Dead Cat Bounce
Demand reversal following the bursting of a bubble*

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Abstract

This is the first paper to theoretically analyze the temporary reversal of the downward trend in financial assets, also known as *dead cat bounce* or *bear market rally*. We show that preferences according to cumulative prospect theory lead an investor to take excessive risk and unprofitable positions in order to recover an initial loss in a declining market. The loss driven behavior results in premature re-entering into the market. We show that heterogeneous investors enter at the same time despite differences in the reference point, wealth and initial loss. The resulting shift in aggregate demand can explain the sudden but temporary reversal common in declining financial markets.

Keywords: Asset pricing; bubbles; cumulative prospect theory; behavioral finance
JEL Classifications: D03, D53, G01, G02

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1 Motivation

During bear markets, in particular after the burst of a bubble, asset prices often experience a temporary reversal of the downward trend followed by a further decline. Investors commonly describe this phenomenon as *dead cat bounce* while the finance literature talks about *bear market rallies* as e.g. Maheu et al. (2012). So far, despite its importance for investors, the *dead cat bounce* has widely been ignored by the theoretical literature.

This paper provides a theoretical explanation for the *dead cat bounce* using insights from the behavioral literature. We consider an investor with preferences according to cumulative prospect theory investing into a single risky asset during a bubble. After realizing a perceived loss following the burst of the bubble, the investor may re-enter after a further fall in the price leading to a reversal of the sell-off. Thereby, our approach does not rely on the arrival of a sequence of important fundamental information or coincided closing of short positions, which are the popular reasons given for the reversal.[1]

Temporary reversals in falling prices are a common feature in capital, commodity and currency markets as shown in Figure 1. In March 2000 the NASDAQ composite index saw the peak of a bubble which had been years in the making. Within a bit more than two months the index lost over a third of its value. In the subsequent 8 weeks the index rose by 33% narrowing the gap to its previous high to 15%. After this short lived recovery, the stock index plummeted once again, reaching its low point over two years later ending up with a total loss of nearly 80% compared to its peak.

The need for a better understanding of such events for investors is clearly given. Additionally, time and again central banks and government bodies intervene in currency crises and stock market bubbles. The coordinated intervention by China's "national team" of state financial institutions investing at least $ 140 billion[2] in the equity market to curb the fall in stock prices has been the latest example. A failure to understand the downward dynamics of these prices can easily lead to a misinterpretation of the success of such measures. If the intervention occurs towards the beginning of a *dead cat bounce*, the upward movement is likely to provide a false sense of success and an early end to supporting measures. Our results even indicate that the timing of the temporary reversal may be triggered by intervention.

Similarly, there is a plethora of examples of central bank intervention in the FX market attempting to smoothen or even reverse the unraveling of currency bubbles. These attempts are more likely to succeed if we understand the dynamics of the currency's depreciation. The temporary reversal of the downward trend is a natural starting point for a more in depth analysis.

We show that (1) the demand can be split into a loss recovery component as well as an investment component. The loss recovery component is the amount the investor needs to invest in order to have a chance of recovering his losses. The investment component is

[1] see e.g. investopedia

[2] This is the estimate in the beginning of August: http://www.ft.com/intl/cms/s/0/ec29a8b2-3bf8-11e5-8613-07d16aad2152.html?ftcamp=engage/email/textlink/subscribers/todaysftheadlines/crm&utm_source=subscribers&utm_medium=email&utm_term=textlink&utm_campaign=todaysftheadlines#axzz3hwsAzC2N

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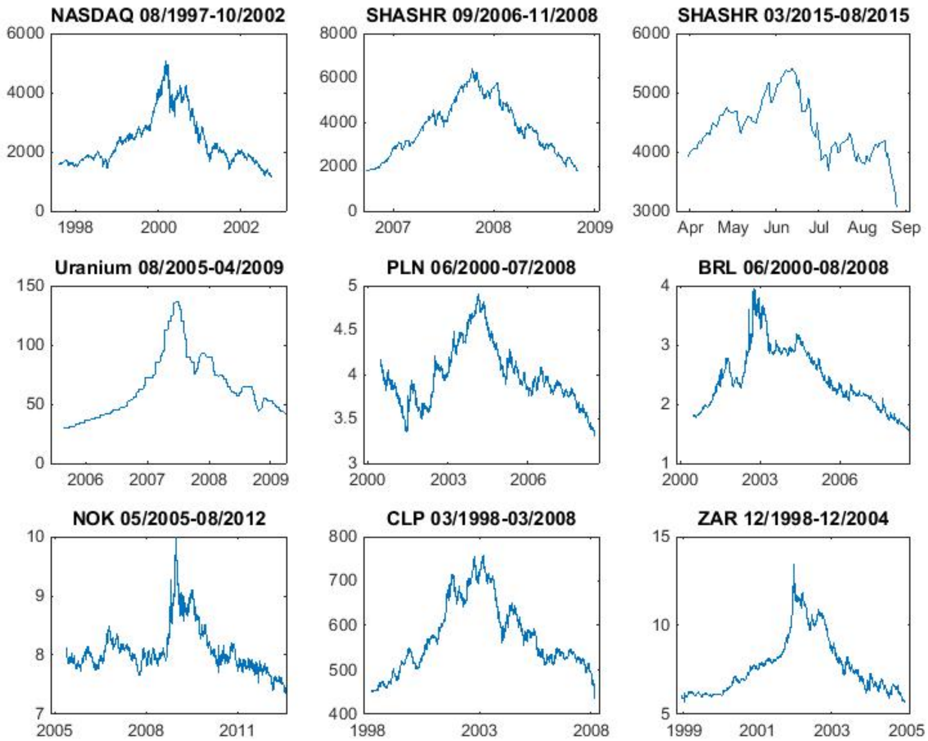
Figure 1: Bursting bubbles followed by bear market rallies



PLN= Polish Zloty, BRL= Brazilian Real, NOK= Norwegian Crown, CLP= Chilean Peso, ZAR= South African Rand

what he is willing to invest beyond this point. (2) The demand is "objectively irrational" meaning that the investor will buy the asset despite negative expected return. Further if he had not suffered an initial loss, he would not enter the market at this point. (3) There is a jump in demand from zero to the desired level once the price falls under a threshold, i.e. demand does not follow a continuous function. (4) This jump is likely to coincide among a large group of investors leading to a large and sudden shift in aggregate demand.

The resulting jump in overall demand explains the strong, sudden reversal in the downward trend. Further, we argue that the reversal can be triggered by seemingly trivial events and that the end of the reversal does not need new fundamental information.

The remainder of the paper is structured as follows. The next section gives a brief overview of the different branches of literature related to this paper. In Section 3, we lay out the assumptions and set-up of the model. Sections 4 and 5 provide the investor's maximization problem as well as an analysis of his decision to re-enter. In Section 6, we analyze the timing when the investor reconsiders his demand and go into the details of the reversal. Section 7 provides a brief summary of the results.

2 Literature

This paper combines several branches of literature such as the ones on bear markets, currency crises, asset pricing and behavioral finance. Yet, the largest contribution is to the literature on financial bubbles. Kaizoji and Sornette (2008) as well as Scherbina and Schlusche (2014) provide concise reviews of prominent approaches on financial bubbles. From their summaries it becomes clear that the existing literature has focused on the build-up and burst of bubbles, ignoring its unraveling.

The wide interest in financial bubbles is not least found in the cost that bubbles can have for the economy as a whole. Jordà et al. (2015) analyze these costs, especially in connection with leverage. Similarly, Brunnermeier and Schnabel (2015) provide insights into the history of bubbles by collecting evidence from the main financial bubbles of the last 400 years. The costs also explain the large literature on central bank intervention. Roubini (2006), Posen (2006), Conlon (2015) are only some examples of a long discussion on whether central banks should burst bubbles. This discussion shows the interest in central bank intervention and the need for a better understanding of its effects. Due to the generality in our approach, our paper also touches on the discussion if central banks should intervene in currency crises and bursting FX bubbles.

In the last years, the research focus on bubbles has shifted from rational investors' behavior towards modeling the actions of noise traders using insights from behavioral research. Scherbina and Schlusche (2014) provide an overview of the recent literature on why financial bubbles occur, highlighting different rational and behavioral approaches. The latter often emphasize the role of herd behavior as in Kaizoji (2010a,b). Brunnermeier (2008) summarizes the different approaches by identifying four types of theoretical models for explaining bubbles.[3] Our model can be seen as the aftermath of his third category. In this class of models, limited arbitrage is caused by rational, well-informed and sophisticated investors' interaction with behavioral traders where the latter are subject to psychological biases. Abreu and Brunnermeier (2003) give an example of such a model. They show why rational arbitrageurs may fail to correct excessive price developments driven by noise traders. Building upon their findings, we show that reversals can happen despite rational arbitrageurs and in absence of fundamental news. Further, we find that the interaction between the inhibited arbitrage in Abreu and Brunnermeier (2003) and our reversal is likely to matter for a better understanding of unraveling bubbles.

The existing behavioral approaches for explaining financial bubbles are quite different from those used in behavioral asset pricing models. Instead of using psychological biases to explain noise trader behavior, the latter focus on optimizing behavioral traders using cumulative prospect theory (CPT) as highlighted by Giorgi and Hens (2006). Introduced by Kahneman and Tversky (1979) and further developed by Tversky and Kahneman (1992), the potential of CPT to explain puzzles in asset pricing has been widely

[3] These four strands of models are: 1) all investors have rational expectations and identical information, 2) investors are asymmetrically informed and the existence of a bubble need not be common knowledge, 3) rational traders interact with behavioral traders and limits to arbitrage prevent rational investors from limiting the price impact of behavioral traders, 4) investors hold heterogeneous beliefs and agree to disagree about the fundamental value.

recognized by now. Barberis (2013) emphasizes this by providing an overview of the contribution of CPT to the asset pricing literature. He also describes the four elements of prospect theory: 1) reference dependence, 2) loss aversion, 3) diminishing sensitivity and 4) probability weighting. Having analyzed the existing literature, he concludes that diminishing sensitivity, i.e. the curvature of the utility function for gains and losses, matters less in financial research.[4] This paper is the first attempt to apply prospect theory to bubbles. In contrast to Barberis (2013), our results are primarily driven by diminishing sensitivity, highlighting their importance in extreme market situations.

Most earlier work in the behavioral asset pricing literature, such as Barberis et al. (2001), assume loss averse investors to be homogeneous. Berkelaar and Kouwenberg (2009) find that the heterogeneity of investors in wealth and reference point matters. This is to some extent in contrast to our results where a difference in wealth or reference value has no effect on the re-entering consideration but only on the amount demanded.

Apart from the literature on financial bubbles, there is a growing literature on bear market rallies. While this is the first theoretical paper on this topic, there has been some empirical work. Using a four state Markov switching model Maheu et al. (2012) divide the history of the S&P500 into bull market, bear market, bull correction and bear market rally. The authors show that the average bear market rally corresponds to a recovery of more than half of the average bear market loss. These results quantify the importance of understanding downward dynamics and specifically bear market rallies. For future empirical work, our results hint that it may be worth differentiating between bear market rallies close to the equilibrium level and those at the height of the unraveling.

Generally, theoretical work on downward dynamics such as bear market rallies is scarce. An exception is the literature on fire sales such as by Miller and Stiglitz (2010). The idea that balance sheet effects matter seems to be largely accepted at least in times of crisis. However, fire sales merely exacerbate the downward pressure. It follows, that in presence of fire sales, the mechanism explaining temporary reversals will have to be even stronger.

3 Model Set-up

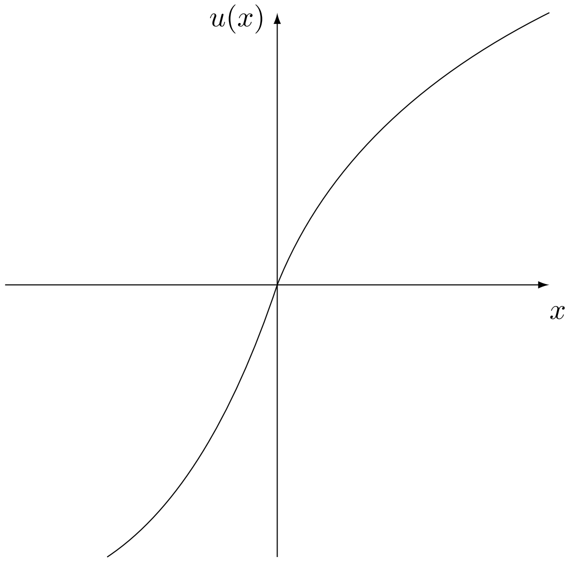
Our model looks at an investor's reevaluation of his portfolio choice during the unraveling of a bubble after suffering an initial loss.

Consider an investor with preferences according to cumulative prospect theory (CPT) as introduced by Kahneman and Tversky (1979). The investor's utility function is split into a concave part for gains with respect to a reference value and a convex part for losses. This implies risk seeking behavior once losses have occurred and risk averse behavior after gains. Figure 2 illustrates the utility function according to cumulative prospect theory.

Using experimental data, Tversky and Kahneman (1992) find that the utility function

[4] This is the case as the existing literature focuses on returns close to the reference value.

