Practica 1 Algorítmica

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Índice general

Capítulo 0

Entorno y automatización

0.1. Hardware

0.1.1. CPU

Datos obtenidos mediante la ejecución de `$ lscpu` en la máquina virtual donde han sido realizadas las ejecuciones.
Todos los ejercicios han sido ejecutados en dicho hardware.

Máquina virtual:
La máquina virtual tiene asignados dos núcleos del procesador.
Arquitectura: `x86_64`
modo(s) de operación de las CPUs: `32-bit`, `64-bit`
Orden de bytes: Little Endian
CPU(s): 2
CPU MHz: 2594.004
Caché L1d: 32K
Caché L1i: 32K
Caché L2: 256K
Caché L3: 6144K

Sistema Anfitrión:
El sistema anfitrión dispone de un microprocesador I7 4720HQ 2.60GHz

0.1.2. Memoria ram

Máquina virtual:
Tiene asignados 2GB de memoria RAM

Sistema Anfitrión:
Dispone de 16GB de memoria RAM

0.2. Software

0.2.1. Compilador

G++ versión 4.8

Opciones de compilado: g++ fichero.cpp o ejecutable -std=gnu++0x

0.2.2. Sistema operativo

Ubuntu 14.04.5 LTS

0.3. Macros y script

Se han diseñado distintos script para automatizar las mediciones de eficiencia, se detallan a continuación.

"Macro.csh"

```
#!/bin/csh -vx

set NOMBRE="Heapsort"

echo "" >> Salida.dat
@ i = 1000

while ( $i < 30000 )
./${NOMBRE} $i >> Salida.dat
@ i += 1000
end

./Macro2.csh
```

Este script realiza la llamada al algoritmo un determinado numero de veces pasandole un parámetro determinado en cada iteración. Tras su finalización se invoca a Macro 2.

"Macro2.csh"

```
#!/usr/bin/gnuplot

set terminal png
set output "Grafica.png"
set xlabel "Tamanio"
set ylabel "Tiempo (seg)"

plot "Salida.dat" title "Heapsort" with points

set output "Hibrida.png"
set xlabel "Tamanio"
set ylabel "Tiempo (seg)"

f(x)=a*x*(log(x)/log(2))+b

fit f(x) 'Salida.dat' via a, b

plot "Salida.dat", f(x) title "Curva ajustada"
```

Este script genera la gráfica a partir de los tiempos obtenidos en la ejecución del algoritmo, genera el ajuste y obtiene las constantes.

Çomparativa.csh"

```
#!/usr/bin/gnuplot

set terminal png

set autoscale                           # Pone la escala en modo automÃ¡tico
unset log
unset label                             # Quita etiquetas anteriores
set xtic auto                           # set xtics automatically
set ytic auto                           # set ytics automatically

set xlabel "Tamanio"
set ylabel "Tiempo (seg)"

set output "O(nLog(n)).png"

set title "Comparacion de algoritmos O(nLog(n))"

plot "Heapsort.dat" title "Heapsort" w lp, "Mergesort.dat" title "Mergesort"
 w lp, "Quicksort.dat" title "Quicksort" w lp

set output "O(n^2).png"

set title "Comparacion de algoritmos O(n^2)"

plot "Burbuja.dat" title "Burbuja" w lp, "Seleccion.dat" title "Seleccion"
 w lp, "Insercion.dat" title "Insercion" w lp

set output "Ordenacion de vectores.png"

set title "Comparacion de algoritmos de ordenacion de vectores"

plot "Heapsort.dat" title "Heapsort" w lp, "Mergesort.dat" title "Mergesort"
 w lp, "Quicksort.dat" title "Quicksort" w lp, "Burbuja.dat" title "Burbuja"
 w lp, "Seleccion.dat" title "Seleccion" w lp, "Insercion.dat" title "Insercion" w lp
```

Este script genera tres comparativas de los argoritmos de ordenación de vectores, una para los de tipo O(N^2), para los de tipo O(nLOG(n)) y para todos ellos.

