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Diana Barro, Elio Canestrelli, Fabio Lanza

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Volatility vs. downside risk: optimally protecting against drawdowns and maintaining portfolio performance

Diana Barro
Elio Canestrelli
Fabio Lanza
University of Venice at Ca' Foscari

Abstract

As a consequence of recent market conditions an increasing number of investors are realizing the importance of controlling tail risk to reduce drawdowns thus increasing possibilities of achieving long-term objectives.
Recently, so called volatility control strategies and volatility target approaches to investment have gained a lot of interest as strategies able to mitigate tail risk and produce better risk-adjusted returns. Essentially these are rule-based backward looking strategies in which no optimization is considered. In this contribution we focus on the role of volatility in downside risk reduction and, in particular, in tail risk reduction.
The first contribution of our paper is to provide a viable way to integrate a target volatility approach, into a multiperiod portfolio optimization model, through the introduction of a local volatility control approach.
Our optimized volatility control is contrasted with existing rule-based target volatility strategies, in an out-of sample simulation on real data, to assess the improvement that can be obtained from the optimization process. A second contribution of this work is to study the interaction between volatility control and downside risk control. We show that combining the two tools we can enhance the possibility of achieving the desired performance objectives and, simultaneously, we reduce the cost of hedging. The multiperiod portfolio optimization problem is formulated in a stochastic programming framework that provides the necessary flexibility for dealing with different constraints and multiple sources of risk.

Keywords
Volatility, tail risk, stochastic programming, risk management.

JEL Codes
C61, C63, D8

Address for correspondence:
Diana Barro
Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta S.Giobbe
30121 Venezia - Italy
Phone: (++39) 041 2346938
Fax: (++39) 041 2349176
e-mail: d.barro@unive.it

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Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta San Giobbe
30121 Venice Italy
Fax: ++39 041 2349210

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1 Introduction

As a consequence of recent market conditions and financial turmoil an increasing number of investors realized the importance of portfolio strategies which allow to better control risk, in particular tail risk, and to reduce drawdowns in portfolio returns. The application of these strategies enhances possibilities of achieving long-term objectives increasing benefits from compounding.

Two features of market crisis periods, which are particularly relevant for risk management, are the breakdown in correlations and the presence of significant spikes in volatilities (see, for example, [27][20]). Moreover, in the present economic context the persistence of low yields on long term bond causes a further source of financial stress for institutional investors, such as pension funds and insurance companies, due not only to the solvability issue but also to regulatory requirements which are increasingly strict and tend to penalize volatile asset holdings.

In this financial context, conventional risk management strategies for asset allocation, relying on diversification and balanced stock/bond approaches, do not prove to be effective. In particular, in severe downturns, diversification in not capable of dealing with loss control. We refer to [1] for a discussion on the benefits and limitations of diversification as a risk management tool in a post-crisis perspective.

These observations motivate investors to consider different protection strategies to effectively control risk and, in particular, risk of extreme losses.

Hedging against tail risk is quite a complex problem since there can be many different causes and a full protection can be highly expensive either due to the costs inherent with the purchase of an insurance over a long period, or due to the implementation of a too defensive investment strategy, missing market upside.

Derivative-based strategies, such as protective put and option-based portfolio insurance, suffer from the first problem; while, active equity management strategies, including defensive equity, low beta alternative investments, constant proportion portfolio insurance and stop loss strategies, may fail to provide interesting return during bull market periods.

A more effective approach should be able to mitigate potential consequences of tail risk events without a significant drag in the portfolio performance.

We note that tail risk events are typically associated with a heightened volatility; moreover, a high volatile environment increases the costs of risk protection strategies. These observations support the idea of using volatility for tail risk mitigation.

Recently, many investment institutions have successfully proposed products based on volatility control strategies, or volatility target approaches, to address periods of severe market distress. These risk-control strategies and indexes are based on portfolio de-leveraging driven by the increase in volatility. The goal is to keep a desired and fixed level of volatility, that is a constant level of risk exposure.

Hocquard et al. [23] proposes an innovative approach, based on the payoff distribution model, to control risks related to higher moments of the return's distribution and to target constant level of portfolio volatility.

Other risk management approaches consider the inclusion of volatility as an asset class in the portfolio, or add overlays to the risky portfolio through derivatives based on the volatility (see, for example, [8][11]). The use of volatility for diversification purposes relies on the inverse relationship between equity returns and their volatility which tend to move

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in opposite directions. Moreover, this negative correlation seems to be stronger in period of market downturns. For empirical evidence and economic foundations of this behavior see, for example, [12][32]. For an extended analysis of the use of volatility derivatives in equity portfolio management we refer to [22].

Furthermore, different contributions have proven the importance of volatility as a risk-conditioning factor in portfolio allocation. Fleming et al. [19] studied the economic value of volatility timing. Busse [10] empirically analyzed the positive correlation between mutual fund returns and volatility timing.

Finally, there is substantial evidence of the so-called low-volatility anomaly, showing that low-volatility portfolios historically have offered higher realized average returns (see, for example, [2][3][4][5][9]).

This anomaly is discussed in Xiong et al. [36] which investigates several risk measures, including volatility and tail risk, and found that tail risk is compensated with higher expected return while volatility seems to be not compensated in equity funds.

Given the importance of volatility in investment decisions (see, for example, [11][19][34]) and the debate on the advantages and shortcomings of volatility (see, for example, [15][16] [36]), in this contribution we focus on the role of volatility in downside risk reduction and, in particular, in tail risk reduction.

We tackle the problem from a dynamic portfolio management perspective, integrating different forms of risk control into a multiperiod optimization problem.

The first contribution of our paper is to provide a viable way to integrate a target volatility approach into the multiperiod portfolio optimization model. This is obtained through the introduction of a local volatility control approach, that is a control of the variability in each node of the scenario tree.

Our optimized volatility control is contrasted with existing rule-based target volatility strategies, in an out-of sample simulation on real data, to assess the improvement which can be obtained from the optimization process.

Controlling the volatility impacts also on the kurtosis of the return's distribution, and, to same extent, it provides a tail risk reduction. However, in our opinion, a stand alone volatility control is not sufficient to prevent the portfolio from suffering huge losses in extreme market downside movements. Thus, we propose to integrate a pure tail risk control term in our portfolio model.

A second contribution of our approach is to study the interaction between volatility control and downside risk control. We show that combining the two tools it is possible to enhance the possibility of achieving long term objectives and simultaneously to reduce the cost of hedging.

To these aims we extend [7], allowing for the simultaneous inclusion of tracking error, volatility controls and tail risk controls. The model considered is characterized by a high degree of flexibility in the choice of the investment goals in terms of returns, downside protection and volatility control. It belongs to in the wide family of asset and liability management models (see, for example, [30][38]) where we have a, possibly stochastic, reference entity, which allows us to include a tracking component and a control on downside risk through the presence of relevant thresholds.

The model can be interpreted in terms of different layers of protection which can be activated to respond to different demands for downside risk protection.

One of the main advantages of our approach is that it can be easily understood, in terms of

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risk management, as consisting of two main parts. The manager first sets a benchmark for the investment process fixing, in this way, a target return distribution for the portfolio and then activates two different forms of controls to properly re-shape this distribution according to risk tolerance or risk protection goals. Acting on volatility and tail risk we are essentially capable of modifying skewness and kurtosis of the return distribution. This allows a global view on the risk profile of the resulting portfolio rather than contrasting return with a particular measure of risk.

Our model supports the idea that a diversification of risk reduction strategies produces a better risk mitigation while controlling the overall cost. Moreover, it provides a framework in which different strategies can be optimally combined to suit the investor's attitude towards different forms of risk (failure to achieve a target return, huge versus moderate magnitude of losses, etc.).

The paper is organized as follows. In Section 2 we present the formulation of our model. Section 3 provides the results of a extensive set of out-of-sample simulation experiments applying the proposed model to real data. We discuss the obtained results in terms of volatility control and downside risk control, focusing on their joint effects. Furthermore, we present the result of performance measurement and sensitivity analysis with respect to different risk attitudes. Section 4 concludes.

2 Model formulation

We analyze a multiperiod problem formulating it as a multistage portfolio management problem in the flexible framework of stochastic programming in which we can easily deal with multiple correlated sources of risk, different risk controls, and dynamic portfolio rebalancing.

In the following, we introduce notation, parameters and variables for our multiperiod optimization problem. The asset allocation decisions are taken at discrete time stages, in which the manager can choose between n risky assets and a riskless one (liquidity component). We consider a multistage stochastic programming problem in arborescent formulation, that is with implicit non-anticipativity constraints.

The tree structure is described through the set of nodes at each time, $\mathcal{K}_t = \{K_{t-1} + 1, \ldots, K_t\}$, $t = 1, \ldots, T$, where T is the time horizon for the investment problem. We denote with $k_t \in \mathcal{K}_t$ a node in the tree at time t, with $t = 1, \ldots, T$, while $k_0 = 0$ denotes the root node.

