Network diversification for a robust portfolio allocation

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Abstract

Portfolio allocation strategies often seek risk budgeting and diversification by relying only on correlation matrices to model relationships between assets. Although this approach can capture, in normal times, most of the dependencies between asset prices, it faces several challenges in terms of noise resistance, capturing non-linear relations that can naturally appear in the market and extreme allocations in long-short portfolio strategies. This paper presents novel network-based strategies that combine equal volatility allocation with network centrality measures to construct efficiently diversified portfolios and deliver stable strategies also suitable for long-short investments. Networks can encode linear and non-linear relationships between asset prices. To encode several layers of information simultaneously multiplex networks - a particular form of a multilayer network - can be deployed. Associated centrality measures can agnostically account for each asset's (ir)relevance in diversifying the risks of the portfolios. The results show that network-based portfolios can outperform several competing alternatives, maintaining a favourable risk characteristic.

Keywords: asset allocation, portfolio construction, graph theory, networks, multilayer networks, eigenvector centrality

JEL Classification: C15, G11, G15 ,G17, G0, G1, E44

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1 Introduction

The financial market is strongly interconnected and characterised by numerous and complex relationships between financial assets. Financial futures provide the investor with a liquid and cost-efficient set of assets that span different markets, sectors, and geographical areas. Therefore, the investor is empowered to create a potentially well-diversified portfolio and adapt it to different market cycles. Risk parity is for example one popular allocation strategy that aims at diversifying the portfolio by distributing the risk contribution equally between its components (cf. Roncalli 2013). One approach is to estimate risk as volatility focusing on the linear dependence of the asset prices to account for their risk contribution. Alternatively, the investor chooses other risk measures if interested in accounting for more non-linear or tail relationships between the assets. A different approach is targeting returns. Time series-momentum strategies, in particular, leverage price trends in the market to choose the allocations and decide whether to hold long or short positions aiming at positive returns (cf. Tobias J Moskowitz, Ooi, and Pedersen 2012). As we will see later, addressing diversification by combining trend-following strategies with risk parity allocations can naturally lead to sudden jumps and overexposure to related assets in alternative cycles of their positions (e.g. one is long and the other short).

In this paper, we propose to account for diversification via the interconnectedness of the individual assets modelled through networks and in particular, correlation networks. Correlation networks of the financial markets have been studied for the last two decades, and they also had an important role in inspiring modern allocation strategies like hierarchical risk parity. Correlation networks can, in fact, reveal structures in the market (like asset classes or sector clustering) from the sole asset returns.

Here we present a novel family of strategies that use network features to enhance diversification. In fact, we notice that the centrality of an asset in the network is related to its contribution to the diversification measure that accounts for the effective number of uncorrelated bets (cf. Meucci 2010). Therefore it is possible to introduce strategies that do not rely on optimization algorithms (usually inclined to amplify noise effects) and that can be safely used in long-short portfolios. Moreover, generalizing the strategies to multiplexes (multilayer networks) can also naturally encode non-linear relationships among the assets. Multiplex-diversification strategies can encode the different market behaviours into a unique framework and address them simultaneously to limit the exposure to risk that linear correlations cannot reveal.

Brief Literature Review

The relationship between assets in a portfolio using financial networks has been studied extensively in the last years starting with Mantegna 1999 who linked the minimum spanning tree (MST) to the financial theory. Onnela et al. 2003 have shown that the assets of the classic Markowitz portfolio (Markowitz 1952) are always located on the outer leaves of the corresponding MST. Pozzi, Di Matteo, and Aste 2013 used filtered graphs such as the minimum spanning tree and the planar maximally filtered graph (PMFG) to show that selecting peripheral assets can improve the risk-return profile of a portfolio. On this basis Peraltaa and Zareei 2016 more formally prove that under plausible assumptions there is a negative relationship between the centrality of the assets and their optimal weights within the Markowitz framework. Furthermore, portfolio selection and allocation methods based on networks and centrality measures have been studied by Baitinger and Papenbrock 2017, Výrost, Lyócsa, and Baumöhl 2019, Konstantinov, Chorus, and Rebmann 2020, Olmo 2021 and even be applied to cryptocurrency portfolios by Giudici, Pagnottoni, and Polinesi 2020.

Original contributions

We introduce a novel group of allocation methods based on weights inverse to an assets centrality and volatility including for example the inverse degree centrality portfolio (IDCP) and the inverse eigenvector centrality portfolio (IECP). These methods allocate a higher weight to peripheral assets in the network and therefore try to establish a robust diversification and aim to prevent a fast spread of financial stress over the portfolio.

First, we apply these new techniques to commonly used filtered graphs such the planar maximally filtered graph (PMFG). Furthermore, we exploit multiplex networks to concurrently incorporate different layers of relations between the assets. A multilayer network - of which a multiplex network is a special case - is a network made up of multiple layers. Each layer represents a given relationship between the vertices of the network. Multilayer networks are able to reproduce different kinds of interaction between the vertices simultaneously and have been useful in several fields, for example, to model how diseases spread to a particular network of contacts. Musmeci et al. 2017 have used this powerful instrument to study the properties of financial networks and identify features that have not been visible in the individual single-layer networks employing the corresponding dependency measures.

Using empirical backtests we show that the proposed network-based allocation concepts can help to improve the performance, the risk-return characteristic and the diversification of a portfolio. Multiplex networks can be useful to add complementary information to the network and improve specific measures such as skewness or tail risk.

2 Diversification and Networks

Diversification is the necessary ingredient that any contemporary portfolio and investment strategy seek. The investor desires strategies that allocate assets in ways that perform in all market regimes. Risk parity strategies (name first introduced by Qian 2005, see Roncalli 2013 for a comprehensive review) allocate the assets controlling their mutual risk exposure and distributing the risk equally between the portfolio components, or the asset classes. Risk exposure is evaluated by also looking at correlation matrices estimated over time-series of the asset returns. A correlation matrix provides a measure of the interconnection between each asset with all the other assets in the portfolio. These quantities are intrinsically dynamical and affected by two concurring phenomena: to make the estimation reliable despite the stochastic fluctuations occurring at different time scales, the operator would need a large amount of data points (longer period of time). However, the market transforms itself continuously and older data points could be less representative for the current situations in the market. This well-known trade-off lead to several approaches and methodologies studied both from academia and practitioners (see e.g. Ledoit and Wolf 2004).

Networks can express a simplified representation of the interactions between entities and thus, potentially provide a more stable perspective. Given a matrix of pairwise correlations between the assets, the interconnectivity of such a mathematical object can be represented as a complete undirected graph: a network where each asset is a vertex and is connected to all other vertices. A portfolio of N assets will have $N\,(N-1)\,/2$ edges. However, to filter out the most important connections, we can enforce sparsity in the network and select only the most "relevant" connections.

2.1 Networks

A network is a graph $G = (V, E)$ defined by a set of vertices $V = \{1, ..., N\}$ and a set of edges $E = \{1, ..., M\}$ connecting pairs of vertices. We consider only undirected graphs, i.e. the order of the two connected vertices is not important. In our framework the vertices correspond to the assets of our portfolio. The edges can be coloured by assigning them a non-negative quantity called weight. The weighted network carries a larger amount of information. We use this concept and assign to each edge a weight which defines how strongly the two assets are connected in a specific relationship.

In the following analysis, we study networks where the weights (in this work interpreted as distances) will be defined using three different dissimilarity measures. The first measure is given by the Gower distance metric applied to the Pearson correlation coefficient $\rho_{i,j}$ which measures the linear relationship between two random variables

$$d^{\rho}_{i,j} = \sqrt{\frac{1}{2}\,(1-\rho_{i,j})}$$

The higher the correlation between two assets the stronger the two assets are connected. The second measure is defined by the Gower distance metric using the Kendall tau rank distance $\kappa_{i,j}$.

$$d^{\tau}_{i,j} = \sqrt{\frac{1}{2}\,(1-\kappa_{i,j})}$$

The Kendall correlation is used to measure the ordinal association between two random variables. For the third dissimilarity measure, we introduce the lower tail dependence coefficient for random variables X and Y

$$\lambda_l = \lim_{t \downarrow 0} \mathbb{P}(X \leq F_X^{-1}(t) | Y \leq F_Y^{-1}(t))$$

with distribution functions $F_X = \mathbb{P}(X \leq x)$ and $F_Y = \mathbb{P}(Y \leq y)$. As shown by Lohre, Rother, and Schäfer 2020 the function of the λ_l coefficient

$$d^{l}_{i,j} = -\log(\lambda_l)$$

is a dissimilarity measure. As estimator for the lower tail dependence coefficient we use the estimator defined by Schmid and Schmidt 2007. The lower tail-dependence is in particular important in periods of market stress when a good and robust diversification is extremely desirable.

2.1.1 Minimum Spanning Tree

A well-known and broadly used graph filtering is the minimum spanning tree (MST). This is a filtered subgraph that connects all vertices while minimizing the total distance. Starting from the complete graph the number of edges is reduced from $N(N-1)/2$ to $N-1$. The MST attempts to retain only the most important edges representing the most significant information of the complete graph.

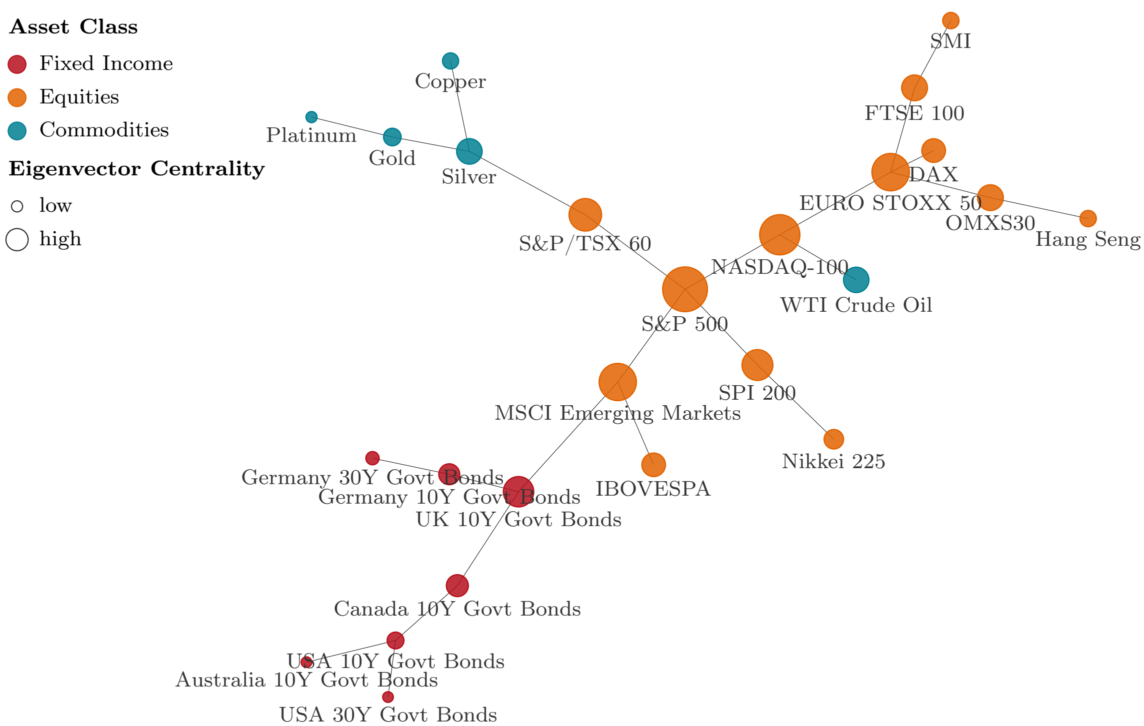

Figure 1: Illustration of a minimum spanning tree for the long-only portfolio.

