Exercise # 1. Numerical methods for ODES.

Alexandre Rodrigues (2039952)

November 21, 2021

Intro

Methods

Answers

Question 1

$$y(t) = e^{-5t} \tag{1}$$

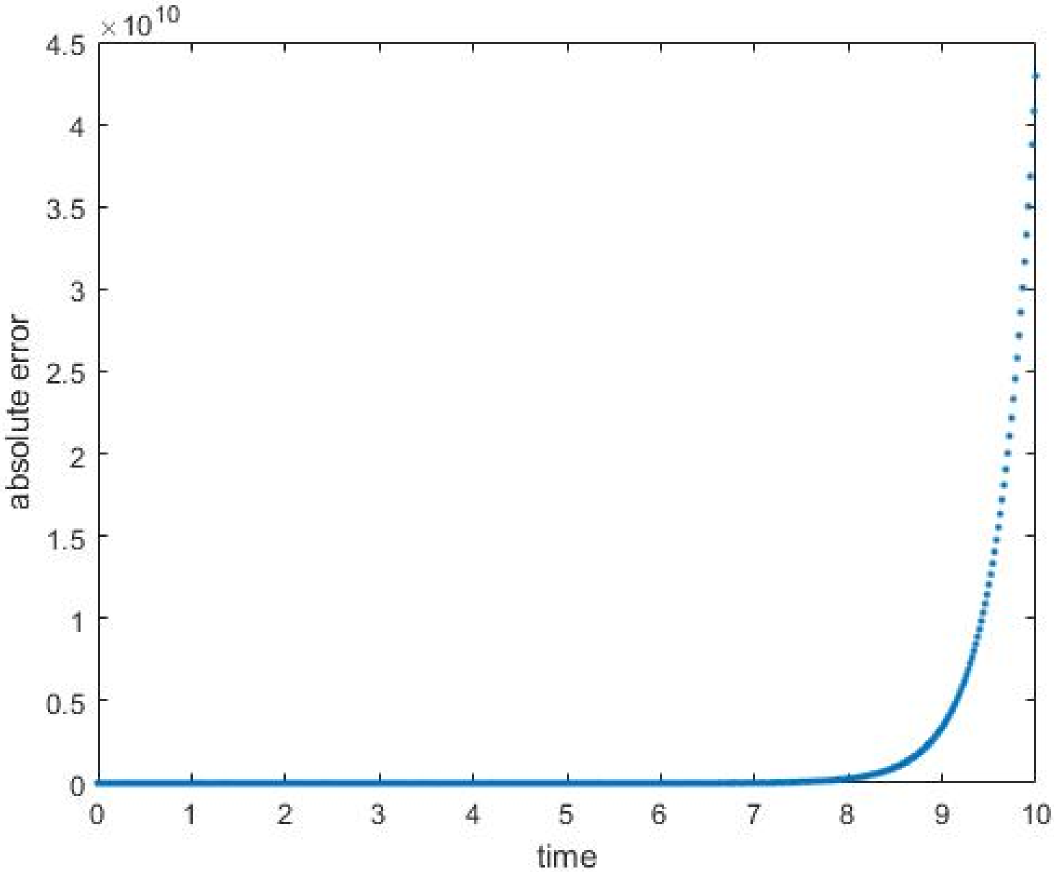

Figure 1: Absolute error in function of time using Forward Euler method to compute y(1)

We got a maximum error of 4.2916×10^{10}...

