Tail Risk Hedging Performance:
Measuring What Counts

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Abstract

We discuss the importance of using proper metrics for measuring the historical performance of tail risk hedging portfolios in particular, and for any strategy with levered payoffs in general. It is our view that simply using historical compounded returns when the payoffs may be multiples of the investment, and ignoring the timing and magnitude of cash flows can potentially paint an inaccurate picture, sometimes grossly so, of the economic value of such strategies. To obtain a more accurate picture that is consistent with the objectives of such strategies, the timing and magnitude of cash flows should be included when analyzing their impact on portfolio construction. While the correct quantitative metrics are obviously critical in measuring the efficacy and reliability of tail hedging strategies, the importance of subjective metrics, ease of implementation, flexibility, and the relevance to underlying objectives of investors is equally important.

Key Takeaways

- Unlike fully-funded strategies, only providing NAV based returns fails to communicate the efficacy of a tail hedge. In order to get a complete view of a tail hedge strategy, NAV, cash flows, and NAV based returns should be presented together
- By performing such analyses, it can be demonstrated that cost-effective tail hedging can provide risk-adjusted return enhancement, rather than being a negative expected return investment
- In addition to quantitative metrics, subjective measurements of the reliability, flexibility, and ease of implementation of tail hedges relative to other alternatives is an important consideration, and can be quantified in terms of a scoring mechanism

As practitioners providing tail risk hedging solutions to investors, we are often asked to provide return time series, and also compounded returns, both hypothetical and actual, for tail risk hedging strategies. While the summary returns data is easily calculated through the calculus of compounding, we have to often explain to investors that the meaning of such data has to be thoroughly understood before it is used in making portfolio decisions. In this paper we will try to give a clear exposition for this need so investors are looking at the correct metrics for evaluating the benefits of such highly convex and non-linear strategies in their portfolios. The need is even more critical today, since current portfolio optimization approaches and software, such as single period optimization using a mean-variance type of approach can give precisely the wrong answer if the correct inputs are not used.

An analogy will make this clear. Rarely, if ever, do homeowners ask their insurance providers to send them the return statistics on their insurance premiums paid. The reason is simple: insurance on homes is bought for its desirable conditional cash-flow characteristics, i.e. even though the insurance premium is expected to be a total loss every year, the relatively small insurance premium paid annually protects the home-owner from a catastrophic loss if the house burns down. Computing the total cumulative return on this premium, as is done for traditional investments might mathematically be sound, but conceptually does not make much sense.

What people who buy home insurance remember is that when their home was damaged, the insurance policy paid off enough to cover the losses. This means that the salient feature of insurance like investments is the *reliability* of the contingent payoffs when they matter. People don't usually complain about the small home insurance premium they pay, and therefore don't compute cumulative (negative) returns on the insurance. For most homeowners, it is a given that home insurance is a cost. In exchange, it allows them to enjoy the home without having to set aside the full value of the house in reserve for replacement costs. Thus the reason why people buy home insurance is because (1) it is cheaper to buy insurance than to set aside a lot of money for a low probability event; (2) if purchased from a reliable party, the insurance pays off when it is needed; and (3) the cash flow is sufficient and satisfactory for the premium cost incurred.

These three reasons are why the authors have never asked our home insurance providers to provide the internal rate of return (IRR) for the years, maybe even decades, that we have been buying home insurance. It is not that the numbers cannot be computed - they can be, but the numbers may lead to the wrong conclusions and erroneous decisions around the value of the insurance policy. If the insurance provider were to come back and report that the cumulative return over the last three decades we have been buying home insurance was -99.9%, what would we do with the information? We suspect that despite this dismal cumulative "performance" of our insurance policy over the last three decades, and with the expectation that it will have exactly the same type of dismal performance over the next thirty years, we would still buy insurance for another year, even though the mathematical expected return on the insurance by itself is a total loss. The reason, obviously, is that having the insurance provides us with positive cash-flows when we need them. This reason is sufficient for us to buy insurance for another year, since we cannot forecast when our homes will need the coverage due to an unseen catastrophic loss. Thus we combine or "aggregate" in the sense of proper mental accounting (Thaler [1999]) to make sense of the negative expected return on the insurance policy by itself.

Focusing on tail hedging of investment portfolios, we argue that there are three primary reasons that support the use of tail hedge overlays (see Bhansali [2014]):

1. Tail hedges deliver marked to market gains during periods of stress.
2. Monetization proceeds come at an opportune time when there is a need for liquidity.
3. Tail hedges improve the overall risk characteristics when combined with a hedged portfolio allowing investors to be more aggressive to achieve higher returns.

A tail hedge overlay is quite similar to our home insurance example. In exchange for a small amount of "premium" spent, the owner of a tail hedge gets protection against a catastrophic market loss. This is the first point above. The premium protects over some unknown event over a fixed horizon, without perfect foresight on when such an event might occur. As future market movements cannot be predicted, it is difficult to pick the optimal time to put on a tail hedge. Similarly, home insurance is typically purchased as soon as one has acquired the underlying property, to protect against the unknown future risk of fires, floods, and other damage.

