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Volatility as an asset class: Holding VIX in a portfolio

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Abstract
Hedging market downturns without sacrificing upside has long been sought by investors. If VIX was directly investable, adding it as a hedge to the S&P 500 would result in significantly improved performance over the equity only portfolio. However, tradable VIX products do not provide the hedge or returns investors seek over long-term horizons. Alternatively, deconstructing VIX to find the key S&P 500 options which drive VIX movements leads to a synthetic VIX portfolio that provides a more effective hedge. Using these options captures correlations and returns similar to VIX, and combined with the S&P 500, outperforms the buy-and-hold index portfolio.

KEYWORDS
portfolio returns, S&P 500 index, VIX index, volatility

JEL CLASSIFICATION
G11; G12

1 | INTRODUCTION

Extreme stock market downturns are the most disconcerting periods for investors since they are risk averse, wish to limit their exposure to volatility, and seek to avoid negatively skewed payoffs (Kumar, 2009). Significant market downturns, such as those experienced from October 2007 to March 2009, resulted in massive losses for most investors as equity indices retreated over 50%. The anxiety of such movements even provoked many investors to sell large portions of equity, mutual fund, and exchange-traded fund (ETF) holdings, thus guaranteeing large losses.[1] The ability to more effectively hedge equity investments with assets that would reduce portfolio downside risk and without giving up upside potential could combat a considerable amount of investor unease.

While several assets exist with returns that are, on average, negatively correlated with equities, these instruments do not appear to provide the desired hedge against market downturns since the return correlations either become positive in times of distress or are to cost prohibitive. For example, during the market collapse in 2008, the value of both equities and commodities fell, though traditionally, commodities are negative beta assets.[2] Bond holdings also declined in value during the 2008 crash, as the risk in commercial borrowing increased while liquidity fell. Many hedge funds designed to cushion losses in equity markets experienced reversals over the 2007–2009 period. Szado (2009) documents the increased correlation of asset returns over the crisis period above the levels seen in the 2004–2006 period, implying that as the need for diversification grew, the ability of many assets to hedge equity holdings shrank at the most inopportune time. Additionally, while tail-oriented products like long puts did very well in crash periods, in periods of stability, such as the recent bull market from 2009 to 2018, they

[1] Mary Pilon, "Many Bought Shares High, Sold Low," *Wall Street Journal*, May 18, 2009.

[2] Numerous commodity indices also retreated by more than 50% of value during the equity market decline of 2007–2009 as inflation ground to a standstill and consumption of raw materials slowed.

have had significant negative performance.[3] So, where do investors turn to manage downside risk beyond having perfect market foresight?

Holding volatility would appear to be the answer. If a portfolio could hold volatility as an asset it may make traditional negative or counter-cyclical investments moot for hedging purposes. This is because of the mean-reverting nature of volatility and its negatively skewed payoff. Thus, investors' interest in the VIX index.

Since the introduction of VIX, it has widely been regarded as the economy's indicator of risk in the equities market. As noted by Whaley (1993, 2000), the VIX index is considered "the investors' fear gauge index." An important byproduct of introducing VIX was the newfound opportunity for investors to trade in futures, introduced in March 2004, and options, introduced in February 2006, thus allowing investors to enter into contracts which generate payoffs specifically related to volatility. Additionally, exchange-traded products have been established, starting in 2009 with the iPath S&P 500 VIX Short-Term Futures ETN (ticker: VXX, SVXY, and others), that offer a more direct way to access volatility as an investment. While it is possible to invest in equity options on the S&P 500 and construct a payoff that would be related to the volatility of the index, these VIX products provide a simple, but not necessarily effective, way to invest in VIX that is attractive for investors looking to hold or sell volatility.

A number of researchers have considered the possibility of hedging portfolios with VIX-mimicking assets. Dash and Moran (2005) initially considered the ability of newly formed VIX-based products to lower portfolio risk. Emerging possibilities then developed for such a strategy, including the use of VIX futures, VIX options, and VIX-based ETNs. Brenner, Ou, and Zhang (2006) introduced an option straddle specifically designed to hedge volatility risk. This instrument is sensitive to volatility innovations and thus useful as a hedge. Windcliff, Forsyth, and Vetzal (2006) consider the variations in the contract designs of volatility derivatives and discuss the difficulties of hedging the returns with such instruments, particularly given delta and delta-gamma hedging techniques. Black (2006) and Moran and Dash (2007) find that adding VIX futures to a passive portfolio can significantly reduce portfolio volatility. VIX's quick movements during risky markets also improve the skewness and kurtosis of the overall portfolios. Briere, Burgues, and Signora (2010) advocate a sliding approach when hedging in which more (fewer) VIX futures contracts are held when VIX levels are notably lower (higher) due to the mean-reverting nature of the index. Jones (2011) and Warren (2012) also advocate using VIX futures only in a tactical manner.

However, it does not appear using these products provides a payoff like that of the VIX index or provides the long-term hedge against increases in volatility that most investors desire. As Szado (2009) notes, exposure to VIX calls and puts, as well as VIX futures, does not directly mimic holdings in the spot levels of VIX given that the mean-reverting nature of the underlying are priced into the derivative values. While H.-C. Chen, Chung, and Ho (2011) suggest VIX futures do enhance the performance of a portfolio of equities, Whaley (2013) points out that trading VIX products result in poor performance. More recently Basta and Molnar (2019) and Bordonado, Molnár, and Samdal (2017) address the ineffectiveness of using VIX and VIX-style products as hedging or speculative strategy.

Given the current evidence that VIX products are not effective hedges leads to a basic question; since VIX is constructed using tradeable assets, should it not be possible to create a portfolio that uses these underlying assets as a hedge in a more effective and profitable way than the current traded products?

To answer this question three steps are taken. First, a portfolio is constructed that holds VIX and the S&P 500, treating VIX as a tradable asset. The returns to this portfolio will reveal whether "investing" in the cash VIX index provides a more effective hedge to long-equity positions, either through lower costs or superior returns, than alternative hedges, such as purchasing index puts. This seems especially relevant given the current low levels of volatility and steepness of the volatility skew. The results show that, if the VIX index were directly investable, holding VIX in a portfolio with the S&P 500 yields positive and significant alphas across all market cycles, and are in direct contrast to the evidence for the tradable VIX assets. VIX futures, ETNs, or single use options do not replicate or provide the hedge that trading the cash VIX index would even when accounting for the level of volatility.

Second, a test to determine the S&P 500 options that drive the changes in VIX is conducted. This test goes beyond Whaley (2009), who sought to clarify the meaning of VIX and discuss its characteristics. Whaley emphasizes that, like the S&P 500 index, the VIX index is not directly investable. While it is quite simple to replicate the payoff of the S&P 500 by holding the 500 underlying stocks in the appropriate proportions (or more simply, via investment in low-cost ETFs), it is difficult or nearly impossible to replicate VIX by holding the underlying S&P 500 options. This is in part because VIX is constructed using call and put out-of-the money options with two maturities closest to 30 days, with weights that are squared. Additionally, these weights change daily. Thus, even if a portfolio were able to hold the correct proportions

[3]Refer to the performance on a put writing strategy by the Asset Consulting Group that noted total return for put writing strategy is 1,153% since 1986. This is also consistent with the findings of Ilmanen (2012) and Litterman (2011).

on a given day, which would require a significant investment in many option contracts, the next day the proportions would change, and the rebalancing costs would be prohibitive. This makes the VIX index, viewed as an asset, practically un-investable. So, it is important to determine the options that are the most correlated to VIX changes, and how these options change with VIX changes through time. The results show that ITM puts have the strongest relationship to VIX changes. This is interesting as ITM puts are not a part of the VIX calculation.

Lastly, by deconstructing VIX into the S&P 500 options that drive the changes in VIX, and using optimal portfolio weights established between the S&P 500 and the VIX index, a hedge portfolio is formed that uses a small number of options, requiring monthly rebalancing, while maintaining a strong correlation to VIX. By deconstructing VIX into the individual option components, it is possible to form a portfolio of liquid S&P 500 options that captures some of the payoff to the VIX index which eludes the ETN, futures, and option contracts on VIX. This synthetic VIX position improves the hedging prospects of an otherwise passive long only equity portfolio. The main benefit comes from significant outperformance in down markets with limited underperformance in up markets.

The rest of the article is as follows. Section 2 presents the data and its sources. Section 3 presents the analysis of the performance of VIX combined with the S&P 500. Section 4 investigates which S&P 500 options best capture VIX changes and how using these options mimic VIX returns as part of a hedge in a long S&P 500 portfolio. Section 5 concludes the paper.

2 | DATA AND METHODOLOGY

The data for the analysis of VIX-based hedging strategies is collected from numerous sources. The actual daily VIX levels are taken from the CBOE.[4] The methodology for VIX construction was amended in September, 2003, though retroactive calculation allows for collection of data beginning in 1990. S&P 500 index options data, SPX, beginning in 1996 and ending in 2017 is collected from Optionmetrics. SPX calls and puts are used to create the synthetic VIX position. The Fama and French (1993) MKT, SMB, and HML factors and the momentum factor (MOM) are provided on Kenneth French's data library website.[5] Additionally, SPX options are used to create at-the-money (ATM) and out-of-the-money (OTM) straddles, which are typically employed as plays on volatility, and whose returns are used as option-related factors in evaluating the results of various strategies. Using the returns from these strategies as additional factors in a risk model can capture the volatility and skew premiums that are present in many hedge funds and option-based strategies. Historical S&P 500 prices and returns are taken from CRSP to calculate the returns of portfolios that hold S&P 500 as the long-equity asset (CRSP Stocks, 1990–2017).