The set of node-specific parameters used in the model construction are

$b(k_t)$, the ancestor node k_t;

π_{k_t}, the probability associated to node k_t;

$r_{i\,k_t}$, the return on risky asset i, for the period $[t-1, t]$ in node k_t, with $i = 1, \ldots, n$;

$r_{l\,k_t}$, the return on risk-less asset, for the period $[t-1, t]$ in node k_t;

x_{k_t}, value of the risky benchmark in node k_t.

with $k_t \in \mathcal{K}_t$ and $t = 1, \ldots, T$.

The set of parameters related to the dynamic rebalancing are

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κ^+, the transaction cost of purchase of risky assets;

κ^-, the transaction cost of sale of risky assets;

and to the control of risk

α_{TE}, coefficient for the tracking error term in the objective function;

α_{VC}, coefficient for the volatility term in the objective function;

α_{TR}, coefficient for the tail risk term in the objective function;

σ, volatility control parameter;

ρ, tail risk parameter.

Let us introduce the set of admissible decisions, $\mathcal{D} = \times_{t=0}^{T} \mathcal{D}_t$ for the multiperiod optimization problem. We denote with $d \in \mathcal{D}$ an admissible decision, where $d = (d_0, d_1, \ldots, d_T)$, with $d_0 = (l_0, q_0, y_0)$ is the set of initial decision variables, and $d_t = (l_t, q_t, y_t, a_t, v_t)$ is the set of decision variables at time t. The decision variables at time 0 are

l_0, the initial cash holdings;

$q_0 = (q_{10}, \ldots, q_{n0})$, with q_{i0} the initial holdings for the i-th risky asset, $i = 1, \ldots, n$;

y_0, the initial value of the portfolio.

The decision variables at time t, $t = 1, \ldots, T$ are

$l_t = (l_{K_{t-1}+1}, \ldots, l_{K_t})$, where l_{k_t} is the cash amount in node $k_t \in \mathcal{K}_t$;

$q_t = (q_{1\,K_{t-1}+1}, \ldots, q_{1\,K_t}, \ldots, q_{n\,K_{t-1}+1}, \ldots, q_{n\,K_t})$, with $q_{i\,k_t}$ the amount invested in the i-th risky asset in node $k_t \in \mathcal{K}_t$;

$y_t = (y_{K_{t-1}+1}, \ldots, y_{K_t})$, the value of the portfolio in node $k_t \in \mathcal{K}_t$;

$a_t = (a_{1\,K_{t-1}+1}, \ldots, a_{1\,K_t}, \ldots, a_{n\,K_{t-1}+1}, \ldots, a_{n\,K_t})$, with $a_{i\,k_t}$ the amount of asset i, $i = 1, \ldots, n$, purchased in node $k_t \in \mathcal{K}_t$;

$v_t = (v_{1\,K_{t-1}+1}, \ldots, v_{1\,K_t}, \ldots, v_{n\,K_{t-1}+1}, \ldots, v_{n\,K_t})$, with $v_{i\,k_t}$ the amount of asset i, $i = 1, \ldots, n$, sold in node $k_t \in \mathcal{K}_t$.

2.1 Objective function

The objective function of our problem integrates three terms which represent different forms of control on different aspects of risk. A first term accounts for the presence of a reference portfolio for the investment process. The benchmark helps in identifying not only a target risk profile for the portfolio but also a performance goal. Secondly, we include a volatility control term which acts reducing the variability of the portfolio values thus stabilizing the wealth path. Finally, we include a tail risk control which aims at preventing the portfolio from experiencing extreme losses.

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These controls can be activate simultaneously as well as separately and properly choosing the weights assigned to each of them we can obtain different trade-offs. This structure of the problem allows us to study the combined effects of different forms of risk control on the shape of the return distribution and to identify the trade-offs between upside capture and different risk controls.

The objective function of the proposed multistage portfolio management problem is

$$\min_{d \in \mathcal{D}} \quad \sum_{t=0}^{T} \sum_{k_t=K_{t-1}+1}^{K_t} \pi_{k_t} \left[\alpha_{TE} \ \phi_{TE}(y_{k_t}, x_{k_t}) + \alpha_{VC} \ \phi_{VC}(y_{k_t}, \sigma) + \alpha_{TR} \ \phi_{TR}(y_{k_t}, \rho) \right] \tag{1}$$

The first term, $\phi_{TE}(y_{k_t}, x_{k_t})$, represents a distance measure between the managed portfolio and the index, and it allows us to include a tracking error component. The second term, $\phi_{VC}(y_{k_t}, \sigma)$, introduces a control for the volatility of the managed portfolio. Finally, the third term, $\phi_{TR}(y_{k_t}, \rho)$, accounts for a control on the extreme downside risk, i.e. tail risk.

In this contribution we choose to use mean absolute deviation as measure of distance for all the terms in the objective function. This choice allows us to accommodate for both symmetric and asymmetric distance measures. Moreover, it allows us to rewrite our multistage stochastic programming problem as a linear stochastic programming problem. Nevertheless, different distance measures can be considered and, in particular, other choices for tail risk measures can be considered still maintaining the linearity of the resulting optimization problem. See [28] for a review of static portfolio optimization models which can be casted as linear programming problems.

In the following, we will discuss in detail each term in the objective function and its specification using mean absolute deviation as distance measure.

2.1.1 Tracking error control

The introduction of the benchmark term helps in defining the framework in which the portfolio management process takes place. The benchmark can be either an equity index, or a reference balanced index or a liquidity reference index according to different goals of the investment. The tracking can be carried out using either a symmetric or asymmetric distance measures.

The use of absolute deviation measures in portfolio model is well known in the literature. For the static case, see, for example, [26][14][25][31][35]. For dynamic replication problems see, for example, [6][17][21]. Finally, we refer to [37] for a detailed discussion on mean absolute deviation models and tracking error models using stochastic programming.

In more detail, in the case of symmetric mean absolute deviation we have

$$\phi_{TE}(y_{k_t}, x_{k_t}) = |y_{k_t} - x_{k_t}| \,. \tag{2}$$

While, if we consider an asymmetric distance measure we penalize only downside deviations as follows

$$\phi_{TE}(y_{k_t}, x_{k_t}) = |y_{k_t} - x_{k_t}|^- = \max\left\{0, x_{k_t} - y_{k_t}\right\}. \tag{3}$$

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2.1.2 Local volatility control

With regards to the second goal, we propose to manage the variability of portfolio values using a local volatility control approach. We consider a corridor built using two path-dependent barriers inside which we allow the portfolio to move. We set a lower barrier $z_{t\,LO}$ and an upper barrier $z_{t\,UP}$ and consider the distances of the managed portfolio from the barriers.

The volatility control term can be written as

$$\phi_{VC}(y_{k_t}, \sigma) = \phi_{LO}(y_{k_t}, z_{t\,LO}, \sigma) + \phi_{UP}(y_{k_t}, z_{t\,UP}, \sigma) \tag{4}$$

where σ is a parameter which sets the width of the corridor. This approach is flexible and allows us to consider also asymmetric distance measures from the barriers modulating the variability control. In particular, we require the portfolio to stay inside the corridor possibly occupying the upper part of it. The control is carried out through all nodes in the event tree thus obtaining a local control at each time step rather than a control on the volatility of the entire period.

We propose to obtain the desired control minimizing negative deviations from the lower barrier using an symmetric distance measure while, at the same time, minimizing distance from the upper barrier using a symmetric measure. This will produce the effect of forcing the portfolio to move inside the corridor above the lower barrier and as close as possible to the upper limit.

$$\phi_{VC}(y_{k_t}, \sigma) = |y_{k_t} - z_{t\,LO}|^{-} + |y_{k_t} - z_{t\,UP}| \tag{5}$$

where $z_{t\,LO}$ and $z_{t\,UP}$ are path-dependent barriers set as follows

$$z_{t\,LO} = y_{b(k_t)} - \sigma \tag{6}$$

$$z_{t\,UP} = y_{b(k_t)} + \sigma \tag{7}$$

where σ is a parameter which determines the width of the corridor and thus the allowed variability for the portfolio values. We can allow for a time varying width parameter, σ_t, accounting for different market volatility levels (for example, forecasted volatility or implied volatility) in order to obtain a more customized control of the variability of the portfolio.

The definition of path-dependent barriers allows the portfolio to follow the trend in the market and capture upside potential when the market is in a positive phase. On the reverse situation, this means that the portfolio can follow also the negative trend of the market and the control on the volatility is not sufficient to prevent it from experiencing extreme losses.

2.1.3 Tail risk control

To avoid extreme downside risk is properly the role of the tail risk control term. To this aim, we introduce, in the third term of the objective function, a shortfall control which penalizes losses in excess of a given threshold. The goal is to minimize, at all time steps, downside deviations from a given threshold which may represent, for example, a maximum acceptable loss.

For the use of an extension of the mean absolute deviation model to penalize larger downside deviations, in the static case, see [29].

