Bull and Bear Markets During the COVID-19 Pandemic *

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Abstract

The COVID-19 pandemic has caused severe disruption worldwide. We analyze the aggregate U.S. stock market during this period, including implications for both short and long-horizon investors. We identify bull and bear market regimes including their bull correction and bear rally components, demonstrate our model's performance in capturing periods of significant regime change, and provide weekly forecasts that improve risk management and investment decisions. An investment strategy that uses out-of-sample forecasts for market states outperforms a buy and hold strategy during the pandemic by a wide margin, both in terms of annualized returns and Sharpe ratios.

Key Words:
COVID-19, equity returns, Markov switching, predictive density, investment strategies

JEL Classification Codes: G1,G17,G11,C32,C58

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1 Introduction

This paper dates and forecasts bull and bear markets for the COVID-19 pandemic period based on aggregate equity return data from 1885-2020. We document market phases prior to the pandemic; and then use both smoothed and out-of-sample forecasts to analyze what has happened to equity markets during 2020. Appying those forecasts to risk management and investment strategies during the pandemic period, demonstrates the value of using a 4-state Markov-Switching (MS) model to direct the dynamics of the distribution of stock market returns.

There are several reasons for using a restricted 4-state Markov-Switching (MS) model, as in Maheu et al. (2012). First, unlike *ex post* dating methods (Pagan & Sossounov 2003, Lunde & Timmermann 2004), the Markov-Switching model treats the market states as latent and provides probability estimates for future states and regimes. This probability framework provides a full specification of the data generating process. As such, it generates probability statements about the market dynamics which are essential for investment and risk management decisions.

Secondly, in contrast to simple specifications which focus on two states of the market, our model allows for four states including bull corrections and bear rallies. Conventional methods of partitioning regimes do not identify intra-regime dynamics that can be very important for forecasts and investment decisions. For example, if a rally (positive sub-trend) starts during a bear regime, what is the probability that it will continue and transition to a bull market regime as opposed to falling back to the bear market state? Will a bull market correction (negative sub-trend) continue to a bear regime or recover to stay in the bull market regime?

Thirdly, higher-order moments of the state-specific distributions also provide useful information – for example, risk assessments associated with different states. They combine to provide the aggregate mixture distribution that governs the market dynamics and the associated predictive mean and density forecasts that are key for decisions.

Finally, the parameters and states are identified with economic restrictions that are consistent with investors attitudes to market phases. The data supports these economically motivated restrictions compared to an unrestricted 4-state model.

We find that the market moved from a bull state at the start of 2020 to a bull correction and quickly to a bear market on February 26. This bear market dominated until June 3 when the market transitioned to a bear rally and has remained in this phase. This is in contrast to the dating methods used in the popular press ("New bull market in stocks could last three years and may produce another 30% in gains, veteran strategist says", Aug 31, 2020, https://www.marketwatch.com/). Those methods focus solely on price trends and ignore risk. Our model classifies the recent market into the bear rally due to the larger variance in returns. This elevated risk is not consistent with past bull market phases that display lower variability.

Traditional 2-regime models using *ex post* dating methods are unable to distinguish between bear rally and bull market states. Our approach does this, in part, due to probability estimates of risk differences across those states. We show the dramatic impact that the COVID-19 pandemic has had on the return distribution and risk measures. The model provides very accurate forecasts of turning points and these would have been available in real-time to investors.

Given the benefits of a full probability model of stock market phases, it is natural to ask whether forecasts from this mixture-distribution model can improve investment and risk management decisions. To this end, we define a pseudo Sharpe ratio to characterize in-sample estimates of the state density parameters. We then extend this measure to an out-of-sample predictive Sharpe ratio, derived from the predictive density of returns which is sensitive to the forecasted market states. This measure can be useful in assessing the risk and return of entering the market.

Several market-timing investment strategies are explored. We show that simple timing rules directing when to exit and enter the market lead to improved investment decisions relative to a buy and hold strategy in 2020. These results are robust to different timing strategies and are a result of the precise turning points our model identifies and forecasts. Each of these market-timing strategies are in real time and would have been available to an investor using the model for forecasts.

We have implemented a number of model comparisons to evaluate the robustness of our results, including log-predictive likelihood values for the alternative models, none of which improve on our proposed model. Details and further robustness tests are reported in the Appendix which also includes additional results, including long-horizon forecasts of state probabilities four weeks ahead throughout 2020 and one year ahead from the final week of our sample.

Our paper is organized as follows. Sections 2 briefly reviews the structure and estimation of the restricted 4-state Markov-Switching bull and bear market model. Section 3 summarizes the data and reports the results for the pre and post COVID-19 periods. Notably, Section 3.3 provides forecasts based on one-week-ahead predictive densities including out-of-sample forecasts of future states and the associated risk and return measures. Section 3.4 reports market-timing strategies that exploit those forecasts. Section 4 reports comparisons of our model to competing models. Section 5 concludes.

2 Model and Estimation

Define log-returns as r_t, $t = 1, \ldots, T$ and $r_{1:t-1} = \{r_1, \ldots, r_{t-1}\}$. Consider the following 4-state Markov-Switching (MS4) model, based on Maheu et al. (2012), for returns:

$$\begin{aligned} r_t|s_t &\sim N(\mu_{s_t}, \sigma^2_{s_t}) \\ p_{ij} &= p(s_t = j|s_{t-1} = i), \quad i = 1, ..., 4, \;\; j = 1, ..., 4. \end{aligned}$$

in which $s_i, i = 1, ..., 4$, denote the latent states, parameterized as Normally distributed with mean μ_{s_i} and variance $\sigma^2_{s_i}$, and $p_{i,j}$ denote the state transition probabilities.

The following restrictions and labels are imposed for identification purposes,

$$\begin{aligned} \mu_1 &< 0 \;\text{ (bear state)}, \\ \mu_2 &> 0 \;\text{ (bear rally state)}, \\ \mu_3 &< 0 \;\text{ (bull correction state)}, \\ \mu_4 &> 0 \;\text{ (bull state)}. \end{aligned}$$

No restriction is imposed on $\{\sigma^2_1, \ldots, \sigma^2_4\}$. Note that there are four states but we refer to two distinct regimes by B_t as

$$\begin{aligned} B_t = 1 \quad &\text{if} \quad s_t = 1 \text{ or } 2 \;\text{ (bear regime)}, \\ B_t = 2 \quad &\text{if} \quad s_t = 3 \text{ or } 4 \;\text{ (bull regime)}. \end{aligned}$$

The transition matrix takes the following form.

$$\boldsymbol{P} = \begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{pmatrix}$$

This specification implies that bear states and bear rally states cannot move to a bull correction and bull states and bull correction states cannot move to a bear rally state. This is done to avoid confounding these states that display common mean trends up or down in prices. For instance, it would be difficult to separate states 1 and 3 which both display negative average growth. The restricted transition matrix also means that the start of a new regime must be from a bear state or a bull state.

To further enforce economic restrictions on this model, the following long-run trends are imposed on the model. Solving for the stationary distribution associated with P we can compute the vector of unconditional state probabilities:

$$\boldsymbol{\pi} = (\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{e} \tag{2.1}$$

where $\boldsymbol{A}' = [\boldsymbol{P}' - \boldsymbol{I},\ \boldsymbol{\iota}]$ and $\boldsymbol{e}' = [0, 0, 0, 0, 1]$ and $\boldsymbol{\iota} = [1, 1, 1, 1]'$. The long-run restrictions are

$$E[r_t|\text{bear regime}, B_t = 1] = \frac{\pi_1}{\pi_1 + \pi_2}\mu_1 + \frac{\pi_2}{\pi_1 + \pi_2}\mu_2 < 0 \tag{2.2}$$

$$E[r_t|\text{bull regime}, B_t = 2] = \frac{\pi_3}{\pi_3 + \pi_4}\mu_3 + \frac{\pi_4}{\pi_3 + \pi_4}\mu_4 > 0. \tag{2.3}$$

We perform posterior simulation with Gibbs sampling steps which reject any draws that violate the parameter restrictions, coupled with the simulation smoother of Chib (1996) to sample the latent state vector. Estimation follows from Maheu et al. (2012) with the same priors employed in our analyses. For the MCMC output simulation, consistent posterior moments or predictive density quantities can be computed and are detailed in Maheu et al. (2012). We collect $30,000$ posterior draws for inference after dropping an initial $5,000$ draws for burn-in.

3 Data and Results

Daily equity capital gains for 1885 - 1927 are from Schwert (1990). Equity data from 1928 - 2020 use the S&P500 index daily adjusted close reported by Yahoo Finance (ĜSPC symbol). Risk-free return data are from the U.S. Department of the Treasury. From these data, weekly continuously compounded returns, scaled by 100, are obtained for 1885 - 2020. Weekly returns are computed using Wednesday data and Thursday if Wednesday data is missing. The last observation is November 25, 2020. In the following we refer to the equity index returns as the S&P500 or the market. A matching weekly realized variance measure RV_t, is computed as the sum of intra-week daily squared returns. Summary statistics for the weekly data are reported in Table 1.

Although, in what follows, our model has been applied to weekly returns associated with a broad equity market index, it could also be applied to other asset classes at the individual security or portfolio level. For example, the latter could include balanced equity - fixed income portfolios and risk-parity portfolios.

Full sample parameter estimates for the four states are found in Table 2. Those estimates include posterior means and 0.95 probability density intervals for μ_i and σ_i associated with each of the four states. Below those estimates, we report a pseudo Sharpe ratio, μ_i/σ_i, for each state i for $i = 1, 2, 3, 4$. This measures the expected return adjusted for risk assuming a zero risk-free rate for each state. The parameter estimates are broadly similar to those in Maheu et al. (2012). The average return in the bear states, -0.94, is more negative than -0.11 in the bull correction phases of the bull regimes. Analogously, the upward trend for returns in bull states (0.52) is stronger than 0.23 in the bear rally phase of the bear regime. Combining the return estimates with state volatilities, the Sharpe ratios are consistent with economic intuition, ranked from highest to lowest in the bull, bear rally, bull correction and bear states respectively. Note that the 0.95 probability density intervals do not overlap for these estimates. For example, the 0.95 probability density interval $(0.06, 0.13)$ associated with the pseudo Sharpe ratio for the bear rally (state 2), is very different than that $(0.35, 0.65)$ for the bull market (state 4). These estimates highlight the fact that a distinction between a bear rally and a bull state is worth modeling.