Table 1 presents the summary statistics for S&P 500 and VIX returns. Panel A shows that S&P 500 returns over the entire sample period average 68 basis points per month, while VIX returns average 163 basis points per month. The standard deviation of the VIX monthly returns is 20.03%, meaning the average VIX returns are statistically indistinguishable from zero. Additionally, the median return is −113 bps, highlighting the positively skewed distribution and the mean-reverting nature of volatility.

The correlation between S&P 500 and VIX returns is −0.64, demonstrating the strong negative relation as should be expected. Limiting the sample to months in which S&P 500 returns are positive yields an average S&P 500 and VIX return of 3.07% and −6.83% per month, respectively. For negative S&P 500 return months, the average S&P 500 and VIX returns are −3.45% and 16.25% per month, respectively. When separating the returns into up and down periods, mean returns for both the S&P 500 and VIX are significantly different from zero, highlighting the importance of analyzing differing market conditions. More interestingly, the absolute value of the mean S&P 500 returns is similar for up and down market conditions, while the positive mean VIX returns are over twice as large during periods of negative S&P 500 movement as the negative mean VIX returns in periods of positive S&P 500 returns. This asymmetric relation between S&P 500 returns and VIX returns shows that volatility has the potential, if used correctly, as a hedging instrument against S&P 500 losses since it returns more in down markets than it loses in up markets.

Panels B and C split the sample into two different time periods, the 1990–2003 and 2004–2017 periods. This is done to demonstrate the stability of the findings over long time periods, even though the average monthly return is highly sensitive to smaller time periods. The returns for the S&P 500 and VIX are not significantly different from zero between the two periods, and this includes accounting for both the up and down S&P 500 months.[6] The VIX average return is

[4]http://www.cboe.com/micro/VIX/historical.aspx.

[5]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. We thank Kenneth French for making this data available.

[6]*t* Tests are all below 1.5.

TABLE 1 Summary statistics

	n	Mean return	Median	STD	SKEW	KURT	Correlation
Panel A: Full sample							
S&P 500	336	0.68%	1.05%	4.10%	−0.619	1.376	
VIX index	336	1.62%	−1.13%	20.03%	1.642	6.679	−0.6404
S&P 500 > 0	213	3.07%	2.43%	2.36%	1.045	0.697	
VIX index	213	−6.83%	−7.47%	12.78%	0.200	0.263	
S&P 500 < 0	123	−3.45%	−2.50%	3.07%	−1.621	3.331	
VIX index	123	16.25%	11.04%	21.86%	1.890	6.876	
Panel B: 1990–2003							
S&P 500	168	0.77%	1.01%	4.35%	−0.475	0.500	
VIX index	168	1.44%	−0.29%	17.46%	0.980	1.949	−0.6287
S&P 500 > 0	105	3.42%	3.03%	2.49%	0.765	−0.009	
VIX index	105	−5.59%	−6.11%	12.65%	0.390	0.571	
S&P 500 < 0	63	−3.66%	−2.69%	2.95%	−1.393	2.119	
VIX index	63	13.14%	9.55%	18.18%	0.963	1.504	
Panel C: 2004–2017							
S&P 500	168	0.60%	1.09%	3.85%	−0.844	2.720	
VIX index	168	1.80%	−1.92%	22.36%	1.907	7.807	−0.6711
S&P 500 > 0	108	2.72%	2.12%	2.19%	1.393	2.061	
VIX index	108	−8.04%	−7.97%	12.84%	0.040	−0.074	
S&P 500 < 0	60	−3.23%	−1.99%	3.20%	−1.894	4.823	
VIX index	60	19.51%	12.98%	24.90%	2.108	7.190	

	VIX + Return	VIX − Return
Panel D: Percentage of monthly outcomes		
S&P 500 + Return	17.70%	45.90%
S&P 500 − Return	28.80%	7.60%

Note: The monthly and median returns, standard deviation (STD), skewness (SKEW), and kurtosis (KURT) for the S&P 500 and the VIX index from 1990 through December 2017, along with the correlation between the two indices. Each panel includes returns separated into up and down S&P 500 months. Panel A presents results for the full sample. Panel B presents results for the 1990–2003 period. Panel C presents results for the 2004–2017 period. Panel D shows the percentage of outcomes where the S&P 500 and VIX Index had both positive, negative, and different signed returns in a given month.

positive and the median negative, and the correlation to the S&P 500 is around −0.65 regardless of the sample period used. The results suggest a stability of the relationship between VIX and the S&P 500.

Panel D breaks the sample into months where VIX and the S&P 500 have different monthly signed returns and months where the signed returns are the same. Almost 75% of the sample has months when the VIX and S&P 500 have different signed returns, with 46% of those when the S&P 500 is up and VIX is down. Just over 17% of the sample has months when both VIX and the S&P are up, which is not unreasonable since large positive movements can result in increases in short-term volatility.[7] The 8% of observations when volatility falls while the S&P 500 falls is unusual, but reflects that volatility and returns are not always negatively correlated.

3 | EFFECTIVENESS OF VIX AS A HEDGE

3.1 | The relationship between the VIX index and the S&P 500

Given the asymmetry in returns found in Table 1, understanding the degree with which VIX moves with positive and negative S&P 500 moves is paramount if the goal is to construct the proper portfolio weights. As such, the following regressions are estimated:

$$r_{VIX,t} = \alpha + \beta r_{S\&P500,t} + \gamma r^2_{S\&P500,t} + \delta VIX_{t-1} + \varepsilon_t, \tag{1}$$

[7]This can happen because call options can be bid resulting in a steepening of the volatility curve resulting in a volatility smile.

$$r_{VIX,t} = \alpha + \beta_1 r^{+}_{S\&P500,t} + \beta_2 r^{-}_{S\&P500,t} + \gamma r^{2}_{S\&P500,t} + \delta VIX_{t-1} + \varepsilon_t, \quad (2)$$

where $r_{VIX,\,t}$ is the return of the VIX index on month t, VIX_{t-1} is VIX index (in % terms) on month $t-1$, $r_{S\&P500,\,t}$ is the return of the S&P 500 on month t, α is a constant, and ε_t is the residual on day t. The prior month VIX is included since the level of volatility at which the return occurs matters. Given the asymmetric relationship between market returns and volatility, the sample is also split into positive and negative S&P 500 returns for Equation (1). Alternatively, in Equation (2), positive and negative S&P 500 moves ($r^{+}_{S\&P500,t}$, $r^{-}_{S\&P500,t}$) are separated and included together. Panel A of Table 2 presents the estimations results.

When the full sample is used, the relation between VIX returns and S&P 500 returns is highly significant, as a 1% change in the S&P 500 results in a 2.98% change in the VIX index.[8] The positive coefficient on $r^{2}_{S\&P500,t}$ shows the concave relationship between volatility and index returns. The negative coefficient on prior month VIX level is expected, as higher levels of prior volatility should result in less positive VIX changes due in part to the effect of mean reversion. When separating the sample into positive and negative S&P 500 returns highlights the asymmetry between VIX and the S&P 500. A 1% fall in the S&P 500 has a minimum of a 3.247% greater change in VIX, depending on VIX level, than when it rises. Additionally, the coefficient on the $r^{2}_{S\&P500,t}$ is negative when looking at only negative S&P 500 returns, suggesting a convex relationship; that as S&P 500 returns get worse, VIX gets even higher. The results from the regression of Equation (2) suggest an even more extreme asymmetric relationship, where the coefficient on negative S&P 500 movements is significant but the coefficient on positive S&P 500 movements is not. In other words, for every −1% move in the S&P 500, the VIX will increase by 5.36%, depending on the level of VIX at that time, but for 1% S&P 500 increases VIX may not move at all.

Using these coefficient estimates from Equation (1), Panel B shows the weight of VIX required in a portfolio long both VIX and S&P 500 to yield a portfolio return of zero when the S&P 500 experiences a 5% move. A 5% loss threshold is used since it is common for long managers and quantitative strategies to enact stop losses or rebalance at or around this level (Khandani & Lo, 2011). The panel shows returns for both a 15 and 28 VIX level, and using the coefficient estimates from the full sample, and the two positive and negative regression specifications. Lower levels of VIX show VIX responding greater to −5% moves, with the negative return specification showing a 33% VIX return at a VIX of 15. At VIX at 28, this move is more muted, as a 5% S&P 500 move is more likely at these levels of volatility. This means a less responsive hedge due to the pull of volatility mean reversion at these higher VIX levels. Translating these values into portfolio holdings show, depending on specification, that the VIX portion of the portfolio ranges between 13% and 30%.[9] On average, over the entire time period, the portfolio would require a weight of 19.9% in VIX to completely offset a loss of 5% in the S&P 500 so that the overall portfolio return is 0%.

3.2 | Incorporating the VIX index in a portfolio

To demonstrate the effectiveness of VIX as a hedge and complementary asset to the S&P 500, a portfolio is constructed that holds a combination of the two through time. The weight in VIX is determined in month t using Equation (1) that uses only the data from months $t-60$ to $t-1$. The VIX weight is calculated at the end of month t based on the portfolio allocation to VIX from the predicted value from Equation (1) for each possible VIX level and S&P return, weighted by the likelihood of each scenario.[10] For example, using the coefficients from the second regression in Table 2, the weight in VIX if the S&P 500 was down 3% and VIX was 20 would be 18%. If this occurred once in the sample, the weight would be one divided by 60, and this weight would be multiplied by 18%. This is done for every outcome and then all outcomes are added together. In the case of positive S&P 500 outcomes, the optimal allocation to the hedge would be 0%. This leads to an average allocation to VIX of 5.05%.