The MST was linked to the financial theory by Mantegna 1999 highlighting the ability of MST to extract a hierarchical organization among the assets. Moreover, a related representation in the form of a hierarchical clustering tree was used by López de Prado 2016 to introduce the innovative hierarchical risk parity allocation method, which will be part of our empirical analysis.

To determine the minimum spanning tree, one can use the Kruskal's algorithm (Kruskal 1956). This algorithm starts with the set of edges from the complete graph. Then, each step adds the edge with the lowest-weight that does not form a cycle to the minimum spanning tree until all vertices are connected.

2.1.2 Planar Maximally Filtered Graph

The minimum spanning tree represents a highly simplified network which might lose too much important information from the complete graph. Another filtered graph that connects all the assets and retains more information than the MST is the planar maximally filtered graph (PMFG) introduced by Tumminello et al. 2005. A planar graph is a graph that can be drawn without overlapping edges (a spanning tree is planar). A maximal one is a planar graph for which the addition of any other edge results in a non-planar graph. The PMFG is a greedy solution of the weighted maximum planar graph problem: find a planar maximally filtered sub-graph such that the total weight is maximized or in our case the total distance is minimized. It is a graph with N vertices and $3N - 6$ edges.

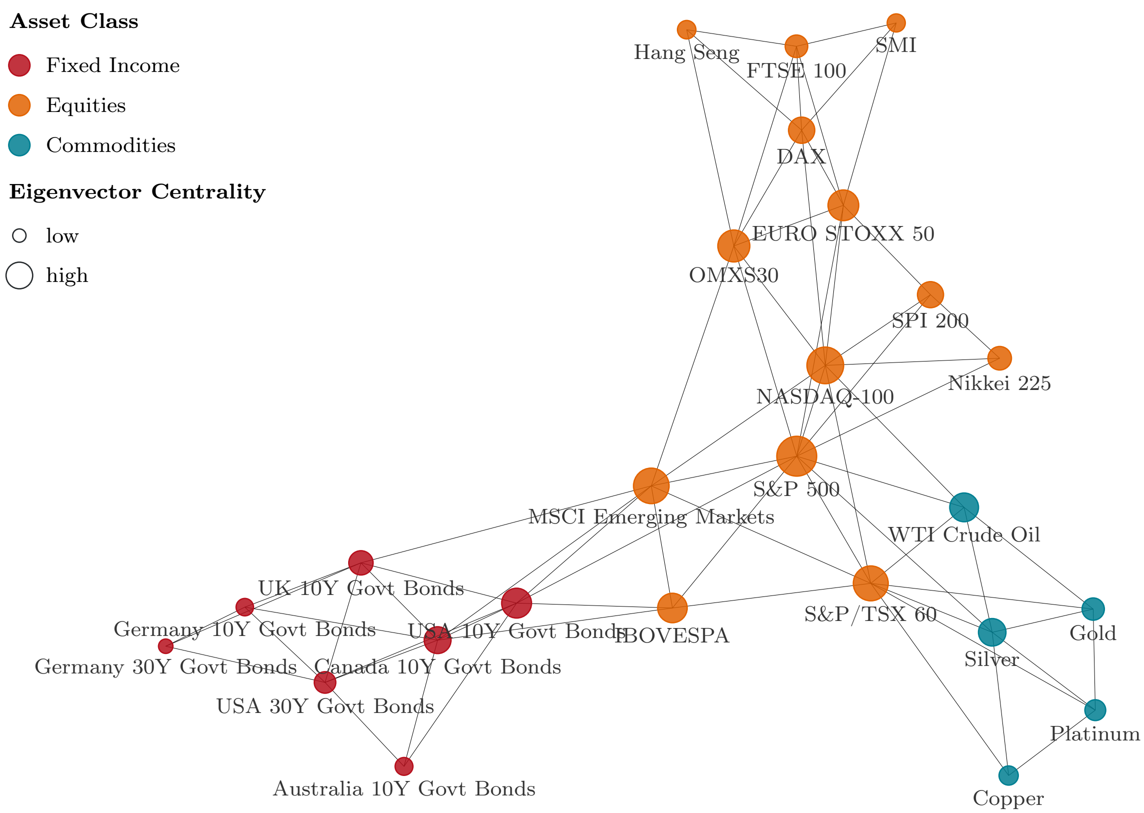

Figure 2: Illustration of a planar maximally filtered graph.

We can understand why a PMFG is a powerful tool to represent the financial market by considering first that it contains the MST as its backbone. Therefore, the graph maintains the hierarchical organization highlighted by the MST. The PMFG, however, includes cliques.[1] It can represent, for example, a group of assets (up to four) whose prices are highly related by encoding them in vertices with a distance of only one

[1]A clique is a subgraph fully connected. For example, a 4-clique in a portfolio network is a group of 4 assets where each asset is connected to all the other assets in the group.

edge from each other. In particular, in a PMFG, each asset participates at least in one 3-clique, while only 3- and 4- cliques are allowed.

We apply the algorithm of Boyer and Myrvold 2004 to extract the planar maximally filtered graph. When the number of assets is large, the filtering method PMFG can be slow. In these cases, one can opt for the Triangulated Maximally Filtered Graph (TMFG), a fast and scalable filtering method introduced by Massara, Di Matteo, and Aste 2016 which provides an approximate solution to the weighted maximum planar graph problem.

2.1.3 Multiplex Network

A network codifies only one particular aspect of a potentially complex relationship between two vertices. However, it might be desirable to simultaneously encode different aspects of the relationship of the vertices. For this case, one can introduce a multiplex network (MPN): a multilayer network where each layer has the same vertices, but the edges of each layer encode different levels of information. Originally introduced in the context of social networks by Goffman 1974 multilayer networks have been effectively applied in various fields and applications, e.g. epidemiology, electrical power grids or transportation.

Musmeci et al. 2017 deployed the multiplex approach to investigate the interplay between linear, non-linear and tail dependencies in a financial network during financial stress periods such as the dotcom bubble or the global financial crisis. They constructed a 4 layer network by means of Pearson, Kendall, Tail and Partial correlation.

In this work we will deploy a 3-layer multiplex network consisting of planar maximally filtered graphs where the vertices of each layer illustrate the same assets and the edges represent the Pearson correlation, the Kendall correlation and the lower tail dependence of the assets. In this way, the multiplex network encodes several perspectives on the portfolio in a unified manner. Including a layer for the lower tail dependency can for example help to increase the diversification in periods of financial stress and therefore reduce the contagion within the portfolio.

2.2 Network Measures

The network structure provides essential information regarding each asset's relative role with respect to the full portfolio. Converting the structure into quantitative measures is an important task of Network Science. The "importance" of a single vertex of a social network has been widely studied, and in the literature, several measures have been introduced to associate a quantitative measure to this concept. Different measures evaluate different ways in which a vertex can be important or marginal within the network. Our aim is to codify the network structure into proper investment choices.

We can divide the network measures into two big classes, the ones exploiting only the network structure, namely the vertices and the connections between them (the edges), and those that use further information attached to vertices and in particular edges, e.g. the weights. However, network measures relying only on the topological information have an important advantage: they are quantities model agnostic. This allows to combine or compare measures solely based on the networks topologies and makes it straightforward to combine quantitative information from different networks as in multiplex networks.

2.2.1 Degree Centrality

Historically the first and a relatively simple measure is the degree centrality (DC). It counts the number of incoming edges of a vertex. In other words, the degree $D(c)$ of asset c counts how many other assets are directly connected to c. It is a local measure, each vertex has its own DC, and it depends only on the subnetwork in the neighbourhood of the vertex.

If we are dealing with MST or PMFG graphs, we know a priori the total number of edges and therefore we can see that a uniform graph, if it exists, would count $D_{MST} = 1 - 1/N$ and $D_{PMFG} = 3 - 6/N$. Because of the properties we mentioned before, each vertex has a minimum degree of 1 for MST and 3 for the PMFG.

An asset in the portfolio network has a higher DC than the uniform value above, if it has a higher number of neighbours in the network, showing a consequent higher number of important correlations with respect to the other vertices.

2.2.2 Eigenvector Centrality

Another widely-used but more sophisticated measure is the eigenvector centrality (EC) firstly introduced by Bonacich 1972 which also counts for the importance of connected vertices. A vertex with few connections could have a high EC if the connected vertices have a high importance themselves. Google's PageRank algorithm (Page et al. 1999) initially used by the search engine, for example, is a variant of the eigenvector centrality.

An important tool to represent quantitatively a network is the so-called adjacency matrix. It encodes in a squared matrix the entire connectivity between the vertices in a network. The adjacency matrix $A = (a_{i,j})$ has $a_{i,j} = 1$ if vertex i is connected to vertex j and $a_{i,j} = 0$ otherwise. From this map, the eigenvector centrality is derived by finding the largest eigenvalue λ such that for a vector $x \in \mathbb{R}^N$

$$\lambda x = Ax$$

The eigenvector centrality for vertex i is then given by the i-th element of the eigenvector x. The Perron-Frobenius theorem ensures the following desirable characteristic: if the graph is strongly connected, i.e. every vertex is reachable from every other vertex, then A has a unique largest real eigenvalue and the corresponding eigenvector can be chosen to have strictly positive values. Since the eigenvector is only unique up to a multiplicative factor, we normalize the centrality such that the sum over all vertices equals 1.

2.2.3 Eigenvector Centrality for multilayer networks

In case of a multilayer network G consisting out of m layers $G_1, ..., G_m$ there is not a single solution to define the eigenvector centrality. There are two intuitive ways (cf. Solá et al. 2013): The first is to consider the projection network $\hat{G} = (V, \hat{E})$ with $\hat{E} = \cup_{i=1}^{m} E_i$. The adjacency matrix $\hat{A} = (\hat{a}_{i,j})$ of the projection network is given by

$$\hat{a}_{i,j} = \begin{cases} 1, & \text{if } a_{i,j}^k \text{ for some } 1 \leq k \leq m \\ 0, & \text{otherwise} \end{cases}$$

where $A^k = (a_{i,j}^k)$ is the adjacency matrix of layer k. The projection network is important when one is focusing on the pure existence of a relationship between two vertices.

In the situation where one would like to take into account if a vertex has more than one or maybe very important relationships encoded by a certain layer it is helpful to consider the uniform eigenvector-like centrality given by the positive and normalized eigenvector of the matrix

$$\tilde{A} = \sum_{i=1}^{k} A^k$$

The centrality of a vertex might be considered higher if a vertex is connected to other vertices in more than one layer. In our context it might be desirable to identify if an asset has a high centrality in more normal market phases (represented by a network where the weights are defined using the Pearson correlation or Kendall τ) and at the same time in a stressed market situation (represented by a layer using the lower tail dependency).