0.4. Medición usando chrono

Todas las mediciones se han realizado usando la libreria chrono de STL.

```
int main(int argc, char * argv[])
{
  if (argc != 2)
    {
      cerr << "Formato " << argv[0] << " <num_elem>" << endl;
      return -1;
    }

  int n = atoi(argv[1]);

  int * T = new int[n];

  assert(T);

  srandom(time(0));

  for (int i = 0; i < n; i++)
    {
      T[i] = random();
    };

  high_resolution_clock::time_point tantes, tdespues;
  duration<double> transcurrido;

  tantes = high_resolution_clock::now();

  burbuja(T, n);

  tdespues = high_resolution_clock::now();

  transcurrido = duration_cast<duration<double>>(tdespues - tantes);
  cout << n << " "  << transcurrido.count() << endl;

  delete [] T;

  return 0;
};
```

Capítulo 1

Análisis de eficiencia teórica

1.1. Algoritmos de orden O(n^2)

Burbuja

$$\sum_{i=0}^{n-2}\sum_{j=0}^{n-i-2} 1 = \sum_{i=0}^{n-2}(n-i-1) = \sum_{i=0}^{n-2} n - \sum_{i=0}^{n-2} i - \sum_{i=0}^{n-2} 1 = n\sum_{i=0}^{n-2} 1 - \sum_{i=0}^{n-2} i - \sum_{i=0}^{n-2} 1 =$$

$$= n\cdot(n-1) - (0+1+2+\ldots+(n-2)) - (n-2+1) =$$

$$= n\cdot(n-1) - \frac{(n-2)\cdot(n-1)}{2} - (n-1) =$$

$$= n^2 - n - \frac{n^2-3\cdot(n)+2}{2} - n + 1 =$$

$$= n^2 - \frac{n^2}{2} + \frac{n}{2} = f(n) \in O(n^2)$$

Tras llevar a cabo los cálculos determinamos que el algoritmo Burbuja posee una eficiencia teórica de tipo O(n^2).

1.2. Algoritmos de orden O(n^3)

Floyd

$$\sum_{i=0}^{n-1}\sum_{i=0}^{n-1}\sum_{i=0}^{n-1} O(1) = \sum_{i=0}^{n-1}\sum_{i=0}^{n-1}(nO(1)) = \sum_{i=0}^{n-1}(O(1)*n*n) = O(1)*n^2*n = n^3O(1) \in O(n^3)$$

Tras llevar a cabo los cálculos determinamos que el algoritmo Floyd posee una eficiencia teórica de tipo O(n^3).

Capítulo 2

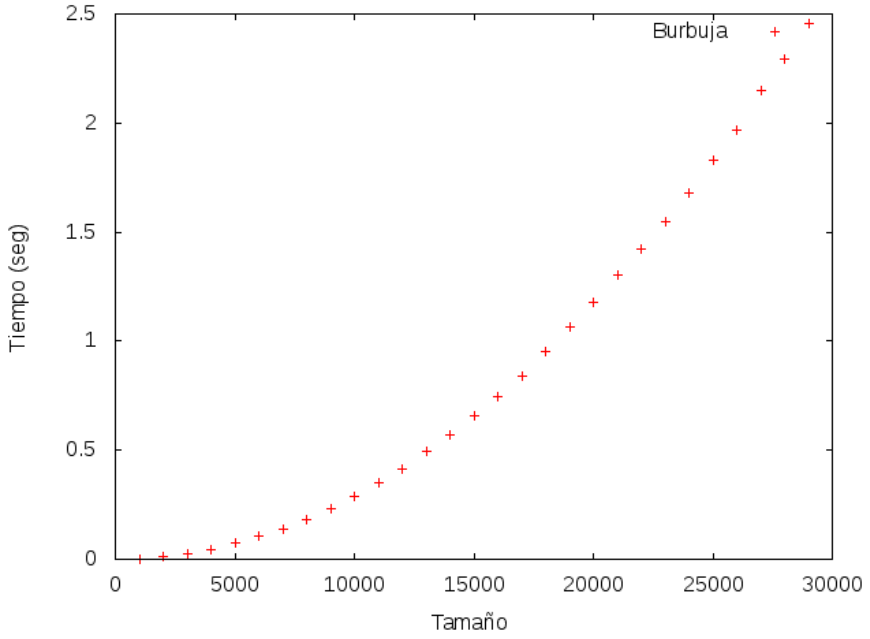
Análisis de eficiencia empírica e híbrida

Se han llevado a cabo determinadas mediciones sobre los distintos algoritmos, a continuación se detallan los resultados obtenidos de dichas mediciones para cada uno de los algoritmos.