The goal is then to assess whether this portfolio can demonstrate the benefit of capturing both the asymmetric relationship between VIX and the S&P 500 as well as minimizing the cost of volatility mean reversion. If the portfolio can accomplish both tasks, it establishes a best-case scenario by which to evaluate the performance of VIX-like assets as a supplement to typical passive investing.

In addition to showing the performance of portfolios that constantly hedge with VIX, three additional portfolios are calculated allowing for the mean-reverting nature of VIX to determine when to remove the hedge. These "threshold"

[8]This result is robust to different time periods and alternative specifications.

[9]Higher levels of VIX require higher allocations to VIX demonstrates that VIX loses its effectiveness as a hedge at higher levels of vol. In other words, its responsiveness to S&P 500 changes is lower.

[10]Likelihood of outcome simply means the chance of having a x% return in the S&P 500 based on the monthly historical distribution of returns.

TABLE 2 Relation between VIX and S&P returns

	Equation (1)			Equation (2)
	Full sample	**Months S&P 500 returns < 0**	**Months S&P 500 returns > 0**	**Full Sample**
Panel A				
α	0.188*** (8.72)	0.307*** (6.76)	0.098*** (3.61)	0.156*** (6.42)
β	−2.98*** (16.02)	−5.621*** (4.60)	−2.374** (2.38)	
$\beta 1$				−5.365*** (6.42)
$\beta 2$				−0.860 (1.17)
γ	18.712*** (6.28)	−1.489 (0.15)	18.483 (1.63)	−0.128 (0.02)
δ	−0.952*** (8.22)	−1.69*** (7.78)	−0.639*** (5.25)	−0.985*** (8.44)
R^2	0.52	0.48	0.21	0.53
Obs.	358	130	228	358

	VIX level					
	15%			28%		
S&P 500 move	**Full sample**	**S&P 500 < 0**	**S&P 500 > 0**	**Full sample**	**S&P 500 < 0**	**S&P 500 > 0**
Panel B						
−5%	24.15%	33.11%		11.77%	11.14%	
5%	−5.66%		−7.08%	−18.03%		−15.39%
Allocation to VIX	17.2%	13.1%		29.81%	30.97%	

Note: Results from the regressions of the Equations (1) and (2) using the entire sample from 1990 to 2017 are presented:

$$r_{VIX,t} = \alpha + \beta r_{S\&P500,t} + \gamma r^2_{S\&P500,t} + \delta VIX_{t-1} + \varepsilon_t,$$

$$r_{VIX,t} = \alpha + \beta_1 r^+_{S\&P500,t} + \beta_2 r^-_{S\&P500,t} + \gamma r^2_{S\&P500,t} + \delta VIX_{t-1} + \varepsilon_t,$$

where $r_{VIX,\,t}$ is the return of the VIX index on month t, VIX_{t-1} is VIX index (in % terms) on month $t-1$, $r_{S\&P500,\,t}$ is the return of the S&P 500 on month t, α is a constant, and ε_t is the residual on month t. Panel A reports the coefficient estimates and robust t-statistics (given in parentheses), where the first three columns are the results of the regressions of Equation (1), and the last column is the result of the regression of Equation (2). Panel B reports the predicted VIX returns using the coefficient of the second and third columns of Panel A, depending on S&P 500 move. The row "Allocation to VIX" presents the necessary proportions of VIX in a portfolio with the S&P 500 that yields a portfolio return of zero when the S&P 500 returns ±5%, given the estimates from each regression. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

strategies remove the VIX index when it goes above a certain level and puts the extra portion back into the index. Since volatility mean-reverts, investing in the VIX index when it is low could be seen as likely to provide protection against volatility increases. DeLisle, Doran, and Peterson (2011), Dennis, Mayhew, and Stivers (2006), Cheng (2019), and Giot (2005) document the asymmetric relationship between VIX and the S&P 500 and specifically show that VIX increases and S&P 500 declines are more strongly correlated than VIX decreases and S&P 500 increases. Whaley (2009) documents the mean-reverting nature of VIX and describes its asymmetric nature such that VIX will rise more (less) dramatically during a stock market decline (rally). Furthermore, Simon (2003) notes the tendency of traders to overvalue (undervalue) the equity market when volatility levels are unusually low (high). Consistent with Daigler and Rossi (2006), evidence suggests that investing in VIX when it is low not only provides a hedge against declines in the S&P 500, but will not proportionally penalize investors when the S&P 500 increases.

To demonstrate how this simple trading rule may reduce the expense of the hedge as it becomes less effective at higher levels of volatility, consistent with the findings of Panel B of Table 2, three threshold levels are considered. The first threshold is the long-run average for the VIX index ($VIX^{(MEAN)}$). Using the mean level as a cutoff is a natural candidate, along with mean plus one ($VIX^{(1+SD)}$) and minus one standard deviation ($VIX^{(1-SD)}$) VIX levels based on the distribution of volatility at a given point at time *t*.

Table 3 presents mean monthly excess returns and the CAPM alpha and beta from the regressions of monthly portfolio returns on the S&P 500 monthly market return. The 4-factor alpha and betas from the regression of monthly returns on the three Fama and French (1993) factors (MKT, SMB, and HML) and a MOM is also shown. It is not expected that the Fama–French factors (henceforth FF) will be significant since the underlying asset is the S&P 500 itself but are included just in case. Additionally, the alpha and market beta of a 6-factor model which includes the returns on an ATM straddle and OTM strangle in addition to the 4-factor model are presented. The long ATM S&P 500 straddle returns capture the option-like payoff imbedded within VIX.[11] The OTM strangle factor is also included to capture additional skewness/gamma not present in the ATM options.

A quick note on why the option returns are included. Option-based plays on volatility produce returns distinct from equity-based strategies (see Broadie et al., 2009; Coval & Shumway, 2001), and including the ATM straddle and OTM strangle factors in the explanatory model delineates the proposed VIX strategies' abnormal returns from those of common equity- and option-based returns. Both option positions payoff symmetrically about the strike price when the underlying experiences high volatility (i.e., both highly positive and highly negative returns increase realized volatility). A variance swap (a forward contract where the settlement is the notional value multiplied by the difference in the realized variance and the variance strike) could have been considered, since it also pays off when realized volatility is high, but these contracts are strictly over-the-counter products.[12] Constructing an ATM straddle or OTM strangle, on the other hand, can be easily done in option exchange markets. For this reason, these option positions are more appealing from a general investor's standpoint. Thus, given similar payouts between exchange-traded options and variance swaps, the straddle and strangle provide a natural factor to control for option-like payoffs that depend on high realized volatility.

The results in panel A of Table 3 show that VIX serves as an effective hedge to holdings in the S&P 500. Using no threshold, a portfolio that holds approximately 5% VIX results in a small but insignificant monthly excess return of eight basis points, or almost 1% a year. The alpha is significant after controlling for CAPM, FF, and option-based factors, and is in excess of 21–48 basis points a month. Unlike the excess returns, the alpha captures the benefit of the asymmetric protection of VIX after netting out the market variability. The FF factors are not significant unlike the option factors. The two factors are highly correlated, explaining why the signs are opposite from one another. The net effect is positively related to the returns, meaning that higher volatility results in better portfolio performance. The increase in the alpha for the 6-factor is because VIX returns aren't purely volatility driven; for example, changes in the volatility skew can increase VIX levels while strangle returns are unchanged.

When controlling for the different thresholds of investments two results come into focus. First, excess returns and alphas for the portfolio are highly dependent on threshold level. Too low a threshold, as is the case for $VIX^{(1-SD)}$, and the hedge is not on enough to be effective. Too high a threshold, as in no threshold case, and the cost of the hedge and the speed of mean reversion at extreme levels of volatility hurt the overall returns. For the $VIX^{(MEAN)}$ and $VIX^{(1+SD)}$ portfolios, the excess returns and alphas are between 20 and 70 basis points a month. So, finding the right VIX level to remove the hedge is essential. Second, CAPM betas steadily decline from the $VIX^{(1-SD)}$, a beta of 1, to the $VIX^{(1+SD)}$ threshold, a beta of 0.81. The $VIX^{(1+SD)}$ threshold has a beta like the no threshold case, meaning that it reduces the market risk in a similar fashion, but without having to hedge at higher volatility levels. This implies at a certain level, hedging S&P 500 price moves with VIX becomes ineffective. This threshold level is quite robust to different allocations to VIX, even if the return profiles are different.[13]

To test for the stability of the results two subperiods, 2007–2011 and 2013–2017, are examined. This is done to capture the high volatility period during the mortgage and sovereign wealth crises, and the subsequent low volatility that has been experienced over this recent 5-year period. Both subperiods show that the $VIX^{(1+SD)}$ portfolio results in significant alphas even after controlling for typical factors. The fact that the alpha is significant in the low volatility period is surprising since the average level of volatility over the past 5 years has been significantly lower than the long-term historical mean. This suggests that holding VIX, even when volatility has long, sustained low periods, has added benefits.

Figure 1a,b demonstrates the performance of these VIX portfolios through time.

[11]ATM straddle returns have been used by Broadie, Chernov, and Johannes (2009) and Coval and Shumway (2001) to help explain option or nonnormal returns. We follow a methodology to construct the factor similar to Broadie et al. (2009) who use a ratio of 1:1 long puts and calls that holds the next-month expiration and rolls over each contract at the end of the month to the following expiration. For example, at the end of January 2008, a call and a put are identified that are closest to ATM and expire in March 2008. These two options are purchased at the ask price at the end of January. These options are then sold at the bid price at the end of February and new ATM options are purchased with April expiration dates.