3 Network-based Allocation Concepts

It is possible to exploit the network structure to improve the investment decisions. In the literature and among practitioners, the network structure has already been used for portfolio selection and portfolio allocation with promising results. Pozzi, Di Matteo, and Aste 2013 use financial networks to build diversified portfolios that outperform out-of-sample. They use the MST and the PMFG to select stocks from peripheral regions and show that portfolios build out of these stocks outperform portfolios build out of more central or randomly chosen stocks. Peraltaa and Zareei 2016 more formally prove that under plausible assumptions there is a negative relationship between the centrality of the assets and their optimal weights within the Markowitz framework. Clemente, Grassi, and Hitaj 2021 transfer the optimization problem of the minimum variance portfolio to financial networks using a clustering coefficient to measure the level of inter-connectivity of an asset within the system. Vỳrost, Lyócsa, and Baumöhl 2019 use sparse graphs such as MST and PMFG to build multi-asset portfolios where the allocated weights fulfil constraints derived from different centrality measures.

3.1 Inverse Degree Centrality Portfolio

The networks presented in the previous section do not carry any univariate information such as the volatility of an asset. The networks only show the interconnection between the components of our portfolio. The starting point for our network-based allocation concepts is given by the well-known naïve risk-parity portfolio (NRP), also known as inverse-volatility or volatility-parity, taking into account only the volatility of each asset. The weights are given by

$$w_i^{NRP} = \frac{\sigma_i^{-1}}{\sum_j \sigma_j^{-1}}$$

For portfolio of uncorrelated assets, NRP allocates the weights such that all assets contribute the same risk to the portfolio.

Similarly, we introduce the inverse degree centrality portfolio (IDCP). Let DC_i denote the degree centrality for asset i within a predefined network. The weights for the IDCP are then defined by

$$w_i^{IDCP} = \frac{(\sigma_i DC_i)^{-1}}{\sum_j (\sigma_j DC_j)^{-1}}$$

The higher the volatility and the higher the centrality of an asset, the lower its allocated weight. In this way, the strategy privileges the less connected, peripheral assets and penalizes the assets with a high level of centrality, aiming at a well-diversified portfolio.

3.2 Inverse Eigenvector Centrality Portfolio

In the same way we define the portfolio weights using the eigenvector centrality for a given network. We multiply the NRP-weights with the inverse eigenvector centrality, i.e.

$$w_i^{IECP} = \frac{(\sigma_i EC_i)^{-1}}{\sum_j (\sigma_j EC_j)^{-1}}$$

where EC_i denotes the eigenvector centrality of asset i. This follows the same logic to assign a higher weight to peripheral assets and achieve a higher and more robust diversification within the portfolio.

The eigenvector centrality is of particular interest for the allocation of a portfolio because it encodes specific information about the portfolio diversification. In fact, the EC is strongly related with the diversification measure *Effective Number of uncorrelated Bets* (ENB), presented by Meucci 2010 (for more details see appendix B). ENB measures the entropy associated to risk contribution of implicit independent factors of an allocation to estimate the effective number of uncorrelated bets. In a portfolio where returns of each asset are scaled to have all the same volatility (like in naïve risk-parity), the relevant object to determine the risk contribution of the assets is the correlation matrix. To compute the ENB, one extracts the implicit uncorrelated factor via principal component analysis: the covariance matrix Λ_Σ for the uncorrelated factors is diagonal and each entry of the diagonal λ_n^2 is the variance of the nth uncorrelated factor. A linear transformation E can map the allocations into the space of the uncorrelated factors $w_F = E^{-1}w$ if E is the linear operator that diagonalizes the covariance matrix Σ

$$E^T \; \Sigma \; E = \Lambda_\Sigma$$

E is a column matrix of eigenvectors $e_1, ..., e_n$ and Λ_Σ a diagonal matrix of the eigenvalues $\lambda_1, ..., \lambda_n$. To understand the relation between EC and ENB, we can think of a correlation network as an approximation of a complete graph where each edge is weighted with the corresponding correlation coefficient $\rho_{i,j}$. In this situation, the adjacency matrix coincides with ρ, the correlation matrix, and the corresponding EC is associated with the eigenvector corresponding to its highest eigenvalue. Therefore, for portfolios where each asset return is scaled by its volatility, the EC corresponds to the riskiest uncorrelated factor.

4 Long-only multi-asset portfolio

The network-based allocation methods are studied in the context of a long-only and a time-series momentum strategy, both denoted in USD. In this chapter we present the results for the long-only multi-asset portfolio. The methods are benchmarked against numerous industry standards such as equal weighting (EW), naïve risk-parity (NRP), equal risk contribution (ERC), minimum variance portfolio (MVP), most diversified portfolio (MDP) and hierarchical risk parity (HRP).

4.1 The dataset and the strategies

The long-only multi-asset portfolio consists of listed futures markets stemming from three different asset classes: commodities, equities and fixed income. We aimed at building a representative and well-diversified multi-asset portfolio covering the most prominent and liquid asset classes. The listed futures offer a very liquid and cost-efficient way to obtain a global multi-asset portfolio. Furthermore, as unfunded instruments they conveniently allow us to dynamically leverage the portfolio and target a certain volatility. The futures P&L is converted into USD on a daily basis. As start date for our index we chose January 1995, because not all and in particular many equity future markets are not available earlier. This ensures that the multi-asset characteristic of the index is homogeneous over time. Still, some of the future series start later than the index start date and then enter into the strategies when available. Table 1 shows the listed futures used for the long-only multi-asset portfolio.

The weights of the constituents in the long-only portfolio are calculated using the different allocation methods presented in the next paragraph 4.2. The futures portfolio is re-balanced every month and leveraged to realize a target volatility of 5% on a daily basis. The target volatility enables us to better compare performances for different periods of time and across different portfolios or allocation methods. We use an exponentially-weighted moving average based on historical returns to forecast the portfolio volatility. Since historical volatility estimators often tend to overshoot the target volatility we take the maximum of two different time windows (20 and 90 business days) which leads to a good match of the target volatility (cf. Deutsche Börse 2018). We reflect transaction costs on a realistic basis by charging 1 future tick whenever the future position changes or a future position is rolled into the next contract.

ID	Constituent	Asset Class	Currency	ID	Constituent	Asset Class	Currency
1	Copper	Commodities	USD	14	OMXS30	Equities	SEK
2	Gold	Commodities	USD	15	S&P 500	Equities	USD
3	Platinum	Commodities	USD	16	S&P/TSX 60	Equities	CAD
4	Silver	Commodities	USD	17	SMI	Equities	CHF
5	WTI Crude Oil	Commodities	USD	18	SPI 200	Equities	AUD
6	DAX	Equities	EUR	19	Australia 10Y Govt Bonds	Fixed Income	AUD
7	EURO STOXX 50	Equities	EUR	20	Canada 10Y Govt Bonds	Fixed Income	CAD
8	FTSE 100	Equities	GBP	21	Germany 10Y Govt Bonds	Fixed Income	EUR
9	Hang Seng	Equities	HKD	22	Germany 30Y Govt Bonds	Fixed Income	EUR
10	IBOVESPA	Equities	BRL	23	UK 10Y Govt Bonds	Fixed Income	GBP
11	MSCI Emerging Markets	Equities	USD	24	USA 10Y Govt Bonds	Fixed Income	USD
12	NASDAQ-100	Equities	USD	25	USA 30Y Govt Bonds	Fixed Income	USD
13	Nikkei 225	Equities	JPY				

Table 1: Constituents of the long-only multi-asset portfolio.

4.2 The allocation methods

Most common portfolio allocation techniques are either based on simple approaches which ignore the correlation between the assets, e.g. equal weights (EW) or naïve risk-parity (NRP), or the allocation techniques are based on more complex approaches depending on forecasting the covariance matrix and calculating its inversion. Examples are minimum variance (MV), most diversified portfolio (MDP) and equally-weighted risk contribution (ERC). Despite the valid mathematical concepts the complex allocation methods often perform poorly out-of-sample due to numerical instabilities and noise (cf. Michaud 1998). Therefore, more recent approaches strike new paths, for example hierarchical risk parity (HRP) aims at uncovering a hierarchical structure within the portfolio and utilize this structure to build a robust allocation.

Let us briefly introduce the allocation methods we use for benchmarking the network-based allocation methods.

Equal weighting (EW)

An equal weighted portfolio gives the same importance to each asset. It is ignoring the volatility of the asset and the dependency structure of the portfolio, i.e. the weights are given by

$$w_i = \frac{1}{N}$$

for each asset i.

Naïve risk-parity (NRP)

A naïve risk-parity or inverse volatility portfolio (cf. Roncalli 2013) weights the assets in inverse proportion to their volatility

$$w_i = \frac{\sigma_i^{-1}}{\sum_j \sigma_j^{-1}}.$$

The method is often called naïve since it fully neglects the dependency structure of the portfolio and allocates the weights only with respect to each assets historical volatility.

Minimum variance portfolio (MVP)

The minimum variance portfolio (cf. Markowitz 1952) is aiming at minimizing the variance of the portfolio. Here we request a full investment, i.e. the absolute weights sum up to 1, together with either a long-only constraint in the long-only portfolio or a constraint given by the sign of a momentum indicator in a momentum strategy. The weights are determined by solving the quadratic optimization problem

$$\arg\min_w \frac{1}{2} w^T \Sigma w$$

respecting the given constraints where Σ denotes the covariance matrix. Minimum variance portfolios often tend to show a high concentration in only a few low-volatility assets.

Most diversified portfolio (MDP)

The most diversified portfolio (cf. Choueifaty and Coignard 2008) is aiming at maximizing the diversification within a portfolio where the diversification is defined as ratio of its weighted average volatility and its volatility. Let $\sigma(w)$ denote the portfolio volatility for a given weight vector (w_i) then the diversification ratio is given by

$$DR(w) = \frac{\sum_i w_i \sigma_i}{\sigma(w)}$$

Again we request a full investment together with either a long-only constraint in the long-only portfolio or a constraint given by the sign of a momentum indicator in a momentum strategy. The weights are determined by solving the quadratic optimization problem

$$\arg\max_w DR(w)$$

respecting the given constraints.

Equal risk contribution (ERC)

The equal risk contribution (cf. Maillard, Roncalli, and Teiletche 2010) is defined such that all assets equally contribute to the portfolio risk, i.e. the risk contribution of asset i

$$RC_i = \frac{w_i (\Sigma w)_j}{\sqrt{w' \Sigma w}}$$

is the same for each asset. The weights are determined by solving the optimization problem

$$\arg\min_w \sum_i \left(\frac{RC_i}{\sqrt{w' \Sigma w}} - \frac{1}{N} \right)$$

As before we suppose a full investment together with either a long-only constraint in the long-only portfolio or a constraint given by the sign of a momentum indicator in a momentum strategy.