The posterior means of the state transition matrix $\boldsymbol{P}$ indicate persistence of bull and bear regimes in that states within the bear regime ($s_t = 1, 2$) and those in the bull regime ($s_t = 3, 4$) are likely to cluster together so that moves between regimes will be infrequent. Interestingly, a bull correction state is much more likely (0.097) to transition back to the bull state than to move to a bear state (0.013); whereas, a bear rally state is more likely (0.019) to transition to the bull state than it is to fall back to a bear state (0.013). Furthermore, regime changes are almost always transitions from a bear rally or from a bull correction. Directly jumping from a bull state to a bear state or vice versa is very rare.

Table 3 reports the unconditional probabilities associated with the four states from which one can compute the unconditional probabilities for the regimes. For example, the bull market regime has a long-run probability of 0.672. Using the formulae in equation 2.2, one can compute that the associated long-run mean of weekly returns in bull regimes as 0.186; whereas, that for bear regimes is -0.069.

3.1 Before COVID-19

Figure 1 displays the cumulative log-return and realized volatility (top), the probability of a bull regime $P(B_t = 2 \mid r_{1:T})$ (middle), and the probabilities of the 4 individual states (bottom) during 2019, the year before the COVID-19 pandemic. Recall that the bull regime combines the probabilities associated with the bull market and bull correction states. Very early in 2019, the market moved from a bear rally state into the bull state. Throughout 2019 the model decisively identifies a bull regime with fluctuations between the bull state and bull corrections.

3.2 After COVID-19

Figure 2 reports the same information as Figure 1 but for the year 2020 during which COVID-19 erupted. The year began with strong evidence of a bull market, although with the probability of a bull correction building. The week of January 29 began a sequence of large negative drops in the market leading to increasing evidence of a transition from a bull market to a bear market. By February 26, the market had transitioned decisively from the bull correction state to the bear market state. By April the probability of the bear market state declined until April 22 revealed a transition to a bear rally state. At the end of our estimation window

(November 25, 2020), the probability of a bear rally state is still very high at 0.824. This is because, even with a visible upward trend in the index since April, the market volatility is still too high and variable to be consistent with a typical bull regime. This conclusion is supported, in part, by the observation that the weekly realized volatility (red line in the top panel) is variable, with spikes (for example, during September and October 2020), and higher on average in comparison to its historical average value 1.94 from Table 1.

3.3 Forecasts

Although one can date stock market cycles after the fact with smoothed probability estimates, it is much more difficult to forecast changes out-of-sample. In this section, we report results for which the model has been estimated at each point t using all past data $1, \ldots, t$ to produce a forecast of the state one week ahead $t+1$.

Figure 3 uses these model forecasts of the state probabilities one week ahead to generate out-of-sample regime forecasts. This figure compares the out-of-sample forecast of the market regime probabilities with the full-sample (smoothed) probability estimates. It shows a relatively accurate week-by-week classification of regimes that was available in real-time to an investor using the model to forecast stock market cycles.

One challenging period for the model forecasts was late August to early September. From July to September the index displayed a strong positive trend, and the model forecasts allocated a nontrivial probability to a bull regime. In hindsight, this episode is precisely identified as a bear regime using the smoothed estimates.

The breakdown of the regime forecasts into the constituent state forecasts one week ahead in seen in Figure 4. Given this additional disaggregation, there is more deviation between the state forecasts and smoothed estimates, but overall there is a strong correspondence between the two. Note again that, with the exception of the July to September period, the forecasts assign the highest probability to the bear rally state rather than a bull state.

The COVID-19 period has had an important impact on several features of the return distribution. For example, Figure 5 displays the one-week-ahead predictive density generated by the model. From the middle of March 2020 there was a dramatic impact that flattened the return distribution for the rest of the year, along with more subtle changes in the location. This implied a sharp increase in risk associated with holding the S&P500 portfolio.

Figure 6 illustrates the one-week-ahead predictive Sharpe ratios defined as $\frac{E(r_t|r_{1:t-1})}{\sqrt{\mathrm{Var}(r_t|r_{1:t-1})}}$. The flattened density of returns in March is accompanied by a sudden drop of the predictive Sharpe ratio. This ratio becomes positive in June and continues to improve over the summer months until a decline in August. For the remainder of our sample, the predictive Sharpe ratio never does attain the values at the start of 2020 before COVID-19 struck.

To illustrate another way, these changes in risk can be clearly seen from Value-at-Risk levels estimated for our model and illustrated in Figure 7. The dashed Value-at-Risk levels obtained from an assumption that returns are normally distributed would significantly understate risk as compared to those levels implied by our model that incorporates a mixture of four state distributions.

3.4 Market-Timing Investment Strategies

In this section, we consider some simple market-timing strategies that exploit the forecasts from our model. All of the investment strategies are based on the out-of-sample predictions of states and regimes; and the simple maxim to buy low and sell high. This can take several forms such as selling at the end of a bull market and buying at the end of a bear market. However, our MS4 model provides much more detailed and useful information. For example, the states that identify increasing prices are state 4 (bull state) and the riskier state 2 (bear rally). Holding the market during periods for which forecasts assign significant probability for these states could be fruitful.

In each case, the investor can buy the market and continue to hold the market; or sell and hold a risk-free asset. No short selling is allowed. Here are the market-timing strategies we consider, in which τ_B and τ_S are threshold probabilities associated with investment decisions for the regime versus the state-directed strategies.

1. Strategy B: buy or continue to hold the market when $P(B_t = 2|r_{1:t-1}) > \tau_B$ and otherwise sell.

2. Strategy S: buy or continue to hold the market if $P(s_t = 2|r_{1:t-1}) > \tau_S$ or $P(s_t = 4|r_{1:t-1}) > \tau_S$ and otherwise sell.

The first strategy B only uses the aggregate regime information associated with the probability forecasts for B_t. The second strategy exploits the positive expected return in both the bear rally ($s_t = 2$) and bull states ($s_t = 4$). We focus on using one cutoff value τ_S for both of those states but this could be generalized and we present some evidence of this below.

Table 4 shows the results for the out-of-sample investment period from January 2, 2020 to November 25, 2020. The total number of transactions required to implement the various strategies for that period are reported in the last column. Since at most 3 or 4 transactions would be required, and transaction costs are very low (for example, 0.3 cents per share on NYSE Arca for the SPDR security that tracks the S&P500 index), the reported returns exclude transaction costs which would be negligible.

The annualized return is 13.1% with a Sharpe Ratio of 0.566 if the investor buys in the first week of 2020 and holds the position until the last week of our data sample (last Wednesday of November). This compares with a hypothetical buy and hold return of 6.46% if an investor held the index for our entire sample from 1885.

As reported in Table 4, using the market-timing strategy B does not perform well in 2020 even with a range of alternative cutoff values, τ_B, for buying and selling. However, exploiting the additional information about states provided by the MS4 model (as in strategy S), yields very positive results. For example, with $\tau_S = 0.5$, the market-timing strategy generates a 22% annualized return with a Sharpe Ratio of 1.203. This investment strategy performs significantly better than the buy and hold strategy.

Figure 8 displays returns for strategy S as a function of different values of τ_S. For most values of $\tau_S \in (0.5, 0.9)$ a positive return is achieved with the best performance for values less than 0.65. Given the relatively short sample associated with the first eleven months of 2020, it is clear that being in the market early (values of τ_S closer to 0.5) to capture bear market rallies is fruitful. As τ_S increases, the length of time in the market decreases so the annualized return decreases and is more variable given the effect of the heightened market volatility during the bear market rally.

One can achieve even larger gains by separating the thresholds in strategy S, that is, relaxing the constraint of a common τ_S associated with both bear rally and bull states. For example, fixing τ_S at 0.5 for the bear rally state (buying or continuing to hold the market if $P(s_t = 2|r_{1:t-1}) > 0.5$); while allowing τ_S to vary for the bull state (buy if $P(s_t = 4|r_{1:t-1}) > \tau_S$ and sell otherwise), generates annualized returns as high as 28%. This is further evidence that the added value associated with using information inherent in the state probabilities and predictive state densities for investment strategies is quite robust.

4 Robustness Results

We have implemented a number of model comparisons to evaluate the robustness of our results. Table 5 reports log-predictive likelihood values for the 2020 data for several alternative models. Included is our proposed 4-state model (MS4), the MS4 model with Student-t innovations, the MS4 model with an unrestricted transition probability matrix, a MS2 model and a GARCH model. None of these specifications improve on our proposed MS4 model. We have reported more details about these model comparisons and further robustness tests in the Appendix.

5 Conclusion

This paper estimates and forecasts bull and bear markets and their component states during the COVID-19 pandemic. We document market phases prior to the pandemic period and analyze what has happened to the U.S. equity market in 2020. We find that the market moved from a bull state at the start of 2020 to a bull correction and quickly to a bear market on February 26. This bear market dominated until April 22 when the market transitioned to a bear rally, remaining in this phase until the current date (end of November, 2020).

We show the dramatic impact of the COVID-19 pandemic on the forecasts of the return distribution and how effective market-timing strategies can exploit model forecasts. Investment strategies which use the conventional bear and bull market regime classifications to forecast turning points do not outperform a buy and hold strategy during 2020. However, using the more detailed information inherent in our 4-state model, particularly the risk differences across states, contributes significantly to market-timing strategies. Using forecasts for market states to guide an investment strategy outperforms buy and hold by a wide margin, both in terms of annualized returns and Sharpe ratios.

The Appendix includes additional results, including long-horizon forecasts, investment returns associated with alternative thresholds for state probability forecasts, and additional details concerning a number of model comparisons which show our results to be robust.