However, unlike home insurance, tail hedging in the financial markets has the two other features listed above. In a period of crisis in the markets, the tail hedges can be sold ("monetized") and the cash can be put to good use, including re-investing in the markets. The parallel in the home example would be an owner being able to monetize the value of the insurance payments and trade them in the markets. Unfortunately home insurance, so far, is not monetizable and tradable in the same form.

The third point above is important because the inclusion of the tail hedge in the portfolio allows investors to build portfolios which have the same or lower downside loss potential as an unhedged portfolio, but also allows them to garner more potential gains. In other words, the tail hedge allows a skewing of the distribution of portfolio returns.[2] There is a parallel for this tilting in the home insurance example. Homeowners routinely select desirable, albeit high risk locations for purchasing homes, for instance in hurricane-prone areas in Florida, or earthquake or fire-hazard zones in California, as long as they are able to purchase insurance against catastrophic losses from these hazards cheaply.

Importantly, without paying attention to the conditional cash flow events that drive the latter two objectives, the returns from the hedges, on their own, would still be very negative over time.

The purpose of this paper is to illustrate with simple, hypothetical examples first, and then with actual experience from managing tail risk hedging strategies, why the cash-flow based analysis is central to insurance type payoffs. While the traditional NAV based fund accounting is not incorrect, we believe it simply does not capture the reasons for including risk mitigation strategies in a portfolio. The leverage afforded by options based tail hedging strategies just magnifies the conceptual incoherence of using traditional performance metrics for measuring the performance of tail hedging strategies without including their value at the total portfolio level. To this end, and to keep the discussion explicit and transparent, we will walk through each of the points above using a backtest of a simple tail hedge strategy, generically referred to hereinafter

[2] See Exhibit 11

and in the charts as the tail hedging strategy or the "LeftTail Strategy"[3], both as a stand-alone portfolio and a portfolio overlay. Further, we will show that only focusing on NAV based returns for tail hedges, as is common practice used for performance measurement for fully-funded strategies, fails to communicate the efficacy of a tail hedge. We believe NAV, cash flows, and NAV based returns should be presented together in order to properly evaluate tail hedge strategies. The examples and the analysis below apply both to left tail (market melt-downs) and right tail (market melt-ups) (see Bhansali [2018]), though our focus here will primarily be on the left tails.

The Potential Wrong Message Sent by NAV Based Returns For Tail Hedges

Traditional NAV based accounting reports net performance returns typically daily or monthly, depending on the fund's investor needs. These returns are often used to calculate compounded historical returns looking back over various prior periods: quarter-to-date, year-to-date, and so on. The methodology is:

$$Return = \frac{PnL}{Starting\ NAV + Subscription}$$

where:

$$Starting\ NAV = Prior\ Ending\ NAV - Redemption$$

$$Ending\ NAV = Starting\ NAV + Subscription + PnL$$

The inception to date or on-going cumulative compounded returns are predicated on the concept that an investor contributes an amount of capital on day one and does nothing throughout the life of the investment. The initial capital and any gains or losses flow directly into the start of the next period, or are invested from period to period at the internal rate of return r_i for each period i,

$$Compounded\ Return = [(1 + r_1) * (1 + r_2) \dots (1 + r_n)] - 1$$

As a shortcut for analysis, fund return streams are provided as a series of per period percentages so investors can simply take the product of their starting capital and the return stream at any point along the series to get an estimate of what their performance may have looked like. Similarly, compounded returns are usually provided so investors can easily calculate what expected performance over a longer period of time has looked like historically. This standard methodology also allows for easy comparison between funds, such as computing Sharpe ratios, volatilities etc. It is common knowledge that this type of analysis can differ from actual returns experienced by a given investor, but it is assumed that the hypothetical investor who has entered the fund on day 1 has re-invested all cash flows back into the fund and thus this

[3] LeftTail Strategy data is sourced from LTA, OptionMetrics and Bloomberg. Each quarter, the tail hedge strategy spends a quarter of the budget on a new 1Y out of the money tail hedge option on the S&P 500 index. If the current value of any tail hedge exceeds 8x its original purchase price, the position will be fully monetized.

representative investor's experience represents the performance of the fund so another investor can make an informed analysis relying on this long term return series.

We begin with two simple scenarios that demonstrate the NAV based accounting methodology. Exhibits 1-2 differ in the volatility of their respective returns, with Exhibit 2 having returns more akin to a highly leveraged overlay protection strategy, such as a tail hedge. The starting capital for both examples is $10. In all the examples, we will compute two returns. The first return, which we call "compounded return", strings together the returns using the compounding formula above. The second return, which we call "dollar return", looks at the total terminal dollar value received, and computes the return of the dollar capital relative to the dollar value initially invested. Note that in both examples, there is no present value factor, since we are computing the ex-post summary return of a time-series of investor experiences.

$$Dollar\ Return = \frac{\sum_i PnL_i}{\sum_i Subscription_i}$$

In both Exhibits, the compounded NAV based return equals the actual dollar return of each investment, which should be no surprise:

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	0.4	10.4	0	4.00%
2	10.4	0	0	10.4	0	0.00%
3	10.4	0	0.2	10.6	0	1.92%
		10	0.6		0	
Compounded Return						6.00%
Dollar Return						6.00%

Exhibit 1: Single Subscription Low Return Volatility

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	1	11	0	10.00%
2	11	0	-1	10	0	-9.09%
3	10	0	7	17	0	70.00%
		10	7		0	
Compounded Return						70.00%
Dollar Return						70.00%

Exhibit 2: Single Subscription High Return Volatility

Note that in both Exhibits and respective calculations above, we are inherently assuming the use of a buy and hold strategy, which is what makes the process of stringing together single period returns to get long term returns possible. If there are no additional cash flows in or out of

the fund during the life of the investment, the compounded return and dollar return will be equal to one another.

Tail hedges, or insurance contracts in general, however, are generally not buy and hold strategies. Tail hedges are intended to be time and event specific and proper utilization of tail hedges require active monetization, whether mandatory, following a rules based approach, or voluntary, as when market tail hedges are sold before expiry thru active management. Further, investors of tail hedge funds typically want to access liquidity provided by a monetization event as soon as possible. This can be via a redemption to either offset losses from the underlying portfolio, or for redeployment into the market and potentially catch a rebound. Finally, it is possible (and in many cases, likely) for the premium in a tail hedge to decay to zero, which means investors may be required to add a subscription in order to extend and maintain the hedge. For these reasons, we must include cash flows in our analysis when looking at fund performance. From our perspective, ignoring the cash-flows can paint an egregiously inaccurate picture of the value of tail hedges, which surprisingly, is not immediately familiar to many professional practitioners in finance, who clearly understand compounding. The reason, as we will show, is that when the cash flows are small compared to the size of the investments, the mismatch between the two measures of return are small, but when the payoffs and cash flows are large compared to the investment, as in the case of premium based hedging strategies, the two measures can diverge substantially, to the point of having opposite signs.

Building on our previous examples, we will see that when cash flows are included, holding return streams constant, compounded returns and dollar returns are no longer equal. Exhibit 3 below has an identical return series to Exhibit 1 and Exhibit 4 has an identical return series to Exhibit 2, except there are redemptions in period 1. However, the actual profit or dollar based return is no longer the same and can begin to diverge quite quickly as shown in Exhibit 4.

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	0.4	10.4	0.4	4.00%
2	10	0	0	10	0	0.00%
3	10	0	0.19	10.19	0	1.92%
		10	0.59		0.4	
Compounded Return						6.00%
Dollar Return						5.92%

Exhibit 3: Low Return Volatility with Redemption

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	1	11	1	10.00%
2	10	0	-0.91	9.09	0	-9.09%
3	9.09	0	6.36	15.45	0	70.00%
		10	6.45		1	
Compounded Return						70.00%
Dollar Return						64.55%

Exhibit 4: High Return Volatility with Redemption

Now that we have set the stage for the main message, we can take the analysis one step further to model returns from funds where redemption or monetization flows are of magnitude that are more in-line with what an investor would expect to receive from a tail hedge. In Exhibit 5, as the size of percentage returns increase, the difference between the two calculations continues to diverge. Note, it's broadly recognized in the industry that a 5x or 500% return is not necessarily considered outsized for a tail hedging portfolio. As a matter of fact, and as described in the next section, a 5x to 10x return on premium deployed is quite within expectations for a typical tail hedging strategy during a market event that the tail hedge is targeting[4].

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	40	50	40	400.00%
2	10	0	0	10	0	0.00%
3	10	0	30	40	0	300.00%
		10	70		40	
Compounded Return						1900.00%
Dollar Return						700.00%

Exhibit 5: Large Tail Hedge Returns with Redemption

Exhibit 6 shows the effects of large negative returns which, again, are expected for any tail hedge portfolio as options decay to zero. Here, the portfolio makes a 5x return in the first period, and the investor redeems the $40 profit. There is no change in value in the second period, and the portfolio loses the majority of its value in the third period. An example of such a payoff profile would be a levered "right tail" call option strategy that pays out when the equity market goes up. If we look at the dollar based return, the investor in this strategy would hypothetically make $31 on a $10 investment. However, because of the effect of the cash flow redemption, the compounded return is -50%. A negative return when the investor made 4x on the initial investment is clearly not representative of the investor's true experience in the example.

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	40	50	40	400.00%
2	10	0	0	10	0	0.00%
3	10	0	-9	1	0	-90.00%
		10	31		40	
Compounded Return						-50.00%
Dollar Return						310.00%

[4] Expected returns should not be considered reliable predictions of future events and should not be relied on as such. Actual realized returns on investments will depend on a variety of factors, such as the value of the assets and market conditions at the time of a transaction, any related transaction costs, and the timing and manner of sale, all of which may differ from the assumptions on which expected returns are based. Actual realized returns on investments may differ materially from any expected returns range presented herein.

