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Title: Heterogeneous Expectations in Asset Pricing:
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Highlights

- We develop and estimate a heterogeneous agent model for the equity market
- Estimation results reveal fundamentalists, chartists, and noise trader behavior.
- Especially switching between strategies helps improve the fit of the model.
- The model is capable of explaining bubbles and crashes in financial markets.

Heterogeneous Expectations in Asset Pricing:
Empirical Evidence from the S&P500

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Abstract

This paper empirically assesses heterogeneous expectations in asset pricing. We use a maximum likelihood approach on S&P500 data to estimate a structural model. Our empirical results are consistent with a market populated with fundamentalists and chartists. In addition, agents switch between these groups conditional on their previous performance. The results imply that the model can explain the inflation and deflation of bubbles. Finally, the model is shown to be in the deterministically stable region, but produces stochastic bubbles of similar length and magnitude as empirically observed.

Keywords: asset pricing, agent based models, fundamental analysis, technical analysis, momentum trading

JEL-codes: G11, G12

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1   Introduction

This paper studies the empirical relevance of heterogeneous agent models (HAMs) in equity markets. Prior literature on HAMs shows that models in which financial markets consist of boundedly rational traders with heterogeneous expectations provide a viable alternative to the representative agent hypothesis and rational expectations models. Such models yield an intellectually satisfying depiction of market behavior and they can explain several stylized facts of financial markets. Such evidence, however, is usually of an analytical and simulation-based nature. The current paper directly takes the model to the data without having to make major adjustments to it. Using a maximum likelihood approach on S&P500 data, we find empirical evidence consistent with heterogeneous expectations and switching between groups. Although it might be possible to get comparable empirical results using more parsimonious statistical models, the current study is an important step in the empirical verification of HAMs, giving both statistical significance and ample economic intuition.

The finance literature has recognized the importance of heterogeneity between investors and its effect on asset pricing. Heterogeneity is introduced as differences in preferences, such as differences in risk aversion, but more often as differences in beliefs (or expectations). As an early example, Lintner (1969) studies aggregation issues in a market where both judgments and preferences differ between investors. The typical HAMs also focus on differences in expectations, but differ from classical asset pricing models in that they move away from the notion of rational expectations and a representative agent, favoring instead a context characterized by boundedly rational agents, as originally proposed by Simon (1957). Of course, this poses more challenges for the economist as there is a wide spectrum of possible boundedly rational behaviours, whereas rational expectations imposes a unique solution. Cutler, Poterba and Summers (1991) show that interactions between rational investors and noise traders following positive feedback strategies – buying when prices rise, selling when prices fall – can reproduce the momentum and mean reversion patterns observed in markets. DeLong, Shleifer, Summers and Waldmann (1990) show that rational traders in such a model can actually destabilize the market by initially driving up prices beyond fundamentals and then later selling out at even higher prices to the feedback traders.

A crucial ingredient of the heterogeneous agent models as introduced by Brock and Hommes (1997, 1998), is the presence of a core of non-rational positive feedback traders – sometimes called chartists – who expect past price changes to continue in the future. This group stands in contrast to a group of fundamentalist traders, who trade upon mean reversion. Fundamentalists trade upon the expectation that the market price will mean revert towards a certain fundamental price, while chartists are technical analysts

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who trade on historical price patterns. The fundamentalists are close to the rational traders of the classical framework[1]; chartists, in contrast, are more akin to momentum traders. Apart from the fundamentalist - chartist distinction, the adoption of HAMs also allows investor behavior to be characterized by the flexibility of agents to switch between these two archetypes conditional on past performance, as introduced by Brock and Hommes (1997). In support of this mechanism, Bloomfield and Hales (2002) present experimental evidence that participants switch between a mean-reverting and a trending regime in forecasting conditional on the most recent realization, even though they know that the underlying process follows a random walk.

The literature on HAMs has gained momentum over the last two decades. Zeeman (1974), in a largely neglected contribution, was amongst the first to point out the potential of HAMs to explain financial market dynamics. The literature expanded rapidly following the contributions of Day and Huang (1990), Kirman (1991), Chiarella (1992), and Lux and Marchesi (1999). The seminal work of Brock and Hommes (1997, 1998) introduced the notion of agents switching strategies and triggered broad interest in this approach to financial market behavior; see Chiarella et al. (2009a) for a recent overview of the relevant literature.

To be consistent with the stylized facts in high frequency data of financial markets, as outlined, for example, by Pagan (1996), the distribution of returns of any worthwhile model of financial markets should display excess kurtosis, heavy tails, and zero predictability in levels, while having strong persistence in the second moment. The heterogeneous agents literature has been especially successful in explaining financial market behavior in terms of the second and higher moments; see, for instance, De Grauwe and Grimaldi (2006). The combination of destabilizing dynamics, in the form of technical analysis, and stabilizing mean reversion dynamics, in the form of fundamental analysis, has proven to be a source of interesting financial market outcomes. For example, Lux (1998) illustrates how a heterogeneous agent model can replicate a number of important financial market phenomena, such as volatility clustering, leptokurtosis, heavy tails, and normalization under time aggregation. He and Li (2007) show that agent heterogeneity, risk-adjusted trend chasing through a geometric learning process, and the interplay of noise and the underlying deterministic dynamics can be the source of power-law distributed fluctuations. In addition, De Grauwe and Grimaldi (2006) and Spronk et al. (2013) document that application of heterogeneous agent models to the foreign exchange market can resolve several of the puzzles in international finance, such as the excess volatility puzzle and the disconnect puzzle. Concerning the level of returns, studies typically

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[1] They are still boundedly rational, though, in that they do not take into account the existence of chartists. However Chiarella et al. (2006b) do consider models where the fundamentalists take account of the existence of chartists and find that the basic dynamic mechanisms at work are not essentially changed.

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focus on the autoregressive behavior of returns; see, for example, the study of Westerhoff (2006).

The above-cited papers generally rely on stochastic simulation techniques to replicate characteristics of financial markets. Studies that actually empirically estimate heterogeneous agent models are less frequently encountered in the literature, however, due to the highly nonlinear structure of the models. As an early example, Frankel and Froot (1990) formulate a fundamentalist - chartist model and verify it empirically. In their setup, however, chartism is represented by a random walk and switching between groups is not permitted, so the model is highly simplified. Vigfusson (1997) focuses on the foreign exchange market, and circumvents the problem of the nonlinear switching mechanism by replacing it with a Markov regime switching approach; Chiarella et al. (2012) apply a similar strategy. Although empirically simplifying matters, this replacement comes at the cost of economic interpretation of the switching function. Westerhoff and Reitz (2003) introduce time variation in the impact of either chartists or fundamentalists conditional on the distance of the market price to the fundamental price, but not due to the relative profitability of the rules. De Zwart et al. (2009) illustrate the economic value of HAMs from an investment perspective. Boswijk et al. (2007) are the first to estimate a fully-fledged switching model by rewriting a HAM as a smooth transition auto-regressive model and estimate it for the S&P500. De Jong et al. (2009, 2010) apply a similar methodology for Asian equity markets and the European Monetary System, respectively. Alfarano et al. (2005) derive and estimate a closed-form solution of the distribution of returns generated by a simple agent based model. Franke (2009) applies the simulated method of moments to estimate a HAM on foreign exchange rate returns. Manzan and Westerhoff (2007) estimate a HAM and focus on its forecasting ability. Franke and Westerhoff (2012) present a method to compare types of nonlinear models, which are non-nested.

This paper extends the literature in a number of important ways. We show that, in contrast to previous empirical work, under slightly simplifying assumptions and marginal additions that the HAM introduced in Chiarella and He (2003) can be estimated directly. Due to the specific setup of the model that includes our modifications, empirical verification of this HAM becomes possible using a quasi maximum likelihood estimation procedure[2]. As such, it represents an important extension of the literature in terms of methodology and evidence. In addition, using the model and estimated parameter set, we present more detailed characteristics of the model in terms of composition of the market, market characteristics in terms of bubbles and crashes, and stability of the estimated model. By doing so we again make

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[2]A quasi maximum likelihood procedure is applied rather than a regular maximum likelihood procedure to retrieve robust standard errors of the estimates because the distribution of residuals is unknown.

