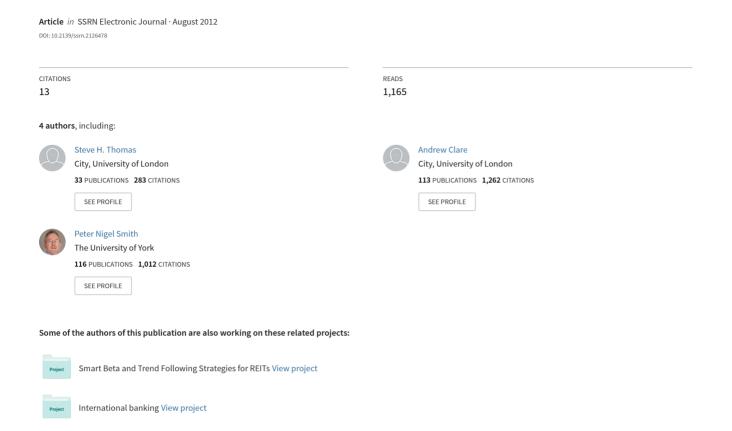
The Trend is Our Friend: Risk Parity, Momentum and Trend Following in Global Asset Allocation



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Abstract

We examine the effectiveness of applying a trend following methodology to global asset allocation between equities, bonds, commodities and real estate. The application of trend following offers a substantial improvement in risk-adjusted performance compared to traditional buy-and-hold portfolios. We also find it to be a superior method of asset allocation than risk parity. Momentum and trend following have often been used interchangeably although the former is a relative concept and the latter absolute. By combining the two we find that one can achieve the higher return levels associated with momentum portfolios but with much reduced volatility and drawdowns due to trend following. We compare the performance of selected strategies using measures based on the utility function of a representative investor. These results reinforce the superiority of combing trend following with momentum strategies. We observe that a flexible asset allocation strategy that allocates capital to the best performing instruments irrespective of asset class enhances this further.

Keywords: Risk parity, trend following, momentum, global asset allocation, equities, bonds, commodities, real estate.

JEL Classification: G10, 11, 12

1. Introduction

Investors today are faced with the task of choosing from a wide variety of asset classes when seeking to invest their money. With electronic trading and the rapid expansion of the Exchange Traded Funds (ETFs) universe, the ability to invest in a vast array of asset classes and instruments both domestically, and overseas, has never been easier. The traditional method of asset allocation of 60% in domestic equities and 40% in domestic bonds and, apart from a little rebalancing, holding these positions indefinitely increasingly appears archaic. Aside from the diversification benefits lost by failing to explore alternative asset classes, Asness et al (2011) argue that this is a highly inefficient strategy since the volatility of equities dominates the risk in a 60/40 portfolio. Instead they suggest that investors should allocate an equal amount of risk to stocks and bonds, to achieve 'risk parity', and show that this has delivered a superior risk-adjusted performance compared to the traditional 60/40 approach to asset allocation. Although, nominal returns have historically been quite low to this strategy, proponents argue that this drawback of constructing a portfolio comprised of risk parity weights can be overcome by employing leverage. Inker (2010), however, argues that the last three decades have been especially favourable to government bonds and that this has generated flattering results for risk parity portfolio construction techniques. For example, in the early 1940's US Treasury yields were very low and in the following four decades delivered cumulative negative returns. Furthermore, critics have also pointed out that when applying risk parity rules investors are effectively taking no account of the future expected returns of an asset class.

There exist other possible rules-based approaches to asset allocation, including those based upon financial market 'momentum' and 'trends', support for both of which can be found in the academic literature, particularly in the case of the former.¹

There now exists quite a substantial literature that finds support for the idea that financial market momentum offers significant explanatory power with regard to future financial market returns. Many studies, such as Jegadeesh and Titman (1993, 2001) and Grinblatt and Moskowitz (2004) have focussed on momentum at the individual stock level, while others such as Miffre and Rallis (2007) and Erb and Harvey (2006) have observed the effect in commodities. Asness et al (2013) find momentum effects within a wide variety of asset

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¹ The importance of technical analysis for fund managers is assessed by Menkhoff (2010)

classes, while King et al (2002) use momentum rules as a means of allocating capital across asset groups. Typical momentum strategies involve ranking assets based on their past return (often the previous twelve months) and then buying the 'winners' and selling the 'losers'. Ilmanen (2011) argues that this is not an ideal approach to investing and that investors would be better served by ranking financial instruments or markets according to rankings based upon their past volatility. Ilmanen suggests that failing to do this leads to the situation where the most volatile assets spend a disproportionate amount of time in the highest and lowest momentum portfolios.

Trend following has been widely used in futures markets, particularly commodities, for many decades (see Ostgaard, 2008). Trading signals can be generated by a variety of methods such as moving average crossovers and breakouts with the aim to determine the trend in the price's of either individual securities or broad market indices. Long positions are adopted when the trend is positive and short positions, or cash, are taken when the trend is negative. Because trend following is generally rules-based it can aid investors since losses are mechanically cut short and winners are left to run. This is frequently the reverse of investors' natural instincts. The return on cash is also an important factor either as the collateral in futures trades or as the 'risk-off' asset for long-only methods. Examples of the effectiveness of trend following are, amongst others, Szacmary et al (2010) and Hurst et al (2010) for commodities, and Wilcox and Crittenden (2005) and ap Gwilym et al (2010) for equities. Faber (2009) uses trend following as a means of informing tactical asset allocation decisions and demonstrates that it is possible to form a portfolio that has equity-level returns with bond-level volatility. Ilmanen (2011) and Friesen et al (2009) offer a variety of explanations as to why trend following may have been successful historically, including the tendency for investors to underreact to news and their tendency to exhibit herding behaviour. Shynkevich (2012) questions the more recent effectiveness of similar rules in the US equity market.

A few studies have sought to combine some of the strategies previously discussed. Faber (2010) uses momentum and trend following in equity sector investing in the United States, while Antonacci (2012) uses momentum for trading between pairs of investments and then applies a quasi-trend following filter to ensure that the winners have exhibited positive returns. The risk-adjusted performance of these approaches has been a significant improvement on benchmark buy-and-hold portfolios.

The aim of this paper is to extend previous work in this area by combining strategies and by applying these strategies in a multi-asset class context. We find that trend following portfolios produce higher Sharpe ratios than comparable, equally-weighted buy and hold portfolios with much lower maximum drawdowns. This is the case both in multi-asset portfolios and within asset classes. Our results show that asset class weightings based on risk parity rules also produce much improved risk-adjusted returns in recent years compared to the same comparable buy and hold portfolios. However, further investigation does reveal that these results are largely due to the outperformance of bonds over other broad asset classes over our sample period. We find that a risk parity approach to investing adds little to performance within asset classes, in sharp contrast to our findings with regard to trend following rules which enhance portfolio performance still further when they are applied within asset class. Our results show that multi-asset class investing using momentum signals, does improve the risk-return characteristics of a multi-asset class portfolio, compared to a buy-and hold equivalent, but not substantially. We also find that combining the momentum based rules, while simultaneously volatility adjusting the weights does not have a significant impact upon performance, but when we combine momentum based rules, whether the weights have been volatility-adjusted or not, with trend following rules we find a substantial improvement in performance, compared with applying just momentum-based rules. We also show how our findings can form part of a flexible asset allocation strategy, where trend following rules are used to rank 95 financial markets according to their volatility-weighted momentum, an approach which has the attractive quality of not requiring any asset allocation weights to be predetermined. This flexible approach to asset allocation produces attractive and consistent risk-adjusted returns. Next, we examine whether the impressive returns generated by some of these strategies could be explained by their exposure to known risk factors. We find that, although the alphas that we calculated were lower than unconditional mean returns, a significant proportion of the return could not be explained with reference to these risk factors. Finally, we assess the ranking of strategy returns using measures which take into account the impact of the higher order moments in returns. In particular we employ both the popular Sortino ratio which compares average returns to a measure of downside risk and a utility function based index which takes into account the impact of skewness and excess kurtosis on the utility of a representative risk averse investor. The Smetters and Zhang general measure shows that risk averse investors benefit significantly from the reduction in negative skewness offered by trend following.

Perhaps the most important implication of the results presented here relates to the degree to which a pure trend following strategy, or one overlaid on a momentum strategy with volatility-adjusted weightings, reduces drawdowns compared to buy and hold benchmark. We believe that such strategies would be ideal for risk averse investors, and perhaps particular for investors in the final years of saving for retirement, or in drawdown, where a drawdown could have a significant impact on their retirement income.

The rest of this paper is organised as follows: in Section 2 we present our data; in Section 3 we present our main results and the methodologies used to produce them; in Section 4 we show how the results in Section 3 can inform a flexible asset allocation strategy; in Section 5 we consider whether the results from some of the key rules-based approaches can be attributed to exposures to known risk factors; in Section 6 we provide a ranking of the strategies based on the utility function of a representative investor and finally, Section 7 concludes the paper.