Exhibit 6: Large Negative Returns

Finally, in Exhibit 7, we show a return stream where the tail hedge value has decayed to zero in the first period resulting in a -100% return. To continue the tail hedge program, a new subscription of $10 is done in the second period and earns a +500% return, followed by a small loss in the third period. The dollar profit net of total subscriptions is positive in this hypothetical example, but the compounded return is -100%. The first period return of zero effectively corrupted the future return stream since all future returns will be multiplied by the initial -100% return. While the compounding based computation is not incorrect, it simply does not communicate the positive dollar returns that were realized in this example. While the likelihood of a fund losing its entire value at a single point in time is unlikely in reality, this example is meant to demonstrate that the simple mathematics of NAV based performance calculations may not always be conceptually representative of the actual value to the investor. Anecdotally, in March of 2020, VIX call option strategies and to a close degree S&P 500 index put options strategy demonstrated almost exactly this type of return profile. While these options had lost close to 100% of their premium value over the last ten plus years, they delivered large enough returns to make up for all the cumulative losses in one episode! An observer looking at the cumulative returns even after the large gain would compute the compounded return of such a strategy to be close to -100%, paying no attention to the gains realized when the hedge was effective. This conclusion would not ascribe any value to the payoffs from the hedge when it was critically valuable to the investor.

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	-10	0	0	-100.00%
2	0	10	50	60	0	500.00%
3	60	0	-5	55	0	-8.33%
		20	35		0	
Compounded Return						-100.00%
Dollar Return						175.00%

Exhibit 7: Loss of Premium over a Single Period

As we hope these examples have shown, we believe providing NAV based returns in isolation for tail hedge funds are not sufficient to provide a clear picture of performance. The inclusion of cash flows and NAV, in the context of the underlying portfolio that is being hedged is necessary; cash-flow magnitude and timing are both important to obtain the correct picture of the tail hedge performance. This is because the NAV and any performance calculation based on it only shows the performance of the non-monetized value remaining in the fund, and the reason tail hedging is implemented is in recognition of the cash-flow that can be monetized and possibly extracted contingent on a large market event.

Now that we have established a clear background for appropriate performance computations, we proceed to use this framework to demonstrate the three main points of this paper outlined in the introduction.

How Properly Accounting For Tail Hedges Provides A More Complete View of Their Value

To help better communicate the historical performance of a tail hedge and what an investor experience might have been, we present our data in a dollar based, normalized manner using a simple rules-based passive tail risk hedging strategy. There is nothing special about the example used here, and we simply picked a conservative representative backtest from 2017-09-18 to 2021-10-31 where the tail hedge strategy purchases 20% OTM puts and monetizes when the options reach an 8x multiple of their respective premium. Other examples abound, and certainly the reader can take our example and apply it to various hypothetical scenarios to judge the pros and cons of the value of tail hedges using the proper framework that includes cash flow magnitudes and timing. More details on the experience of actual tail hedge funds through COVID-19 and prior are presented in a recent paper on monetization strategies that use fund data (see Bhansali et. al. [2020]).

To make the computations tractable and transparent, we made the following assumptions in our calculations:

1. Portfolios have a starting value of $100
2. The sum of all subscriptions for the life of the tail hedging strategy are normalized to a $2 annualized "spend". This means that on average, the cost of the hedge was 2% per year.
3. Month-end returns on the NAV of the tail hedge strategy assume all inflows for premium cash flows occurred at the beginning of the month and all outflows from monetization occurred at the end of the month

Exhibit 8 shows the normalized market value of the tail hedging strategy which is displayed as NAV in blue. The S&P 500 (market) is shown in grey and set to begin at a value of $100. Cash flows are shown as bars where subscriptions are red and redemptions are green.

We observe that the performance of the tail hedge relative to the S&P 500 shows marked to market gains during two periods of market stress where the grey line has the most significant declines. In December of 2018, the S&P 500 declined just under 10% while the return of the tail hedging strategy for the same month was 79.89% (Appendix Exhibit 12). During the COVID crisis of March 2020, the SPX dropped over 12% and the hedging strategy's return was 203.46%.

Subscriptions occur incrementally throughout the life of the strategy as options decay and additional cash is needed to put on new positions. There is a single, but significant monetization event during the COVID crisis when the 8x multiple threshold is reached.

Exhibit 8: LeftTail Strategy Market Value and Cash Flows
Source: LongTail Alpha, OptionMetrics, Bloomberg

When we compound the historical month-end NAV returns of the tail hedge, we get a value of -99.99% suggesting that the tail hedge lost the full amount of total subscriptions in the hedging strategy (Appendix Exhibit 12). This would seem like dismal performance for a strategy to most investors. However, applying the dollar return methodology paints a very different picture of the costs versus benefits. When we look at the sum of premium cash flows and monetization cash flows in Exhibit 9, we see that the hedging strategy spent a total of $10.00 in subscriptions and earned $5.10 in redemptions for a total net loss of $4.90, or 49%. Although this is still a net negative dollar amount, it is a much smaller actual loss compared to the -99.99% calculated from the compounded return. Many investors may not mind having a 49% loss on a small amount of premium in order to experience a substantial payoff when the markets are crashing and there is widespread panic and distress. Trying to appraise the value of the strategy from compounded returns would possibly have distracted attention from the magnitude and timing of the desirable contingent payoff.

