

Exercise # 1. Numerical methods for ODES.

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Intro

Methods

Answers

Question 1

$$y(t) = e^{-5t} \quad (1)$$

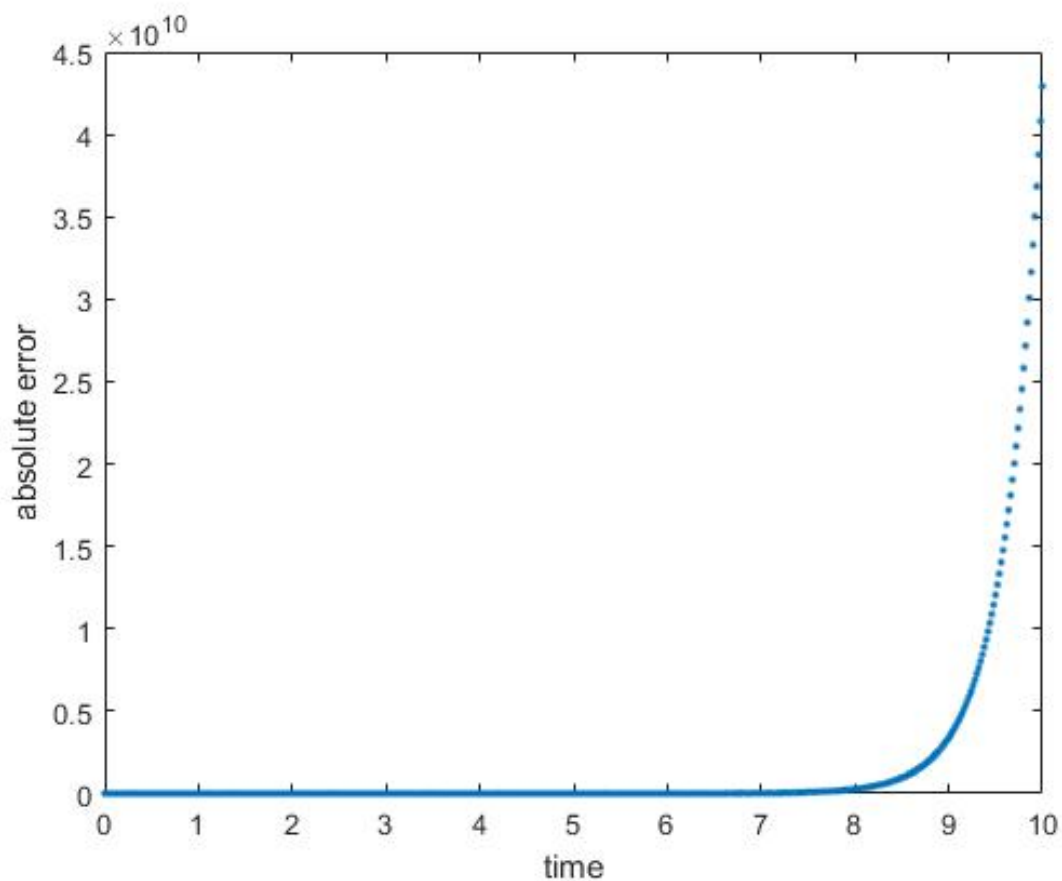


Figure 1: Absolute error in function of time using Forward Euler method to compute $y(1)$