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To this aim we consider a threshold, ρ, and an asymmetric distance measure which accounts for negative deviations from it. The resulting distance measure is

$$\phi_{TR}(y_{k_t}, \rho) = |y_{k_t} - \rho|^{-} = \max\{0, \rho - y_{k_t}\} \tag{8}$$

The model can easily accommodate for a time varying threshold, ρ_t, thus allowing for a risk tolerance levels changing with the market conditions.

The proposed approach to deal with different form of risk can be easily casted into the optimization framework and provides a direct interpretation in terms of replication of the benchmark, portfolio variability and downside deviation from the loss threshold.

Given the scenario tree description of the uncertainty and the arborescent formulation of the stochastic programming problem, the mean absolute deviation measures become LP computable. Applying a widely used technique in mathematical programming, we can write the absolute value as the sum of two positive variables, denoting the positive and negative deviations, respectively; in this way the problem can be rewritten as a linear programming problem, adding the necessary constraints. In more detail, we define

$$\theta^{+}_{k_t} = |y_{k_t} - x_{k_t}|^{+} = \max\{0, y_{k_t} - x_{k_t}\} \tag{9}$$

$$\theta^{-}_{k_t} = |y_{k_t} - x_{k_t}|^{-} = \max\{0, -y_{k_t} + x_{k_t}\} \tag{10}$$

$$\eta^{-}_{k_t} = |y_{k_t} - z_{t\,LO}|^{-} = \max\{0, -y_{k_t} + z_{t\,LO}\} \tag{11}$$

$$\gamma^{+}_{k_t} = |y_{k_t} - z_{t\,UP}|^{+} = \max\{0, y_{k_t} - z_{t\,UP}\} \tag{12}$$

$$\gamma^{-}_{k_t} = |y_{k_t} - z_{t\,UP}|^{-} = \max\{0, -y_{k_t} + z_{t\,UP}\} \tag{13}$$

$$\nu^{+}_{k_t} = |y_{k_t} - \rho|^{+} = \max\{0, y_{k_t} - \rho\} \tag{14}$$

$$\nu^{-}_{k_t} = |y_{k_t} - \rho|^{-} = \max\{0, -y_{k_t} + \rho\} \tag{15}$$

The resulting linear objective function is

$$\min \quad \sum_{t=0}^{T} \sum_{k_t=K_{t-1}+1}^{K_t} \pi_{k_t} \left[\alpha_{TE}\left(\theta^{+}_{k_t} + \theta^{-}_{k_t}\right) + \alpha_{VC}\left(\eta^{-}_{k_t} + \gamma^{-}_{k_t} + \gamma^{+}_{k_t}\right) + \alpha_{TR}\ \nu^{-}_{k_t}\right] \tag{16}$$

The proposed model is flexible and the tracking term can be substituted with a pure performance goal as well as the tail risk control term can be relaxed and, properly setting the relevant loss levels, can act as a downside risk control reducing the stress on extreme losses. We choose to include a tracking error term, instead of a return goal, since, in our opinion, this adds a further dimension to the risk control process and it can be of interest for institutional investors.

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2.2 Constraints

At each point in time, $t = 1, \ldots, T$, the admissible decision set $\mathcal{D}_t^{(1)}$ is defined through the following constraints.

- Asset balance constraints

$$
\begin{aligned}
& q_{i\,0} = \bar{q}_i && (17)\\
& q_{i\,k_t} = (1 + r_{i\,k_t}) \left[q_{i\,b(k_t)} + a_{i\,b(k_t)} - v_{i\,b(k_t)} \right] && (18)\\
& i = 1, \ldots, n. && (19)
\end{aligned}
$$

- Cash balance constraints

$$
\begin{aligned}
& l_0 = \bar{l} && (20)\\
& l_{k_t} = (1 + r_{l\,k_t}) \left[l_{b(k_t)} - \sum_{i=1}^{n} \kappa^+ \, a_{ib(k_t)} + \sum_{i=1}^{n} \kappa^- \, v_{ib(k_t)} \right] && (21)
\end{aligned}
$$

- Wealth constraints

$$
\begin{aligned}
& y_0 = l_0 + \sum_{i=1}^{n} q_{i\,0} && (22)\\
& y_{k_t} = l_{k_t} + \sum_{i=1}^{n} q_{i\,k_t} && (23)
\end{aligned}
$$

- Short sale and borrowing constraints

$$
\begin{aligned}
& l_0 \geq 0 && (24)\\
& q_{i\,0} \geq 0 && (25)\\
& l_{k_t} \geq 0 && (26)\\
& q_{i\,k_t} \geq 0 && (27)\\
& a_{i\,k_t} \geq 0 && (28)\\
& v_{i\,k_t} \geq 0 && (29)\\
& && (30)
\end{aligned}
$$

Moreover, the choice of mean absolute deviation and mean absolute downside deviation as distance measures in the objective function allows us to recast the problem into a linear optimization problem provide that we add the following constraints.

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- Constraints for LP computability

$$\theta^+_{k_t} - \theta^-_{k_t} = y_{k_t} - x_{k_t} \tag{31}$$
$$\eta^+_{k_t} - \eta^-_{k_t} = y_{k_t} - z_{t\,LO} \tag{32}$$
$$\gamma^+_{k_t} - \gamma^-_{k_t} = y_{k_t} - z_{t\,UP} \tag{33}$$
$$\nu^+_{k_t} - \nu^-_{k_t} = y_{k_t} - \rho \tag{34}$$
$$\theta^+_{k_t} \geq 0 \tag{35}$$
$$\theta^-_{k_t} \geq 0 \tag{36}$$
$$\eta^+_{k_t} \geq 0 \tag{37}$$
$$\eta^-_{k_t} \geq 0 \tag{38}$$
$$\nu^+_{k_t} \geq 0 \tag{39}$$
$$\nu^-_{k_t} \geq 0 \tag{40}$$
$$\tag{41}$$

In particular, given the definition of $z_{t\,LO}$ and $z_{t\,UP}$, constraints (32) and (33) can be written as

$$\eta^+_{k_t} - \eta^-_{k_t} = y_{k_t} - y_{b(k_t)} + \sigma \tag{42}$$
$$\gamma^+_{k_t} - \gamma^-_{k_t} = y_{k_t} - y_{b(k_t)} - \sigma. \tag{43}$$

2.3 Volatility constraints

Furthermore, to better analyze the role of volatility control in downside protection we consider also a different way of controlling local volatility. To this aim, we introduce a second model in which the control is pursued using a set of constraints rather than through the inclusion of a term in the objective function. The resulting model has a tracking goal as well as a goal of minimizing tail risk while satisfying the volatility constraints.

The admissible decision set at time t for this model as $\mathcal{D}^{(2)}_t = \mathcal{D}^{(1)}_t \bigcap \mathcal{D}^*_t$, where $\mathcal{D}^*_t$ is defined by a set of constraints to control local volatility as follow

$$|y_{k_t} - y_{b(k_t)}| \leq s \tag{44}$$

where s is the parameter that characterizes the desired level of control. The inequality constraints are then transformed into pairs of inequalities as follows

$$y_{k_t} - y_{b(k_t)} \geq -s \tag{45}$$
$$y_{k_t} - y_{b(k_t)} \leq s \tag{46}$$

The resulting optimization problem is characterized by the presence of a tracking error term and a tail risk term in the objective function (as described in previous section) and by the additional set of constraints (45)-(46) to control volatility. The constraints guarantee a stricter control since they cannot be violated by the optimal solution at any degree, and this

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allows us to check for the loss in terms of upside potential without the need of trading it off with the volatility control objective. Moreover, this approach allows us to compare the costs for tail risk hedging with and without the presence of a control on volatility.

3 Empirical analysis

In order to asses the effects of the combined presence of the three forms of risk control in the proposed model we carry out an out-of-sample simulation of the management process on real data.

We focus on the relationship between downside protection and volatility control and consider different configurations for the objective function and the constraints. In particular, we study the following models

- Model 1: Tracking error control, volatility control and tail risk control are included in the objective function ($\alpha_{TE} \neq 0$, $\alpha_{VC} \neq 0$, $\alpha_{TR} \neq 0$ and admissible decision set $\mathcal{D}_t^{(1)}$, for $t = 1, \ldots, T$).
- Model 2: Tracking error and tail risk controls are included in the objective function, while volatility control is introduced in the constraints ($\alpha_{TE} \neq 0$, $\alpha_{VC} = 0$, $\alpha_{TR} \neq 0$ and admissible decision set $\mathcal{D}_t^{(2)}$, for $t = 1, \ldots, T$).

For each model we consider different specifications of the objective function considering different trade-offs between the risk controls.

3.1 Data description and scenario generation

We have considered a weekly dataset from January 1, 2000 to December 27, 2013. The benchmark is the S&P500 Index, while the risky assets used for the tracking problem are the S&P Sector Indexes. In table 1 we provide a summary of the statistics for the benchmark and the sector indexes in the period considered. The overall period negative skewness and the excess of kurtosis suggest the usefulness of risk mitigation strategies.