Figure 2: CPT preference utility function



is best described as

$$u(x) = \begin{cases} u^+(x) & \text{if} \quad x \geq 0 \\ u^-(-x) & \text{if} \quad x < 0 \end{cases} \tag{1}$$

$$u^+(x) = x^\alpha \quad \text{for} \quad x \geq 0 \quad \text{and} \quad u^-(-x) = -\lambda(-x)^\beta \quad \text{for} \quad x < 0.$$

In line with the experimental evidence by Tversky and Kahneman (1992), we assume $0.5 < \alpha = \beta < 1$.[5] Tversky and Kahneman (1992) find a loss aversion parameter of $\lambda \approx 2.25$. A higher loss aversion parameter λ leads to a stronger punishment for losses. For simplicity and without change in the results,[6] we set $\lambda = 1$.

Assumption 1. *The investor's utility function is given by*

$$u(x) = \begin{cases} x^\beta & \textit{if} \quad x \geq 0 \\ -(-x)^\beta & \textit{if} \quad x < 0. \end{cases}$$

with $0.5 < \beta < 1$.

Following Kahneman and Tversky, x is given by the change of wealth relative to a reference value Y_0. The change in wealth is dependent on the investor's demand for a single risky asset. Without loss of generality, we assume that the risk-less alternative yields zero interest. The risky asset has two possible outcomes. With probability π, the asset yields a high final value Y_g. With probability $(1 - \pi)$, the asset realizes a final value $Y_b < Y_g$. We assume that, in a bubble setting, the good state is less likely:

[5] It is worth mentioning, that while Bernard and Ghossoub (2010) depend on $\alpha < \beta$ for an interior solution in a model similar to ours, this assumption is not necessary in our setting.

[6] The implications of this simplification are analyzed in Appendix I.

Assumption 2. *The probability of the positive outcome is given by*

$$\pi < 0.5$$

Though common, the setting with binary outcomes is easily criticized. We are arguing that the specification is more appropriate in bubble settings than in a standard market situation. Due to their nature, bubble assets are likely to be perceived as having quasi binary outcomes.[7]

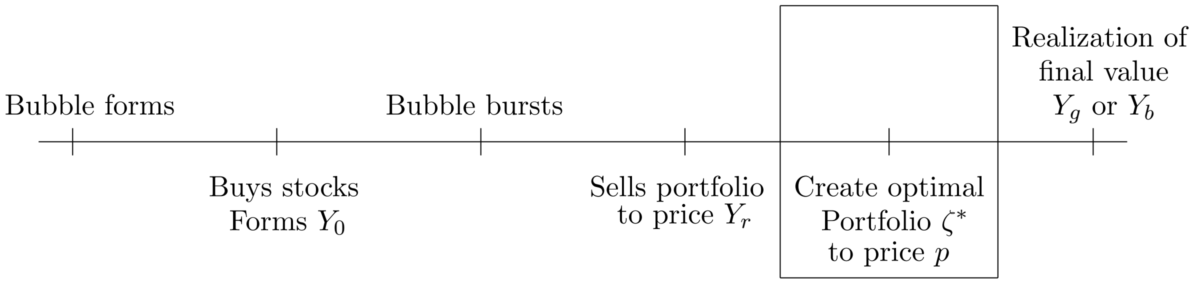
In line with Kahneman and Tversky (1979), the probability π is subjective. Agents perceive probabilities in a distorted way where low probabilities are seen as larger, while larger probabilities are undervalued. In this setting, there is so far no need to specify what the objective probabilities are.

Figure 3 illustrates the chronological setting of the model. While the bubble grows, the investor enters the market buying one unit of the asset.

Assumption 3. *The investor invests all his disposable wealth into the risky asset when first entering.*

The investor now forms his reference value Y_0. In contrast to the existing literature, we refrain from imposing a functional form on the reference value.[8] We can think of Y_0 as the price that the investor is convinced the asset will reach.

Figure 3: Time line



The box surrounds the area explicitly incorporated into the model. The investor's actions are shown below the line, while the market development is above.

The burst of the bubble is assumed to be caused by a change in market expectation, leading to the subjective expected value

$$E(v_s) := \pi Y_g + (1 - \pi) Y_b. \tag{2}$$

After the bubble bursts, the investor sells all his assets to price $Y_r < Y_0$. Consequently, he realizes a loss with respect to his reference value of $(Y_0 - Y_r)$. By assumption,

[7]Consider the example of Internet stocks in the 1990s. Investors had the hope to "find the next Amazon" leading to large gains. At the same time, there was a large risk of bankruptcy for firms which did not become profitable.

[8]Most authors like Barberis et al. (2001), Gomes (2005) and Bernard and Ghossoub (2009) use the risk free rate as the growth rate for the reference value, which makes it easier to solve the models. One advantage of our approach is that we do not depend on this assumption.

the CPT investor is not able to engage in short selling. This is a resonable assumption when seeing the CPT investor as less sophisticated.

Assumption 4. *No short selling by the CPT investor.*

From this point onwards, the CPT investor is loss-driven. He is willing to take large risks hoping to recover his losses. After a further fall in the price to p, the investor decides on his new portfolio ζ. The timing of the decision is hence exogenous[9] while the amount invested is endogenous. A larger ζ implies a larger exposure to the risky asset. Due to the short selling constraint, $\zeta \geq 0$ must hold. Further, given that the investor sold his holdings to price Y_r the investor faces a budget constraint[10] of $\zeta p \leq Y_r$. Consequently, the bounds are given by

$$0 \leq \zeta \leq \frac{Y_r}{p}. \tag{3}$$

$p < Y_r$ implies that the upper bound is larger or equal to one and increasing with a lower price p. Finally, the investor will realize either Y_g or Y_b. Without loss of generality we set $Y_b > 0$. This allows us to interpret Y_g, Y_0, Y_b, Y_r and p as price levels.

Assumption 5. *The hierarchy of price levels as explained above is summarized as*

$$Y_g > p > Y_b \qquad \textit{and} \qquad Y_0 > Y_r > p$$

We can limit ourselves to cases where the investment yields a negative subjective expected value. The limitation is justified as we are considering reversals at the height of the unraveling. If the asset yielded a positive return, the investment would be attractive for risk neutral investors as well. In this case, there would be no further downward pressure in the price and the reversal would not be temporary. Put differently, without selling pressure the asset would be close to its equilibrium price. This implies that the reconsideration would occur towards the end of the unrevealing rather than at its height.

Assumption 6. *The risky asset has a negative expected return*

$$\pi(Y_g - p) - (1 - \pi)(p - Y_b) < 0.$$

Assumption 6 provides a lower bound for the price when considering to re-enter, given by:

$$p > \pi Y_g + (1 - \pi)Y_b.$$

[9]In this model, the timing when the investor reconsiders his portfolio is exogenous. This is a strong simplification as it implies the absence of intertemporal optimization. However, the setting is sufficient to show how the willingness of the investor depends on the price of the asset and to demonstrate the discontinuous reaction in demand resulting from CPT preferences.

[10]This budget constraint results from Assumption 3 that the investor is fully invested at the height of the bubble. Given the utility function, he is more likely to invest an amount smaller than unity. In that case the budget constraint is relaxed. Consequently, this budget constraint is the minimum budget constraint.

Given our Assumptions 1 and 2 that $\beta < 1$ and $\pi < 0.5$, Assumption 6 also implies that an investor with CPT preferences who has not suffered an initial loss would not invest into the risky asset, as

$$\pi(Y_g - p)^\beta - (1-\pi)(p - Y_b)^\beta < 0. \tag{4}$$

The implications of Assumption 6 for the reversal are discussed in more detail in Section 6.

4 The Investor's Maximization

Let us now turn to the investor's maximization problem when choosing the optimal portfolio ζ. There are two cases which need to be considered.

I) The investor is able to recover his initial losses with respect to his reference value in the good state.

II) Even with the good outcome, the investor cannot recover his initial loss with respect to his reference value. This implies that the whole maximization takes place in the convex part of the utility function.

It is crucial to keep in mind that the utility in the good state depends on the endogenous variable ζ. This implies that it depends on the size of ζ whether the agent can recover his losses and hence which case we need to consider. The cases are endogenous.

4.1 Case I: Complete Recovery of Losses Possible

For now, let us assume that the investor makes an overall gain if the outcome is positive, i.e. Case I. It follows that given the utility function in Assumption 1, the investor's subjective expected utility when creating his optimal portfolio is given by

$$\begin{aligned} E(U) &= \pi u^+(x_g) + (1-\pi) u^-(x_b) \\ &= \pi (x_g)^\beta - (1-\pi)(-x_b)^\beta . \end{aligned} \tag{5}$$

Following the setting above, x_g and x_b are given by initial loss $Y_0 - Y_r$ and the gain and loss from the reinvestment, respectively:

$$x_g = \zeta(Y_g - p) - (Y_0 - Y_r) > 0 \quad \text{and} \quad x_b = \zeta(Y_b - p) - (Y_0 - Y_r) < 0.$$

It follows that

$$\begin{aligned} E(U) &= \pi[\zeta(Y_g - p) - (Y_0 - Y_r)]^\beta - (1-\pi)[-(\zeta(Y_b - p) - (Y_0 - Y_r))]^\beta \\ &= \pi[\zeta(Y_g - p) - (Y_0 - Y_r)]^\beta - (1-\pi)[\zeta(p - Y_b) + (Y_0 - Y_r)]^\beta \end{aligned} \tag{6}$$

Loss recovery in the good state implies therefore that $\zeta(Y_g - p) - (Y_0 - Y_r) \geq 0$. This in turn, leads to the condition of a minimum investment into the asset necessary for Case I to hold. This is given by

$$\zeta_{Min} = \frac{Y_0 - Y_r}{Y_g - p} \tag{7}$$

The minimum investment is needed to have a chance to recover the initial losses. It represents the ratio of the initial loss with respect to the reference value and the potential gain by investing into the risky asset. Together with the upper bound, the minimum ζ for Case I leads to

$$\begin{aligned} \frac{Y_0 - Y_r}{Y_g - p} \leq \zeta \leq \frac{Y_r}{p} \\ p \leq Y_g \frac{Y_r}{Y_0} \end{aligned} \tag{8}$$

It is important to note that this is independent of the bad outcome. Hence, the case differentiation is not connected to the expected value but only to the size of the positive outcome.

The investor's maximization problem is given by

$$\max_{\zeta} E(U), \tag{9}$$

leading to the following proposition:

Lemma 1. *Given Assumptions 1, 3, 5 and 6: For all parameters within Case I, the investor invests a positive amount given by ζ^* up to his budget constraint, where*

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} \tag{10}$$

with $\Omega = \frac{Y_g - p}{p - Y_b} > 0$, $\Phi = \frac{\pi}{1-\pi} > 0$ and $\gamma = \frac{1}{1-\beta} > 2$. He does so despite negative expected returns.

Proof. See Appendix A.

Ω gives the ratio of the outcome in the good state and the absolute of the outcome in the bad state. $\Omega > 1$ implies that the gains in the good state exceed the losses in the bad state. Φ gives the ratio of the subjective probability for the good state and the subjective probability of the bad state. $\Phi < 1$ implies that the good state is perceived as less likely than the bad state. Given that $\pi < 0.5$, we find $\Phi < 1$. Following the definitions above, $\Omega\Phi$ is the probability weighted outcome ratio. A $\Omega\Phi < 1$ implies a negative subjective expected value, which is equivalent to Assumption 6.

We can interpret γ as the elasticity of intertemporal substitution. A larger β implies a higher elasticity of intertemporal substitution and hence, lower cost for a suboptimal distribution of consumption across periods. In this setting, this is equivalent to lower cost for an uneven distribution across the states of nature.

Lemma 1 shows the importance of loss recovery. When incentivized by even a remote chance to recover losses, the investor will invest a positive amount even if he expects negative returns.

Corollary 1. *The optimal investment can be decomposed into a loss recovery component ζ_{Min} and an investment component IC:*

$$\zeta^* = \zeta_{Min} + IC, \tag{11}$$

where

$$\zeta_{Min} = \frac{Y_0 - Y_r}{Y_g - p} > 0 \quad \text{and} \quad IC = \frac{\Omega^{\gamma-2}\Phi^{\gamma}(1+\Omega)(Y_0 - Y_r)}{(1-\Omega^{\gamma-1}\Phi^{\gamma})(p - Y_b)} > 0. \tag{12}$$

The recovery component is what is necessary in order for the investor to have a chance to recover his initial losses. The investment component is given by the remainder.

Proof. See Appendix B.

The loss recovery component ζ_{Min} results from risk seeking behavior i.e. the convex part of the utility function. The investment component, in contrast, results from the concave part which corresponds to risk aversion.