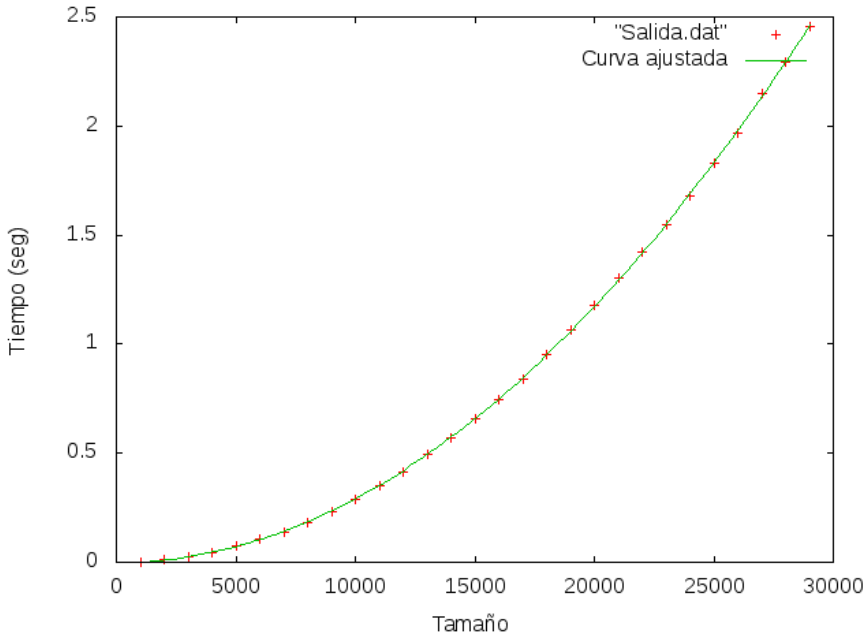
2.1. Algoritmos de orden $O(n^2)$

2.1.1. Burbuja

Eficiencia empírica



Eficiencia híbrida

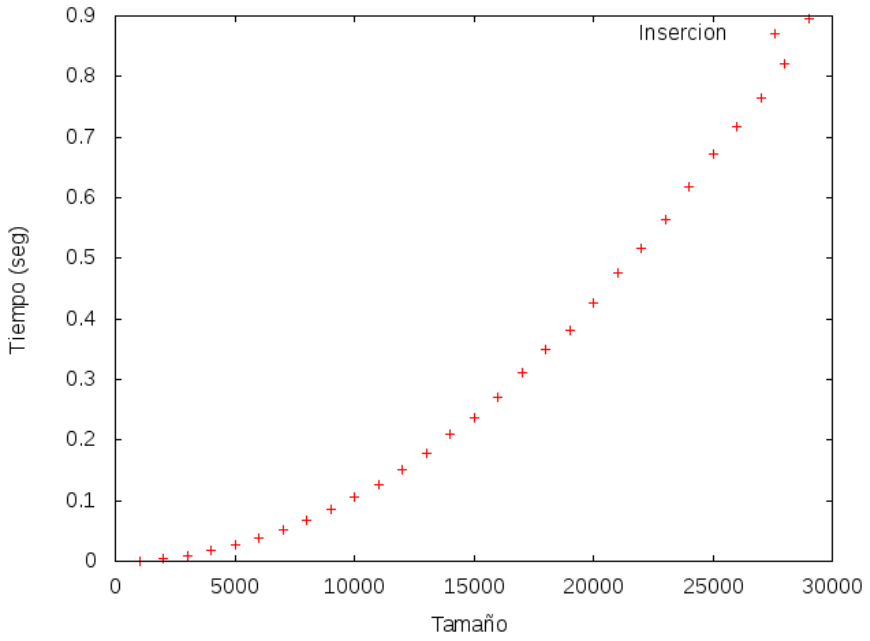



```
*******************************************************************************
Sun Mar 11 18:45:29 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 29
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 4.46438e+18       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 2.26518e+08

initial set of free parameter values

a               = 1
b               = 1
c               = 1

After 12 iterations the fit converged.
final sum of squares of residuals : 0.000702233
```

