2 d) Método de transformação de variáveis para

$$p_{k,\lambda}(x) = \frac{x^{k-1}\lambda^{k}}{\Gamma(k)} e^{-\lambda x}$$

$$\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx$$ Funçao gama

$$\gamma(k,x) = \int_0^x \frac{y^{k-1}}{\Gamma(k)} e^{-y} dy$$ Função gama incompleta

Função de distribuição cumulativa

$$F(x) = \int_0^x p_{k,\lambda}(y)\, dy$$

$$u = F(x)$$

$$x = F^{-1}(u)$$

$$= \int_0^x \frac{y^{k-1}\lambda^{k}}{\Gamma(k)} e^{-\lambda y} dy$$

substituição

$$z = \lambda y$$

$$dz = \lambda dy$$

$$= \int_0^{\lambda x} \left(\frac{z}{\lambda}\right)^{k-1} \frac{\lambda^{k}}{\Gamma(k)} e^{-z} \frac{dz}{\lambda}$$

$$F(x) = \int_0^{\lambda x} \frac{z^{k-1} e^{-z}}{\Gamma(k)} dz = \gamma(k, \lambda x)$$

gammainc

$$u = \gamma(k, \lambda x)$$

$$\gamma^{-1}(k,u) = \lambda x \Rightarrow x = \frac{1}{\lambda}\gamma^{-1}(k,u)$$

gammaincinv

x = (1/lambda)*gammaincinv(u,k)

Exercicio 6.

$$Q_X(k) = \overline{e^{ikX}} = \int_{-\infty}^{+\infty} e^{ikx} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2} + ikx} dx$$

$$\frac{(x-\mu)^2}{2\sigma^2} + ikx = \frac{(x-a)^2}{2\sigma^2} + b \qquad \begin{cases} a = ? \\ b = ? \end{cases}$$

$$\frac{x^2 - 2\mu x + \mu^2 + 2ik\sigma^2 x}{2\sigma^2} = \frac{x^2 - 2ax + a^2 + 2b\sigma^2}{2\sigma^2}$$

$$x^2 - (2\mu - 2ik\sigma^2)x + \mu^2 = x^2 - 2ax + a^2 + 2b\sigma^2$$

$$\begin{cases} a = \mu - ik\sigma^2 \\ \mu^2 = a^2 + 2b\sigma^2 \Rightarrow b = \frac{\mu^2 - a^2}{2\sigma^2} \end{cases}$$

$$a^2 = \mu^2 - 2i\mu k\sigma^2 + (-i)^2 k^2 \sigma^4$$

$$= \frac{2i\mu k\sigma^2 - k^2\sigma^4}{2\sigma^2} = i\mu k - \frac{k^2\sigma^2}{2}$$

$$Q_X(k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^b \underbrace{\int_{-\infty}^{+\infty} e^{-\frac{(x-a)^2}{2\sigma^2}} dx}_{\sqrt{2\pi\sigma^2}}$$

$$P_\sigma(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

$$\int_{-\infty}^{+\infty} P_\sigma(x)\, dx = 1$$

$$\boxed{\int_{-\infty}^{+\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \sqrt{2\pi\sigma^2}}$$

Funcao Caracteristica

$$Q_X(k) = e^b = e^{i\mu k - \frac{k^2\sigma^2}{2}}$$

$$Q_X(k) \xrightarrow[k\to\infty]{} 0$$