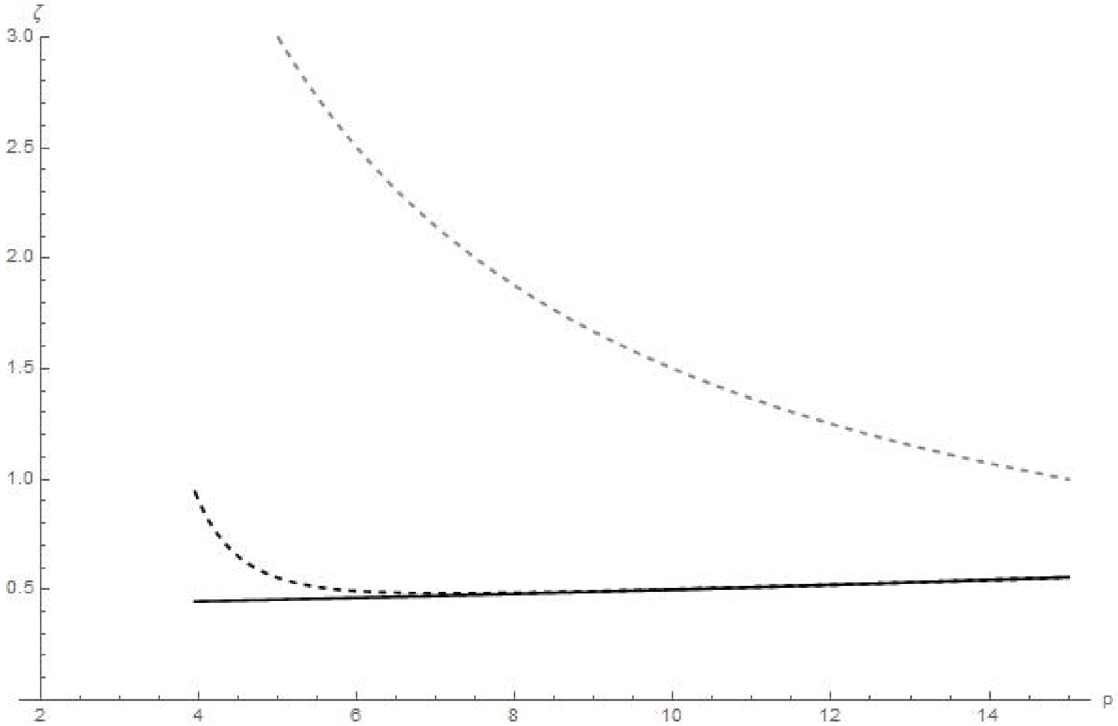
As mentioned before, the optimization problem is only correctly specified if the optimal ζ^* in Equation (10) fulfills the minimum ζ condition in Equation (7). Equation (11) shows that this condition is fulfilled for all ζ^* as $IC > 0$. Let us now take a closer look on the effect of the price p. Given that we are analyzing the unraveling of a bubble, it makes sense to look at what happens when the price falls further. We can show that:

Corollary 2. *A lower price leads to a larger investment component and a lower loss recovery component of ζ^*.*

Proof. See Appendix C.

The reasoning is that the risky asset becomes more profitable, due to the lower price. Hence, less investment is needed to recover prior losses. At the same time, a more lucrative investment becomes more attractive and the investment component rises. We are mostly interested in the overall effect on the optimal demand which results from the interplay of the two effects as illustrated in Figure 4. The solid black line is the minimum ζ_{Min} as in Condition (7). The dashed black line illustrates the optimum ζ^* as in Equation (10) and the dashed gray line shows the upper bound for ζ given by the budget constraint. The decline in the loss recovery component dominates for high prices as the investment remains unprofitable. However, for sufficiently low prices, the investment component takes over and rapidly increases with falling prices driving the overall demand up.

Figure 4: Minimum, Maximum and Optimal ζ



The solid black line is the minimum ζ_{Min} as in Condition (7). The dashed black line illustrates the optimum ζ^* as in Equation (10). The dashed gray line shows the upper bound for ζ given by the budget constraint. The parameter values are given by: $Y_g = 60$, $Y_r = 15$, $Y_0 = 40$, $Y_b = 1$, $\pi = 0.05$ and $\beta = 0.8$. As in the analysis we set $\lambda = 1$.

Figure 5 illustrates the effect of a change in the parameter values of π, Y_g, Y_0 and Y_r, respectively. In the upper two cases, an increase in π and Y_g, respectively, leads the risky asset to be more lucrative. Hence, for this specification the investment component drives the demand even for higher values of p. In the bottom left cases, an increase in the reference value leads to an upward shift of both minimum and optimal demand. This is the case as the investor is loss driven and a larger loss leads to a higher willingness to invest. Similarly, a larger loss due to a lower Y_r shifts both minimum and optimal demand up. Additionally, the budget constraint is more restricting due to the lower recovery.

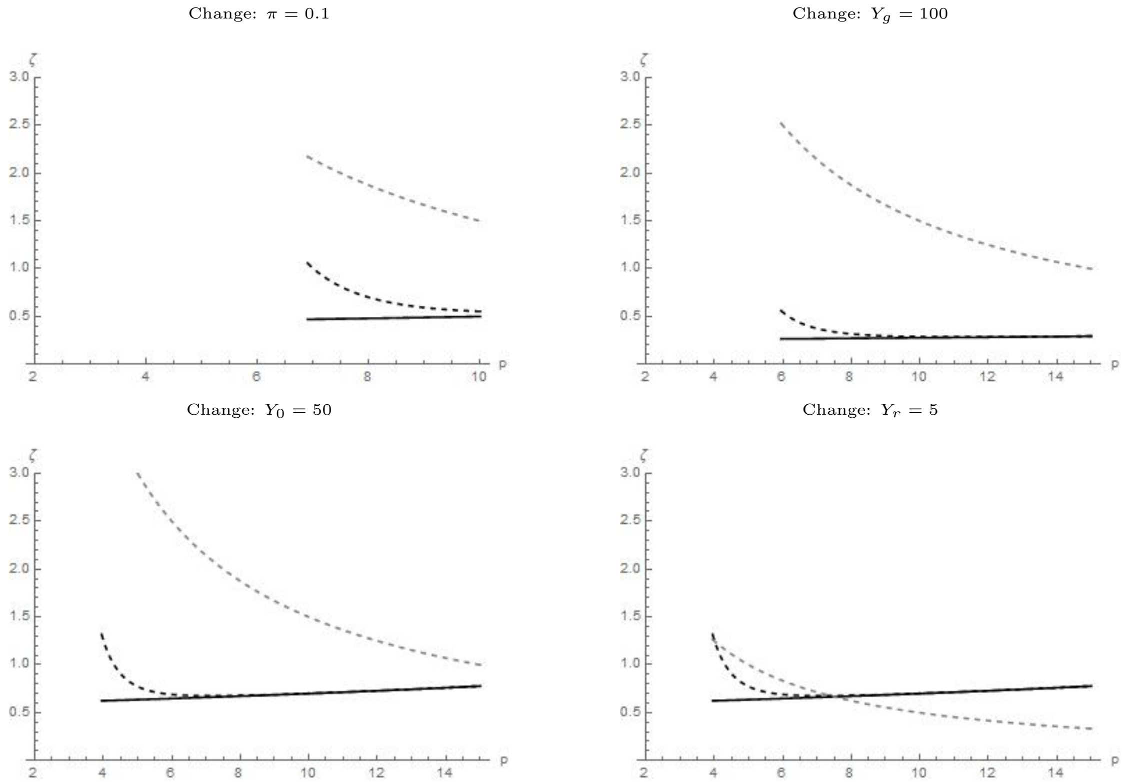
4.2 Case II: Only Partial Recovery of Losses Possible

We now turn to the case, where it is impossible for the investor to recover his losses fully. The expected utility function is consequently given by

$$\begin{aligned} E(U) &= \pi u^-(x_g) + (1-\pi)u^-(x_b) \\ &= -\pi[-\zeta(Y_g - p) + (Y_0 - Y_r)]^\beta - (1-\pi)[\zeta(p - Y_b) + (Y_0 - Y_r)]^\beta. \end{aligned} \tag{13}$$

The investor's optimization in Case II leads to the following lemma:

Figure 5: Minimum, Maximum and Optimal ζ



The images illustrate ζ as in Figure 4 for different parameter values. The solid black line is the minimum ζ_{Min} as in Condition (7). The dashed black line illustrates the optimum ζ^* as in Equation (10). The dashed gray line shows the upper bound for ζ given by the budget constraint. The 4 graphs illustrate the effect of a change in π, Y_g, Y_0 and Y_r, respectively, compared to the benchmark case in Figure 4.

Lemma 2. *Given Assumptions 1, 2, 3, 4, 5 and 6: Whenever the investor has no chance to regain his losses he either invests all he can or nothing.*

Proof. See Appendix D.

The intuition for this result is straightforward. The investor is risk seeking in his optimization in Case II. If the terms of the investment are unacceptable, the investor will not invest anything, for example when the investment implies a certain loss. If the terms are acceptable, the investor will invest all his disposable wealth into the asset in order to have a chance to get as close as possible to recovering his prior losses. He perceives the riskiness of the asset positively, as an increase in the spread of good and bad states gets him, *ceteris paribus*, closer to recovering his losses.

Combining Lemmas 1 and 2 leads to the following. If the investor chooses to re-enter the market, he will invest ζ^* up to his budget constraint. The investor will only re-enter however, if the utility from investing is larger than the utility from abstaining from the market.

5 The Decision to Re-enter

The focus of our analysis is the investor's willingness to re-enter the market given that he reconsiders his absence from the market at a specific point in time. In order to determine this, we need to compare the investor's utility from remaining outside the market and re-entering.[11]

An investor compares re-entering and investing ζ^* to the certain loss when abstaining the market. For re-entering to be optimal, the following condition needs to be fulfilled

$$E(U(\zeta^*)) \geq E(U(\zeta = 0)).$$

Lemma 3. *Given Assumptions 1 and 6: The condition for re-entering and hence for positive demand is given by*

$$\pi \left(\frac{IC}{\zeta_{Min}}\right)^{\beta} \left[\left(\frac{1}{\Omega^{\beta}\Phi}\right)^{\gamma} - 1\right] - 1 \leq 0. \tag{14}$$

Proof. See Appendix E.

Lemma 3 describes the moment when an investor is willing to re-enter the market. Any investor, for whom this condition is not fulfilled, will not re-enter the market upon reconsidering his portfolio. It is important to keep in mind that this model focuses on a bear market. Consequently, we are interested in what happens when the price falls and whether this leads to a sudden increase in demand. Lemma 1 and Lemma 3 lead to the following conclusion:

Proposition 1. *When p falls, the "potential" demand, i.e. the desired amount invested upon reconsideration, jumps from zero to the optimal level.*

Proof. See Appendix F.

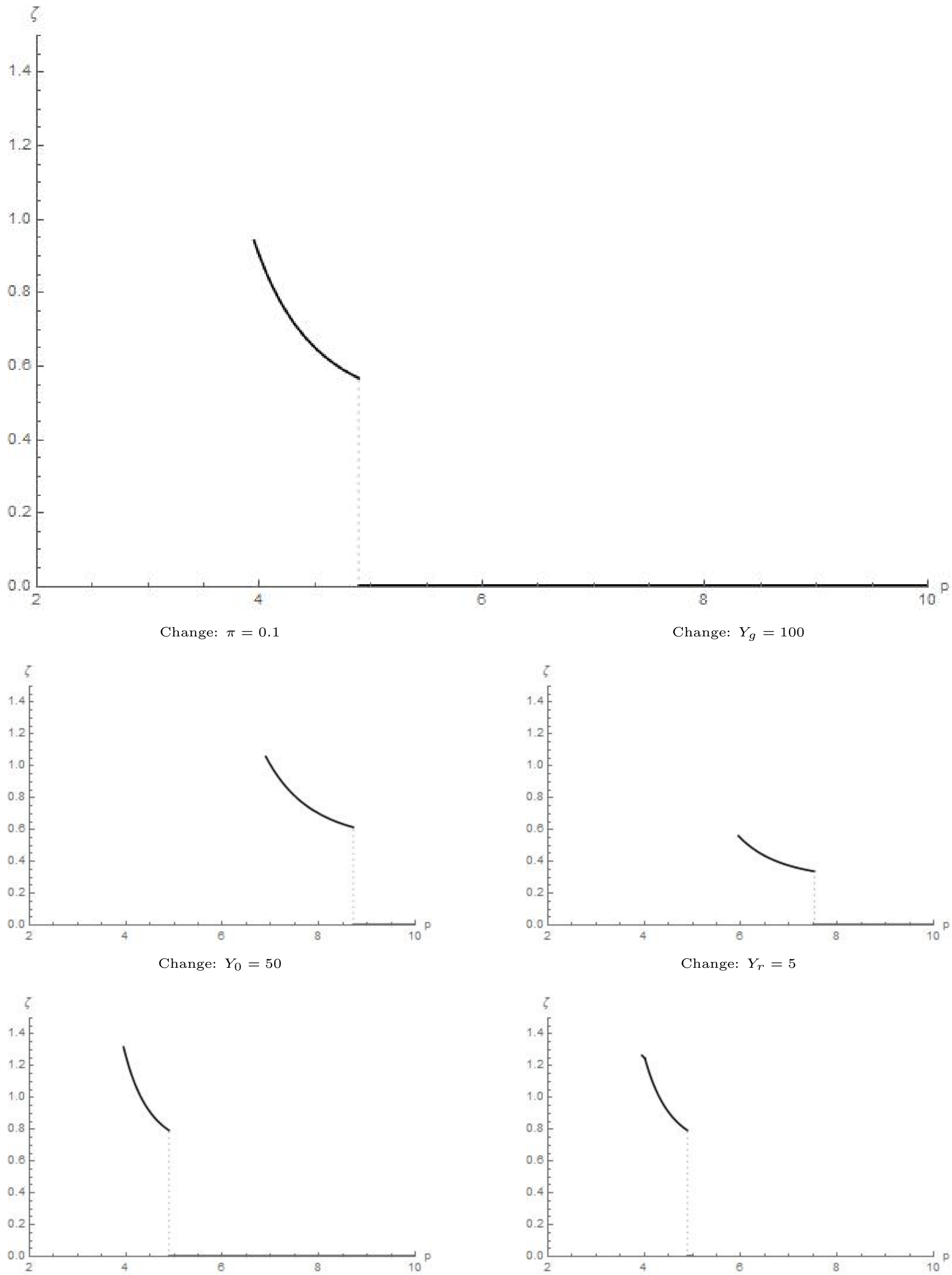
We argue that this sudden jump can be responsible for the temporary reversal of the downward trend.