[12]See Carr and Wu (2009) for details on synthesizing and pricing variance swaps.

[13]Results are available upon request.

TABLE 3 VIX index and S&P 500 portfolio returns

	Excess return	CAPM	4-Factor	6-Factor	Excess return	CAPM	4-Factor	6-Factor	Excess return	CAPM	4-Factor	6-Factor	Excess return	CAPM	4-Factor	6-Factor	6-Factor 2007–2011	6-Factor 2013–2017
α	0.08	0.21***	0.21***	0.48***	0.03	0.03	0.03	0.06	0.21**	0.27***	0.27***	0.45***	0.26**	0.39***	0.37***	0.78***	0.64***	1.94***
	(1.12)	(4.02)	(4.03)	(9.48)	(0.63)	(0.76)	(0.74)	(1.26)	(2.43)	(3.28)	(3.21)	(4.79)	(2.52)	(4.25)	(4.12)	(8.52)	(3.67)	(7.91)
β_{MKT}		0.81***	0.81***	0.86***		0.99***	0.99***	1.01***		0.91***	0.89***	0.96***		0.81***	0.81***	0.91***	0.86***	0.66***
		(66.93)	(66.3)	(71.04)		(104.13)	(101.91)	(87.7)		(46.32)	(45.44)	(42.45)		(38.19)	(37.94)	(41.28)	(18.57)	(5.71)
β_{SMB}			0.00	0.00			0.00	0.00			−0.02	−0.01			−0.03	0.00	−0.04	0.05
			(1.71)	(0.59)			(0.18)	(0.11)			(0.79)	(0.43)			(1.06)	(0.14)	(0.49)	(0.57)
β_{HML}			0.00	0.00			0.00	0.00			0.01	0.01			0.05	0.05	0.12	0.09
			(0.67)	(0.79)			(0.27)	(0.23)			(0.35)	(0.41)			(1.72)	(2.02)	(2.24)**	(1.01)
β_{MOM}			0.00	0.00			0.00	0.00			0.02	0.01			0.01	0.01	0.05	−0.03
			(0.13)	(0.01)			(0.69)	(0.51)			(0.98)	(0.78)			(0.56)	(0.53)	(1.74)	(0.47)
β_{ATM}				−0.83**				−0.71**				−2.23***				−2.61***	−3.06**	0.04
				(2.54)				(2.31)				(3.66)				(4.41)	(2.05)	(0.03)
β_{OTM}				1.21***				0.39**				1.51***				2.39***	2.35***	2.73***
				(7.11)				(2.43)				(4.76)				(7.74)	(3.18)	(3.67)
Obs.		263	263	263	263	263	263	263	263	263	263	263	263	263	263	263	60	60
R^2		0.94	0.95	0.96		0.98	0.98	0.98		0.89	0.89	0.90		0.85	0.85	0.89	0.94	0.65

Note: Monthly excess returns and coefficient estimates for portfolios that hold the S&P 500 and hedge using the VIX index from 1990 to 2017 are shown. Four portfolios are tested. The first uses VIX and always holds the index (no threshold case). The next three use a threshold level of VIX where if VIX is above the threshold, the portion of the portfolio that is in VIX is removed and re-invested back into the S&P 500. The first threshold is the long-run average for the VIX index ($VIX^{(MEAN)}$), the second and third are the mean plus one ($VIX^{(1+SD)}$) and minus one standard deviation ($VIX^{(1-SD)}$) based on the distribution of volatility at a given point at time *t*. Results are also present for the ($VIX^{(1+SD)}$) portfolio separated into two time periods; a high volatility period, 2007–2011, and a low volatility period 2013–2017. The α for each portfolio are the excess returns (portfolio—S&P 500), CAPM alpha, 4-factor alpha from the regression of monthly returns on the three Fama and French (1993) factors (MKT, SMB, and HML) and a MOM, and the 6-factor alpha from the regression of monthly returns including the 4-factor model and returns on a ATM S&P 500 straddle and 5% OTM S&P 500 strangle. Robust *t*-statistics are given in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Abbreviations: ATM, at-the-money; MOM, momentum factor; OTM, out-of-the-money.

Assuming an initial investment of one million dollars, Figure 1a shows portfolios that invest in VIX and the S&P 500 from the 1990 to 2017 period, and use an appropriate threshold, have final dollar values that are almost double that investment in the S&P 500 index alone. This performance discrepancy, shown in Figure 1b through the excess returns, is driven by periods of heightened volatility, such as 2000, 2008, and even through short-term shocks with periods of quick mean reversion, like those of August 2015. The excess returns also show how sensitive the returns are to the threshold levels. The no threshold portfolio hardly benefits from periods of high volatility because it fails to remove the hedge and suffers when volatility reverts back to lower levels. This demonstrates the power of adding VIX to a portfolio to capture the asymmetry in performance between volatility and the S&P 500 by effectively timing when to have the hedge on.

Yet these results are not replicated in practice and are in direct contrast with the returns of existing VIX tradable products. Alexander, Kapraun, and Korovilas (2015), Basta and Molnar (2019), Bordonado et al. (2017), and others have

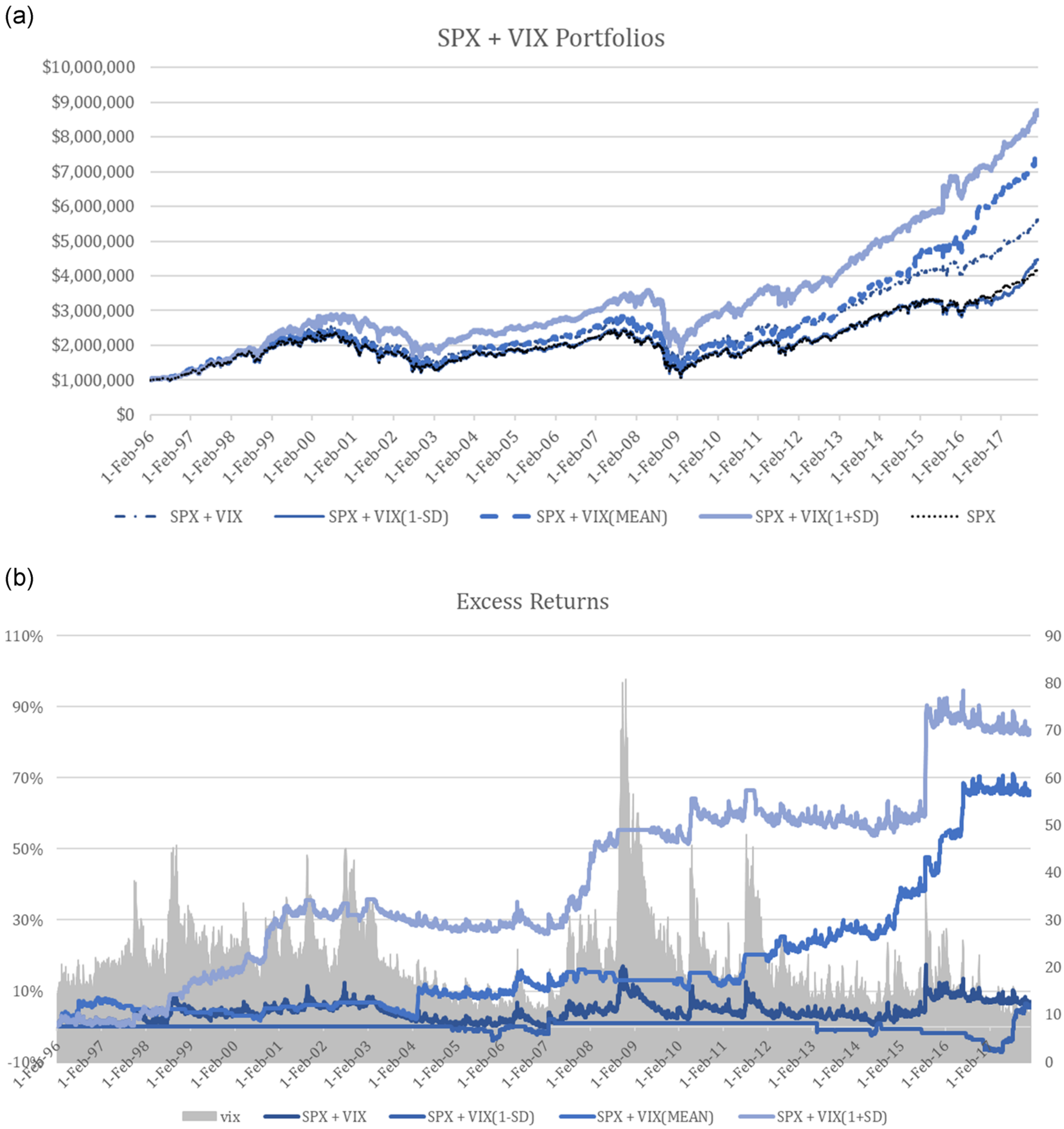

FIGURE 1 Theoretical portfolio incorporating VIX index as a hedge. (a) The performance of investment strategies incorporate the holding of the VIX index as an asset is tracked. The results present the dollar returns for a portfolio that hold VIX unconditionally (SPX + VIX), when VIX is below the historical mean (SPX + VIX$^{(\text{MEAN})}$), mean plus one standard deviation (SPX + VIX$^{(1\ +\text{SD})}$), mean minus one standard deviation (SPX + VIX$^{(1\ -\ \text{SD})}$), and the buy-and-hold S&P 500 portfolio (SPX). The starting value of the portfolio is $1,000,000. (b) Excess return above the S&P 500 for all portfolios along with the VIX index [Color figure can be viewed at wileyonlinelibrary.com]

examined in detail the performance of many VIX products and documented the underperformance.[14] To confirm these findings and make direct comparisons, similar portfolios to the VIX portfolio are formed at the mean plus one standard deviation threshold using VIX futures, VXX, and other volatility instruments. For brevity, the results are summarized.

