Asset Bubbles and Inflation as Competing Monetary Phenomena

by Guillaume Plantin*

Abstract. In a model with multiple price-setting equilibria with varying price rigidity à la Ball and Romer (1991), a central bank using a Taylor rule may inadvertly create asset bubbles instead of reaching its inflation target regardless of the value of the natural rate. These monetary bubbles differ from natural ones in three important ways: i) They do not push up the interest rate no matter their size and thus earn low returns themselves; ii) They burst when inflation picks up; iii) They always crowd out investment by draining resources from the most financially constrained agents.

Introduction

An increasing number of observers contend that the very accommodative monetary policies that have prevailed in advanced economies since 2008 have had the unintended consequences of blowing asset bubbles instead of spurring much needed real investment. This view has undoubtedly gained significant traction since the housing boom that preceded the 2008 crisis, and even more so since the Covid-19 crisis.[1] The proponents of this view accordingly worry that these bubbles may burst as inflation picks up and monetary policy tightens, thereby generating severe financial instability.[2]

This narrative is commonly dismissed by economists and policy makers as not grounded in theory. To be sure, a sizeable literature studies the interplay of monetary policy and bubbles.[3]

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[1]Examples abound, for example, in the blogosphere: "Pandemic-Era Central Banking Is Creating Bubbles Everywhere" (https://www.bloomberg.com), "The Fed Has Created A Monster Bubble It Can No Longer Control" (https://seekingalpha.com), "The Fed Is Creating A Monster Bubble" (https://www.forbes.com/), "Fed Trying To Inflate A 4th Bubble To Fix The Third" (https://seekingalpha.com),"The Fed's Corporate Bond Buying Is Stoking Bubble Fears" (https://www.cnbc.com),...

[2]See, e.g., "Stock Market Bubble Will Burst And Inflation Will Follow", "Will Higher Inflation End U.S. Asset Bubbles?" (https://www.forbes.com).

[3]Following Bernanke and Gertler (2001), contributions include Gali (2014), Ikeda and Phan (2016), Allen et al. (2018), Dong et al. (2020), and Asriyan et al. (2021).

Bubbles in this literature are not a monetary phenomenon, however. They arise because the natural interest rate—the one that would prevail in the presence of flexible prices—is sufficiently low to make bubbles sustainable. This literature then studies how monetary policy should be modified to take these "natural" bubbles into account.

By contrast, this paper introduces bubbles as *a pure monetary phenomenon*. Such "monetary" bubbles are the unintended consequence of monetary policy in an economy in which bubbles would be impossible were prices flexible and monetary policy deprived of real effects. Beyond fitting a widespread narrative, monetary bubbles have three important features that distinguish them from natural ones:

1. *Monetary bubbles do not lift interest rates nor asset returns.* Natural bubbles push the interest rate up, the more so the larger they are. By contrast, the interest rate remains low in the presence of monetary bubbles no matter their size. In particular, monetary bubbles themselves earn a low expected return.

2. *Monetary bubbles and large CPI inflation are mutually exclusive.* Natural bubbles and CPI inflation can have any arbitrary joint dynamics in our setup. By contrast, monetary bubbles and CPI inflation are mutually exclusive and cannot jointly occur in equilibrium. As an example, we construct a sunspot equilibrium in which monetary bubbles burst when inflation (stochastically) picks up.

3. *Monetary bubbles always crowd investment out.* Whereas natural bubbles may (or may not) crowd investment in by alleviating financial constraints, monetary bubbles are always substitutes to investment.

Given these properties, monetary bubbles offer a natural rationalization of the common narrative that low policy rates may backfire into "bad" bubbles. Their low return and negative impact on investment, which are two sides of the same coin, fit well in an environment in which business investment has yet remained subdued despite low rates and compressed risk and liquidity premia. Their incompability with CPI inflation and with resulting higher policy rates also resonates with current concerns about market crashes following bouts of inflation.

We seek to write down the simplest model in which such monetary bubbles may arise. We import two standard New Keynesian ingredients, imperfectly competitive price setters and the conduct of monetary policy via an interest-rate rule, in an economy plagued by three frictions—market incompleteness, a financial friction, and a monetary friction. Each of these frictions plays the following role.

Market incompleteness. Market incompleteness in the form of overlapping generations opens up the possibility of equilibria with rational bubbles.

Financial friction. Proceeds from investments cannot be entirely pledged to outside financiers. Such limited pledgeability implies that natural bubbles may crowd investment in as they alleviate financial constraints.

Monetary friction. Firms must incur a fixed menu cost if they seek to update their prices. As in the seminal paper of Ball and Romer (1991), menu costs open up the possibility of multiple equilibria with varying levels of price rigidity. Such multiplicity occurs when, as is the case in our economy, the gain to an individual firm from updating its price over the statu quo is larger when other firms update their prices than when they stick to the statu quo (strategic complementarity in price setting).

Our main insights are best described by comparing equilibria with different levels of price rigidity. First, there exists a flexible-price equilibrium in which all firms find it optimal to incur the menu cost at each date and adjust their prices. Monetary policy perfectly pins down inflation in this case and has no real effects. The real rate of the economy is its natural rate, and thus there may be bubbles if and only if this natural rate is sufficiently small. As is standard, such natural bubbles when feasible lift the interest rate. Their impact on investment is ambiguous due to the financial friction. They might crowd investment in if the higher interest rate improves the net worth of financially constrained agents, as, e.g., in Farhi and Tirole (2012a) or Martin and Ventura (2012).[4]

There also exists a rigid-price equilibrium because firms find updating their individual prices too costly when the overall price level is constant, making fixed prices a self-justified phenomenon. The logic of the interest-rate feedback rule then implies that the policy rate, which is also the real rate given price rigidity, is below the natural rate. Thus there can be rational bubbles, deemed monetary bubbles, in the rigid-price equilibrium even though the natural rate is too large for natural bubbles to arise in the flexible-price equilibrium. Absence of arbitrage implies that these monetary bubbles earn the policy rate no matter their size. Since monetary bubbles unlike natural ones do not affect the interest rate, something else has to give in equilibrium so that they can squeeze in. We show that monetary bubbles drain resources from financially constrained agents with investment opportunities, thereby eroding their net wealth, towards savers so that the latter can purchase the bubbles. Monetary bubbles are thus always detrimental to investment in a model in which natural ones, when they are feasible, may by contrast be beneficial.

In sum, we show that a standard monetary friction, a menu cost, can create endogenous instability both in goods markets by inducing endogenous changes in price flexibility, and in capital markets by unlocking the possibility of bubbles even (but not only) when a high natural rate precludes them. There are many equilibria in our economy as multiplicity in goods markets compounds with that in assets markets. All the equilibria with monetary bubbles however share important features:

[4] As is well-known, this contrasts with rational bubbles in a frictionless environment that always crowd out investment.

i) the same low real interest rate; ii) the same below-target CPI inflation; iii) the same subdued investment expenditures.

In order to give a more concrete illustration of this endogenous instability of goods and assets prices, we construct a particular sunspot equilibrium in which the economy stochastically switches from a rigid-price regime to a flexible-price one. The economy starts out with fixed prices. The interest rate is low. A stochastic monetary bubble grows that diverts savings from investment. At a stopping time at which firms coordinate on adjusting their prices, the bubble bursts, inflation picks up, and both nominal and real rates increase. This occurs in an economy that faces no fundamental uncertainty nor uncertainty regarding the conduct of monetary policy.

Related Literature

A large and growing literature explores the empirical plausibility of menu costs as a significant source of price rigidity. Reviewing it is beyond the scope of this paper. Yet, the insight pioneered by Ball and Romer (1991) that menu costs may generate multiple equilibria has been much less explored. This paper is to my knowledge the first to stress that such multiplicity may go beyond inflation dynamics and pave the way to asset bubbles as a monetary phenomenon.