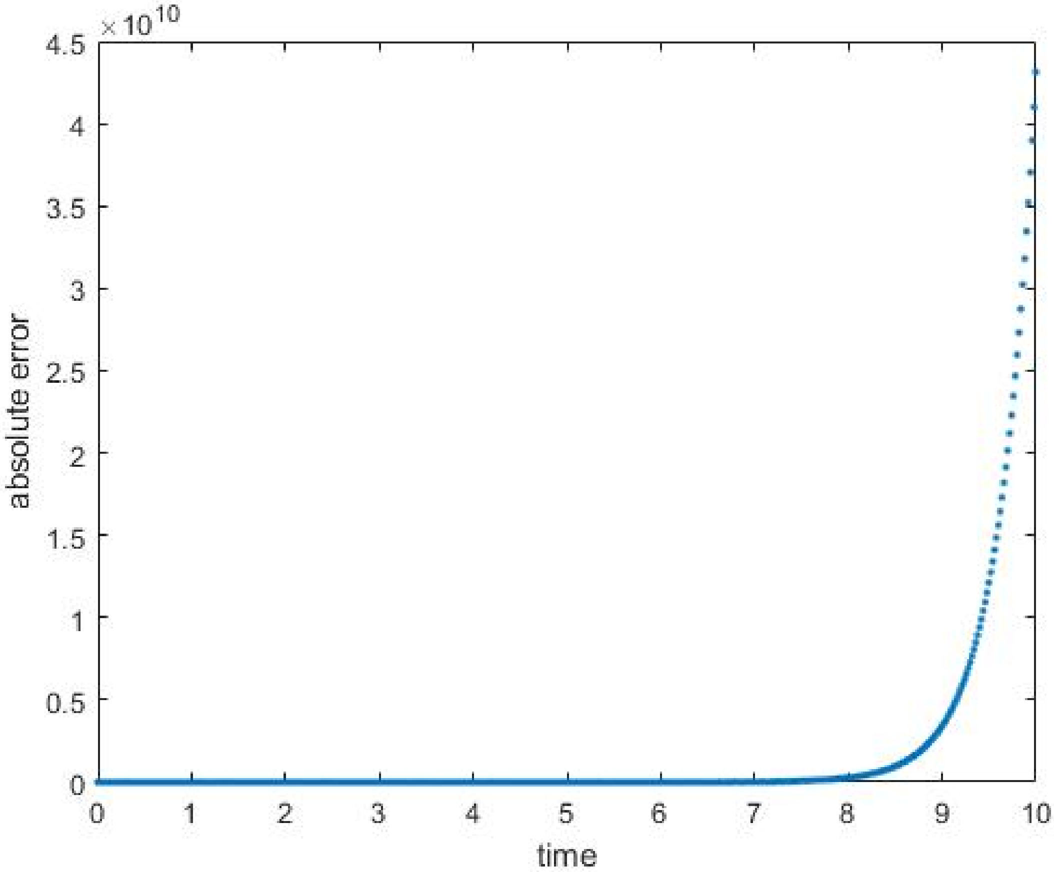

Figure 2: Absolute error in function of time using RK4 method to compute y(1)

We got a maximum error of 4.3146×10^{10}...

Comment the different behavior observed by the numerical method.

The Simpson's method has an empty stability region as proved by: ... We can notice the difference in the initial conditions in our results. The FE calculation for y(2) is better then the RK4 calculation given the best final error. This is, although, not that relevant, the difference is of about $0.5 \times 10^{-10}\%$.

Question 2

The exact solution can be found as:

$$y(t) = \frac{1}{10t + 1} \tag{2}$$

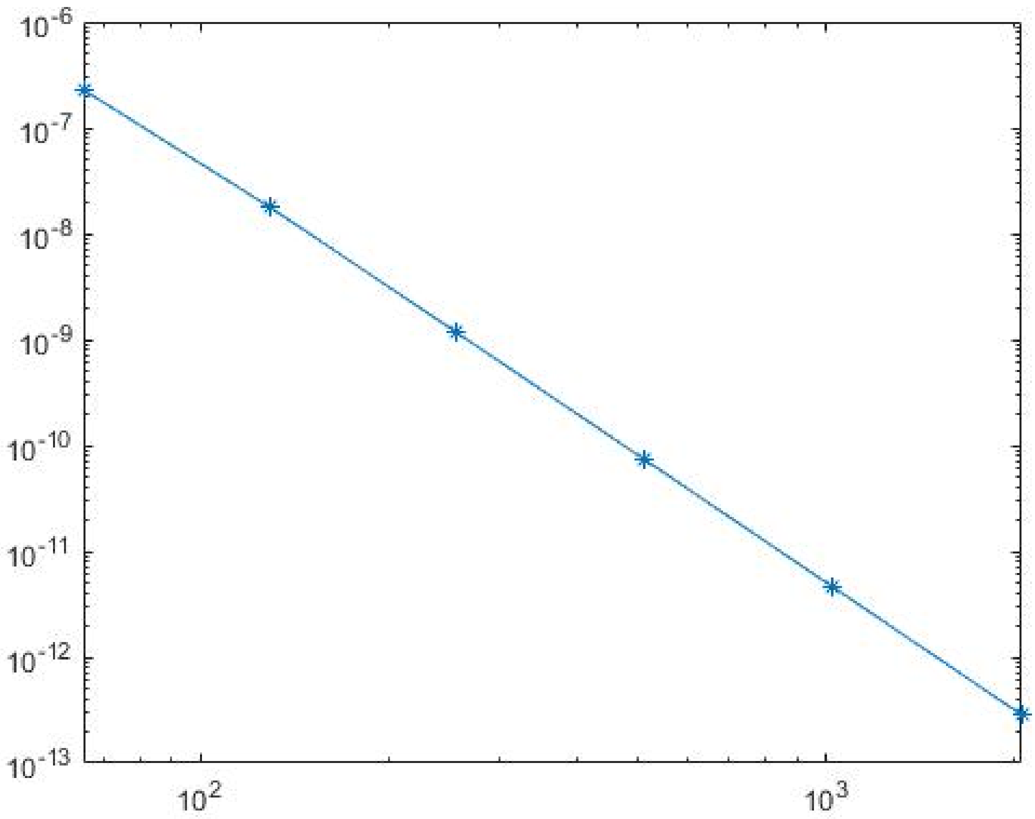

Figure 3: LogLog plot of the error as a function of the number of steps.