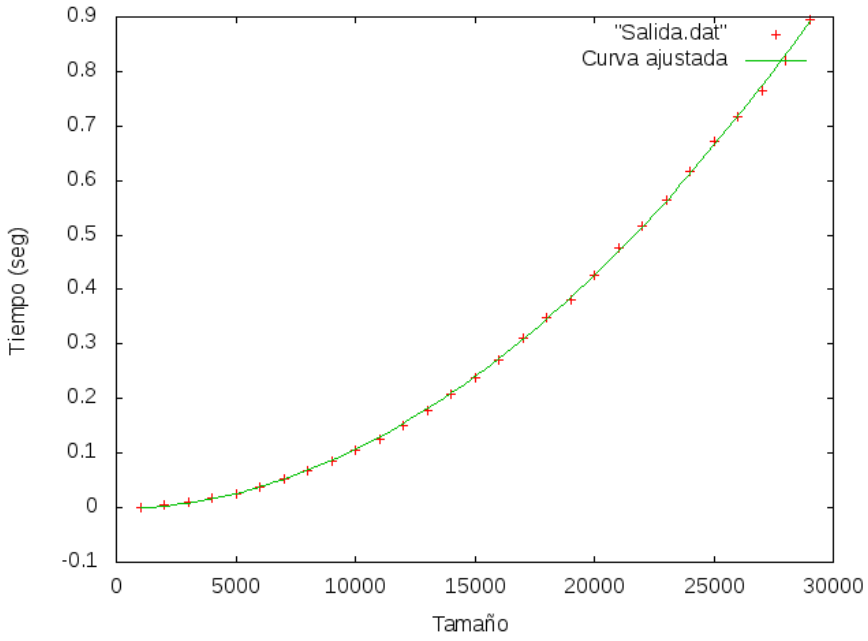
```
rel. change during last iteration : -1.80811e-08

degrees of freedom    (FIT_NDF)                        : 26
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00519701
variance of residuals (reduced chisquare) = WSSR/ndf   : 2.70089e-05

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 2.92569e-09      +/- 1.544e-11    (0.5278%)
b               = 1.70151e-07      +/- 4.774e-07    (280.6%)
c               = -0.00254704      +/- 0.003107     (122%)


correlation matrix of the fit parameters:

                a      b      c
a                1.000
b               -0.970  1.000
c                0.770 -0.882  1.000
```

2.1.2. Inserción

Eficiencia empírica



Eficiencia híbrida

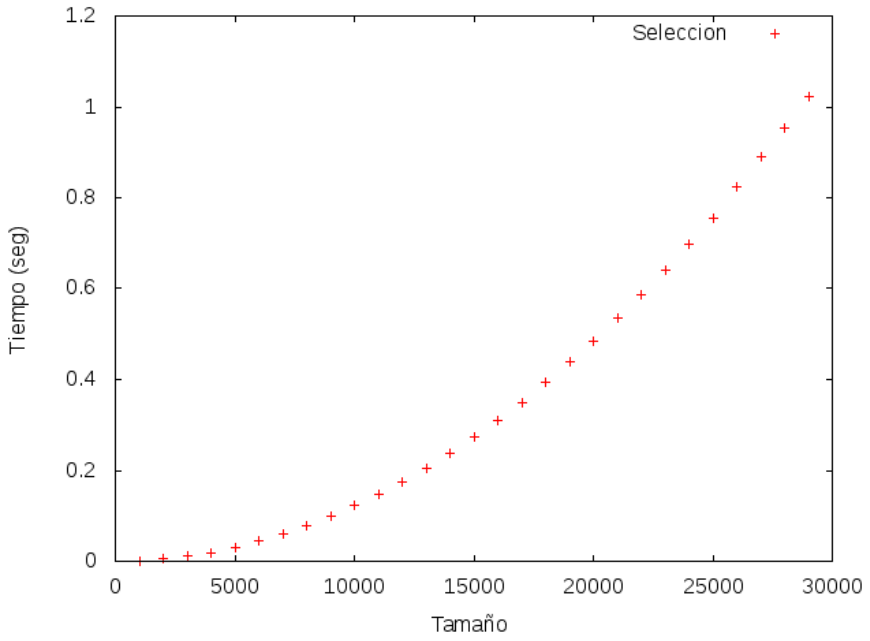



```
*******************************************************************************
Sun Mar 11 18:46:31 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 29
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 4.46438e+18       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 2.26518e+08

initial set of free parameter values

a               = 1
b               = 1
c               = 1

After 12 iterations the fit converged.
final sum of squares of residuals : 0.000442853
```

