

The Low-Risk Effect, from Betting Against Beta to Betting Against Correlation

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January 13, 2022

Abstract

The aim of this work is to analyze the so-called "Low Risk Effect" and the evolution of the risk-reward relationship in time. Perhaps one of the milestones of the whole modern finance has been the investigation and the debate about the positive relationship between risk and reward in asset allocation, but are we sure that this theoretical paradigm is able to provide also empirical evidence? Starting from the "father" of Modern Portfolio Theory, passing through Sharpe's CAPM and Black 1972, we analyzed in deep the so-called "Low Risk Effect" and the debate between leverage constraints and behavioural theories. We concluded our journey decomposing Betting Against Beta (Asness, Frazzini and Pedersen) and analysing their last contribution: Betting Against Correlation (BAC), a factor that goes long low correlation stocks and shorts high correlation ones. Starting from the BAC methodology framework, we decided to create some modifications in order to test the goodness of the model in terms of performance against the reference index. Finally we tried to implement a profitable strategy for S&P500 over the time interval 2003-2021, evidencing the phases of negative correlated stocks and arriving to define strategy's sector composition. To conclude our work we performed a sectorial analysis in which we investigated the composition of Long/Short portfolios for our best strategy Correlation Weighted qBAC, trying to evidence the main drivers for strategy's performance and critical issues in the last few years. To go further we built a "walking correlation analysis" that resulted useful to observe the dynamic evolution of stocks correlation in time both against the market and within the sector.

1 Introduction to the Low Risk Effect

On 1952 ¹, Henry Markowitz laid the foundations for the whole modern portfolio analysis, trying to solve the problem that would have afflict a great number of economists in time: the relationship between risk and reward in capital assets. In supporting the concept of diversification as fundamental tool in portfolio construction and asset allocation, Markowitz² pointed out two specific features: uncertainty and correlation among stock returns, stating that if we consider rational behaviours, investors

will always choose the portfolio characterized by the highest return for a fixed level of risk. A decade after Markowitz's intuition, Sharpe ³ and Treynor ⁴ created the basis of Capital Asset Pricing Model (CAPM), a milestone of the whole modern financial theory. Starting from the model of rational investor behaviour and utility maximization, Sharpe operated to identify an appropriate measure of risk for capital assets and to define the equilibrium relationship between expected return and risk. Assuming ⁵:

¹H. Markowitz, "Portfolio Selection", *Journal of Finance*, Vol. 7, No 1 (1952)

²H. Markowitz, "Efficient Diversification" (1959)

³W. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", *J. of Finance* (1964)

⁴J. Treynor, "Market Value, Time, and Risk" (1961)

⁵F. Black, M. Jensen and M. Scholes, "The Capital Asset Pricing Model: Some Empirical Tests" (1972)

- 1 Risk-averse investors, wishful to maximize their expected utility within an homogeneous time horizon, solely choosing on the basis of mean and variance.
- 2 Absence of taxes and transaction costs.
- 3 Homogeneous knowledge of parameters of the joint probability distribution of returns.
- 4 Ability to borrow or lend at a given riskless rate of interest.

It has been possible to define the relationship between the expected risk premiums of assets and their *systematic risk*:

$$E(\tilde{R}_i) = E(\tilde{R}_M)\beta_i \quad (1)$$

The equation (1) contains the feature that will be the pivotal point of the whole analysis: the "Beta", representing the sensitivity of the asset's returns to the market or simply the component of "systematic" (non-diversifiable) risk of any asset i and defined as follows:

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)}$$

Recurring to (1) it is then possible to state that expected excess return is directly proportional to its β and to define α_i as

$$\alpha_i = E(\tilde{R}_i) - E(\tilde{R}_M)\beta_i$$

From the abovedescribed CAPM's Beta directly descends the concept of **Security Market Line (SML)**. Nothing but the relationship between risk and return for securities, SML puts in relation the expected return with the β , simply defining a risk-reward equation. Because of this relationship it is possible to assert that the risk premium of an asset $[E(R_i) - r_f]$ is directly proportional to market portfolio β and risk premium:

$$E(R_i) - r_f = \beta[E(R_M) - r_f] \quad (2)$$

Figure 1 shows the graphical representation of the SML:

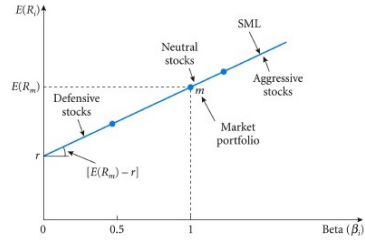


Figure 1: The Security Market Line

The SML presents a trade-off between systematic risk and return for any i -th asset, then offering a theoretical risk-adjusted price equilibrium. Thus, assets characterized by fair price lie exactly on the SML and then, in correspondence of the SML, the risk is appropriately remunerated by the expected return. For this reason, securities that are plotted above the SML are considered *undervalued*, while securities lying below it are *overvalued*. The gap between the actual return and the theoretical expected return is the α .

The model here presented represents surely a key point, but it has not been exempt from critics and broad discussions. For example, Black, Jansen and Scholes ⁶ (BJS) tested empirically, for the 1926-1966 interval, the model provided by CAPM providing a complete description of the structure of security returns. Confirming Pratt's intuition ⁷ BJS find out that from 1931 to 1965 low-beta stocks performed better than CAPM predicts, in the US stock market. Black ⁸ essentially called into question a specific assumption of CAPM, according to which investors should be able to lend or borrow at a risk free rate, deriving a SML sensibly flatter than the one predicted by Sharpe-Litner-Mossin. The main issue with CAPM, both for academics and practitioners, has been the use of a single source of risk (β) to describe the relationship between capital assets' risk and return. Starting from this spark, some researchers started to study alternative models, known as *multifactor methods*, that are able to capture and incorporate micro and macroeconomic variables, such as inflation, oil prices or GDP growth. Following this hint, the Arbitrage Pricing Model came on stage:

$$E(R_i) = r_f + \beta_{i1}\lambda_1 + \dots + \beta_{ik}\lambda_k \quad (3)$$

where β_i represents the asset's sensitivity to a specific factor and λ_k is known as " k -factor"

⁶F. Black, M. Jensen and M. Scholes, "The Capital Asset Pricing Model: Some Empirical Tests" (1972)

⁷S. Pratt, "Relationship between Viability of Past Returns and Levels of Future Returns for Common Stocks" (1967)

⁸F. Black, "Capital Market Equilibrium with Restricted Borrowing", *The Journal of Business*, Vol. 45, No 3 (1972)

⁹E. Fama and K. French, "Common Risk Factors in the Returns on Stocks and Bonds", (1993)

risk premium". More recently the contribution provided by Fama and French states that the positive relationship represented by the slope of SML disappeared during the 1963-1990 time intervals, here the researchers' need to find additional variables to be inserted in the model. In 1993, Fama and French⁹ extended their asset pricing theory through the so-called "*Three Factor Model*", with the result that a market factor combined with the two proxies seems to provide a good explanation of cross-sectional average returns:

- **Size:** **SMB** (Small minus Big), the monthly difference between the returns on a small stocks portfolio and a big stocks one.
- **Book-to-market:** **HML** (High minus Low), the monthly difference between the returns on high book-to-market value portfolio and low book-to-market value one.
- **Market factor:** **Excess Market Return** ($R_M - r_f$). Where R_M is the return on a market portfolio and r_f one-month bill rate used as risk free rate.