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the connection to the existing literature on HAMs in terms of stability and various types of market behavior.

The estimation results reveal that both fundamentalists and chartists are present in the S&P500. Consistent with simulation evidence in the HAM literature, fundamentalists are stabilizing while chartists are destabilizing. The introduction of switching between groups is particularly beneficial for the explanatory power of the model. The model in this paper is able to explain the inflation and deflation of bubble-like situations, such as the IT bubble. Finally, we show that the model has a stable underlying deterministic dynamics and produces highly similar dynamics as empirically observed.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the data and methodology. Section 4 presents our estimation results and Section 5 the simulation results. Section 6 concludes.

2   The Model

Typically, traders within a HAM solve a standard portfolio optimization problem by distributing their wealth between a risky asset and a risk-free asset[3]. By means of a mean-variance optimization procedure, traders determine their demand for the risky asset. Preferences are assumed to be homogeneous across traders. Demand for the risky asset is then a function of the expected future price of the risky asset. It is in the formation of these expectations that the HAM approach differs from the standard one as the expectation is either of the fundamentalist or the chartist type. Traders subsequently submit their demand to a Walrasian auctioneer, who sets the market clearing price for the risky asset. After each period, traders evaluate whether the fundamentalist or the chartist strategy has been more profitable. The relative profitability of the fundamentalist (chartist) rule then determines the probability of using the fundamentalist (chartist) rule in the next period and investors reallocate their portfolio accordingly. This process is repeated each period.

The heterogeneous agents literature frequently speaks of fundamentalists and chartists. Chartists are akin to momentum or positive feedback traders, while fundamentalists are contrarian by nature. On the rationality spectrum, fundamentalists are closer to the traditional fully rational traders while chartists are closer to noise traders. Limits to arbitrage, see Barberis and Thaler (2003), are therefore embedded within the model as chartists can drive out fundamentalists by creating bubble-like situations.

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[3]Generalization to similar heterogeneous agent models with multiple risky asset markets is interesting but empirically restrictively complicating because the number of (unidentified) coefficients would increase substantially; see Chiarella et al (2007) for a theoretical approach to a multi-asset framework.

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In this study, our model is based on the Chiarella and He (2003) model, which contains many of the traditional HAM features as just described. The added value of this model vis-à-vis the traditional HAM literature is based on its market microstructure and the functional form of the chartists. These two features make this particular model suitable for empirical estimation purposes. In particular, the model moves away from the assumption of a Walrasian auctioneer and introduces a more realistic market microstructure in the form of a market maker[4]. Although still stylized in its implementation, the setup with a market maker is a step forward compared to that of the more unrealistic Walrasian auctioneer. Regarding the chartist rule, the typical functional form of the chartist expectation function is replaced with a more realistic one. The typical functional form of the chartist group in HAMs is a simple $AR(1)$ process. This is a satisfactory assumption for stability analysis in a simulation setup. However, given the empirical lack of autocorrelation in returns, it is necessary to introduce a more sophisticated technical rule in an estimation study. Here, the chartists are assumed to use a moving average rule that replaces the $AR(1)$ configuration[5]. A problem with the moving average rules is that the choice of lag length remains arbitrary. Therefore, the long-run moving average is assumed to be given by a geometric decay process (GDP). Theoretically, the GDP incorporates unbounded memory but the relative weight on past observations is regulated by a memory parameter, the decay rate, which can be estimated. In the same paper, Chiarella and He (2003) show that the GDP is a reasonable approximation for moving average strategies.

In order to make the model suitable for estimation, we introduce a number of additional changes to the traditional HAM. Contrary to Chiarella and He (2003) we assume homogeneous risk aversion across groups and zero information costs. Given that individuals can switch between different agent types and risk aversion is an individual character trait, homogeneous risk aversion is not a strong assumption. Furthermore, Chiarella and He (2003) show that differences in risk aversion between groups can be an additional source of bifurcation, which we want to avoid in the current study as it adds additional nonlinearity in the estimation process. Also, given that all groups are aware of the fundamental value, switching does not demand any additional skill or knowledge so that switching occurs at zero information costs.

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[4] In a recent survey, Chiarella et al. (2009a) give an overview of the literature combining heterogeneous agent models with market microstructure considerations. O'Hara (1995) notes that there is only one market in the world using the auctioneer mechanism: the silver market in London.

[5] See also Brock et al. (1992) for an empirical application of these technical rules and Chiarella et al. (2006) for the implications of moving average rules in heterogeneous agent models. Hommes et al. (2005) provide evidence of the existence of fundamentalists and chartists in an experimental setup.

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Consistent with the Brock and Hommes (1998) framework, traders optimize their portfolio by allocating total wealth between a risky asset and a risk free asset. Let p_t be the (ex-dividend) price of the risky asset at time t, y_t be the (assumed commonly known) dividend process, and g_t be the expected growth rate of dividends, which is determined conditional on information available at period t. The latter definition implies

$$\mathbb{E}_t(y_{t+1}) = (1 + g_t)y_t. \tag{1}$$

In turn, consistent with the Gordon growth model (see Gordon, 1962), the fundamental price is given by

$$\overline{p}_t = \frac{1 + g_t}{\rho_t - g_t} y_t, \tag{2}$$

where ρ_t is the required expected return on equity conditional on the information available at time t. We regard $\bar{p}_t$ as the rational expectations fundamental price, which is assumed common to all traders in the market. For both g_t and ρ_t we assume that agents update their estimate of the true value each period as more data become available; the calculation of these quantities will be discussed in more detail in Section 3. We write the model in price deviations from the fundamental price, that is,

$$x_t = p_t - \overline{p}_t. \tag{3}$$

The excess return in terms of price deviations is then given as

$$R_t = x_t - Rx_{t-1} + \delta_t, \tag{4}$$

where $R = 1 + r$, in which r is the risk-free rate, and δ_t is the error made in forecasting next period's fundamental price, that is

$$\delta_{t+1} = \overline{p}_{t+1} + y_{t+1} - \mathbb{E}_t[\overline{p}_{t+1} + y_{t+1}], \tag{5}$$

so that R_t is a standard measure of return, and

$$\mathbb{E}_t[\delta_{t+1}] = 0. \tag{6}$$

All groups hold the same expectation about the dividend process, and thus all agents have the same expectation about the mean of the fundamental price.

The market is populated by two groups of traders; fundamentalists and chartists. Fundamentalists focus solely on the market price in relation to its fundamental value, while chartists are technical analysts, who focus only on past prices and price changes.