It is important to highlight that this fixed menu cost is the only source of equilibrium multiplicity that we focus on. In particular, the resulting multiplicity in inflation dynamics is unrelated to that possibly generated by interest-feedback rules (Benhabib et al., 2001a,b, 2002a,b). It would actually still prevail under any other modeling of monetary policy.[5] The Taylor rule here only has the implication that given a positive inflation target, the policy rate is lower than the natural rate when the economy is in the rigid-price equilibrium. This unlocks the possibility of monetary bubbles even when the natural rate is large.

This paper also contributes to the literature that studies the effect of bubbles on investment, in particular in the presence of financial constraints (Caballero et al., 2006; Farhi and Tirole, 2012a; Martin and Ventura, 2012; Hirano and Yanagawa, 2016). We introduce simple New Keynesian ingredients in a related environment that enables us to compare how natural and monetary bubbles affect investment.

This paper also has connections to the literature that studies interest-rate policies as a tool to mitigate financial-market imperfections (Benmelech and Bergman, 2012; Caballero and Simsek, 2020; Diamond and Rajan, 2012; Farhi and Tirole, 2012b). This literature has emphasized how subsidizing the interest rate and financial repression may backfire into various forms of excessive risk taking. To our knowledge, we are the first to show that such excessive risk taking may

[5] Ball and Romer (1991) for example obtain this multiplicity in their original paper in which monetary policy consists in controlling money supply in the presence of a cash-in-advance constraint.

materialize into “bad” rational bubbles.

Finally, it is interesting to relate this paper to the intermediary asset pricing literature pioneered by He and Krishnamurthy (2012, 2013). In this literature, negative shocks to sophisticated investors’ wealth negatively affects all asset prices. The very distinct impacts of natural and monetary bubbles on entrepreneurs’ net worth is also the main driver of their respective properties here.

The paper is organized as follows. Section I sets up the model. Sections II, III, and IV respectively study the flexible-price, fixed-price, and general versions of it, and Section V concludes.

I. Setup

Our model introduces New Keynesian ingredients in a simple economy in which the limited pledgeability of future cash flows may lead to the emergence of bubbles that crowd investment in, as in Farhi and Tirole (2012a) or Martin and Ventura (2012).

Time is discrete and indexed by $t \in \mathbb{N}$. The economy is populated by private agents—households and entrepreneurs, and by a monetary authority. All agents use the same currency as a unit of account only (“cashless economy”). Private agents consume a final good that they produce out of a continuum of intermediate goods indexed by $i \in [0,1]$ using the technology

(1)
$$C_t = \left(\int_0^1 C_{i,t}^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$. The date-t price of intermediate good i is denoted P_t^i, and P_t denotes the price of the final good—the “price level”.

Entrepreneurs. At each date, a unit mass of entrepreneurs are born and live for two dates. They consume only when old, at which time they are risk neutral. Entrepreneurs are endowed with a production technology and with an investment technology.

Production technology. Each date-t young entrepreneur $i \in [0,1]$ owns a technology that transforms L units of date-t labor into αL units of the date-t intermediate good i, where $\alpha > 0$. The technology fully depreciates after one production cycle.

Investment technology. Each date-t young entrepreneur owns a technology that transforms x date-t consumption units into ρx date-$t+1$ consumption units, where $\rho > 1$.

Households. A unit mass of households are born at each date, and live for two dates. Households supply labor to firms when young. They rank bundles (C_Y, C_O, L) of consumption when young, consumption when old, and labor according to the criterion

$$u(C_Y) + \beta C_O - \frac{\gamma L^2}{2}, \tag{2}$$

where $\beta \in (0, 1)$, $\gamma > 0$, and u' exists and is a decreasing bijection over $(0, +\infty)$.

Monetary authority. The monetary authority announces at each date t the gross nominal interest rate R_t at which it is willing to borrow from and lend to private agents between t and $t+1$. It sets R_t applying the interest-rate rule

$$R_t = r_t \Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{1+\psi}, \tag{3}$$

where r_t is the natural rate—households' rate of intertemporal substitution in the flexible-price version of the model, $\psi, \Pi^* > 0$, and $\Pi_t = P_t / P_{t-1}$ is the (gross) rate of inflation between $t-1$ and t. We will take $P_{-1} > 0$ as exogenously given.

Frictions. The economy is plagued by two frictions, a financial one and a monetary one.

Assumption 1. ***(Financial friction: Limited pledgeability)*** *An entrepreneur can divert all or part of the proceeds from her investment and consume a fraction* $1 - \lambda$ *of the diverted proceeds, where* $\lambda \in (0, 1)$.

This financial friction will induce credit rationing that may give rise to bubbles under some circumstances despite dynamic efficiency ($\rho > 1$). The second friction is a nominal rigidity such that monetary policy may have real effects:

Assumption 2. ***(Monetary friction: Menu cost)*** *Young date-*t *entrepreneur* $i \in [0, 1]$ *must incur a fixed cost equal to* F *units of labor, where* $F \geq 0$*, in order to change the price of intermediate good* i *from the statu quo* P^i_{t-1} *to a new value.*

The usual broad interpretation of the menu cost F is that it stands for the costs of information collection and decision making incurred by an entrepreneur unwilling to stick to the statu quo.[6] We posit to fix ideas that $P^i_{-1} = P_{-1}$ for all $i \in [0, 1]$.

Finally, we impose the parameter restriction

$$u' \left(\frac{\alpha^2 \beta \rho}{\gamma} \right) \leq \beta \rho, \tag{4}$$

and will explain its role in due course.

[6] See Alvarez et al. (2011) for an explicit modelling of costly information collection and menu costs in a price-setting problem.

Discussion. Two comments are in order. First, overlapping generations are only a simple way to generate the incompleteness that allows for rational bubbles. Less stylized (and less tractable) alternatives would of course be available (Aiyagari, 1994; Bewley, 1986; Woodford, 1990, e.g.). As in Martin and Ventura (2012) or Farhi and Tirole (2012a), the concept of generation in this setup should not be interpreted literally: Time elapsing between two dates is much shorter than 75 years. Assuming the same short-lived agents with simple preferences as in these papers enables us to introduce in the simplest fashion our novel insights on the joint instability of goods and assets markets. A particular gain from such simple preferences is that they enable us to solve for the equilibrium without resorting to log-linearization nor any other approximation. This will be important for the analysis in the presence of multiple equilibria in Section IV.

Second, as in the seminal paper of Ball and Romer (1991), we rely on a simple fixed menu cost to generate multiple equilibria with varying price rigidity. We could alternatively borrow from the literature that generates such multiplicity out of informational frictions (Amador and Weill, 2010; Gaballo, 2017, e.g.). We leave this exciting route for future research.

II. Flexible-price model

This section shuts down the menu-cost friction by positing that $F = 0$. Monetary policy thus has no real effects, and interest-rate rule (3) only serves to pin down inflation. Section A derives the unique non-bubbly perfect-foresight equilibrium. Section B then studies the existence and properties of natural bubbles.

A. Non-bubbly equilibrium

We define a perfect-foresight equilibrium as a situation in which private agents optimize with perfect foresight, markets clear, and $\log \Pi_t$ is bounded.[7]

This economy with flexible price admits a unique perfect-foresight equilibrium without bubbles that we now solve for. Standard arguments detailed in Appendix A.1 imply that given Taylor rule (3), any perfect-foresight equilibrium with non-exploding inflation must be such that inflation is equal to Π^*. Real equilibrium variables can then be determined as follows. Appendix A.1 shows that a non-bubbly perfect-foresight equilibrium must be a steady state, and so we drop the time subscript here for simplicity.[8] Appendix A.1 also shows that profit maximization by entrepreneurs

[7] This latter restriction to non-exploding inflation is only meant to address the well-known criticism of the elusive terminal condition that applies to *any* model of inflation determination with a Taylor rule (Castillo-Martinez and Reis, 2019, e.g.).