2. Data

To investigate the possible value in risk parity, momentum and trend following approaches to asset allocation we consider five broad market asset classes as represented by well known financial market indices. These five major asset classes are: developed economy equities (MSCI World), emerging market equities (MSCI Emerging Markets), government bonds (Citigroup World Government Bond Index), commodities (DJ-UBS Commodity Index) and real estate (FTSE/EPRA Global REIT Index). The indices representing each of these broad asset classes are available in a total return format. Basic descriptive statistics of these indices are presented in panel A of Table 1. In addition to using these broad financial market indices, for each of these asset classes we also collected individual, country level index data or, in the case of commodities, data on individual commodities. These sub-components of the main asset classes are also available in total return terms. We collected both sets of data to see whether the rules that we explore here are best applied at the higher asset class level, or whether applying them at a more disaggregated manner should be preferred. The developed economy equity market indices that we collected were all produced by MSCI. They are the country level MSCI indices for: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, Canada, United States, Australia, Hong Kong, Japan, New Zealand and Singapore. We collected equivalent MSCI indices for a set of emerging economy equity

indices, which included indices for: Brazil, Chile, Colombia, Mexico, Peru, Poland, South Africa, Turkey, China, India, Indonesia, Korea, Malaysia, Philippines, Taiwan and Thailand. We collected country level government bond indices, produced by Thomson Financial, for the following countries: Australia, Germany, Canada, France, Ireland, Japan, Netherlands, Austria, Sweden, Switzerland, United Kingdom, United States, Denmark, Belgium, Spain, Italy, New Zealand, Finland and Norway. We collected a set of commodity indices produced by DJ-UBS indices which included those representing the following commodities: Aluminium, Coffee, Copper, Corn, Cotton, Crude Oil (WTI), Gold, Heating Oil, Lean Hogs, Live Cattle, Natural Gas, Nickel, Silver, Soybeans, Soybean Oil, Sugar, Unleaded Gas, Wheat, Zinc, Cocoa, Lead, Platinum and Tin. Finally, we collected country level REIT indices produced by FTSE/EPRA for the following countries: Australia, Belgium, France, Germany, Hong Kong, Italy, Japan, Netherlands, Singapore, Sweden, Switzerland, United Kingdom and United States. In total we collected index total return data on 24 developed economy markets, 16 emerging economy equity markets, 19 government bond markets, 23 commodities markets and 13 country level real estate markets. All index data are end of month, denominated in US dollars and span the period from January 1993 to December 2011.

We use the indices described above to calculate the monthly returns necessary for both momentum-based and volatility-based rankings, and also for assessing the subsequent performance of each strategy. The trend following rules are however, based upon price index levels rather than being derived from returns. The trend following signals are calculated based on the price indices of the Developed Equity, Emerging Equity and Real Estate indices. Excess return indices are used for the same purpose to give the signal for Commodities (to take account of backwardation/contango in markets), while we use total return indices for the government bond indices because of a lack of price historic information on the indices of this asset class.

3. Results

3.1. Trend following and risk parity applied to the five broad asset classes

We first examine the five broad asset class indices. Panel A of Table 1 shows the performance of these during 1994-2011. Compound returns range from approximately 5% to 7% although on a risk-adjusted basis bonds were the clear winner with a Sharpe ratio of 0.66 compared to 0.1 to 0.2 for other assets. All of the latter also experienced a drawdown in

excess of 50% during the sample period whereas bonds never had a drawdown of more than 5%.

The performance statistics presented in the left-hand column of Panel B of Table 1 are generated by a portfolio with 20% invested in each of the five broad asset classes with monthly rebalancing. This portfolio has better risk-adjusted performance than all of the individual asset classes (shown in Panel A of Table 1) with the exception of bonds. The maximum drawdown of this equally-weighted portfolio remains close to 50% though and the portfolio is negatively skewed, that is, it is more volatile than average when losing money and less volatile than average when making money. The other columns in Panel B of this Table show performance statistics for trend following versions of the equally-weighted portfolio. That is, we apply a trend following rule for each asset class using varying signal lengths. In applying these trend following rules we follow the method of Faber (2007). More precisely, if the price of the asset class index is above its x-month moving average then we say that the asset class is in an uptrend and it is purchased, if not already held. However, if the price is below this x-month moving average then the asset is said to be in a downtrend and the asset is sold and the proceeds invested in US 3-month Treasury Bills. Signals are determined on an end-of-month basis. Consistent with Faber (2007), no short-selling is permitted and no transactions costs are deducted. Finally, each asset class has an equal weight. In the case where all five asset class signals are positive then the portfolio is 100% invested, equally across each asset class, that is, 20% in each asset class. However if, for example, four of the signals are positive and one negative, then 20% of the portfolio is invested in the four asset classes with the positive signal, 20% is invested in US Treasury bills, and 0% in the asset class with the negative signal. Our results show that for a variety of signal lengths, returns are higher and volatilities lower than the comparable equally-weighted portfolio without trend following applied. Consequently Sharpe ratios are much improved and maximum drawdowns are subdued too. This superior risk-adjusted performance is a consequence of the trend following rules keeping investors out of markets during the most severe declines when volatility is at its highest. The less negative skew on these portfolios is also worthy of note, which is particularly true at shorter signal lengths and supports the findings of Koulajian and Czkwianianc (2011).

The final Panel of Table 1 displays the results of a risk parity method of asset allocation, applied to the five broad asset classes. Following the method of Asness et al (2011), portfolio

weights are proportional to the inverse of observed volatility. More specifically, we calculate the asset class volatilities using one year's worth of data, and then calculate the weights from these volatilities. This process is repeated at the end of each month. In the (unlikely) event that the calculated volatilities of each asset class are identical, the return on the portfolio over the next month would be identical to the return generated by the equally-weighted portfolio described in Panel B. Our results show that the level of return of the risk-parity portfolio is similar to that of this equally-weighted portfolio but with approximately half the volatility. And so risk parity appears to add value, compared with an equally-weighted portfolio of these broad asset classes. However, all of the trend following portfolios in the Panel B demonstrate higher risk-adjusted returns and much lower drawdowns, though.

These results suggest that both trend following and risk parity rules can add value to a multi-asset class portfolio over time. The far-right column of Panel C, shows the results of applying both sets of rules, that is, the performance statistics of a risk parity portfolio that adopts trend following too. The investment weights are the same as the standard risk parity portfolio but, crucially, if the trend (using only a 10-month moving average, consistent with Faber (2007)) is negative in a particular asset class its risk parity weight is allocated to T-bills instead. So if all asset classes are in an uptrend, then the weights of the portfolio for the following month would be identical to those of the 'risk parity' portfolio. This approach produces a much improved set of performance statistics over the pure risk parity approach; Sharpe ratio is in excess of 1.0, compared to 0.6 for the risk parity approach and the maximum drawdown is less than 5%, compared to over 20% for the risk parity approach. Furthermore, in Sharpe ratio terms, this combination of risk parity and trend following produces performance statistics that are superior to the pure trend following portfolios described in panel B of the Table.

3.2 Trend Following applied within the broad asset classes

Thus far we have looked at broad indices to examine the merits of trend following. The next logical step is to consider whether, by decomposing an index into its constituents, and applying trend following to these individually, improves the level of performance. For instance, whilst there may be some periods when all components are either in uptrends or downtrends, there are also likely to be periods when there the performance of subcomponents of the broad asset classes diverge. By only being long the uptrending components it may be possible to outperform the benchmark.

Table 2 reports the performance of trend following within each asset class, where the approach is comparable to the one used to produce the performance statistics for panel B of Table 1. The equally-weighted portfolio is the base case whereby each component of the asset class is given the same investment weight with rebalancing occurring on a monthly basis. All the trend following portfolios are formed on the same basis except that during any downtrends the allocation to that sub-component is invested in US T-Bills. The first point of note is how the base case non-trend-following portfolios are generally an improvement on the broad asset class indexes shown in Table 1 as one moves away from market-cap weightings. In other words, equally weighting the sub-components, rather than market value weighting them, as is typically the case with broad financial market indices, would have generally produced superior performance over this sample period. Only in the case of the bond asset class is the broad index superior to the equally-weighted sub-components.

The trend following portfolios show considerable risk-adjusted performance improvements compared to their equally-weighted portfolio comparators. The only exception is again the bonds category where we observe little difference. Faber (2007) highlights how a trend following portfolio will underperform a buy-and-hold portfolio during major bull markets. This is the scenario largely witnessed for bonds during the period of study (with the exception of some of the peripheral European nations in very recent years). The other asset classes have experienced one or more periods of stress in the past 20 years, for example, the dot-com crash for equities, the \$10 per barrel oil in the late 1990's as part of multi-decade bear market in commodities, the property collapse in credit crunch of 2008, etc. In each of these remaining asset classes we see higher returns from trend following in the region of 1-3% per annum, however, the most noticeable factor is the dramatic reduction in volatility, by around 40-50% of the equally-weighted portfolios. This in turn leads to much higher Sharpe ratios and much lower experienced drawdowns. In terms of signal length, it is not apparent that there is much difference in risk-adjusted performance. The most noticeable difference, again consistent with Table 1, is that skewness becomes more positive as the signal length is shortened. The downside to shorter signals in reality is that more transactions will be required and thus additional associated costs incurred.