	Premium Cash Flow	Monetization Cash Flow
9/18/2017	2.00	
12/14/2017	0.50	
3/15/2018	0.50	
6/14/2018	0.50	
9/20/2018	0.50	
12/21/2018	0.50	
3/14/2019	0.50	
6/20/2019	0.50	
9/19/2019	0.50	
12/19/2019	0.50	
3/16/2020		-5.10
3/19/2020	0.50	
6/18/2020	0.50	
9/17/2020	0.50	
12/18/2020	0.50	
3/18/2021	0.50	
6/17/2021	0.50	
9/16/2021	0.50	
Total	**10.00**	**-5.10**

Exhibit 9: LeftTail Strategy Cash Flows
Note: Assumes an annualized Premium Cash Flow budget of $2 to protect $100.

How Combining With the Underlying Portfolio Demonstrates Total Portfolio Gains

In this section we demonstrate that the timing and magnitude of the cash flows also plays an important role when combining a tail hedge with the overall portfolio, in terms of increasing long term risk adjusted expected returns. Some of this was discussed in theory and practice in Bhansali and Davis [2010]).

We will proceed with the same tail hedging strategy, but use it as an overlay alongside a base portfolio solely comprised of the S&P 500 Index. We will also increase the date range of the simulated backtest from 1996-01-02 to 2021-10-31 to get a longer term view. As before, the tail hedge market value and cash flow amounts are normalized so that the annualized sum of all subscriptions is $2 per year. Our total portfolio will have a starting value of $100, and all subscriptions will be funded from this amount as they are needed for the tail hedge portfolio. Similarly, any redemptions will be reinvested back into the S&P 500 Index.

Looking at the aforementioned largest S&P 500 Index declines during our sample time period, we can see that the overlay portfolio had a marked improvement over the S&P 500 Index alone. As shown in Exhibit 10, the hedged portfolio relative to the index alone had an improved drawdown of 1.34% in December 2018, and 6.06% in March 2020.

	S&P 500 with LeftTail Strategy	S&P 500
…		
10/31/2018	-6.10%	-6.94%
11/30/2018	1.25%	1.79%
12/31/2018	-7.84%	-9.18%
…		
1/31/2020	-0.21%	-0.16%
2/29/2020	-6.76%	-8.41%
3/31/2020	-6.45%	-12.51%
…		

Exhibit 10: S&P 500 with LeftTail Strategy Overlay
Source: LongTail Alpha

Exhibit 11 shows several total portfolio level statistics for the two strategies. Focusing on the full history of the strategy in the CAGR (Cumulative Annualized Growth Rate) column, we can see that the hedged portfolio underperformed by about 57bp on an annualized basis. However, the hedged portfolio had almost 11% less of a drawdown, an improved left tail skew and a lower volatility. This improvement in the overall risk characteristics when compared to the index portfolio alone allows the investor to be more aggressive in order to achieve higher returns. If the two portfolios were normalized to target the same 15% volatility, the hedged portfolio would outperform by around 80bp on an annualized basis. In other words, for the same amount of risk, the investor is able to generate more long term returns. This idea has been discussed by us in previous papers on how monetization and re-investment based on even very simple rules can result in substantial long term increase in risk-adjusted returns (see Bhansali et. al. [2020]). The results of this simple aggregation exercise suggest that with the proper and consistent accounting, where the hedges are combined with the underlying equity portfolio, makes the tail hedged exposure to the market a candidate for benchmarking liquid equity market exposure when drawdown risk is a concern. While this is an interesting idea, as of this writing the lack of uniformity in underlying tail hedging strategies has made it hard for investors to identify what a proper tail hedged equity benchmark should look like. We believe that over time more investors will see the value of aggregation and improved risk adjusted performance to adopt equity market benchmarks with built in tail hedges, and standardization will likely develop.

	Total Return	CAGR	CAGR 15% Vol	Max Drawdown	Calmar Ratio	Monthly Sharpe	Monthly Vol (ann.)
S&P 500 with LeftTail Strategy	547.08%	7.49%	8.69%	-45.07%	0.17	0.63	12.69%
S&P 500	641.93%	8.06%	7.89%	-56.78%	0.14	0.58	15.21%

	Monthly Skew	Monthly Kurt	Best Day	Worst Day	Best Month	Worst Month
S&P 500 with LeftTail Strategy	-34.00%	0.3	6.77%	-6.84%	10.12%	-12.94%
S&P 500	-61.00%	1.13	11.58%	-11.98%	12.68%	-16.94%

Exhibit 11: Overlay Summary Statistics
Source: LongTail Alpha

Discussion

So far we have highlighted the shortcomings of relying only on traditional, NAV based metrics of performance for tail risk hedging strategies. In the process, we have argued that cash flow characteristics, including both magnitude and timing, provide a more accurate picture of the true economic value of hedging strategies. One purpose of quantifiable metrics for performance measurement is to create an estimate of the reliability of the hedging strategy ex ante. The payoffs are a highly complex function of the underlying portfolio of hedging securities, the price initially paid to acquire them, the monetization approach, and many other aspects. Therefore forecasting the payoffs in the future are dependent on these assumptions. While historical performance of a hedge might shed some light on the expected future performance of the hedge, one has to be careful not to extrapolate past history for a strategy that is significantly different in its design. By (1) carefully selecting design elements that reflect true portfolio needs, (2) stress shocking a portfolio of hedging securities that are consistent with the design elements, and (3) simultaneously running back-test simulations for a more complete picture of the hedge strategy's efficacy can be obtained.