Fundamentalists expect the market price to move towards its fundamen-

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tal, or equivalently, the deviation of the market price x_t to revert to zero. The expectations of the fundamentalists can thus be written as

$$\mathbb{E}_{f,t}(x_{t+1}) = x_t - \alpha x_t = (1 - \alpha)x_t, \tag{7}$$

where $\alpha \in [0, 1]$ and $1 - \alpha$ measures the fundamentalists' perceived speed of mean reversion of the market price towards the fundamental price. Combining (7) with (4), we can write the expected return of the fundamentalists as

$$\mathbb{E}_{f,t}(R_{t+1}) = (1 - R - \alpha)x_t. \tag{8}$$

Chartists, on the other hand, focus only on (changes in) past market prices. To be more specific, they use a moving average rule in which expectations are formed by comparing the short-run moving average of prices with the long-run moving average of prices; see Zhu and Zhou (2009) for empirical implications and evidence of moving average rules in equity markets. As we have already stated, instead of imposing an arbitrary lag length for the moving average process, we endogenize it by using a geometric decay process (GDP). As such, the expectations of chartists can be written as[6]

$$\begin{aligned} \mathbb{E}_{c,t}(x_{t+1}) &= x_t + d(x_t - \tau_t) \tag{9} \\ \tau_t &= \omega\tau_{t-1} + (1 - \omega)x_t, \end{aligned}$$

where τ_t is the geometric decay function or long-run geometric moving average, and $\omega \in [0, 1]$ is the decay rate or the memory. The larger is ω, the more the past price deviations influence the current trend and the less the most recent realization. The parameter d measures the direction and degree to which chartists respond to a deviation between the market price and its long-run moving average. If $d > 0$, chartists push the price further away from the GDP; $d < 0$, on the other hand, implies contrarian expectations as the market price is expected to move towards its long-run moving average. Taking (4) into account, the expected returns of chartists are given by

$$\mathbb{E}_{c,t}(R_{t+1}) = (1 - R)x_t + d(x_t - \tau_t). \tag{10}$$

Fundamentalists and chartists are assumed to be mean variance optimizers, so that the optimal demand for the risky asset by group h at time t is given by

$$z_{h,t} = \frac{\mathbb{E}_{h,t}(R_{t+1})}{\gamma\sigma^2}, \tag{11}$$

where γ is the risk aversion parameter (assumed to be common to both

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[6]Note that "momentum trader" would be a more accurate description of the behavior than "chartist". However, we choose to follow the literature in referring to the group of technical analysts as "chartists".

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groups) and σ^2 is the perceived risk associated with investing in the risky asset given by $V(p_{t+1} + y_{t+1})$ with V the expected variance operator. Using (8) and (10) for the fundamentalist and chartist groups, (11) becomes, respectively,

$$z_{f,t} = \frac{(1 - R - \alpha)x_t}{\gamma\sigma^2} \qquad (12)$$

and

$$z_{c,t} = \frac{(1 - R)x_t + d(x_t - \tau_t)}{\gamma\sigma^2}, \qquad (13)$$

in which $z_{f,t}$ is fundamentalist demand and $z_{c,t}$ is chartist demand for the risky asset in period t.

The sum of chartist and fundamentalist demand for the risky asset equals the total market demand traded via the market maker. The market maker takes an offsetting position so as to clear the market. Conditional on the position that has to be taken, that is, conditional on total market demand, the market price announced for trading in the following period is adjusted accordingly. Using μ to denote the corresponding speed of price adjustment, the dynamical system describing the behavior of the market maker can be written as

$$x_{t+1} = x_t + \mu\left(n_{f,t}z_{f,t} + n_{c,t}z_{c,t}\right) + \eta_{t+1}, \qquad (14)$$

where $n_{h,t}$ is the fraction of agents in group h ($h = f, c$) at time t satisfying $n_{f,t} + n_{c,t} = 1$ for any t, and $\eta_{t+1} = -(\overline{p}_{t+1} - \overline{p}_t) + v_{t+1}$ where v_{t+1} represents the IID noisy demand of any other types of traders in the market as well as shocks to the supply of stocks[7]. Note that we refrain from taking into consideration any microfoundation of the market maker, such as, for example, the dynamics of her or his inventory[8]. As such, our market maker scenario represents a highly stylized form of market microstructure. We define the relative difference between the size of the fundamentalist and chartist groups as

$$m_t = n_{f,t} - n_{c,t}, \qquad (15)$$

so that $m_t \in [-1, 1]$. The dynamical system can be written as

$$x_{t+1} = x_t + \frac{\mu}{2}\left[(1 + m_t)z_{f,t} + (1 - m_t)z_{c,t}\right] + \eta_{t+1}. \qquad (16)$$

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[7]Note that $E[-(\overline{p}_{t+1} - \overline{p}_t)] < 0$ because of the positive growth rate g_t. This would imply that the inventory of the market maker continuously decreases. This is partly offset by the, on average, positive growth in the supply of stocks (i.e., the number of outstanding stocks) captured by ν_{t+1}. Empirically, though, we find that the mean of η_{t+1} is not significantly different from zero.

[8]See Zhu et al. (2009) for a specification that does take into account the dynamics of the market maker's inventory.

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The distribution of agents across the fundamentalist and chartist groups is conditional on past performance. Performance is measured by profit, which is given by the ex-post excess return earned by each group. This return is calculated as the optimal demand for the risky asset multiplied by the actual price change, that is,

$$\pi_{h,t} = (x_t - Rx_{t-1} + \delta_t)z_{h,t-1}. \tag{17}$$

Agents are assumed to update their beliefs conditional on the profitability of the two rules based on discrete choice probabilities as introduced by Manski and McFadden (1981) and by Brock and Hommes (1997, 1998) in the heterogeneous agent literature. We set

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\begin{aligned}
m_t &= \frac{\exp(\beta\pi_{f,t}) - \exp(\beta\pi_{c,t})}{\exp(\beta\pi_{f,t}) + \exp(\beta\pi_{c,t})} \\
&= \tanh\left(\frac{\beta}{2}\left[\pi_{f,t} - \pi_{c,t}\right]\right) \\
&= \tanh\left(\frac{\beta}{2}[x_t - Rx_{t-1} + \delta_t][z_{f,t-1} - z_{c,t-1}]\right),
\end{aligned}
\tag{18}
$$

where β is the intensity of choice, which measures the speed of traders' reaction to differences in profitability between the groups[9]. The cost for both groups is set equal to zero as both trading rules are simple to apply without any costly data. The hyperbolic tangent function[10] translates the profit difference between the two groups into a difference in weight between minus one and plus one. When the profit difference is large and positive (negative), which is when the fundamentalists (chartists) make sufficiently more profit, more agents will use the fundamentalist (chartist) rule and m_t goes to plus one (minus one). In other words, traders use a positive feedback rule in choosing their strategy. The intensity of choice parameter β measures the sensitivity of agents to profit differences. The higher is β, the quicker agents will respond to a difference in profit by switching to the other group. At $\beta = 0$, there is no switching and agents are distributed equally across groups. At the other extreme, when $\beta \to \infty$, there is infinite sensitivity to the differences in performance and either all agents are fundamentalists or all agents are chartists. The sluggishness in switching is a typical behavioral aspect and the core of the model. The intensity of choice can also be interpreted as the *status quo* bias of investors, it measure the extent to which they adhere to

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[9]Chiarella and He (2003) introduce a cost to using a certain strategy, with the cost for the fundamental strategy higher than for the chartist strategy. This causes m_t to decrease with a constant fraction, which is conditional on the cost difference. We assume that both strategies are equally costly as this makes the model more parsimonious.

[10]Note that $\tanh(x) = \frac{\exp(2x)-1}{\exp(2x)+1}$. Whereas the original Brock and Hommes (1997, 1998) model specifies the weights using a logit function, we use the *tanh* function. Combined with the terms $(1 + m_t)$ and $(1 - m_t)$ in (16), though, this is mathematically equivalent.

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a strategy that is performing worse; see Kahneman et al. (1982).

Combining the market maker with the agents, the dynamics driving prices can be written as a stochastic dynamical system. After simplifying notation and rearranging terms, the system can be written as

$$x_{t+1} = x_t + \frac{\mu}{2\gamma\sigma^2} \left[2(1-R)x_t - (1+m_t)\alpha x_t + (1-m_t)d(x_t - \tau_t)\right] + \eta_{t+1},$$
(19)

$$\tau_{t+1} = \omega\tau_t + (1-\omega)x_{t+1},$$
(20)

$$m_{t+1} = \tanh\left(\frac{\beta}{2\gamma\sigma^2}(x_{t+1} - Rx_t + \delta_{t+1})(-\alpha x_t - d(x_t - \tau_t))\right).$$
(21)

The model represented by the stochastic dynamic system (19)-(21) is the model we estimate and evaluate in the remainder of the paper. The next section describes our choice of the fundamental price $\bar{p}_t$ and outlines our estimation procedure.