The important debate around the Three Factors Model and its assumptions has been the push for the formulation of the more recent *Five Factor Model*, provided by Fama and French in 2014¹⁰ and still debated. Starting from the concept and formulation of the *Dividend Discount Model* (DDM), the two researchers have been able to show analytically the relation between stock average returns and two new variables: *profitability* and *investments*.

Positive, Flat or Negative? In oppositions to the traditional theory, some practitioners studied in deep the slope of SML in the effort to demonstrate that the risk-reward relationship is not positive or flat, but even negative. In 1972, Haugen and Heins¹¹ laid the foundations for this innovative idea: they essentially documented a negative relationship between risk and realized return for the 1926-1971 period in both US Stock and Bond Markets. The beginning of the 2000s has been characterized by several studies based on H&H results, such as Blitz and Van Vliet¹² and Clarke, de Silva and Thorley¹³ presented empirical evidence that stocks characterized by low volatility generate higher risk-adjusted returns and then high risk stock

significantly underperform, over the 1986-2006 period. In 2012, Baker and Haugen¹⁴ tested the above mentioned phenomenon, both on developed and emerging markets over the 1990-2011 period. Stocks are ranked by volatility and divided into *deciles* for each local market, re-ranked at the beginning of each month up to the end of the interval. In *Figure 2* we can easily observe the interval year-by-year results for annual return and volatility:

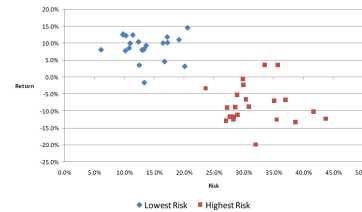


Figure 2: Annual Return vs Volatility, Lowest Decile and Highest Decile 1990-2011

Graphically it results evident that there is a negative relationship (negative slope) between risk and returns with lowest-risk stocks clearly outperforming highest-risk ones. The result is even more undeniable in *Figure 3*, where we can observe the rolling 3-year difference between low volatility and high volatility:

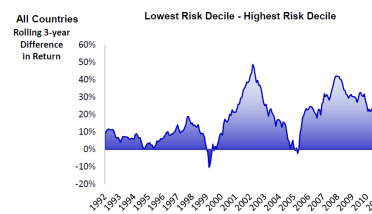


Figure 3: Rolling 3-year Difference, Lowest Risk Decile - Highest Risk Decile

Absolutely deserving is the study provided by Cowan and Wilderman¹⁵ who focused their attention on the so-called "*Beta Puzzle*", evidencing that portfolios composed by low beta stocks historically matched or beaten the market, while higher risk stocks significantly underperformed. *Figure 4* shows the performance of low and high beta U.S. stocks for the period 1969-2011:

¹⁰E. Fama and K. French, "A Five-Factor Asset Pricing Model" (2014)

¹¹R. Haugen and A. Heins, "On the Evidence Supporting the Existence of Risk Premiums in the Capital Market"

¹²D. Blitz and P. Van Vliet, "The Volatility Effect: Lower Risk Without Lower Return" (2007)

¹³R. Clarke, H. de Silva and S. Thorley, "Minimum-Variance Portfolios in the U.S. Equity Market" (2006)

¹⁴N. Baker and R. Haugen, "Low Risk Stocks Outperform within All Observable Markets of the World" (2012)

¹⁵D. Cowman and S. Wilderman, "Re-Thinking Risk: What the Beta Puzzle Tells Us about Investing" (2011)

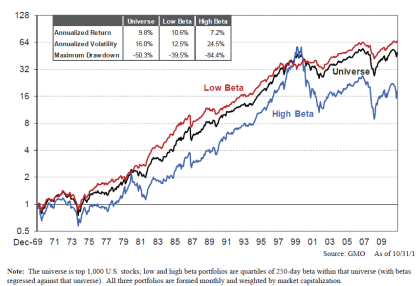


Figure 4: Beta Puzzle - U.S.

Results show that the portfolio composed by low beta stocks realized an annualized return of 10.6% against a return of 7.2% of high beta ones, outperforming the market. The only exception is in the closeness of the *Dot-com* bubble, such as for the 3-years rolling difference in Figure 2. Also in this case empirical test seems to overturn the positive relation between risk and return. The contribution provided by Bob Haugen¹⁶ represents a real turning point in terms of modern finance and investments, but what are the causes that has been able to determine a negative relationship between risk and return in time? In 2011, Baker, Bradley and Wurgler¹⁷ tried to adapt fundamental principles of *behavioural finance* to explain the low volatility anomaly, obtaining that some agents manifest a preference for lotteries and bias of representativeness and overconfidence tend to lead to an extra demand of high risk stocks. Remaining on agents behaviours, enlightening is the contribution provided by Société Générale¹⁸, Dylan Grice and Albert Edwards evidence that high risk is able to produce in investors' mind high excitement; because of this, quality stocks are usually seen as boring, and "boring" is systematically underpriced.

Shifting to deviations from market efficiency, Baker, Bradley and Taliaferro¹⁹ argued that inefficiencies need to be considered at different levels. In this sense, they decomposed the low-risk anomaly into micro and macro components, finding that the procedure has both theoretical and empirical implications. The introduction of behavioural assumptions and market inefficiency seems to invert the classical risk-reward relationship. Baker and Haugen even affirmed

that "*the basic pillar of finance has fallen*", evidencing the necessity of using behavioural models characterized by market inefficiencies. But how to Ride β Anomaly? In this sense, Bernstein²⁰ provides a complete and comprehensive analysis on "*Beta anomaly*", trying to develop some portfolio strategies and exploit the phenomenon and claiming that low-beta stocks provide similar average returns with respect to high-beta ones, but recurring to much lower volatility. Bernstein provides two main explanations of low-beta anomaly: one related to behavioral finance and preferences for *lottery-like* returns and the other related to the theory of leverage constraints. Citing Cowan and Wilderman, it states that high-beta stocks are traded *at premium* because they offer a sort of "*leverage with protection*" (or implicit leverage) due to their convex payoff, similar to that of an option:

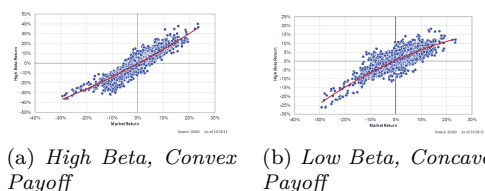


Figure 5: High vs Low Beta Payoff

Considering options' terminology, high beta payoff seems to have the same characteristics of owning the market plus a call on the market: in a rally investor gets the extra payoff of the call, while in decline he loses one the call premium. Conversely, low beta is similar to owning the market and selling a call on the market, such as the so-called *covered call strategy*. The convexity appears clearly an advantage for investors, but not all that glitters is gold: convexity does not come for free and this characteristic is generally expensive, tending to result overpriced. The main consequence of different payoff structures is that the spread between high- and low beta portfolios is deeper in case of bull market rather than in bear one, as we can see in Figure 6:

¹⁶Many researches and datasets are available at: <http://www.lowvolatilitystocks.com/>

¹⁷M. Baker, B. Bradley and J. Wurgler, "Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly" (2011)

¹⁸Why we overpay for excitement (and the secret pleasure of being boring), SG, 4 April 2012

¹⁹M. Baker, B. Bradley and R. Taliaferro, "The Low-Risk Anomaly: A Decomposition into Micro and Macro Effects", (2014)

²⁰Quantitative Research: The Low-Beta Anomaly-Easy to Demonstrate, Harder to Explain, Difficult to Exploit (2012)

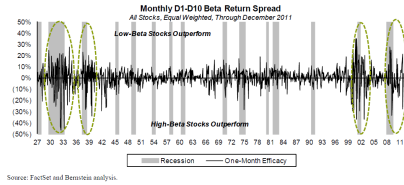


Figure 6: Low vs High Beta Spread, 1927-2011

High beta outperformances seem to be correlated with low quality rallies and characterized

by a string magnitude when they occurs but, in general, low beta stocks tend to overperform high beta ones, as previously formalized. The best way to exploit the anomaly consists then in a long-only strategy consisting of investments in low beta stocks to obtain a higher cumulative return over the long-run period.

Many researchers and practitioners are still working on methodologies and strategies to exploit the anomaly, we will try to provide our view and possible strategies later on.

2 From Betting Against Beta to Betting Against Correlation

In 2014 Andrea Frazzini and Lasse Pedersen presented a new dynamic model of **leverage constraints** starting from the assumption that, for U.S. stock market, the slope of SML is too flat than the one predicted by CAPM. They began with the basic assumption that, according to CAPM, economic agents should prefer the portfolio with the highest expected return per unit of risk and leverage or de-leverage it to meet their risk preference. Nevertheless many investors, such as individuals and mutual funds, are not able to do so, constrained in the leverage they can take. This phenomenon is then able to generate an overweight for risky assets. Their model can be seen as an extension of Black (1972) study and it is essentially based on five proposition. **First Proposition:** the model presumes a flatter SML than the one presented by CAPM, the slope depends on the tightness of the funding constraints across the agents. **Second Proposition:** the model is able to provide positive average return, depending positively on the ex ante tightness of constraints and beta spread. **Third Proposition:** the model predicts that in case of pressure on funding liquidity constraints the BAB factor provides negative returns, because of the increase in expected future returns. As a consequence of this propositions, when the funding liquidity risk is high all the betas are compressed toward one, **Forth Proposition.** It is possible to affirm that investors without constraints prefers low-beta stocks and possibly apply leverage; while, on the other hand, more-constrained agents tend to overweight, and consequently prefer, high-beta stocks, as suggested by the **Fifth Proposition.** Starting from the abovementioned assumptions,

the authors constructed a factor that goes long on low-beta assets and shorts high-beta ones; ranking securities of each asset class on their estimated beta and assigned to one portfolio (high- or low-beta). Portfolios are rebalanced every calendar month.

From the evidence provided by Betting Against Beta factor, Asness, Frazzini, Gormsen and Pedersen (2017)²¹ operate to solve the dispute of academics and practitioners around the underlying economic drivers of the low risk anomaly. The debate is essentially whether the low-risk effect is due to leverage constraints or driven by behavioural effects; in the first case the risk should be measured using *systematic risk*, in the other it should be measured using *idiosyncratic risk*. Starting from the analytical definition of Beta:

$$\beta_i = correlation_{i,M} \times \frac{volatility_i}{volatility_M}$$

the authors have been able to decompose the BAB factor in two new specific factors: betting against correlation (**BAC**) and betting against volatility (**BAV**). BAC goes long stocks with low correlation to the market and shorts stocks with high correlation, while BAV goes short and long based on the volatility. Decomposing BAB Factor, it is then possible to state that:

$$BAB_t = \alpha_0 + \alpha_1 BAC_t + \alpha_2 BAV_t + \epsilon_t$$

For each sub-set considered, the BAC factor has been constructed as follows:

- 1 At the beginning of each month stocks are ranked in ascending order based on their estimated volatility, then their are assigned to one between the five *quintiles*.

²¹C. Asness, A. Frazzini, N. Gormsen and L. Pedersen, "Betting Against Correlation: Testing Theories of the Low-Risk Effect" (2017)

- 2 Within each quintile stocks are ranked based on their correlation and assigned to one of the two portfolios (low and high correlation).

As for BAB factor, both portfolios are levered, or delevered, to one and in each one stocks are weighted with respect to their correlation and rebalanced every calendar month. In this way we obtain, for each quintile, a self-financing BAC portfolio that goes long on low correlation and shorts on high one. Our overall BAC factor is the equal-weighted average of the five betting against correlation factors. Formally, let's consider the portfolio weights:

$$x_H^q = k^q(z^q - \bar{z}^q)^+$$

$$x_L^q = k^q(z^q - \bar{z}^q)^-$$

where z^q is the $n(q) \times 1$ vector of ranked correlations within each quintile $q = 1, 2, 3, 4, 5$, \bar{z}^q is obtained as $1'_{n(q)} z^q / n(q)$, with $n(q)$ the number of securities in each quintile and $1'_{n(q)}$ vector of ones. The excess return of BAC in each quintile is obtained as:

$$r_{t+1}^{BAC(q)} = \frac{1}{\beta_{t,q}^{L,q}}(r_{t+1}^{L,q} - r_f) - \frac{1}{\beta_{t,q}^{H,q}}(r_{t+1}^{H,q} - r_f)$$

where $r_{t+1}^{L,q} = r_{t+1}^{q'} x_L^q$ and $r_{t+1}^{H,q} = r_{t+1}^{q'} x_H^q$ are respectively the returns of low high-correlation portfolios and $\beta_{t,q}^{L,q}$ and $\beta_{t,q}^{H,q}$ the corresponding betas. Then, the return to the final BAC factor is obtained as:

$$r_{t+1}^{BAC} = \frac{1}{5} \sum_{q=1}^5 r_{t+1}^{BAC(q)}$$

Simultaneously, the BAV factor is constructed similarly to betting against correlation, the only difference is that stock are first sorted on correlation rather than on volatility. In this way we obtain:

$$r_{t+1}^{BAV} = \frac{1}{5} \sum_{q=1}^5 r_{t+1}^{BAV(q)}$$

What we have seen up to now are the local factors, Asness et al. provide also global factors, obtained as the weighted average by ex-ante market cap of national portfolios:

$$r_{t+1}^{BAC,global} = \sum_{k=1}^K \frac{\pi_t^k}{\sum_j \pi_t^j} r_{t+1}^{BAC,k}$$

$$r_{t+1}^{BAV,global} = \sum_{k=1}^K \frac{\pi_t^k}{\sum_j \pi_t^j} r_{t+1}^{BAV,k}$$