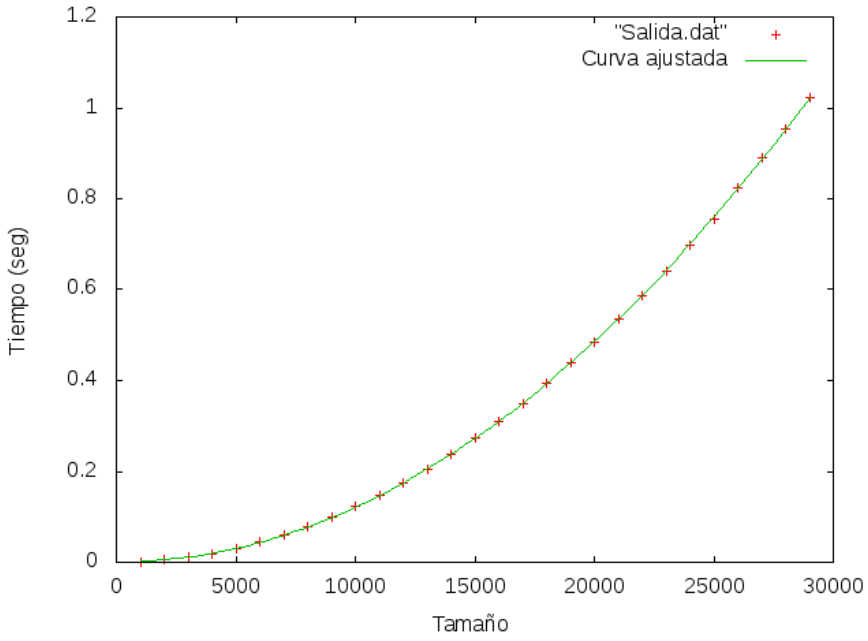
```
rel. change during last iteration : -2.86602e-08

degrees of freedom    (FIT_NDF)                        : 26
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00412708
variance of residuals (reduced chisquare) = WSSR/ndf   : 1.70328e-05

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 1.04643e-09      +/- 1.226e-11    (1.172%)
b               = 4.84082e-07      +/- 3.791e-07    (78.31%)
c               = -0.00235619      +/- 0.002467     (104.7%)


correlation matrix of the fit parameters:

                a      b      c
a                1.000
b               -0.970  1.000
c                0.770 -0.882  1.000
```

2.1.3. Selección

Eficiencia empírica



Eficiencia híbrida



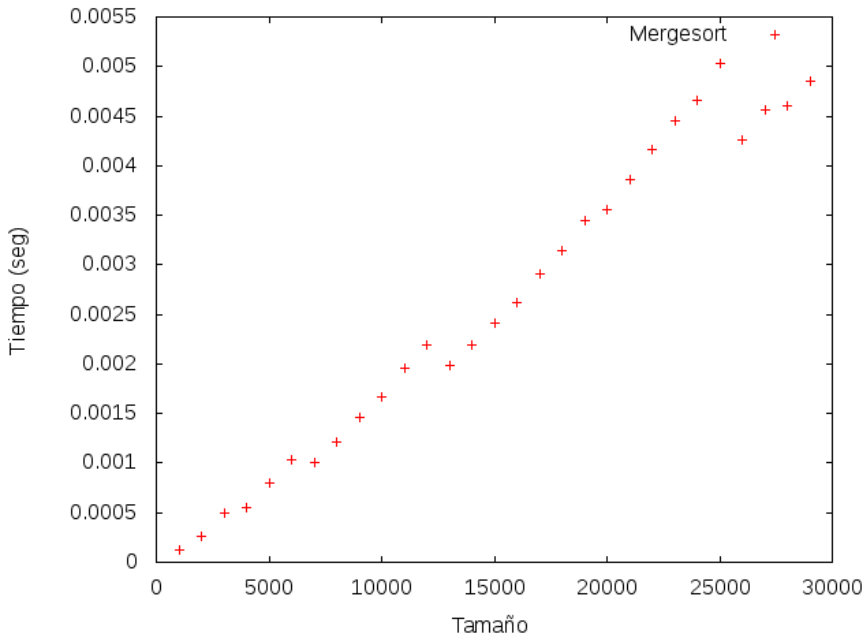

```
*******************************************************************************
Sun Mar 11 18:47:58 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 29
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 4.46438e+18       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 2.26518e+08

initial set of free parameter values

a               = 1
b               = 1
c               = 1

After 12 iterations the fit converged.
final sum of squares of residuals : 5.40929e-05
```

```
rel. change during last iteration : -2.33028e-07

degrees of freedom    (FIT_NDF)                        : 26
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00144239
variance of residuals (reduced chisquare) = WSSR/ndf   : 2.0805e-06

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 1.22159e-09      +/- 4.286e-12    (0.3508%)
b               = -2.19171e-07     +/- 1.325e-07    (60.45%)
c               = 0.00108918       +/- 0.0008623    (79.17%)


correlation matrix of the fit parameters:

                a      b      c
a                1.000
b               -0.970  1.000
c                0.770 -0.882  1.000
```