Table 3 displays the performance of a multi-asset portfolio with 20% assigned to each, broad asset class, but with the trend following rule applied to the components of each of these broad

asset classes, that is, we decompose each asset class into its components and then apply the trend following rules applied to produce the performance statistics in Table 2. We can see that this yields a return regardless of signal length of just over 10%, an annualized volatility of approximately 7.5% and a maximum drawdown less than 9%. Again, this is a substantial improvement on the equally-weighted base case portfolio, whose performance statistics are shown in the first column of Table 3, where no trend following rules have been applied. In addition, we observe an improvement in risk-adjusted returns compared to the broad trend following asset class models in Table 1. This indicates that splitting an asset class into its component parts adds value.

3.3 Risk Parity applied within the broad asset classes

Having shown that decomposing an asset class into sub-components and then applying trend following rules to these individual sub-components can improve the risk return characteristics of a multi-asset class portfolio, we now consider whether the same approach improves riskreturn outcomes using risk parity rules. Panel A of Table 4 shows the performance of risk parity within an asset class. We can compare the results in panel A of Table 4 with the related equally-weighted portfolios for each asset class presented in Table 2. When we do this we observe very little difference in risk-adjusted performance. For example, Panel A of Table 2 shows that the Sharpe ratio of developed economy, equally-weighted portfolio is 0.23; the risk parity-weighted equivalent portfolio has a Sharpe ratio of 0.26 (column 1, panel A, Table 4). Whilst one may argue that developed equity markets have similar risk characteristics, and thus risk parity can only offer minimal improvements, this is not the case in commodity markets. Ilmanen (2011) describes how natural gas and heating oil have exhibited considerably more volatility historically than soybeans and gold, and yet we still find minimal improvement from risk parity. But the Sharpe ratio for the equally-weighted portfolio of commodities (column 1, panel D, Table 4) is 0.28, which is identical to the Sharpe ratio calculated for the risk parity-weighted commodities portfolio, shown in Panel A of Table 4. Panel B of Table 4 reports the application of risk parity weights in conjunction with a 10-month trend following signal. As in Panel C of Table 1, we find that risk-adjusted performance improves markedly with the additional trend following filter. Returns are higher and volatilities lower in all cases albeit only marginally in the case of bonds. Comparing the risk parity trend following results to the equal weighted ones in Table 3 we observe little difference in performance.

The implication of these results appear to be that risk parity has been exceptionally successful in recent times due to the impressive risk-adjusted returns of bonds which make up substantial portions of these portfolios; and that, in contrast to trend following techniques, has very little to add within asset classes.

3.4 Momentum

The momentum effect of buying 'winners' and selling 'losers' has been well established in the financial literature by, amongst others, Jeegadeesh and Titman (1993) for equities and Miffre and Rallis (2007) for commodities². We now examine momentum in a multi-asset context. Remaining consistent with our previous results, we eschew short selling and thus look to hold portfolios of 'winners'. The formation of portfolios within each asset class is somewhat complicated by having unequal numbers of instruments, for example, we have 24 Developed Equity market indices but only 16 Emerging Equity market indices. For this reason we focus on the 'top half' or 'top quarter' of winning markets within each asset class rather than at a prescribed number. All momentum rankings are calculated based on the prior 12-month return.

Table 5 reports the performance of momentum-based rules within each of the five asset classes. Firstly we note that the overall level of return is typically higher than for an equally-weighted portfolio (see Table 2) of all markets within the asset class. This is particularly true for commodities, where the momentum-based average return is 12.28%, compared with 7.04% for the equally-weighted equivalent. Sharpe ratios are also generally higher although these remain below the equivalents for trend portfolios. A comparison of panels A and B of the Table show that there is relatively little performance difference between choosing the top 25% of winners and choosing the top 50% of winners. The far-right column of the Table shows the performance of a portfolio with 20% in each of the five asset class momentum portfolios with monthly rebalancing. Again, this is an improvement on the base case equally-weighted portfolio in Table 1 with superior risk-adjusted performance, however, it produces inferior performance statistics to the trend following approach in Table 3. The main downside, to the momentum strategy is the large maximum drawdown in excess of 40% that an investor would have had to endure.

² An alternative method for evaluating the success of momentum strategies is presented by Banerjee and Hung (2011).

3.5 Combining Momentum with trend following

Thus far we have observed that applying both trend following and momentum individually are means of obtaining improved performance on traditional buy-and-hold portfolios, though the performance enhancement is greater in the case of the former. We now consider if they can be used in combination to enhance multi-asset class, risk-adjusted returns further.

Momentum is a relative concept in that there is always a portfolio of a winners and a portfolio of losers. Trend following, by contrast, is an absolute concept (if based on clearly defined rules) whereby all, some or none of the considered asset classes can be in an uptrend or a downtrend. This raises the possibility of having a momentum portfolio of winners in a downtrend that is they are falling in price, just more slowly than the losers, and *vice versa*. To this extent, combining momentum and trend following has some attractions since it ensures assets are both winners and in an uptrend. From the perspective of an investor that does not short sell, it also ensures that there is minimal exposure to the effects of 'momentum crashes' as described by Daniel and Mozkowitz (2013) since 'downtrending winners' are not held and the loser portfolio has not been sold short either.

There are two different methods of combining trend following and momentum. One is the approach of Faber (2010) who uses the trend following signal of a broad equity market index to determine whether to buy or sell a momentum portfolio of equity sectors. This method, which we call a trend following asset class filter, has a binary outcome in terms of the asset allocation with either 100% investment in the risk assets or 0%. The alternative approach is the one of individual trend following used by ap Gwilym et al (2010) whereby each single component of the momentum portfolio has the trend following rule applied to it.³

Table 6 presents the results of combining momentum and trend following. Panels A and B show the top half and top quarter momentum portfolios for each asset class with the application of a trend following asset class filter using a 10-month signal. We observe that risk-adjusted returns are improved for four of the five asset classes compared to Table 5. For example, Panel A of Table 5 shows that the Sharpe ratio of the 'top half momentum rule' applied to developed economy equities is 0.38; for the same asset class Panel A of Table 6 shows that the 'top half momentum plus trend following asset class filter' produces a much

³ A further combination examined by Fuertes et al (2010) is of momentum and term structure strategies.

improved Sharpe ratio of 0.92. Furthermore, maximum drawdowns are also reduced (from 56.02% to 15.43% in the case of Developed economy equities) while the skew of the portfolios becomes more positive (from -0.76 to -0.28 in the case of Developed economy equities). The far-right column of Table 6 again reports the statistics for a portfolio made up of 20% in each of the five momentum ranked and trend filtered asset classes, rebalanced monthly. These too show a substantial improvement on the equivalents in Table 5. Sharpe ratios are between 0.3 and 0.5 higher than for the portfolios formed only on the basis of the momentum rule, the skew is approximately zero and the maximum drawdown for the top half portfolio is under 10%.

Panels C and D show the results from combining the two momentum rules with the trend following rule applied within asset classes. There appears little to choose between this and the broad asset approach. Results are somewhat improved for REITs and bonds but worse for equities. Similar performance is also observed for the multi-asset portfolios. When we compare the investment experience of these with the trend following only portfolios from Table 3 we find that the addition of momentum increases the level of return by 1.5% to 2.5% per annum but this comes at the expense of higher volatility. Sharpe ratios for the top half portfolios are marginally higher than comparables in Table 3, whilst the top quarter values are around 0.1 lower.

3.6 Volatility-adjusted momentum and trend following

Ilmanen (2011) makes the case for adjusting momentum rankings to take account of the volatility of each asset. It is argued that without this consideration that the most volatile assets spend a disproportionate amount of time in the top and bottom momentum ranking categories. We calculate volatility-adjusted momentum rankings by dividing the prior twelve month total return by the realised volatility over the same period and then ranking in the standard fashion.

Table 7 shows the results of volatility-adjusted momentum ranking within each asset class. Compared with the standard results in Table 5 we observe very little difference. Returns and volatilities are very similar and the combined portfolios in the far-right column have almost identical Sharpe ratios to their volatility-unadjusted equivalents. For example, the Sharpe ratio of the 'top half, momentum ranked' portfolio of developed economy equities is 0.38

(Panel A, Table 5), compared with a value of 0.34 produced by the 'top half volatility-adjusted, momentum ranked' technique for the same markets.