The size of the jump in demand is given by the optimal demand in Equation (10) from Lemma 1 as the investor has no holding prior to re-entering.

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^{\gamma}\Phi^{\gamma}}{\Omega - \Omega^{\gamma}\Phi^{\gamma}}$$

Figure 6 depicts Proposition 1 for different parameter values. The solid line is given by the investor's demand. The jump in "potential" demand occurs once Condition (14) is fulfilled. In the first specification, the investor switches from zero demand to over 56% of what he held when the bubble burst. For lower prices, the demand rises continuously. The discontinuity in the demand function already hints the potential for swift upward

Figure 6: Proposition 1: The jump in demand



The images illustrate the optimal investment for different parameters. The gray line is the minimum ζ_{Min} as in Condition (7). The dashed black line illustrates the optimum ζ^* as in Equation (10). The dashed gray line shows the upper bound for ζ given by the budget constraint. The first image illustrates the benchmark case: $Y_g = 60$, $Y_0 = 40$, $Y_r = 15$, $Y_b = 1$, $\beta = 0.8$ and $\pi = 0.05$. As in the analysis we set $\lambda = 1$. The subsequent images result from a change in pi, Y_g, Y_0 and Y_r, respectively.

movements in the price. From one moment to the other, an investor is willing to invest large amounts into an asset which he sold off before.

The four additional graphs highlight the *ceteris paribus* effect of an increase in the probability of the good state π, a better outcome in the good state Y_g, a higher reference value Y_0 and a decrease in the recovery price Y_r. Doubling the probability π leads to a similar size in the jump, yet the jump occurs earlier, i.e. for higher p. This is intuitive as a higher π implies a more profitable investment. The same is true for an increase in the good outcome. In the latter case however, the investment reaches only around 34% of the prior holdings.

The two bottom graphs show an increase in the reference value and a decrease in the recovery rate, respectively. Both are equivalent to a higher loss with respect to the reference value and hence have the same effect. The only difference is that a change in Y_r lowers the budget constraint, restricting the slope of the demand for low p. The reaction to a change in the initial loss leads us to the main result of the paper. Using Lemmas 1 and 3 we can show that:

Proposition 2. *The decision to re-enter is independent of the initial loss; however the size of the potential investment positively depends on it.*

Proof. See Appendix G.

A group of investors with similar expectations about the asset's true value are willing to enter the market at a similar time, independently of their prior losses and reference value. The size of each of the investor's investments will however differ. The current disposable wealth of the investor is given by Y_r. The above implies that the timing of the reinvestment is also independent of the disposable wealth.[12]

Our results agree with Berkelaar and Kouwenberg (2009) on that the heterogeneity of investors with regard to wealth and reference value matters. In our setting however, the implications are quite different. While the heterogeneity is important to explain the size of individual demand, the timing is not affected. This implies that from a macro perspective, the heterogeneity matters much less as the individual demand jumps at the same time leading to a jump in aggregate demand. Given the importance of the timing, we discuss this in more detail in the next section.

6 Equilibrium Price

The above analysis has so far considered the CPT investor in isolation. In this section, we take the previous results further by looking at the interaction of multiple CPT and risk-averse (RA) investors. The model proposed here is not a pure equilibrium model, but describes how the market reaches its equilibrium.

[11] In the following sections we are focusing on interior solutions for the optimal investment, i.e. cases not bound by the budget constraint. Bound cases are considered in Appendix H.

[12] This is true, as long as the disposable wealth is sufficient to allow for the optimal investment. If this is not given, the re-entering will be delayed.

The layout of the model is summarised as follows. In period 0, the CPT investors update their rational expectations about the pay-offs of the asset and trade with the RA investors before the latter update their own expectations. The RA investors do not know the exact demand function of the CPT investors. Therefore, in period 1 the RA investors submit sell orders anticipating the maximum possible demand of the CPT investors to a given price. The anticipated maximum demand, however, assumes the demand function of the CPT investors to be continuous.[13] This creates a disparity between the anticipated demand and the actual demand due to the jump in the latter. Consequently, there are two possible cases. If the actual demand falls short of the anticipated maximum demand, the RA investors submit further sell orders in the next periods until the market is in equilibrium.[14] If the actual demand exceeds the anticipated demand, this implies that the orders submitted by the RA investors imply too low a price. It follows that the price will rise in the following period in order to bring the market into equilibrium. This reversal in the price is what we describe as a dead cat bounce. In what follows, we go into the details of the model.

There is a set of CPT and RA investors, each holding part of the asset.

Assumption 7. *The total amount of investors and assets is normalised to unity. n investors are risk-averse investors (RA), while $1 - n$ are CPT investors, following the optimisation behaviour described above. At the start of the bear market, a total amount x of the asset is held by the RE and $1 - x$ is held by the CPT investor.*

This allows us to differentiate between different asset classes and gives an idea, in which markets we would expect a dead cat bounce to appear. For simplicity, we also normalise the updated pay-off of the asset to

Assumption 8. *The pay-off of the asset in the good state is normalised to 1, while the bad state's pay-off is set to 0.*

$$Y_g = 1 \qquad \textit{and} \qquad Y_b = 0 \tag{15}$$

Furthermore,

Assumption 9. *We assume that all CPT investors receive information that the asset is overvalued before the RA investors do. The CPT investors are hence able to sell all their holdings to price p_0 to the RA investors, where p_0 is determined by the RA investors expectations before receiving the update.*

The results do not necessarily depend on the strict form of this assumption, but it helps to reduce the number of additional parameters. After learning about the updated pay-off structure of the asset, the RA investors' demand for the asset is determined by the maximisation of their utility function.

[13] One option is that the RA investors simply ignore the influence of the perceived loss by the CPT investors. We keep the function general in order not to pose strong restrictions.

[14] One can think of this as the RA investors submitting orders with a fix price and quantity.

Assumption 10. *Each RA investor maximizes a CRRA utility function given by*

$$U_{RA} = \pi(X_g)^\alpha + (1-\pi)(X_b)^\alpha, \tag{16}$$

where $1-\alpha$ is the degree of risk aversion of the RA investor, X_g is the wealth in the good state and X_b is the wealth in the bad state.

In contrast to the CPT, the CRRA utility function is taking into account the total wealth, rather than the change in wealth.

Assumption 11. *The total wealth of the RA investor in the good state is given by*

$$\begin{aligned} X_g &= \frac{x}{n}Y_g + \Delta(Y_g - p) + W + (Yg - p_0)\frac{1-x}{n} \\ &= \frac{x}{n} + \Delta(1-p) + W + (1-p_0)\frac{1-x}{n} \\ &= \frac{1}{n} + \Delta(1-p) + W - p_0\frac{1-x}{n}. \end{aligned} \tag{17}$$

In the bad state, this is equivalent to

$$\begin{aligned} X_b &= \frac{x}{n}Y_b + \Delta(Y_b - p) + W + (Y_b - p_0)\frac{1-x}{n} \\ &= -\Delta p + W - p_0\frac{1-x}{n} \end{aligned} \tag{18}$$

The total wealth in the good state decomposes as follows: $\frac{x}{n}Y_g$ is the final pay-off of the initial holdings of the individual RA investor. It is given by the total amount of the asset held by RA investors x, divided by the number of RA investors n. The amount purchased by an individual RA investor is given by Δ. p corresponds to the price he is paying. W is given by the exogenous, independent wealth of the RA investor and $(Yg-p_0)\frac{1-x}{n}$ is the profit from buying all of the CPTs' asset holdings. In the bad state, the total wealth is derived equivalently.

The maximisation of the RA investors' utility results in an optimal change in demand Δ^*. For this analysis, it makes sense to consider the total supply.

Lemma 4. *The total supply is given by the total amount of assets to be sold by the n RA investors*

$$\begin{aligned} S(p) = -n\Delta^* &= -n\frac{(W - p_0\frac{1-x}{n})(\Omega^\epsilon\Phi^\epsilon - 1) - \frac{1}{n}}{p(\Omega + \Omega^\epsilon\Phi^\epsilon)} \\ &= \frac{1 - (Wn - p_0(1-x))(\Omega^\epsilon\Phi^\epsilon - 1)}{p(\Omega + \Omega^\epsilon\Phi^\epsilon)}, \end{aligned} \tag{19}$$

where $\epsilon = \frac{1}{(1-\alpha)}$ is one over the risk aversion parameter. The supply function $S(p)$ is a concave function increasing in price p.

We do not restict short selling for RA investors, as they are likely to be more sophisticated. However, such a restriction would not significantly alter the results.

When choosing how much to supply, the RA investors anticipate the total maximum demand for a given price and supply the anticipated (maximum) equilibrium amount. The logic is as follows. In a given period, the suppliers observe the demand to the current price. The highest demand they deem possible in the next period would follow a function which is zero at the current price but increases as the price falls in the next period. Why would the RA investors go for the maximum demand? Assume that the RA investors collectively under-anticipated the total demand. In this case, each of the suppliers would have had an incentive to supply more. In contrast, if the RA investors assumed the total demand to be too high, none of the suppliers would have profited from supplying less, as the price will simply fall in the next period and the excess supply will be met by the resulting higher demand. The RA investors, therefore supply in order to meet the maximum demand they deem possible. Thereby, the RA investors are more careful than in standard equilibrium models.[15]

Assumption 12. *Given a price p_t in the current period and a price p_{t+1} in the next period, the anticipated maximum demand is given by $\Upsilon_{p_t}(p_{t+1})$, with $\Upsilon_{p_t}(p_t) = 0$, $\frac{\partial \Upsilon_{p_t}(p_{t+1})}{\partial p_{t+1}} < 0$ and $\frac{\partial^2 \Upsilon_{p_t}(p_{t+1})}{\partial p_{t+1} \partial p_{t+1}} > 0$. Once the suppliers observe a positive demand, they can anticipate the actual demand.*[16]

If the actual demand is equal to the anticipated maximum demand, the market will be in equilibrium. This is the standard case considered in equilibrium models. If the demand remains non-positive, the suppliers will repeat the step above, using the anticipated maximum demand function $\Upsilon_{p_{t+1}}(p_{t+2})$. If the demand is positive but below the supply, it is optimal for the suppliers to lower the price further. They will lower it more cautiously however taking into account the current level of demand.

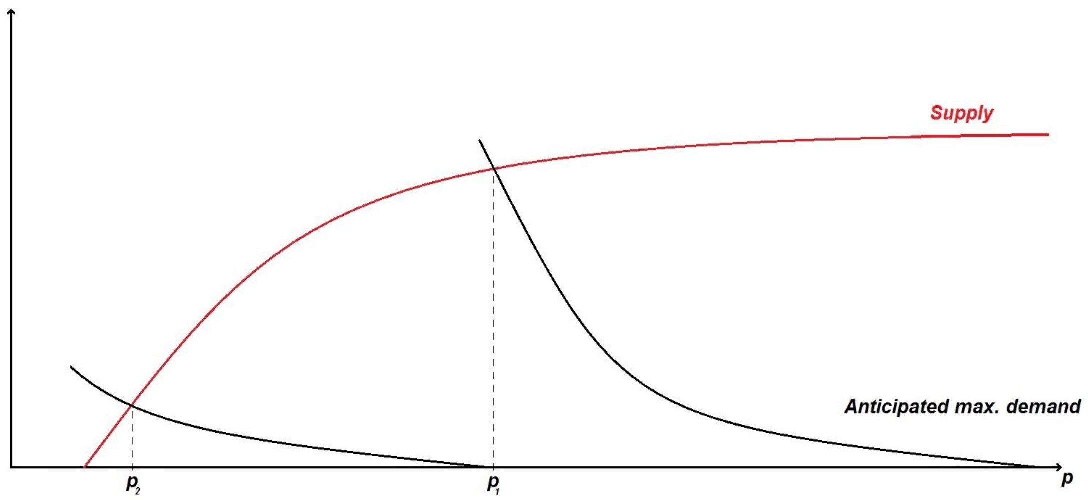
Figure 7 illustrates the procedure described above. The x-axis shows the price. As the price falls, the anticipated maximum demand is increasing until it reaches the supply function at price p_1. This is the maximum price, which the suppliers expect to lead to an equilibrium. They hence choose to supply the amount $S(p_1)$ in this period. As there is no demand, the suppliers use the anticipated maximum demand function once more in the next period, leading to the supply of $S(p_2)$.

This describes the development of the supply by the RA investors. In the next step, let us consider the equilibrium in the market given by the (actual) aggregated demand and the total supply. The total demand of the CPT investors given Assumptions 7, 8

[15] In order to not make it optimal to submit sell orders for the whole supply, we could assume that there are marginal costs to submitting orders. This would lead the suppliers to, again, go for the anticipated maximum demand, given that they are confident enough about this being the maximum.

[16] The definition of $\Upsilon_{p_t}(p_{t+1})$ is kept fairly independent on purpose in order to keep the results general but interpretable.