3 Data and Estimation Methodology

We estimate the model introduced in the previous section for the S&P500. However, before we are able to do so, we need to define a fundamental price so that we can form the fundamentalists' expectation. Furthermore, we have to make some technical adjustments to the model to be able to estimate it.

From Datastream, we collect monthly data on prices (that is, closing prices for each first trading day of the month) and earnings[11] from January 1970 to October 2012, yielding 514 observations. The index itself does not generate earnings; as such, we use the price earnings calculation for the S&P500 from Datastream, which is a weighted average of the price earnings ratios of the constituent stocks within the index. Datastream takes the most recently announced earnings. Because not all companies publish earnings at the same date, the index earnings not only change once per quarter, but continuously. Datastream publishes an updated number every Wednesday. Stocks included in the S&P500 trade on either the NYSE or the NASDAQ. Our model is appropriate for this index as both exchanges make use of a type of market maker in the trading process[12]. We follow Boswijk et al. (2007) and form the fundamental price by use of (2). The required return on assets ρ_t consists of a capital gain plus a yield component. Furthermore, consistent with (2), the (long-run) capital gain is equal to the growth in earnings, such that $\rho_t = g_t + \mathbb{E}_t(y_{t+1}/p_{t+1})$. As a result, the denominator

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[11]Note that we shall use earnings instead of dividends as a measure of y_t since earnings data are less sensitive to management choices compared to dividends data.

[12]In fact, both exchanges use "specialists" to provide liquidity. At the NYSE, there is a single exchange member, while the NASDAQ employs several competing market makers.

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of (2) is equal to the expected earnings yield, $\rho_t - g_t = \mathbb{E}_t(y_{t+1}/p_{t+1})$. The resulting fundamental value equals

$$\bar{p}_t = \frac{(1 + g_t)}{\mathbb{E}_t(y_{t+1}/p_{t+1})} y_t.$$

The earnings growth g_t is calculated as the average realized growth rate up to period t. Similarly, the expected earnings yield $\mathbb{E}_t(y_{t+1}/p_{t+1})$ is calculated as the average realized earnings yield up to period t. The stochastic discount factor is therefore completely driven by earnings as suggested by Fama and French (2002)[13]. The resulting real price, real fundamental price, and difference between them are depicted in Figure 1.

Figure 1: Price and Fundamental Price

This figure depicts the price p_t, fundamental price $\bar{p}_t$, and their difference x_t. The left-hand axis is the scale for p_t and $\bar{p}_t$; the right-hand axis is the scale for x_t.
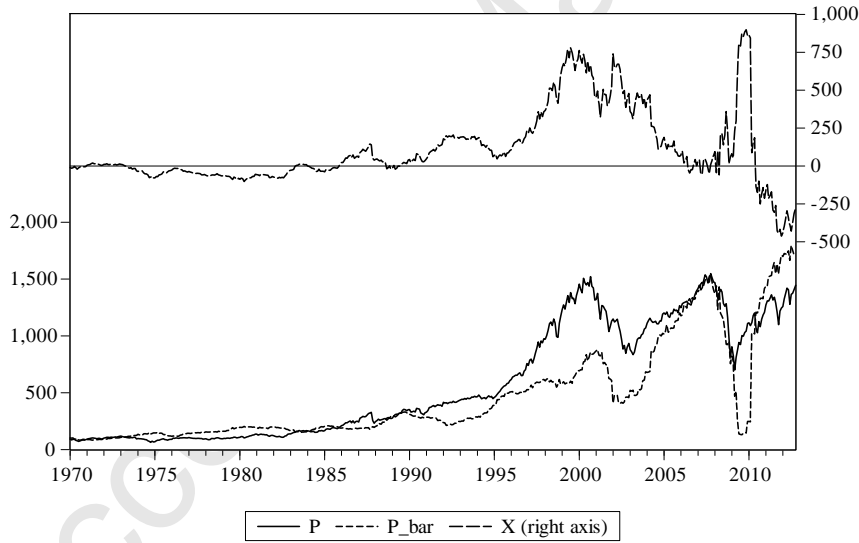
One sees from Figure 1 that the market price p_t stays relatively close to the fundamental price $\bar{p}_t$ up until 1995, after which the IT bubble inflates the difference. Only in 2006 does the fundamental value catch up with the market price. A striking observation is that the market is highly overvalued relative to our fundamental in 2009. In contrast to the late 1990's, it is not

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[13]Note that the fundamental value we define does not necessarily need to represent the actual fundamental value of equity. Rather, it should represent the fundamental value as perceived by the agents in the model because this determines their expectations.

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the market price p_t that increases excessively, but the fundamental price $\overline{p}_t$ that drops sharply due to falling earnings resulting from the global financial crisis. Both the market price and financial price have gone up since then, ending in an undervaluation at the end of the sample. Table 1 presents the descriptive statistics, confirming the graphical results.

Table 1: Descriptive Statistics

The table presents the descriptive statistics of p_t, $\overline{p}_t$, x_t, and $x_t - x_{t-1}$. AC represents the autocorrelation in levels, and Abs - AC the autocorrelation in absolute values.

	p_t	$\overline{p}_t$	x_t	$x_t - x_{t-1}$
Mean	581.96	472.83	109.13	-0.525
Median	380.29	274.22	27.61	1.139
Max.	1547.04	1795.69	898.16	312.07
Min.	63.39	83.86	-463.11	-756.13
St.Dev.	491.97	436.48	245.77	57.42
Skew	0.524	1.397	1.130	-4.396
Kurt	1.654	3.897	4.300	66.047
AC	0.994	0.984	0.970	-0.047
Abs - AC	0.994	0.984	0.966	0.271
Obs	513	513	513	513

The structural model given by (19)-(21) is not fully identified empirically. In particular, the behavioral coefficients of the market maker and traders μ and β cannot be estimated directly as they serve as scaling factors for the coefficients α and d in the demand functions. Also, we can only observe the ratios of the market maker's impact and the intensity of choice over risk aversion times the volatility, that is, $\mu/2\gamma\sigma^2$ and $\beta/2\gamma\sigma^2$, respectively. To resolve this issue, we propose the following simplification. First, R is assumed to be equal to one, which is a safe assumption given the monthly frequency[14]. Second we redefine α and d as not solely being a reaction to the mispricing and moving average, respectively, but as an aggregate measure of traders' risk tolerance *and* their reaction to the market. As such, α and d are market impact coefficients. The term $\eta_{t+1} = c + \varepsilon_{t+1}$, where c captures any fundamental drift and ε_{t+1} is a noise process with mean of 0. Finally, δ_{t+1} is the error made in forecasting the fundamental value; by (6), we can drop it from the empirical model without loss of generality. The resulting econometric model reads

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[14]The risk-free rate for the United States is of the order of magnitude of 0.1% per month, so $R \approx 1.001$. Experiments using the government bond yield as exogenous variable do not qualitatively affect the empirical results.

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$$x_{t+1} - x_t = c + \mu\left[-(1 + m_t)\alpha x_t + (1 - m_t)d(x_t - \tau_t)\right] + \varepsilon_{t+1}, \qquad (22)$$

$$\tau_{t+1} = \omega\tau_t + (1 - \omega)x_{t+1}, \qquad (23)$$

$$m_{t+1} = \tanh[\beta(x_{t+1} - Rx_t)(-\alpha x_t - d(x_t - \tau_t))]. \qquad (24)$$

We indirectly estimate the impact of the market maker μ and the intensity of choice β in combination with the demand functions. The parameters governing the switching between groups, β, and the speed of price adjustment of the market maker, μ, are not uniquely identified in (22) to (24). However, because α and d appear in both (22) and (24), all parameters can be retrieved in a two-step procedure. The structural coefficients μ, β, α, and d are not individually identified, but they are combined. We can observe them indirectly by estimating the fully identified reduced-form system. The reduced form econometric model is

$$x_{t+1} - x_t = c - (1 + m_t)\alpha^* x_t + (1 - m_t)d^*(x_t - \tau_t) + \varepsilon_{t+1}, \quad (25)$$

$$\tau_{t+1} = \omega\tau_t + (1 - \omega)x_{t+1}, \qquad (26)$$

$$m_{t+1} = \tanh\left[(x_{t+1} - x_t)(-\alpha^{**}x_t - d^{**}(x_t - \tau_t))\right], \qquad (27)$$

where $\alpha^* = \mu\alpha$, $d^* = \mu d$, $\alpha^{**} = \beta\alpha$, and $d^{**} = \beta d$.