Table 8 presents the results of volatility-adjusted momentum weighting within each asset class combined with the ten month trend following rule. These results are comparable to those presented in Table 6, where no volatility adjustment is applied to the momentum weights. A comparison of the two tables shows, that volatility-adjusting the momentum weights offers some small improvement here. Sharpe ratios are marginally higher and the combined portfolios are an improvement on their unadjusted counterparts. For example, the Sharpe ratio of the 'top quarter, momentum ranked portfolio with individual trend following' applied to developed economy equities without the volatility adjustment is 0.71 (panel D, Table 6), but when the volatility adjustment is applied, the Sharpe ratio rises to 0.81 (panel D, Table 8).

4. Flexible Asset Allocation

To this point we have considered forming portfolios either within an asset class, on a risk parity basis or using an equally-weighted model, i.e. 20% in each asset class. We have used the market as a guide in terms of the assets to include in these portfolios based on momentum and trend following rules. In this section of the paper we extend this approach to allow the market to guide the asset allocation decision further. We now rank all ninety-five of the markets by volatility-adjusted momentum with no differentiation made with respect to the asset class to which they belong. We present results based on holding the top 5 winning markets (equally-weighted), as well as the top 10, 15, 20, 25, 30, 40, and 50 markets (for a portfolio with a relatively small number of positions (13 or less), this means that it could be comprised entirely of one asset class).

The benefit of this flexible approach to asset allocation is that it removes any prejudices from the portfolio composition. For instance, if one thinks that commodities are a poor investment because the roll yields have been negative for periods of time in recent years then this should show up in the momentum rankings and the allocation to them will be reduced as a result. One is not required to make a judgement about whether government bond yields are too low to represent any kind of long-term value, or if they represent an excellent investment because we are on the brink of a deflationary collapse, etc.

Table 9 displays the results of this flexible volatility-adjusted, momentum strategy. Firstly we can see that the average return for any portfolio comprising 30 positions or fewer is around 14% per annum. This compares with an average return of 8.6% for the equally weighted portfolio of all markets shown in the far-right column of the same table. In the range of 20-50 positions we find that the volatility of the flexible momentum portfolio is actually lower than for the equally-weighted portfolio of all markets, producing Sharpe ratios ranging from 0.65 to 0.84. The optimum number of positions on a risk-adjusted basis appears to be between 15 and 30, although these portfolios suffered maximum drawdowns of 29.0% to 33.5% which again is less than the equally-weighted case although perhaps too high for conservative investors.

Previously in this paper we have seen how the addition of trend following to momentum portfolios has improved their performance. Table 10 reports the performance of a flexible momentum approach with individual trend following (10-month signal) applied to each instrument. Firstly, we note that returns are slightly higher by around 1% per annum compared to the non-trend following results in Table 9. The table shows that the equally-weighted portfolio return approximately 10% per year while momentum portfolios with 15 to 30 positions return around 14.6% - 15.6% pa. Interestingly, we find that risk-adjusted performance improves with the number of positions up to 15 and then levels out at a Sharpe ratio of slightly less than 1.0. This level of Sharpe ratio is very similar to produced by the equally-weighted trend following rules reported in Tables 3 (without any momentum), 6 and 8. The application of momentum with trend following thus appears to increase the level of return compared to just trend following on its own but comes at the expense of higher volatility. To this extent momentum portfolios with the application of a trend following overlay appear to produce a higher beta version of the basic trend following method.

Figure 1 shows a comparison between the rolling 3-year annualized returns of the 20 position flexible momentum with trend following portfolio and an equally-weighted portfolio of all 95 markets without any trend following. Firstly we note that the former never has a losing three-year period and, in all but one short period, the annual return is in excess of 5%. In general, the returns of the flexible momentum portfolio are nominally higher during periods when the equally-weighted returns are also high. This is unsurprising since the momentum strategy can only select the best of what is available. If the environment is generally one of low returns then outsized gains are unlikely to be achieved. We also notice that significant periods of

relative outperformance to the flexible strategy occur when the non-trend following portfolio is under stress. For instance there appears to be a sizeable gap in performance between 2000 and 2003 and between 2009 and 2011.

Two big differences between the results presented in Tables 9 and 10 are the maximum drawdowns and the skew. Consistent with our earlier findings, trend following substantially reduces volatility and drawdowns. For example, a 15 position volatility-adjusted momentum portfolio, with trend following, experienced its maximum drawdown of 20.8%, compared with a maximum drawdown of 33.5% produced by the same approach, but without trend following. The skew of the former portfolio is also less negative at -0.02 compared with -0.31 for the latter. Figure 2 shows how the asset allocation of this 20 position, flexible multi-asset momentum portfolio with trend following varies over time. Firstly, no single asset class appears to dominate over the sample period. Developed equities have a large presence in the late 1990's while bonds have large weightings during the early 2000's after the dot-com crash and during the credit crisis in the late 2000's where cash levels also rise. We also note the large exposure to REITs as property was booming in the mid-2000's. Emerging equities make appearances periodically, but perhaps not as much as one would expect given the rise of the BRICs. We attribute this to the generally high levels of volatility that this asset class displays (see Table 1) and thus the lower adjusted rankings these achieve.

5. Risk Adjustment

The properties of returns presented thus far refer to unconditional returns from risk parity, trend following and momentum strategies. In this section we examine whether these returns are explained by widely employed risk factors. For clarity, we examine the returns from particular strategies and present the results in Table 11. In the table: EW represents the returns on a portfolio consisting of all 95 markets and commodities with equal weighting; TF represents the returns generated by applying the 12-month trend following filter shown in Table 3 (last column); MOM EW represents the returns generated by equally weighted momentum portfolio shown in Panel A of Table 5 (last column); represents the returns from the momentum strategy; MOM VW represents the returns generated by momentum strategy, where the momentum weights are volatility adjusted and the number of positions in the portfolio was 15 (column 3, Table 9); and TF & MOM VW represents the momentum strategy where weights are volatility-adjusted and where a trend following filter is applied to the individual markets (Table 10).

For each of these strategies, we examine estimates of alphas after regressing the returns from the strategies on two sets of risk factors. The first set of risk factors are those of Fama and French (1992): MKT which represents the excess return on the US equity market, SMB which is designed to capture small stock risk relative to large stocks, and HML which captures the premium on high book to market value stocks relative to low book to market value stocks. We add to these three factors the momentum factor suggested by Carhart (1997), UMD. The second set of risk factors are a wider set of 'market' risk factors which are: the excess return from the Goldman Sachs Commodity Market Index (GSCI); the return on the MSCI world equity market index (MSCI); the return on the Barclays Aggregate Bond Index (BAR); the return on the Dow-Jones UBS Commodity futures index (DJUBS). We add to these the five hedge fund factors of Fung and Hsieh (2001): the PTFS Bond (SBD), Currency (SFX), Short-term Interest Rate (SIR), commodities (COM) and Stock Index (STK) look back straddle returns⁴. These are risk factors identified by Asness at al. (2013) and Menkhoff et al. (2012) as significant in the context of a range of markets.

The results of these two sets of regressions are shown in Table 11 where Newey and West (1987) t-statistics are shown in square brackets. However, for purposes of comparison, the first column of Panel A in Table 11 shows the raw, average monthly returns for the five strategies; the Newey West t-statistics show that all are highly, and significantly different from zero. Looking across all of the strategy returns and risk factors, there is little evidence that exposure to these factors is able to account for the returns from the strategies. A comparison of the estimated alphas from the two risk adjustment regressions shown in Panels A and B show that the alphas remain large and significantly larger than zero in comparison to the raw, average returns. For example, the average return for the TF & MOM VW strategy is 1.365% per month; the Fama and French adjusted alpha is just over 1.00% per month. We also find that the Fama-French factors are jointly significantly different from zero in all cases judging by the significance of the F-statistics shown in the final column of the table. This is due to the contribution of the excess market return and, perhaps unsurprisingly, to the return to the Cahart momentum factor (UMD) which are both positive and individually significantly different to zero. The alphas calculated using the wider set of market factors (Panel B) also remain highly and statistically different from zero; the estimated alpha for the TF & MOM

⁴ Data for these risk factors can be found at http://faculty.fuqua.duke.edu/dah7/DataLibrary/TF-FAC.xls.

VW strategy is estimated to be 1.26% per month. The world equity market return and aggregate commodity market futures returns have a positive and significant effect as do the short-term interest rate and stock market hedge fund look back straddle factors. These positive relationships imply that the strategies we examine are providing a hedge against the risks that these factors represent.

The analysis of risk explanations for the trend following and momentum returns that we have found therefore suggests that while risk factors can provide a statistically significant contribution, there remains a significant unexplained alpha which is at least two-thirds of the level of the raw excess returns.