Figure 7: The anticipated maximum demand



The red line illustrates the supply function $S(p)$. The black line illustrates the development of the anticipated maximum demand in two steps.

and 9 is given by

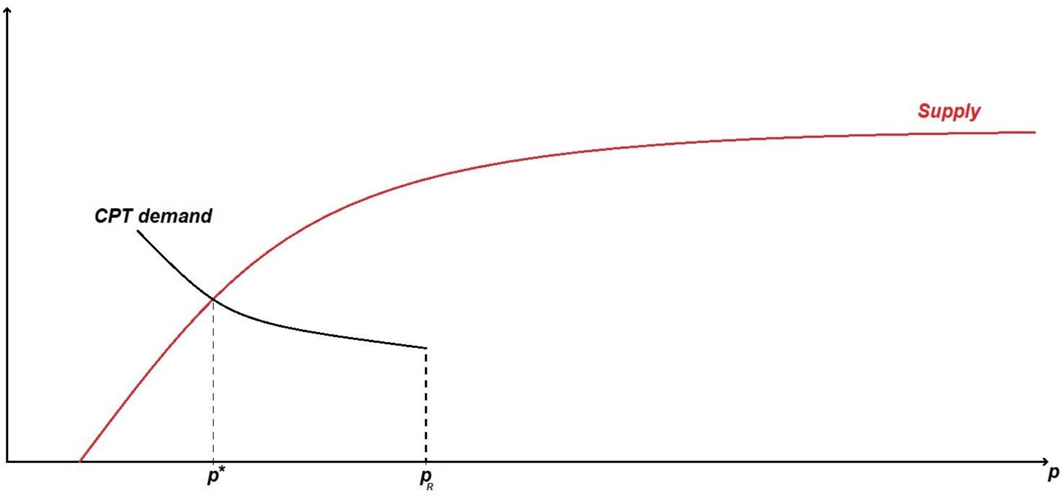
$$D(p) = (1-n)\zeta^*(p) = (1-n)\left(\frac{1-x}{1-n}\right)\left(\frac{Y_0 - p_0}{p}\right)\frac{(1+\Omega^\gamma\Phi^\gamma)}{(\Omega - \Omega^\gamma\Phi^\gamma)}$$
$$= (1-x)\left(\frac{Y_0 - p_0}{p}\right)\frac{(1+\Omega^\gamma\Phi^\gamma)}{(\Omega - \Omega^\gamma\Phi^\gamma)}. \tag{20}$$

The efficent price p^* is hence given by $D(p^*) = S(p^*)$. Figure 8 illustrates the equilibrium. We denote the price at the re-entry of the CPT investors by p_R. As highlighted in Proposition 2, this point is independent of the initial loss and will hence be the same for heterogeneous investors.

The next part combines the anticipated maximum demand and the actual demand function. Figure 9 illustrates an example where the model exhibits a dead cat bounce. At the first intersection of the anticipated maximum demand and the supply function, at price p_1, the actual demand is zero. At the second intersection at p_2, the actual demand exceeds the anticipated demand, due to the jump at $p_R > p_2$. It follows that in period 1, there is no trade and the price is set to p_1. In period 2, the price falls to price p_2 and $\Upsilon_{p_1}(p_2)$ units are traded. As demand exceeds supply at this point, the price will increase over the next periods until it reaches the equilibrium level p^*. Consequently, after falling to p_2, the price experiences a reversal rising by $p^* - p_2$. This is the size of the dead cat bounce.

Similarly, figure 10 shows an example, which does not lead to a dead cat bounce. In period 1, the price reaches p_1. The actual demand given by the black line is not equal to the supply, however, it is positive. Consequently, $D(p_1)$ of the asset is traded. Having now observed the actual demand, the suppliers will supply only as much in the

Figure 8: The equilibrium price



The red line illustrates the supply function $S(p)$. The black line illustrates the aggregated optimal demand by the CPT investors.

next period as is necessary to meet the actual demand at price p^*. There is hence, no overshooting in the price and no reversal.

Generalising this, we find a clear condition for the dead cat bounce to appear. Whether there is a dead cat bounce, depends on the relative size of the equilibrium price p^*, the re-entry price p_R, the point where the supply function is zero $S^{-1}(0)$ and the intersection of the anticipated maximum demand with the supply function (p_t). This is illustrated in figure 11.

Lemma 5. *There exists a dead cat bounce, iff the intersection of the anticipated maximum demand with the supply function falls between the price for zero demand and the equilibrium price:*

$$S^{-1}(0) < p_t < p^* \tag{21}$$

There is no dead cat bounce, if the intersection of the anticipated maximum demand with the supply function falls between the equilibrium price and the re-entry price:

$$p^* \leq p_t \leq p_R \tag{22}$$

The size of the dead cat bounce is determined by the distance between the equilibrium price and the intersection: $p^* - p_t$.

The first implication we can draw from this is that the size of the dead cat bounce cannot exceed $p^* - S^{-1}(0)$. Apart from this, the results are very sensitive to the starting value when the RA investors begin to anticipate the CPT investors' demand as well as the functional form of the anticipated demand. For this reason, it is more meaningful to state the results in terms of probabilities and expectations dependent on the starting value. This model extension leads to a series of testable predictions.

Figure 9: Example with Dead Cat Bounce

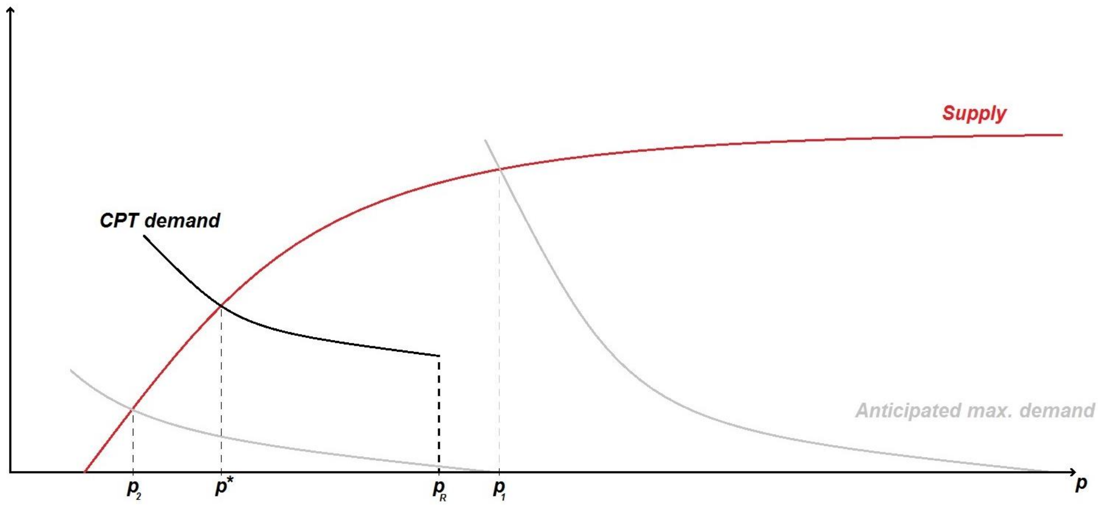
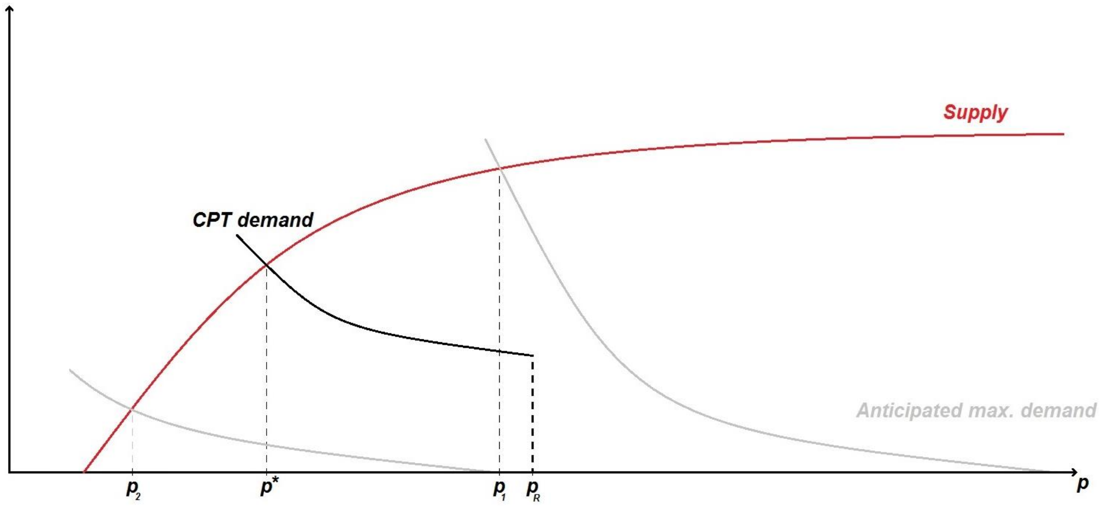


Figure 10: Example without Dead Cat Bounce



The red line illustrates the supply function $S(p)$. The black line illustrates the aggregated optimal demand by the CPT investors. The grey line illustrates the development of the anticipated maximum demand in two steps.

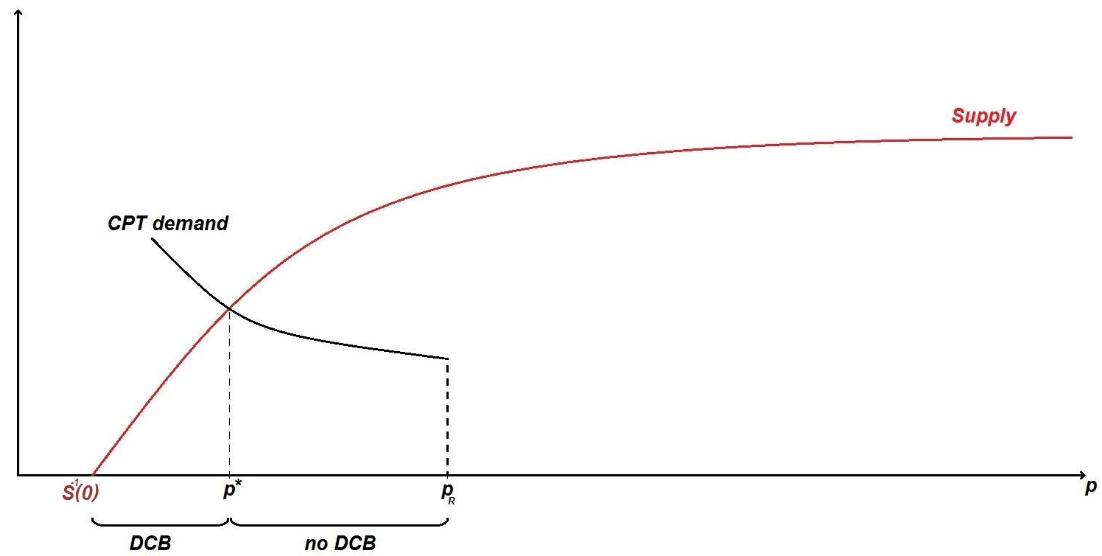
Proposition 3. *Given the interaction of multiple CPT and RA investors:*

(a) The higher the reference value Y_0 of the CPT investors, the higher is the probability of a dead cat bounce and the larger its expected size.

(b) The larger the share of RA investors n, the smaller is the possible dead cat bounce.

(c) A lower share of RA investors n or a lower share x of initial holdings held by

Figure 11: Condition for a Dead Cat Bounce



The red line illustrates the supply function $S(p)$. The black line illustrates the aggregated optimal demand by the CPT investors. If the price p_t falls in the area marked *DCB*, this will lead to a dead cat bounce. Similarly, if p_t lies in the *no DCB* area, there will be no dead cat bounce.

RA investors increases the probability that the dead cat bounce happens in a later period.

Proof. On the reference value Y_0:
It is straightforward to show that

$$\frac{\partial D(p)}{\partial Y_0} = \frac{(1-x)}{p}\frac{(1+\Omega^\gamma \Phi^\gamma)}{\Omega - \Omega^\gamma \Phi^\gamma} > 0 \tag{23}$$

$$\frac{\partial p_R}{\partial Y_0} = 0 \tag{24}$$

$$\frac{\partial S(p)}{\partial Y_0} = 0. \tag{25}$$

Given this, the equilibrium price p^* increases while the rest remains the same, leading to a larger dead cat bounce as $\frac{\partial (p^* - p_t)}{\partial Y_0} = \frac{\partial p^*}{\partial Y_0} > 0$. Furthermore, as p^* increases, $S^{-1}(0)$ and p_R remaind the same. As illustrated in figure 11, the area which leads to a DCB hence increases and the area for no DCB shrinks.

On the share of RA investors n:
An increase in the share of RA investors has no effect on the actual demand function

$$\frac{\partial D(p)}{\partial n} = 0$$

it is therefore reasonable to assume that the curvature of the anticipated demand function does also not react. As

$$\frac{\partial S(p)}{\partial n} = \frac{w(1-\Omega^{\epsilon}\Phi^{\epsilon})}{p(\Omega+\Omega^{\epsilon}\Phi^{\epsilon})} \qquad \text{and} \qquad \frac{\partial^2 S(p)}{\partial n \partial p} > 0, \tag{26}$$

the supply curve increases with n for high prices and decreases for low prices (positive expected return or $1 < \Omega\Phi$). Let us consider the case where there exists a DCB, i.e. $p^* > p_t$. We know from above that $\frac{\partial D(p)}{\partial n} = \frac{\partial \Upsilon_{p_t}(p_{t+1})}{\partial n} = 0$, consequently, the change in the intersection of $S(p)$ with $D(p)$ and $\Upsilon_{p_t}(p_{t+1})$, respectively, given a higher n depends purely on the change in the supply function. Given that $p^* > p_t$ and $\frac{\partial^2 S(p)}{\partial n \partial p} > 0$, it must be true that $\frac{\partial(p^*-p_t)}{\partial n} < 0$. It follows that the size of the DCB $p^* - p_t$ decreases as n increases, if there is a DCB.