Hierarchical risk parity (HRP)

The hierarchical risk parity approach goes back to López de Prado 2016. The standard HRP approach uses a hierarchical clustering algorithm to group assets with a similar risk profile together. It reveals a nested cluster structure that is often visualised using a so-called dendrogram. This is done by defining a distance metric based on the pairwise correlation of the assets. Applying a hierarchical cluster algorithm provides a hierarchical clustering tree. The cluster tree is used to quasi-diagonalise the covariance matrix. Finally, the weights are calculated for each of the assets using a recursive bi-sectioning procedure of the reordered covariance matrix. We start at the top of the tree and with a weight of 1 for each asset. Then we divide the assets into two equal subsets ("bi-sectioning") and rescale the weights by multiplying each weight with the inverse proportion of its subsets variance. Both subsets are divided again, and the weights are rescaled respectively. Recursively the final weights are thereby derived. More details about the algorithm can be found in appendix A.

The hierarchical structure of the HRP approach can result in a more robust investment performance that is less prone to noise. The robustness of the approach was confirmed by Jaeger et al. 2021 using a block-bootstrapping resampling method. HRP was found to show better risk-adjusted returns and lower maximum drawdowns compared to NRP and ERC. There are many variations of the original approach. A schematic overview can be found in Schwendner et al. 2021.

4.3 Results

In this chapter we discuss the results of the backtests and present the most prominent performance metrics and concentration metrics. Furthermore we analyse the robustness of the strategies using 1000 simulated backtests generated by a block-bootstrapping method.

For the discussions we focus on the following 4 network approaches:

- IDCP^{ρ}: the inverse degree centrality portfolio applied to the planar maximally filtered graph (PMFG) based on Pearson correlation.
- IECP^{ρ}: the inverse eigenvector centrality portfolio applied to the PMFG based on the Pearson correlation.
- IECP^{l}: the inverse eigenvector centrality portfolio applied to the PMFG based on the lower tail dependence.
- $\text{IECP}^{\rho,\tau,l}$: the inverse eigenvector centrality portfolio applied to the multiplex network with three PMFG-layers defined by the Pearson correlation, Kendall's tau and the lower tail dependence. The eigenvector centrality of the multiplex network is determined by the uniform eigenvector-like centrality.

Figure 3 shows the performance and the drawdowns of the unfunded futures portfolio with an initial investment of 100 for selected allocation methods.

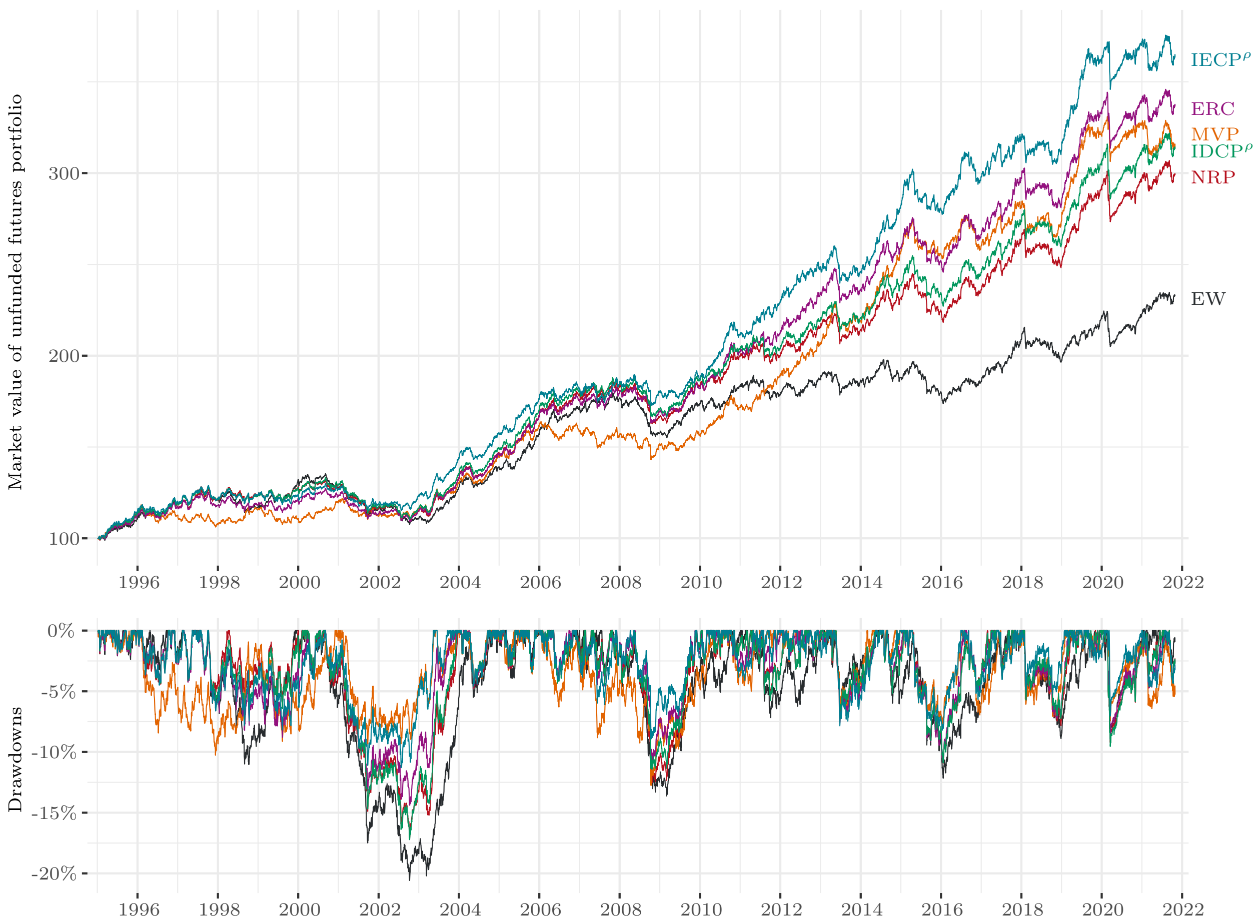

Figure 3: Market value and drawdowns of a subset of the long-only unfunded futures portfolios.

Table 2 presents the most common performance and risk metrics for the multi-asset portfolio as well as the transaction costs. The results are also visualized in Figure 4.

	Other Allocations						Network-based Allocations			
Metric	EW	NRP	MVP	MDP	ERC	HRP	IDCP$^\rho$	IECP$^\rho$	IECP^l	IECP$^{\rho,\tau,l}$
CAGR	3.19%	4.17%	4.35%	4.48%	4.62%	4.36%	4.35%	4.92%	4.56%	4.79%
Volatility	4.83%	4.90%	5.08%	5.06%	4.96%	4.98%	4.92%	4.95%	4.95%	4.95%
Skewness	-0.49	-0.52	-0.52	-0.44	-0.48	-0.41	-0.48	-0.49	-0.41	-0.45
Sharpe Ratio	0.66	0.84	0.85	0.87	0.92	0.86	0.87	0.98	0.91	0.96
Sortino Ratio	1.05	1.35	1.36	1.42	1.49	1.41	1.40	1.59	1.47	1.55
Calmar Ratio	0.16	0.25	0.35	0.39	0.33	0.51	0.26	0.44	0.33	0.38
CVaR(95%)	-10.6%	-9.0%	-5.8%	-8.0%	-7.7%	-4.1%	-8.6%	-5.8%	-6.8%	-6.0%
Max Drawdown	-20.6%	-17.1%	-12.8%	-11.7%	-14.4%	-8.7%	-17.2%	-11.3%	-14.2%	-12.8%
Transaction Costs (p.a.)	0.12%	0.16%	0.25%	0.29%	0.20%	0.23%	0.20%	0.23%	0.25%	0.22%

Table 2: Performance metrics for the long-only multi-asset portfolio.

IECP$^\rho$ shows the best performance, Sharpe ratio and Sortino ratio followed by IECP$^{\rho,\tau,l}$, ERC and

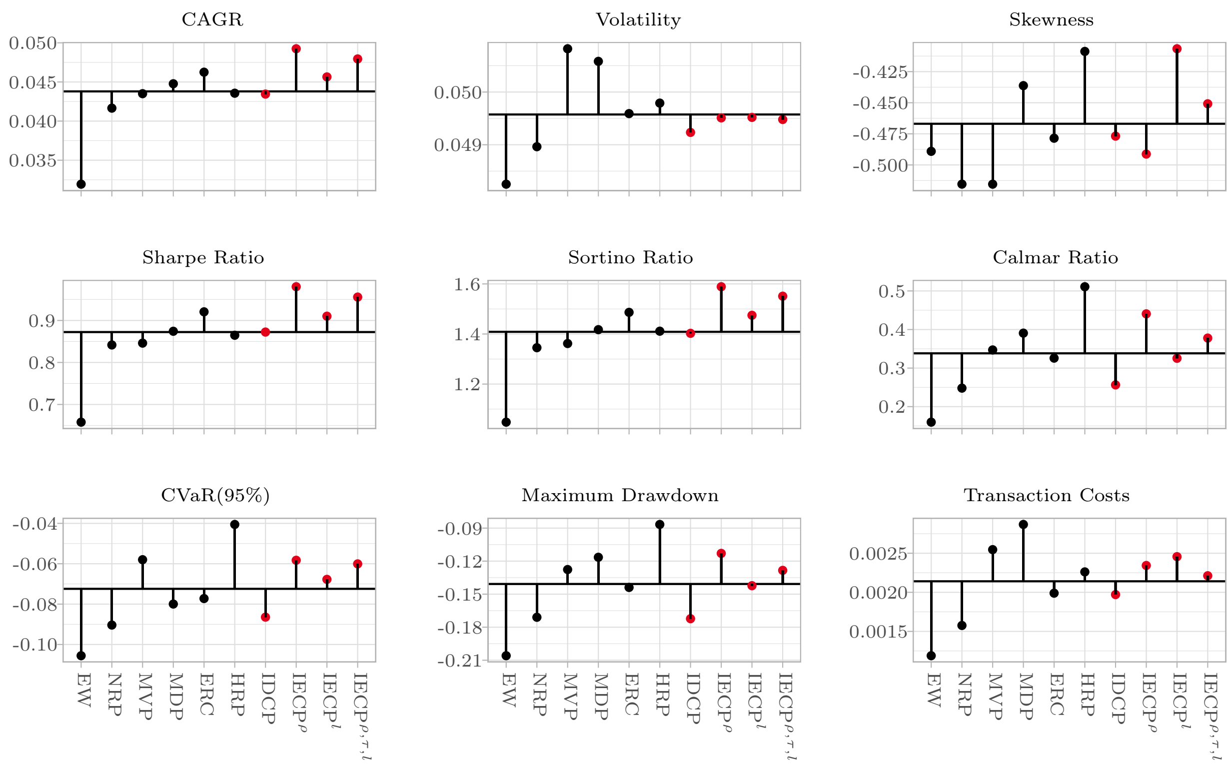

Figure 4: Performance metrics for the long-only multi-asset portfolio. In red, the performances for network-based strategies.

IECP^l. In general, these three metrics show a very similar ranking since the target volatility mechanism standardizes the volatility of the portfolios over time. The worst performances are shown by EW and NRP.