where π_t^k is the market capitalization of a country k at time t . To sort the stocks within each quintile and then to construct the factors, it has been necessary to estimate beta, correlation and volatility for all stocks. The estimation method for the beta is the same of Frazzini and Pedersen (2014). They essentially estimate pre-ranking betas form rolling regression of excess returns on market excess returns, according to Merton (1980) whenever it is possible, they recur to daily data in order to improve accuracy. The estimated beta for the i -th security is obtain as:

$$\hat{\beta}_i^{ts} = \hat{\rho}_{i,m} \frac{\hat{\sigma}_i}{\hat{\sigma}_m}$$

where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ are respectively the estimated volatility of the i -th stock and the market, while $\hat{\rho}_{i,m}$ is their correlation. Correlation seems to move more slowly than volatilities, than the authors estimated volatilities recurring to a one-year rolling windows of one-day log-returns and five-year rolling windows of overlapping three-days log-returns for the correlation. Lastly, to reduce the influence of outlier they shrink the time-series estimated beta $\hat{\beta}_i^{ts}$ toward the cross-sectional mean β^{xs} :

$$\hat{\beta}_i = x_i \hat{\beta}_i^{ts} + (1 - x_i) \beta^{xs}$$

setting the shrinkage factor ²² $x_i = 0.6$ and the cross-sectional mean $\beta^{xs} = 1$.

Decomposing the betting against beta in two factors, indeed BAC and BAV, the authors have been able to separate two components of risk: BAV factor is closely related to idiosyncratic risk and represents a pure volatility bet, while BAC is linked to systematic risk. Again, at the beginning of each calendar month stocks are sorted based on their ex ante volatility and then conditionally on ex ante correlation, in this way they are assigned to one of five volatility quintiles. Within each quintile stocks are assigned to one of five correlation portfolio, based on NYSE breakpoints. As written before, portfolios are weighted based on their market cap, refreshed and rebalanced every calendar month.

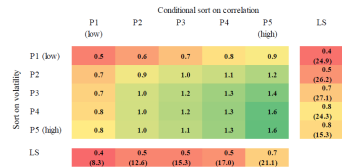


Figure 7: CAPM Beta, 1930-2015

²²The choice of the shrinkage factor does not influence the way in which stocks are sorted into portfolios

Figure 7 represents CAPM betas, as we can easily observe ex post beta increases with both ex ante volatility and ex ante correlation.

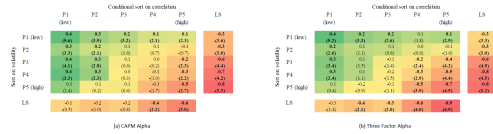


Figure 8: CAPM vs Three Factor Alpha, 1930-2015

Focusing on the intercept of the regression, Figure 8 provides a comparison between (a) the CAPM alpha and (b) the Three-Factor alpha²³, in both cases we can observe that alpha decreases as correlation or volatility increase. Considering the *long/short* portfolios we can see that the separate effects of volatility and correlation on risk-adjusted return results to be significant for many of the cases, with a particularly strong effect for correlation. The methodology of construction and the dynamic behind the BAC factor should results more clear, now we are going to focus on the performance of betting against correlation factor. In each volatility quintile, is constructed a rank-weighted BAC factor and then an overall factor is built as the average of these. In Figure 9 and 10 we can observe the performances of these factors:

3 Testing BAC Investment Strategy

According to the evidence provided by Ansess, Frazzini, Gormsen and Pedersen we decide to remain in the spectrum of leverage constraints theories and in particular on **Betting Against Correlation** factor. Our work consists of an empirical investigation on Betting Against Correlation factor, in which the strategy has been manipulated and tested, trying to maximize its performances. In particular, we provide two different strategies, **qBAC** and **dBAC**, composed by *long/short* portfolios based on stock correlations against S&P 500 over the time interval 2003-2021, testing in deep each strategy for two different stocks weight methodologies,

²³Market (MKT), size (SMB) and value (HML)

Volatility quintile	1	2	3	4	5	BAC
Excess return	0.55 (4.32)	0.86 (5.52)	0.82 (8.31)	1.03 (5.88)	1.48 (3.78)	0.97 (6.74)
Alpha	0.39 (3.56)	0.63 (5.50)	0.87 (4.38)	0.68 (4.08)	1.25 (4.96)	0.79 (5.45)
MKT	-0.14 (-5.1)	-0.05 (-1.9)	0.05 (1.5)	0.09 (2.2)	0.13 (2.14)	0.02 (0.5)
SMB	0.62 (16.6)	0.61 (35.7)	0.58 (12.7)	0.58 (10.1)	0.61 (7.1)	0.60 (13.6)
HML	0.12 (2.3)	0.17 (3.2)	0.26 (8.6)	0.31 (3.9)	0.23 (1.9)	0.22 (3.5)
RMW	0.02 (0.4)	0.05 (0.9)	0.17 (2.5)	0.16 (1.9)	0.28 (2.2)	0.02 (0.5)
CMA	0.08 (1.0)	0.08 (0.9)	0.18 (1.9)	0.04 (0.4)	0.01 (0.0)	0.08 (0.8)
SR	0.60	0.90	0.87	0.81	0.80	0.93
R2	0.52	0.80	0.62	0.59	0.72	0.79
R2	0.34	0.31	0.25	0.18	0.13	0.27
# obs	630	630	630	630	630	630

Figure 9: U.S. sample, 1963-2015

Volatility quintile	1	2	3	4	5	BAC
Excess return	0.27 (2.08)	0.64 (4.37)	0.64 (3.98)	0.70 (3.95)	1.15 (4.55)	0.68 (4.48)
Alpha	0.11 (0.99)	0.40 (3.39)	0.27 (1.84)	0.24 (1.48)	0.82 (3.39)	0.27 (2.77)
MKT	0.01 (0.4)	0.07 (2.0)	0.15 (3.7)	0.18 (4.1)	0.21 (3.27)	0.12 (3.5)
SMB	0.45 (11.9)	0.71 (11.5)	0.75 (10.6)	0.85 (10.8)	1.11 (9.6)	0.81 (12.6)
HML	0.10 (1.4)	0.14 (1.9)	0.26 (2.9)	0.23 (2.4)	0.05 (0.4)	0.16 (1.9)
RMW	0.18 (2.2)	0.31 (3.3)	0.49 (6.6)	0.71 (2.3)	0.41 (2.5)	0.16 (4.3)
CMA	0.10 (1.2)	0.09 (0.9)	0.08 (0.7)	0.11 (0.8)	0.15 (0.8)	0.10 (1.0)
SR	0.41	0.80	0.79	0.78	0.90	0.89
R2	0.21	0.69	0.40	0.32	0.73	0.60
R2	0.33	0.31	0.29	0.30	0.23	0.35
# obs	306	306	306	306	306	306

Figure 10: Global sample, 1990-2015

Figure 9 shows the results in the U.S. market for the 1963-2015 time interval. In particular, it is possible to observe that BAC has a statistically significant alpha with respect to all the Five-Factor model explanatory variables provided by Fama and French (2015), both within each quintile as well as for the overall BAC factor. Globally for the 1990-2015 interval, Figure 10, the researchers obtain the same result in terms of alphas for the overall BAC factor, while within the volatility quintiles they are not all statistically significant.

composition of S&P 500 logically changed during the period and then we decide to rebalance the index composition every calendar year, then also considering acquired and delisted stocks until possible to reduce the risk of incurring in the so-called "**survivorship bias**". For the entire universe, we calculated our three key variables: Adjusted Beta ²⁴, Standard Deviations and Correlation monthly on a 104 week basis.