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Table 1: Weekly Return Statistics

N	Mean	Mean($RV^{.5}$)	Skewness	Excess Kurtosis
7064	0.125	1.938	-0.565	8.007

Table 2: Posterior Estimates

	mean	95% DI
bear μ_1	-0.94	(-1.09, -0.79)
bear rally μ_2	0.23	(0.14, 0.32)
bull correction μ_3	-0.11	(-0.21, -0.02)
bull μ_4	0.52	(0.42, 0.64)
σ_1	5.60	(5.21, 6.03)
σ_2	2.44	(2.27, 2.61)
σ_3	1.85	(1.69, 2.04)
σ_4	1.09	(0.97, 1.21)
μ_1/σ_1	-0.17	(-0.20, -0.14)
μ_2/σ_2	0.10	(0.06, 0.13)
μ_3/σ_3	-0.06	(-0.12, -0.01)
μ_4/σ_4	0.49	(0.35, 0.65)

$$\text{Transition matrix } \boldsymbol{P} = \begin{pmatrix} 0.906 & 0.092 & 0 & 0.002 \\ 0.013 & 0.968 & 0 & 0.019 \\ 0.013 & 0 & 0.891 & 0.097 \\ 0.001 & 0 & 0.122 & 0.876 \end{pmatrix}$$

Table 3: Unconditional State Probabilities

	mean
bear π_1	0.084
bear rally π_2	0.245
bull correction π_3	0.356
bull π_4	0.316

Table 4: Investment Returns: 2020

	Return	Sharpe Ratio	number of transactions
Strategy B[a]: $\tau_B = 0.5$	-0.009	-0.048	3
Strategy S[b]: $\tau_S = 0.5$	0.220	1.203	4
Buy-and-hold	0.131	0.566	0

The returns are annualized. Transaction costs are 0.3 cents per share.
[a] Buy if $P(B_t = 2 \mid r_{1:t-1}) > \tau_B$ and sell otherwise.
[b] Buy if $P(s_t = 2 \mid r_{1:t-1}) > \tau_S$ or $P(s_t = 4 \mid r_{1:t-1}) > \tau_S$. Sell otherwise.

Table 5: Log-Predictive Likelihood: 2020

MS4	MS4t	MS4 Unrestricted	MS2	GARCH
-127.6	-127.9	-128.7	-132.8	-146.3

Figure 1: Estimates for 2019

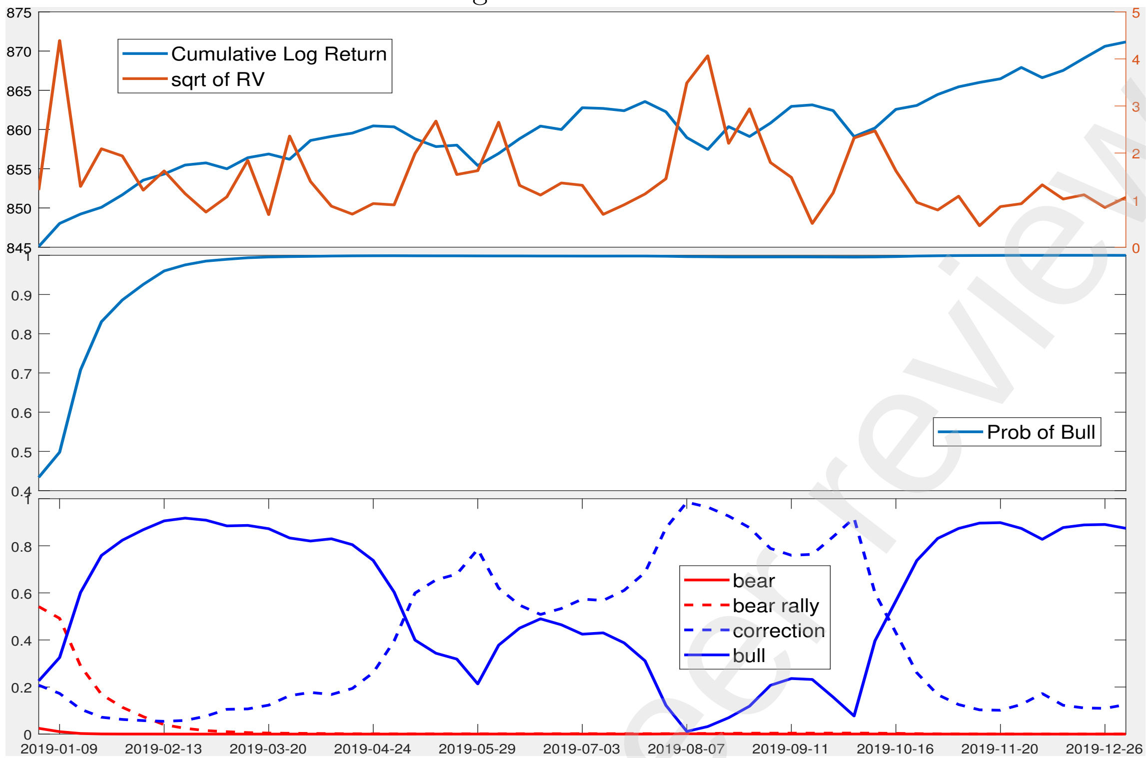
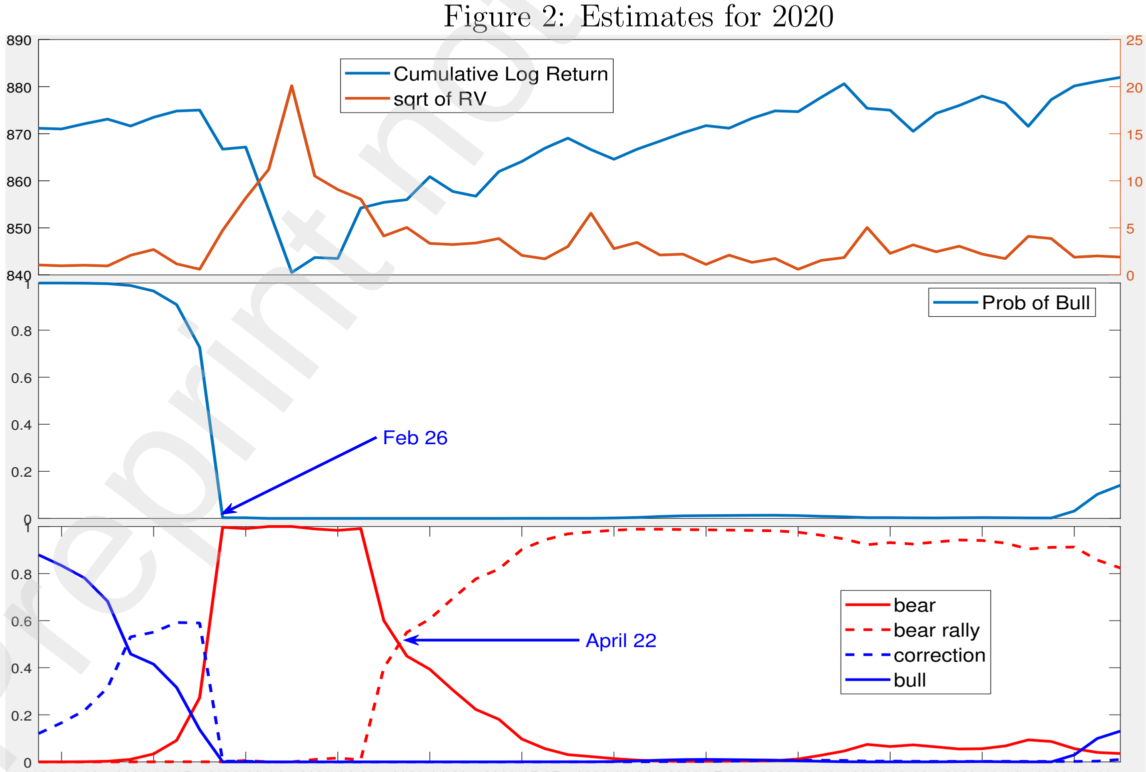