Portfolios which hold VIX futures substantially underperform those which hold the cash VIX by a difference of 44 basis point per month using the 4-factor model for the 2006–2017 period, and more important, underperform the S&P 500 index itself. So, similar to other's findings, the results show that while VIX futures can dampen periods of high volatility, the overall costs are prohibitive. CAPM, 4-factor, and 6-factor alphas of portfolios hedged with S&P 500 puts, VIX-call hedged portfolios, and VXX show very similar results and significance to those of the VIX futures.[15] While it is entirely possible that the pricing and day-to-day management of these assets are erasing any theoretical improvements for investors, the goal of the paper is not to understand why these products seem to perform so poorly.[16] The goal here is to determine if there is a better alternative. So, the next section will begin to dissect VIX to find out if, and how, there is a more effective way to use it in hedging market downturns.

4 | REPLICATING THE VIX INDEX

4.1 | Understanding the drivers of VIX change

While numerous studies have analyzed the impact of the VIX, or changes in VIX, on the returns of equity portfolios, few have considered what drives the actual changes in VIX itself. Whaley (2009) examines the effect of changes in the S&P 500 index on VIX changes and finds an asymmetric response to those changes. This is not surprising and is confirmed by the results of DeLisle et al. (2011). Recent analysis has focused on price settlement around expiration by Griffin and Shams (2017) but that has less to do with hedging than market manipulation.[17] To this point, there has been no examination based on the actual options that cause VIX to change on a day-to-day basis. Since the VIX is constructed using the two options with maturities closest to 30 days,[18] and all out-of-the money puts and calls on the S&P 500, it is worth testing which options drive the movements of VIX. Since there is an asymmetric response between changes in the S&P 500 and VIX, it is highly likely that puts versus calls, options further out-of-the money, and options closer to expiry are responsible for the changes.

Before deriving an empirical specification to test the relationship between S&P 500 options and VIX changes, the methodology used to calculate the VIX index is briefly summarized. The specification for VIX is given by[19]

$$\sigma_j^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q\left(K_i\right) - \frac{1}{T}\left[\frac{F_j}{K_0} - 1\right]^2, \tag{3}$$

$$VIX = 100\sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right]\right\}\frac{N_{365}}{N_{30}}}, \tag{4}$$

where K is the strike price of the options that are OTM up to options closest to ATM,[20] ΔK is the difference in strike prices, $Q(K)$ is the midpoint price of the option at strike K, T_1 is the expiration of the option with maturity closest to below 30 days, T_2 is the expiration of the option with maturity closest to above 30 days, N is the minutes to expiration and F represents the ATM forward price of the index. Using all OTM strikes available with non-zero bid prices and excluding strikes with positive bid prices that follow two zero bid strikes, Equation (3) constructs a σ_j^2 for both the options, while Equation (4) weights the σ_j^2 to create a 30-day measure of implied volatility.

[14]For further research see also Shu and Zhang (2012) and Zhang, Shu, and Brenner (2010).

[15]All results are available upon request.

[16]For issue related to the pricing a tracking of these products, see Y. Chen and Tsai (2017); Frijns, Tourani-Rad, and Webb (2016); Gehricke and Zhang (2018); and Tang and Xiaoqing (2019).

[17]Their finding shows that deep OTM SPX options can affect VIX calculations at expiration. This is unique to the day of expiration and settlement and is driven by significant purchases done Tuesday before expiration. So, while it is true that these OTM options can significantly affect valuation- it appears to be isolated specifically around options that expire in 1 day and is a temporary affect that is only experienced at market open on VIX settlement.

[18]In October 2014 the VIX calculation started using weekly options instead of only front-month and next-month traditional 3rd-Friday expiries.

[19]A *white paper* available from CBOE at http://www.cboe.com/micro/vix/vixwhite.pdf shows the construction of the VIX index.

[20]Options that has a bid price equal to zero (i.e., no bid) are excluded. Once two options with consecutive strike prices are found to have zero bid prices, no options with lower strikes are considered for inclusion.

Although both equations are straightforward to calculate and recreate the current level of VIX, the nonlinear nature makes replicating the payoff to the VIX index difficult to accomplish. The weight on each option is a function of a root-weighted time variable, which changes daily, making rebalancing costs prohibitive while creating indivisible option units. However, it is possible to unwind the formula into smaller components, allowing for a test of the effect of each option, or the changes in the prices and moneyness of each option, on the changes in VIX.

Taking the natural log of Equation (4) removes the nonlinear term so the equation becomes linear and allows for a simple regression to assess the impact of the individual options on the change in VIX. Since changes are of interest, testing the difference between $\ln(VIX_t) - \ln\,(VIX_{t-1})$, is equivalent to testing $\ln\left(\frac{VIX_t}{VIX_{t-1}}\right)$, or the percentage change in VIX each day. The key explanatory variables to include are whether the option was a call or a put, the time to expiration of the option, the price change, the strike price or moneyness change, and an interaction term between these variables. It is necessary to incorporate an interaction term, since the first term in Equation (2) is multiplicative in these variables.

The following specifications are run, controlling for time variation:

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t} + \varepsilon_t, \tag{5}$$

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t} + \Delta P_{\kappa,\phi,\tau,t} \times CD_t + \Delta KS_{\kappa,\phi,\tau,t} \times CD_t + \varepsilon_t, \tag{6}$$

where $\Delta P_{\kappa,\phi,\tau,t}$ is the change in price of ϕ(call/put) option (×100) with a strike of K and maturity τ at time t. ΔKS represents the change in moneyness of the option, where moneyness is equal to S/K. DM is the time to maturity of the option, and I is the interaction of all three variables. CD is a dummy variable equal to one if the option is a call and zero if the option is a put. Equation (5) examines the effect of price and moneyness changes of all options and calls and puts separately on VIX changes. Equation (6) interacts the call dummy variables with change in price and moneyness variables to capture differences between call and put options. Each specification clusters on the date to avoid overstating the t-stats. The results over the sample period 1996 through 2017 are shown in Table 4.

Ten estimations are conducted. The first seven use Equation (5) and the final three use Equation (6). The first regression utilizes the full sample and the next six examine only puts or calls, and then puts or calls at VIX levels above and below the long-run mean. The final three estimations using Equation (6) look at the full sample then considers just positive or negative VIX changes only.

The first three regressions reveal the power and significance of the change in option price and moneyness on VIX change. The first regression shows that option price changes are significant and positively related to VIX changes. For each dollar increase in put prices, the change in VIX is approximately 3.05%. When separating by puts and calls, the ΔP coefficient is highly significant for put price changes only. This implies that not only are put price changes responsible for the VIX change, but S&P 500 options closer to ATM will have greater impact on VIX because of their higher delta and greater sensitivity to S&P 500 price moves.

The coefficient on ΔKS is negative and significant in all specifications. It is more negative in the call only specification, meaning moneyness changes for calls impact VIX changes more than puts. This coincides with the findings of Griffin and Shams (2017) who find that deep OTM calls are used to manipulate the settlement price of VIX. To understand the degree of impact of a moneyness change, take the example of both a put and call option with a strike of 1,000. Using the coefficients from the second and third regressions, if the S&P 500 changes $\pm$50 points, the moneyness change is -0.05 for both options, translating to 29% (15%) impact on VIX by the call (put). Options close to expiry have a greater impact on VIX changes but its significance and impact is small, as is the interaction term. Initially, it appears that close to ATM put price moves and OTM call moneyness moves are the biggest marginal contributors to the changes in VIX.

When the sample is separated by VIX levels and option type in the fourth through seventh regressions, the effects of price and moneyness changes on VIX changes are affirmed. The coefficient on put price changes when VIX is below the long-run mean is 0.32 higher than when VIX is above the long-run mean, translating to 3.2% greater price effects on VIX per dollar of price. This also holds for the moneyness changes as well. Using the prior example with the SPX at 1,000, the change in SPX price and the change in the moneyness of the call option has a 26% greater impact on VIX when it is below the long-run mean. A smaller effect is observed for the puts moneyness as well, but there is a greater combined impact as the interaction term for puts is negative. This demonstrates similar implications as the first three regressions but highlights a stronger effect for both price and moneyness changes when VIX is below its historical mean.