[8] This stems from absence of capital accumulation and quasi-linear preferences shutting down any connection between dates other than through expectations.

when setting the prices of intermediate goods implies that the real wage w must satisfy:

(5) $$w = \frac{\alpha(\epsilon - 1)}{\epsilon}.$$

Denoting r the real interest rate, households solve:

(6) $$\max_{C_Y, C_O, L} u(C_Y) + \beta C_O - \frac{\gamma L^2}{2}$$

s.t.

(7) $$C_Y + \frac{C_O}{r} = wL,$$

where $C_Y, L \geq 0$.[9] Optimal labor supply yields

(8) $$wu'(C_Y) = \gamma L,$$

and the Euler equation is

(9) $$u'(C_Y) = \beta r.$$

Combining (5), (8), and (9) yields the following expression for households' savings $wL - C_Y$:

(10) $$wL - C_Y = \delta(\epsilon - 1)r - \phi(\beta r),$$

where

(11) $$\delta \equiv \frac{\alpha^2 \beta(\epsilon - 1)}{\epsilon^2 \gamma}, \phi \equiv (u')^{-1}.$$

Combining (5), (8), and (9) again yields entrepreneurs' profit from producing the intermediate good,

(12) $$(\alpha - w)L = \delta r,$$

that they entirely save since they consume only when old. Adding up (10) and (12) yields a simple expression of aggregate savings as a function of the interest rate:

(13) $$S(r) \equiv \delta \epsilon r - \phi(\beta r).$$

Entrepreneurs invest $I = 0$ in their investment technology if $r > \rho$, and $I = +\infty$ if $r \leq \lambda\rho$. For $r \in (\lambda\rho, \rho)$ they invest I such that both their incentive-compatibility constraint and the

[9]For brevity we do not impose $C_O \geq 0$. Alternatively one could endow households with a sufficiently large income when old (e.g., from selling labor when old as well).

participation constraint of households bind.[10] Incentive compatibility requires that they hold a stake larger than $1-\lambda$ in their projects, and so investment size solves

$$\frac{\lambda \rho I}{r} = I - \delta r. \tag{14}$$

Condition (14) states that the funds $I - \delta r$ borrowed from households by entrepreneurs—equal to total investment I minus entrepreneurs' own resources δr—must be equal to the pledgeable part of the investment's payoff $\lambda \rho I$ discounted at r. This implies a simple function of investment as a function of r:

$$I(r) \equiv \frac{\delta r^2}{r - \lambda \rho}. \tag{15}$$

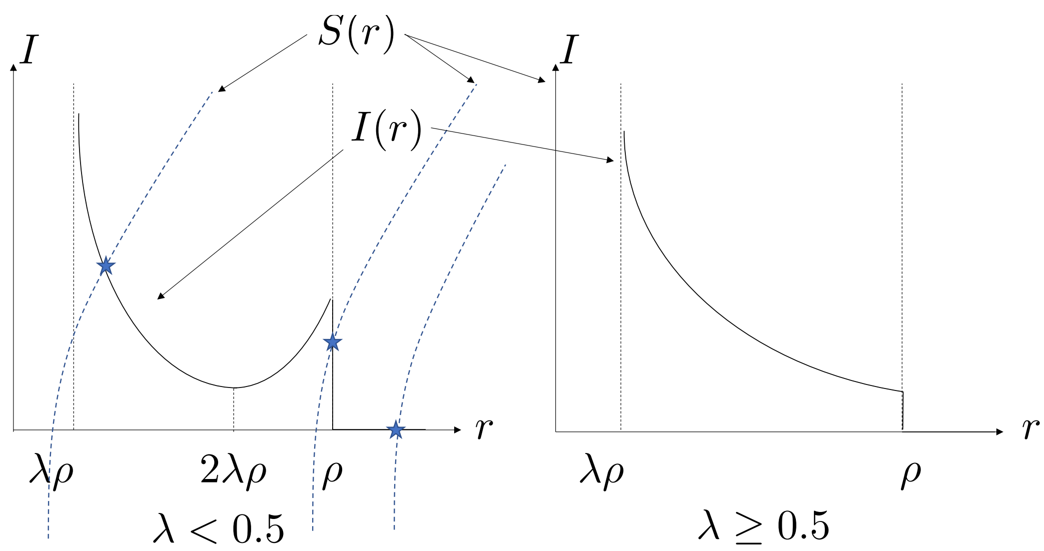

Figure 1: Investment and interest rate in the non-bubbly equilibrium

Figure 1 depicts the graph of $I(r)$ which is strictly decreasing over $(\lambda\rho, \rho]$ if $\lambda \geq 1/2$ (right-hand panel), and admits a unique minimum at $2\lambda\rho$ otherwise (left-hand panel). Two forces compete in shaping entrepreneurs' investment capacity $I(r)$. This capacity is driven both by their net worth δr and by the leverage ratio $r/(r - \lambda\rho)$ that applies to it. As r increases, so does their net worth since this spurs labor supply and thus entrepreneurs' profits.[11] This also negatively affects their

[10] Appendix A.1 details the solution to this standard optimal-contracting problem.

[11] In Farhi and Tirole (2012a), entrepreneurs' net worth increases with respect to the interest rate because they must store an exogenous endowment at this rate before investing. Their endowment endogenously increases with respect to r here.

leverage ratio as they can finance with external funds a fraction at most equal to $\lambda\rho/r$, decreasing in r, of the proceeds from investment. If $\lambda \geq 0.5$, the negative leverage effect always more than offsets the net worth effect. If entrepreneurs are more constrained ($\lambda < 0.5$), the positive net-worth effect dominates for sufficiently high rates, that is, when $r \in [2\lambda\rho, \rho]$. The reason is that the marginal positive effect of an increasing rate on entrepreneurs' net worth is constant, whereas the marginal negative effect on leverage decreases with respect to r, and so the former may prevail for a sufficiently large rate.

Savings equal investment in equilibrium. The three dashed lines in Figure 1 illustrate three possibilities for the graph of aggregate savings $S(r)$ depending on parameter values. In any case, market clearing pins down a unique equilibrium (r, I). It may be such that $r \leq \rho$ and $I = I(r)$ given by (15), or such that $r > \rho$ and $I = 0$. In this latter case entrepreneurs find loans to households preferable to investment. There is also an intermediate situation in which $r = \rho$ and entrepreneurs' incentive-compatibility constraint is slack ($I < I(r)$). The following proposition collects these results and offers a simple comparative statics property.

Proposition 1. ***(Non-bubbly flexible-price equilibrium)*** *There exists a unique non-bubbly equilibrium with perfect foresight. If $r < \rho$, investment is constrained:* (14) *is binding. If $r > \rho$, entrepreneurs lend to households rather than invest. It may also be that $r = \rho$ and* (14) *is slack.*

Output $\alpha L = \delta\epsilon r$, interest rate r, and investment I increase with respect to λ other things being equal.

Proof. See Appendix A.1. □

As the pledgeability λ of entrepreneurs' ventures decreases, this reduces their ability to lever up their net wealth, which both reduces their investment capacity and raises the price of storage vehicles (depresses the interest rate) as the supply of such vehicles by entrepreneurs shrinks. Lower returns on savings in turn reduce life-long returns from supplying labor which depresses output.

B. Bubbly steady state

A necessary and sufficient condition for bubbly equilibria to exist in this flexible-price economy is that the real rate r of the non-bubbly equilibrium be strictly smaller than the unit growth rate of the economy. Provided $\rho\lambda < 1$, other parameters can clearly be such that this is the case. It is worthwhile stressing that, as in Farhi and Tirole (2012a), the economy is always dynamically efficient as the overall return on assets is $\rho > 1$. The low return on external funds induced by financial frictions may create space for bubbles, though.

We focus here on the "bubbly steady state": the (unique) situation in which a constant-size bubble is refinanced at a unit interest rate.[12] This bubbly steady state suffices to illustrate the properties of bubbles in the flexible-price model that we want to contrast with that of the "monetary bubbles" introduced later in Section III.