Figure 2: Estimates for 2020



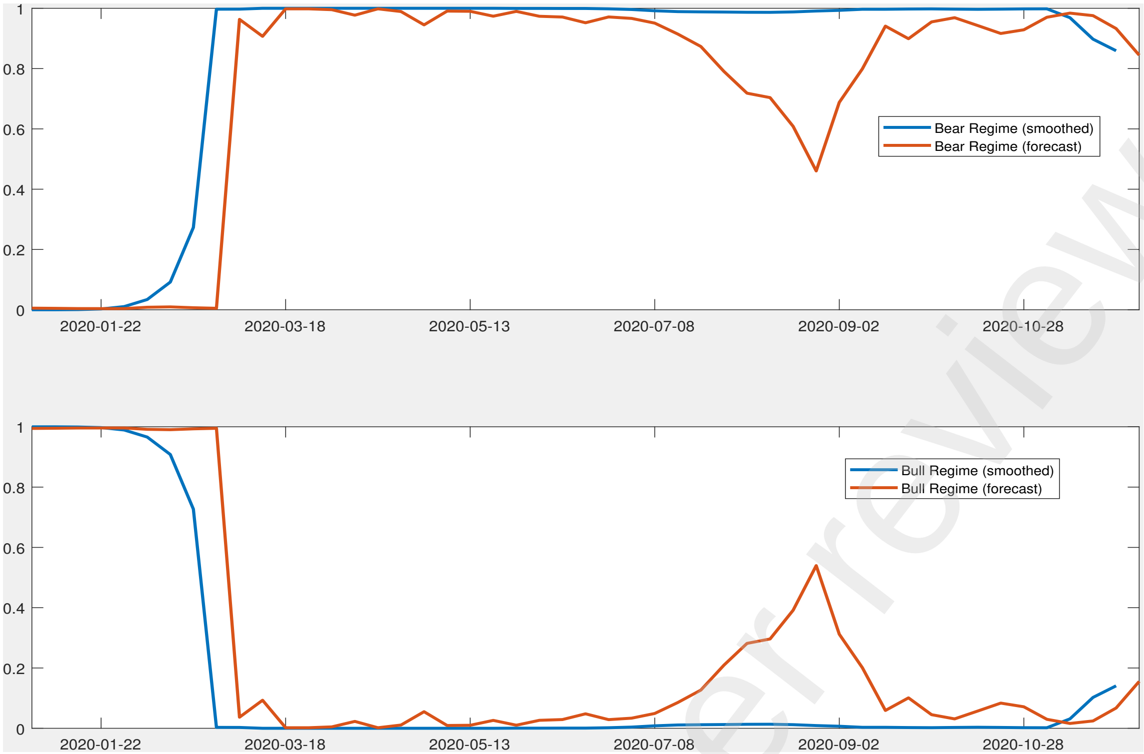


Figure 3: Out-of-sample: One-week-ahead Regime Probability Forecasts

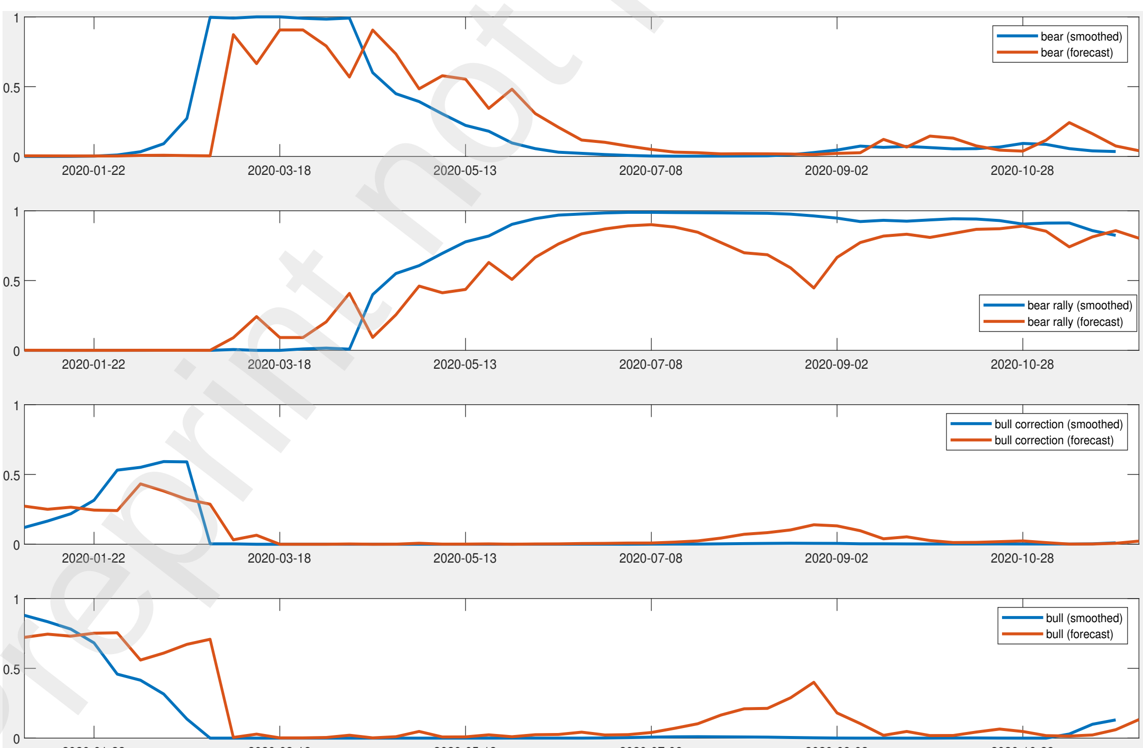


Figure 4: Out-of-sample: One-week-ahead State Probability Forecasts

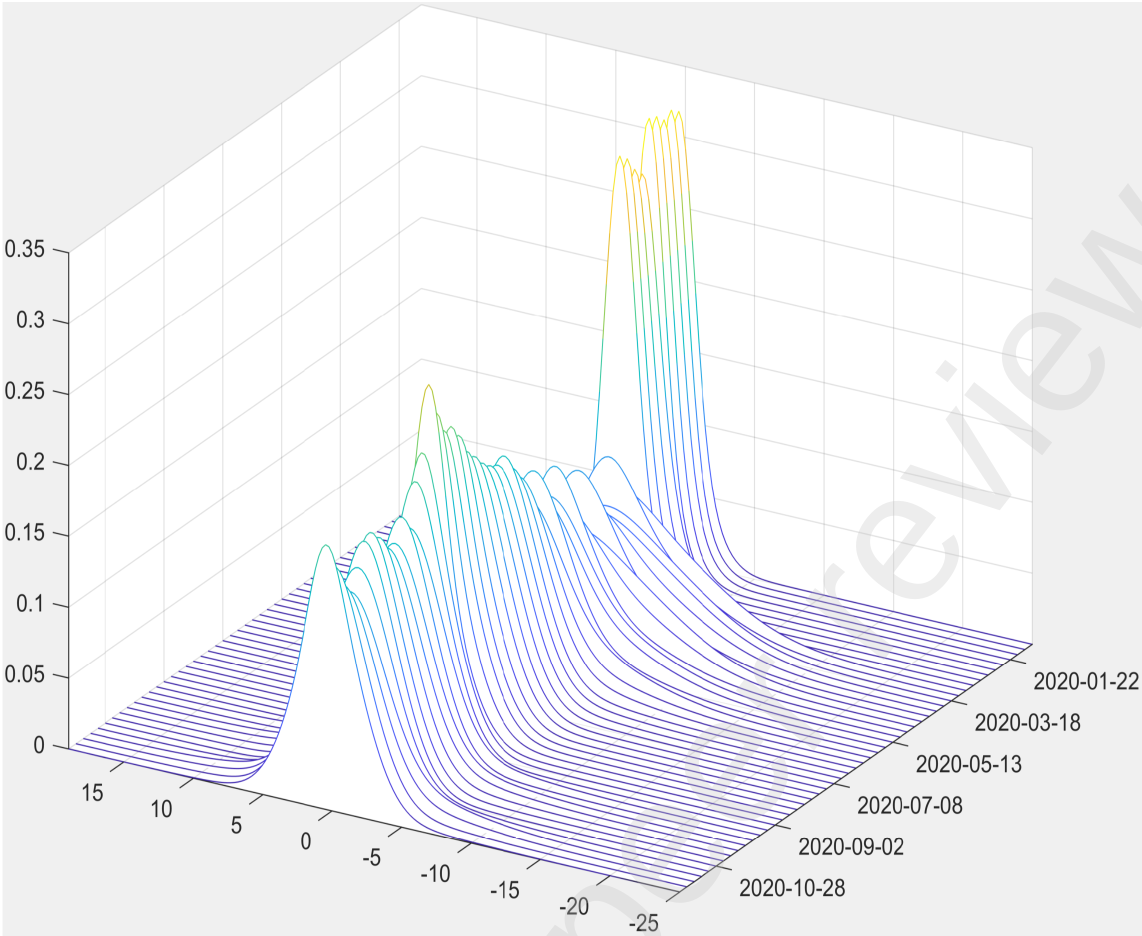
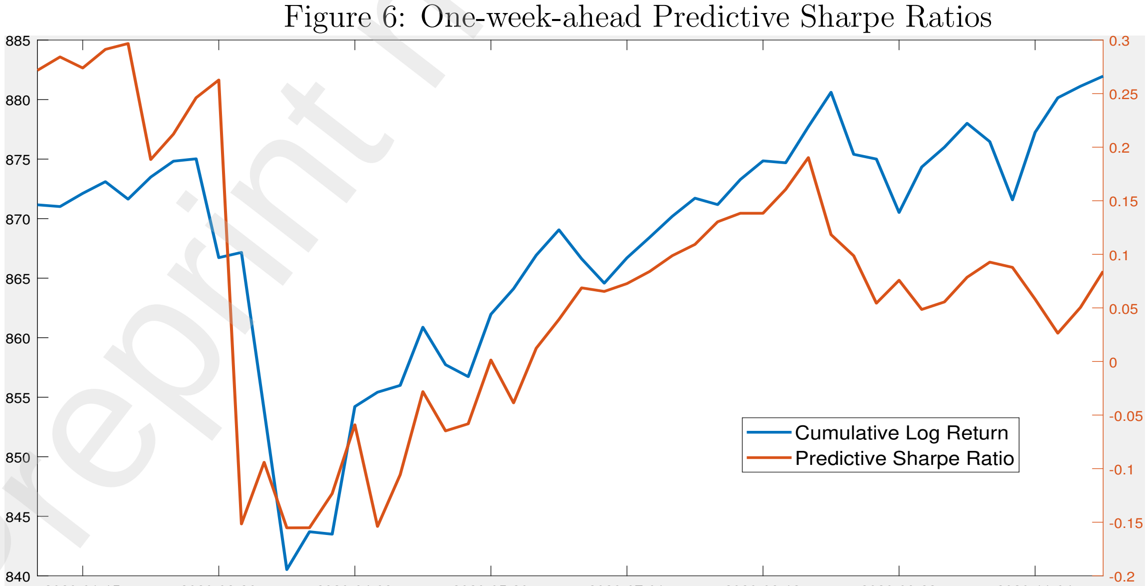


Figure 5: Out-of-sample: One-week-ahead Predictive Densities

The X-axis is a grid of possible return values. The Y-axix is the time. The Z-axis is the probability density function values.

Figure 6: One-week-ahead Predictive Sharpe Ratios



The Predictive Sharpe ratio is defined as the ratio of predictive mean and standard deviation, $\frac{E(r_t|r_{1:t-1})}{\sqrt{\text{Var}(r_t|r_{1:t-1})}}$.