TABLE 4 VIX factor regression

	ΔVIX	Puts only ΔVIX	Calls only ΔVIX	Puts only VIX > LRM ΔVIX	Puts only VIX < LRM ΔVIX	Calls only VIX > LRM ΔVIX	Calls only VIX < LRM ΔVIX	ΔVIX	ΔVIX > 0	ΔVIX < 0
ΔP	0.305***	0.849***	0.054	0.731***	1.051***	0.124**	0.36***	0.835***	0.815***	0.538***
	(13.94)	(23.6)	(1.29)	(18.69)	(20.57)	(2.18)	(11.3)	(23.85)	(12.28)	(13.76)
ΔKS	−3.963***	−2.988***	−5.83***	−1.957***	−4.382***	−4.557***	−9.761***	−2.993***	−2.184***	−1.705***
	(28.58)	(27.73)	(26.78)	(17.31)	(30.21)	(21.88)	(28.99)	(27.66)	(13.91)	(17.19)
DM	−0.04***	−0.056***	−0.048***	−0.166***	−0.053***	−0.083**	−0.072***	−0.046***	−0.08***	0.009
	(2.99)	(4.28)	(2.72)	(4.51)	(3.94)	(2.09)	(4.68)	(3.39)	(4.17)	(0.66)
$\Delta P \times CD$								−0.85***	−0.862***	−0.267***
								(13.91)	(10.01)	(6.08)
$\Delta KS \times CD$								−2.833***	−2.082***	−1.912***
								(−17.55)	(−9.85)	(−13.26)
I	−0.094	−0.832***	−0.088	−0.622***	−2.872***	0.174	2.603***	−0.63***	−0.131*	−0.464***
	(0.72)	(5.14)	(0.34)	(4.88)	(6.15)	(0.79)	(5.17)	(7.14)	(1.91)	(4.64)
Constant	0.006***	0.008***	0.006***	0.02***	0.009***	0.009**	0.01***	0.007***	0.042***	−0.033***
	(3.43)	(4.51)	(2.69)	(4.77)	(5.01)	(2.31)	(4.68)	(3.86)	(19.6)	(18.21)
R^2	0.472	0.520	0.559	0.607	0.568	0.634	0.645	0.513	0.411	0.311
Obs.	1,451,269	1,037,555	413,714	159,941	877,614	110,734	302,980	1,451,269	683,357	767,912

Note: Results from the following regressions are shown:

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t} + \varepsilon_t$$

and

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t} + \Delta P_{\kappa,\phi,\tau,t} \times CD_t + \Delta KS_{\kappa,\phi,\tau,t} \times CD_t + \varepsilon_t,$$

where $\Delta P_{\kappa,\phi,\tau,t}$ is the change in price of ϕ(call/put) option (scaled by 100) with a strike of K and maturity τ at time t. ΔKS represents the change is moneyness of the option, where moneyness is equal to S/K. DM is the time to maturity of the option, and I is the interaction of all three variables. CD is a dummy variable equal to one if the option is a call and zero if the option is a put. The first regression examines the effect of price and moneyness changes of calls and puts separately on VIX changes. The second regression interacts the dummy variables with change in price and moneyness variables to capture differences between call and put options. The first seven columns show the results of the regression of the first equation and the final three columns show the results of the regression of second Equation (6). LRM is the long-run mean of VIX. Each specification clusters the standard errors by date to avoid overstating the t-statistics. t-Statistics are given in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

When all options are grouped together, but include the dummy variables for the calls, the results and conclusions are supported. The eighth regression coefficients are very similar to the findings in the second and third regressions. The last two specifications account for up and down VIX movements and again show a stronger price and moneyness effect when VIX moves up, highlighting the asymmetric relationship between the S&P 500 and VIX.

These findings, combined with put-call parity, can now be used to estimate which options have the most impact on VIX. Specifically, even if OTM calls are important drivers of VIX changes, through the put-call parity relationship, the results for OTM calls can be extended or replaced by ITM puts. Heretofore, given than more in-the-money put prices changes, and more out-of-the-money call moneyness changes, are more impactful on VIX changes, then it must be the case that ITM puts have the strongest relationship to VIX changes, even though these options aren't directly used in the VIX calculation.

As an example, to show how ITM puts are related to VIX, examine what happened on August 3, 2010. The S&P 500 closed at 1,120.46, falling 5.4 points, and VIX closed at 22.63, up 0.62. Based on the coefficients of the eighth regression, the option that would have the most impact on VIX changes would be the September 1195 put. On this day the option price increased by \$6.2 while the moneynesss changed by −0.00452. The maturity factor had little impact as did the interaction term. Overall, the correlation between price changes of this option and VIX from the beginning to the end of the month was 0.75. This highlights how the results of the regression can be used to guide which options, on average, have the most impact on VIX changes, or are most correlated to VIX changes. That ITM puts are the most responsible/related to VIX changes may be surprising since the typical hedging strategy is to use ATM or OTM puts to hedge against S&P 500 down markets.

Given these findings, and because many investors are interested in a portfolio that hedges against downturns in the market, holding a portfolio that is long ITM S&P 500 puts would appear to be a natural fit. However, as shown in Table 5, and documented by many others, buying puts is expensive.[21] Thus, it seems necessary to not only go long puts, but sell either OTM puts or corresponding calls to offset the cost. To confirm this, the performance of various puts and calls at different strikes are assessed before forming any spread combination. The performance of each option is calculated by entering into the option that expires in month $t+1$ at the beginning of the month t and closing the option at the end of month t. This option best captures the relative time period for the options used in the VIX calculation. Alternatively, it would have been possible to keep track of multiple maturities that are used in the VIX calculation, but focusing only on one option keeps the analysis simple and reduces the expense of rebalancing these maturities within month. Panel A of Table 5 shows the returns to puts and calls ranging from 5% ITM to 5% OTM by 2.5% increments from 1996 through 2017.

It is immediately obvious that regardless of the option chosen, whether it is a put or a call, or at a certain strike, that consistent option buying results in negative average returns. This holds over monthly, weekly, or daily investment time horizons.[22] This is reassuring as it has been consistently shown that selling options should result in a positive return and is typically associated with the volatility/insurance premium (see Bakshi & Kapadai, 2003; Carr & Wu, 2009). The average and median dollar monthly returns are negative for all the options, with OTM puts ($S/K=1.05$) having the highest percentage loss (−\$5.27/\$14.96 = 35.2%) and lowest probability of finishing with a positive return (18.6%). The daily correlations between the price change of the options and VIX changes are above 0.8 for all the puts, and between −0.38 and −0.61 for the calls. When the sample investment is restricted to only invest in the option if VIX is below the mean plus one standard deviation, there is little change in the correlation and cost of the option, but there is significant performance improvement for the puts. This result is consistent with the evidence shown in Tables 2 and 3. The ITM put options ($S/K=0.95$) percentage return improved from −7.1% to −5.1%. By comparison, call performance got worse. OTM call ($S/K=0.95$) returns fell from −19% to −27%. While the direction of the performance improvement is not surprising, the magnitude may be, and is useful in the construction of a better tradable VIX alternative and hedging instrument.

Call and put spreads are shown in Panel B of Table 5. The performance improvement of buying a spread versus buying an option outright is clear. Take the case of the ITM-OTM put spread. The average dollar loss for this investment per month is −\$0.31 which is a marked improvement over the outright position. This gain is coming from the greater decay from the OTM position relative to the ITM but without the expense of giving up correlation to VIX. When the investment is restricted to just below the mean plus one standard deviation levels for VIX, the average dollar return is positive, \$0.41. The ITM-OTM spread outperforms the ATM-OTM spread, which again is a function of the lower decay in the ITM option. For the call spreads, the ITM-OTM spread outperforms the ATM-OTM spread, and performs worse when the investment is restricted to when VIX is below the mean plus one standard deviation.

[21]For example, Aït-Sahalia, Wang, and Yared (2001); Bakshi and Kapadia (2003); Bollen and Whaley (2004); Bondarenko (2003); Coval and Shumway (2001); Jackwerth (2000); and Liu, Pan, and Wang (2005) generally find that the historical costs of puts, particularly OTM and ATM puts, are too expensive to be justified.

[22]Results available upon request.

TABLE 5 Monthly option returns

	ATM	$S/K = 0.95$	$S/K = 0.975$	$S/K = 1.025$	$S/K = 1.05$	ATM	$S/K = 0.95$	$S/K = 0.975$	$S/K = 1.025$	$S/K = 1.05$
Panel A										
Average Cost	\$32.57	\$77.99	\$50.55	\$21.63	\$14.96	\$33.44	\$7.69	\$17.00	\$55.36	\$79.89
% Cost of S&P 500	2.55	5.84	3.86	1.73	1.22	2.65	0.68	1.43	4.25	6.04
Avg. Dollar Return	−\$6.62	−\$5.57	−\$6.33	−\$6.11	−\$5.27	−\$2.40	−\$1.48	−\$2.11	−\$1.91	−\$0.93
Median Dollar Return	−\$14.58	−\$12.00	−\$15.05	−\$10.95	−\$7.20	−\$5.75	−\$1.85	−\$5.80	−\$4.13	−\$1.65
\$ Skew	1.640	0.730	1.090	2.208	2.582	0.783	2.309	1.334	0.408	0.165
Correlation to VIX	0.831	0.815	0.823	0.833	0.826	−0.559	−0.384	−0.476	−0.611	−0.648
% Positive Return	29.3	38.0	33.8	23.6	18.6	39.9	24.7	33.8	45.2	49.0
VIX < (Mean + SD)										
Average Cost	\$30.69	\$78.51	\$49.48	\$19.54	\$12.98	\$31.54	\$5.77	\$14.75	\$54.47	\$80.18
% Cost of S&P 500	2.26	5.60	3.58	1.45	0.97	2.36	0.47	1.16	3.98	5.79
Avg. Dollar Return	−\$5.63	−\$4.03	−\$5.08	−\$5.22	−\$4.44	−\$3.08	−\$1.58	−\$2.61	−\$2.67	−\$1.79
Median Dollar Return	−\$13.25	−\$9.38	−\$12.75	−\$9.95	−\$6.90	−\$6.10	−\$1.75	−\$5.85	−\$4.25	−\$2.05
\$ Skew	1.658	0.631	1.015	2.361	2.934	0.837	2.583	1.431	0.440	0.194
Correlation to VIX	0.831	0.818	0.827	0.827	0.816	−0.564	−0.318	−0.449	−0.631	−0.676
% Positive Return	30.1	39.3	34.5	24.0	18.3	38.4	22.3	31.9	44.5	48.0