h	**error**
3.125000×10^{-2}	2.291844×10^{-7}
1.562500×10^{-2}	1.785763×10^{-8}
7.812500×10^{-3}	1.160234×10^{-9}
3.906250×10^{-3}	7.312862×10^{-11}
1.953125×10^{-3}	4.579586×10^{-12}
9.765625×10^{-4}	2.863750×10^{-13}

The error reduces with the increase in the number of steps due to the decrease of h as expected in theory. ...

Question 3

Question 4

Stability for RK4

As found and explained in this course unit slides, we know that the maximum value of h can be related to the largest modulus eigenvalue with the following relation:

$$h\lambda > -2 \tag{3}$$

We found λ using `lambda = -eigs(A,1,'lm')` to be $\lambda = -7.8388262 \times 10^4$. This gives us a theoretical value $h_{max} = 2.55140238 \times 10^{-5}$.

I tested various values of h around this value. I found that the method produced error, i.e. NaN values for $h > 3.5 \times 10^{-5}$. This although does not show the real point of instability. The error increases to very significant values of around 0.2 for $h >$.

Method	**Number of steps**	**Error**	**CPU time (secs)**
ODE45	9445	1.155269×10^{-5}	$8.791882s$
CN	100	4.467899×10^{-3}	$208.038764s$
CN	1000	4.441078×10^{-4}	$514.197773s$
CN	10000	4.438412×10^{-5}	$3086.958397s$
BDF3	100	4.482679×10^{-3}	$188.455409s$
BDF3	1000	4.442484×10^{-4}	$557.765280s$
BDF3	10000	4.438552×10^{-5}	$3399.768021s$

Question 5

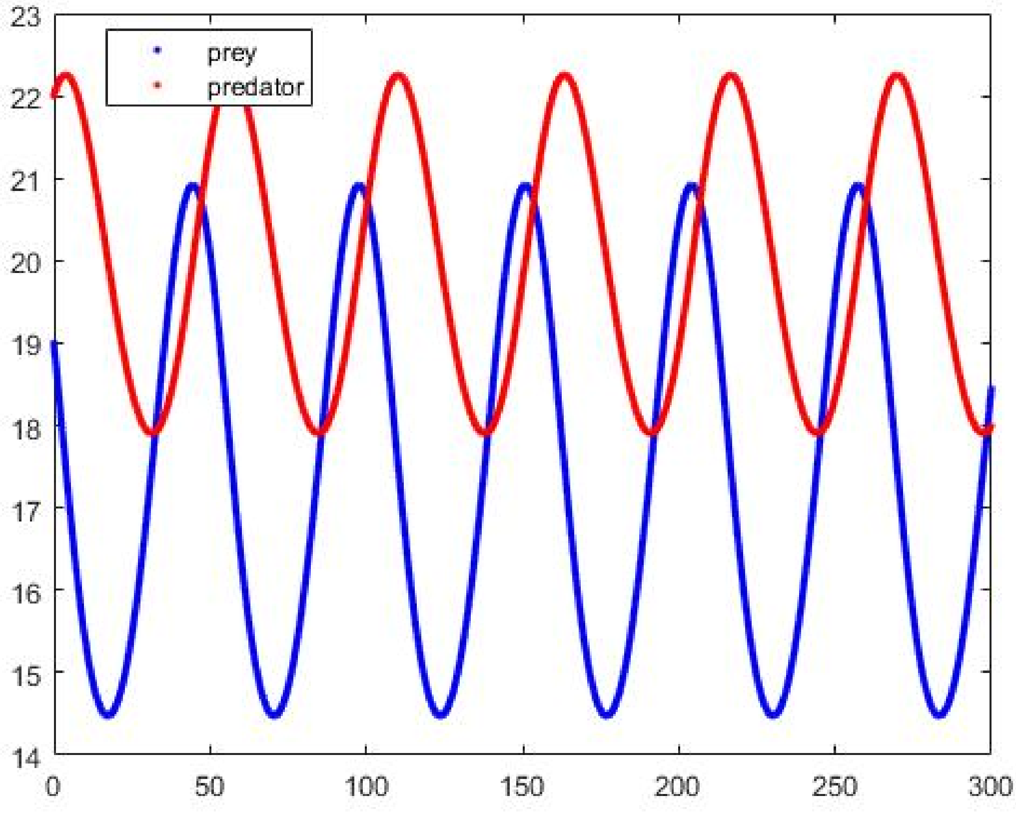

Figure 4: Evolution of the number of preys and predators.

Results

Outputs