As we need to estimate four reduced form coefficients and we have four unknowns, and the structural parameters α and β are present in both (19) and (21), all coefficients can be retrieved; see, for example, Nieberding (2006) who describes a similar issue surrounding the identification of linear supply and demand functions. Because we are dealing with estimated coefficients with an accompanying standard error, the structural coefficients α, d, μ, and β cannot be found analytically and thus have to be retrieved numerically. The coefficients α, d, μ, and β are found by numerically minimizing with respect to them the loss function $\mathbb{Z}$, given by

$$\mathbb{Z} = \frac{|\alpha^* - \mu\alpha|}{|\alpha^* + \mu\alpha|} + \frac{|d^* - \mu d|}{|d^* + \mu d|} + \frac{|\alpha^{**} - \beta\alpha|}{|\alpha^{**} + \beta\alpha|} + \frac{|d^{**} - \beta d|}{|d^{**} + \beta d|}. \qquad (28)$$

The numerators in the four elements of (28) represent the differences between the estimated reduced-form coefficients α^*, d^*, α^{**}, and d^{**} and the underlying structural coefficients α, d, μ, and β. We divide by the sum of these relations because of the difference in order of magnitude between the coefficients, especially α and β. Not doing so could lead to a suboptimal result in the numerical optimization procedure as excessive weight could be given to coefficients with a large absolute value. Starting values for α and d in optimizing (28) are the estimates obtained from the restricted version of the model. For the market maker impact we use $\mu = 1$ as starting value.

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The starting value for β is subsequently derived from the definition of the coefficients, $\frac{\mu}{\beta} = \frac{\alpha^*}{\alpha^{**}}$.

We use maximum likelihood to estimate the model; specifically, given that the empirical distribution of ε_t is unknown, quasi maximum likelihood is applied to retrieve robust standard errors of the estimated coefficients[15]. Boswijk et al. (2007) estimate their model using nonlinear least squares; this is theoretically also possible for our model, but we decided to use maximum likelihood as this allows us to use standard likelihood ratio tests[16]. Due to its nonlinear nature, the model is sensitive to initial values; this is also illustrated theoretically in Chiarella and He (2003). To assure ourselves that the iterative optimization procedure of the maximum likelihood estimation does not converge to a local optimum, we undertake the following steps. First, we estimate a restricted version of the model with fixed market maker speed of adjustment and without switching between groups. That is, $\mu = 1$ and $m_t = m = 0$[17]. This restricted model can be estimated using OLS and is therefore not sensitive to initial values; it gives initial values for c, α^*, d^*, and ω. The starting value for τ_t is set equal to the first observation of x_t in the data set. Subsequently, we estimate the full model given by (25)-(27). We perform a grid-search over different initial values for α^{**}, and d^{**} to make sure that we select the global optimum of the likelihood function. A likelihood ratio test between the restricted setup of the model with $\mu = 1$ and $m_t = 0$ and the unrestricted setup of the model indicates the joint significance of the market maker and the switching rule. The first year of data is not used in the estimation because of initialization of the model; hence, it is estimated on a total of 502 montly observations.

The next section presents the estimation results for the model given by (25)-(27).

4 Results

4.1 Estimation Results

This section presents the results of the estimation of the heterogeneous agent model with a market maker given in Section 2 using the methodology outlined in Section 3. To illustrate the importance of the model's different ele-

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[15]To account for possible heteroskedasticity in the errors alternatively, we also estimated the model imposing a $GARCH$ structure, by defining $x_t = \ln(p_t) - \ln(\overline{p}_t)$, and by defining $\overline{p}_t = \frac{1}{12} \sum_{i=0}^{11} p_{t-i}$. The results are qualitatively robust to both alternatives in the sense that we consistently find stabilizing fundamentalists, trend chasing chartists, and the switching model that delivers the best fit. We decided to present the results using the robust standard errors because this configuration is closests to the theoretical model.

[16]Estimation results using non-linear least square are highly similar; results available on request.

[17]This scenario corresponds to an infinite status quo bias or zero intensity of choice, $\beta = 0$.

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ments, it is estimated both with and without a number of different restrictions. The analyses are repeated for two sub-samples, 1971 - 1991 and 1992 - 2012, because of the relatively long period and the higher volatility in the latter part of the sample (see Figure 1). The results for the different model specifications estimated over the full sample period are presented in Table 2; these are the reduced form results, the structural parameters are presented later in Table 4.

Table 2: Estimation Results for the Full Sample

The table presents the estimation results of different versions of the model given by (25)-(27) for the full sample period. Column (A) is the full model, Column (B) introduces the restrictions $\mu = 1$ and $\beta = 0$, Column (C) examines $\mu = 1$, $\beta = 0$, and $d^* = 0$, Column (D) sets $\mu = 1$, $\beta = 0$, and $\alpha^* = 0$. LLR-A is the test statistic of the likelihood ratio test versus the full model; LLR-B the statistic for comparison between restricted versions of the model. Standard errors are in parentheses, and *, **, and *** respresent significance at the 10%, 5%, and 1% level, respectively.

	(A)	(B)	(C)	(D)
c	2.083	3.022	2.286	-0.531
	(3.669)	(4.779)	(4.126)	(3.130)
α^*	0.030***	0.030**	0.025**	
	(0.006)	(0.013)	(0.010)	
d^*	0.051***	0.044		0.011
	(0.012)	(0.036)		(0.052)
ω	0.919***	0.786***		0.711
	(0.009)	(0.240)		(1.269)
α^{**}	0.002			
	(0.002)			
d^{**}	-0.003			
	(0.004)			
logl	-2737.28	-2746.90	-2747.58	-2750.47
Obs	502	502	502	502
LLR-A		19.24***	20.60***	26.38***
LLR-B			1.36	7.14***

The results for the full model, shown in Column (A), are first and foremost consistent with the hypothesis that fundamentalists and chartists are active in the S&P500. The fundamentalists' mean reversion coefficient α^* is positive and significant. A speed of mean reversion α^* of 0.030 implies that price is expected to revert to its fundamental at 3.0% per month. The estimate of the decay rate ω is highly significant and close to one (though significantly below one). This indicates that there is a long memory in the moving average rule, or, in other words, that the trend that the chartists are using moves relatively slowly and hence the chartists put relatively little weight on the most recent observation. The estimate of the parameter d^*

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indicates that chartists are significantly present and they are destabilizing in that they extrapolate trends, as hypothesized. The coefficient of 0.051 implies that chartists extrapolate the existing gap between x_t and its trend with 5.1% per month.

The coefficients α^{**} and d^{**} are both not significantly different from zero[18]. More important for determining the added value of the switching mechanism, therefore, is to compare the results for the full model in Column (A) to those of the restricted models in the remaining columns. Column (B) presents the estimation results for the model with the restrictions $\mu = 1$ and $\beta = 0$. The results for the model without switching in Column (B) are highly comparable to the results for the switching model in Column (A) with respect to α^*, d^*, and ω, but with the notable exception that d^* is no longer significant. That is, chartists are no longer present in the market. Generally, the estimates are somewhat weaker in the sense that the significance levels are larger in Column (A) compared to Column (B). As a result, the log likelihood decreases significantly after removing switching[19]. This constitutes significant evidence in favor of the added value of the market maker and the switching between groups. The model in Column (C) is restricted further to only embed fundamentalists. The fundamentalist coefficient α^* is again positive and significant; the model delivers significantly less performance than the full model in (A), but the fit is statistically equal to the model without switching in (B) judging by the LLR-B statistic. The model in Column (D) is restricted to only embed chartists. This version of the model does not yield any significant coefficients. The model fit is significantly worse than that of the models in columns (A) and (B).