6. Assessing the Value of Strategy Returns for Investors

The analysis in this paper has demonstrated that the risk parity strategy out performs an equally-weighted approach in terms of both average returns and Sharpe ratio and both in raw and risk factor adjusted terms. Even more clear-cut is the improvement in terms of raw returns of employing a trend following or momentum strategy or a combination of the two. However, much of the improvement in average returns achieved by the momentum strategy is at the cost of increased downside risk. Average raw or risk-factor adjusted returns or the Sharpe ratio do not provide a metric suitable for comparisons of strategy performance where skewness or higher moments are significant. In this section we evaluate strategy returns using measures that take into account higher moments and, more importantly, provide a direct connection to the utility function of investors.

The first measure that we consider is that discussed by Sortino and Price (1994). The Sortino ratio is constructed as the ratio of the average excess return divided by the standard deviation of negative returns. It offers an atheoretic focus on the size of returns relative to downside risk which penalises strategy returns with significant negative skewness. Whilst the Sortino index allows for downside risk, it is not directly connected to investor preferences. The second measure for ranking investments that we consider is the generalized measure proposed by Smetters and Zhang (2013). They show that in order to be valid for non-Normal distributions of returns, any measure cannot be independent of investor preferences. The version that we report is that based on the power utility function where the utility function can be written as:

$$U(r) = \frac{(1+r)^{1-\gamma}}{1-\gamma}$$

Where γ is the (constant) coefficient of relative risk aversion. This is a suitable utility function because of its wide use in asset pricing studies and, as Barroso and Santa-Clara (2013) point out, it is sensitive to higher order moments in returns. The form of the generalised measure that we report is given by:

$$SZ4 = \frac{(1+r)^{-\gamma}}{\gamma(1+r)^{-(1+\gamma)}} \left(\frac{SR^2}{2} + \frac{\gamma(1+\gamma)}{6} . SR^3 . Skew - \frac{\gamma(1+\gamma)(2+\gamma)}{24} . SR^4 . (Kurt - 3) \right)$$

Where SR is the Sharpe Ratio, Skew is skewness and Kurt is kurtosis of the return series concerned. We are required to calibrate the coefficient of relative risk aversion γ which we set at 4 following Bliss and Panigirtzoglu (2004). The ranking results that we find are not sensitive to variations in this parameter⁵. We also compute the SZ3 measure which includes only the first three moments excluding the contribution of kurtosis.

Values for the Sortino index and the Smetters-Zhang generalised measure for the strategy returns examined in Section 5 are given in Table 12. These measures are indices and we assess their ranking and relative size as one would the Sharpe ratio. The values for these measures show the pronounced improved performance of trend following and momentum over the equally-weighted strategy. However, most striking is the sharply improved performance of the trend following and combined momentum and trend following strategies with their low maximum drawdown and mild positive skewness when compared to the sharply negatively skewed momentum returns. The differences shown in the SZ3 measure are more marked when kurtosis is considered as well in the SZ4 measure. These rankings are consistent with that provided by the Sharpe ratio but are much more pronounced in scale. They demonstrate that trend following should be strongly favoured over momentum by risk averse investors.

⁵ Results for risk aversion equal to 5 and 10 are available from the authors. Both provide more substantial differences in performance but with the same ranking of strategies.

7. Conclusions

We have studied a number of different approaches to global asset allocation. We observed that a basic risk-parity approach outperformed an equally-weighted methodology across five major asset classes by offering a similar return but with approximately half the volatility. The success of this strategy is in part due to the outstanding risk-adjusted returns of bonds over the period of study. When we examined risk parity within an asset class we observed little difference with equally-weighted portfolios.

Another improvement on an equally-weighted buy-and-hold asset allocation was to use trend following. A simple rule was employed that switched out of risk assets and into cash when the former were in a downtrend. Consistent with Faber (2009), we find this approach gives rise to substantially enhanced risk-adjusted returns in a multi-asset portfolio. Unlike risk parity, we note that trend following also offers improved performance within four of the five asset classes we consider. Perhaps the greatest benefit of trend following is the reduction in volatility that accrues to this approach by being out of markets during substantial periods of decline. This in turn leads to huge reductions in the maximum drawdown an investor would experience. We show that this reduced negative skewness is also heavily favoured by risk averse investors.

Momentum has been well documented as an anomaly in the financial literature. We observe that momentum exists within a variety of asset classes, both adjusted and unadjusted for volatility. Pure momentum portfolios have a tendency though, to still experience relatively large drawdowns. One way to overcome this is to combine them with a trend following methodology, either based on the trend of the asset class or the individual instrument. Portfolios that combine trend following and momentum show much improved risk-adjusted performance, smaller drawdowns and less negative skew that the latter alone. We note though that while these combined strategy portfolios have higher nominal returns than trend following alone, they do not display any improvement in risk-adjusted returns. The suggestion is thus that adding momentum increases the beta compared to the basic trend following portfolio. There is also some evidence that this also results in improved higher order behaviour when viewed from the perspective of a risk averse investor with constant relative risk aversion preferences.

We have offered a flexible asset allocation strategy. A wide selection of instruments from a variety of asset classes were ranked according to their volatility-adjusted momentum and before a trend following filter was applied. By choosing only the winning markets it was possible to achieve a high level of return with lower volatility than a developed equity index. The benefit of this approach is that one makes no judgements about the appropriate allocation to each asset class, instead the market makes the decision itself.

Finally, we examined whether the impressive returns generated by some of these strategies could be explained by their exposure to known risk factors. Although, the alphas that we calculated were lower than unconditional mean returns, a significant proportion of the return could not be explained with reference to these risk factors.

Our results show then that a pure trend following strategy, or one overlaid on to a momentum strategy with volatility-adjusted weightings, produces much lower drawdowns than a comparable buy and hold strategy. In addition to improving the utility of a representative risk averse investor, in a world of heterogeneous investors, the substantial reduction in the drawdown has important implications for very risk averse investors, for example, investors who are nearing retirement. If one is looking to sell an investment portfolio in order to buy an annuity a large drawdown just prior to the purchase could dramatically affect future living standards. To avoid such a shock using conventional asset allocation techniques, which might involve gradually moving out of high risk assets like equities, into low risk assets prior to retirement, clearly involves in the investor having to accept much lower returns in order to keep possible drawdowns to an acceptable level. This in turn reduces the purchasing power of the portfolio at retirement. The trend following multi-asset portfolio improves on this.

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Table 1: Performance statistics based on five broad asset classes (1994-2011)

This table presents performance statistics for: the five broad asset class categories (Panel A); for the equally-weighted return on these broad asset classes (Panel B, column 1); for the trend following portfolios based on these broad asset classes with varying trend following signal lengths, (Panel B, columns 2 to 5); for a portfolio comprising the five broad asset classes where the weights are determined by risk parity rules, where volatility has been calculated using 12 months of return data (Panel C, column 2); and for a portfolio comprised of the five main asset classes, where their weights were determined by risk parity rules with a trend following overlay (Panel C, column 2). The "risk off" asset class used in the portfolios that are constructed using trend following rules is US T-Bills. The performance statistics of the portfolios presented in Panels B and C were all based on monthly rebalancing.

Panel A: Benchmark Returns					
	D E '	Emer.	D 1		DEM
	Dev. Equity	Equity	Bonds	Comms.	REITs
Annualized Return (%)	6.09	5.48	5.12	6.14	7.04
Annualized Volatility (%)	15.72	24.54	3.07	15.98	18.88
Sharpe Ratio	0.19	0.10	0.66	0.19	0.21
Max. Monthly Return (%)	11.32	17.14	3.44	13.00	20.70
Min. Monthly Return (%)	-18.93	-28.91	-1.89	-21.28	-27.85
Maximum Drawdown (%)	53.65	61.44	4.69	54.26	67.20
Skew	-0.72	-0.71	-0.01	-0.62	-0.97
Panel B: Equal Weight Model					
	Equal Wt	Trend F	ollowing (signal length,	, months)
		6	8	10	12
Annualized Return (%)	6.71	8.35	9.11	9.16	8.73
Annualized Volatility (%)	12.65	6.92	7.01	7.03	6.93
Sharpe Ratio	0.28	0.76	0.86	0.86	0.81
Max. Monthly Return (%)	10.21	7.61	6.75	6.75	6.22
Min. Monthly Return (%)	-18.99	-6.55	-6.55	-6.55	-6.55
Maximum Drawdown (%)	46.60	10.27	6.86	7.41	9.85
Skew	-1.07	-0.05	-0.14	-0.23	-0.44
Panel C: Risk Parity					
	Risk Parity	RP TF			
Annualized Return (%)	6.78	7.61			
Annualized Volatility (%)	6.13	4.17			
Sharpe Ratio	0.60	1.08			
Max. Monthly Return (%)	3.96	3.80			
Min. Monthly Return (%)	-8.40	-4.92			
Maximum Drawdown (%)	20.46	4.92			
Skew	-1.01	-0.60			

Table 2: Trend following model by asset class (1994-2011)

This table presents performance statistics for the subcomponents of each broad asset class. Column 1 presents the performance statistics for a equally-weighted portfolios of the sub-components of each broad asset class category. Columns 2 to 5 presents performance statistics for portfolios formed with the asset class sub components using trend following rules with a range of signal lengths, and where the "risk off" asset is US T-Bills. The performance statistics are all based on monthly rebalancing.