Overall, all methods match the target volatility very good even though MVP and MDP tend to slightly overshoot the volatility target of 5%. This is not surprising since both tend to allocate high weights on few assets, show an overall high leverage and may exhibit instabilities as discussed earlier.

HRP shows the lowest drawdown, however IECP$^\rho$ and IECP$^{\rho,\tau,l}$ show low drawdown results as well. This is consequently reflected in a similar positive ranking for the Calmar ratio. The particular good skewness of IECP^l and with lesser extent for IECP$^{\rho,\tau,l}$ highlights that the incorporating of the lower tail dependency can shape the portfolio distribution in a positive way. The maximum drawdown is rather mediocre. To further quantify the amount of tail risk we calculated the conditional value at risk (CVaR) with a confidence level of 95% on an empirical basis. The transaction costs of the network approaches are in line with other methods like ERC and HRP but better than MVP and MDP.

Figure 5 shows the average allocation per asset class for each strategy. It is interesting to notice that HRP, similarly to MVP, has a larger exposure towards fixed income assets. On the other hand, the network and multiplex approaches have reduced exposure to fixed income assets, a behaviour that is closer to risk parity strategies.

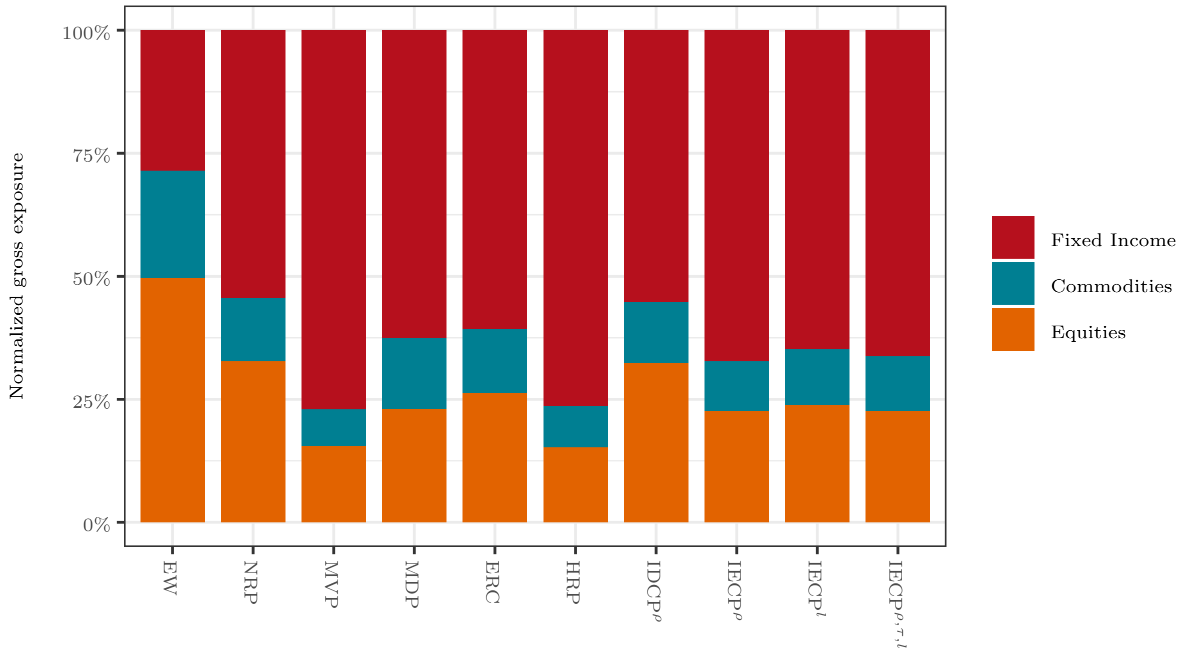

Figure 5: Average normalized gross exposure per asset class for long-only strategies. MVP and HRP show a higher average exposure towards fixed income assets.

How well diversified are the network portfolios?

There are a few measures aiming at quantifying the degree of diversification. The most common ones are the maximum risk contribution (in terms of volatility), the diversification ratio, the concentration ratio and the effective number of uncorrelated bets. More details about the concentration measures can be found in the appendix. Some of the allocation methods are specifically designed to optimize certain concentration measures. ERC shows the most diversified risk budgets, and therefore exhibits the smallest maximum risk. By construction, MDP maximizes the diversification ratio. Choueifaty, Froidure, and Reynier 2011 introduced the concept of the concentration ratio and show that NRP minimizes its value. Although not considered in this work, it is worth to mention that Lohre, Opfer, and Ország 2014 constructed the diversified risk parity portfolio (DRP) which is optimizing the effective number of uncorrelated bets.

	Other Allocations						Network-based Allocations			
Metric	EW	NRP	MVP	MDP	ERC	HRP	IDCP$^\rho$	IECP$^\rho$	IECP^l	IECP$^{\rho,\tau,l}$
Maximum Risk (Avg.)	0.64%	0.36%	1.59%	0.71%	0.21%	0.94%	0.49%	0.75%	0.71%	0.61%
Diversification Ratio (Avg.)	2.05	2.4	2.33	2.79	2.65	2.18	2.49	2.48	2.51	2.53
Concentration Ratio (Avg.)	0.06	0.04	0.14	0.09	0.05	0.07	0.05	0.06	0.06	0.05
Uncorrelated Bets (Avg.)	2.06	4.24	7.45	7.42	5.83	6.3	4.69	6.21	5.98	6.1

Table 3: Concentration metrics for the long-only multi-asset portfolio.

Table 3 shows the concentration metrics for the different allocation methods. We calculate the diversification metrics on a monthly basis and then take the average of those values. The network-based approaches

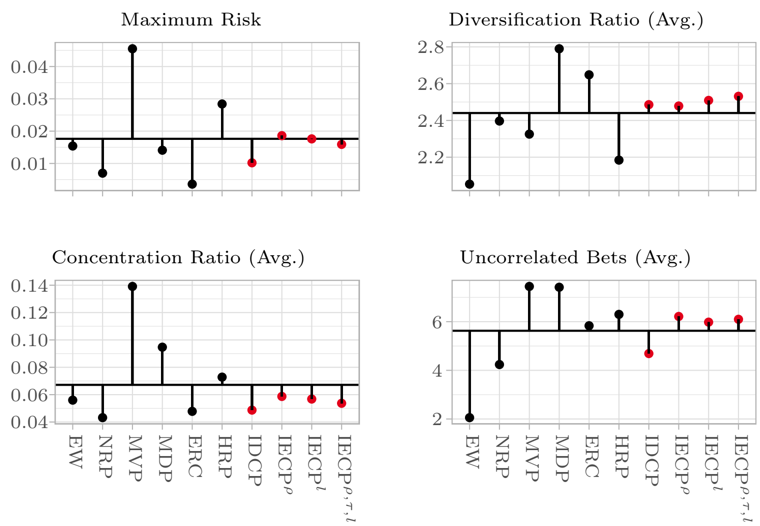

Figure 6: Concentration metrics for the long-only multi-asset portfolio. In red, the concentrations for network-based strategies.

show overall good results for all concentration metrics. In particular they show a high diversification ratio and a high number of uncorrelated bets.

Robustness of the strategies

To get a deeper understanding about the robustness of the backtest results we analysed the allocation methods on an additional dataset of 1000 scenarios generated via block bootstrapping. This helps to better assess the significance of the backtest and to understand how much the results are driven by randomness or the choice of the asset universe.

For this purpose we build blocks with a fixed length of 20 business days and a random starting point in time from the futures return time series where we restrict the data on the period when all 25 assets are available. The scenarios are constructed by sampling the blocks with replacement to reconstruct a time series with the same length as the original time series. For these scenarios we then calculated the resulting portfolio time-series and the performance measures.

Figure 7 shows density plots illustrating comparisons of the results for selected allocation methods against IECP$^\rho$, the inverse eigenvector centrality portfolio based on the PMFG using the Pearson correlation. The density plots of the compound annual growth rate show a similar shape for all presented methods with NRP slightly lagging behind. The volatility plots highlight the robustness of the IECP$^\rho$ approach. Consequently, this then leads to small out-performance in the Sharpe ratio even though we do not display the results here since the shapes are very similar to the CAGR. Furthermore, IECP$^\rho$ shows the best results for the maximum drawdown.

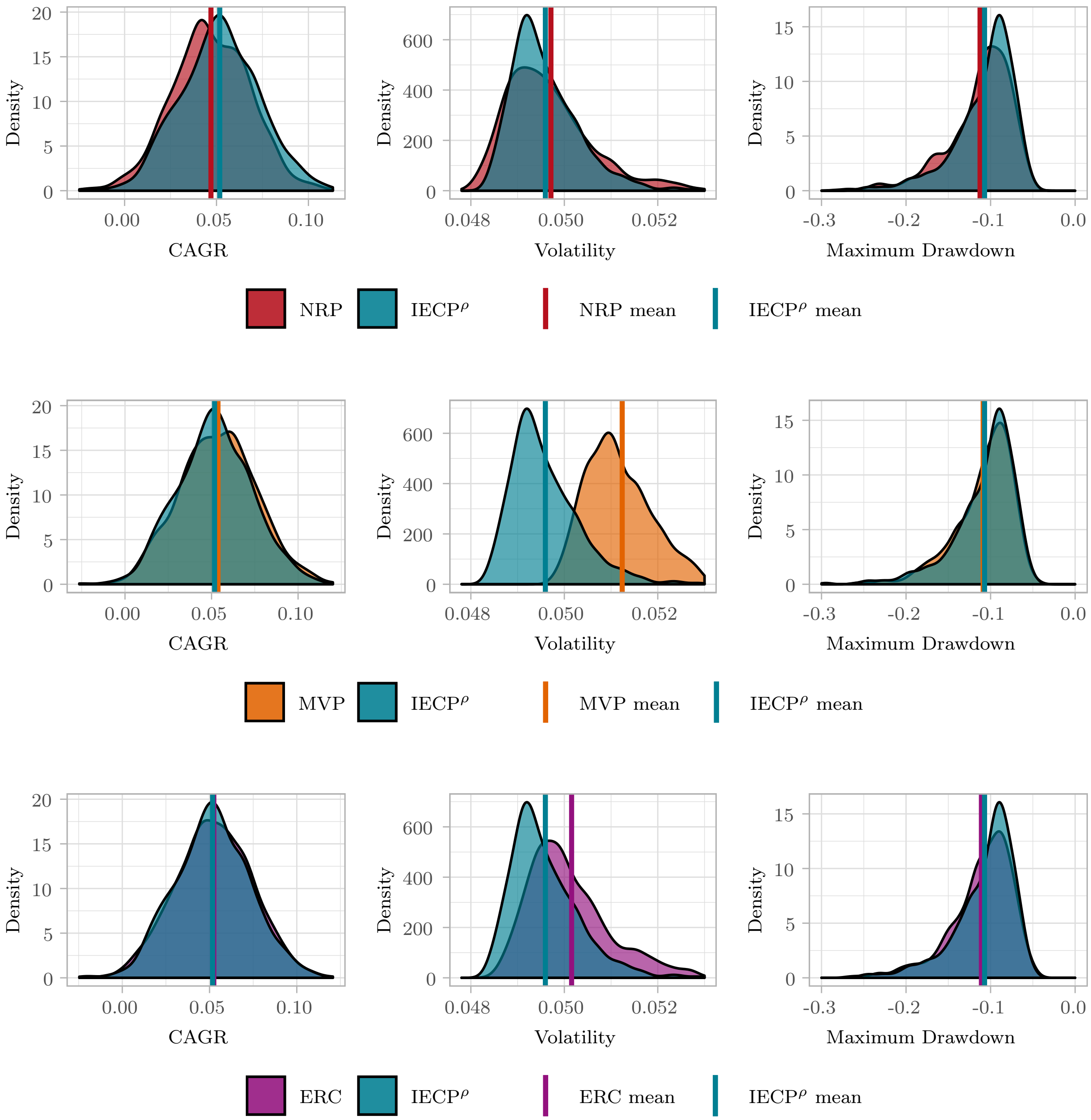

Figure 7: Density plots showing performance measure evaluated for 1000 block-bootstrapped scenarios.