Hence, the overall conclusion is that both chartists and fundamentalists are present in the market, and that switching adds significantly to the fit of the model. The fundamentalists are especially important in describing market dynamics.

Table 3 presents estimation results for the two separate sub-samples. The results for the two subsamples are qualitatively similar to those of the full sample estimation. In the left panel of Table 3, we find significant ev-

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[18]This is caused by the functional form of the $tanh$ function in which they are embedded. The expression is a transformation of $\mathbb{R} \to [-1, 1]$. As such, there can be variation in the profit difference $\pi_{f,t} - \pi_{c,t}$ when m_t is equal to one or minus one; variation in the profit functions in these regions has no effect on m_t. Therefore, the standard error of estimation becomes large and the estimates not significant. This is consistent with the findings of Boswijk et al. (2007), who also find significant evidence for the existence of fundamentalists and chartists but cannot reject the null hypothesis of zero intensity of choice. Teräsvirta (1997) shows that the significance of the intensity of choice β is not important as long as the groups are significant and the model fit increases after introducing switching.

[19]Boswijk et al. (2007), consistent with the smooth transition autoregression (STAR) literature, argue that one should conduct a bootstrap F-test in order to reject linearity in such models. In our case, however, this does not hold as the coefficients α and d are still identified under the null hypothesis of linearity, that is, $\beta = 0$, which is not the case in standard STAR models.

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Table 3: Estimation Results for the Subsamples

The table presents the estimation results of different versions of the model given by (25)-(27) for the two subsamples. Column (A) is the full model, Column (B) introduces restrictions $\mu = 1$ and $\beta = 0$, Column (C) examines $\mu = 1$, $\beta = 0$, and $d^* = 0$, Column (D) sets $\mu = 1$, $\beta = 0$, and $\alpha^* = 0$. LLR-A is the test statistic of the likelihood ratio test versus the full model; LLR-B the statistic for comparison between restricted versions of the model. Standard errors are in parentheses, and *, **, and *** respresent significance at the 10%, 5%, and 1% level, respectively.

	1971 - 1991				1992 - 2012			
	(A)	(B)	(C)	(D)	(A)	(B)	(C)	(D)
c	0.131	0.262	0.477	-0.503	3.698	9.169	5.839	-1.560
	(0.850)	(0.835)	(0.836)	(0.811)	(6.424)	(9.806)	(8.188)	(6.375)
α^*	0.028***	0.016	0.006		0.029**	0.044	0.032*	
	(0.009)	(0.019)	(0.012)		(0.011)	(0.029)	(0.019)	
d^*	0.035	0.056		0.038	0.050**	0.054		0.206
	(0.024)	(0.076)		(0.085)	(0.021)	(0.052)		(1.995)
ω	0.873***	0.826***		0.793*	0.920***	0.840***		-0.125
	(0.036)	(0.256)		(0.414)	(0.015)	(0.265)		(1.187)
α^{**}	0.194				0.002			
	(0.680)				(0.002)			
d^{**}	-0.428				-0.003			
	(1.479)				(0.004)			
logl	-954.84	-958.62	-959.34	-959.12	-1458.56	-1462.87	-1463.51	-1465.09
Obs	251	251	251	251	252	252	252	252
LLR - A		7.56**	9.00*	8.56*		8.62**	9.90*	13.06***
LLR - B			1.44	1.00			1.28	4.44**

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idence for stabilizing fundamentalists. The chartist coefficient d^*, however, is (marginally) insignificant. This is explained by the fact that x_t stays relatively close to its fundamental value throughout this subsample. The results from the restricted versions of the model in Columns (B), (C) and (D) indicate that it is especially the switching that is important for model fit. The fit of the full model in Column (A) is significantly higher than that of the restricted models. The fit of the models in columns (B)-(D) do not differ significantly from each other.

The results for the second sub-sample on the right hand side of the table again give significance for both fundamentalists and chartists in the full version of the model in Column (A). Hence, the significance of chartists in the full-sample results are driven by the second part of the sample period, which is characterized by multiple bubble episodes. Column (B) on the right hand side of Table 3 again illustrates the importance of switching for the fit of the model. The likelihood value of the restricted model in Column (B) is significantly lower than in Column (A). In addition, both fundamentalist and chartist coefficients are no longer significant. This indicates that the switching mechanism separates the two groups in the data. Columns (C) and (D) are similar to those for the full sample period: only fundamentalists add explanatory power when considered in isolation.

Table 4: Implied Structural Coefficients

The table presents the structural parameters from the model given by (19)-(21) implied by the estimation results of the reduced-form model given by (25)-(27).

	1971 - 2012	1971 - 1991	1992 - 2012
α	0.00981	0.17665	0.01036
d	0.09877	0.04915	0.10259
μ	1.26070	0.33635	1.13471
β	0.43907	0.54375	0.69162

Table 4 presents the structural parameters derived from the estimated reduced-form models. The implied coefficients are of the expected sign and magnitude, and are consistent throughout the different sample periods. The fundamentalists play a stabilizing role throughout the sample, $\alpha > 0$. This effect is stronger in the first subsample than in the second, which is explained by the bubble episodes in the latter part of the sample. The chartists are destabilizing throughout the sample, $d > 0$. This effect is stronger in the second subsample. The impact of the market maker μ is positive and somewhat smaller for the first subsample sample. Given the sign of the intensity of choice β, the switching function operates as a positive feedback rule, in other words, agents switch towards the more profitable rule. The estimated sensitivity to profit difference is largest for the second subsample. This can be explained by the much higher volatility in prices themselves during the build-up and decline of the IT bubble and is consistent with the estimation

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results in the right panel of Table 3^{20}. All in all, the in-sample estimation results reveal that both groups are active in trading S&P500 stocks and that there is significant switching between the chartist and fundamentalist groups.

4.2   Switching between Strategies

To get a better understanding of the mechanics in the model, this subsection studies the estimated weights and their characteristics.

Figure 2 illustrates the behavior of the distribution of agents over the two groups m_t. The histograms on the left-hand side all show spikes at -1, 0, and $+1$, that is, the market is typically either fully fundamentalist, fully chartist, or split evenly between these two groups. This distribution is especially pronounced in the full sample period. The scatter plots of m_t as a function of the profit difference on the right-hand side display a distinct S-shape, which indicates that traders switch from fundamentalism to chartism and vice-versa when there is a perceived profit opportunity. For the full sample we observe a somewhat less sharp S-shape, which is caused by the somewhat lower estimate of β. This also explains the spike at zero in the histogram for the full sample.

Figure 3 displays the evolution of m_t versus x_t. The left hand figure clearly indicates a strong increase in switching in the second half of the sample. Switching is more agressive in our case compared to, for example, Boswijk et al. (2007). This is explained by the higher data frequency data. Note, however, that this does not necessarily imply that individuals often switch from one group to the other; it is the composition of the market as a whole that changes. To make trends in the switching more clear, the right hand figure displays the 12-months moving average of m_t. The figure indicates that there is a strong relation between the x_t and m_t: whenever x_t is moving towards (away from) zero, that is, when price is moving towards (away from) its fundamental value, m_t is moving upwards (downwards), or the fraction of fundamentalists (chartists) increases. This is exactly the behavior expected from the agents in the model. That is, fundamentalists bring price back to its fundamental value, while chartists extrapolate the current trend.
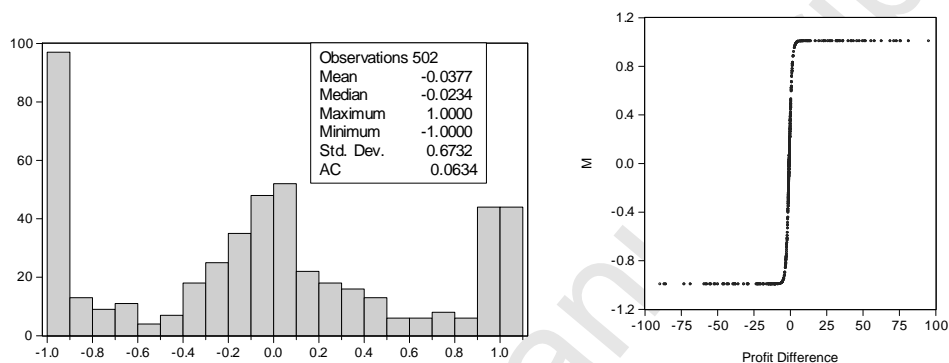
Clearly visible in the figure are the two bubble periods toward the end of the sample. During the late 1990's, m_t is below zero for a prolonged period and at its all-time low at the start of 1998, meaning that chartists are dominating the market. As a result, price is driven far away from its fundamental, or x_t is driven up. The IT bubble is corrected by a strong increase in fundamentalism, illustrated by an increase in m_t to its all-time high in 2002.