Since $\rho > 1$, the non-bubbly perfect-foresight equilibrium, if it is such that $r < 1$, must also be such that investment is given by entrepreneurs' binding financial constraint:

$$I(r) = \frac{\delta r^2}{r - \lambda\rho} = \delta\epsilon r - \phi(\beta r), \tag{16}$$

and so the bubbly steady state features a bubble with size B that solves

$$\frac{\delta}{1 - \lambda\rho} = \delta\epsilon - \phi(\beta) - B. \tag{17}$$

Condition (17) states that investment equals savings net of investment in a bubble B at the equilibrium unit interest rate.[13] The right-hand panel in Figure 2 shows bubble size B as the wedge between savings and investment at $r = 1$.

The remainder of the paper deems "natural" these bubbles that arise because the natural interest rate r is smaller than one, as opposed to the purely monetary bubbles studied in Section III that will grow (and burst) despite higher natural rates.

Output is unambiguously higher in the bubbly steady state than in the non-bubbly equilibrium as it increases to $\delta\epsilon$ from $\delta\epsilon r$. Both capital and labor share increase as well since their relative contributions to output remain constant. Comparing investment at bubbly and non-bubbly steady states, Figure 2 shows that the rise of a bubble always crowds investment out when $\lambda \geq 0.5$ but crowds it in if $\lambda < 0.5$ and r is sufficiently large.[14] The intuition is simply that the raise in interest rate caused by the bubble has a negative impact on entrepreneurs' leverage ratio that may or may not offset the increase in their own resources.

[12] There are of course a plethora of bubbly dynamics whereby an initial bubble keeps shrinking as it is refinanced at interest rates strictly smaller than one. Also, bubbles may burst stochastically and new bubbles may stochastically arise at each date.

[13] To fix ideas, we suppose that the bubble is initialy issued by old households at date 0.

[14] The left-hand panel in Figure 2 illustrates crowding out when the non-bubbly steady state is at E and crowding in when it is at E'.

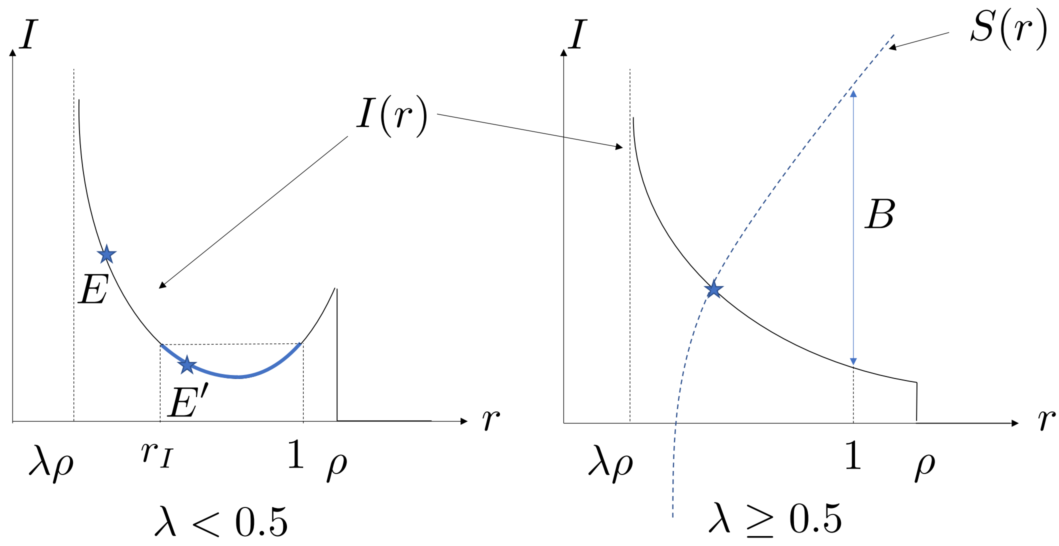

Figure 2: Investment in the bubbly steady state

Proposition 2. ***(Natural bubbles and investment)*** *The bubbly steady state exists if and only if the non-bubbly equilibrium is such that $r < 1$. Output is higher under the bubbly steady state than under the non-bubbly equilibrium. So is investment if and only if $2\lambda\rho < 1$ and $r \geq r_I$, where $r_I < 1$ solves $I(r_I) = I(1)$.*

Households' utility is higher in the bubbly steady-state than in the non-bubbly equilibrium whereas that of entrepreneurs may or may not be higher.

Proof. See Appendix A.2. □

Since output is larger in the presence of a bubble and the proceeds from investment are indeterminate, the effect of the bubble on total income per period (the sum of the two) is also ambiguous. It is easy to show that anything can go. In any case, given the purpose of the paper, the important result in this section is that natural bubbles can be either good or bad for investment and for entrepreneurs' welfare. We now show that bubbles as a pure monetary phenomenon are by contrast always detrimental to investment and entrepreneurs.

III. Fixed-price model

This section studies the case in which F is sufficiently large given other parameters that the prices of intermediary goods are fixed, and so $P_t = P_{-1}$ for all t. To be sure, this is an unrealistic polar case, yet we dwell on it because it delivers important insights about monetary bubbles in the simplest setting. Section A first characterizes the (unique) non-bubbly perfect-foresight equilibrium in this fixed-price model. Section B introduces and studies monetary bubbles. The remainder of the paper restricts the analysis to the case in which $\epsilon \geq 2$.[15]

A. Non-bubbly equilibrium

In order to describe non-bubbly equilibria with fixed prices, it is useful to start from the non-bubbly equilibrium with flexible prices (Proposition 1). The real block of this flexible equilibrium is fully characterized by five variables (C_Y, L, I, w, r) that satisfy in turn five relations in the case of binding entrepreneurs' financial constraints:

$$w = \alpha\frac{\epsilon - 1}{\epsilon}, \tag{18}$$

$$wu'(C_Y) = \gamma L, \tag{19}$$

$$u'(C_Y) = \beta r, \tag{20}$$

$$wL - C_Y + (\alpha - w)L = I = \frac{(\alpha - w)Lr}{r - \rho\lambda}. \tag{21}$$

Equation (18) stems from optimal price setting by intermediate-good producers, (19) and (20) respectively reflect households' optimal supply of labor and savings, and (21) states that total savings equal investment given in turn by a binding financial constraint.

With fixed prices, the producers of intermediate goods merely accommodate demand as long as this does not lead to negative profits, and so (18) no longer necessarily holds. The real rate $\hat{r}$ is however given by the nominal one. From (3) and $\Pi_t = 1$, it is equal to

$$\hat{r} = \frac{r}{(\Pi^*)^\psi}, \tag{22}$$

where r is the natural rate of the economy. Overall, relative to the flexible-price case, only four values (C_Y, L, I, w) need to be solved for to characterize the equilibrium, and they satisfy the four conditions given by $\{(19); (20); (21)\}$. The proof of Proposition 3 below shows that these admit a unique solution.

Let (C_Y^R, L^R, I^R, w^R) denote the equilibrium variables associated with a given policy rate $\hat{r}$—that is, the (unique) solutions to $\{(19); (20); (21)\}$ for a rate equal to $\hat{r}$.[16] It is interesting to notice that the same real variables (C_Y^R, L^R, I^R, w^R) would obtain as the real block of the flexible-price equilibrium in an economy in which the elasticity of substitution ϵ is replaced by a "shadow"

[15]This corresponds to a realistic restriction to a labor share larger than 50%.