As illustrated in figure 11, if $p_R \geq p_t \geq p^*$, there is no DCB. A $p_t > p_R$ would imply that there is no trade in this period and the suppliers adjust their supplies according to p_{t+1}. This p_{t+1} can then either fall in the DCB or no DCB area. Let us consider the effect of a change of p_t on the possibility of a DCB occurring in a later period. A DCB can only occur in a later period $(t+1,...)$ if $p_t > p_R$. Hence, an increase of p_t over this threshold would make it possible for a DCB to occur in the next period. A decrease in p_t would not affect the probability of a DCB in the next period as long as $p_t < p_R$. As mentioned before,

$$\frac{\partial S(p)}{\partial n} = \frac{w(1-\Omega^{\epsilon}\Phi^{\epsilon})}{p(\Omega+\Omega^{\epsilon}\Phi^{\epsilon})} \tag{27}$$

As $\frac{\partial S(p)}{\partial n} > 0$ for negative expected return $(1 > \Omega\Phi)$, the supply function in this region shifts upwards. This implies, that the anticipated maximum demand and the supply function intersect for a small p_t if they intersect in this region. It must hence be true that $\frac{\partial p_t}{\partial n} < 0$ for negative expected return. If it holds that $\pi(1-p_R)-(1-\pi)p_R < 0$, i.e. if the CPT investors are willing to re-enter the market with negative expected returns, then $\frac{\partial S(p_R)}{\partial n} > 0$. Further this implies that a lower n could push p_t over the threshold p_R and thereby delay a potential DCB. Consequently, a lower n can increase the probability that the DCB occurs in a later period.

The same logic also applies to a change in the initial share x of the asset held by RA investors.

$$\frac{\partial S(p)}{\partial x} = \frac{p0(1-\Omega^{\epsilon}\Phi^{\epsilon})}{p(\Omega+\Omega^{\epsilon}\Phi^{\epsilon})} \tag{28}$$

As $\frac{\partial S(p)}{\partial x} > 0$ for negative expected return $(1 > \Omega\Phi)$, it must be true that $\frac{\partial p_t}{\partial x} > 0$ in this case. Hence, a higher x could push p_t over the threshold p_R and thereby delay a potential DCB. $\square$

Proposition 3a) states, a higher reference value of the CPT investors, e.g. due to more optimism at the build up of the bubble, increases the probability of a dead cat bounce as well as its expected size. This is due to the same effect observed in Proposition 2. A higher reference value leads to a higher perceived loss and thereby to a larger demand upon re-entering. The larger jump in demand means, that the difference between the anticipated maximum demand and the actual demand is larger. This leads to a larger reversal and also increases its probability.

According to Proposition 3b), a larger share of RA investors reduces the possible dead cat bounce. The reason for this is that the amount of the asset held by each RA investor $\frac{x}{n}$ is smaller compared to his wealth w. Due to this, the RA investors are overall willing to take more risk. This increases the supply for the asset by RA investors when the asset has negative expected returns in which case the RA investors engage in shortselling. This makes the aggregate supply more price sensitive in this area. Consequently, the difference between the prices for which the supply meets the anticipated maximum demand and the actual demand shrinks and the possible dead cat bounce shrinks as well.

Similarly, Proposition 3b) states a lower share of RA investors decreases the supply of the asset and increases the price where the supply meets the anticipated maximum demand for a given period. This in turn increases the probability that the CPT investors do not yet re-enter the market as the price is too high. Consequently, the probability that, if there is going to be a dead cat bounce, we see it in a later period increases. If the RA investors held a lower share before the beginning of the bear market (a lower x), this decreases each RA investors total wealth equally in the good and in the bad state as can be seen in Assumption 11. Due to their risk aversion, this implies that a lower initial share x leads to a larger punishment for the difference in outcomes. This decreases the amount a RA investor is willing to sell in excess of his holdings (short selling). This in turn increases the price for which the supply curve and the anticipated maximum demand intersect. As above, this is equivalent to saying the probability that there is a dead cat bounce in a later period increases, if there is going to be a dead cat bounce at all.

7 Conclusion

Unraveling bubbles in capital, currency and commodity markets often experience temporary reversals of the downward trend, also known as dead cat bounce or bear market rally. To our knowledge, this is the first paper to offer an explanation for a phenomenon which is largely recognized in financial markets.

According to investopedia, a dead cat bounce "can be a result of traders or investors closing out short positions or buying on the assumption that the security has reached a bottom"[17]. In this paper, we show how preferences according to cumulative prospect theory (CPT) can explain the temporary reversal of the downward trend in unraveling bubbles. CPT preferences lead the investor to take high risk and unprofitable investments in the hope to recover losses experienced after the burst of the bubble. This leads to a jump in demand for the individual investor. The decision to re-enter is independent

[17] Source: http://www.investopedia.com/terms/d/deadcatbounce.asp#ixzz3m19rFFWb

of the investor's reference value and prior loss. Due to this, we find that a group of investors with the same expectations with regard to the asset will enter at the same time, leading to a large aggregate jump in demand.

Furthermore, we find several testable results using an extended model. A lower share of unsophisticated investors (CPT investors) is associated with a smaller dead cat bounce. A higher share of unsophisticated investors or a higher share of holdings by unsophisticated investors during the peak of the bubble increase the probability that a price reversal happens at a later point in time, i.e. that there is a longer period between the peak of the bubble and the dead cat bounce. Finally, the larger the optimism during the build-up of the bubble, the higher the probability of a dead cat bounce and the larger its expected size.

This paper provides a new mechanism explaining the temporary reversal of the downward trend in unraveling bubbles. There are several other papers that have shown mechanisms which are likely to interact with the one above. Some, such as fire sales as described by Miller and Stiglitz (2010) work against the reversal and hence highlight the size of the demand necessary to match the downward pressure. Others, such as herd behavior and limited arbitrage are likely to support the reversal once it started. Kaizoji and Sornette (2008) and Kaizoji (2010a,b) analyze the effect of herd behavior in financial markets. On the one hand, herding puts additional pressure on the price once it starts falling, on the other hand, it is also likely to amplify the reversal. Abreu and Brunnermeier (2003) show that arbitrageurs may forgo betting against a bubble when uncertain about the timing of the burst. This bubble riding behavior is likely to appear during the reversal as well, prolonging its existence further.

Further research is clearly necessary in order to better understand the downward dynamics in financial markets. More specifically, our results provide several areas for further research. Firstly, a more dynamic setting is necessary to see how the mechanism shown in this paper interacts with the intertemporal optimization as well as investors herding and fire sales. A further analysis in a multi-asset setting is also likely to provide valuable insights. Additionally, it is crucial that we get better insights into the empirical side of the reversal, dead cat bounce or bear market rally in the aftermath of financial bubbles. The existing research on bear market rallies is limited to the stock market. The commonness of the phenomenon in FX as well as commodity markets raises the question to what extent the reversals differ from one another. Our analysis also indicates that reversals may differ depending on whether they occur in the middle of the fall in prices or mark the end of the unraveling.

This paper is a first step in the theoretical analysis of unraveling bubbles. We believe that these downward dynamics deserve much more attention and promise valuable insights for policymakers, researchers and investors.

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A Proof of Lemma 1

The maximization problem in Case I is given by

$$\max_{\zeta} E(U), \tag{29}$$

which leads to the first order condition

$$\begin{aligned}\frac{\partial E(U)}{\partial \zeta} =& \pi\beta[\zeta(Y_g - p) - (Y_0 - Y_r)]^{\beta-1}(Y_g - p) \\ & - (1-\pi)\beta[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-1}(p - Y_b) = 0 \end{aligned} \tag{30}$$

$$\begin{aligned}\Rightarrow \quad \Omega\Phi[\zeta(Y_g - p) - (Y_0 - Y_r)]^{\beta-1} &= [\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-1} \\ \Omega\Phi[\zeta(p - Y_b) + (Y_0 - Y_r)]^{1-\beta} &= [\zeta(Y_g - p) - (Y_0 - Y_r)]^{1-\beta} \\ \Omega^{\gamma}\Phi^{\gamma}[\zeta(p - Y_b) + (Y_0 - Y_r)] &= [\zeta(Y_g - p) - (Y_0 - Y_r)] \\ \zeta((Y_g - p) - (p - Y_b)\Omega^{\gamma}\Phi^{\gamma}) &= (Y_0 - Y_r)(1 + \Omega^{\gamma}\Phi^{\gamma}) \\ \zeta^* &= \frac{(Y_0 - Y_r)}{(p - Y_b)}\frac{1 + \Omega^{\gamma}\Phi^{\gamma}}{\Omega - \Omega^{\gamma}\Phi^{\gamma}}\end{aligned}$$

with $\Omega = \frac{Y_g - p}{p - Y_b} > 0$, $\Phi = \frac{\pi}{1-\pi} > 0$ and $\gamma = \frac{1}{1-\beta} > 2$.
The second order condition is given by

$$\begin{aligned}\frac{\partial^2 E(U)}{\partial\zeta\partial\zeta} =& \pi\beta(\beta-1)[\zeta(Y_g - p) - (Y_0 - Y_r)]^{\beta-2}(Y_g - p)^2 \\ & - (1-\pi)\beta(\beta-1)[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-2}(p - Y_b)^2 < 0\end{aligned}$$

$$\begin{aligned}\Rightarrow \quad \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}[\zeta(p - Y_b) + (Y_0 - Y_r)] &> [\zeta(Y_g - p) - (Y_0 - Y_r)] \\ \zeta(\Omega - \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}})(p - Y_b) &< (Y_0 - Y_r)(1 + \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}).\end{aligned}$$

Using ζ^* from the first order condition we get

$$\begin{aligned}\frac{Y_0 - Y_r}{p - Y_b}\frac{1 + \Omega^{\gamma}\Phi^{\gamma}}{\Omega - \Omega^{\gamma}\Phi^{\gamma}}(\Omega - \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}})(p - Y_b) &< (Y_0 - Y_r)(1 + \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}) \\ \frac{1 + \Omega^{\gamma}\Phi^{\gamma}}{\Omega - \Omega^{\gamma}\Phi^{\gamma}}(\Omega - \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}) &< (1 + \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}) \\ -\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}} + \Omega^{\gamma+1}\Phi^{\gamma} &< \Omega^{(1+\frac{2}{2-\beta})}\Phi^{\frac{1}{2-\beta}} - \Omega^{\gamma}\Phi^{\gamma} \\ \Omega^{\gamma}\Phi^{\gamma} &< \Omega\end{aligned}$$

It is straightforward to show, that this second order condition is equivalent to Inequality (4) which follows from Assumption 6:

$$\begin{aligned}
\Omega^\gamma \Phi^\gamma &< \Omega \\
\Phi \Omega^\beta &< 1 \\
\frac{\pi}{1-\pi}\left(\frac{Y_g - p}{p - Y_b}\right)^\beta &< 1 \\
\pi(Y_g - p)^\beta - (1-\pi)(p - Y_b)^\beta &< 0
\end{aligned}$$

It follows that ζ^* is the optimal demand under Case I.

Finally, let us consider the upper bound. Given the single extreme value shown by the first order condition, the optimal value must be given by the upper bound, whenever ζ^* exceeds the upper bound given by the budget constraint $\frac{Y_r}{p}$.

It follows for Case I, that the optimal demand is either given by ζ^* or full investment. ■

B Proof of Corollary 1

Corollary 1 follows from a simple transformation of Equation (10):

$$\begin{aligned}
\zeta^* &= \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} \Omega \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} \frac{1 + \Omega^\gamma \Phi^\gamma - \Omega^{\gamma-1}\Phi^\gamma + \Omega^{\gamma-1}\Phi^\gamma}{1 - \Omega^{\gamma-1}\Phi^\gamma} \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} \left(1 + \frac{\Omega^{\gamma-1}\Phi^\gamma(1+\Omega)}{1 - \Omega^{\gamma-1}\Phi^\gamma}\right) \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} + \frac{(Y_0 - Y_r)}{(Y_g - p)} \frac{\Omega^{\gamma-1}\Phi^\gamma(1+\Omega)}{1 - \Omega^{\gamma-1}\Phi^\gamma} \\
&= \zeta_{Min} + \frac{(Y_g - p)\Omega^{\gamma-2}\Phi^\gamma(1+\Omega)(Y_0 - Y_r)}{(p - Y_b)(1 - \Omega^{\gamma-1}\Phi^\gamma)(Y_g - p)} \\
&= \zeta_{Min} + \frac{\Omega^{\gamma-2}\Phi^\gamma(1+\Omega)(Y_0 - Y_r)}{(1 - \Omega^{\gamma-1}\Phi^\gamma)(p - Y_b)} \\
&= \zeta_{Min} + IC
\end{aligned}$$

Further, given Inequality (4), we can show that

$$\begin{aligned}
\pi(Y_g - p)^\beta - (1-\pi)(p - Y_b)^\beta &< 0 \\
\Phi \Omega^\beta &< 1 \\
\Omega^{\gamma-1}\Phi^\gamma &< 1
\end{aligned}$$

This implies that $IC > 0$ must hold. ■

C Proof of Corollary 2

The loss recovery component $\zeta_{Min} = \frac{(Y_0 - Y_r)}{(Y_g - p)}$ is given by Equation (7). It follows

$$-\frac{\partial \zeta_{Min}}{\partial p} < 0.$$

Hence, as the price p falls, the loss recovery component declines as well.