5 Time-series momentum portfolio

In this section, we consider a time-series momentum strategy and analyse how well network-based allocation methods perform in a long-short context.

The fundamental idea of momentum strategies is given by the assumption that asset returns show a positive serial correlation, i.e. there exists periods of ongoing positive or negative price trends. Momentum strategies are designed to take a long position in assets that performed well and take a short position in assets that performed poorly over a predefined historical time period. It has been shown (Jegadeesh and Titman 1993, Tobias J Moskowitz, Ooi, and Pedersen 2012 or Asness, Tobias J. Moskowitz, and Pedersen 2013) that (time-series or cross-sectional) momentum strategies may produce positive returns even though there are theoretical contradicting arguments like the hypothesis of efficient markets.

In the literature different momentum indicators can be found. Typically, they distinguish from each other by using different time-periods for defining the momentum signal, but it is also common to incorporate more sophisticated methods for example so-called break-out rules which try to reduce noise and extract more pronounced signals.

The coexistence of long and short positions is sometimes attended by extreme negative correlations in the portfolio. This could for example happen when there is a short position in the US 10Y Govt Bond future and a long position in the US 30Y Govt bond future at the same time. In this case traditional covariance-based allocation methods like MVP, MDP or ERC might run into optimization problems and they are prone to attribute extreme weights to highly negative correlated assets since they erase each other's risk. In fact, when two strongly correlated assets have opposite positions (long vs short), their risk contributions can, in principle, cancel each other out, leading the strategies to pic very large and opposite weights (cf. figure 11). The measured volatility of such portfolio tends to be very low and thus demands a very high leverage to obtain the target volatility. Economically, this means taking an extreme and probably unintended counter-position in two assets with a correspondingly high spread risk. In addition, this can lead to exceptionally high transaction costs. This problem could be avoided by adding ad hoc constraints but in general this would contradict the fundamental ideas of these concepts.

5.1 The dataset and the strategies

For the time-series momentum strategy we chose a broader portfolio of 35 futures starting in 1988. Table 4 shows an overview of the assets included. The start date is chosen such that a majority of the considered assets are available at start. For the momentum strategy we attach a high importance to build a well-diversified portfolio consisting of numerous and highly liquid assets due to otherwise high transaction costs.

In this work we focus on the allocation concepts rather than on finding the best momentum indicator, therefore we use a more basic momentum strategy. Our time-series momentum strategy works as follows: for each constituent a momentum indicator is calculated. The indicator calculates the price change over 4

ID	Constituent	Asset Class	Currency	ID	Constituent	Asset Class	Currency
1	Brent Crude Oil	Commodities	USD	19	MSCI Emerging Markets	Equities	USD
2	Copper	Commodities	USD	20	NASDAQ-100	Equities	USD
3	Gasoil	Commodities	USD	21	Nikkei 225	Equities	JPY
4	Gold	Commodities	USD	22	OMXS30	Equities	SEK
5	Natural Gas	Commodities	USD	23	S&P 500	Equities	USD
6	Silver	Commodities	USD	24	S&P/TSX 60	Equities	CAD
7	WTI Crude Oil	Commodities	USD	25	SMI	Equities	CHF
8	AUD/USD	Currencies	USD	26	SPI 200	Equities	AUD
9	CAD/USD	Currencies	USD	27	STOXX Europe 600	Equities	EUR
10	EUR/USD	Currencies	USD	28	Australia 10Y Govt Bonds	Fixed Income	AUD
11	GBP/USD	Currencies	USD	29	Canada 10Y Govt Bonds	Fixed Income	CAD
12	JPY/USD	Currencies	USD	30	Germany 10Y Govt Bonds	Fixed Income	EUR
13	DAX	Equities	EUR	31	Germany 30Y Govt Bonds	Fixed Income	EUR
14	EURO STOXX 50	Equities	EUR	32	Switzerland 10Y Govt Bonds	Fixed Income	CHF
15	FTSE 100	Equities	GBP	33	UK 10Y Govt Bonds	Fixed Income	GBP
16	Hang Seng	Equities	HKD	34	USA 10Y Govt Bonds	Fixed Income	USD
17	IBOVESPA	Equities	BRL	35	USA 30Y Govt Bonds	Fixed Income	USD
18	KOSPI 200	Equities	KRW				

Table 4: Constituents of the time-series momentum portfolio.

different past periods (we use 65, 125, 185 and 250 business days). We use 4 different historical periods to further diversify the strategy. Whenever the momentum for at least 3 periods is positive the indicator of the constituent gets a value of 1. Whenever the momentum is negative for at least 3 periods the indicator of the constituent gets a value of -1. In case of levelling out momentum signals the respective constituent is not included in the final portfolio. The signals and the weights are calculated on a monthly basis at the beginning of each month.

Analog to the long-only portfolio the weights of the constituents are calculated using the different allocation methods. Let us explain in more detail how we handle the signal in the different allocation methods. For EW and NRP we simply multiply the weight as described in 4.2 with the signal. For MVP, MDP and ERC we incorporate the constraint stemming from the signal

$$w_i > 0, \quad \text{if signal}_i = 1$$
$$w_i < 0, \quad \text{if signal}_i = -1$$

directly as a constraint into the respective optimization problem. For HRP and the network-based methods we invert the returns used for generating the clustering tree and the networks respectively if assets show a negative trend signal. In this case, HRP tends to form clusters grouping together assets belonging to the same asset class and showing the same sign of the signal. Similarly, in the networks (MST or PMFG) the same assets are linked closely together.

Again the futures portfolio is leveraged to realize the target volatility of 5% on a daily basis and transaction costs are reflected by charging 1 future tick size whenever the future position changes or a future position is rolled into the next contract.

5.2 Results

Figure 8 shows the performance and the drawdowns of the unfunded futures portfolio with an initial investment of 100 for selected allocation methods.

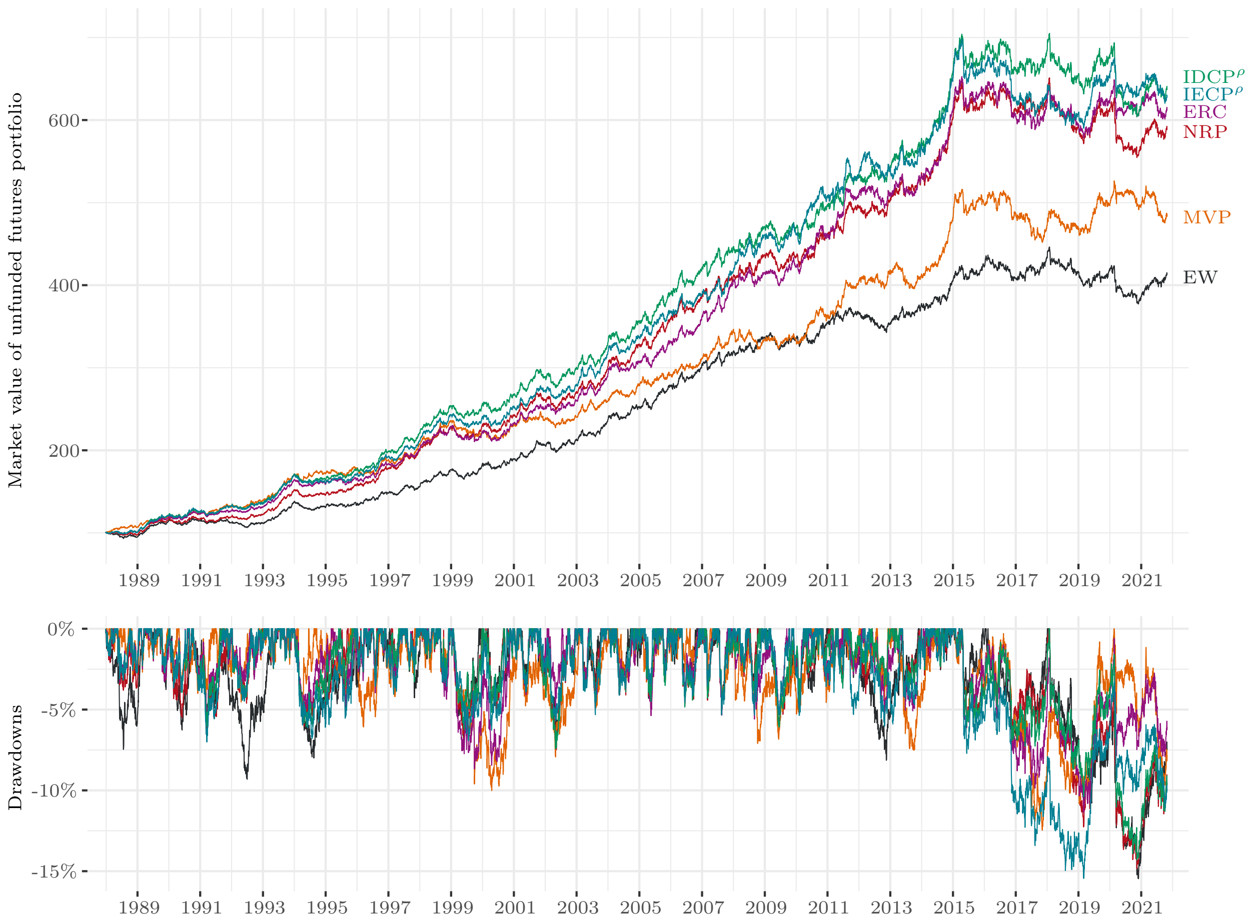

Figure 8: Market value and drawdowns of a subset of the unfunded time-series momentum futures portfolios.

Until 2015 the strategy shows a remarkably good performance but in recent years the strategy performs rather poorly. This behaviour is in line with industry benchmarks, see for example the Barclay CTA Index and BTOP50 Index or the Societe Generale CTA Index and TREND Index which track the performance of a number of "Managed Futures" strategies, often employing trend following strategies.[2] The whitepaper from Abbey Capital 2019 discusses different theories questioning if there is a structural change in the market causing the under-performance. Daniel and Tobias J. Moskowitz 2016 analysed momentum portfolios for

[2] Please note that these are total-return indices and do not aim at a target volatility of 5%.

a period of almost 100 years and show that there have been longer periods where momentum strategies under-performed dramatically (see also Hurst, Ooi, and Pedersen 2017).