A key point in multiperiod portfolio management problems is a proper description of uncertainty. The stochastic component of the model in our problem is represented by the vector of returns for the index and the risky assets. In order to include such a component into the optimization model we need to reduce its description to a proper scenario tree. This step usually involves, first, the estimation of a model to describe the multivariate risk process and then a suitable procedure to obtain a discrete representation of it. There is a clear trade-off between the accuracy of the representation and the computational tractability of the resulting optimization problem. Many recent contributions in the related literature deal with the issue of optimally reducing the dimension of a scenario tree still maintaining as much information as possible.

In this contribution, we assume stationarity of the process and apply historical bootstrapping to generate future returns. We partially account for correlation among assets and between each asset and the risky benchmark in an indirect way preserving the co-movements between the returns, since the random sequences are applied across all the risky assets and the benchmark to extract the sampled returns.

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	Mean	Std. Dev.	Ann.Std.Dev.	Skewness	Ex. Kurtosis
S&P 500	0.0003	0.0263	0.1898	-0.8248	6.7530
COND	0.0008	0.0318	0.2295	-0.4074	4.9501
CONS	0.0009	0.0205	0.1482	-1.3324	11.9840
ENRS	0.0014	0.0341	0.2461	-1.2118	8.0374
FINL	-0.0001	0.0420	0.3033	-0.0638	11.6080
HLTH	0.0008	0.0242	0.1747	-1.0336	9.1043
INDU	0.0007	0.0306	0.2210	-0.5292	4.0025
INFT	-0.0003	0.0391	0.2825	-0.5651	3.5599
MATR	0.0007	0.0346	0.2496	-0.4906	2.8557
TELS	-0.0009	0.0307	0.2218	-0.4530	5.6900
UTIL	0.0003	0.0272	0.1966	-1.2399	8.1627

Table 1: Descriptive statistics (mean, standard deviation, annualized standard deviation, skewness and excess of kurtosis) for the index and the portfolio assets, January, 2000 - December, 2013.

Many other approaches based on different multivariate risk models can be applied to generate scenarios. We refer to [13] for an overview on several available techniques for scenario tree generation and reduction, and for the proposal of alternative approaches to path-dependent scenario generation for financial problems.

3.2 Historical out-of-sample backtest

Each experiment has been carried out using an out-of-sample rolling simulation procedure. We consider a 50-week management period with weekly rebalancing. At each decision stage we generate a scenario tree and we solve the optimization problem. The first period optimal portfolio is then evaluated on a mark-to-market basis using realized returns and the process is carried over to the next step.

The management exercise has been carried out considering three different subperiods in the available dataset to study the performances of the proposed models under different market conditions.

- First period: from October 7, 2005 to September 22, 2006.
- Second period: from September 7, 2007 to August 22, 2008.
- Third period: from August 7, 2009 to July 23, 2010.

The first period is characterized by a limited drop of the index at the beginning, followed by a rising market with a relevant drawdown at the end of the period. In the second period considered the index experiences a moderate gain at the beginning, followed by two consistent declines in the remaining part of the sample. Finally, the third period represents a phase of rising market with a more significant positive performance.

In the periods considered we observe the presence of significant volatility, changing market conditions and significant drops in the value of the index. Due to this conditions they can

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represent interesting test periods for our model since we aim at obtaining a portfolio in which both variability and extreme losses are controlled, still keeping capability of participating in upside market potential.

3.3 Volatility control

The first contribution of our paper is to integrate the volatility control into the portfolio optimization process. To analyze the effectiveness of the proposed strategies we study the behavior of the returns distribution obtained under the two models, for different values of the parameters.

Furthermore, we provide comparisons of our optimized volatility control strategies with a rule based non-optimized target volatility strategy which systematically adjusts exposure to the benchmark in order to maintain a pre-specified level of risk. This approach is essentially a dynamic portfolio allocation between a risky asset (the index) and a riskless one (cash), such that the overall risk of the portfolio (as measured by volatility) is kept constant through time. These strategies have become particularly popular in recent years and many different financial corporations provide products which implement this approach. In the following, we briefly present a rule-based target volatility strategy and apply it to the available data for the S&P 500 Index.

We denote with σ_{target} the desired level of volatility for the managed portfolio, and with $\hat{\sigma}_t$ an estimate of the volatility of the risky component representing expected future market conditions. The exposure to equity at time t, denoted with w_t, is defined as the ratio between the target volatility and the forecasted volatility

$$w_t = \min\{\sigma_{target}/\hat{\sigma}_t, 1\} \tag{47}$$

where the exposure is limited to 1 to avoid leverage when the volatility is too low with respect to the target. If the realized volatility is lower than the target volatility the exposure is increased, whereas, if the realized volatility is higher than the target the allocation in the risky asset is reduced.

Future volatility cannot be perfectly forecasted, however, there is evidence that most assets exhibit volatility clustering. This means that recent realized volatility may provide significant information on future risk in the near term. As a consequence future volatility is usually estimated from historical equity returns on a trailing basis using for example exponentially weighted moving average estimators.

In figure 1 we present the weekly realized volatility for the S&P 500 Index, computed on a trailing basis along the entire dataset, using 5, 10 and 20 weeks, (first plot); and we present the time-varying exposures to the index, obtained for different levels of targeted annualized volatility ($TV = 5\%$, 10%, 15%, 25%,) under two different hypothesis for the computation of the realized volatility, 5-week and 10-week trailing basis (second and third plot, respectively).

We recall from table 1 that the annualized volatility of the index for the entire sample period is approximately 19%. Lower level of target volatility (5% and 10%), actually de-leverage the portfolio partially allocating it into the riskless asset. A target level of annualized volatility of 25% results in a portfolio, almost always, entirely invested in the risky component.

In figure 2 we present portfolio values obtained, along the entire sample period, for different levels of targeted volatility. We compare them with the portfolio fully invested in the index. The realized volatility is computed from historical data on a 10-week trailing

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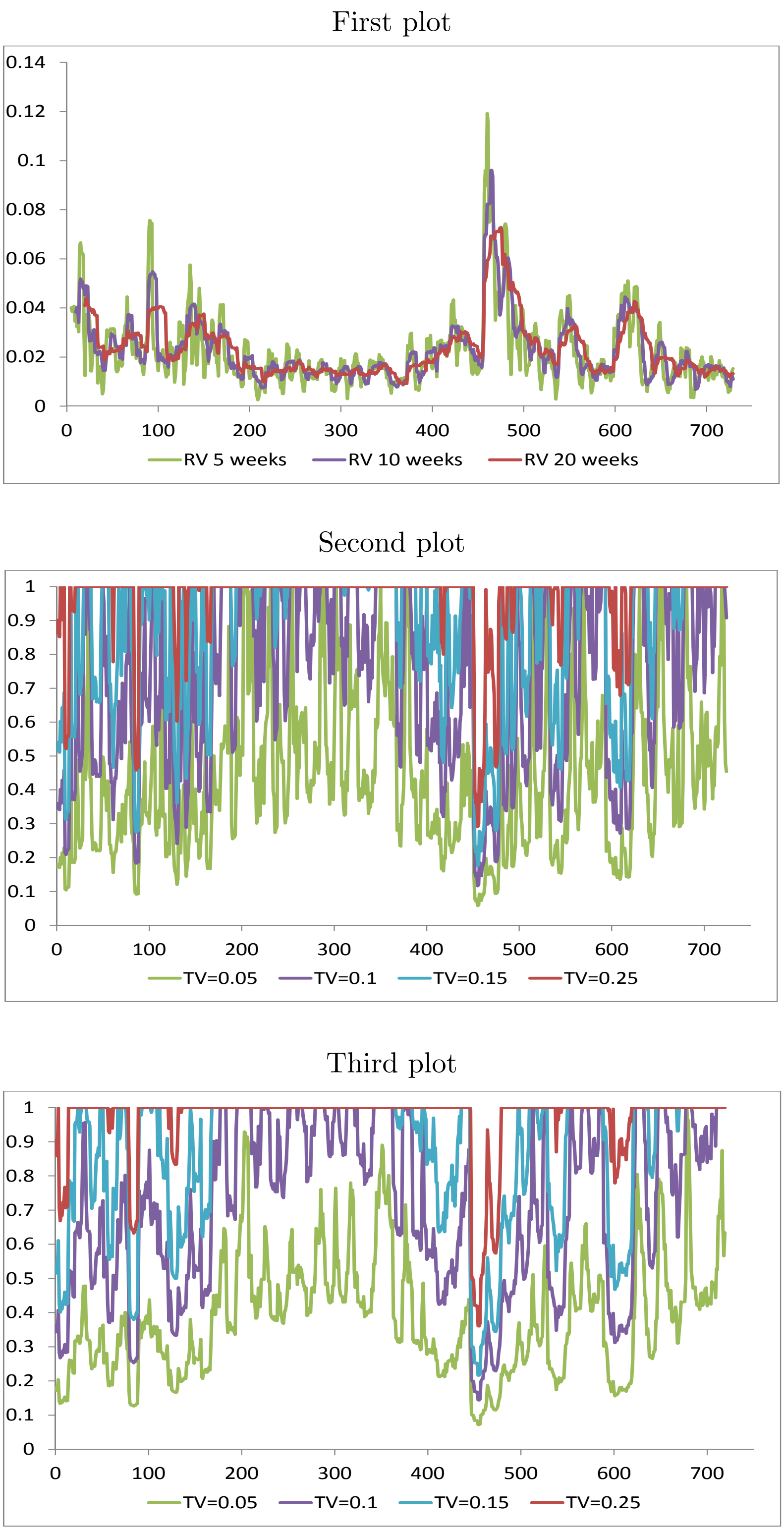

Figure 1: S&P 500 weekly realized volatility computed on a trailing basis, using 5, 10 and 20 weeks (first plot). Time-varying exposures in the S&P 500, for different levels of targeted (annualized) volatility $TV = 5\%,\ 10\%,\ 15\%,\ 25\%,$ using a 5-week (second plot) and a 10-week (third plot) realized volatility January, 2000 - December, 2013.