From the proof in Appendix B it is clear that we can write IC as

$$IC = \frac{(Y_0 - Y_r)}{(Y_g - p)} \frac{\Omega^{\gamma-1}\Phi^{\gamma}(1+\Omega)}{1 - \Omega^{\gamma-1}\Phi^{\gamma}}$$

For simplicity, let us denote

$$\Psi = \frac{\Omega^{\gamma-1}\Phi^{\gamma}(1+\Omega)}{1 - \Omega^{\gamma-1}\Phi^{\gamma}}$$

It follows that $IC = \zeta_{Min}\Psi$. We can analyze the components separately

$$\begin{aligned}
-\frac{\partial \Psi}{\partial p} &= -\frac{\partial \Omega}{\partial p}\Phi^{\gamma}\left(\frac{\left(\gamma\Omega^{\gamma-1} + (\gamma-1)\Omega^{\gamma-2}\right)\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right) + \left(\Omega^{\gamma} + \Omega^{\gamma-1}\right)(\gamma-1)\Omega^{\gamma-2}\Phi^{\gamma}}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2}\right) \\
&= -\frac{\partial \Omega}{\partial p}\frac{\Phi^{\gamma}}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2}\left((\gamma-1)\Omega^{\gamma-2} + \Omega^{\gamma-1}\left(\gamma - \Omega^{\gamma-1}\Phi^{\gamma}\right)\right) \\
&= -\frac{\partial \Omega}{\partial p}\frac{\Phi^{\gamma}\Omega^{\gamma-2}}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2}\left(\gamma - 1 + \gamma\Omega - \Omega^{\gamma}\Phi^{\gamma}\right). \qquad (31)
\end{aligned}$$

As $\frac{\partial \Omega}{\partial p} < 0$, $\gamma > 2$ and Inequality (4) holds, we find that

$$-\frac{\partial \Psi}{\partial p} > 0. \qquad (32)$$

Hence, a fall in the price leads to an increase in Ψ. As $-\frac{\partial \zeta_{Min}}{\partial p} < 0$, this implies two contrary forces within the investment component. We can show that

$$\begin{aligned}
-\frac{\partial IC}{\partial p} &= -\frac{\partial \zeta_{Min}}{\partial p}\Psi - \zeta_{Min}\frac{\partial \Psi}{\partial p} \qquad (33) \\
&= -\frac{Y_0 - Y_r}{\left(Y_g - p\right)^2}\Phi^{\gamma}\frac{\left(-2(\gamma-1)\Omega^{\gamma} - (\gamma-2)\Omega^{\gamma-1} - \Omega^{2\gamma-2}\Phi^{\gamma} - \Omega^{\gamma+1}\left(\gamma - \Omega^{\gamma-1}\Phi^{\gamma}\right)\right)}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \qquad (34) \\
&> 0. \qquad (35)
\end{aligned}$$

A lower price leads to a larger investment component as the effect on Ψ dominates the effect on ζ_{Min}. ■

D Proof of Lemma 2

The first order condition for the investor's maximization problem in Case II is given by

$$\begin{aligned}\frac{\partial E(U)}{\partial \zeta} =&\pi\beta[-\zeta(Y_g-p)+(Y_0-Y_r)]^{\beta-1}(Y_g-p)\\&-(1-\pi)\beta[\zeta(p-Y_b)+(Y_0-Y_r)]^{\beta-1}(p-Y_b)=0\end{aligned}$$

$$\begin{aligned}\Rightarrow\quad \Phi\Omega[\zeta(p-Y_b)+Y_0-Y_r]^{1-\beta} &= [-\zeta(Y_g-p)+Y_0-Y_r]^{1-\beta}\\ (\Phi\Omega)^{\gamma}[\zeta(p-Y_b)+Y_0-Y_r] &= [-\zeta(Y_g-p)+Y_0-Y_r]\\ \zeta[(Y_g-p)+(p-Y_b)(\Phi\Omega)^{\gamma}] &= (Y_0-Y_r)[1-(\Phi\Omega)^{\gamma}]\\ \hat{\zeta}^* &= \frac{Y_0-Y_r}{p-Y_b}\frac{1-(\Phi\Omega)^{\gamma}}{\Omega+(\Phi\Omega)^{\gamma}}.\end{aligned} \tag{36}$$

The second order condition is given by

$$\begin{aligned}\frac{\partial^2 E(U)}{\partial\zeta\partial\zeta} =&-\pi\beta(\beta-1)[-\zeta(Y_g-p)+(Y_0-Y_r)]^{\beta-2}(Y_g-p)^2\\&-(1-\pi)\beta(\beta-1)[\zeta(p-Y_b)+(Y_0-Y_r)]^{\beta-2}(p-Y_b)^2<0.\end{aligned}$$

Taking into account the result for the first order condition in Equation (36) we get

$$\begin{aligned}-\Phi\Omega^2\left[-\hat{\zeta}^*(Y_g-p)+(Y_0-Y_r)\right]^{\beta-2} &> \left[\hat{\zeta}^*(p-Y_b)+(Y_0-Y_r)\right]^{\beta-2}\\ -\Phi\Omega^2\left[-\frac{Y_0-Y_r}{p-Y_b}\frac{1-(\Phi\Omega)^{\gamma}}{\Omega+(\Phi\Omega)^{\gamma}}(Y_g-p)+(Y_0-Y_r)\right]^{\beta-2} &>\\ &\left[\frac{Y_0-Y_r}{p-Y_b}\ \frac{1-(\Phi\Omega)^{\gamma}}{\Omega+(\Phi\Omega)^{\gamma}}(p-Y_b)+(Y_0-Y_r)\right]^{\beta-2}\\ -\Phi\Omega^2\left[\frac{1-(\Phi\Omega)^{\gamma}+\Omega+(\Phi\Omega)^{\gamma}}{\Omega+(\Phi\Omega)^{\gamma}}\right]^{2-\beta} &> \left[\frac{-\Omega+(\Phi\Omega)^{\gamma}\Omega+\Omega+(\Phi\Omega)^{\gamma}}{\Omega+(\Phi\Omega)^{\gamma}}\right]^{2-\beta}\\ -\Phi\Omega^2[1+\Omega]^{2-\beta} &> [(\Phi\Omega)^{\gamma}(1+\Omega)]^{2-\beta}\\ -1 &> (\Phi\Omega)^{\gamma}\Omega^{-1}\end{aligned} \tag{37}$$

From Equation (37) it becomes clear that the second order condition for a local maximum cannot be fulfilled as the right-hand-side cannot be negative. Hence, there is no local maximum for the expected utility in Case II.

It is straightforward to see that the second order condition for a local minimum must be fulfilled as $-1<(\Phi\Omega)^{\gamma}\Omega^{-1}$. Hence Equation (36) always describes the minimum of the subjective expected utility. Consequently, the optimal demand is given by the corner solutions.

By Assumption 6 the investment has a negative subjective expected return, i.e. $\Phi\Omega<1$. It follows that the minimum is given by a positive investment, so that there are feasible ζ to both sides of the minimum. This and the fact that Equation (36) describes the only extreme point imply that the investor needs to decide between investing the minimal amount $\zeta = 0$, i.e. abstaining from the market, and investing the maximum amount $\zeta = \frac{Y_r}{p}$.

The intuition here is as follows. On the one hand, a higher investment leads to higher expected losses, which makes investments less attractive. On the other hand, a larger investment implies more risk. The risk seeking preferences of the investor lead the asset to be more attractive. Depending on which of the effects is stronger, the investment is either zero or equal to the budget constraint.

As shown above, the investor in Case II either invests fully into the risky asset or abstains from the market. ■

E Proof of Lemma 3

For the investor, the crucial question is whether to abstain or to invest. Consequently, it comes down to whether the utility of buying ζ^* is larger or equal than the utility of not investing.

Re-entering with ζ^* implies full recovery of the initial loss as in Equation (5) while abstaining from the market implies a certain loss. In order for re-entering to be optimal, it must thus hold that:

$$E(U(\zeta^*)) \geq E(U(\zeta = 0))$$

$$\pi \left[\zeta^*(Y_g - p) - (Y_0 - Y_r)\right]^\beta - (1-\pi)\left[\zeta^*(p - Y_b) + (Y_0 - Y_r)\right]^\beta \geq -(Y_0 - Y_r)^\beta$$

$$\pi \left[\Omega \frac{1+\Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} - 1\right]^\beta - (1-\pi)\left[\frac{1+\Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} + 1\right]^\beta \geq -1$$

$$\pi \left[\frac{\Omega + \Omega^{\gamma+1}\Phi^\gamma - \Omega + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta - (1-\pi)\left[\frac{1+\Omega^\gamma \Phi^\gamma + \Omega - \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta \geq -1$$

$$\pi \left[\frac{(1+\Omega)\,\Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta - (1-\pi)\left[\frac{1+\Omega}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta \geq -1$$

$$\pi \left[\frac{(1+\Omega)\,\Omega^{\gamma-1} \Phi^\gamma}{1 - \Omega^{\gamma-1} \Phi^\gamma}\right]^\beta - (1-\pi)\left[\frac{1+\Omega}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta \geq -1$$

$$\pi \left(\frac{IC}{\zeta_{Min}}\right)^\beta - (1-\pi)\left[\frac{1+\Omega}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta \geq -1$$

$$\pi \left(\frac{IC}{\zeta_{Min}}\right)^\beta - (1-\pi)\left[\frac{(1+\Omega)}{\Omega\left(1 - \Omega^{\gamma-1}\Phi^\gamma\right)} \frac{\left(\Omega^{\gamma-1}\Phi^\gamma\right)}{\left(\Omega^{\gamma-1}\Phi^\gamma\right)}\right]^\beta \geq -1$$

$$\left(\frac{IC}{\zeta_{Min}}\right)^\beta \left[\pi - (1-\pi)\left(\frac{1}{\Omega\Phi}\right)^{\beta\gamma}\right] \geq -1$$

$$\left(\frac{IC}{\zeta_{Min}}\right)^\beta \left[\pi - (1-\pi)\Phi\left(\frac{1}{\Omega^\beta\Phi}\right)^{\gamma}\right] \geq -1$$

$$\pi \left(\frac{IC}{\zeta_{Min}}\right)^\beta \left[\left(\frac{1}{\Omega^\beta\Phi}\right)^{\gamma} - 1\right] - 1 \leq 0$$

This condition must be fulfilled for a positive demand in case of an interior solution.

■

F Proof of Proposition 1

Proposition 1 consists of two components. First, lower price leads to a positive demand by the investor. Additionally, Proposition 1 states that there is a jump in the demand.

From Lemma 3 we know that positive demand for an investor within Case I is dependent on Condition (14) being fulfilled:

$$\pi \left(\frac{IC}{\zeta_{Min}}\right)^\beta \left[\left(\frac{1}{\Omega^\beta\Phi}\right)^{\gamma} - 1\right] - 1 \leq 0.$$

As in Appendix C, we denote $\Psi = \frac{IC}{\zeta_{Min}}$ leading to

$$\pi \Psi^\beta \left[\left(\frac{1}{\Omega^\beta\Phi}\right)^{\gamma} - 1\right] - 1 \leq 0.$$

To see how a lower p influences this condition, we look at the negative derivate of the left-hand-side with respect to p.

$$-\frac{\partial LHS}{\partial p} = \pi\beta \frac{\left(\Omega^{-\beta\gamma}\Phi^{-\gamma} - 1\right)}{\Psi^{1-\beta}} \frac{\partial \Psi}{\partial p} - \frac{\pi\Psi^{\beta}\beta\gamma}{\left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right)} \frac{\partial \Omega}{\partial p}$$

The first derivate of Ψ is given by Equation (31)

$$\frac{\partial \Psi}{\partial p} = \frac{\partial \Omega}{\partial p} \frac{\Phi^{\gamma}\Omega^{\gamma-2}}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \left(\gamma - 1 + \gamma\Omega - \Omega^{\gamma}\Phi^{\gamma}\right)$$

It follows that

$$\begin{aligned}
-\frac{\partial LHS}{\partial p} &= -\pi\beta \frac{\partial \Omega}{\partial p} \left(\frac{\left(\Omega^{-\beta\gamma}\Phi^{-\gamma} - 1\right)}{\Psi^{1-\beta}} \frac{\Omega^{\gamma-2}\Phi^{\gamma}}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma}\right) - \frac{\Psi^{\beta}\gamma}{\left(\Omega^{-\beta\gamma}\Phi^{-\gamma}\right)} \right) \\
&= -\frac{\pi\beta}{\Psi^{1-\beta}} \frac{\partial \Omega}{\partial p} \left(\frac{\left(\Omega^{-1} - \Omega^{\gamma-2}\Phi^{\gamma}\right)}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma}\right) - \Psi\gamma \left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right) \right) \\
&= -\frac{\pi\beta}{\Psi^{1-\beta}} \frac{\partial \Omega}{\partial p} \left(\frac{\left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma}\right)}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)\Omega} - \frac{\left(\Omega^{\gamma-1}\Phi^{\gamma}(1+\Omega)\right)}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)} \gamma \left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right) \right) \\
&= -\frac{\partial \Omega}{\partial p} \frac{\pi\beta \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma} - \gamma \left(\Omega^{\gamma}\Phi^{\gamma} + \Omega^{\gamma+1}\Phi^{\gamma}\right) \left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right)\right)}{\Psi^{1-\beta}\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)\Omega} \\
&= -\frac{\partial \Omega}{\partial p} \frac{\pi\beta \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma} - \gamma(1+\Omega)\right)}{\Psi^{1-\beta}\left(\Omega - \Omega^{\gamma}\Phi^{\gamma}\right)} \\
&= \frac{\partial \Omega}{\partial p} \frac{\pi\beta \left(1 + \Omega^{\gamma}\Phi^{\gamma}\right)}{\Psi^{1-\beta}\left(\Omega - \Omega^{\gamma}\Phi^{\gamma}\right)}
\end{aligned}$$

Given that $\frac{\partial \Omega}{\partial p} < 0$, $\Psi = \frac{IC}{\zeta_{Min}} > 0$ and $\Omega > \Omega^{\gamma}\Phi^{\gamma}$, we can conclude that

$$-\frac{\partial LHS}{\partial p} < 0.$$

A decline in the price p lowers the left-hand-side of Condition (14) and hence, makes it more likely to be fulfilled. Lemma 3 states, that whenever this condition is fulfilled, the investor demands a positive amount.