[16]The superscript R stands for "rigid".

elasticity η that solves (18) given the fixed-price wage w^R. The variables (C_Y^R, L^R, I^R, w^R) solve indeed

$$w^R = \alpha \frac{\eta - 1}{\eta}, \tag{23}$$

$$w^R u'(C_Y^R) = \gamma L^R, \tag{24}$$

$$u'(C_Y^R) = \beta \hat{r}, \tag{25}$$

$$I^R = \frac{\delta(\eta)(\hat{r})^2}{\hat{r} - \lambda\rho} = \delta(\eta)\eta\hat{r} - \phi(\beta\hat{r}), \tag{26}$$

where $\delta(x) = \alpha^2\beta(x-1)/(x^2\gamma)$.

Otherwise stated, monetary policy in the fixed-price case has the same real effects as if the central bank could select the elasticity of substitution ϵ in the flexible-price case. This sharp characterization of the real effects of monetary policy will turn useful when analyzing monetary bubbles. The following proposition summarizes these insights. Define

$$\underline{r} \equiv \max\left\{\lambda\rho; \inf\left\{x \mid \frac{\alpha^2\beta x}{\gamma} \geq \phi(\beta x)\right\}\right\}. \tag{27}$$

Notice that $\underline{r}$ does not depend on ϵ, and that (4) implies that the inequality in (27) is satisfied at ρ.

Proposition 3. ***(Non-bubbly equilibrium with fixed prices)*** *For any $\hat{r} \in (\underline{r}, \rho)$, there exists a unique non-bubbly perfect-foresight equilibrium with fixed prices. There exists a decreasing one-to-one mapping $\eta()$ between $(\underline{r}, \rho)$ and $(\eta(\rho), +\infty)$ such that the real block of this equilibrium is identical to that of the non-bubbly flexible-price equilibrium of an economy with elasticity of substitution $\eta(\hat{r})$ and otherwise identical parameters $(\alpha, \rho, \lambda, \beta, \gamma, \phi(.))$.*

Proof. See Appendix A.3. □

Again, Proposition 3 states that the selection of a nominal interest rate in the fixed-price case is identical to that of an elasticity of substitution in the flexible-price case as far as the real block of the model is concerned. Output and investment have relatively simple expressions in this non-bubbly equilibrium.

Corollary 4. ***(Output and investment in the non-bubbly case)*** *In this fixed-price equilibrium, output αL and investment I are the following functions of $\hat{r}$:*

$$\alpha L = \frac{\alpha^2\beta\rho\lambda}{2\gamma}\left(1 + \sqrt{1 + \frac{4\gamma\phi(\beta\hat{r})(\hat{r} - \rho\lambda)}{\alpha^2\beta^2\rho^2\lambda^2}}\right), \tag{28}$$

$$I = \alpha L - \phi(\beta\hat{r}). \tag{29}$$

Two values $\underline{r} < \hat{r} < \hat{r}' < \rho$ may lead to the same output. In this case investment is strictly higher under $\hat{r}'$.

Proof. See Appendix A.4. □

An inflation target Π^* strictly larger than 1 implies that the real rate $\hat{r}$ is smaller than the natural rate r: The interest-feedback rule dictates monetary easing when inflation is below target. It is transparent from the expressions in Corollary 4 that if u is sufficiently close to linear (-ϕ' large), then output and investment decrease with respect to the interest rate over $[\hat{r}, r]$. Thus, the monetary easing induced by below-target inflation spurs investment and output in this case.

B. Monetary bubbles

The necessary and sufficient condition for bubbles to rise in this fixed-price model is that the policy rate $\hat{r} = r/(\Pi^*)^\psi$ be strictly smaller than 1. This of course can always be the case as soon as $\underline{r} < 1$ regardless of the value of the natural rate r. One only needs $(\Pi^*)^\psi$ to be sufficiently large other things being equal. The following proposition highlights two distinctive properties of such monetary bubbles relative to natural ones.

Proposition 5. ***(Monetary bubbles are always bad for investment and entrepreneurs)*** *There exist fixed-price equilibria with bubbles if $\underline{r} < \hat{r} < 1$. In such equilibria, bubbles earn an expected return $\hat{r}$. Output and households' utility are higher than in the non-bubbly equilibrium, whereas investment and entrepreneurs' utility are lower.*

Proof. See Appendix A.5. □

There are two major differences between the natural bubbles that arise in the flexible-price case and these monetary bubbles when prices are fixed:

1. Natural bubbles raise the interest rate whereas monetary bubbles do not affect it since the monetary authority controls it. Monetary bubbles thus earn low returns themselves.

2. Whereas natural bubbles may be either substitute or complement to investment, and either good or bad for entrepreneurs (Proposition 2), a monetary bubble always crowds out investment and reduces entrepreneurs' utility.

In the current environment of low real rates and subdued business investment, monetary bubbles therefore seem to be of a more plausible nature than natural ones. These two features of monetary bubbles are two sides of the same coin that can be explained as follows. A useful way to compare natural and monetary bubbles consist in studying their respective impacts on prices and quantities in the capital market. Consider first natural bubbles. Equilibrium in the capital market in the flexible-price non-bubbly equilibrium implies:

$$\delta(\epsilon)\frac{r^2}{r-\lambda\rho} = \delta(\epsilon)\epsilon r - \phi(\beta r), \tag{30}$$

where $\delta(x) = \alpha^2\beta(x-1)/(x^2\gamma)$ is decreasing over $[2, +\infty)$ whereas $x\delta(x)$ is increasing, and r is the natural rate. The left-hand side of (30) is entrepreneurs' investment and the right-hand one is aggregate savings. The presence of a natural bubble b leaves ϵ of course unchanged but affects the equilibrium interest rate, which jumps to a value $r' > r$ such that[17]

$$\frac{\delta(\epsilon)r'^2}{r'-\lambda\rho} = \delta(\epsilon)\epsilon r' - \phi(\beta r') - b. \tag{31}$$

Output, total savings, and entrepreneurs' savings all increase in the presence of a bubble. As seen in Section II, investment may or may not increase depending on whether the leverage effect more than offsets this.

Consider then monetary bubbles. Equilibrium in the capital market in the fixed-price non-bubbly equilibrium implies:

$$\frac{\delta(\eta(\hat{r}))(\hat{r})^2}{\hat{r}-\lambda\rho} = \delta(\eta(\hat{r}))\eta(\hat{r})\hat{r} - \phi(\beta\hat{r}). \tag{32}$$

Unlike in the flexible-price case, the presence of a bubble b now leaves the interest rate $\hat{r}$ unchanged. Conversely, the "shadow" elasticity of substitution $\eta(\hat{r})$ is replaced by a new elasticity of substitution η^b such that the capital-market clears

$$\frac{\delta(\eta^b)\hat{r}^2}{\hat{r}-\lambda\rho} = \delta(\eta^b)\epsilon^b\hat{r} - \phi(\beta\hat{r}) - b. \tag{33}$$

It must be that $\eta^b > \eta(\hat{r})$.[18] The proof of Proposition 5 shows that this entails in turn that output and the labor share increase with a bubble $b > 0$, whereas the capital share and investment must be lower in the presence of the bubble than in its absence. Intuitively, it is possible to squeeze bubbles on top of investment projects only if households overall have more investable funds. This must come at a reduction in the capital share relative to the non-bubbly equilibrium ($\eta^b > \eta(\hat{r})$) which always negatively affects investment because it reduces entrepreneurs' net wealth.

In sum, a compact way of stating the difference between natural and monetary bubbles is that the former affect r whereas the latter affect ϵ, and this shapes their respective impacts on the economy.

[17] r' was equal to 1 in Section II that studied the bubbly steady state.
[18] This stems from the LHS of (33) being decreasing in η and the RHS increasing.

IV. Endogenous price flexibility: Asset bubbles versus inflation

This section studies the general model. As a first step, we show that there exists a range of menu costs such that the flexible and fixed-price equilibria coexist. We then show that this multiplicity can manifest itself through interesting sunspot equilibria whereby a stochastic asset bubble first grows and then gives rise to CPI inflation as it bursts.