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[20]The full-sample coefficients for α and β are not in between those for the sub-samples. This is not strictly necessary due to the nonlinear structure of the model.
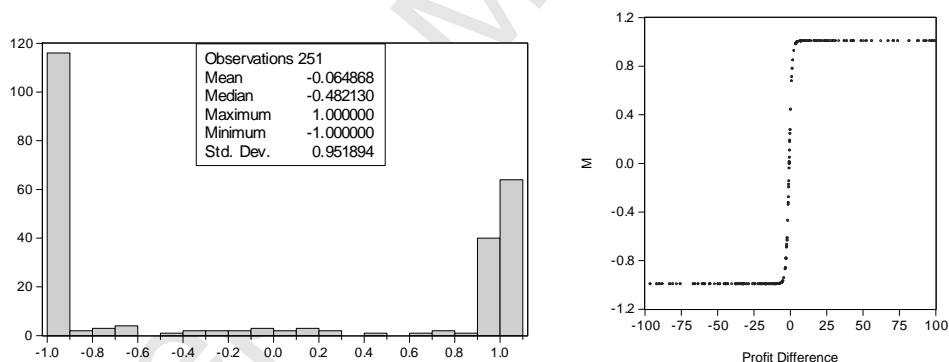
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Figure 2: Estimated Weights

The figure presents histograms and descriptive statistics of the estimated weights m_t (left figures) and the relation between the weight m_t and the profit difference $\pi_{f,t} - \pi_{c,t}$ (right figures).

1971 - 2012

Observations	502
Mean	-0.0377
Median	-0.0234
Maximum	1.0000
Minimum	-1.0000
Std. Dev.	0.6732
AC	0.0634

1971 - 1991

Observations	251
Mean	-0.064868
Median	-0.482130
Maximum	1.000000
Minimum	-1.000000
Std. Dev.	0.951894

1992 - 2012

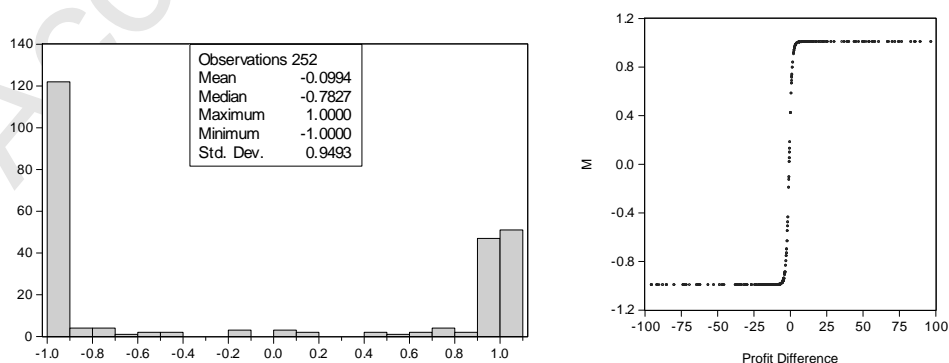| Observations	252
Mean	-0.0994
Median	-0.7827
Maximum	1.0000
Minimum	-1.0000
Std. Dev.	0.9493

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Figure 3: Estimated Weights and Price Deviations for the Full Sample
The figure presents the estimated weights (right scale) and the price deviation from its fundamental x_t (left scale). The left hand side figure displays the monthly weights m_t whereas the right hand side figure displays 12-months moving averages of m_t.
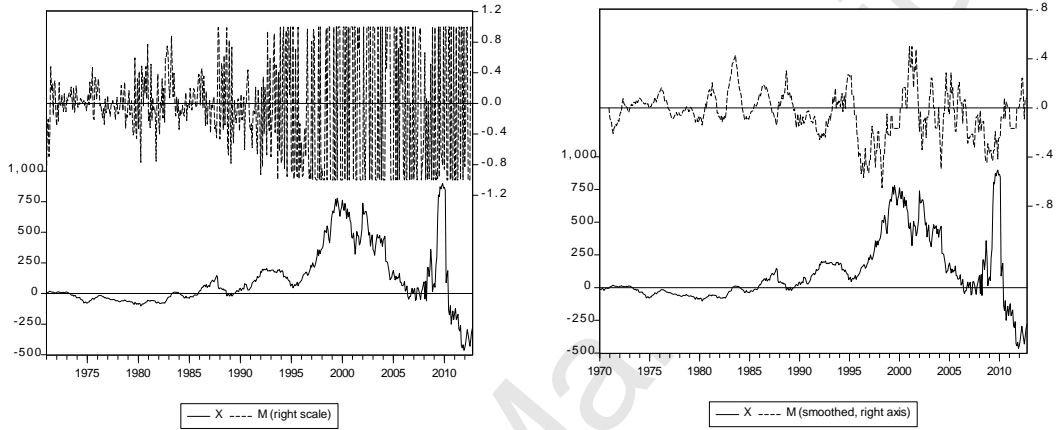
Interestingly, the recent credit crisis shows a completely different mechanism. In contrast to the IT bubble, the fundamental price increased together with the market price before the outbreak of the crisis. In addition, the fundamental value drops together with the market price starting in 2007. As a result, x_t remains close to zero and we observe no particularly large changes in the composition of the market. It is only in 2009 that we observe that chartists are introducing an upward trend in the market price, which is enforced by the sharp decrease in the fundamental price. This is illustrated by a sharp drop in m_t. These observations are similar to the results in De Jong et al. (2010), who focus on the European Monetary System. Hence, the inflation and deflation of bubbles and crises can be explained directly by our heterogeneous agent model in combination with switching weights.

5   Investor Behavior

In this section we study the stability of the nonlinear system as it is estimated in Section 4. First we look at the deterministic skeleton by means of a deterministic simulation setup, that is, we run a simulation without stochastic noise, in order to shed light on the behavior of agents and to verify whether the market is deterministically stable in the long run. Next, we run a stochastic simulation to study the HAM's stochastic characteristics.

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Parameter settings of the simulations are given by the total sample estimates from Table 3; the starting values are set to the medians of the empirically observed values. The complete set of parameters and starting values are presented in Table 5. Since the estimated intercept c is not significantly different from zero, we put it equal to zero in the simulations. As a result, the equilibrium value of the model is zero as well, which can be seen by fulfilling $x_t = x$ in (25).

Table 5: Simulation Coefficients and Starting Values
The table presents the coefficient set and starting values used in both the deterministic and stochastic simulations.

| Coefficients		Starting values	
c	0.000	x_0	27
α^*	0.030	τ_0	27
d^*	0.051	m_0	0
ω	0.919		
α^{**}	0.002		
d^{**}	0.003		

5.1 Deterministic Behavior

Chiarella and He (2003) study extensively the dynamic properties of the HAM with a market maker, and determine the stability regions of the parameter set. To determine the stability of the system estimated for the S&P500, we run a deterministic simulation. To be more specific, we simulate the system given by (25)-(27) without noise, so that $\varepsilon_t = 0$. Figure 4 shows the first 150 periods of the simulation.