As stated above, we built two main strategies: a *quartile Betting Against Correlation* (**qBAC**) and a *decile Betting Against Correlation* factor (**dBAC**); then we tested both the strategies by manipulating their stock weights in order to check them and to provide a confirmation to the evidence presented by Asness et al in their work.

Considering our first strategy, qBAC, it results structured as follows:

- 1 Stocks are ordered in an ascending way based on their *Standard Deviation* (STD).
- 2 The ordered sample set is divided into *quartiles*.
- 3 Each quartile is again re-ordered in an ascending way, but based on stocks' *Correlation* (CORR).
- 4 Within each quartile a *long/short portfolio* is constructed. It goes long low-CORR stocks and shorts high-CORR stocks (we used the median as threshold).
- 5 Stocks are weighted according two different methodologies: **Correlation Weighted and Equally Weighted**, rebalanced every calendar month.
- 6 We computed returns and adjusted returns, compounded, for each long/short portfolio and we constructed the overall BAC factor for each calendar month.

Following the above mentioned procedure we have been able to obtain our qBAC strategies, characterized by different weights. Even if we previously tested both qBAC and dBAC strategies, in this paper we decided to consider only the best performing and more robust one: qBAC.

Before we consider the strategies results it can be useful to focus on stocks weights methodology we used. According to Asness et al, we decided to study a methodology that is able attribute a sort of premium, in terms of portfolio weight, to those stocks that are characterized by low correlation values. Moreover, the authors argued that best performances are driven by

"*rank-basis weights*" rather than "*equal-weight methodologies*". Our methodology is slightly different from the one provided by the practitioners, but it is clearly based on the same intuition.

As written before, we decided to build a **Correlation weighted** strategy in which short portfolio weights x_S are constructed as follows:

$$x_{i,S} = \frac{\hat{Corr}_{i,S}}{\sum_i \hat{Corr}_{i,S}} \quad (4)$$

In this way, higher weights are attributed to higher correlation stocks for the short leg. On the other hand a manipulation has been necessary for long portfolio weights x_L . Applying the same methodology we would attribute higher weights to higher correlation stocks (within the long side); to avoid this phenomenon we then compute the weights recurring to the reciprocal of correlations:

$$\hat{Corr}_{i,L}^{-1} = \frac{1}{\hat{Corr}_{i,L}} \quad (5)$$

and then we computed single weights as;

$$x_{i,L} = \frac{\hat{Corr}_{i,L}^{-1}}{\sum_i \hat{Corr}_{i,L}^{-1}} \quad (6)$$

Portfolios are rebalanced every calendar months and weights within each leg of L/S portfolios sum up to 1, then we can consider our strategy as *Dollar Neutral*.

Once strategy weights are obtained we calculated the returns, and subsequently the overall BAC factor, for our models. Let's define r_i the single stocks weekly return, then monthly returns are defined as:

$$R_i^* = \left(\prod_{h=1}^k (1 + r_i) \right) - 1 \quad (7)$$

where k is the number of weeks in a single month. Thus, the single stock weighted return is:

$$R_i = R_i^* x_i \quad (8)$$

and consequently the single portfolio return will be:

$$R_p = \sum_{i=1}^n R_i \quad (9)$$

where $p \in (L; S)$. Moreover we decided to compute three different kind of strategy returns: a simple L/S portfolio return obtained as difference between the long side and the short one,

²⁴Adjusted Beta = (0.66) * Raw Beta + (0.33) * 1

a return adjusted for the risk and an adjusted excess return²⁵, respectively:

$$R_{L,S} = R_L - R_S \quad (10)$$

$$R_{L,S} = \frac{1}{\beta_L} R_L - \frac{1}{\beta_S} R_S \quad (11)$$

$$R_{L,S} = \frac{1}{\beta_L} (R_L - r_f) - \frac{1}{\beta_S} (R_S - r_f) \quad (12)$$

The overall BAC factor is computed as the simple mean of single BAC factors (single L/S portfolios). **Problems and Criticality, Negative Correlations.** Before to proceed with the analysis of results we considered necessary to evidence a criticality manifested during our study. Manipulating our dataset, we found out empirically evidences of a topic that has been amply debated during the most recent years: the pattern of volatility (Std), and consequently of correlation (Corr) in time.

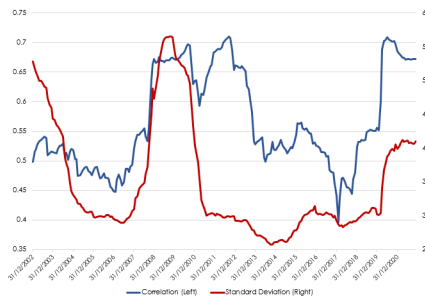


Figure 11: Correlation vs Std, 2003-2021

The right axis corresponds to Std, while the left one to correlation. We can clearly distinguish a similar path in terms of direction during the 2003-2021 interval, with obviously abnormal spikes near financial market sell-offs (Dot-com Bubble, Lehman Crisis and Covid-19 Pandemic in our sample), but different magnitudes during 2020 bear market. While correlation reached its peak in 0.7 area, standard deviation is still well below the Dot-com and Lehman market crashes. Focusing then on the correlation side, we decided to investigate on extreme values that characterize our sample set. Analysing the data we evidenced negative correlations near to specific economic events, as shown in Figure 12:

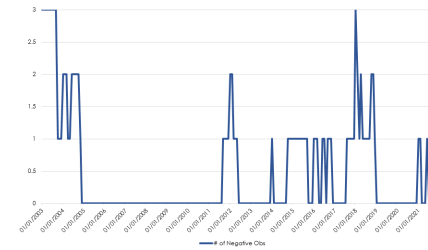


Figure 12: Negative Correlations, Monthly Observations 2003-2021

The graph shows the number of negative correlated stocks each month for the time interval 2003-2021. The observations are not common in the period considered, but it is possible to observe a clear increase in frequency of negative observations during the 2017-19 subset. Hypothesising that the movement and the presence of negative correlated stocks could be ascribed both to company-specific and to macro facts we decided to go further, extracting the stocks characterized by such phenomenon. The table in Figure 13 evidences the companies characterized by negative correlation during the time interval 2003-2021, with the percentage of negative observations expressed in terms of total negative monthly observations.