We got a maximum error of $4.2916 \times 10^{10} \dots$

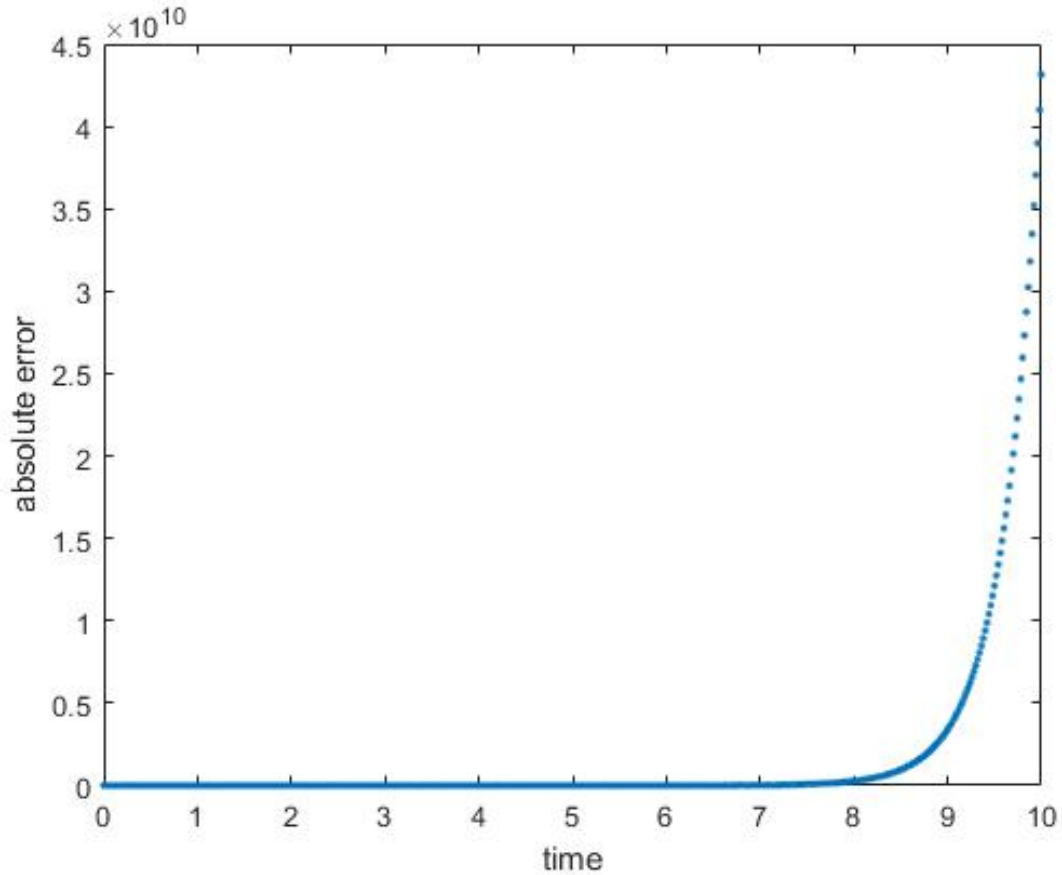


Figure 2: Absolute error in function of time using RK4 method to compute $y(1)$

We got a maximum error of $4.3146 \times 10^{10} \dots$

Comment the different behavior observed by the numerical method.

The Simpson's method has an empty stability region as proved by: ... We can notice the difference in the initial conditions in our results. The FE calculation for $y(2)$ is better then the RK4 calculation given the best final error. This is, although, not that relevant, the difference is of about $0.5 \times 10^{-10}\%$.

Question 2

The exact solution can be found as:

$$y(t) = \frac{1}{10t + 1} \quad (2)$$

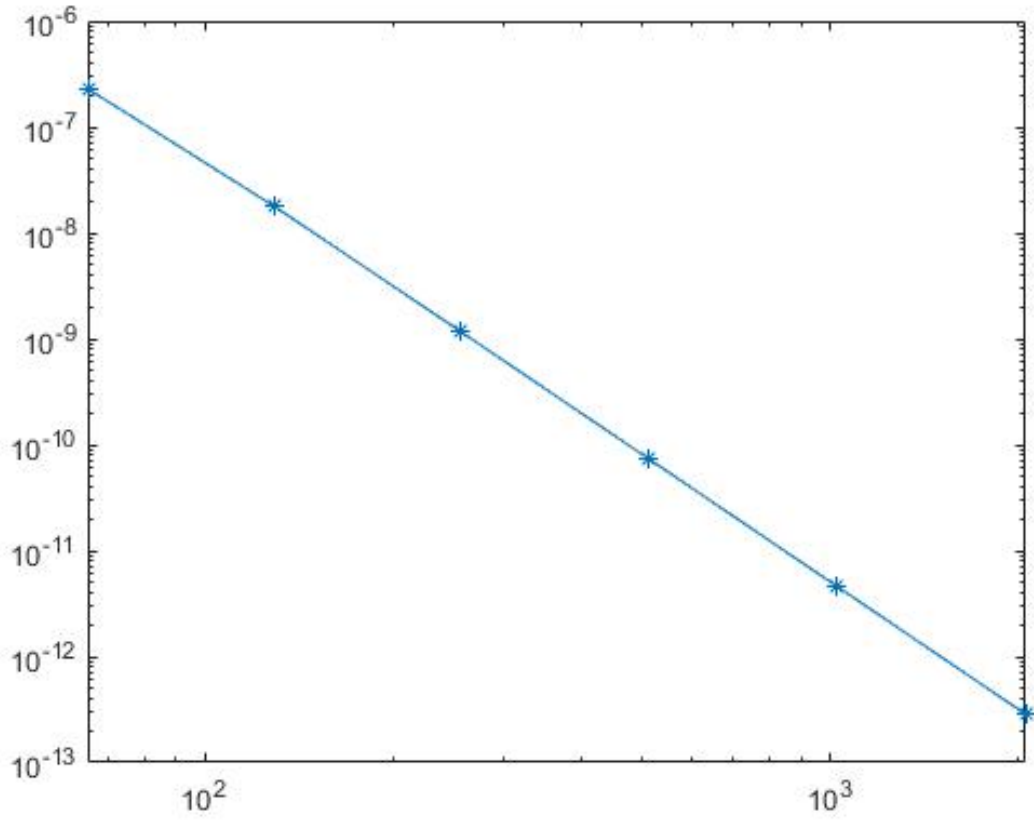


Figure 3: LogLog plot of the error as a function of the number of steps.

h	error
3.125000×10^{-2}	2.291844×10^{-7}
1.562500×10^{-2}	1.785763×10^{-8}
7.812500×10^{-3}	1.160234×10^{-9}
3.906250×10^{-3}	7.312862×10^{-11}
1.953125×10^{-3}	4.579586×10^{-12}
9.765625×10^{-4}	2.863750×10^{-13}

The error reduces with the increase in the number of steps due to the decrease of h as expected in theory. ...

Question 3

Question 4

Method	Number of steps	error	CPU time elapsed
ODE45	??	3.125000×10^{-2}	2.291844×10^{-7}
CN	100	1.562500×10^{-2}	1.785763×10^{-8}
CN	1000	7.812500×10^{-3}	1.160234×10^{-9}
CN	10000	3.906250×10^{-3}	7.312862×10^{-11}
BDF3	100	1.953125×10^{-3}	4.579586×10^{-12}
BDF3	1000	9.765625×10^{-4}	2.863750×10^{-13}
BDF3	10000	9.765625×10^{-4}	2.863750×10^{-13}

Question 5

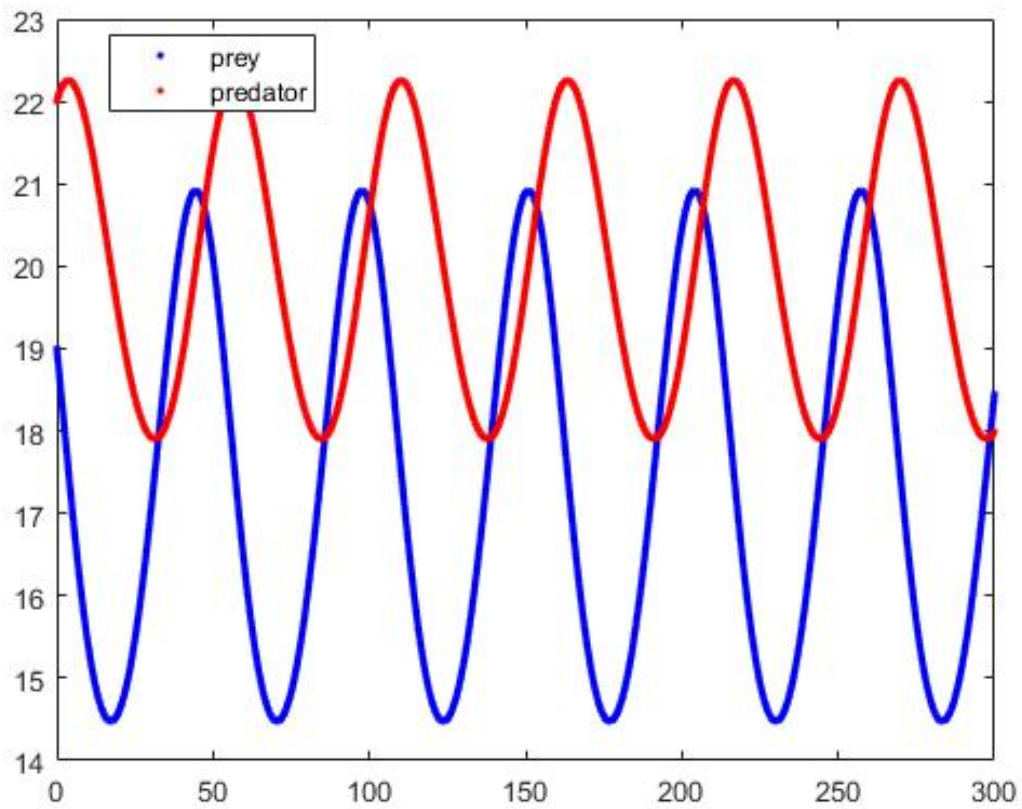


Figure 4: Evolution of the number of preys and predators.

Results

Outputs