The first conclusion to be drawn from Figure 4 is that the model with the current set of coefficients is stable; x_t converges to the fixed equilibrium point. This is consistent with the results of He and Li (2007, 2008), who find that the parameter set of a HAM that produces long memory is generally in the stable region.

In the early periods, m_t jumps from its starting value of zero to 0.0225, in other words, the fundamentalist presence increases. This is induced by the fact that initial demand by chartists is equal to zero since $x_0 = \tau_0$. As a result, fundamentalists completely determine the price change and consequently make the biggest profit. In subsequent periods m_t gradually moves towards zero as chartists, like fundamentalists, forecast a price decline because $\tau_t > x_t$ and $x_t > 0$. Since both groups forecast the correct direction of change, they eventually share the market fifty-fifty. Stability in this setup is guaranteed by the fact that fundamentalists will push x_t toward zero, while τ_t responds slowly to changes in x_t; as a result, x_t is always closer to zero than τ_t.

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Figure 4: Deterministic Simulation
The figure illustrates the behavior of x_t, τ_t, and m_t
in a deterministic simulation setup.
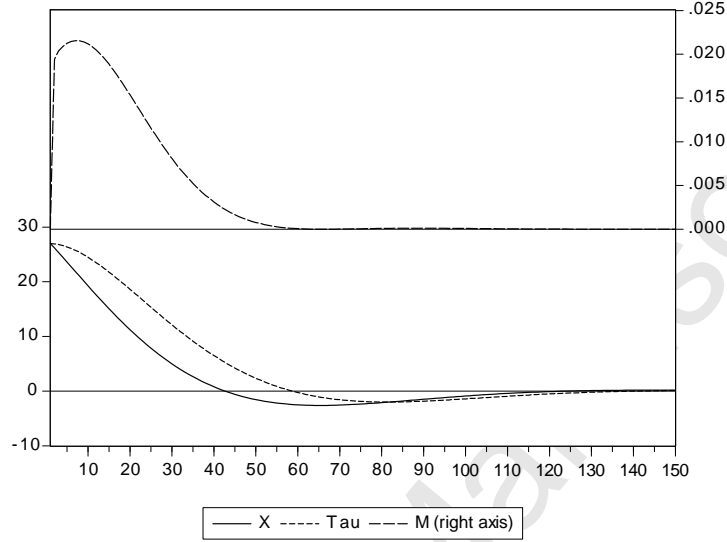
5.2 Stochastic Behavior

By including noise, it becomes possible to study the stochastic properties of the model. To conduct this analysis, we run a simulation of 1,000 periods and examine the properties of the generated series. Noise is introduced by generating a random normal process ε_t, with the same variance as the residuals from the full-sample estimation of the full model. Figure 5 presents one typical series.

Panel (A) of Figure 5 displays properties of the full sample of simulated x_t together with the weight difference m_t. We note that x_t oscillates around zero throughout the sample, while the price deviation from its fundamental can be large (up to 800 in this particular example) and long-lasting (typically up to a hundred periods). It is important to note that the model is determining the long-run behavior of the market and not the residual term. In other words, the explanatory power of the model as estimated in the previous section is substantial. The weights appear to bounce up and down from minus one to plus one; not a single group is dominant for long periods on end. Note that this does not imply that individuals change strategy and trade accordingly on a weekly basis, as weights do not indicate behavior of individuals, but rather the contemporaneous composition of the market. Panel (B) of Figure 5 confirms this impression by presenting a close-up of periods 1 to 100 from Panel (A). The chartist geometric decay process τ_t,

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Figure 5: Stochastic Simulation
The figure presents one simulated series of the model. Panel (A) represents the time-series of simulated m_t (right axis) and x_t (left axis). Panel (B) is a close-up of Panel (A) of periods 1 until 100, with τ_t included, and Panel (C) presents the histogram of simulated weights m_t.

(A)

(B)

(C)
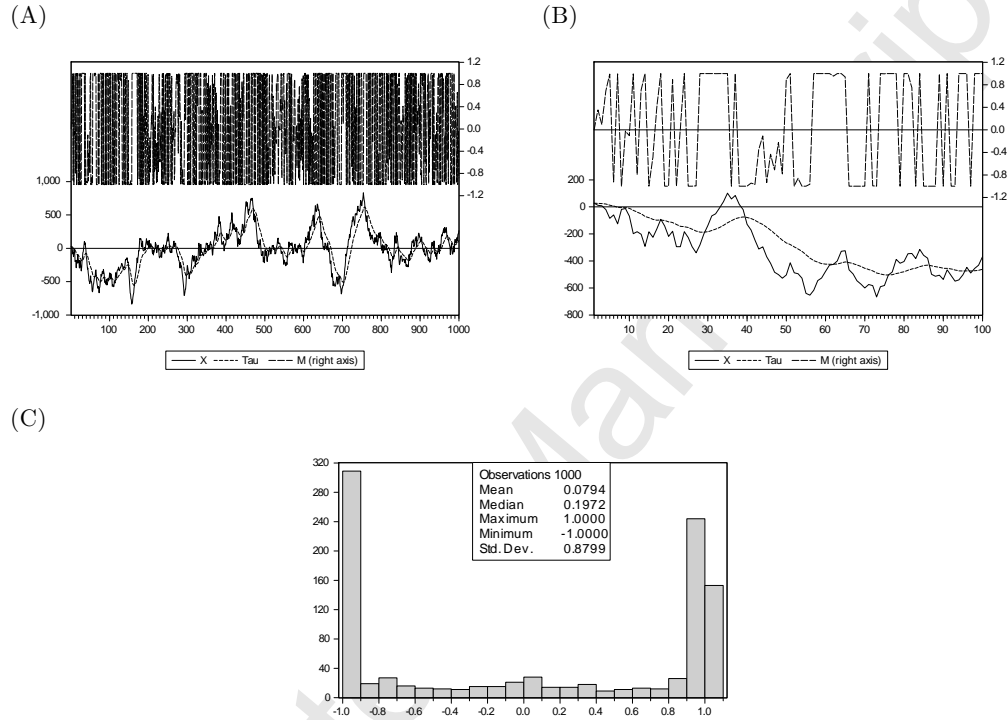
which now becomes visible, is a smoothed value of the average of x_t. Large swings in m_t coincide with changes in x_t itself. There are periods in which chartists or fundamentalists are dominant for several months. For example, around period 30 and between periods 60 and 70 fundamentalists dominate for a number of months as the market moves towards its fundamental value (i.e., x_t moves towards zero). In Panel (C) of Figure 5 we can see the histogram of weights m_t. Comparable to the estimated weights in the previous section, the two spikes around minus one and plus one are again visible.

6   Conclusion

Warren Buffet stated during a shareholder meeting of Berkshire Hathaway in 2006 that *"[...] it's like most trends: At the beginning, it's driven by fundamentals, then speculation takes over. As the old saying goes, what the wise man does in the beginning, fools do in the end. With any asset class*

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that has a big move, first the fundamentals attract speculation, then the speculation becomes dominant. The heterogeneous agent literature attempts to capture this market characteristic of asset pricing by introducing a more realistic and empirically sound alternative to the classical representative agent model with rational expectations. A number of papers in the heterogeneous agents literature have shown that such models are capable of replicating the stylized facts of financial market data. However, until recently, this result was mainly established using simulation techniques. This paper has attempted to bring a heterogeneous agent model to the data in search of direct empirical support. For the S&P500, we find significant evidence of the presence of fundamentalists, who expect mean reversion, and chartists, who are destabilizing vis-à-vis a geometric decay function. In addition, we show that the model fit benefits especially from allowing agents to switch between the chartist and fundamentalist groups based on their relative profitability. By using this setup, the model is able to explain bubble situations, such as the IT bubble in the late 1990's.

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