However, there is more to the performance measurement exercise than just simple arithmetic computations, which we will discuss in this section. While market based tail risk hedging is not insurance per se, we can take some cues from the insurance and re-insurance markets on how that industry deals with similar performance measurement issues.

In a recent survey by J.D. Power [2020], the following five additional factors were mentioned as important to maintaining lasting relationships with homeowners who are insuring themselves against loss: interaction, policy offerings, price, billing process, policy information, and claims. The reliability of claims payments in the event of a loss is obviously critical, and we can reasonably conclude that for every investor looking for a tail hedge, a high degree of comfort in the tail hedge doing what it is designed to do is probably at the top of the list of metrics. But as for a homeowner, a simple process that makes the tail hedge transparent and easy to monitor, monetize and manage is also important. Tail hedging is a solution that best serves its purpose when customized towards solving specific portfolio construction problems. Thus tail hedging only makes sense when the risks of the underlying portfolio and the investor utility function are understood. In this context, and as discussed above, it is important for investors to aggregate the hedge with the underlying portfolio exposures that are being hedged to extract the full utility of the hedge.

Other alternatives should also be considered as well. For instance, diversification strategies should always be considered when available for endogenous portfolio risk mitigation.

An allocation to fixed income or even alternatives such as trend following have demonstrated the ability to reduce portfolio risk in many macroeconomic environments, without the complications of having a different, cash flow based measurement approach within the portfolio. However, at the time of this writing, the low yield levels and perception of inflation risks on the horizon has created the concern that fixed income might not be a very potent diversifier until yields have risen to long term historical levels. And the basis risk or delayed response from alternatives such as trend following makes these strategies somewhat less reliable against sharp market shocks. Our belief is that all diversification strategies have a place in investment portfolios, and proper attention to their purpose, cost, and implementation details are just as important as the ability to measure ex-post performance. For options based tail hedging, which doesn't fit neatly into the box of fully funded traditional or alternative strategies, we believe that the framework has to be expanded in the manner discussed in this paper.

Conclusions

We discuss the importance of using a more comprehensive set of metrics for measuring the performance of tail risk hedging portfolios in particular, and any strategy with levered payoffs in general. Using historical compounded returns when the payoffs are multiples of the premium, and such payoffs are withdrawn, can paint an inaccurate picture, sometimes grossly so, of the economic value of such strategies. The reason, as discussed above, is that the timing and magnitude of the cash flows matter immensely, and when such cash flows are withdrawn, the impact on the compounded returns has to be corrected for the value added from the cash flows. Otherwise one can arrive at exactly the wrong conclusions regarding the efficacy of these strategies.

Further, we demonstrated that when the cash flows generated during market crises are re-invested in the markets, the strategy can increase the long term risk-adjusted returns, rather than reducing the returns of the portfolio as one would expect from the negative expected return of owning an insurance policy. We hope that we have demonstrated that tail hedges can be value-additive to portfolios by providing protection during market events, providing liquidity, and improving overall risk characteristics. However, to properly evaluate the performance of a tail hedge or other high payoff strategies, the full set of measurement tools must be applied. In particular, we conclude that one cannot just limit the analysis to compounded hypothetical returns that don't pay attention to cash-flow magnitude and timing alone.

Beyond the quantitative metrics that investors obviously have to apply to gauge the value of any strategy, tail hedging requires attention to other very important features, some of which are somewhat subjective but equally important. The robustness and reliability of hedges, which is the main reason to engage in hedging activity in the first place, has to be evaluated based on proper, customized portfolio design.