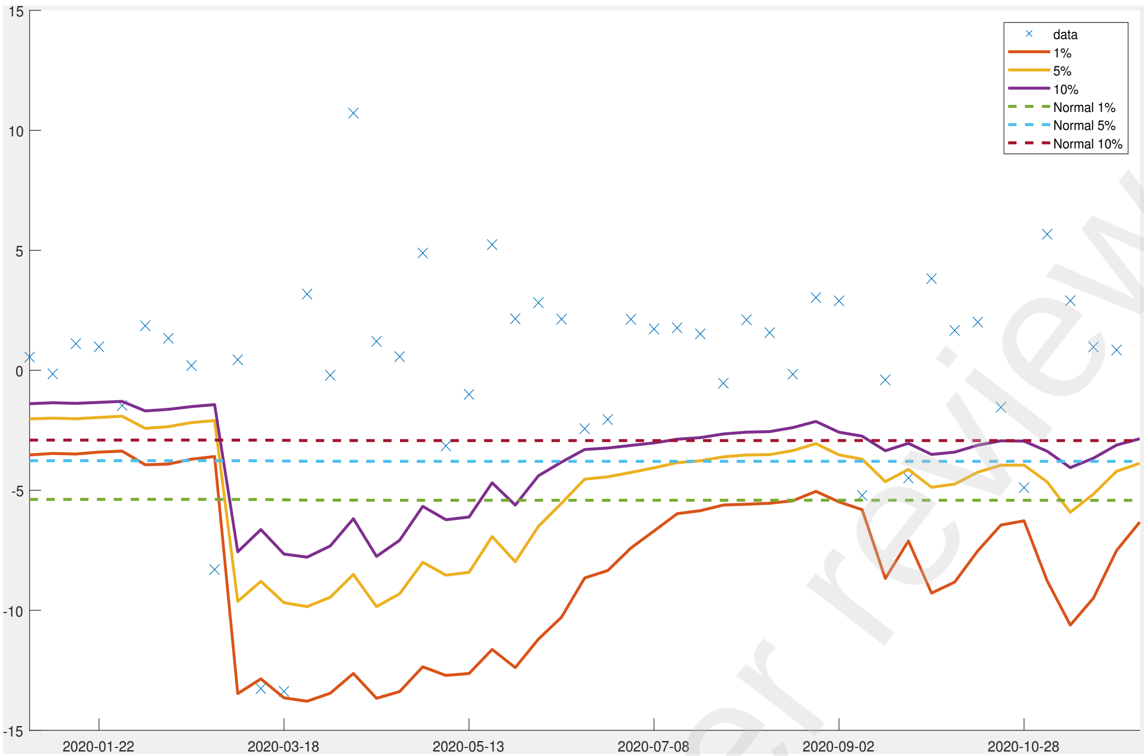
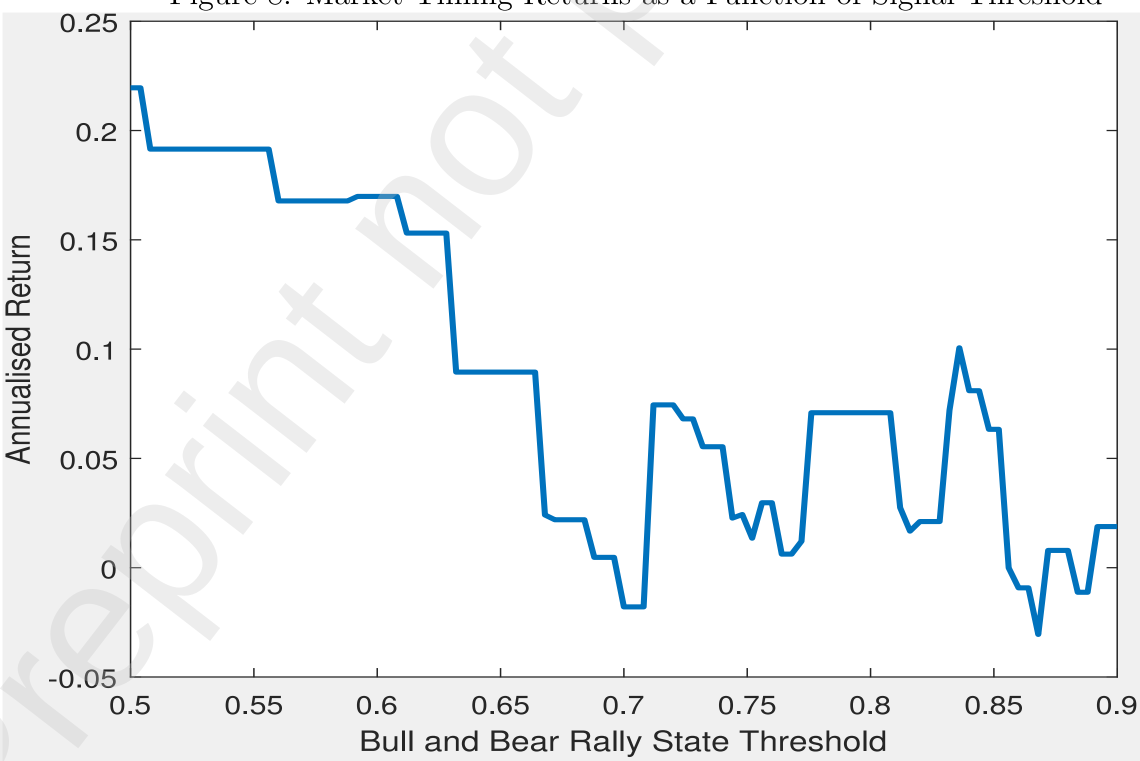


Figure 7: Out-of-sample: One-week-ahead Value-at-Risk Forecasts

Figure 8: Market-Timing Returns as a Function of Signal Threshold



The blue line is the return in 2020 till the end of the sample period as a function of τ_S for the investment strategy S: buy or continue to hold the market if $P(s_t = 2|r_{1:t-1}) > \tau_S$ or $P(s_t = 4|r_{1:t-1}) > \tau_S$ and otherwise sell.

6 Appendix: Robustness and Additional Results

As reported in the paper, we have implemented a number of model comparisons to evaluate the robustness of the results. In this Appendix we provide more details, for example, how the alternative models date the turning points in 2020. Generally, there is a close correspondence with the MS4 model. One notable exception is a 2-state Markov switching model (MS2). The MS2 classifies the summer months of 2020 as a bull state while our MS4 identifies this period as a bear market rally. This could be considered as a drawback of the 2-state model in that it does not allow for intra-regime dynamics.

6.1 Predictive Likelihood Comparisons

Figure 9 illustrates differences in the cumulative predictive likelihood associated with alternative models for year 2020, using the GARCH(1,1) model as the benchmark value. The values at time t are log predictive Bayes factor as $\log\left[\frac{p(r_{2020 \text{ until } t}|r_{\text{before } 2020}, \text{M})}{p(r_{2020 \text{ until } t}|r_{\text{before } 2020}, \text{GARCH(1,1)})}\right]$, where M indicates various alternative models including the MS4. According to Kass & Raftery (1995), a value $\frac{p(r|M_0)}{p(r|M_1)}$ that is larger than 5 indicates strong data evidence to support model M_0. The 2020 data clearly favours the MS4 model against the GARCH(1,1). In addition, having a Student-t distribution for each state in the MS4 model does not provide any additional value, as is confirmed by a comparison of Figure 10 with the state probabilities associated with the MS4 model reported in Figure 2 in the paper.

6.2 MS4 versus GARCH

Figure 11 shows the smoothed standard deviations from the MS4 versus a GARCH(1,1) model. The MS4 model picks up the volatility surge quickly at the beginning of 2020 and does not have the long tapering-off period associated with the GARCH model. The implications for Value-at-Risk are illustrated in Figure 12. The quick adjustment of the Value-at-Risk estimates generated by the MS4 model match the high potential losses after March 2020. This adjustment is substantially delayed and less flexible using a GARCH(1,1) model. The latter is due to the single exponential decay parameter in the GARCH model which invokes higher persistence and less flexible adjustment to shocks.

6.3 MS4 versus MS2

Figures 13 and 14 show the probability of a bull market regime inferred from our 4-state model versus a simple 2-state model for the period 2019-2020. There is one striking difference between these two models in the period from April to September 2020. While the MS4 model classifies this period as a bear regime (in a bear rally state), the MS2 model signals a bull regime. This difference is due to the 2-state model not having the structure to incorporate intra-regime dynamics. The September and October evidence from the 2-state model had the bull market probability plunge to the bottom with small humps, while the 4-state model always estimated that the bull regime had not been confirmed yet.

6.4 Long-Horizon Forecasts

The paper focused on one-week-ahead forecasts from the MS4 model and how they might be used for investment decisions. The model estimates also have implications for longer-run behaviour of stock market returns. One application of long-horizon forecasts is to look at 4-week-ahead forecasts through 2020 as illustrated in Figures 15 and 16. As in the 1-week-ahead forecasts discussed in the paper, the changing shape of the density forecasts will have significant implications for time-varying Value-at-Risk forecasts.

Being stationary, our MS4 model implies that any long-horizon predictions converge to the implied stationary distribution. Figures 17 and 18 report one-year-ahead probability forecasts for states and associated regimes looking forward from our most recent observations at the end of November 2020. What is notable is that the transition back to *normal* times in the form of the long-run values of the states is slow. Even if the effects of COVID-19 were to disappear today, given the persistence identified with market phases our model predicts that it would take months for the stock market to return to the long-run (unconditional) state and market-regime probabilities.

6.5 Investment Returns

The investment strategies reported in the main text showed that for the 2020 time period a market-timing strategy that exploited the MS4 state probability forecasts generated a 22% annualized return with a Sharpe Ratio of 1.203, significantly better than the buy and hold strategy. Similar strategies could also be applied to other time periods, such as, the financial crisis period. Using Strategy S with fixed threshold values, $\tau_S = 0.5$, also resulted in an improvement in the Sharpe Ratio and returns relative to buy and hold over that 2007:07 to 2009:12 period.

As discussed in the paper, one can achieve even larger gains by separating the thresholds in strategy S, that is, relaxing the constraint of a common τ_S associated with both bear rally and bull states. For the 2020 period corresponding to Table 2 in the paper, Figure 19 reveals that fixing τ_S at 0.5 for the bear rally state (buying or continuing to hold the market if $P(s_t = 2|r_{1:t-1}) > 0.5$); while allowing τ_S to vary for the bull state (buy if $P(s_t = 4|r_{1:t-1}) > \tau_S$ and sell otherwise), generates annualized returns as high as 28%. This is further evidence that the added value for investment strategies, associated with using information inherent in the state probabilities and predictive state densities, is robust.