	Put spreads		Call spreads					
	P95-P105	ATM-P105	C105-C95	CATM-C95	1st and 4th	1st and 3rd	2nd and 4th	2nd and 3rd
Panel B: Spreads								
Average Cost	\$63.02	\$17.60	\$72.20	\$25.75	\$37.27	−\$9.18	−\$8.15	−\$54.60
% cost of S&P 500	4.62	1.34	5.36	1.97	2.65	−0.74	−0.64	−4.02
Avg. Dollar Return	−\$0.31	−\$1.36	\$0.54	−\$0.92	\$0.62	−\$0.85	−\$0.43	−\$1.90
Median Dollar Return	−\$3.55	−\$6.15	\$3.95	−\$2.44	−\$0.18	−\$7.77	−\$4.05	−\$9.55
\$ Skew	0.172	0.977	−0.173	0.385	−0.051	0.172	0.172	0.428
Correlation to VIX	0.663	0.681	−0.662	−0.590	0.644	0.663	0.662	0.676
% Positive Return	46.8	35.0	53.6	46.4	49.4	46.4	46.4	42.6
VIX < (Mean + SD)								
Average Cost	\$65.52	\$17.71	\$74.41	\$25.77	\$39.75	−\$8.89	−\$8.06	−\$56.70
% cost of S&P 500	4.63	1.29	5.32	1.89	2.74	−0.69	−0.60	−4.04
Avg. Dollar Return	\$0.41	−\$1.19	−\$0.21	−\$1.50	\$1.91	\$0.62	\$0.31	−\$0.98
Median Dollar Return	−\$0.95	−\$5.40	\$1.45	−\$3.47	\$1.90	−\$2.40	−\$1.40	−\$7.13
\$ Skew	0.106	0.959	−0.105	0.478	−0.129	0.105	0.104	0.374
Correlation to VIX	0.709	0.737	−0.707	−0.619	0.685	0.709	0.708	0.726
% Positive Return	48.9	35.8	51.5	43.7	52.0	48.5	48.5	44.1

Note: Monthly average and median dollar returns of S&P 500 puts and calls ranging from 5% ITM to 5% OTM by 2.5% increments from 1996 through 2017 are shown. The monthly performance of each option is calculated by entering into the option that expires in month $t + 1$ at the beginning of the month t and closing the option at the end of month t. Moneyness is equal to S/K where S is the sport price of the S&P 500, and K is the strike of the option. *Average Cost* is the average price of the option contract over the sample period. *% Cost of the S&P 500* is cost of the option divided by S&P 500 price. *% Positive Returns* is the percentage of months that the option position results in a positive monthly return. Panel A shows the returns to individual option positions. Panel B shows spread returns, where P95-P105 is an 5% ITM-5% OTM put spread and 1st and 4th is long the P95-P105 spread and short the CATM-C95 call spread. Each panel includes the returns for all time periods and the returns when VIX is below its long-run mean plus one standard deviation (VIX < (Mean + SD))
Abbreviations: ATM, at-the-money; OTM, out-of-the-money.

The final step is to combine the two spreads; buying the put and selling the call spread. Based on the individual spreads, buying the ITM-OTM put spread and selling the ATM-OTM call spread should result in the best performance. When combined with the VIX level threshold, this investment not only has positive average returns, \$1.91, but median returns (\$1.90) as well. It maintains a high level of correlation to VIX (0.685) and has an average cost similar to buying an ATM put outright.

These results, and combined with the results of Table 2, suggest the best way to form a portfolio that replicates VIX and has lower cost, will buy monthly an ITM-OTM put spread and sell an ATM-OTM call spread. This will allow the portfolio to benefit from volatility shocks by having a put spread while also having relatively low cost from selling a call spread. This call spread not only helps to fund the volatility and price protection, but when combined with the net long-equity position will have a payoff structure similar to a covered call. However, this means that the portfolio is susceptible to underperformance if there are big upward moves in the market, but as a hedging instrument, this structure initially appears to be a better alternative to existing VIX-based products.

4.2 | Performance of VIX replication (VIX^{o+})

To test the effectiveness of this structure, the option position is incorporated in a portfolio in a similar fashion to the cash VIX in Section 3.2, and tradable VIX-based products. Again, the reason for this simple structure, the use of a single maturity, and the monthly rollover, was not only for the ease in understanding, but its ability to be used in practical applications. The investment in this long ITM-OTM put spread and short ATM-OTM call spread (VIX^{o+} henceforth) will be used in two ways. The first portfolio will always invest in the hedge and the second will only invest when the VIX index is below the mean plus one standard deviation level ($VIX^{o+(1+SD)}$). The portfolio will hold the equivalent number of options corresponding to 100 shares in the index, while holding a percentage of the portfolio in the options dictated by the regression in Equation (1). Similar to the process for the allocation to the cash VIX, the weight in VIX^{o+} for all possible return outcomes, both negative and positive, leads to an allocation of 5.05% to the hedge. So, for a $1 mm portfolio, $50,500 would be used to pay for the cost of VIX^{o+}. There is no need to worry about the margin since the short call is covered. This is done every month when VIX is below the threshold, otherwise 100% of the portfolio is in the S&P 500.

Figure 2 shows the returns to a portfolio that initially invests $1,000,000 in the S&P 500 only, the S&P 500 and $VIX^{(1+SD)}$, the S&P 500 and the VIX^{o+}, and the S&P 500 and the $VIX^{o+(1+SD)}$, starting January 1996 and ending in December 2017.

The annualized monthly return to the $VIX^{o+(1+SD)}$ portfolio is 10.1% with a volatility of 15.2%. This return is 1.8% higher than the S&P 500 buy-and-hold portfolio while being 3.7% less risky, even after accounting for bid-ask spreads and transaction costs. It underperforms the VIX portfolio by 1.3%, which is expected, since the options embed premium that are not in the VIX index.

The excess return $VIX^{o+(1+SD)}$ portfolio relative to the S&P 500 comes from significant outperformance in negative markets, 18% on average, without giving up much in positive markets, −5% on average. For example, in 2000 and 2008 the

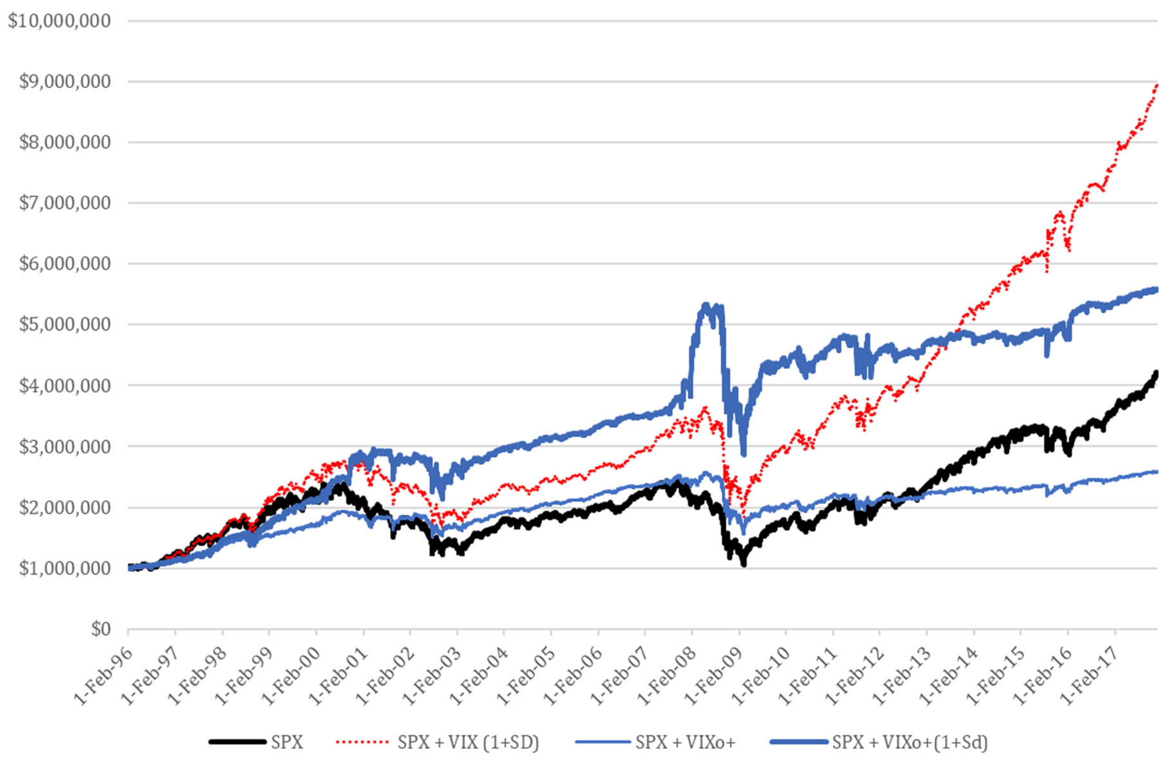

FIGURE 2 VIX^{o+} portfolio returns. The value of portfolios that consist of the buy-and-hold S&P 500, the S&P 500, and VIX using the mean plus one standard deviation threshold ($SPX + VIX^{(1+SD)}$), the S&P 500 and option position with no threshold (VIX^{o+}), and the S&P 500 and option position using the mean plus one standard deviation threshold ($VIX^{o+(1+SD)}$) is tracked. The results are show for the 1996–2017 period. The starting value of the portfolio is $1,000,000 [Color figure can be viewed at wileyonlinelibrary.com]