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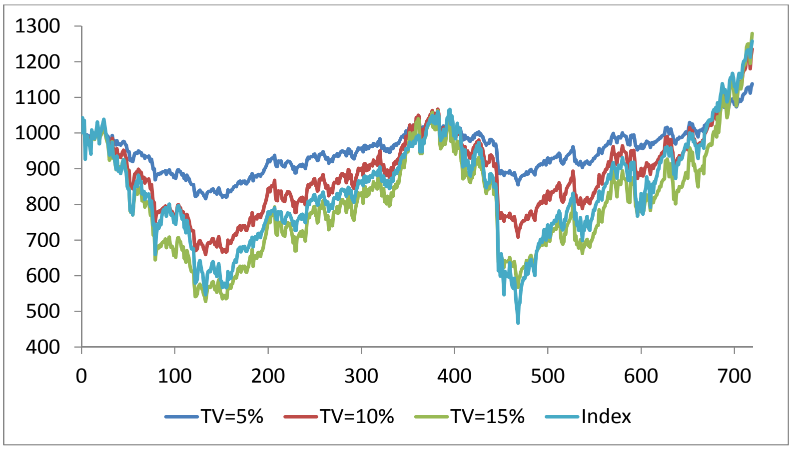

Figure 2: Comparison between the portfolio fully invested in the index and portfolio values for rule-based target volatility approach, for different values of the (annualized) targeted volatility ($TV = 5\%,\ 10\%,\ 15\%$). January, 2000 - December, 2013.

basis. We can observe that the portfolio with a targeted volatility of 5% (annualized) is able to achieve a significant reduction of risk, with respect to the index. Nevertheless, passively following the index it is subject to a relevant amount of losses.

In figure 3 we present the results from the application of Model 2, in which the local volatility control is obtained with the inclusion of volatility constraints. The variability reduction is analyzed for different values of the parameter s. The second period (second plot in figure 3) is characterized by a consistent variability and a huge drop in the value of the index. We note that setting a very strict control, that is $s = 5$, we obtain a portfolio which never falls below the initial value. Allowing for a larger and larger control ($s = 15,\ 25$), the acquired degrees of freedom are used to better pursue the tracking goal and the resulting portfolio more closely resembles the index, up to the value $s = 1000$ which essentially corresponds to the unconstrained case. This is confirmed also in a period in which the index experiences a positive trend. In this case, see third plot in figure 3, the stricter control prevents the portfolio from obtaining all the upside potential from the market.

In figure 4 we present the results obtained applying the volatility control through Model 1, in which the tracking error control, the volatility control and the tail risk control are simultaneously present in the objective function. The three goals are equally weighted ($\alpha_{TE} = \alpha_{VC} = \alpha_{TR}$) and we consider different values of the control parameter, $\sigma = 0.25,\ 1,\ 5,\ 10$. Again, we present the results obtained in the three periods considered. We can observe that in these cases the upside participation in the market is generally more consistent when compared to results obtained with Model 2.

3.4 Tail risk control

To study the effects of the presence of the tail risk term we compare the behavior of a portfolio obtained considering only tracking error and tail risk controls with the model in which we include also volatility control.

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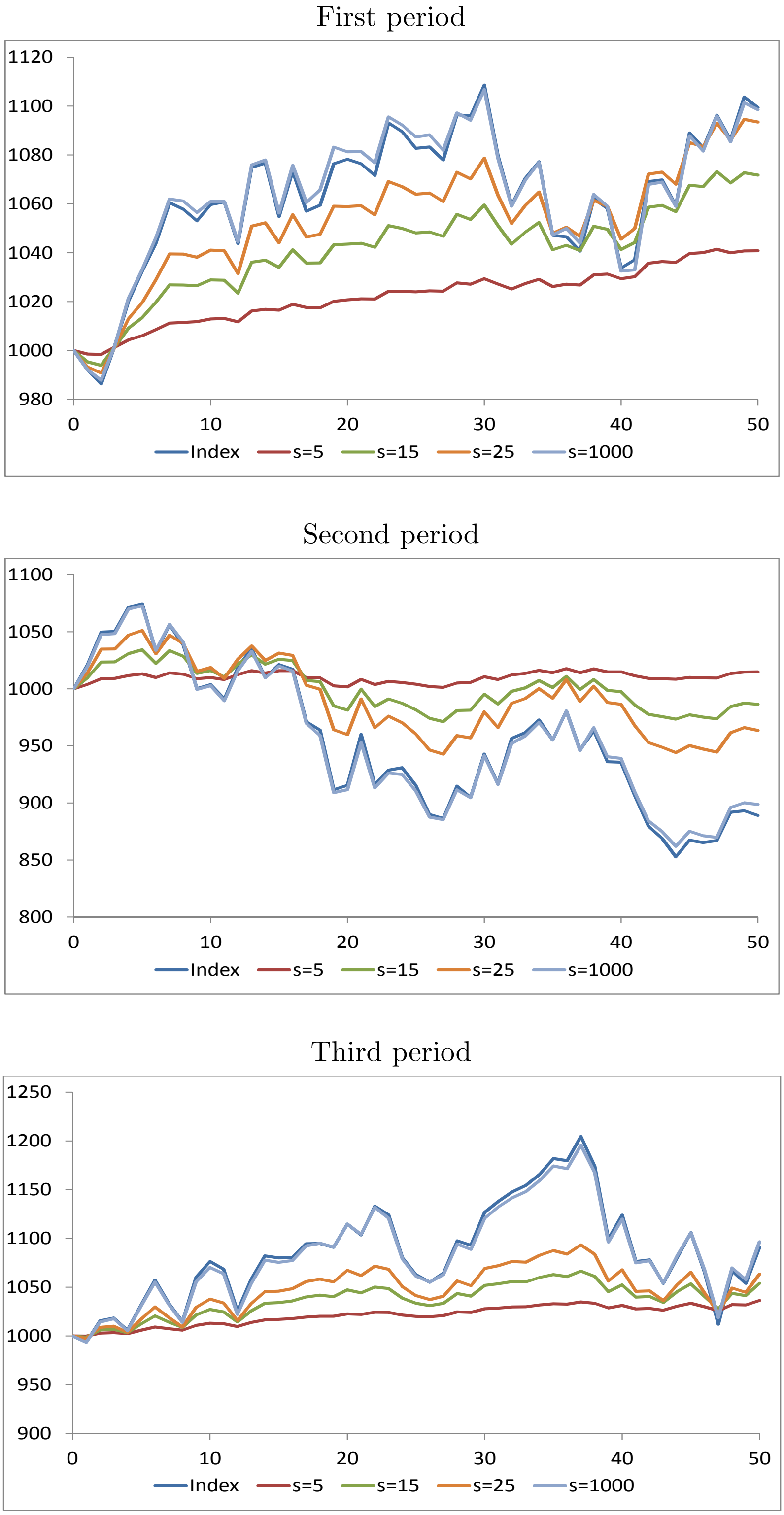

Figure 3: Comparison between the portfolio fully invested in the index and portfolio values in the case of local volatility control through constraints (Model 2), for different values of the parameter s (different lines in the same plot) and different market periods (different plots).