A. Multiple equilibrium levels of price rigidity

Recall that the parameters that determine the real block of the flexible-price model studied in Section II are $(\epsilon, \alpha, \rho, \lambda, \beta, \gamma, \phi(.))$, whereas the ones that shape the fixed-price equilibrium are $(\alpha, \rho, \lambda, \beta, \gamma, \phi(.), \hat{r})$, where $\hat{r} = r/(\Pi^*)^{\psi}$. For brevity, we assume for the remainder of the paper that these parameters are such that:

$$\underline{r} < \hat{r} < 1 < r < \rho. \tag{34}$$

This restricts the analysis to situations in which: i) the flexible-price equilibrium (Proposition 1) is such that entrepreneurs are financially constrained; ii) there are no equilibria with natural bubbles; iii) there exists a fixed-price equilibrium as in Proposition 3; iv) there exists fixed-price equilibria with monetary bubbles. We also suppose that output (28) and investment (29) decrease with respect to the interest rate over the range $[\hat{r}, r]$, so that output and investment are larger in the fixed-price (non-bubbly) equilibrium than in the flexible-price one other parameters than F being equal. Finally, we assume for simplicity that the menu cost F corresponds to a non-pecuniary disutility for entrepreneurs and thus does not reduce their resources when sunk.[19]

To be sure, the restriction to $r > 1$ is only meant to sharpen our results by exhibiting bubbles in an economy in which the natural rate precludes them. All the results in this section carry over in the case $r \leq 1$ corresponding to economies such that "$r < g$". The only difference is that equilibria with natural bubbles would be sustainable as well.

Proposition 6. ***(Multiple equilibrium levels of price rigidity)*** *Holding all parameters $(\epsilon, \alpha, \rho, \lambda, \beta, \gamma, \phi(.), \hat{r})$ fixed, there exists Π^*, ψ and $0 < \underline{F} < \bar{F}$ such that if $F \in [\underline{F}, \bar{F}]$, both the non-bubbly flexible-price equilibrium in Proposition 1 and the non-bubbly fixed-price equilibrium in Proposition 3 can be sustained. There also exist equilibria with monetary bubbles.*

Proof. We let Y^F and w^F denote the respective output and real wage in the flexible-price equilibrium described in Proposition 1, and Y^R and w^R their counterparts in the rigid-price equilibrium in

[19]This implies that the equilibrium when $F = 0$ (Section II) can be readily used when prices are flexible without adjusting entrepreneurs' resources for sunk menu costs.

Proposition 3. The fixed-price equilibrium can be sustained if entrepreneur $i \in [0,1]$ born at date t finds it preferable to leave the price of good i unchanged at $P^i_{t-1} = P_{-1}$ and save F to optimal pricing when other agents behave as described in Proposition 3. Formally, it must be that

$$\max_{P^i} \left\{ Y^R \left(\frac{P_{-1}}{P^i} \right)^{\epsilon} \left(P^i - \frac{W^R}{\alpha} \right) \right\} - FW^R \leq Y^R \left(P_{-1} - \frac{W^R}{\alpha} \right), \tag{35}$$

which can be rewritten after optimizing over P^i

$$F \geq \left(\frac{\alpha(\epsilon-1)}{Y^R} \right)^{\epsilon-1} \left(\frac{Y^R}{w^R \epsilon} \right)^{\epsilon} + Y^R \left(\frac{1}{\alpha} - \frac{1}{w^R} \right). \tag{36}$$

Similarly, the flexible-price equilibrium can be sustained if entrepreneur $i \in [0,1]$ born at date t finds it preferable to optimize the price of the intermediate good P^i_t rather than leave it unchanged at P^i_{t-1} and save F when other agents behave as described in Proposition 1:

$$\max_{P^i} \left\{ Y^F \left(\frac{P_t}{P^i} \right)^{\epsilon} \left(P^i - \frac{W^F_t}{\alpha} \right) \right\} - FW^F_t \geq Y^F \left(\frac{P_t}{P^i_{t-1}} \right)^{\epsilon} \left(P^i_{t-1} - \frac{W^F_t}{\alpha} \right)^{+}, \tag{37}$$

which can be rewritten after optimizing over P^i

$$F \leq \left(\frac{\alpha(\epsilon-1)}{Y^F} \right)^{\epsilon-1} \left(\frac{Y^F}{w^F \epsilon} \right)^{\epsilon} - Y^F (\Pi^*)^{\epsilon-1} \left(\frac{1}{w^F} - \frac{\Pi^*}{\alpha} \right)^{+}. \tag{38}$$

From (36) and (38), the set of menu costs for which both equilibria coexist is an interval with nonempty interior if and only if

$$\begin{aligned} \left(\frac{\alpha(\epsilon-1)}{Y^R} \right)^{\epsilon-1} \left(\frac{Y^R}{w^R \epsilon} \right)^{\epsilon} + Y^R \left(\frac{1}{\alpha} - \frac{1}{w^R} \right) < \left(\frac{\alpha(\epsilon-1)}{Y^F} \right)^{\epsilon-1} \left(\frac{Y^F}{w^F \epsilon} \right)^{\epsilon} \\ - Y^F (\Pi^*)^{\epsilon-1} \left(\frac{1}{w^F} - \frac{\Pi^*}{\alpha} \right)^{+}. \end{aligned} \tag{39}$$

By assumption monetary easing expands output: $Y^R > Y^F$. Furthermore, equations (19) and (20) together with $Y = \alpha L$ imply that $Y^R/w^R = Y^F/[w^F(\Pi^*)^{\Psi}] < Y^F/w^F$. Together with $w^R \leq \alpha$, this implies that (39) holds as soon as the second term on the RHS is sufficiently large, that is, given that $w^F = \alpha(\epsilon-1)/\epsilon$, if Π^* is not too small relative to $\epsilon/\epsilon - 1$. Economically, this restriction ensures that an entrepreneur has incentives to change its price in the flexible-price regime because the increase in the nominal wage reduces too much its margin otherwise.

Notice that this restriction on Π^* is feasible holding parameters $(\epsilon, \alpha, \rho, \lambda, \beta, \gamma, \phi(.), \hat{r})$ fixed because ψ can be chosen so that $\hat{r}$ is fixed given r and Π^*. This restriction on Π^* is only a rather strong sufficient condition ensuring that (39) holds. Whether (39) actually holds without it, at least for some range of parameters, is an open question that we have not been able to tackle analytically, and that is of limited interest given the qualitative nature of the analysis.

Monetary bubbles. If (39) holds for the non-bubbly fixed-price equilibrium it therefore also holds for sufficiently small monetary bubbles as the right-hand side is unchanged whereas the left-hand side can be made arbitrarily close to its value in the non-bubbly case. □

Proposition 6 states that other real parameters being equal, there exists a range of values for the menu cost F such that both flexible-price and rigid-price equilibria can be sustained. Such a range of menu costs can exist if and only if each entrepreneur finds it more valuable to adjust its price when fellow entrepreneurs do so (in the flexible-price equilibrium) than when they do not (in the rigid-price equilibrium), or, when its differential gain from optimal pricing over the statu quo is larger when other firms price optimally. The essential economic force ensuring that this is the case is the fact that when prices are rigid and the real rate is thus low, the capital share is smaller than when they are flexible. This more than offsets the fact that output is overall larger when prices a rigid. This makes the potential gains from changing its price smaller to an entrepreneur in a rigid-price environment than to an entrepreneur in a flexible-price one.

This economy has a plethora of equilibria because it compounds multiplicity in goods and assets markets. If agents coordinate on rigid prices for goods, this unlocks the possibility of monetary bubbles via a low interest rate. All equilibria share the common property, though, that CPI inflation and asset bubbles do not jointly occur.

B. An equilibrium with unstable goods and assets prices

More generally, this multiplicity of equilibria epitomizes instability in both goods and asset markets when prices are strategic complements. This last section offers a more concrete illustration of this instability. We construct a particular sunspot equilibrium that comprises two phases. During the first phase, prices are rigid and a stochastic monetary bubble grows. At the random date at which this phase ends, the bubbles bursts, the economy reverts back to the flexible-price equilibrium, and sticks to it forever. Inflation picks up and both real and nominal interest rates increase.

