variable importance based on the out-of-sample JMA prediction: Full results.

Daily			Weekly			Monthly		
Predictor	PI	avg.coef.	Predictor	PI	avg.coef.	Predictor	PI	avg.coef.
ATMIV	98.86	0.524	ACWI	86.96	-0.043	VIX	99.17	1.071
BAS	87.05	0.073	FINCON	83.22	-0.007	ATMIV	91.45	-0.028
VIX	99.77	1.101	HSX	85.62	-0.045	OilVol	88.74	0.015
HSX	87.91	-0.052	VIX	99.71	1.293	News	94.63	0.049
SPX	75.27	-0.013	ATMIV	93.45	0.269	PCR_EQT	73.58	-0.067
HSXvol	72.18	0.061	SPX	72.10	0.031	SURP_CN	83.56	0.041
FINCON	73.50	0.006	HSXvol	70.01	0.045	SPX	77.13	0.004
ACWI	75.27	-0.020	SURP_EU	80.87	-0.084	SURP_EU	87.82	-0.119
OilVol	75.81	-0.002	OilVol	83.16	-0.057	HSXvol	70.47	0.020
HYBSpread	68.81	0.045	GSV	58.49	0.051	FINCON	71.61	-0.001
SPXETFtdv	73.90	-0.055	Rkurt	70.93	-0.043	GSV	76.70	0.079
Earnnews	73.27	0.046	HYBSpread	58.49	0.041	Earnnews	65.98	0.025
News	78.44	0.055	IVslope1	63.18	0.037	ACWI	72.70	-0.013
SURP_EU	66.61	-0.051	EMBISpread	64.75	-0.017	SURP	85.53	0.017
EMBISpread	60.09	-0.028	Stdret	55.15	0.023	IVslope2	55.35	-0.001
NIKKEI	59.95	-0.023	IVslope2	75.39	-0.033	IVslope1	62.38	0.022
GSV	46.46	0.033	STR	61.52	-0.032	Abtv	54.92	-0.021
STR	60.15	-0.019	News	81.42	0.065	STOXXvol	69.38	-0.026
Rkurt	65.04	-0.062	STOXX	49.80	0.038	Stdret	68.12	0.020
IVslope1	45.60	0.006	SURP	67.21	-0.019	PCR_SPX	53.32	0.022
TERM	58.89	0.025	PCR_EQT	62.92	-0.002	HYBSpread	55.12	0.034
PCR_EQT	39.57	0.013	BAS	46.48	0.038	HSX	47.66	-0.017
STOXX	62.52	0.035	NIKKEI	37.88	-0.030	BAS	48.26	0.008
UKX	32.42	-0.022	SPXETFtdv	60.78	-0.024	EPU	66.44	-0.025
SURP	54.97	-0.022	Earnnews	59.03	0.034	Rskew	52.77	-0.015
Dollarind	41.85	-0.039	Abtv	58.38	0.003	Rkurt	61.92	-0.038
Totdvol	55.77	0.043	STOXXvol	52.54	0.009	Amihud	48.80	-0.006
STOXXvol	54.12	-0.027	SURP_CN	63.89	0.019	NIKKEIvol	46.63	0.058
SCX	42.08	-0.006	SCX	32.59	0.015	NIKKEI	32.33	-0.015
Abtv	58.81	0.020	Amihud	44.05	0.005	EMBISpread	42.60	-0.018
EPU	41.02	0.019	CRBCMDT	55.92	0.009	CRBCMDT	48.68	0.019
Stdret	26.42	0.004	Totdvol	52.92	0.016	STR	52.83	-0.030
Rskew	39.25	-0.040	EPU	43.45	0.001	STOXX	43.62	0.009
TED	39.39	-0.005	PCR_SPX	39.91	0.005	SPXETFtdv	37.25	-0.002
IVslope2	41.60	-0.002	NIKKEIvol	36.19	0.016	Gold	45.91	0.016
PCR_SPX	34.22	0.012	Rskew	41.62	-0.020	Totdvol	33.96	0.005
CRBCMDT	45.77	-0.001	UKX	35.31	-0.013	Dollarind	29.33	-0.012
NIKKEIvol	18.58	0.016	Dollarind	26.76	-0.019	TERM	43.62	-0.018
Amihud	29.19	0.007	DEF	13.87	0.003	SCX	36.16	0.016
SURP_CN	25.27	-0.018	WTI	23.38	0.004	TED	31.88	-0.001
DEF	37.14	-0.017	TERM	22.24	0.006	DEF	12.61	0.008
Gold	16.47	-0.002	Gold	22.67	0.015	UKX	18.55	-0.009
WTI	23.64	0.015	TED	31.99	0.002	WTI	17.12	0.001