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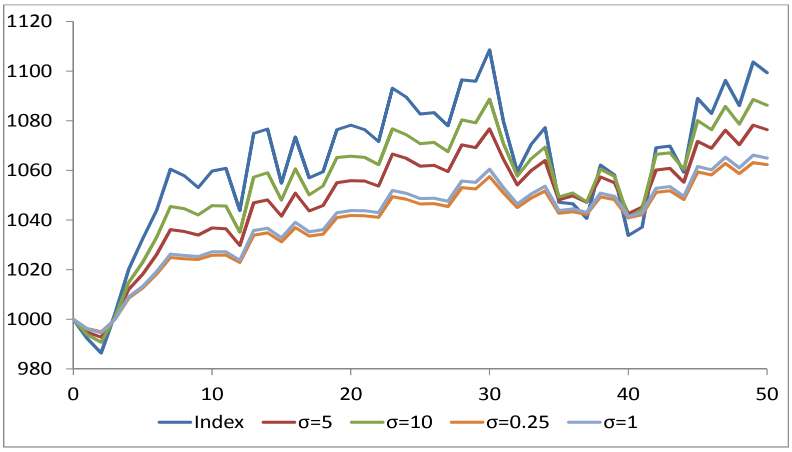
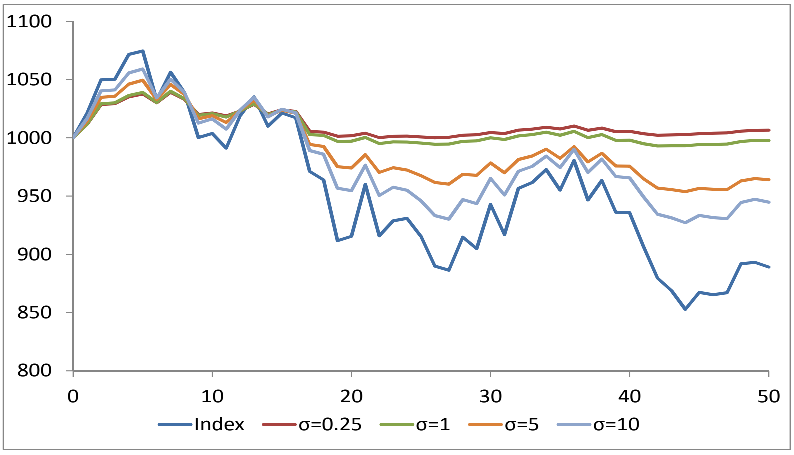
First period



Second period



Third period

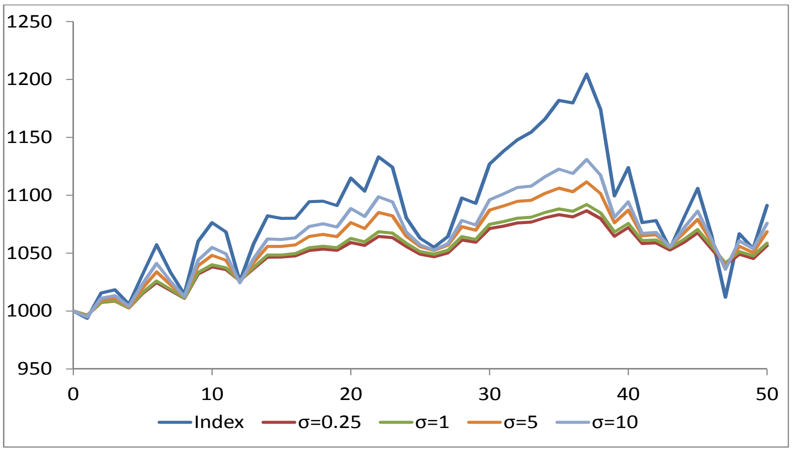


Figure 4: Comparison between the portfolio fully invested in the index and portfolio values in the case of local volatility control in the objective function (Model 1), for different values of the parameter σ (different lines in the same plot) and different market periods (different plots).

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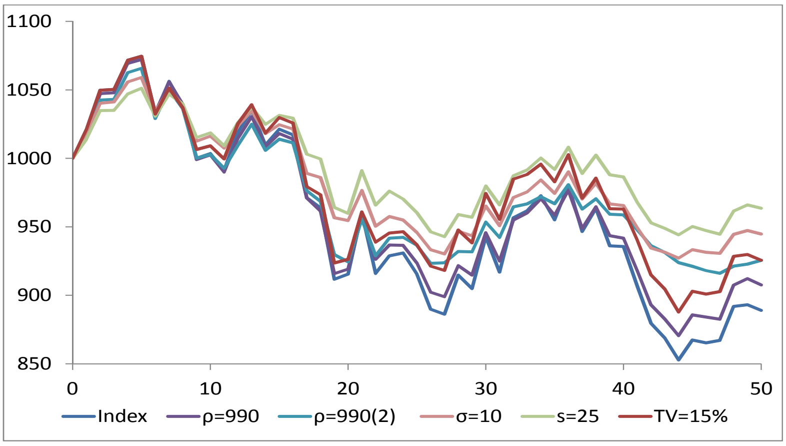

Figure 5: Comparison of portfolio values for different tail risk weights with portfolios which include also volatility control (Model 1 $\sigma = 0.25$, Model 2 $s = 5$). Loss threshold $\rho = 990$. Second period.

Figure 5 exhibits the behavior of portfolios with tail risk control with and without volatility control. In more detail, we compare the following portfolios

- model 1 with $\alpha_{TR} = \alpha_{TE}$ and $\alpha_{VC} = 0$;
- model 1 with $\alpha_{TR} = 2\alpha_{TE}$ and $\alpha_{VC} = 0$;
- model 1 with $\alpha_{TE} = \alpha_{TR} = \alpha_{VC}$ with $\sigma = 10$;
- model 2 with $\alpha_{TE} = \alpha_{TR}$, volatility constraints with $s = 25$
- target volatility approach with $TV = 15\%$;
- the portfolio fully invested in the index.

We consider two models with tail risk control and no volatility control, in the second we doubled the weight coefficient for the tail risk term in the objective function with respect to the weight assigned to the tracking goal, to stress the goal for downside protection. The loss threshold is set equal to $\rho = 990$.

We can observe that including the volatility controls, or assigning more weight to the downside protection, results in effective risk mitigation, but it also reduces the performance of the portfolio in the first part of the period when the index presents positive results.

The target volatility strategy reduces the risk in the tail, but it is less effective than the optimized strategies.

3.5 Joint effects

To analyze the joint effects of volatility and tail risk controls we consider the behavior of the entire return distribution for the managed portfolio. The distribution for the index portfolio represents the target distribution of the pure tracking error problem and is the one with the

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fattest tails (both left and right ones) and the higher variability, indicating possibilities of huge losses as well as huge gains, for all the periods considered (see figure 6).

This analysis allows us to shed light on how the risk controls introduced affect the shape of the return distribution, in particular with respect to tail behavior (thickness and length) and asymmetry.

The obtained result confirm that introducing risk controls produces a significant change in the shape of the portfolio distribution and we can see the trade-off between the upside capture and the mitigation of downside risk.

We compare results obtained through Model 1 and Model 2 (optimized risk controls), for different sets of parameters, with the target volatility approach (non-optimized).

Figure 6 presents the empirical portfolio return distribution for the portfolio fully invested in the index, for the portfolio obtained applying Model 1, with parameters $\sigma = 0.25$ and $\sigma = 1$, for the portfolio obtained with Model 2 for $s = 5$ and finally for the portfolio obtained with the non-optimized target volatility approach for $TV = 5\%$, along the three periods considered.

In table 2 we summarize the statistics for the distributions for all periods and all portfolios. All the risk management strategies considered are effective and, when compared with the index, produce a risk mitigation both in terms of volatility and in terms of tail risk. The cost of this is visible in the reduction of the right tail of the distribution, upside capture, which is more consistent for the portfolio of Model 2. In particular, we can observe that in the second period the optimized portfolios achieve a positive mean return.

To evaluate the hedging power of the proposed strategies versus their costs, in terms of reduced upside potential, in figure 7 we present the cumulative performances of the considered strategies along the three periods. We observe the markedly positive performance obtained by the optimized strategies in the second period, when compared with the benchmark. This result can be contrasted with lower than the benchmark returns in the first and third period.

3.6 Performance evaluation and sensitivity analysis

To evaluate the performance of the proposed strategies we consider classical performance measures, such as the Sharpe ratio and the Information ratio, which use mean and standard deviation to summarize the return and the excess return distribution, respectively, see table 2. The portfolios obtained from Model 1 and Model 2 provide better risk-adjusted performances in all the three cases considered and they outperform the target volatility strategy which provides a moderate improvement over the index in terms of risk-adjusted returns.

According to the Sharpe ratio and the Information ratio the best portfolio, in the first and third period, is obtained with Model 2 ($s = 5$). The portfolios which include the stricter volatility control, either in the objective function ($\sigma = 0.25$,) or in the constraints ($s = 5$), are the only ones which manage to produce a positive Sharpe ratio in the second period.

But, since we want to evaluate the implications of our risk control strategies in terms of reduction in the downside and possibility of maintaining upside capture, we need performance measures which guarantee a degree of flexibility in dealing with the concept of risk and reward, and which allow us to include information on the whole distribution. To this aim, we consider the Omega ratio [24] and the Φ ratio ([18]).