Figure 9: Cumulative Log Predictive Likelihoods

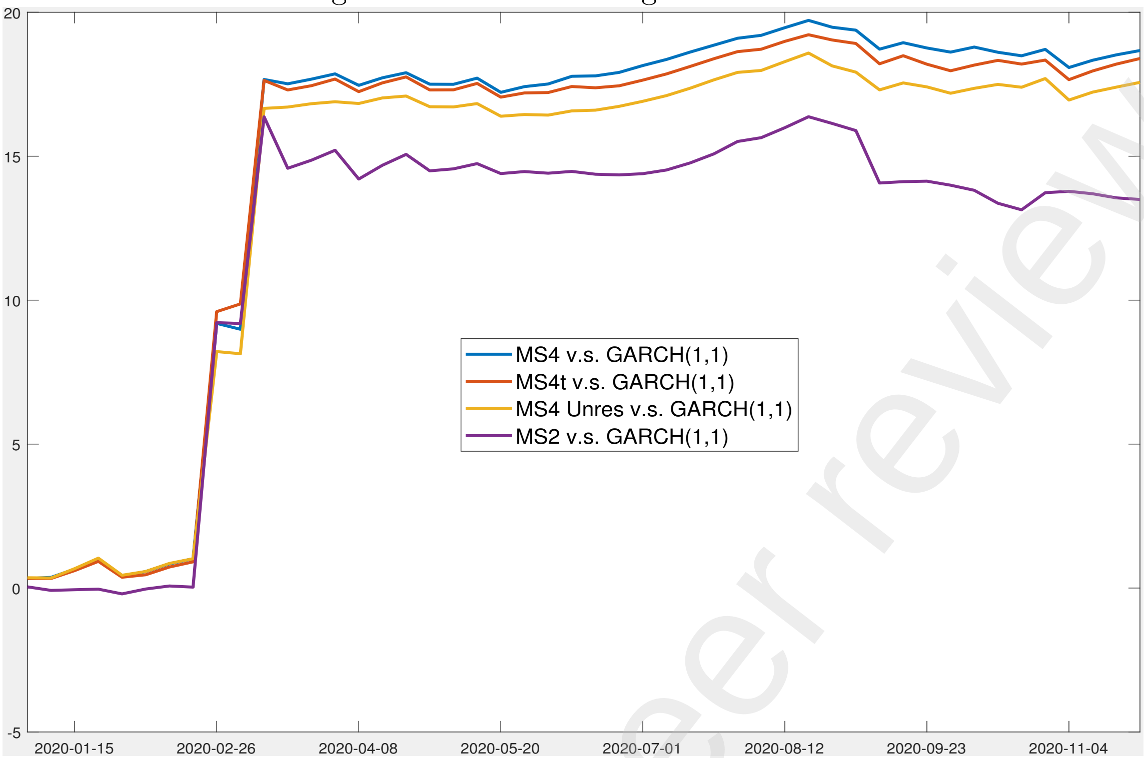
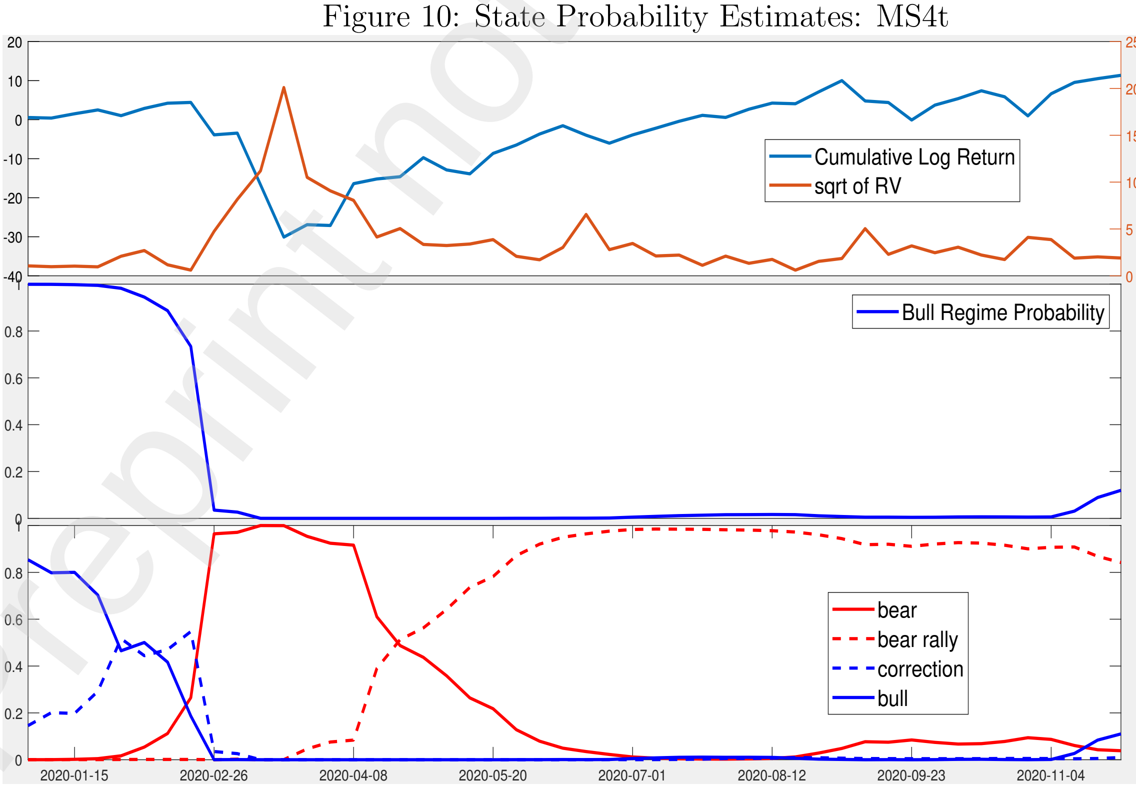


Figure 10: State Probability Estimates: MS4t



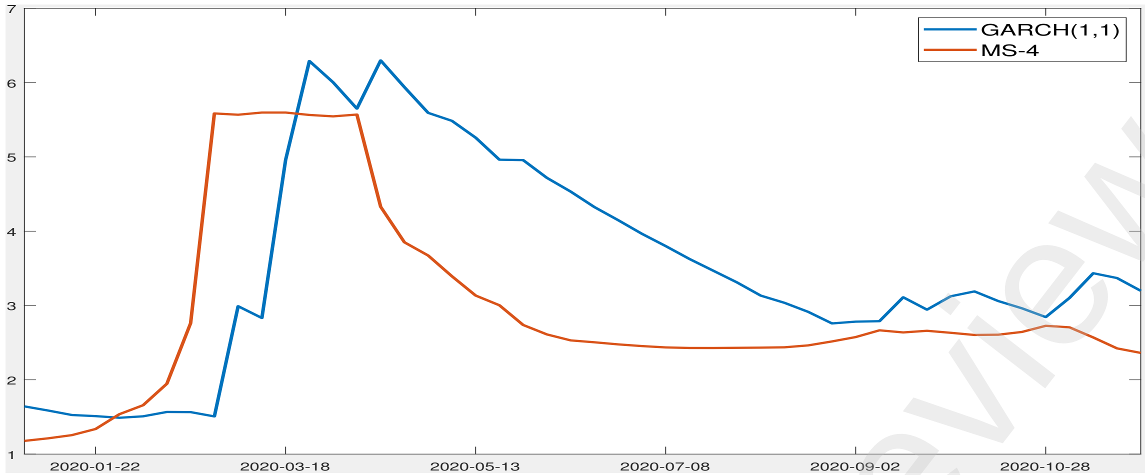


Figure 11: Standard Deviations from GARCH(1,1) and MS4 in 2020.