	Equal Wt	Trend Fo	ollowing (si	gnal length	, months
		6	8	10	12
Panel A: Developed Equity					
Annualized Return (%)	7.42	10.28	11.18	10.99	10.61
Annualized Volatility (%)	18.58	10.35	10.13	10.07	10.00
Sharpe Ratio	0.23	0.69	0.80	0.78	0.75
Max. Monthly Return (%)	14.55	13.25	9.58	9.58	7.87
Min. Monthly Return (%)	-24.54	-9.21	-10.13	-10.13	-10.13
Maximum Drawdown (%)	60.68	15.09	12.25	11.50	11.39
Skew	-0.85	0.17	-0.14	-0.24	-0.39
Panel B: Emerging Equity					
Annualized Return (%)	10.01	13.25	12.74	12.63	12.34
Annualized Volatility (%)	24.27	14.02	14.06	13.80	13.74
Sharpe Ratio	0.28	0.72	0.68	0.69	0.67
Max. Monthly Return (%)	19.02	16.11	15.88	13.37	12.65
Min. Monthly Return (%)	-28.57	-11.05	-11.05	-11.05	-11.05
Maximum Drawdown (%)	56.95	18.31	18.91	25.36	22.88
Skew	-0.62	0.71	0.67	0.40	0.40
Panel C: Bonds					
Annualized Return (%)	8.10	8.23	8.33	8.39	8.31
Annualized Volatility (%)	9.77	9.67	9.61	9.57	9.56
Sharpe Ratio	0.51	0.53	0.54	0.55	0.54
Max. Monthly Return (%)	9.42	9.16	9.16	9.16	9.16
Min. Monthly Return (%)	-8.72	-9.00	-8.94	-8.50	-8.50
Maximum Drawdown (%)	20.85	19.62	19.11	19.35	19.54
Skew	0.01	-0.02	-0.01	0.02	0.04
Panel D: Commodities					
Annualized Return (%)	7.04	8.68	8.63	7.95	8.02
Annualized Volatility (%)	14.06	8.64	8.58	8.39	8.43
Sharpe Ratio	0.28	0.64	0.64	0.58	0.58
Max. Monthly Return (%)	13.26	11.12	10.45	9.84	10.65
Min. Monthly Return (%)	-21.16	-7.91	-8.22	-8.22	-8.22
Maximum Drawdown (%)	47.32	13.01	16.11	14.97	16.02
Skew	-0.83	0.23	0.03	-0.04	0.05
Panel E: REITs					
Annualized Return (%)	8.42	11.48	10.65	10.31	9.91
Annualized Volatility (%)	18.23	9.37	9.25	9.23	9.19
Sharpe Ratio	0.29	0.89	0.82	0.78	0.74
Max. Monthly Return (%)	15.96	10.82	10.82	10.82	9.71
Min. Monthly Return (%)	-26.77	-8.77	-8.77	-8.77	-8.77
Maximum Drawdown (%)	62.16	8.77	8.77	9.88	9.79
Skew	-0.66	0.42	0.31	0.07	-0.14

Table 3: Applying trend following within each broad asset class (1994-2011)

This table presents performance statistics for portfolios that have a default weighting of 20% to each of the broad asset classes described in Table 1. Column 1 presents the performance statistics for an equally weighted portfolio of the five broad asset classes (20% in each asset class). Columns 2 to 5 present the performance statistics for trend following portfolios, for a range of trend following signal lengths, where: the maximum that can be invested in any one of the broad asset classes is 20%; trend following rules have been applied to each of the sub-components of the main asset classes; and where the "risk off" asset class is US T-Bills. The performance statistics of all the portfolios are based on monthly rebalancing.

		Trend Following (signal length, months)					
	No TF	6	8	10	12		
Annualized Return (%)	8.81	10.65	10.58	10.33	10.10		
Annualized Volatility (%)	14.10	7.71	7.61	7.44	7.44		
Sharpe Ratio	0.40	0.98	0.98	0.97	0.94		
Max. Monthly Return (%)	12.05	10.16	8.28	7.43	7.43		
Min. Monthly Return (%)	-21.95	-5.55	-5.40	-5.92	-6.30		
Maximum Drawdown (%)	46.37	7.73	7.73	8.19	8.87		
Skew	-1.03	0.45	0.33	0.08	0.03		

Table 4: Risk parity and trend following within broad asset classes (1994-2011)

Panel A of this table presents performance statistics for portfolios that have been constructed by applying risk parity rules to the sub components of the broad asset classes, where volatility has been calculated using 12 months of return data. Panel B of this table presents performance statistics for portfolios that have been constructed by applying risk parity rules to the sub components of the broad asset classes, where volatility has been calculated using 12 months of return data, with the addition of a trend following rule, with a signal length of 10 months and where the "risk off" asset class is US T-Bills. The performance statistics of all the portfolios are based on monthly rebalancing.

	Dev. Equity	Emer. Equity	Bonds	Comms	REITs
Panel A: Risk Parity Only	1	1			
Annualized Return (%)	7.72	9.58	8.14	6.75	9.11
Annualized Volatility (%)	17.75	22.92	9.56	13.06	17.00
Sharpe Ratio	0.26	0.28	0.53	0.28	0.35
Max. Monthly Return (%)	14.09	16.27	9.16	13.33	15.67
Min. Monthly Return (%)	-23.05	-27.98	-8.84	-20.83	-27.15
Maximum Drawdown (%)	59.14	55.56	20.95	45.65	58.87
Skew	-0.88	-0.76	0.00	-0.97	-0.90
Panel B: Risk Parity & Trend Fo	ollowing				
Annualized Return (%)	10.88	12.43	8.43	8.22	11.01
Annualized Volatility (%)	9.82	13.18	9.44	8.02	8.83
Sharpe Ratio	0.79	0.71	0.56	0.64	0.90
Max. Monthly Return (%)	9.53	11.45	9.10	10.72	10.53
Min. Monthly Return (%)	-9.91	-9.43	-8.25	-8.10	-7.97
Maximum Drawdown (%)	10.90	24.43	19.54	14.99	8.65
Skew	-0.32	0.30	0.05	0.03	0.09

Table 5: Momentum within asset class (1994-2011)

This Table presents the performance statistics of portfolios formed on the basis of each asset class sub-components' performance momentum. The portfolios in Panel A are constructed by performance ranking the sub-components using 12 moths of return data and then by investing in the top 50% of sub-component performers, that is, the top half of 'winners'. Panel B is constructed in the same way but where the portfolio comprises the top 25% of 'winners'. The performance statistics of all the portfolios are based on monthly rebalancing. NB: the portfolios do not consist of short positions in 'losers'.

	Dev. Equity	Emer. Equity	Bonds	Comms.	REITs	Equal Mom.
Panel A: Momentum - Top Half						
Annualized Return (%)	9.87	9.80	8.73	12.28	10.15	10.89
Annualized Volatility (%)	18.00	24.64	10.14	16.11	17.34	13.71
Sharpe Ratio	0.38	0.27	0.55	0.57	0.41	0.57
Max. Monthly Return (%)	12.66	19.67	10.91	15.37	16.21	11.00
Min. Monthly Return (%)	-21.52	-30.05	-8.49	-21.32	-24.18	-20.53
Maximum Drawdown (%)	56.02	59.80	20.99	50.45	56.01	43.83
Skew	-0.76	-0.73	0.25	-0.72	-0.58	-1.03
Panel B: Momentum - Top Quarter						
Annualized Return (%)	10.66	6.56	9.25	14.46	8.68	10.90
Annualized Volatility (%)	18.81	27.03	9.92	20.21	18.72	14.48
Sharpe Ratio	0.40	0.13	0.62	0.56	0.30	0.54
Max. Monthly Return (%)	12.70	23.75	10.71	15.91	13.82	10.46
Min. Monthly Return (%)	-20.84	-35.46	-7.55	-25.90	-26.28	-21.08
Maximum Drawdown (%)	58.58	64.21	18.00	47.09	56.16	45.12
Skew	-0.72	-0.66	0.29	-0.59	-0.63	-0.94

Table 6: Momentum and Trend Following within Asset Class (1994-2011)

This Table presents the performance statistics of portfolios formed on the basis of each asset class sub-components' performance momentum. The portfolios in Panels A and C are constructed by performance ranking the sub-components using 12 months of return data and then by investing in the top 50% of sub-component performers, that is, the top half of 'winners'. Panels B and D are constructed in the same way but where the portfolio comprises the top 25% of 'winners'. In panels A and B a trend following filter, based on a 10 month signal, is applied to the indicated broad asset class; in the event that a broad asset class is estimated to be in a downtrend the asset class' default holding of 20% is placed in the "risk off" asset class US T-Bills. The portfolio statistics presented in Panels C and D have been generated by applying a trend following filter based on a 10 month signal applied to each sub component of the five broad asset classes, and where the "risk off asset" class is again US T-Bills. In all four panels the maximum holding of any broad asset class is 20%. The performance statistics of all the portfolios are based on monthly rebalancing. NB: the portfolios do not consist of short positions in 'losers'.