The Omega ratio represents a ratio of the cumulative probability of an investment's outcome above a given threshold level, to the cumulative probability of an investment's

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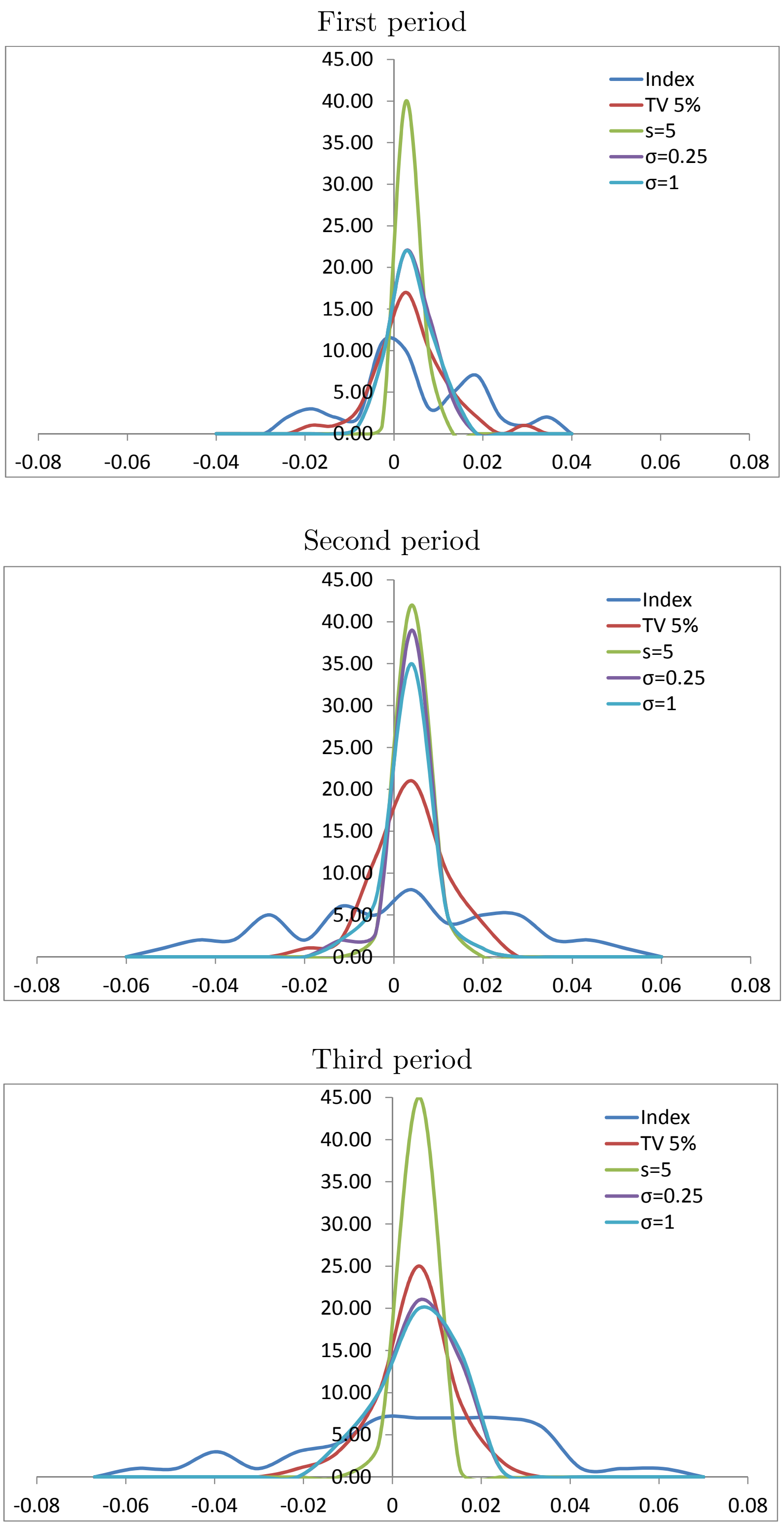

Figure 6: Portfolio distribution (kernel density estimate) following different strategies (different lines in the same plot) applied in different market periods (different plots).

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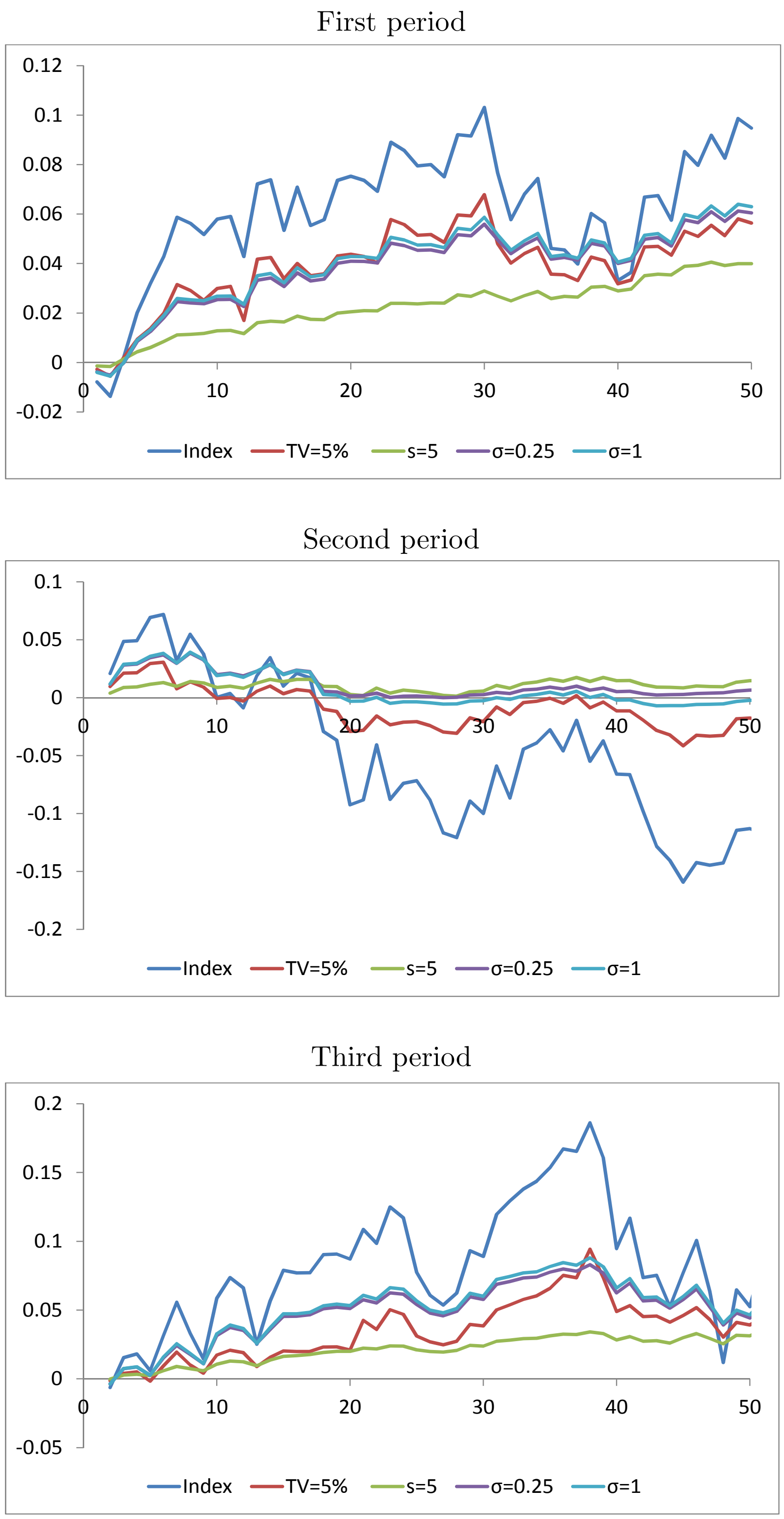

Figure 7: Cumulative return for different strategies (different lines in the same plot) applied in different market periods (different plots).

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(a) First period					
	Index	TV=5%	s=5	$\sigma = 0.25$	$\sigma = 1$
Mean	0.0018	0.0011	0.0007	0.0012	0.0012
Std.dev	0.0140	0.0078	0.0017	0.0045	0.0049
Skewness	-0.0388	0.2907	0.3712	0.1882	0.1710
Ex.Kurtosis	-0.3281	1.3822	-0.1521	-0.4276	-0.4094
Sharpe Ratio	0.1348	0.1440	0.4457	0.2658	0.2552
Information Ratio	0.0306	0.0344	0.1510	0.0724	0.0684

(b) Second period					
	Index	TV=5%	s=5	$\sigma = 0.25$	$\sigma = 1$
Mean	-0.0023	-0.0004	0.0002	0.0001	-0.0001
Std.dev	0.0250	0.0081	0.0029	0.0051	0.0057
Skewness	-0.1014	-0.3059	-0.1565	-0.2162	-0.4125
Ex.Kurtosis	-0.6298	0.2030	-0.3881	4.0993	3.3381
Sharpe Ratio	-0.0941	-0.0495	0.0982	0.0257	-0.0079
Information Ratio	-0.0173	-0.0093	0.0212	0.0051	-0.0015

(c) Third period					
	Index	TV=5%	s=5	$\sigma = 0.25$	$\sigma = 1$
Mean	0.00174	0.0009	0.0007	0.0010	0.0011
Std.dev	0.02538	0.0092	0.0024	0.0076	0.0081
Skewness	-0.52934	-0.3107	-0.0851	-0.0252	-0.0486
Ex.Kurtosis	0.09851	0.7556	-0.3853	-0.2012	-0.2806
Sharpe Ratio	0.06877	0.0966	0.2950	0.1441	0.1397
Information Ratio	0.01360	0.0201	0.0757	0.0325	0.0313

Table 2: Descriptive statistics (mean, standard deviation, skewness, and excess Kurtosis), and performance measure (Sharpe ratio and Information ratio) for different strategies (columns) and market periods (panels (a), (b) and (c)).

outcome below the threshold. Expected returns are divided into two parts, returns above the expected rate (the upside) and those below it (the downside), gains and losses, respectively. It is a measure of performance that does not assume a normal distribution of returns and that takes into account the whole distribution without the need to estimate sample moments. We recall the definition from [24]

$$\Omega(r) = \frac{\int_r^{+\infty}(1 - F(x))dx}{\int_{-\infty}^{r} F(x)dx} \tag{48}$$

where F is the distribution function of returns, and r is the loss threshold.