BAC Ticker	Name	Sector	Industry	% of Negative Obs.
NEM US Equity	NEWMONT CORP	Materials	Metals & Mining	29.3%
KDP US Equity	KEURIG DR PEPPER INC	Consumer Staples	Beverages	9.3%
RTN US Equity	RAIOTECH COMPANY	Industrials	Aerospace & Defense	8.3%
NOC US Equity	NORFOLK GUERMAN CORP	Industrials	Aerospace & Defense	8.3%
DLB787D US Equity	MOTOROLA MOBILITY HOLDINGS L	Information Technology	Tech Hardware & Semiconductors	8.3%
CMG US Equity	CHORDS MEDICAL DRILL INC	Consumer Discretionary	Hotels, Restaurants & Leisure	4.6%
TXU US Equity	ENERGY FUTURE HOLDINGS CORP	Utilities	Electric Utilities	3.7%
MHS US Equity	MEDCO HEALTH SOLUTIONS INC	Health Care	Health Care Facilities & Svcs	2.8%
TIF US Equity	KEURIG GREEN MOUNTAIN INC	Consumer Staples	Food Products	1.9%
GMCR US Equity	TRIPADVISOR INC	Communication Services	Interactive Media & Services	1.9%
DXC US Equity	DXC TECHNOLOGY CO	Information Technology	IT Services	1.9%
SO US Equity	SOUTHERN CO/HE	Utilities	Electric Utilities	1.9%
VNT US Equity	VONER CORP	Information Technology	Electronic Equipment, Instrume	1.9%
NE US Equity	KROGER CO	Consumer Staples	Food & Staples Retailing	1.9%
NVTL US Equity	NETEL COMMUNICATIONS INC-A	Communication Services	Telecommunications	0.9%
ALLE US Equity	ALLEGION PLC	Industrials	Building Products	0.9%
ICBA US Equity	ICBA INC	Information Technology	IT Services	0.9%

Figure 13: Negative Correlation Stocks 2003-2021

We can clearly observe that Cyclical (Industrials and Consumer Discretionary) and Commodities (Materials) are the main sectors characterized by the phenomenon of negative correlation, with Newmont Corp, a gold producer with proven and probable gold reserves of 94.2 million ounces and significant operations the United States, Canada and LATAM representing more than a third of total negative monthly observations. Negative observations are limited with respect to our total sample set, nonetheless they determine an important impact on stock weights methodology, increasing sensibly portfolio concentration and strategy net exposure to company (and industry) specific risks. In case

²⁵We decided consider annual average on US 10y Treasury yield

of Newmont Corp., the frequency of negative observations could be a direct consequence of Gold-Equity negative correlation that characterized the 2014-2020 time interval, then influencing strategy's sector allocation.

The topic of negative observations is then strictly related to the performance, in case of negative correlations we decided to adopt a specific procedure to adjust stock weights within portfolios long legs:

- If the negative correlation is close to zero and not persistent in time we substitute the value with the average of the two closest positive observations.
- If the negative correlation is particularly deep (we fixed a threshold of -0.1), we substitute it with an arbitrary positive value of 0.001 (positive, but extremely close to zero).

The manipulation has been necessary in order to obtain positive weights for all the stocks in the long portfolios and to respect the dollar neutrality assumption of the strategy. Secondly we decide to fix a *cap* for stock weights within each portfolio, redistributing the extra-weight to other stocks. The reason is that a correlation close to zero, or even negative, is able to determine an extra-exposure to the related stocks, operating against the differentiation principle and consequently increasing the risk; then we set:

- 10% of maximum weight in *quartiles*.

Empirical results. Passing through, now we are able to show our strategies results for 2003-2021 time interval. During our analysis we will always considered **Risk-Adjusted Returns** for the strategy, dividing the obtained price returns for each BAC factor for the respective portfolios' beta in order to consider returns per unit of risk. In terms of monthly performances, we obtained:

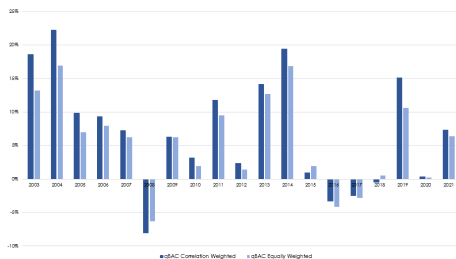


Figure 14: Strategies Annual Returns, 2003-2021

Graphically, both the methodologies show a positive performance during the time interval, with limited a limited drawdown (well below -10%) even during the Great Financial Crisis in mid-2000s. Considering separately *qBAC Correlation Weighted* and *qBAC Equally Weighted* strategies, we can observe that the first one provides a compound return of 372.9% for the time interval, while the other strategy shows a return of 207.8%, confirming the intuition that lower correlated stocks should deserve a premium in terms of allocation. On the other hand, Equally Weighted qBAC seems to provide even a lower volatility of returns, with a maximum drawdown of -6% and a monthly standard deviation of 1.90%.

qBAC Correlation Weighted		qBAC Equally Weighted	
Arithmetic Average	0.61%	Arithmetic Average	0.34%
Standard Error	0.15%	Standard Error	0.13%
Median	0.74%	Median	0.40%
Standard Deviation	2.29%	Standard Deviation	1.90%
Variance	0.05%	Variance	0.04%
Kurtosis	0.93	Kurtosis	1.09
Skewness	-0.11	Skewness	-0.21
Range	15.30%	Range	12.65%
Minimum	-6.93%	Minimum	-6.02%
Maximum	8.37%	Maximum	6.6%
Compound Return	372.9%	Compound Return	207.8%
Sum	138.4%	Sum	76.9%
# of obs	228	# of obs	228

Figure 15: Strategies Comparison, 2003-2021

Focusing on returns distribution, Correlation Weighted qBAC strategy shows a negative skewness of -0.11 and a kurtosis of 0.93, while Equally Weighted qBAC is characterized by a kurtosis of 1.09 and a negative skewness of -0.21.

Comparing BAC strategies risk adjusted returns with S&P 500 Index in *Figure 15*, we will try to extract the main drivers of our empirical results:



Figure 16: Strategies Compounded Returns, 2003-2021

And in terms of annual risk adjusted returns:

	Gbac Weights		S&P 500 Index
	Correlation	Equal	
2003	18.6%	13.2%	26.4%
2004	22.2%	16.9%	9.0%
2005	9.8%	7.0%	3.0%
2006	9.3%	7.9%	13.6%
2007	7.3%	6.3%	3.5%
2008	-8.1%	-6.3%	-38.5%
2009	6.3%	6.2%	23.5%
2010	3.1%	1.9%	12.8%
2011	11.7%	9.5%	0.0%
2012	2.3%	1.4%	13.4%
2013	14.1%	12.7%	29.6%
2014	19.4%	16.9%	11.4%
2015	0.9%	1.9%	-0.7%
2016	-3.3%	-4.2%	9.5%
2017	-2.5%	-2.8%	19.4%
2018	-0.5%	0.6%	-4.2%
2019	15.1%	10.6%	28.9%
2020	0.4%	0.2%	16.3%
2021	7.3%	6.4%	26.9%

Figure 17: Annual Risk Adjusted Returns, 2003-2021

4 Sector Analysis

The second part of our study consists of an analysis from a sectoral perspective. We will focus our attention only on our best strategy, **Correlation Weighted qBAC**, in order to point out long and short portfolios' components. After that we will compute stocks correlation within each sector following the objective of mapping both market and sector correlations and analysing their walk in time. Stocks have been grouped in four macro sectors ²⁶:

- **Cyclical:** Consumer Discretionary, Industrials and Information Technology.
- **Not-Cyclical:** Communication Services, Consumer Staples, Healthcare and Utilities.
- **Financials**
- **Commodities:** Energy and Materials.