Appendix

Month End	Month Start	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
9/30/2017	9/1/2017	0.00	2.00	-0.32	1.68	0.00	-16.11%
10/31/2017	10/1/2017	1.68	0.00	-0.69	0.99	0.00	-40.92%
11/30/2017	11/1/2017	0.99	0.00	-0.35	0.64	0.00	-35.47%
12/31/2017	12/1/2017	0.64	0.50	-0.25	0.89	0.00	-22.07%
1/31/2018	1/1/2018	0.89	0.00	-0.33	0.55	0.00	-37.55%
2/28/2018	2/1/2018	0.55	0.00	0.36	0.92	0.00	65.69%
3/31/2018	3/1/2018	0.92	0.50	0.24	1.66	0.00	16.79%
4/30/2018	4/1/2018	1.66	0.00	-0.65	1.01	0.00	-39.20%
5/31/2018	5/1/2018	1.01	0.00	-0.32	0.69	0.00	-31.54%
6/30/2018	6/1/2018	0.69	0.50	0.06	1.25	0.00	5.20%
7/31/2018	7/1/2018	1.25	0.00	-0.54	0.72	0.00	-42.80%
8/31/2018	8/1/2018	0.72	0.00	-0.21	0.50	0.00	-29.80%
9/30/2018	9/1/2018	0.50	0.50	-0.09	0.91	0.00	-9.17%
10/31/2018	10/1/2018	0.91	0.00	0.88	1.79	0.00	96.87%
11/30/2018	11/1/2018	1.79	0.00	-0.52	1.28	0.00	-28.79%
12/31/2018	12/1/2018	1.28	0.50	1.42	3.20	0.00	79.90%
1/31/2019	1/1/2019	3.20	0.00	-2.17	1.02	0.00	-67.98%
2/28/2019	2/1/2019	1.02	0.00	-0.43	0.59	0.00	-42.11%
3/31/2019	3/1/2019	0.59	0.50	-0.24	0.86	0.00	-21.72%
4/30/2019	4/1/2019	0.86	0.00	-0.32	0.54	0.00	-37.02%
5/31/2019	5/1/2019	0.54	0.00	0.33	0.87	0.00	61.18%
6/30/2019	6/1/2019	0.87	0.50	-0.50	0.87	0.00	-36.20%
7/31/2019	7/1/2019	0.87	0.00	-0.18	0.69	0.00	-20.79%
8/31/2019	8/1/2019	0.69	0.00	0.16	0.86	0.00	23.82%
9/30/2019	9/1/2019	0.86	0.50	-0.20	1.16	0.00	-14.56%
10/31/2019	10/1/2019	1.16	0.00	-0.40	0.76	0.00	-34.67%
11/30/2019	11/1/2019	0.76	0.00	-0.28	0.48	0.00	-37.21%
12/31/2019	12/1/2019	0.48	0.50	-0.21	0.76	0.00	-21.71%
1/31/2020	1/1/2020	0.76	0.00	-0.05	0.71	0.00	-6.50%
2/29/2020	2/1/2020	0.71	0.00	1.76	2.47	0.00	246.59%
3/31/2020	3/1/2020	2.47	0.50	6.05	9.03	5.10	203.46%
4/30/2020	4/1/2020	3.93	0.00	-2.62	1.31	0.00	-66.76%

5/31/2020	5/1/2020	1.31	0.00	-0.75	0.55	0.00	-57.77%
6/30/2020	6/1/2020	0.55	0.50	-0.25	0.80	0.00	-24.03%
7/31/2020	7/1/2020	0.80	0.00	-0.34	0.46	0.00	-42.94%
8/31/2020	8/1/2020	0.46	0.00	-0.15	0.31	0.00	-32.67%
9/30/2020	9/1/2020	0.31	0.50	-0.06	0.75	0.00	-7.15%
10/31/2020	10/1/2020	0.75	0.00	0.13	0.88	0.00	17.30%
11/30/2020	11/1/2020	0.88	0.00	-0.57	0.31	0.00	-64.52%
12/31/2020	12/1/2020	0.31	0.50	-0.11	0.70	0.00	-13.60%
1/31/2021	1/1/2021	0.70	0.00	0.22	0.92	0.00	31.58%
2/28/2021	2/1/2021	0.92	0.00	-0.29	0.64	0.00	-31.12%
3/31/2021	3/1/2021	0.64	0.50	-0.48	0.66	0.00	-42.09%
4/30/2021	4/1/2021	0.66	0.00	-0.20	0.46	0.00	-30.80%
5/31/2021	5/1/2021	0.46	0.00	-0.11	0.35	0.00	-23.51%
6/30/2021	6/1/2021	0.35	0.50	-0.17	0.68	0.00	-20.06%
7/31/2021	7/1/2021	0.68	0.00	-0.07	0.61	0.00	-9.89%
8/31/2021	8/1/2021	0.61	0.00	-0.16	0.45	0.00	-25.67%
9/30/2021	9/1/2021	0.45	0.50	0.24	1.19	0.00	24.63%
10/31/2021	10/1/2021	1.19	0.00	-0.56	0.63	0.00	-47.33%

Month End	Month Start	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
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11/30/2017	11/1/2017	0.99	0.00	-0.35	0.64	0.00	-35.47%
12/31/2017	12/1/2017	0.64	0.50	-0.25	0.89	0.00	-22.07%
1/31/2018	1/1/2018	0.89	0.00	-0.33	0.55	0.00	-37.55%
2/28/2018	2/1/2018	0.55	0.00	0.36	0.92	0.00	65.69%
3/31/2018	3/1/2018	0.92	0.50	0.24	1.66	0.00	16.79%
4/30/2018	4/1/2018	1.66	0.00	-0.65	1.01	0.00	-39.20%
5/31/2018	5/1/2018	1.01	0.00	-0.32	0.69	0.00	-31.54%
6/30/2018	6/1/2018	0.69	0.50	0.06	1.25	0.00	5.20%
7/31/2018	7/1/2018	1.25	0.00	-0.54	0.72	0.00	-42.80%
8/31/2018	8/1/2018	0.72	0.00	-0.21	0.50	0.00	-29.80%
9/30/2018	9/1/2018	0.50	0.50	-0.09	0.91	0.00	-9.17%
10/31/2018	10/1/2018	0.91	0.00	0.88	1.79	0.00	96.87%