Table 5 presents the common performance metrics and a visualization of the metrics can be found in graphic 9.

	Other Allocations						Network-based Allocations			
Metric	EW	NRP	MVP	MDP	ERC	HRP	IDCP$^{\rho}$	IECP$^{\rho}$	IECP^{l}	IECP$^{\rho,\tau,l}$
CAGR	4.28%	5.38%	4.77%	4.30%	5.50%	5.20%	5.62%	5.58%	4.98%	5.56%
Volatility	4.99%	5.04%	5.23%	5.20%	5.10%	5.10%	5.07%	5.09%	5.04%	5.05%
Skewness	-0.36	-0.43	-0.25	-0.27	-0.30	-0.35	-0.43	-0.33	-0.28	-0.32
Sharpe Ratio	0.85	1.05	0.90	0.82	1.06	1.00	1.09	1.08	0.97	1.08
Sortino Ratio	1.39	1.71	1.50	1.36	1.76	1.65	1.78	1.78	1.62	1.80
Calmar Ratio	0.28	0.36	0.39	0.42	0.49	0.36	0.39	0.36	0.49	0.38
CVaR(95%)	-5.95%	-6.16%	-6.27%	-5.83%	-5.38%	-5.73%	-6.31%	-5.51%	-5.04%	-5.21%
Max Drawdown	-15.5%	-14.8%	-12.5%	-10.5%	-11.4%	-14.6%	-14.3%	-15.5%	-10.3%	-14.6%
Transaction Costs (p.a.)	0.32%	0.34%	0.67%	0.74%	0.52%	0.45%	0.41%	0.47%	0.46%	0.45%

Table 5: Performance metrics for the time-series momentum portfolio.

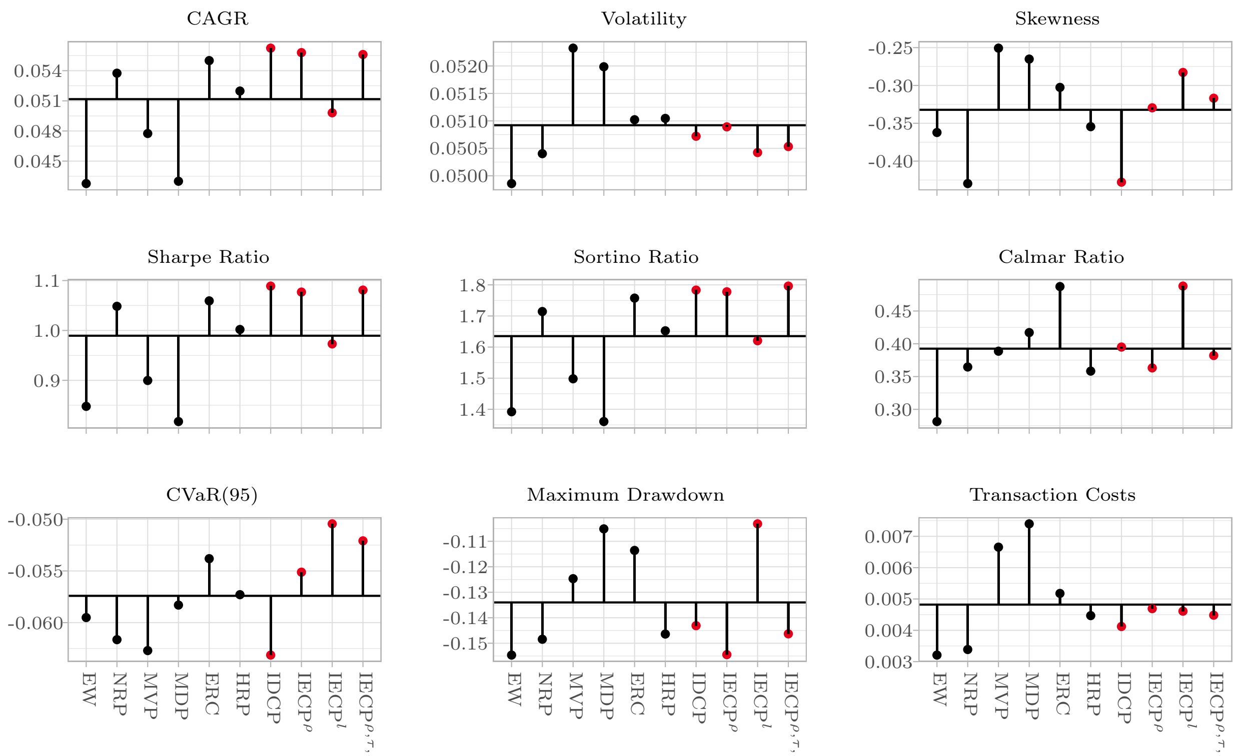

Figure 9: Performance metrics for the time-series momentum portfolio. In red, the performances for network-based strategies.

The network-based approaches IDCP$^{\rho}$, IECP$^{\rho}$ and IECP$^{\rho,\tau,l}$ show the best performance, Sharpe ratio and Sortino ratio followed by NRP. It is noticeable that NRP yields a much better performance than in the long-only portfolio compared to the other methods. Contrary to the long-only portfolio, one of the main drivers for the performance is the difference in the transactions costs. As expected and explained earlier, MVP, MDP or ERC show by far the highest transaction costs partly due to the optimization problems when facing high negative correlations. The volatility target is now slightly overshot by all methods except EW and again MVP and MDP overshoot the most. IECP^l together with MVP, MDP and ERC show the lowest drawdown, best Calmar ratio and best values for skewness.

The average allocation per asset class for each strategy is shown in figure 11. Again HRP and MVP show a similar shape and attribute a high exposure to fixed income and FX, the asset classes showing the lowest volatility. The network and multiplex approaches show a behaviour similar to the risk parity strategies and attribute more weight to commodities and equity.

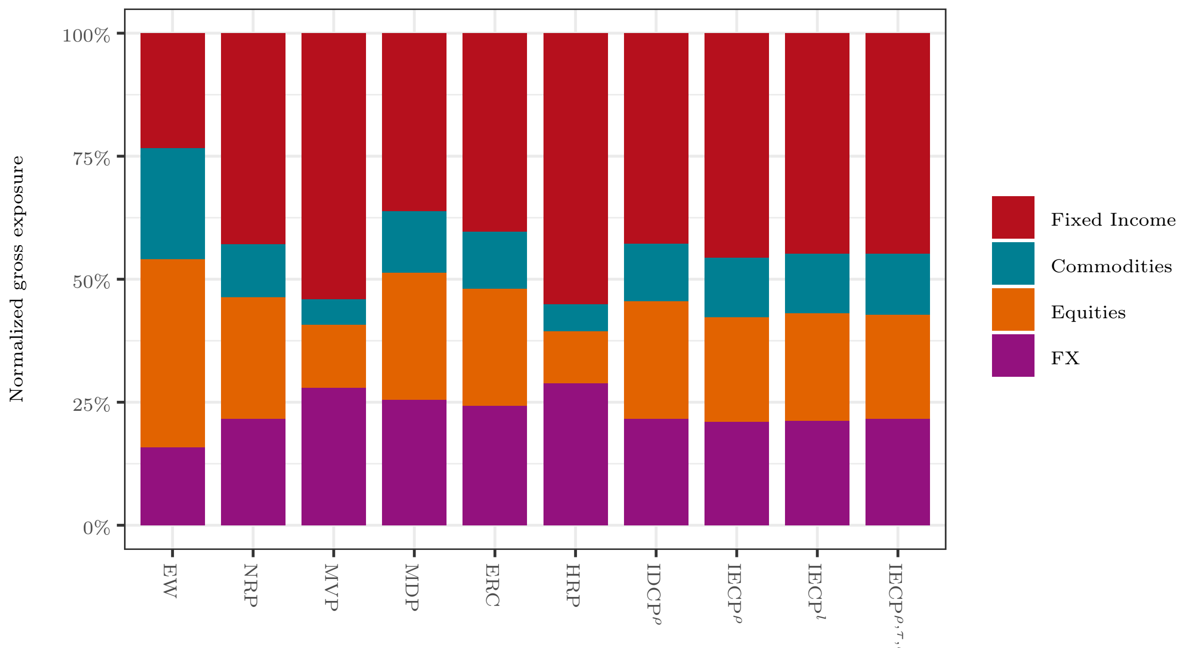

Figure 10: Average normalized gross exposure per asset class for time-series momentum portfolio. MVP and HRP show a higher average exposure towards low-volatility assets.

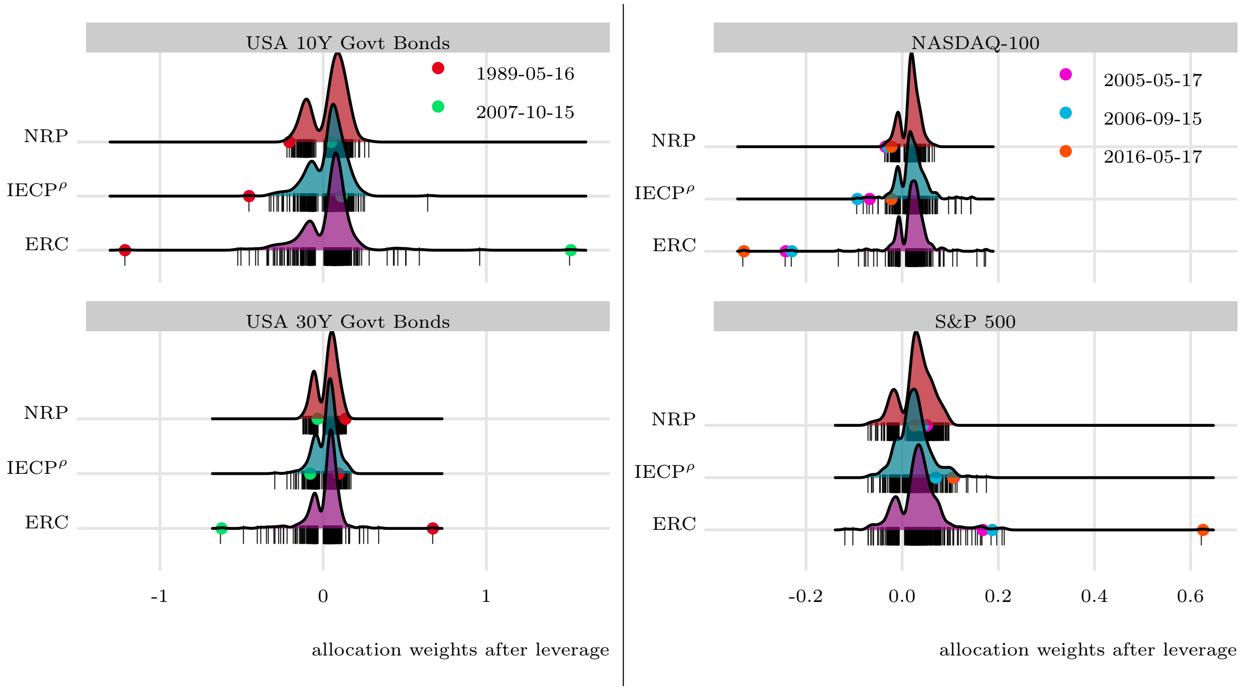

Figure 11: Mid of month positions for NRP, ERC, and IECP$^\rho$ for four selected assets. On the left, US fixed income futures, on the right, US equities, NASDAQ and S&P 500. Highlighted are some rare but extreme positions that ERC strategy can select when the signal assign opposite positions (short vs long) to two (or more) strongly correlated assets. The network-based strategies that we introduce do not suffer from this issue. Differently from NRP, exploiting the information coming from the cross-asset correlation, they are able to take larger positions.

