Exercise # 1. Numerical methods for ODES.

Alexandre Rodrigues (2039952)

November 18, 2021

Intro

Methods

Answers

Question 1

$$y(t) = e^{-5t} \tag{1}$$

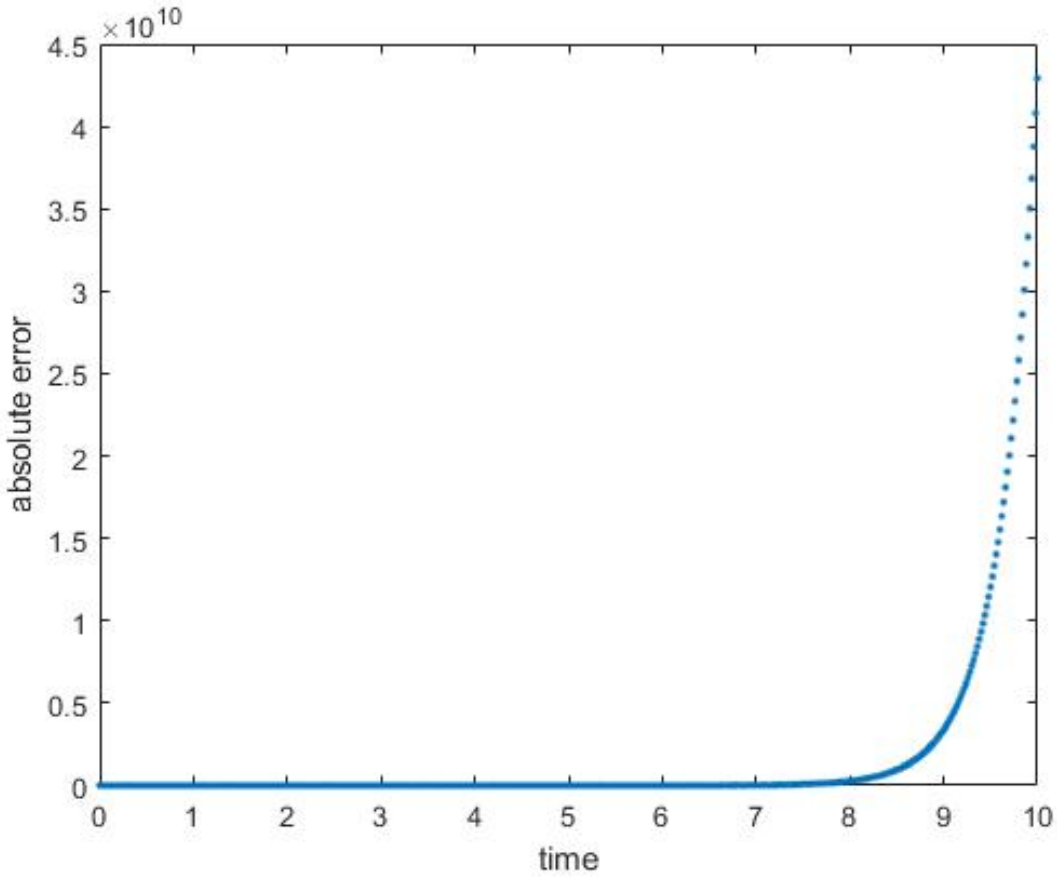

Figure 1: Absolute error in function of time using Forward Euler method to compute y(1)

We got a maximum error of 4.2916×10^{10}...