	Dev.	Emer.				Equal				
	Equity	Equity	Bonds	Comms.	REITs	Mom.				
Panel A: Momentum Only - Top H										
Annualized Return (%)	14.41	14.18	7.31	13.10	9.93	12.27				
Annualized Volatility (%)	12.34	17.21	9.60	12.97	11.81	8.72				
Sharpe Ratio	0.92	0.64	0.44	0.77	0.58	1.05				
Max. Monthly Return (%)	11.84	19.67	10.91	15.37	13.59	8.66				
Min. Monthly Return (%)	-15.43	-15.68	-8.49	-16.78	-9.96	-9.10				
Maximum Drawdown (%)	15.43	30.31	17.48	22.66	14.58	9.13				
Skew	-0.28	0.37	0.30	-0.30	0.21	0.03				
Panel B: Momentum Only - Top Quarter, TF Asset Class Filter										
Annualized Return (%)	15.26	12.96	7.81	15.45	8.05	11.76				
Annualized Volatility (%)	13.48	19.26	9.40	16.74	13.49	10.34				
Sharpe Ratio	0.90	0.51	0.50	0.74	0.37	0.84				
Max. Monthly Return (%)	12.33	23.75	10.71	15.91	13.82	8.68				
Min. Monthly Return (%)	-16.47	-19.37	-7.55	-16.27	-11.71	-8.92				
Maximum Drawdown (%)	16.47	38.22	14.27	24.52	32.58	21.78				
Skew	-0.27	0.51	0.37	-0.15	0.20	-0.04				
Panel C: Momentum Only - Top H	lalf, Individu	al TF								
Annualized Return (%)	13.22	13.45	8.66	13.48	10.39	12.34				
Annualized Volatility (%)	12.90	18.32	10.01	13.14	11.67	9.43				
Sharpe Ratio	0.78	0.56	0.55	0.79	0.62	0.98				
Max. Monthly Return (%)	11.84	19.67	9.46	12.67	11.02	9.03				
Min. Monthly Return (%)	-15.43	-15.68	-8.49	-14.25	-9.26	-7.91				
Maximum Drawdown (%)	19.34	32.70	22.01	24.53	14.16	11.17				
Skew	-0.37	0.30	0.20	-0.17	-0.03	0.07				
Panel D: Momentum Only - Top Q	uarter, Indi	vidual TF								
Annualized Return (%)	13.51	11.67	8.87	15.73	10.37	12.77				
Annualized Volatility (%)	14.74	21.52	9.91	17.66	14.59	11.01				
Sharpe Ratio	0.71	0.40	0.58	0.71	0.50	0.88				
Max. Monthly Return (%)	12.33	23.75	9.59	14.43	13.82	9.29				
Min. Monthly Return (%)	-16.47	-19.37	-7.55	-15.38	-11.71	-8.92				
Maximum Drawdown (%)	25.04	35.26	20.59	26.29	25.67	15.69				
Skew	-0.38	0.24	0.20	-0.18	0.13	0.04				

Table 7: Volatility-adjusted momentum within asset class (1994-2011)

This Table presents the performance statistics of portfolios formed on the basis of each asset class sub-components' performance momentum. The portfolios in Panel A are constructed by performance ranking the sub-components of each asset class using 12 months of return data standardized by the prior 12-month volatility and then by investing in the top 50% of performers, that is, the top half of 'winners'. Panel B is constructed in the same way but where the portfolio comprises the top 25% of 'winners'. In both panels, the "winning" sub-asset classes are equally weighted. The 'Equal Momentum' column reports the performance of a strategy that invests 20% in each of the five asset class momentum portfolios. The performance statistics of all the portfolios are based on monthly rebalancing. NB: the portfolios do not consist of short positions in 'losers'.

	Dev. Equity	Emer. Equity	Bonds	Comms.	REITs	Equal Mom.
Panel A: Momentum - Top Half						
Annualized Return (%)	9.35	10.95	9.02	11.82	10.44	11.07
Annualized Volatility (%)	18.54	25.50	10.05	16.28	17.05	13.96
Sharpe Ratio	0.34	0.31	0.59	0.54	0.43	0.57
Max. Monthly Return (%)	15.02	20.01	10.91	15.37	16.21	11.37
Min. Monthly Return (%)	-26.03	-31.58	-8.49	-21.04	-24.91	-22.03
Maximum Drawdown (%)	61.28	61.42	20.77	48.65	55.87	45.22
Skew	-0.90	-0.65	0.25	-0.61	-0.57	-1.11
Panel B: Momentum - Top Quarter						
Annualized Return (%)	11.09	7.05	8.77	14.07	9.85	11.10
Annualized Volatility (%)	18.79	26.82	9.62	20.28	18.02	14.49
Sharpe Ratio	0.42	0.15	0.59	0.54	0.37	0.55
Max. Monthly Return (%)	13.28	22.21	10.49	16.63	15.09	11.69
Min. Monthly Return (%)	-27.68	-31.33	-7.17	-25.90	-26.28	-22.89
Maximum Drawdown (%)	61.74	68.12	16.96	49.50	52.81	46.35
Skew	-1.03	-0.65	0.25	-0.48	-0.56	-1.05

Table 8: Volatility-Adjusted momentum and trend following within asset class

This Table presents the performance statistics of portfolios formed on the basis of past performance over the previous 12 months. The portfolios in Panels A and C are constructed by performance ranking the subcomponents within each asset class using 12 months of return data standardized by the prior 12-month volatility and then by investing in the top 50% of sub-component performers, that is, the top half of 'winners'. Panels B and D are constructed in the same way but where the portfolio comprises the top 25% of 'winners'. In panels A and B a trend following filter, based on a 10 month signal, is applied to the indicated broad asset class; in the event that a broad asset class is estimated to be in a downtrend the asset class' default holding of 20% is placed in the "risk off" asset class, US T-Bills. The portfolio statistics presented in Panels C and D have been generated by applying a trend following filter based on a 10 month signal applied to each sub-component of the five broad asset classes, and where the "risk off" asset class is again US T-Bills. In all four panels the reported portfolios are equally weighted. The 'Equal Momentum' column reports the performance of a strategy that invests 20% in each of the five asset class momentum portfolios. The performance statistics of all the portfolios are based on monthly rebalancing. NB: the portfolios do not consist of short positions in 'losers'.

	Dev. Equity	Emer. Equity	Bonds	Comms.	REITs	Equal Mom.				
Panel A: Momentum Only - Top H										
Annualized Return (%)	14.42	15.18	7.51	12.50	10.41	12.47				
Annualized Volatility (%)	11.84	17.02	9.50	12.96	11.13	8.47				
Sharpe Ratio	0.95	0.71	0.46	0.72	0.66	1.10				
Max. Monthly Return (%)	11.45	19.71	10.91	15.37	13.59	8.19				
Min. Monthly Return (%)	-13.84	-14.11	-8.49	-16.36	-9.95	-8.21				
Maximum Drawdown (%)	13.84	27.84	17.12	22.06	12.37	8.21				
Skew	-0.21	0.60	0.32	-0.20	0.34	0.03				
Panel B: Momentum Only - Top Quarter, TF Asset Class Filter										
Annualized Return (%)	15.59	13.25	7.29	15.54	8.61	12.64				
Annualized Volatility (%)	12.62	18.06	9.11	16.46	12.53	9.13				
Sharpe Ratio	0.99	0.56	0.46	0.76	0.44	1.04				
Max. Monthly Return (%)	13.28	22.21	10.49	16.63	13.82	9.56				
Min. Monthly Return (%)	-13.97	-15.17	-7.17	-16.94	-12.73	-8.57				
Maximum Drawdown (%)	13.97	31.61	13.66	27.11	31.03	8.57				
Skew	-0.01	0.34	0.35	-0.06	0.15	0.11				
Panel C: Momentum Only - Top H	alf, Individu	al TF								
Annualized Return (%)	12.87	14.29	8.85	13.19	10.72	12.47				
Annualized Volatility (%)	12.56	18.38	9.93	13.30	11.21	9.34				
Sharpe Ratio	0.78	0.61	0.58	0.76	0.68	1.00				
Max. Monthly Return (%)	11.45	19.24	9.16	12.67	11.02	8.67				
Min. Monthly Return (%)	-13.84	-14.11	-8.49	-13.83	-7.84	-7.57				
Maximum Drawdown (%)	22.69	36.09	21.68	23.87	10.45	10.40				
Skew	-0.35	0.36	0.20	-0.14	0.08	0.06				
Panel D: Momentum Only - Top Q	uarter, Indiv	idual TF								
Annualized Return (%)	14.41	10.60	8.51	16.48	11.30	12.94				
Annualized Volatility (%)	14.01	20.67	9.60	17.80	13.90	10.79				
Sharpe Ratio	0.81	0.36	0.56	0.75	0.59	0.91				
Max. Monthly Return (%)	13.28	22.21	9.59	16.63	13.82	9.44				
Min. Monthly Return (%)	-13.97	-15.17	-7.17	-16.94	-11.96	-8.57				
Maximum Drawdown (%)	24.56	43.13	19.39	26.23	25.73	13.82				
Skew	-0.14	0.08	0.18	-0.05	0.06	0.08				

Table 9: Volatility-Adjusted Momentum across Asset Classes (1994-2011)

This Table presents the performance statistics of portfolios formed on the basis of each asset class sub-components' performance momentum. The portfolio formation process was applied to all 95 individual sub-components , regardless of their asset class. The portfolios are constructed by performance ranking the sub-components using 12 months of return data standardized by the prior 12-month volatility and then by investing in the top five performers (column 1), the top ten performers (column 2), etc. Positions are equally-weighted within the portfolio. The performance statistics of all the portfolios are based on monthly rebalancing. NB: the portfolios do not consist of short positions in 'losers'.