With *Figure 18* we provide an aggregate representation of our strategy qBAC for 2017-2019 interval. Long (blue) and short (red) legs are obtained as the average of single quartile portfolios in the reference calendar year:

²⁶According to GICS Sector framework

The strategy clearly outperformed S&P 500 Index until the end of 2016, showing an impressive risk adjusted performance against both the reference index and the equally weighted methodology. The massive presence of central banks on financial markets in the last few years and the consequent impressive rally of Information Technology stocks and FAANG (before and after the pandemic outbreak) could represent important drivers, with a potential impact on stocks' correlation both against the market and within their sector. To investigate the potential drag of the strategy we decided to analyze in deep the sector allocation of qBAC factors and the dynamic evolution of correlations in time.

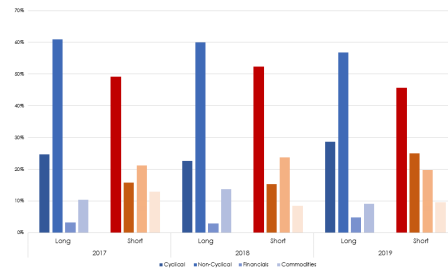


Figure 18: Long vs Short Macro Sector Composition, 2017-2019

The sector allocation shows a clear polarization in terms of exposure, with long portfolios mainly characterized by not-cyclical stocks and a greater short exposure to Cyclical and Financials. Trying to disaggregate the previous graph we decided to provide deeper detail of sector allocation, focusing directly on GICS Sectors for the period 2017-2019:

	2017		2018		2019	
	Long	Short	Long	Short	Long	Short
Consumer Discretionary	14.3%	11.6%	8.9%	15.4%	18.2%	5.8%
Information Technology	3.5%	17.9%	7.8%	17.1%	5.4%	19.1%
Industrials	6.9%	19.6%	5.9%	19.9%	5.3%	20.8%
Communication Services	3.4%	8.1%	5.0%	5.2%	5.6%	5.5%
Health Care	17.5%	4.8%	13.8%	7.0%	4.9%	17.8%
Consumer Staples	12.4%	1.1%	12.2%	0.3%	12.1%	1.1%
Utilities	18.0%	0.0%	18.6%	0.0%	20.3%	0.0%
Real Estate	9.7%	1.9%	10.4%	2.7%	14.0%	0.8%
Financials	3.2%	21.2%	2.9%	23.8%	4.9%	19.8%
Energy	6.0%	7.0%	7.9%	3.4%	5.3%	4.6%
Materials	4.4%	5.9%	5.9%	5.1%	3.9%	5.0%

Figure 19: Long vs Short Sector Composition, 2017-2019

As we can see, the not-cyclical exposure of the long side portfolios is essentially due to the important presence of defensive sectors such as Utilities(18% to 20.2%), Health Care and Consumer Staples. On the other side, Utilities are totally absent in short portfolios, structurally exposed to high beta stocks (Industrials, Tech and Financials). For the purpose of our analysis, we then decided to specifically analyze correlation distribution, over the 2017-2019 interval, for Utilities and Information Technology sectors:

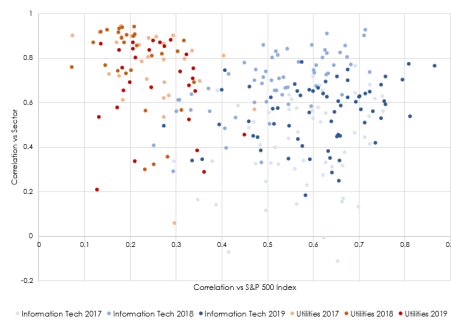


Figure 20: Correlation Comparison, Utilities vs Information Technology, 2017-2019

The scatter plot evidences a clear polarization between the two sectors considered, with Utilities typically characterized by a correlation with S&P 500 Index in the range of 0.15-0.35 while Information Technology seems to be more dispersed above 0.5 in terms of market correlation. This evidence is able to explain the structural polarization between sectors that determined the underperformance of qBAC strategy with respect to the reference index. In this way, the strategy failed to generate alpha in the most recent years by falling in a under-diversified sector allocation. Proceeding deeper in our sector analysis, we decided to remain at GICS Sector level (*Appendix*). By computing correlations within each sector index and against the market, we create a "correlation walk" for each sector; in this way it has been possible to observe a dynamic evolution of correlations in time. Following the long bull market on US equities, only temporarily interrupted by Covid-19 pandemic crisis, it is possible to observe a common path in terms of correlation evolution across sectors, with market correlation higher than historical average for almost all the sectors considered and a sensibly contraction in correlation within sector. The obvious consequence is an extremely difficulty for active investors to extract alpha within sectors, with a clear advantage for pas-

sive instruments replicating the whole market. *Figure 21* provide a graphical representation of the above mentioned phenomenon:

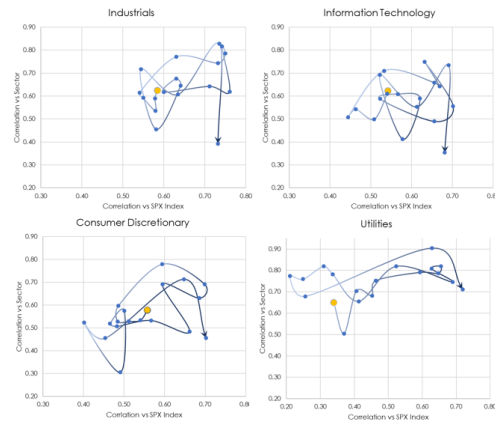


Figure 21: Walking Correlation for Industrials, Information Technology, Real Estate and Utilities, 2003-2021

In our "correlation walk analysis" the only exception is represented by the Energy Sector, a clear laggard in terms of performance in the last few years which is still showing correlation levels higher than historical averages, both against the market and within the sector:

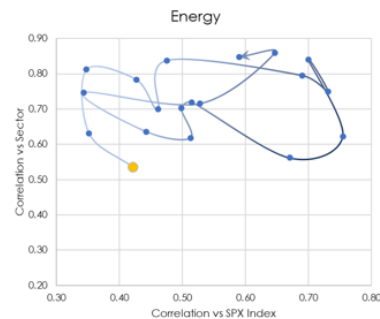


Figure 22: Walking Correlation for Energy Sector, 2003-2021

Our "walking correlation analysis" find evidence in the wide underperformance of Energy sector with respect to other main sector and to S&P 500 in 2017-2021 time interval, as shown in *Figure 3*:

BBG	Name	Price Return	Total Return
XLK US Equity	TECHNOLOGY SELECT SECT SPDR	256.5%	282.9%
XLI US Equity	INDUSTRIAL SELECT SECT SPDR	70.1%	86.5%
XLF US Equity	FINANCIAL SELECT SECTOR SPDR	68.0%	85.5%
XLV US Equity	UTILITIES SELECT SECTOR SPDR	47.4%	73.5%
XLP US Equity	CONSUMER STAPLES SPDR	49.1%	70.8%
XLE US Equity	ENERGY SELECT SECTOR SPDR	-26.3%	-6.3%
SPX Index	S&P 500 INDEX	112.9%	133.3%

Figure 23: SPDR Sectors performance, 2017-2021

5 Conclusions

Starting from the “father” of Modern Portfolio Theory, Henry Markowitz, we analysed Sharpe’s CAPM and its definition of Security Market Line as proxy of the risk-reward relationship. With Black 1972 the debate about the correctness of CAPM prediction and the slope of SML took place. This event represented a real turning point in terms of asset allocation methods, determining the introduction of new sets of variables, such as for Arbitrage Pricing Theory and Three, a subsequently Five, Factor Model by Fama-French. Analyzing the so called “Low Risk Effect” we tried to provide the reader some empirical studies and outcomes able to support the presence of the anomaly. Innovative and challenging the contribution provided by Bob Haugen in 1972 who overturned the risk-reward relationship affirming that the slope of SML is neither positive nor flat but even negative. The debate about the Low Risk Effect is still alive and mainly driven by two different currents: leverage constraints theory and behavioural effects. Remaining on leverage constraints, we focused on the contribution provided by Asness, Frazzini and Pedersen with their study on Betting Against Beta, a strategy that goes long low beta stocks and shorts high beta ones. The natural development of BAB has been “Betting Against Correlation: Testing Theories of the Low Risk Effect”, our specific field of analyses. According to the potential of BAC factor and methodology, we decided to remain in the strategy framework, manipulating the original model to study different outcomes and performances. In this sense we built two strategies based on the above

mentioned framework: qBAC and dBAC, testing them for different stocks weights methodologies. As expected, both strategies characterized by correlation-weights performed better than “Equally Weighted” ones, evidencing how low correlated stocks should deserve a premium in terms of portfolio exposure. Focusing only on the best performing strategy, Correlation Weighted qBAC, we backtested the model for 2003-2021 time interval, wondering if the massive presence of central banks on financial markets in the last few years has been able to influence stock correlation both against the market and within each sector. An important aspect that we evidenced has been the presence of negative correlation stocks in long side portfolios, whose appearance seems to be driven by both macro and company-specific drivers. The phenomenon could determine a significant impact in terms of optimal sector allocation and, in our opinion, could be considered as a starting point in terms of further investigations. To conclude our work we performed a sectorial analysis in which we investigated the composition of Long/Short portfolios in the sub-interval 2017-2019, showing a deep polarization between sector correlation, causing a significant underperformance of the BAC strategy against S&P 500. To conclude we created a “walking correlation analysis” to reach a dynamic evolution in time by considering market and sector correlation. Latest analysis and evidences a common path across almost all the sectors in the last few years, with the only exception represented by the Energy sector, the real laggard of the last few years also in terms of market performance.

6 Appendix: Walking Correlation Analysis

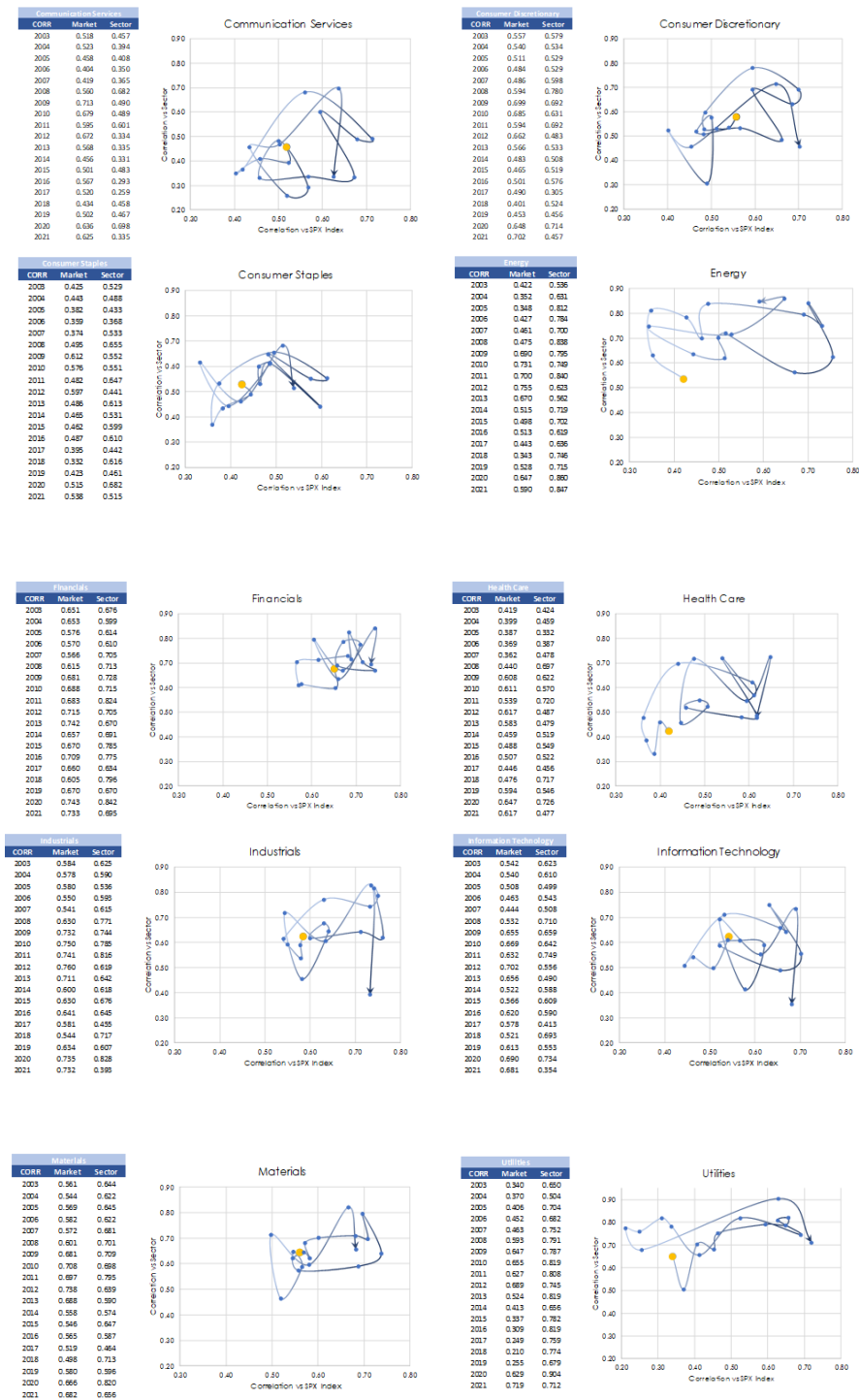


Figure 24: Walking Correlation Analysis, 2003-2021