Furthermore, to evaluate the performances of the proposed strategies, as well as, to carry out a sensitivity analysis with respect to different attitudes towards downside risk and volatility, we need measures which are able to deal with favorable and unfavorable outcomes from a reference threshold in different ways.

To this aim we consider risk-adjusted performance ratios based on one-sided risk measures built using one-sided moments of different orders, see [18]. An interesting property of these

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$\Phi_r^{p,q}$	p	q	Interpretation
$\Phi^{1,1}(r)$ (Omega ratio)	1	1	Small and large deviations are equally weighted
$\Phi^{1,2}(r)$ (Upside potential ratio)	1	2	Large losses are not desired
$\Phi^{0.5,1}(r)$	0.5	1	Relative preference for small gains
$\Phi^{1.5,1}(r)$	1.5	1	Relative preference for large gains

Table 3: Left and right order parameters considered for the Φ ratio and their interpretation in terms os attitude toward risk.

measures is that they can emphasize either small or large deviations from the benchmark according to the investors preferences, allowing to account for different risk/reward trade-offs.

Given any $p > 0$, $q > 0$ and a threshold level b, following [18], we define the performance ratio based on one-sided risk measures of a strategy with total return X as

$$\Phi_b^{p,q} = \frac{E^{1/p}\left[\{(X-b)^+\}^p\right]}{E^{1/q}\left[\{(X-b)^-\}^q\right]} \tag{49}$$

where p and q are the parameters defining the right and left orders, respectively, of the performance ratio. This measure is a reward-to-variability index computing the ratio between the favorable events and the unfavorable ones, properly weighted. The higher the value of the ratio, the more preferable the strategy.

Given the threshold level b, the choice of the parameters p and q depends on the desired relevance given to the magnitude of deviations. The left order q reflects the investor's attitude towards falling below b. If small deviations below the threshold are acceptable when compared to large deviations (catastrophic losses), then a large value ($q > 1$) for the left order is appropriate. While, a value $q < 1$ for the left order is more suitable if the investor is concerned with failing the target but without particular regard to the amount.

The right order p accounts for the investor's appreciation for outcomes above the threshold. A high value for p ($p > 1$) describes attitude towards exceptional performance rather than moderate gains ($p < 1$).

We note that, properly choosing the parameters, the Omega Ratio can be obtained as a particular case of the Φ Ratio and we have $\Phi_r^{1,1} = \Omega(r)$. Furthermore, if we set $(p, q) = (1, 2)$ we obtain the Upside Potential Ratio by [33]. We refer to [18] for more details on the Φ ratio and for a discussion on the relation between expected utility theory and one-sided risk-adjusted performance measures.

We consider different values for the left and right orders since we are interested in analyzing the relative properties of our strategies in terms of risk and reward. In table 3 we summarize the values of the parameters considered.

In more detail, $\Phi^{1,1}(r)$ corresponds to the Omega ratio and since both orders are equal to one, small and large deviations from the benchmark are equally weighted. $\Phi^{1,2}(r)$ gives the Upside potential ratio in this case, the risk measure is the semi-standard deviation and the reward measure is the expected excess of return.

Moreover, we consider $\Phi^{0.5,1}(r)$ which accounts for moderate attitude towards gains, and $\Phi^{1.5,1}(r)$ which, on the other hand, set a relative preference for extreme gains while, in both cases, small and large negative deviations are treated equally ($q = 1$). Note that when $p < q$, the chance of having huge losses (even if with low probability) is not desirable.

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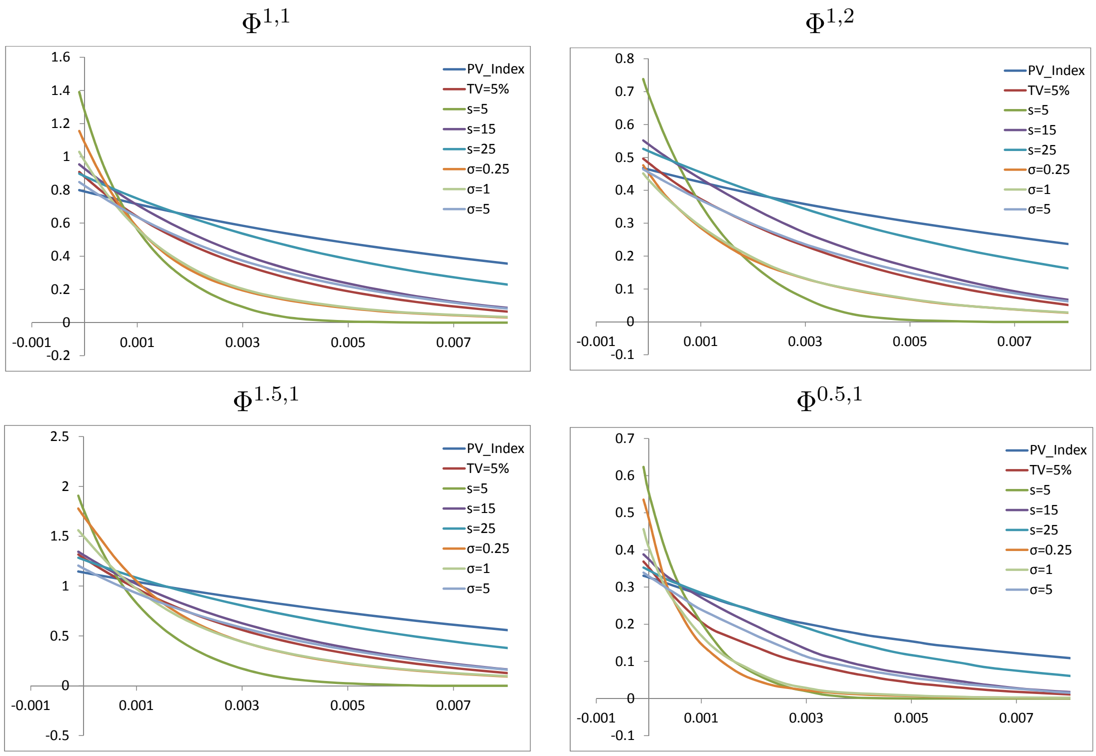

Figure 8: $\Phi^{p,q}$ ratio as function of the threshold level, for different values of p and q (different plots) and different risk control strategies applied during the second period (different lines in each plot).

In figure 8 we present the results of the $\Phi^{p,q}(r)$ ratios, as a function of the threshold level, for different values of the right and left orders. We compare the optimized volatility control strategies with the rule-based target volatility approach and the index in the second period considered in our application.

The steeper the $\Phi^{p,q}$ ratio function is the lower the probability for extreme returns. We note that the stricter control on the variability, obtained with the inclusion of constraints on the local volatility (Model 2 with $s = 5$), results in a more effective protection and the strategy is preferred to the others for low and moderate target returns while if the target return is set at an higher level more riskier strategies are preferred. The indifference point (i.e the intersection with the other curves) depends on the level of the left and right orders, the higher the aversion for (moderate and huge) losses the higher the threshold return which provides indifference.

The results obtained applying the models proposed in this contribution, confirm that they, not only allow for an explicit control of the volatility (through the objective function or the constraints), but also succeeded in improving the return distribution profile of the managed portfolio. In particular, they provide more attractive upside capture and a better downside protection.

The rule based target volatility approach allows for downside risk protection but, in our opinion, does not provide sufficient flexibility in controlling the risk-reward profile of the

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portfolio. This is due mainly to the fact that tail risk is mitigated in an indirect way reducing variability and this has an impact also on the right tail of the distribution.

4 Conclusion

In this contribution we propose a multiperiod portfolio optimization model which includes local volatility control and tail risk control. The performance goal is set through the inclusion of reference benchmark which is dynamically tracked.

The model can be easily interpreted, in terms of risk management process, as consisting of two main parts. First, we define a benchmark for the investment process fixing, in this way, a target return distribution for the portfolio. Then, we activate two different forms of controls to properly re-shape the managed portfolio resulting distribution according to the risk tolerance or risk protection goals. This corresponds to the decomposition of the risk control process into different layers which can be activated separately of jointly.

The results obtained from the application of the model on real data confirm that the proposed approach is effective in obtaining a tailored risk mitigation providing the desired trade-off between risk protection and return achievements; and support the idea that diversifying risk reduction strategies allows to obtain a more efficient control of risk.

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