

Exercise # 2. Iterative Methods For Linear Systems.

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Question 1

My implementation is slower to converge...

Question 2

The spectral condition number of A is

$$k = \frac{\lambda_{\max}(A)}{\lambda_{\min}(A)} \quad (1)$$

nx	h	k	sqrt(k)	iters CG	iters IC(0)	iters IC(10⁻²)	iters IC(10⁻³)
102	1.0000×10^{-4}	6.0107×10^3	77.5288	283	87	45	17
202	2.5000×10^{-5}	2.3810×10^4	154.3039	532	159	78	30
402	6.2500×10^{-6}	9.4770×10^4	307.8473	948	282	137	53
802	1.5625×10^{-6}	3.7814×10^5	614.9304	1792	533	258	97

Table 1: all data

nx	h	iters CG	iters IC(0)	iters IC(10⁻²)	iters IC(10⁻³)
102	1.0000×10^{-4}	283	87	45	17
202	2.5000×10^{-5}	532	159	78	30
402	6.2500×10^{-6}	948	282	137	53
802	1.5625×10^{-6}	1792	533	258	97

Table 2: Iterations for each value of nx

$$h = \frac{1}{N} = \frac{1}{(nx-2)^2}$$

Question 3

show theoretically ?? only 1 iter??

Matlab PCG without preconditioning, My PCG with L as identity matrix

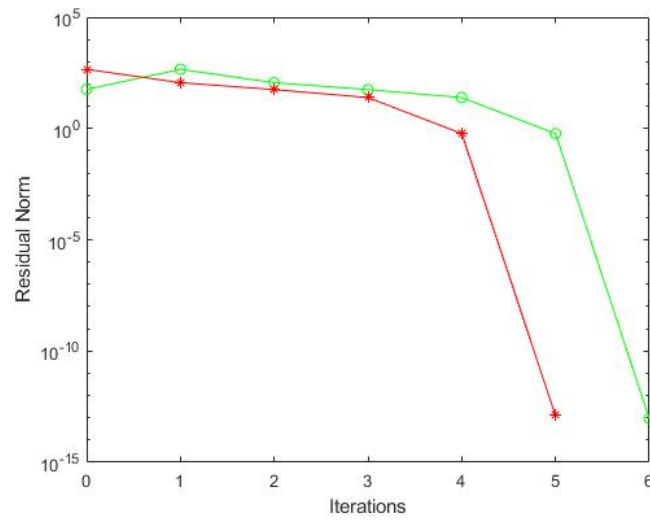


Figure 1: semilogy plot ex3 no preconditioning

Method	Iterations	Final Residual	Computational Time
Matlab PCG	6	9.2128×10^{-14}	0.021s
My PCG	5	1.2744×10^{-13}	0.012s

Table 3: Iterations for each value of nx, no preconditioning

Question 4

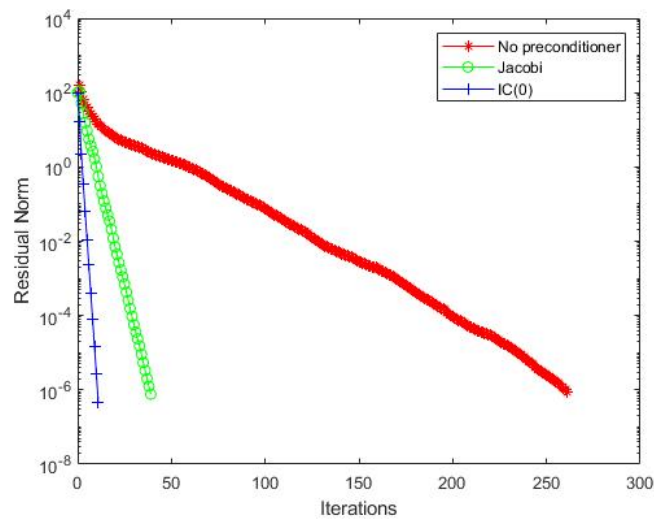


Figure 2: semilogy plot ex4

Question 5

Question 6

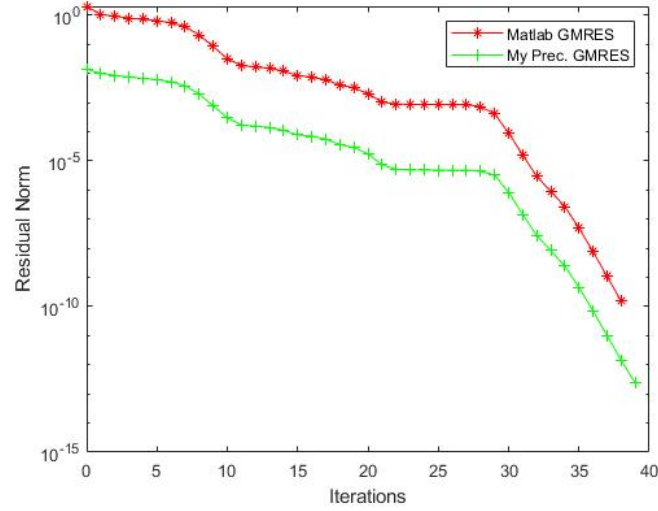


Figure 3: semilogy plot ex6

With L

Method	Iterations	Final Residual	Computational Time
GMRES	1	3.8481×10^{-16}	0.027s
My PCG	1	1.7554×10^{-17}	0.003s