Notes. This table reports the PI and average coefficient based on the out-of-sample JMA prediction. The table is sorted by the average rank of PI, AAC, and ΔR_q^2. Owing to the similarity, we omit the results for MMA prediction.

Table B.4. Variable importance based on the out-of-sample BMA prediction: Full results.

Daily		Weekly		Monthly	
Predictor	PIP	Predictor	PIP	Predictor	PIP
BAS	0.583	HSX	0.495	VIX	0.803
HSX	0.565	HSXvol	0.338	News	0.782
ATMIV	0.635	STR	0.270	OilVol	0.738
HSXvol	0.330	GSV	0.283	PCR_EQT	0.486
VIX	0.708	NIKKEI	0.200	ATMIV	0.355
NIKKEI	0.207	VIX	0.886	SURP_EU	0.730
HYBSpread	0.210	SURP_EU	0.568	HSXvol	0.286
Earnnews	0.404	IVslope1	0.289	SURP_CN	0.574
EMBISpread	0.190	BAS	0.229	Earnnews	0.474
SPX	0.125	ATMIV	0.307	SURP	0.616
GSV	0.141	Stdret	0.176	GSV	0.457
SPXETFtdv	0.211	HYBSpread	0.167	IVslope2	0.204
UKX	0.147	IVslope2	0.306	HSX	0.115
TERM	0.204	Rkurt	0.228	IVslope1	0.250
STR	0.168	OilVol	0.368	PCR_SPX	0.223
SCX	0.149	EMBISpread	0.171	NIKKEI	0.092
PCR_EQT	0.156	Earnnews	0.318	BAS	0.099
IVslope1	0.124	SURP	0.355	SPX	0.081
SURP_EU	0.189	News	0.651	HYBSpread	0.201
EPU	0.190	PCR_EQT	0.225	ACWI	0.103
ACWI	0.082	ACWI	0.091	FINCON	0.085
OilVol	0.127	FINCON	0.108	Stdret	0.326
FINCON	0.091	Abtv	0.201	EPU	0.397
SURP	0.227	SCX	0.107	NIKKEIvol	0.317
News	0.376	SPX	0.093	Gold	0.142
Stdret	0.093	STOXX	0.081	Abtv	0.158
Abtv	0.180	CRBCMDT	0.184	EMBISpread	0.084
TED	0.136	Amihud	0.107	Dollarind	0.077
Rkurt	0.202	SPXETFtdv	0.092	Amihud	0.149
NIKKEIvol	0.080	PCR_SPX	0.117	CRBCMDT	0.187
PCR_SPX	0.101	UKX	0.116	Rskew	0.095
SURP_CN	0.155	EPU	0.236	Rkurt	0.163
CRBCMDT	0.123	SURP_CN	0.225	TED	0.098
Dollarind	0.123	NIKKEIvol	0.173	STR	0.252
IVslope2	0.097	STOXXvol	0.111	STOXXvol	0.201
Amihud	0.068	WTI	0.115	SPXETFtdv	0.063
Totdvol	0.114	Gold	0.119	DEF	0.060
WTI	0.102	DEF	0.065	STOXX	0.091
STOXX	0.071	Totdvol	0.076	TERM	0.126
STOXXvol	0.076	TED	0.100	WTI	0.095
Gold	0.075	Dollarind	0.072	Totdvol	0.064
DEF	0.094	Rskew	0.069	SCX	0.080
Rskew	0.079	TERM	0.068	UKX	0.079

Notes. This table reports the posterior inclusion probability (PIP) based on the out-of-sample BMA prediction. The table is sorted by the average rank of PIP and ΔR_q^2.