Formally, we suppose in this section that the conditions stated in Proposition 6 hold. Let $p \in (0,1)$ such that[20]

$$\underline{r} < \left(p + \frac{1-p}{\Pi^*}\right)\hat{r} < p. \tag{40}$$

Consider a stochastic process $(\tilde{\Omega}_t)_{t\geq 0}$ such that $\Omega_0 = 1$. At each subsequent date $t \geq 1$, $\tilde{\Omega}_t$ remains equal to 1 with probability p, or snaps to 0 with probability $1-p$, in which case it stays equal to

[20]Such a p exists since $\underline{r} < \hat{r} < 1$.

this value forever after. The realizations of $\tilde{\Omega}_t$ are public information. We also construct a strictly positive sequence $(b_t)_{t\in\mathbb{N}}$ such that $b_{t+1} = \hat{r}[1 + (1-p)/(p\Pi^*)]b_t < b_t$.

Proposition 7. ***(An equilibrium with unstable goods and assets prices)*** *If b_0 is sufficiently small other things being equal, there exists a sunspot equilibrium such that:*

- *As long as $\tilde{\Omega}_t = 1$, prices are rigid and agents trade a monetary bubble with date-t value b_t. The policy rate is $\hat{r}$, the real rate $\hat{r}[p + (1-p)/\Pi^*]$.*
- *At the stopping time τ such that $\tilde{\Omega}_\tau = 0$, the bubble bursts, prices becomes flexible, and CPI inflation jumps to Π^* and then stays at this level forever, so that the policy rate becomes $r\Pi^*$ and the real rate becomes r.*

Proof. See Appendix A.6. □

This sunspot equilibrium offers a concrete illustration of the far-reaching instability induced by fixed menu costs. The multiplicity of perfect-foresight equilibria stated in Proposition 6 translates here into purely endogenous risk added to the prices of goods and that of assets in an economy that faces no fundamental uncertainty, nor any uncertainty regarding monetary policy. First, strategic complementarity in the setting of goods prices creates room for stochastic changes in inflation. Second, the initial episode of nominal rigidity comes with a low real rate that unlocks the possibility of monetary bubbles. Even though the particular sunspot equilibrium studied here is only a possibility among many others, it shares with any equilibrium such that the economy reverts to flexible prices at some point the feature that monetary bubbles must be stochastic, and burst no later than at this point. If the bubble was attached to a particular asset, as it bursts right when the real rate increases, the bubble would amplify the impact of the variations of the interest rate on the asset price.[21]

These particular equilibrium dynamics echo some of the narratives in the popular press along the lines that "all the liquidity created by central banks only feeds asset bubbles". In the first phase of the equilibrium, a bubble grows. Investment and inflation remain subdued despite a low policy rate. The start of the second phase mirrors the current concern that the Fed response to a bout of inflation may "burst bubbles".

V. Conclusion

Using a simple model with textbook New Keynesian ingredients—a Taylor rule and an imperfectly competitive intermediate sector, this paper illustrates the widespread narrative that accommodative

[21] It would be straightforward to add a "tree" to which bubbles are attached, as in Tirole (1985).

monetary policy may backfire into bubbles that crowd out investments with superior returns. Bubbles as pure monetary phenomena starkly differ from natural ones in three interesting ways. First, they are compatible with an environment of low expected returns and earn low expected returns themselves regardless of their size. Second, they burst when CPI inflation picks up. Finally, unlike natural bubbles that may be either good or bad for investment, monetary bubbles always hurt the most productive but constrained agents of the economy by diverting resources away from them.

One interesting route for future research consists in analyzing a version of our model in which sufficient heterogeneity across firms warrants equilibrium uniqueness in price setting. Still, strong strategic complementarities would have an important multiplier effect. We conjecture that there could be room for episodes of sticky prices and stochastic monetary bubbles during regimes of monetary easing, provided these regimes end in a stochastic fashion and keep a sufficiently high probability of continuation at each point in time.

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Appendix

A.1. Proof of Proposition 1

Optimal consumption expenditures. As is well-known, optimal allocation of date-t consumption across intermediate goods yields a demand for good $i \in [0,1]$ $Y_t^i = Y_t(P_t/P_i)^\epsilon$, where Y_t is the consumption of the final good, and an equilibrium relation $P_t = (\int_0^1 P_t^{i\,1-\epsilon} di)^{1/1-\epsilon}$.

Price level. Combining the Euler equation under perfect foresight and the interest rule yields for all $t \geq 0$

$$\Pi_{t-1}^{1+\psi} = \Pi_t \Pi^{*\psi}. \tag{A.1}$$

The only price path that satisfies this and does not lead to exploding inflation rates is thus such that $\Pi_t = \Pi^*$ for all $t \geq 0$.

Entrepreneurs. Entrepreneurs produce and invest optimally. Regarding optimal production first, entrepreneur $i \in [0,1]$ at date t, taking the nominal wage W_t as given, posts the price P_i that solves

$$\max_{P_i} P_i Y_t^i - \frac{W_t Y_t^i}{\alpha}, \tag{A.2}$$

where $Y_t^i = Y_t(P_t/P_i)^\epsilon$. The first-order condition and $P_i = P_t$ in equilibrium yields the real wage

$$w_t = w = \frac{\alpha(\epsilon - 1)}{\epsilon}. \tag{A.3}$$

and so the profit rate per unit of labor is

$$\mu_t = \mu = \alpha - w = \frac{\alpha}{\epsilon}. \tag{A.4}$$

Regarding optimal investment, given a real rate r_t and their profit from production μL_t, date-t entrepreneurs choose their own investment a in their technology, the total investment size I, and a stake in the proceeds R_E that solve

$$\max_{\{a,I,R_E\}} \{R_E + r_t(\mu L_t - a)\} \tag{A.5}$$

s.t.

$$\rho I - R_E \geq r_t(I - a), \tag{A.6}$$

$$R_E \geq (1-\lambda)\rho I, \tag{A.7}$$

$$a \in [0, \mu L_t], I \geq a, \tag{A.8}$$

where (A.6) is the participation constraint of the households and (A.7) the incentive-compatibility constraint of the entrepreneurs. The former constraint can be rewritten as $R_E - r_t a \leq (\rho - r_t)I$, implying that $I = a = R_E = 0$ if $\rho < r_t$. It also implies that if $\rho > r_t$, then entrepreneurs maximize I. Combining (A.7) and (A.6) yields $(r_t - \lambda\rho)I \leq r_t a$. Thus the program has no solution if $r_t \leq \lambda\rho$. Otherwise, $a = \mu L_t$, $R_E = (1-\lambda)\rho I$, and $I = \mu L_t/(r_t - \lambda\rho)$. Finally, if $\rho = r_t$, then any $I \in [0, \mu L_t/[\rho(1-\lambda)]]$ solves the program with any $R_E = \rho a \geq (1-\lambda)\rho I$.

Households. Date-t households can trade bonds with entrepreneurs or/and the central bank and so given a real rate r_t they solve:

(A.9)
$$\max_{C_Y, C_O, L} u(C_Y) + \beta C_O - \frac{\gamma L^2}{2}$$

s.t.

(A.10)
$$C_Y + \frac{C_O}{r_t} = wL,$$

where $C_Y, L \geq 0$. Optimal labor supply yields

(A.11)
$$wu'(C_Y) = \gamma L,$$

and optimal consumption yields

(A.12)
$$u'(C_Y) = \beta r_t.$$

As seen in the body of the paper, this yields that households' savings are $\delta(\epsilon - 1)r_t - \phi(\beta r_t)$ and that of entrepreneurs δr_t.