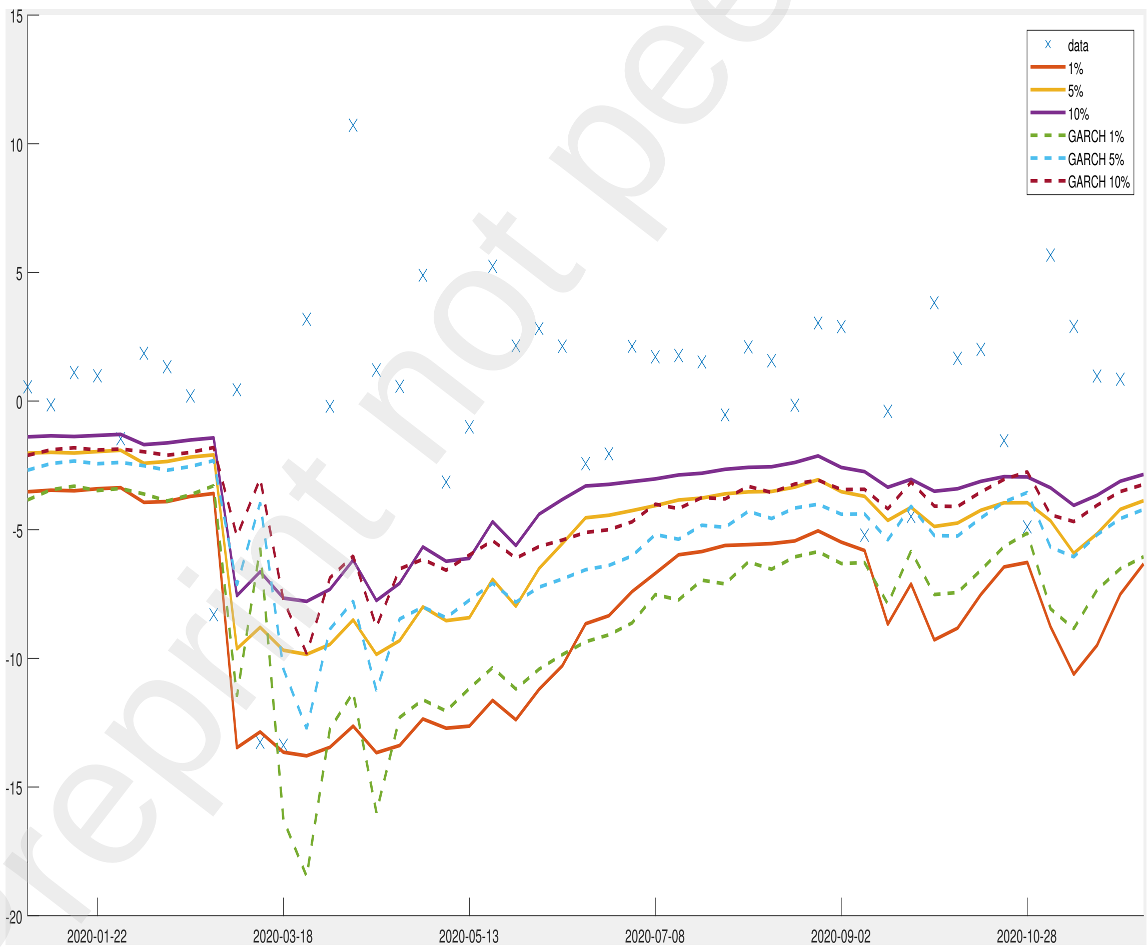


Figure 12: Out-of-sample: One-week-ahead Value-at-Risk Forecasts

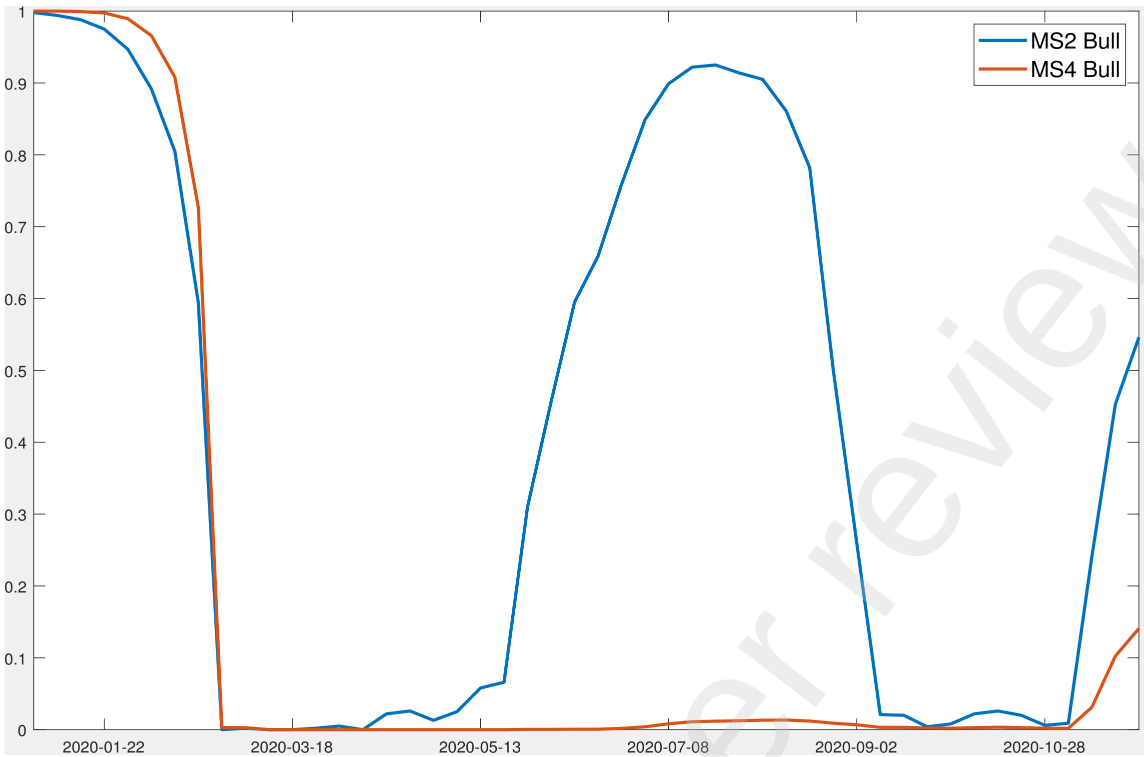


Figure 13: Bull Regime Probability MS2 vs MS4 Model

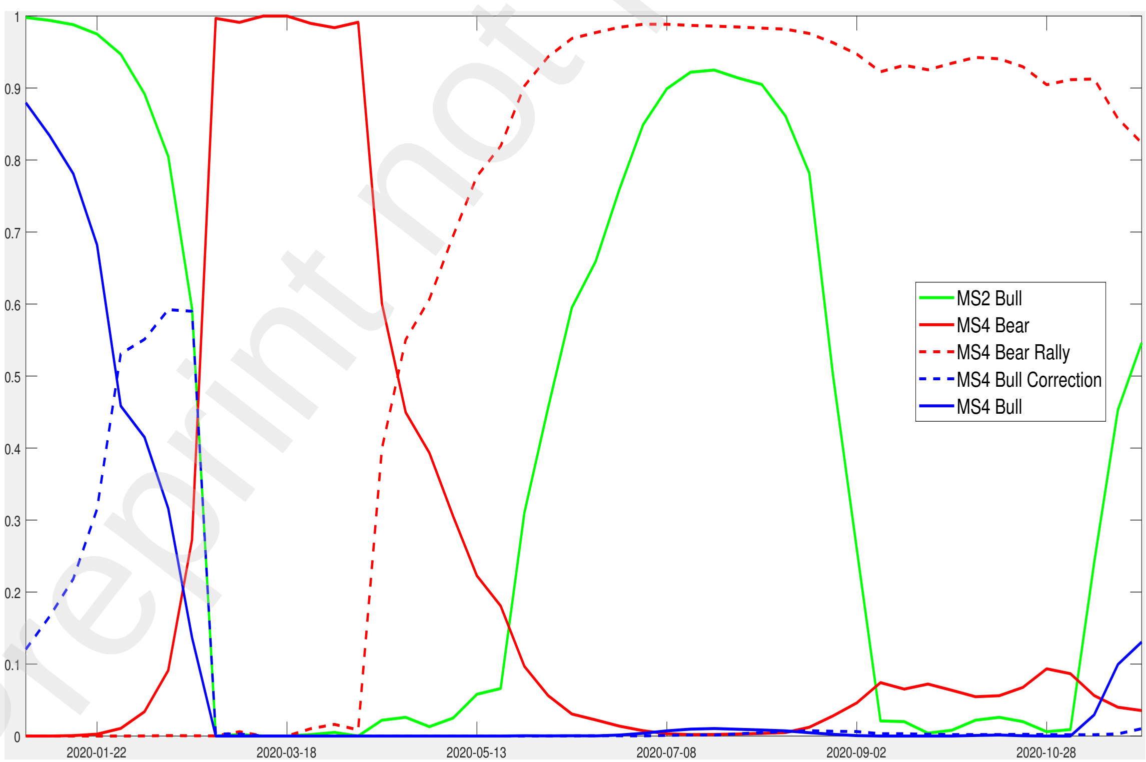


Figure 14: Bull Regime Probability MS2 & State Probabilities MS4

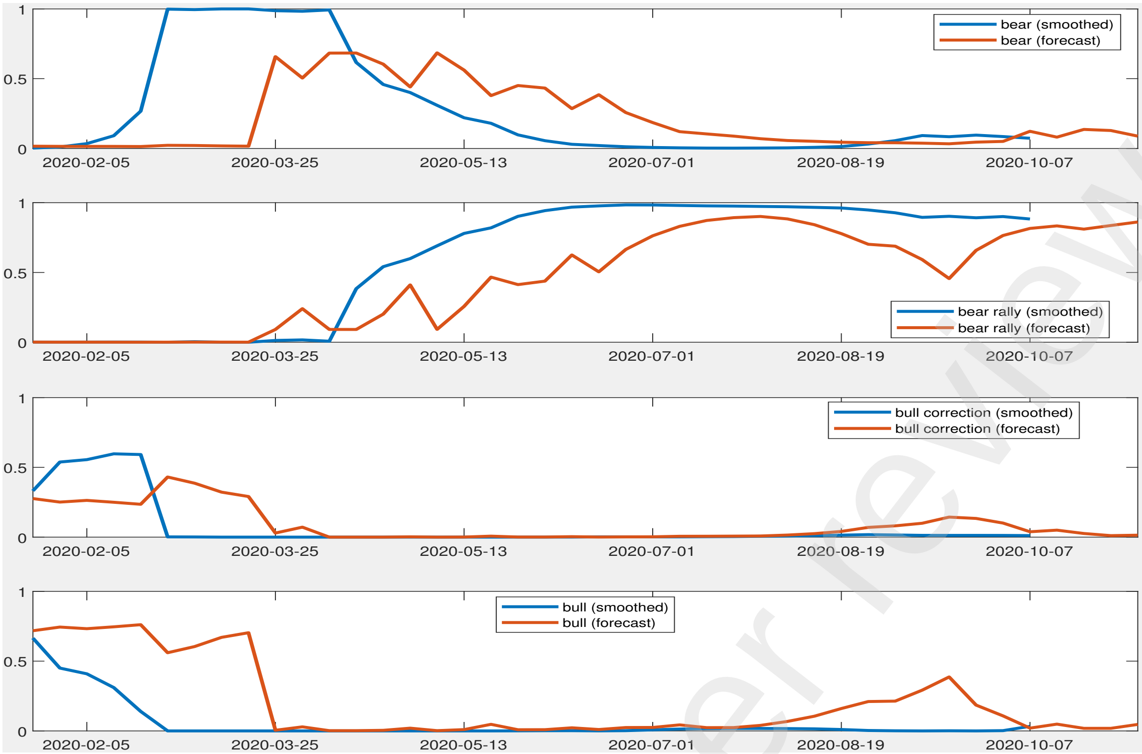


Figure 15: Out-of-sample: States 4 weeks ahead

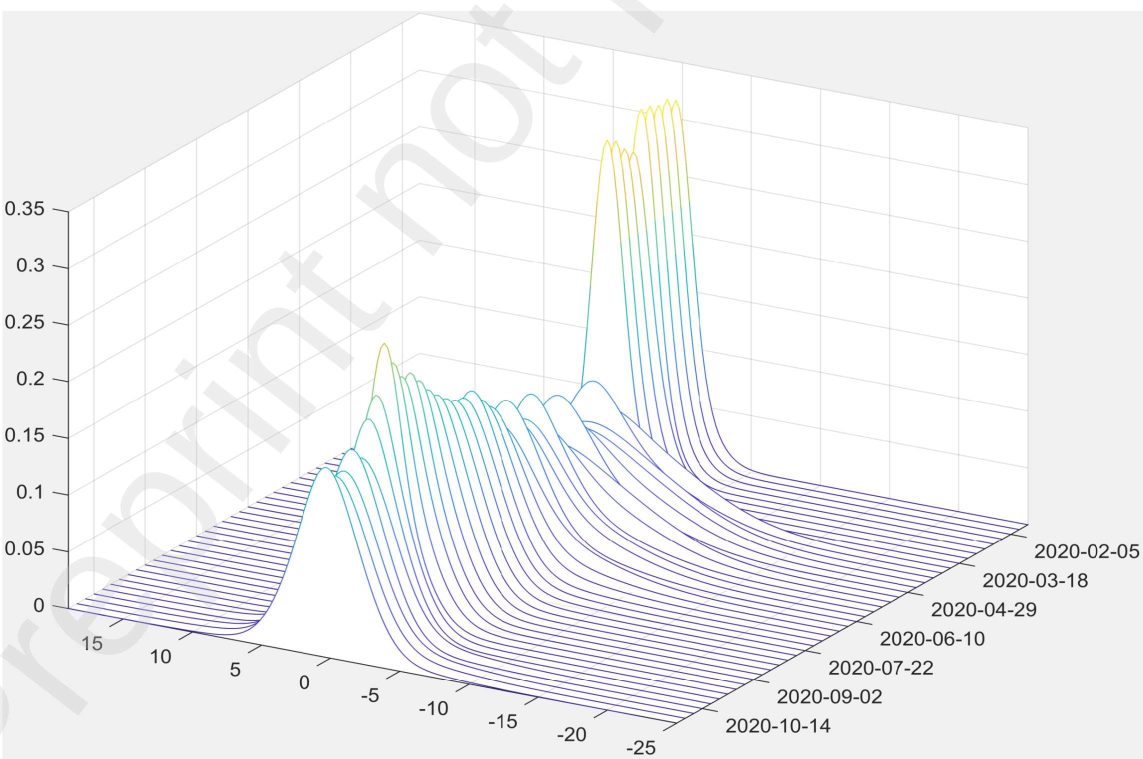


Figure 16: Out-of-sample: Predictive Density 4 weeks ahead

Figure 17: States Estimates for 2020 and One-year Forecasts