$VIX^{o+(1+SD)}$ portfolio outperformed the S&P 500 by 45% and 34%, respectively. By limiting significant portfolio drawdowns, allows the $VIX^{o+(1+SD)}$ portfolio to benefit through time by maintaining a higher capital base even while being outperformed during positive S&P 500 markets. The $VIX^{o+(1+SD)}$ even outperformed the cash VIX during high volatility periods (tech bubble crash (2000–2002) and early 2008) because the put option positions were able to retain value even as the VIX index fell. So, puts outperformed during these periods as the calls brought the overall level of VIX down. However, since 2011, the $VIX^{+o(1+SD)}$ position has underperformed the S&P 500 on a return basis (but not risk adjusted). This is because covered calls have performed quite poorly. Obviously, removing the hedge at optimal times makes a significant difference, as the VIX^{o+} portfolio, that always has the hedge on, underperforms the S&P 500, and maybe highlights why investors are reluctant or have difficulty hedging. For most investors, hedging is a problem of cost and time; it is difficult to find the right hedge and how long to leave that hedge on. Yet it is not just about the timing, as demonstrated by the poor performance of the existing tradable products using the same on/off threshold levels. That is what the $VIX^{o+(1+SD)}$ portfolio demonstrates. It is a function of first, the right kind of assets and second, when to remove those assets so the hedge does not become overly expensive. The downside to this is that removing a hedge will inevitably miss some market shocks.

The tests of the abnormal monthly performance of $VIX^{o+(1+SD)}$ and VIX^{o+} are shown in Table 6. The alphas from the CAPM, 4-factor, and 6-factor models are significant in all specifications at at least the 5% level for $VIX^{o+(1+SD)}$. For VIX^{o+} the alphas are positive but insignificant. The ATM and OTM factors are significant, which is not surprising given that the options are used in the construction of the portfolio. The direction and power of the factors are similar to the $VIX^{(1+SD)}$ results, implying $VIX^{o+(1+SD)}$ has a similar skewness and kurtosis response as the VIX index. However, the coefficient on the MKT factor is a lot lower than the coefficient on MKT in the VIX regression, coming from put outperformance in down markets as well as the calls limiting the S&P 500 upside potential. This suggests that the replicating portfolio does a good job of capturing the hedging benefits of holding the VIX index even if it does limit some upside.

Using different levels of VIX as the threshold level has a minimal effect on the overall return to the replicating portfolio as long as the threshold level of VIX is between the mean level of VIX to the mean plus-two-standard deviation threshold. If a threshold VIX level below the mean is used, for example, then the hedge does not capture large declines in the S&P 500 and becomes moot as a hedge. Similarly, if the threshold is above mean VIX levels plus two standard deviations, the hedge becomes too expensive to hold long-term, causing significant portfolio drag. This highlights not only the importance of finding the right hedging instrument, but the timing as well. Using the simple mean-reverting properties of VIX, and the long-term levels as guidance, provides an excellent approach to managing portfolio risk.

TABLE 6 Synthetic VIX and S&P 500 portfolio returns

	VIX^{o+}				$VIX^{o+(1+SD)}$			
	Mean	**CAPM**	**4-Factor**	**6-Factor**	**Mean**	**CAPM**	**4-Factor**	**6-Factor**
α	0.384***	0.104	0.085	0.050	0.797***	0.505***	0.443***	0.398**
	(2.90)	(1.59)	(1.31)	(0.79)	(4.20)	(3.41)	(3.06)	(2.59)
β_{MKT}		0.439***	0.436***	0.491***		0.459***	0.458***	0.548***
		(29.06)	(28.65)	(32.48)		(13.37)	(13.53)	(14.79)
βATM				−0.042***				−0.067***
				(10.27)				(6.72)
β_{OTM}				0.016***				0.027***
				(7.75)				(5.15)

Note: Monthly excess returns and coefficient estimates for portfolios that hold the S&P 500 and hedge using the VIX index from 1990 to 2017 are shown. Two portfolios are tested that include the long ITM-OTM puts spread and the short ATM-OTM call spread (VIX^{o+}) with the S&P 500. The first uses VIX^{o+} and always holds the option position (no threshold case). The second uses a threshold level of VIX where if VIX is above the mean plus one standard deviation ($VIX^{o+(1+SD)}$), the portion of the portfolio that is in VIX is removed and re-invested back into the S&P 500. The α for each portfolio are the excess returns (portfolio—S&P 500), CAPM alpha, 4-factor alpha from the regression of monthly returns on the three Fama and French (1993) factors (MKT, SMB, and HML) and a MOM, and the 6-factor alpha from the regression of monthly returns including the 4-factor model and returns on a ATM S&P 500 straddle and 5% OTM S&P 500 strangle. Robust *t*-statistics are given in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Abbreviations: ATM, at-the-money; MOM, momentum factor; OTM, out-of-the-money.

4.3 | Hedging challenges

There is a specific downside to this approach; removing the hedge at high levels of VIX. $\text{VIX}^{o+(1+SD)}$ cannot capture the extreme positive skewness and kurtosis in the VIX index and is subject to situations where volatility remains high for extended periods. Hedging extreme skewness and kurtosis in VIX, which typically coincides with sharp and extended declines in the S&P 500, is very costly and not worth implementing consistently over a long period. For example, from September through November of 2008, when the S&P 500 fell over 400 points and VIX increased from a level of 20 to the high 80s, the $\text{VIX}^{o+(1+SD)}$ would have offset some but not all of the losses. This also would have occurred in March and September/October of 2001, July 2002, May 2010, and August 2011, when VIX spiked and the S&P fell sharply. In total, the hedge portfolio would not be able to cover significant market losses of greater than 5% in 28 months in the sample. This does not mean the portfolio underperformed, but it is not be able to capture the full extent of the market losses.

It is not clear whether having a hedged portfolio that can result in positive returns in these extreme situations is worthwhile in the long-run. The average loss in these 28 months for the S&P 500 is −7.82% while the VIX^{o+} portfolio's loss is −3.73% and the $\text{VIX}^{o+(1+SD)}$ portfolio is −5.97%. That is still a significant reduction in the drawdown without the added expense that can mute normal returns. These tail risks, if hedged consistently, would cost more than the total gains from the extreme event. This is partially revealed in the VIX^{o+} portfolio returns, as VIX^{o+} performs 4% better than the S&P 500 over these −5%+ months but underperforms compared to the S&P 500 through time. This leaves the portfolio manager with a difficult choice, namely, whether to buy expensive insurance or expose the portfolio to significant jump/shock risk. Since the $\text{VIX}^{o+(1+SD)}$ portfolio captures the asymmetric relationship between volatility and returns, it can capture most of the downside risk investors wish to hedge. The fact that the portfolio does not capture the extreme negative skewness in the S&P 500 is a tradeoff to hedging most, but not all, of the downside risk.

A potential alternative to hedging this risk is reverting to VIX calls during these high volatility periods. Adjusting the portfolio to incorporate VIX calls as a hedge when VIX is above the mean plus one standard deviation, instead of the VIX^{o+} position, can capture the extreme negative skewness without the cost of S&P 500 puts. Using the calls at these levels is preferable because the cost of the calls does not dampen the potential returns from volatility mean reversion. Take for example investing in 5% OTM VIX calls when the VIX is above the mean plus one standard deviation levels. The average return for these calls is 89% in these periods, where on average, investing in these calls every month would return −5.7%. The return is driven by the volatility moves in October 2008 and September 2011, whereas most other occurrences of high volatility periods result in a net loss. Additionally, it would not capture Flash Crash in May 2010 or the volatility spike of February 2018. So, while this particular approach can provide these extreme payoffs, it is limited to almost one-off scenarios, which highlights the challenges of capturing "Black Swan" events.

5 | CONCLUSION

The results illustrate the feasibility and effectiveness of attempting to hold volatility as an asset class to avoid market shortfalls by hedging downside risk. Since the current asymmetric relationship between the VIX index and the S&P 500 generates the strongest negative correlations when the market is falling, holding the VIX is a natural candidate for hedging market risk. In fact, the results show that if VIX were investable, a portfolio comprised of VIX and the S&P 500 would provide returns and risk levels that far outpace the traditional buy-and-hold portfolio.

Of course, these initial results are hypothetical, and more so, not perfectly replicable via investable VIX products because of multiple issues, such as pricing in anticipated VIX mean reversion and the inability to arbitrage pricing differences between cash VIX and VIX futures. As others have shown, a portfolio holding tradable VIX assets in combination with the S&P 500 significantly underperforms.

Understanding what is responsible for the changes in VIX is critical to assessing what drives the asymmetric relationship between market prices and volatility. The options most correlated to VIX changes are ITM puts, even though ITM puts are not a part of the VIX calculation. With this understanding, a portfolio can be formed that holds a combination of long and short S&P 500 option contracts that best captures the increase in VIX while keeping expenses low. This option portfolio is quite liquid and performs well in capturing the increases in VIX and falls in the S&P 500 without proportionally penalizing the portfolio when the market increases. The aspect it cannot capture is the extreme positive skewness in volatility. This again highlights the difficulty in systematically hedging potential "Black Swan" events beyond market timing or incurring significant long-term costs.

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DATA AVAILABILITY STATEMENT

All data that support the findings of this study are available from the CBOE and Optionmetrics. The data that support the findings of this study are available from the corresponding author upon reasonable request. Restrictions apply to the availability of these data that were used under license for this study.

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