Let us now consider the second component. Lemma 1 implies $\zeta^* > 0$. Given that an abstention from the market leads to zero demand, there must be a jump in demand once Condition (14) is fulfilled. ■

G Proof of Proposition 2

Following Lemma 1, the optimal demand by the investor is given by

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^{\gamma}\Phi^{\gamma}}{\Omega - \Omega^{\gamma}\Phi^{\gamma}}.$$

The demand is consequently positively dependent on the prior loss with respect to the reference value $Y_0 - Y_r$.

Lemma 3 provides the condition for re-entering

$$\pi \left(\frac{IC}{\zeta_{Min}} \right)^{\beta} \left[\left(\frac{1}{\Omega^{\beta} \Phi} \right)^{\gamma} - 1 \right] - 1 \leq 0$$

Keeping in mind that $\frac{IC}{\zeta_{Min}} = \frac{\Omega^{\gamma-1}\Phi^{\gamma}(1+\Omega)}{1-\Omega^{\gamma-1}\Phi^{\gamma}}$, this condition is independent of the prior loss $Y_0 - Y_r$.

It follows that the decision whether to re-enter is independent of the prior loss and the reference value, while the amount demanded increases linearly. ■

H Corner solutions in the optimization

In the following analysis we focus on the case where the budget constraint is binding i.e. the investor cannot invest his optimal amount.

In Case I, the investor is still able to recover his initial loss despite the budget constraint. In Case II, he is unable to recover the loss fully. In either of the two, he has a lower utility compared to the unbound case. *Ceteris paribus*, he will therefore not re-enter at the marginal point.

The investor will re-enter if and only if the utility of re-entering exceeds the utility of not investing. For Case I this implies the following condition:

$$\begin{aligned} E\left(u^{+}\left(\zeta = Y_r/p\right)\right) &\geq E\left(u^{-}\left(\zeta = 0\right)\right) \\ \pi \left[\frac{Y_r}{p}(Y_g - p) - (Y_0 - Y_r) \right]^{\beta} - (1-\pi) \left[\frac{Y_r}{p}(p - Y_b) + (Y_0 - Y_r) \right]^{\beta} &\geq -\left[Y_0 - Y_r\right]^{\beta} \\ -\pi \left[\frac{Y_r Y_g}{p} - Y_0 \right]^{\beta} + (1-\pi) \left[Y_0 - \frac{Y_r Y_b}{p} \right]^{\beta} &\leq \left[Y_0 - Y_r\right]^{\beta} \end{aligned} \tag{38}$$

To see the reaction of the condition to a further fall in the price, consider the negative first derivative of the left-hand-side.

$$\begin{aligned} -\frac{\partial LHS}{\partial p} &= -\pi\beta \left[\frac{Y_r Y_g}{p} - Y_0 \right]^{\beta-1} \left(-\frac{Y_r Y_g}{p^2} \right) + (1-\pi)\beta \left[Y_0 - \frac{Y_r Y_b}{p} \right]^{\beta-1} \frac{Y_r Y_b}{p^2} \\ &> 0 \end{aligned} \tag{39}$$

A lower price increases the left-hand-side of Condition (38). Hence a lower p can lead the condition to be fulfilled.

We can show the same for Case II:

$$E\left(u^{-}\left(\zeta=Y_r/p\right)\right)\geq E\left(u^{-}\left(\zeta=0\right)\right)$$

$$-\pi\left[-\frac{Y_r}{p}(Y_g-p)+(Y_0-Y_r)\right]^{\beta}-(1-\pi)\left[\frac{Y_r}{p}(p-Y_b)+(Y_0-Y_r)\right]^{\beta}\geq-\left[Y_0-Y_r\right]^{\beta}$$

$$\pi\left[1-\frac{Y_r}{p}\frac{Y_g-p}{Y_0-Y_r}\right]^{\beta}+(1-\pi)\left[1+\frac{Y_r}{p}\frac{p-Y_b}{Y_0-Y_r}\right]^{\beta}\leq 1 \tag{40}$$

$$\begin{aligned}-\frac{\partial LHS}{\partial p}=&\pi\beta\left[1-\frac{Y_r}{p}\frac{Y_g-p}{Y_0-Y_r}\right]^{\beta-1}\left(\frac{Y_r}{(Y_0-Y_r)}\frac{Y_g}{p^2}\right)\\&+(1-\pi)\beta\left[1+\frac{Y_r}{p}\frac{p-Y_b}{Y_0-Y_r}\right]^{\beta-1}\left(\frac{Y_r}{(Y_0-Y_r)}\frac{Y_b}{p^2}\right)\\&>0\end{aligned} \tag{41}$$

A fall in the price may be sufficient for an unbound investor to re-enter the market, while an investor bound by his budget constraint would not invest. When the price falls further, the investor's budget constraint relaxes and the investment becomes more profitable. Hence, the investor will enter the market for lower p, i.e. at a later point in time.

It is straightforward to see, that both Conditions (38) and (40) depend on the initial loss and hence, the reference value and the disposable wealth. This has to be the case, as the budget constraint is given by the disposable wealth and the case differentiation depends on the initial loss.

Whenever Conditions (38) and (40), respectively, are fulfilled, the investor re-enters the market. In the cases we consider here, the investor dedicates all his disposable wealth to the investment whenever he re-enters. This implies that the demand jumps from zero to the budget constraint Y_r/p. Hence, we still experience a sudden increase in demand.

Figure 12 illustrates the case where the investor has no chance to fully recover his initial losses. This case is identical to the one depicted in Figure 6, only that we have a larger initial loss due to a higher reference value Y_0 and a lower recovery price Y_r.

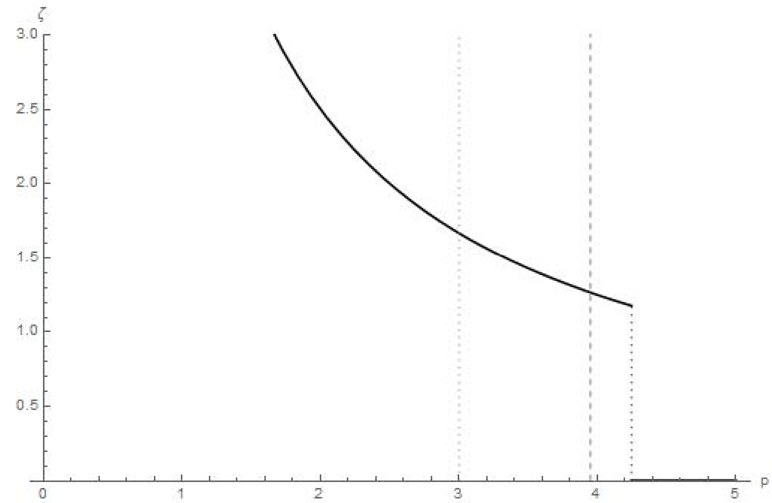
Again, the solid line is given by the demand while the dashed line illustrates Assumption 6. Additionally, the dotted line illustrates Condition (8). For a price p to the left of this line, the initial loss can be recovered and hence we enter Case I. At this point, the investor continues investing all his disposable wealth.

Due to the higher loss, the investor is willing to invest a much higher amount. At the marginal point, the highest price he is willing to invest, he is demanding nearly 120% of his possessions before the burst.

I Change in results if $\lambda \neq 1$

Here, we outline the implications of a loss aversion parameter λ different from unity. As mentioned in Section 3, the experimental evidence indicates $\lambda \approx 2.25$, however, we will show that this does not change our central results.

Figure 12: The jump in demand in Case II



The solid line shows the demand. The dashed line illustrates Assumption 6 which is only fulfilled to the right of the line. For p to the left of the dotted line, the investor can recover his initial loss, i.e. Case I. The parameters are chosen as follows: $Y_g = 60$, $Y_0 = 100$, $Y_r = 5$, $Y_b = 1$, $\beta = 0.8$ and $\pi = 0.05$. As in the analysis we set $\lambda = 1$.

The investor's expected utility, equivalent to Equation (6), is now given by

$$E(U) = \pi[\zeta(Y_g - p) - (Y_0 - Y_r)]^\beta - (1-\pi)\lambda[\zeta(p - Y_b) + (Y_0 - Y_r)]^\beta$$

leading to

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \hat{\Phi}^\gamma}{\Omega - \Omega^\gamma \hat{\Phi}^\gamma}$$

with $\Omega = \frac{Y_g - p}{p - Y_b} > 0$, $\hat{\Phi} = \frac{\pi}{1-\pi}\frac{1}{\lambda} > 0$ and $\gamma = \frac{1}{1-\beta} > 0$.

Note that the results is the same as in Equation (10) apart from $\hat{\Phi}$. $\hat{\Phi}$ is the ratio of the subjective probability for the good state and the subjective probability of the bad state (Φ) weighted by the inverse of the loss aversion parameter. The higher the loss aversion parameter, the lower the optimal demand.

The second order condition is accordingly given by

$$\Omega^{\gamma-1}\hat{\Phi}^\gamma < 1$$

or

$$\pi(Y_g - p)^\beta - (1-\pi)\lambda(p - Y_b)^\beta < 0$$

and must be fulfilled as Inequality (4) is given.

It follows, that the decomposition in Corollary 1 remains valid only that

$$\zeta^* = \zeta_{Min} + \hat{IC}$$

with $\hat{IC} = \zeta_{Min}\hat{\Psi}$ and $\hat{\Psi} = \frac{\Omega^{\gamma-1}\hat{\Phi}^{\gamma}(1+\Omega)}{1-\Omega^{\gamma-1}\hat{\Phi}^{\gamma}} > 0$.

Corollary 2 remains valid as

$$
\begin{aligned}
-\frac{\partial \hat{IC}}{\partial p} &= -\frac{\partial \zeta_{Min}}{\partial p}\hat{\Psi} - \zeta_{Min}\frac{\partial \hat{\Psi}}{\partial p} \\
&= -\frac{Y_0 - Y_r}{(Y_g - p)^2}\hat{\Phi}^{\gamma}\frac{\left(-2(\gamma-1)\Omega^{\gamma} - (\gamma-2)\Omega^{\gamma-1} - \Omega^{2\gamma-2}\hat{\Phi}^{\gamma} - \Omega^{\gamma+1}\left(\gamma - \Omega^{\gamma-1}\hat{\Phi}^{\gamma}\right)\right)}{\left(1-\Omega^{\gamma-1}\hat{\Phi}^{\gamma}\right)^2} \\
&> 0.
\end{aligned}
$$

In case one, the loss aversion parameter appears in both states of the expected utility as both states lead to a loss with respect to the reference value. It follows that

$$
\begin{aligned}
E(U) &= \pi u^{-}(x_g) + (1-\pi)u^{-}(x_b) \\
&= -\pi\lambda[-\zeta(Y_g - p) + (Y_0 - Y_r)]^{\beta} - (1-\pi)\lambda[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta}
\end{aligned}
$$

The optimization leads to the minimum

$$
\hat{\zeta}^{*} = \frac{Y_0 - Y_r}{p - Y_b}\frac{1-(\Phi\lambda\Omega)^{\gamma}}{\Omega + (\Phi\lambda\Omega)^{\gamma}}
$$

which is equivalent as the one in Equation (36). Similarly the second order condition is the same as well. Hence, the results for Case II do not change.

The decision to re-enter given by Lemma 3 is now defined by

$$
\begin{aligned}
E(U(\zeta^{*})) &\geq E(U(\zeta = 0)) \\
\pi\left[\zeta^{*}(Y_g - p) - (Y_0 - Y_r)\right]^{\beta} - (1-\pi)\lambda\left[\zeta^{*}(p - Y_b) + (Y_0 - Y_r)\right]^{\beta} &\geq -\lambda(Y_0 - Y_r)^{\beta} \\
\hat{\Psi}^{\beta}\left[\frac{\pi}{\lambda} - (1-\pi)\left(\frac{1}{\Omega^{\gamma}\hat{\Phi}^{\gamma}}\right)^{\beta}\right] &\geq -1 \\
(1-\pi)\hat{\Phi}\hat{\Psi}^{\beta}\left[1 - \frac{1}{\hat{\Phi}}\left(\frac{1}{\Omega^{\gamma}\hat{\Phi}^{\gamma}}\right)^{\beta}\right] &\geq -1 \\
(1-\pi)\hat{\Phi}\hat{\Psi}^{\beta}\left[\left(\frac{1}{\Omega^{\beta}\hat{\Phi}}\right)^{\gamma} - 1\right] - 1 &\leq 0
\end{aligned}
$$

While the expression in Lemma 3 changes, it is straightforward to see that the conclusions from it given by Propositions 1 and 2 remain valid.

Hence, we can conclude that the results are not affected by the simplification of setting the loss aversion parameter $\lambda = 1$.