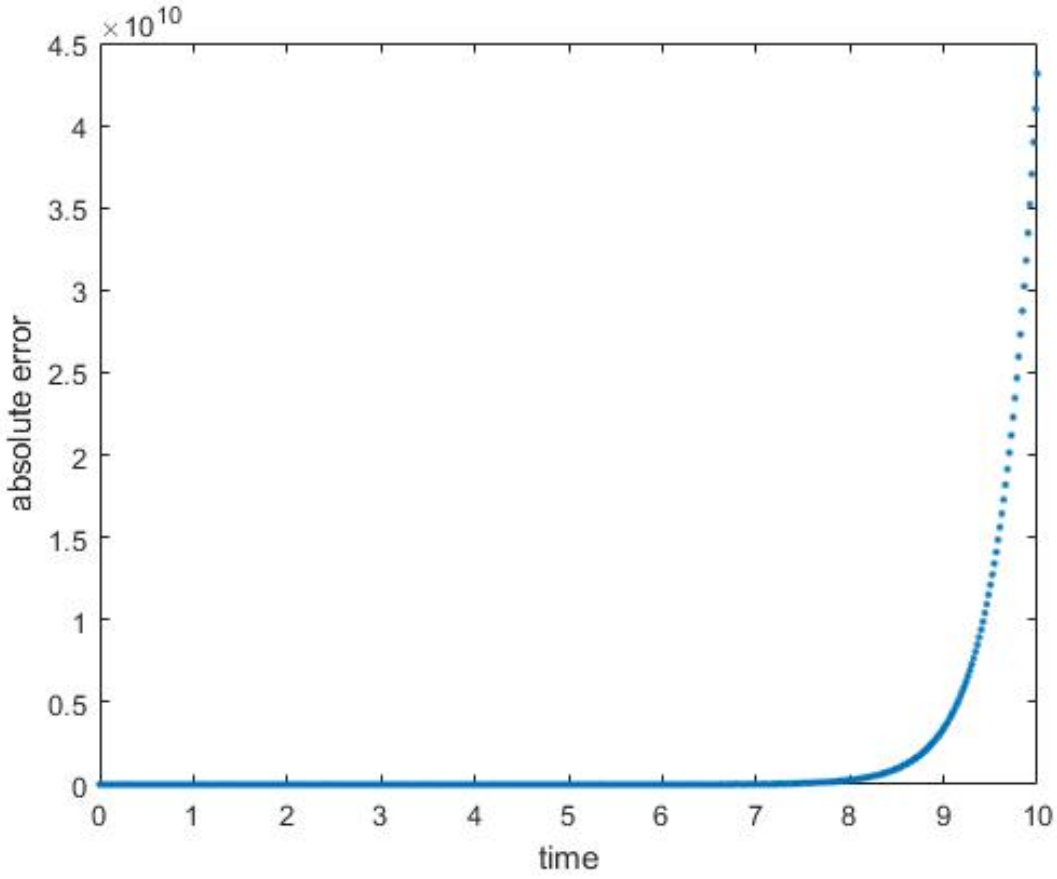

Figure 2: Absolute error in function of time using RK4 method to compute y(1)

We got a maximum error of 4.3146×10^{10}...

Comment the different behavior observed by the numerical method.

The Simpson's method has an empty stability region as proved by: ... We can notice the difference in the initial conditions in our results. The FE calculation for y(2) is better then the RK4 calculation given the best final error. This is, although, not that relevant, the difference is of about $0.5 \times 10^{-10}\%$.

Question 2

The exact solution can be found as:

$$y(t) = \frac{1}{10t + 1} \tag{2}$$

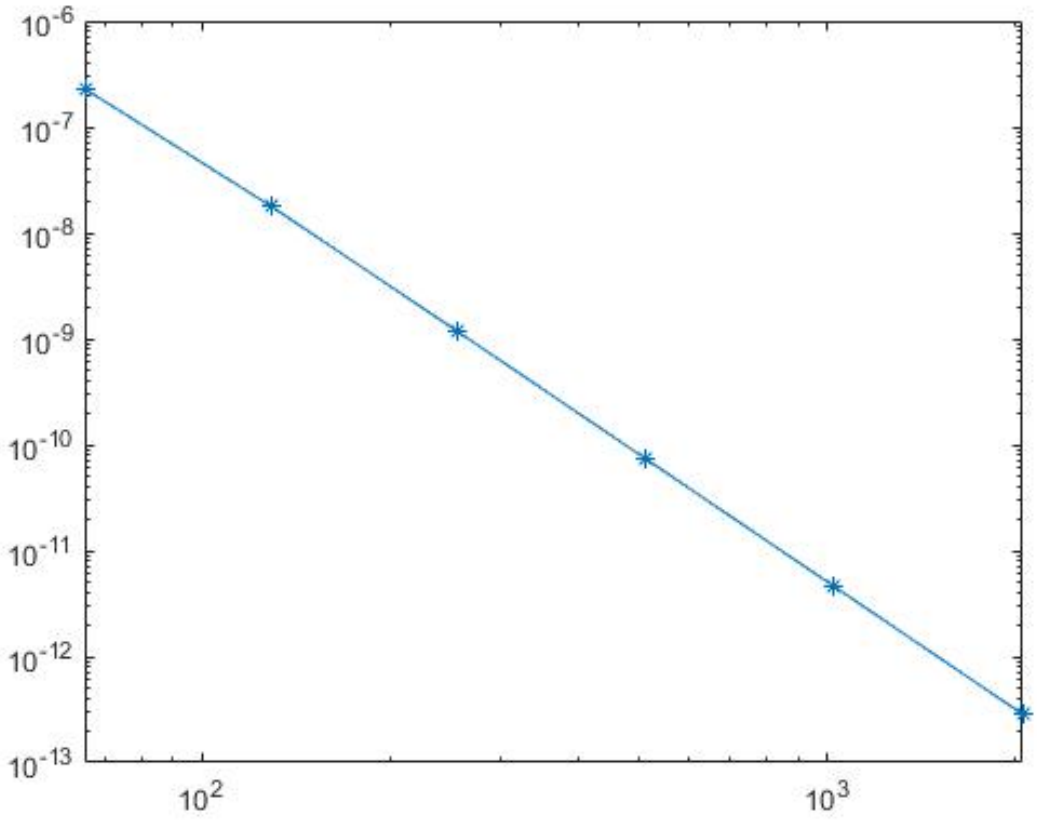

Figure 3: LogLog plot of the error as a function of the number of steps.

h	**error**
3.125000×10^{-2}	2.291844×10^{-7}
1.562500×10^{-2}	1.785763×10^{-8}
7.812500×10^{-3}	1.160234×10^{-9}
3.906250×10^{-3}	7.312862×10^{-11}
1.953125×10^{-3}	4.579586×10^{-12}
9.765625×10^{-4}	2.863750×10^{-13}

The error reduces with the increase in the number of steps due to the decrease of h as expected in theory. ...

Question 3

Question 4

Question 5

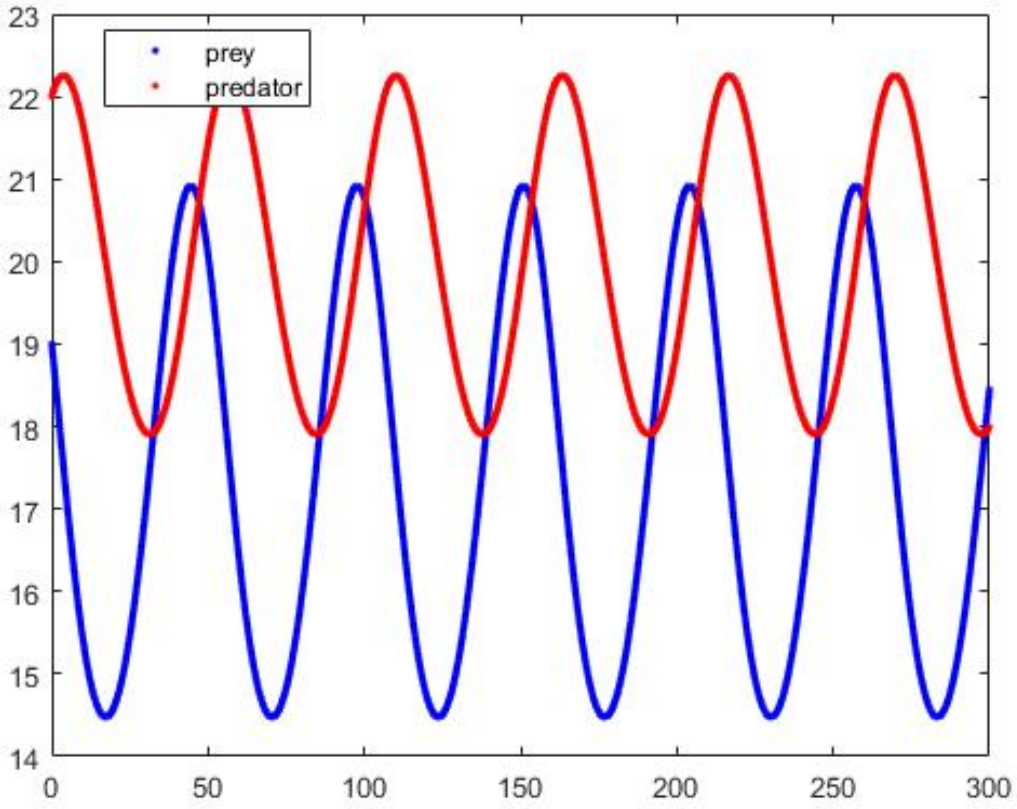

Figure 4: Evolution of the number of preys and predators.

Results

Outputs