From Walras' Law, equilibrium only requires that the bond market clears. A zero net position of the monetary authority implies that there are three possible situations:

- $I_t = 0$ and $r_t = r = \phi(\beta r)/(\delta\epsilon) > \rho$;
- $r_t = r \in (\rho\lambda, \rho)$ and $I(r) = \delta\epsilon r - \phi(\beta r)$, where $I(r)$ is given by (15);
- $r_t = \rho$ and $I \in [0, I(\rho)]$.

Notice in particular that $I(r) = \delta\epsilon r - \phi(\beta r)$ admits a unique solution because the RHS has a larger slope than the LHS for $r \leq \rho$.

The comparative statics w.r.t. to λ directly result from the RHS of $I(r) = \delta\epsilon r - \phi(\beta r)$ being increasing in r, independent of λ whereas the LHS increases with respect to λ and its graph crosses that of the RHS from above.

A.2. Proof of Proposition 2

All that is left from the body of the paper is the impact of the bubble on utilities. The only impact of bubbles on households' decision making is that they face a higher interest rate, which increases their utility from the application of the envelope theorem to their program (A.9). Entrepreneurs' utility is

$$(\rho - r)I = \frac{(\rho - r)\delta r^2}{r - \lambda\rho}. \tag{A.13}$$

If the bubbles reduces I then it reduces their utility since it also raises r. Consider the case in which $\lambda < 0.5$ and $r = 2\lambda\rho$. In this case, entrepreneurs' utility is higher in the presence of a bubble if and only if:

$$\frac{(\rho - 1)\delta}{1 - \lambda\rho} \geq 4\rho^2(1 - 2\lambda)\delta\lambda, \tag{A.14}$$

which holds if λ is sufficiently close to 0.5 all else equal.

A.3. Proof of Proposition 3

Suppose that $\hat{r} = r/(\Pi^*)^{\psi} \in (\underline{r}, \rho)$. A fixed-price equilibrium is a situation in which private agents optimize with perfect foresight and markets clear. Such an equilibrium must be such that the real rate is $\hat{r}$, and thus such that entrepreneurs are financially constrained given that $\hat{r} < \rho$. As a result, the four variables (C_Y, L, I, w) characterize an equilibrium if they solve:

$$wu'(C_Y) = \gamma L, \tag{A.15}$$

$$u'(C_Y) = \beta\hat{r}, \tag{A.16}$$

$$wL - C_Y + (\alpha - w)L = I = \frac{(\alpha - w)L\hat{r}}{\hat{r} - \rho\lambda}. \tag{A.17}$$

(A.15) and (A.16) state that workers optimally save and work. Entrepreneurs' optimal investment and capital market clearing yield (A.17). We show that this system admits a unique solution. Conditions (A.15) and (A.16) yield C_Y and w as functions of L and $\hat{r}$. Injecting these in (A.17) yields in turn

$$\frac{\gamma L^2}{\beta} - \alpha\rho\lambda L - \phi(\beta\hat{r})(\hat{r} - \rho\lambda) = 0, \tag{A.18}$$

which has a unique positive solution in L. Entrepreneurs do not lose money when hiring L if $w \leq \alpha$, or $\gamma L/(\beta\hat{r}) < \alpha$. This is true because the LHS of (A.18) is strictly positive for $L = \alpha\beta\underline{r}/\gamma$ and $\hat{r} > \underline{r}$.

The "shadow" elasticity of substitution $\eta(\hat{r})$ associated with $\hat{r}$ solves $w = \alpha(\eta(\hat{r}) - 1)/\eta(\hat{r})$. This clearly implicitly defines a continuous function of $\hat{r}$, and so its image of $(\underline{r}, \rho)$ is an interval. In the flexible-price environment in Section II, every $\epsilon > 1$ is associated with a unique non-bubbly equilibrium given other parameters. This implies that $\eta(.)$ must be monotonic. Furthermore, $r \to \underline{r}$ when $\epsilon \to +\infty$ other things being equal in these equilibria, implying that $\eta(.)$ must be decreasing.

A.4. Proof of Corollary 4

Letting $y = \hat{r}(\eta(\hat{r}) - 1)/\eta(\hat{r})$, one can write the capital-market equilibrium condition as

$$y^2 - \rho\lambda y - \frac{\gamma}{\alpha^2\beta}\phi(\beta\hat{r})(\hat{r} - \rho\lambda) = 0, \tag{A.19}$$

which has a unique positive solution in y, a function of $\hat{r}$ equal to $\rho\lambda(1 + \sqrt{1 + 4\gamma\phi(\beta\hat{r})(\hat{r} - \rho\lambda)/[\beta(\alpha\rho\lambda)^2]})/2$. The output is equal to $\delta(\eta(\hat{r}))\eta(\hat{r})\hat{r} = \alpha^2\beta y/\gamma$ and investment is output minus $\phi(\beta\hat{r})$, which yields the expressions in the corollary.

That $\phi(\beta\hat{r})(\hat{r} - \rho\lambda)$ need not be monotonic over $(\underline{r}, \rho)$ implies that two rates may yield the same output. That $\phi(\beta\hat{r})$ is decreasing implies that the largest of the two generates more investment.

A.5. Proof of Proposition 5

In the absence of a bubble, the implicit elasticity of substitution $\eta(\hat{r})$ is the unique solution in x to

$$\frac{\delta(x)\hat{r}^2}{\hat{r} - \lambda\rho} = \delta(x)x\hat{r} - \phi(\beta\hat{r}), \tag{A.20}$$

where $\delta(x) = \alpha^2\beta(x-1)/(x^2\gamma)$ is decreasing over $[2, +\infty)$ whereas $x\delta(x)$ is increasing. Suppose $\hat{r} < 1$. We show that there can be bubbles during the crisis by constructing one that arises at date 0 with real value b_0 (issued by old agents to fix ideas) and then grows at the deterministic rate $\hat{r}$ forever. There are of course many other feasible bubbly paths but we are only interested in exhibiting a simple one here. For $b_0 > 0$ sufficiently small, there exists by continuity a unique $\epsilon_0 > \eta(\hat{r})$ such that:

$$\frac{\delta(\epsilon_0)\hat{r}^2}{\hat{r} - \lambda\rho} = \delta(\epsilon_0)\epsilon_0\hat{r} - \phi(\beta\hat{r}) - b_0. \tag{A.21}$$

This stems from the LHS of (A.20) being continuously decreasing with respect to x and its RHS continuously increasing.

One can then for each $t \geq 0$ define $b_{t+1} = \hat{r}b_t < b_t$ and ϵ_{t+1} that solves (A.21) in which b_{t+1} replace b_0. That $\epsilon_t > \eta(\hat{r})$ for all t implies that output and the labor share are larger in the presence of this bubble than in the non-bubbly equilibrium. That the value of each side of (A.21) shrinks in the presence of a bubble also shows that investment (the LHS) is smaller. So must be the capital share given that entrepreneurs' leverage ratio, that depends only on the unaffected interest rate, is constant. The interest rate $\hat{r}$ being unaffected by the bubble also implies that the utility of households increases because of their higher income, and that that of entrepreneurs, equal to $(\rho - \hat{r})I$, decreases.

A.6. Proof of Proposition 7

At the date τ at which $\Omega_\tau = 0$, the economy can revert to the non-bubbly flexible price equilibrium from Proposition 6 as the situation is the same as that at date 0 of the perfect-foresight model.

Consider now the stochastic phase before $\Omega_\tau = 0$. Entrepreneurs being risk neutral and workers being risk neutral when old, the expected interest rate drives their decisions as in the perfect-foresight equilibrium. Given the risk that prices become flexible at the next date with probability $1 - p$ the expected interest rate is $\hat{r}[p + (1-p)/\Pi^*]$. The bubble must earn this rate on average but may burst next date with probability p, implying that it grows at the rate $\hat{r}[1 + (1-p)/(p\Pi^*)]$ as long as $\tilde{\Omega}_t = 1$.