Table 4: Iterations for each value of nx

GMRES without preconditioning, My PCG with L as identity matrix

Method	Iterations	Final Residual	Computational Time
GMRES	6	3.0413×10^{-14}	0.102s
My PCG	5	1.0468×10^{-13}	0.012s

Table 5: Iterations for each value of nx, no preconditioning

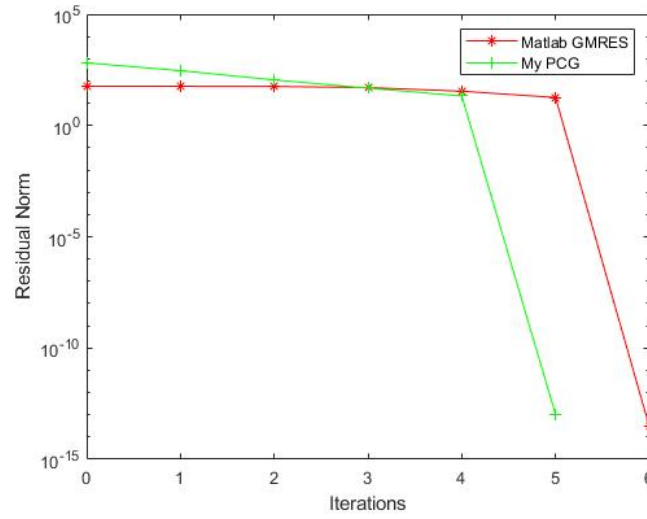


Figure 4: semilogy plot ex6, GMRES without preconditioning, My PCG with L as identity matrix

Question 7

Restart	Iterations	Final Residual	Computational Time
10	1149	9.6915×10^{-13}	2.235s
20	739	9.6140×10^{-13}	1.443s
30	88	6.7203×10^{-13}	0.242s
50	41	4.8414×10^{-13}	0.135s

Table 6: Iterations for each value of nx, no preconditioning

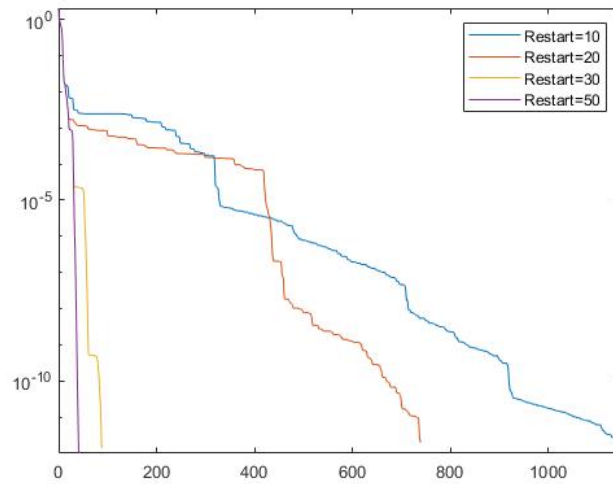


Figure 5: semilogy plot ex7

Question 8

Tolerance	Iterations	Tprec	Tsol	Ttotal	Final Residual	rho
2×10^{-2}	1983	39.59s	77.65s	117.24s	9.9053×10^{-13}	0.4537
1×10^{-2}	691	36.46s	26.63s	63.09s	9.7254×10^{-13}	0.5807
3×10^{-3}	247	40.67s	11.04s	51.71s	9.1709×10^{-13}	0.9401
1×10^{-3}	102	37.61s	5.63s	43.24s	8.7501×10^{-13}	1.4544
1×10^{-4}	34	42.44s	2.93s	45.37s	4.5169×10^{-13}	3.5140
1×10^{-5}	16	76.50s	2.49s	78.99s	4.9947×10^{-13}	9.0720

Table 7: Iterations for each value of nx, no preconditioning

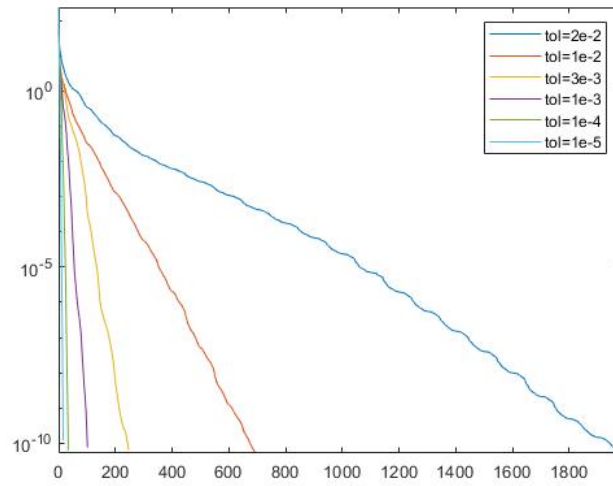


Figure 6: semilogy plot ex8