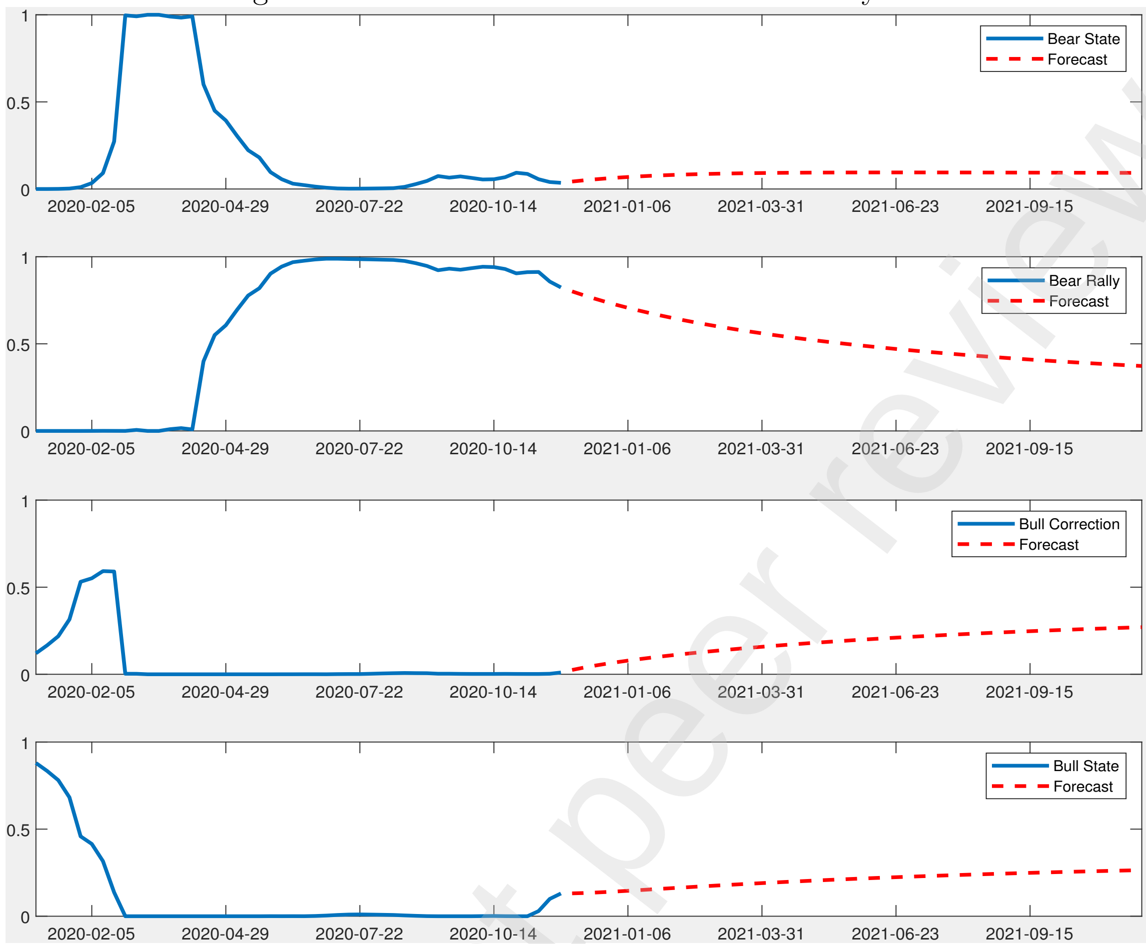


Figure 18: Regime Estimates for 2020 and One-Year Forecasts

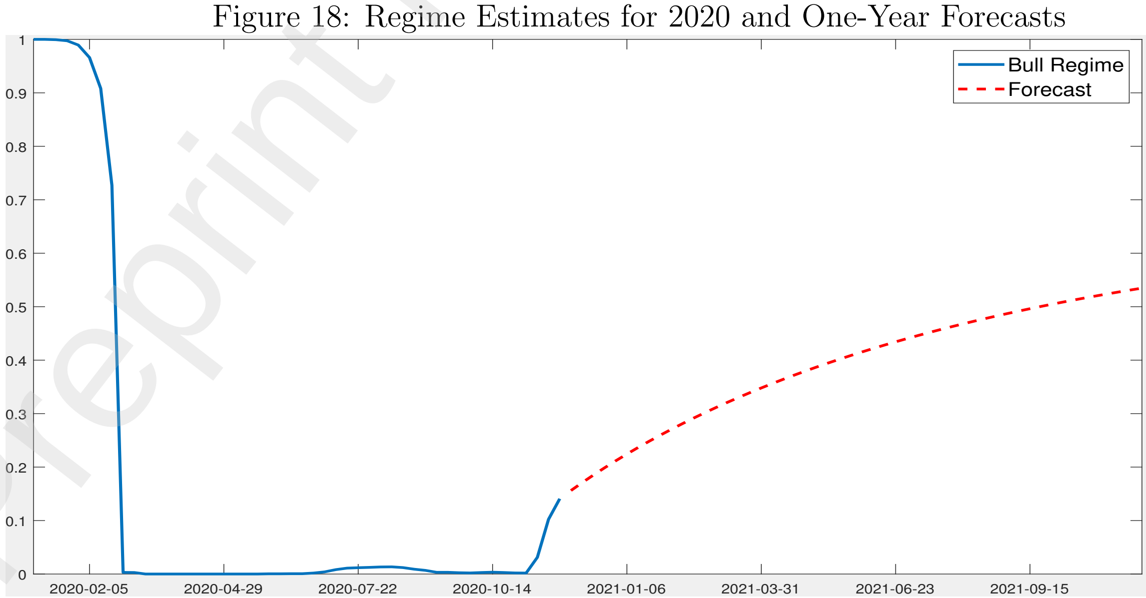
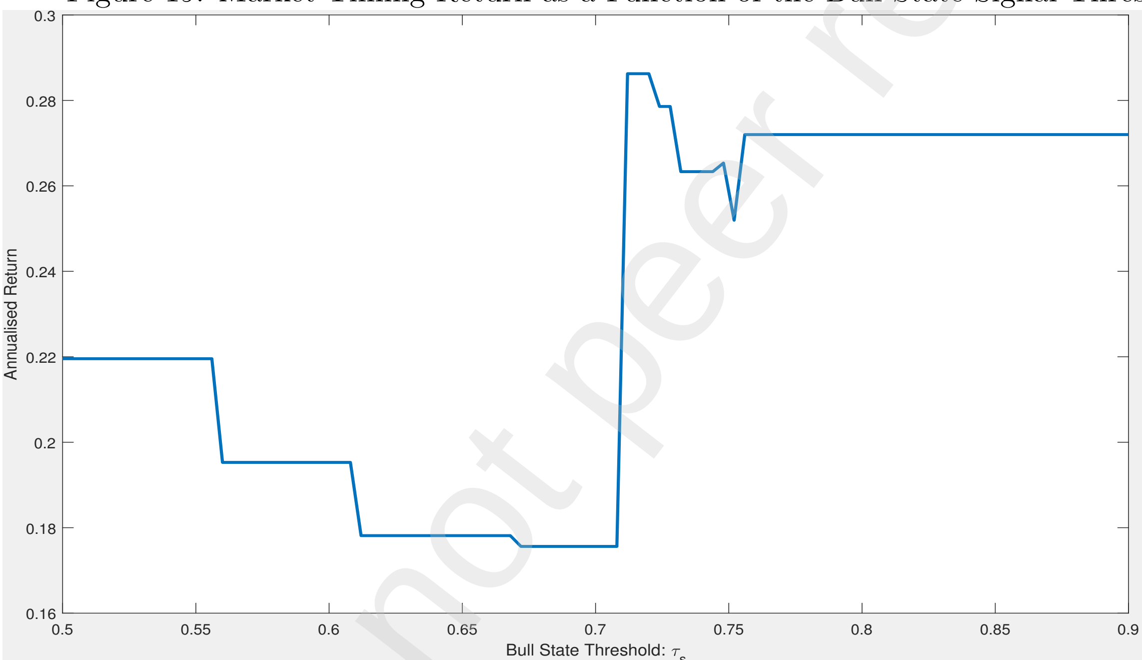


Figure 19: Market Timing Return as a Function of the Bull State Signal Threshold



The blue line is the return in 2020 till the end of the sample period as a function of τ_S for investment strategy S: buy or continue to hold the market if $P(s_t = 2|r_{1:t-1}) > 0.5$ or $P(s_t = 4|r_{1:t-1}) > \tau_S$ and otherwise sell.