				Num	ber of Posi	tions			
	5	10	15	20	25	30	40	50	All
Annualized Return (%)	13.89	14.32	14.62	14.40	14.09	13.96	12.97	11.65	8.64
Annualized Volatility (%)	18.38	15.00	14.18	13.64	13.14	13.01	12.90	13.05	13.72
Sharpe Ratio	0.59	0.75	0.81	0.83	0.84	0.83	0.76	0.65	0.40
Max. Monthly Return (%)	20.58	11.34	11.38	12.52	11.17	10.11	9.95	10.27	11.84
Min. Monthly Return (%)	-12.77	-15.40	-15.67	-14.37	-14.36	-14.79	-15.67	-18.80	-21.54
Maximum Drawdown (%)	35.67	35.70	33.51	32.33	29.02	30.65	35.69	41.21	45.48
Skew	0.15	-0.18	-0.31	-0.33	-0.40	-0.41	-0.59	-0.86	-1.08

Table 10: Volatility-Adjusted Momentum and Trend Following across Asset Classes (1994-2011)

This Table presents the performance statistics of portfolios formed on the basis of each asset class sub-components' performance momentum. The portfolio formation process was applied to all 95 individual sub-components , regardless of their asset class. The portfolios are constructed by performance ranking the sub-components using 12 months of return data standardized by the prior 12-month volatility and then by investing in the top 5 performers (column 1), the top ten performers (column 2), etc. The positions within the portfolios are equally weighted. However, the weight of any sub-component of the portfolio is set to 0.0% if that sub component is determined to be in a negative trend, where ten months of prior price data are used to determine the nature of the trend. The proportion allocated to that market is then allocated instead to the "risk off" asset, US T-Bills. The performance statistics of all the portfolios are based on monthly rebalancing. NB: the portfolios do not consist of short positions in 'losers'.

		Number of Positions									
	5	10	15	20	25	30	40	50	All		
Annualized Return (%)	14.75	15.17	15.69	15.65	14.68	14.60	13.66	12.57	10.02		
Annualized Volatility (%)	17.81	14.09	13.01	12.44	11.97	11.65	10.80	9.99	7.08		
Sharpe Ratio	0.65	0.86	0.97	1.01	0.97	0.99	0.98	0.95	0.98		
Max. Monthly Return (%)	20.58	11.50	11.56	12.52	11.17	10.11	9.31	8.29	7.27		
Min. Monthly Return (%)	-12.77	-11.02	-11.70	-11.46	-9.72	-9.17	-8.94	-8.71	-5.59		
Maximum Drawdown (%)	28.27	26.52	20.80	18.34	16.57	15.78	13.55	13.83	8.68		
Skew	0.23	0.06	-0.02	0.01	-0.04	-0.01	-0.04	-0.05	0.13		

Table 11: Alpha calculations for a selection of investment strategies (1994 to 2011)

This table presents the unconditional mean returns (column 1, panel A) "Average", generated by the different investment strategies: EW represents the returns on a portfolio consisting of all 95 markets and commodities with equal weighting; TF represents the returns generated by applying the 12-month trend following filter shown in the final column of Table 3; MOM EW represents the returns generated by equally weighted momentum portfolio shown in Panel A of Table 5 (last column); represents the returns from the momentum strategy; MOM VW represents the returns generated by momentum strategy, where the momentum weights are volatility adjusted and the number of positions in the portfolio was 15 (column 3, Table 9); and TF & MOM VW represents the momentum strategy where weights are volatility-adjusted and where a trend following filter is applied to the individual markets (Table 10). Panel A also reports the results of regressing the returns from these strategies using Fama and French (1992) three factors, MKT, SMB and HML, plus Carhart's (1997) momentum factor, UMD. Panel B reports the results of regressing the returns from these strategies against a set of wider risk factors described in Section 5 of this paper. Newey and West (1997) t-statistics are shown in square brackets. Prob F is based upon a F-statistic for the test of the joint significance of the independent regressors.

Panel A	Average	Alpha	MKT	SMB	HML	UMD						Prob F
EW	0.861	0.467	0.657	0.0935	0.163	-0.0242						0
	[2.61]	[2.32]	[10.7]	[1.79]	[3.43]	[0.92]						
TF	0.892	0.668	0.286	0.0547	0.0955	0.0758						0
	[5.51]	[5.18]	[7.07]	[1.61]	[2.00]	[3.71]						
MOM EW	1.283	0.785	0.679	0.0387	0.0375	0.2596						0
	[4.06]	[3.35]	[11.4]	[0.57]	[0.62]	[5.32]						
MOM VW	1.285	0.845	0.563	0.0518	0.101	0.217						0
	[4.39]	[3.88]	[8.52]	[0.88]	[1.94]	[4.81]						
TF & MOM VW	1.365	1.003	0.433	0.0327	0.0572	0.229						0
	5.51]	[5.16]	[6.78]	[0.56]	[0.99]	[5.13]						
Panel B		Alpha	GSCI	MSCI	BAR	DJUBS	SBD	SFX	SIR	STK	COM	Prob F
EW		0.362	-0.0108	0.0502	0.0286	0.0369	-0.326	0.612	-1.42	0.764	-1.1	0
		[2.84]	[3.58]	[20.9]	[3.37]	[9.34]	[0.45]	[1.00]	[2.97]	[0.87]	[1.45]	
TF		0.777	-0.00657	0.0209	0.0165	0.0193	-1.36	0.581	-0.623	2.68	1.29	0
		[5.67]	[1.92]	[5.83]	[2.00]	[3.85]	[2.17]	[0.93]	[1.18]	[3.20]	[1.29]	
MOM EW		0.852	-0.0151	0.0399	0.0344	0.0478	-2.37	0.556	-2.34	1.68	2.71	0
		[3.49	[1.79]	[5.74]	[1.80]	[4.55]	[1.09]	[0.65]	[3.64]	[1.02]	[1.07]	
MON VW		1.08	0.00628	0.0364	0.0399	0.0284	-0.143	1.73	-2.34	3.26	1.49	0
		[4.52]	[1.03]	[5.97]	[2.58]	[3.76]	[0.10]	[2.05]	[3.38]	[2.06]	[0.66]	
TF & MOM VW		1.26	0.00248	0.0283	0.0181	0.02	-1.68	2.1	-1.35	4.38	2.27	0
		[5.16]	[0.41]	[4.07]	[1.02]	[2.49]	[1.19]	[2.40]	[1.60]	[2.75]	[0.99]	

Table 12: Ranking measures for a selection of investment strategies (1994 to 2011)

This table presents performance ranking measures for five investment strategies. SR is the Sharpe ratio, Sortino is the sortino index, SZ3 and SZ4 are the generalised measures proposed by Smetters and Zhang (2013) for up to three and up to four moments for each return, respectively: EW represents the returns on a portfolio consisting of all 95 markets and commodities with equal weighting; TF represents the returns generated by applying the 12-month trend following filter shown in the final column of Table 3; MOM EW represents the returns generated by equally weighted momentum portfolio shown in Panel A of Table 5 (last column); represents the returns from the momentum strategy; MOM VW represents the returns generated by momentum strategy, where the momentum weights are volatility adjusted and the number of positions in the portfolio was 20 (column 4, Table 9); and TF & MOM VW represents the momentum strategy where weights are volatility-adjusted and where a trend following filter is applied to the individual markets (Table 10).

	Average	SR	Sortino	SZ3	SZ4
EW	8.722	0.410	0.653	-0.0466	-0.125
TF	10.325	0.972	2.106	0.201	2.976
MOM EW	13.386	0.665	1.166	-0.0868	0.428
MOM VW	14.392	0.828	1.578	-0.0815	1.095
TF & MOM VW	15.654	1.009	2.145	0.1561	3.540