11/30/2018	11/1/2018	1.79	0.00	-0.52	1.28	0.00	-28.79%
12/31/2018	12/1/2018	1.28	0.50	1.42	3.20	0.00	79.90%
1/31/2019	1/1/2019	3.20	0.00	-2.17	1.02	0.00	-67.98%
2/28/2019	2/1/2019	1.02	0.00	-0.43	0.59	0.00	-42.11%
3/31/2019	3/1/2019	0.59	0.50	-0.24	0.86	0.00	-21.72%
4/30/2019	4/1/2019	0.86	0.00	-0.32	0.54	0.00	-37.02%
5/31/2019	5/1/2019	0.54	0.00	0.33	0.87	0.00	61.18%
6/30/2019	6/1/2019	0.87	0.50	-0.50	0.87	0.00	-36.20%
7/31/2019	7/1/2019	0.87	0.00	-0.18	0.69	0.00	-20.79%
8/31/2019	8/1/2019	0.69	0.00	0.16	0.86	0.00	23.82%
9/30/2019	9/1/2019	0.86	0.50	-0.20	1.16	0.00	-14.56%
10/31/2019	10/1/2019	1.16	0.00	-0.40	0.76	0.00	-34.67%
11/30/2019	11/1/2019	0.76	0.00	-0.28	0.48	0.00	-37.21%
12/31/2019	12/1/2019	0.48	0.50	-0.21	0.76	0.00	-21.71%
1/31/2020	1/1/2020	0.76	0.00	-0.05	0.71	0.00	-6.50%
2/29/2020	2/1/2020	0.71	0.00	1.76	2.47	0.00	246.59%
3/31/2020	3/1/2020	2.47	0.50	6.05	9.03	5.10	203.46%
4/30/2020	4/1/2020	3.93	0.00	-2.62	1.31	0.00	-66.76%
5/31/2020	5/1/2020	1.31	0.00	-0.75	0.55	0.00	-57.77%
6/30/2020	6/1/2020	0.55	0.50	-0.25	0.80	0.00	-24.03%
7/31/2020	7/1/2020	0.80	0.00	-0.34	0.46	0.00	-42.94%
8/31/2020	8/1/2020	0.46	0.00	-0.15	0.31	0.00	-32.67%
9/30/2020	9/1/2020	0.31	0.50	-0.06	0.75	0.00	-7.15%
10/31/2020	10/1/2020	0.75	0.00	0.13	0.88	0.00	17.30%
11/30/2020	11/1/2020	0.88	0.00	-0.57	0.31	0.00	-64.52%
12/31/2020	12/1/2020	0.31	0.50	-0.11	0.70	0.00	-13.60%
1/31/2021	1/1/2021	0.70	0.00	0.22	0.92	0.00	31.58%
2/28/2021	2/1/2021	0.92	0.00	-0.29	0.64	0.00	-31.12%
3/31/2021	3/1/2021	0.64	0.50	-0.48	0.66	0.00	-42.09%
4/30/2021	4/1/2021	0.66	0.00	-0.20	0.46	0.00	-30.80%
5/31/2021	5/1/2021	0.46	0.00	-0.11	0.35	0.00	-23.51%
6/30/2021	6/1/2021	0.35	0.50	-0.17	0.68	0.00	-20.06%

7/31/2021	7/1/2021	0.68	0.00	-0.07	0.61	0.00	-9.89%
8/31/2021	8/1/2021	0.61	0.00	-0.16	0.45	0.00	-25.67%
9/30/2021	9/1/2021	0.45	0.50	0.24	1.19	0.00	24.63%
10/31/2021	10/1/2021	1.19	0.00	-0.56	0.63	0.00	-47.33%

Exhibit 12: LeftTail Strategy NAV Returns
Source: LongTail Alpha, OptionMetrics, Bloomberg

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Overlay market values are for illustrative, informational purposes only. They are computed using actual fund flows and net of fees performance. All cash flows are reflective of the strategy's cash flows, but normalized to a $2 per year annualized sped. This implied that the total notional value being hedged is $100 with a $2 annual implementation cost.

Tail Risk hedging strategies are generally designed to protect against large unexpected financial market moves. The concept is to sacrifice a portion of return each year in order to protect a portfolio against a sharp adverse market meltdown or meltup. Tail Risk hedging strategies purchase out of the money options and option structures. In exchange for the leverage offered by these options, an investor is explicitly taking the risk that the total value of the premium spent on purchasing the options or options structures decays to zero.

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Max Drawdown is the worst peak to through return since inception.

The skewness of a dataset measures the degree of distortion from the symmetrical bell curve in a probability distribution and can be calculated by subtracting the mode from the mean and dividing the difference by the standard deviation

Volatility is the standard deviation of returns annualized.

CAGR 15 Vol (Cumulative Annual Growth Rate) represents the return since inception annualized assuming the returns were scaled to achieve a 15% annualized volatility.

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