How well diversified are the network portfolios?

Having a look at the diversification measures, we first have to state that for a portfolio containing negative weights the diversification ratio and the concentration ratio are not defined. Nevertheless the number of uncorrelated bets can be determined. The network-based approaches show good results even though the other correlation-based methods MVP and MDP show the best results here.

	Other Allocations						Network-based Allocations			
Metric	EW	NRP	MVP	MDP	ERC	HRP	IDCP$^\rho$	IECP$^\rho$	IECP^l	IECP$^{\rho,\tau,l}$
Maximum Risk (Avg.)	0.82%	0.39%	1.42%	3.73%	0.19%	1.21%	0.52%	0.92%	0.87%	0.77%
Uncorrelated Bets (Avg.)	4.05	6.5	12.08	11.8	10.11	10.7	7.16	8.37	8.19	8.37

Table 6: Concentration metrics for the time-series momentum portfolio.

We do not perform a robustness analysis for this strategy because the block-bootstrapping method could destroy existing trends. Therefore, the explanatory power of such analysis would be questionable.

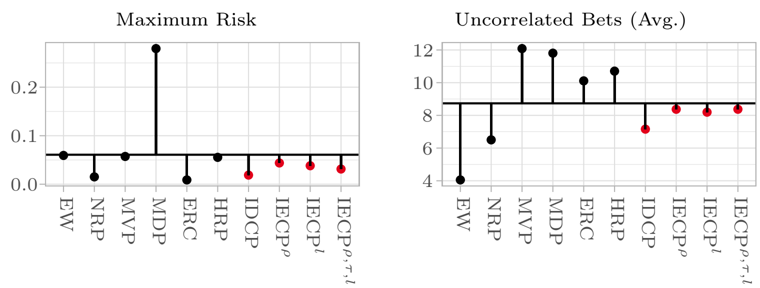

Figure 12: Concentration metrics for the time-series momentum portfolio. In red, the concentrations for network-based strategies.

6 Extensions and outlooks

In this work, we presented a set of strategies to build a well-diversified portfolio. However, the proposed strategies can be easily extended to tackle particular needs of the investor or to explore different levels of stability to noise. We now present a few non-exhaustive examples of how different choices can be made to extend or generate new strategies. We retain ourselves to provide these further extensions full account to avoid the risk of tightening their performances to the historical data we use for the backtests (Bailey et al. 2014).

Besides the choice of the portfolio universe, one can opt for alternative measures of dependencies between asset prices to tackle different diversification aspects. Measures can be either symmetric (e.g. Spearman's rank correlation) or non-symmetric (like partial correlations Kenett et al. 2010). Non-symmetric measures would lead to dependency relations represented by directed networks. In this case, the centrality measures proposed in section 2.2 can be easily adapted to deal with unidirectional edges.

Correlation is not causation, and sometimes the dependency between prices can be known a priori or estimated with other methods instead of correlations. In these cases, the investor can apply the network-based allocation concepts we propose in Sec. 3 to the customized network. For example, in a pure equity universe sector information like the GICS sector index can easily be encoded into a graph (see e.g. Pacreau, Lezmi, and Xu 2021), or an investor can be aware of the supply-chain that relates different assets. In this case, the investor has already a directed network encoding a type relationship between the assets which can be used alone or as a specific layer in the multiplex-approach described in Sec. 2.1.3. In case the network is not strongly connected the construction of the network can be constructed by including an edge when the relationship measure between two assets exceeds a certain threshold. Moreover, the investor can be interested in reducing the sparsity of the filtered graph. In this case, one can opt for a Maximally Filtered Clique Forest (MFCF), an algorithm for learning complex networks introduced by Massara and Aste 2019. Furthermore, the multiplex approach allows to go beyond ordinary networks and encode hyper-graphs (relationships that involve more than two vertices). In fact, one can cluster a group of assets as a complete graph in a layer of the multiplex and create one layer for each cluster.

When it comes to the asset allocation strategy, the investor can also choose different centrality measures (Katz' centrality, betweenness centrality, expected force etc.); however, one needs to pay attention to their meaning in the context of asset allocation and portfolio diversification. For example, betweenness centrality may be an important measure for supply-chain networks. However, its inverse may not be optimal to diversify the portfolio.

Multiplexes and the centrality measures in Sec. 2.1.3 naturally extend to more fine-grained control of the importance of each layer: one can introduce a weight vector that attributes a specific importance to each layer. This could be useful if one considers one layer as particularly important.

An entire range of possibilities comes from colouring the networks, that means associating a specific value, a weight, to each edge. One can think of the (uncoloured) networks proposed in this work as coloured ones but with each edge colour set to one. A variant of the degree centrality measure for these networks is called weighted degree centrality (wDC). In this case, the edges incident to a vertex do not count as 1, but they are weighted by the edge's weight (cf. Barrat et al. 2004). A note of caution here: the graphs we introduced in the previous sections are filtered graphs. The edges are selected when the distance between two vertices is the shortest (while maintaining the topological constraint). Applying the same distance which has been used to generate the filtered graph would generate results that change the intuition behind this measure. Some adaptations of the degree centrality measures to weighted graphs can be found in Opsahl, Agneessens, and Skvoretz 2010.

This non-comprehensive list of extensions highlights how the family of strategies we introduced to tackle diversification is not tightened to a specific vision of diversification. Diversification measures in portfolio management are not uniquely defined and can encode different perceptions of diversification. Networks and multiplexes can lead to novel measures of diversification by assessing the diversity in their relationship (see, e.g. Carpi et al. 2019) or in different statistical price behaviours as employed in this paper.

7 Conclusions

Networks provide a powerful tool to describe relationships between individual assets and illustrate how contagion effects can spread out over financial markets. Furthermore multiplex networks form a valuable extension to simultaneously encode several different aspects of the complex relationship between assets. Starting from the complete graph filtered graphs can uncover and at the same time retain the most important relationships within a portfolio. These sub-graphs seem to provide more stable structures than the complete graph or the full correlation matrix and therefore this property can be utilized to build out-of-sample well-diversified and robust portfolios.

Building upon the existing approaches of modelling financial networks we propose and analyse a new group of network-based allocation concepts. These methods can be deployed not only to long-only portfolios but also to long-short portfolios where correlation-based allocation methods may run easily into extreme positions. Moreover, we transfer the ideas of multiplex networks to portfolio allocation to better reproduce the complex dependency structure within the financial network. This can help to reduce for example the risk of contagion in periods of financial stress where traditional correlations may lose explanatory power. Our backtests suggest that the proposed methods have the potential to outperform competing traditional portfolio allocation techniques. They can improve diversification and provide downside protection in tail events.

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A Hierarchical Risk Parity

The HRP algorithm is composed of three major steps:

1. Definition of distances and tree clustering: from the correlation matrix $\rho_{i,j}$ a distance matrix is defined using the Gower metric
$$d_{i,j} = \sqrt{\frac{1}{2}\left(1 - \rho_{i,j}\right)}$$
Next, construct an Euclidean distance matrix between assets by
$$\hat{d_{i,j}} = \sqrt{\sum_{n=1}^{N} \left(d_{n,i} - d_{n,i}\right)^2}$$
Based on this distance matrix a hierarchical clustering algorithm is used to reveal a hierarchical clustering tree which groups assets with a similar risk profile together.

2. Quasi-diagonalization of the covariance matrix: the cluster tree is used to re-order the assets such that assets with a similar profile are closer to each other. This leads to a quasi-diagonalization of the correlation matrix.

3. Recursive Bisection: the final step of the algorithm calculates the weight for each of the assets using a recursive bi-sectioning procedure of the reordered covariance matrix. We start at the top of the tree and with a weight of 1 for each asset. Then we evenly divide the assets into two subsets ("bi-sectioning") and determine a factor α for the first subset and $1 - \alpha$ for the second subset
$$\alpha = 1 - \frac{\sigma^2(w_1)}{\sigma^2(w_1) + \sigma^2(w_2)}$$
$\sigma^2(w_i)$ denotes the portfolio variance of the subset assuming a minimum variance allocation within the subset. Next, rescale the weights by multiplying each weight with the corresponding factor. Both subsets are divided again, and the weights are rescaled respectively. Successively the final weights are derived.

More details about the HRP algorithm can be found in López de Prado 2016, Lohre, Rother, and Schäfer 2020 and Jaeger et al. 2021.

B Concentration Measures

In this section we give more details about the concentration measures employed in this work.

Maximum risk contribution

The metric shows the highest amount of risk attributed to one individual asset during the time window under consideration.

Diversification ratio

The diversification ratio (DR) measures the volatility reduction in a portfolio originating from diversification effects (cf. Choueifaty and Coignard 2008). The ratio is defined as the ratio of the weighted average of volatilities divided by the portfolio volatility

$$DR(w) = \frac{\sum_i w_i \sigma_i}{\sqrt{w' \Sigma w}}$$

The diversification ratio is strictly greater than 1 except for the case all assets are perfectly correlated when it equals 1.

Concentration ratio

The diversification ratio can be decomposed (cf. Choueifaty, Froidure, and Reynier 2011) into

$$DR = (\rho(1 - CR) + CR)^{-1/2}$$

with a volatility-weighted average correlation ρ and a volatility-weighted concentration ratio (CR) defined by

$$CR(w) = \frac{\sum_i (w_i \sigma_i)^2}{\left(\sum_i w_i \sigma_i\right)^2}$$

The decomposition shows: by adding less correlated assets or by removing highly correlated assets from the portfolio the diversification ratio is improved.

Effective number of uncorrelated bets

The number of uncorrelated bets was introduced by Meucci 2010. Deduced from a principal component analysis (PCA) the portfolio is decomposed into a number of uncorrelated sources of risk - the so-called principal portfolios. The authors define the independent factors as

$$w_F = E^{-1} w$$

where E is the linear operator that diagonalizes the covariance matrix Σ

$$E^T \ \Sigma \ E = \Lambda_\Sigma.$$

E is a column matrix of eigenvectors $e_1, ..., e_n$ and Λ_Σ a diagonal matrix of the eigenvalues $\lambda_1, ..., \lambda_n$. The normalized volatility contribution of each factor is

$$p_n = \frac{w_n^2 \lambda_n^2}{w^T \ \Sigma \ w}, \qquad \sum_k p_k = 1$$

The number of uncorrelated bets is then defined by the exponential of the negative Shannon's entropy:

$$ENB_1 (w, G) = \exp\left(-\sum_k p_k \ln p_k\right)$$