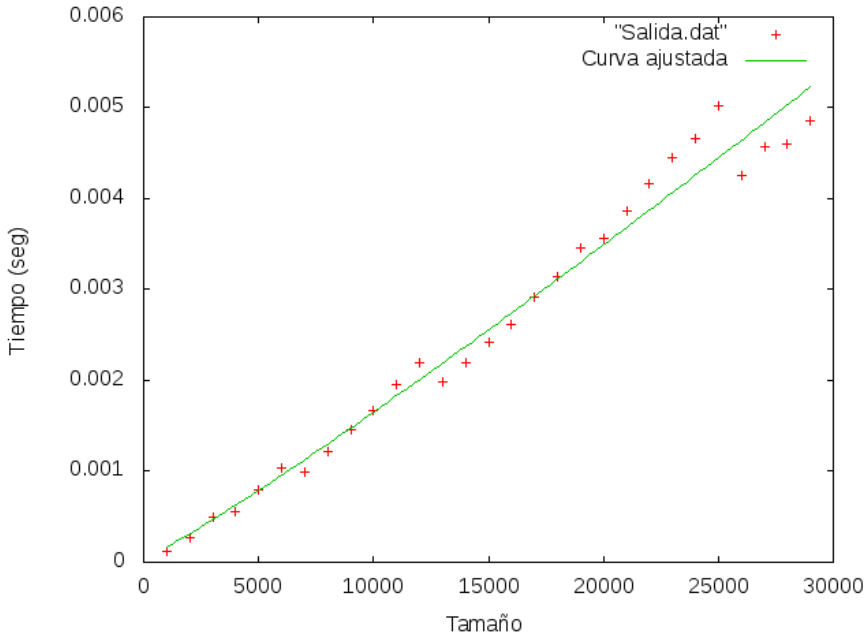
2.2. Algoritmos de orden O(n log n)

2.2.1. Mergesort

Eficiencia empírica



Eficiencia híbrida

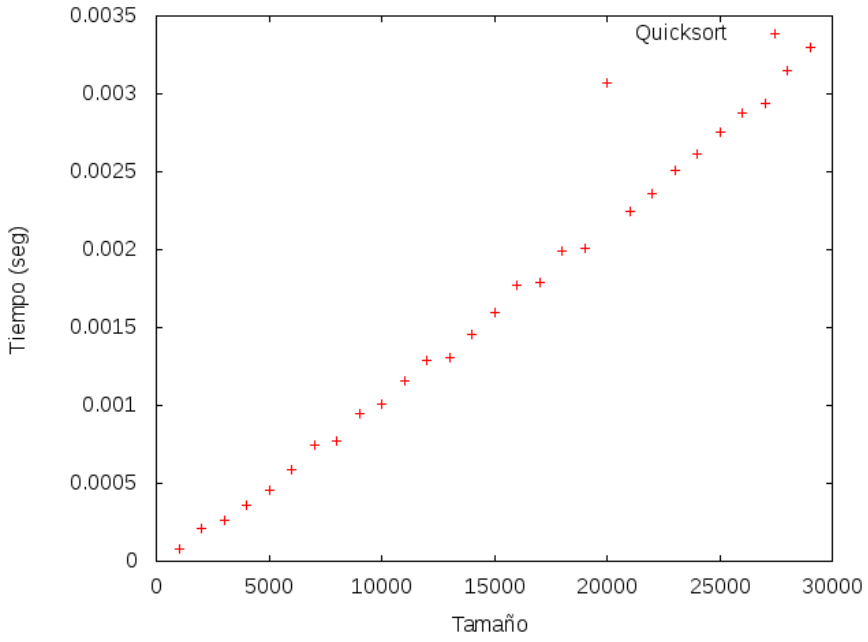



```
*******************************************************************************
Sun Mar 11 18:46:44 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 29
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 1.76788e+12       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 174587

initial set of free parameter values

a               = 1
b               = 1

After 9 iterations the fit converged.
final sum of squares of residuals : 1.53265e-06
rel. change during last iteration : -9.71919e-09
```

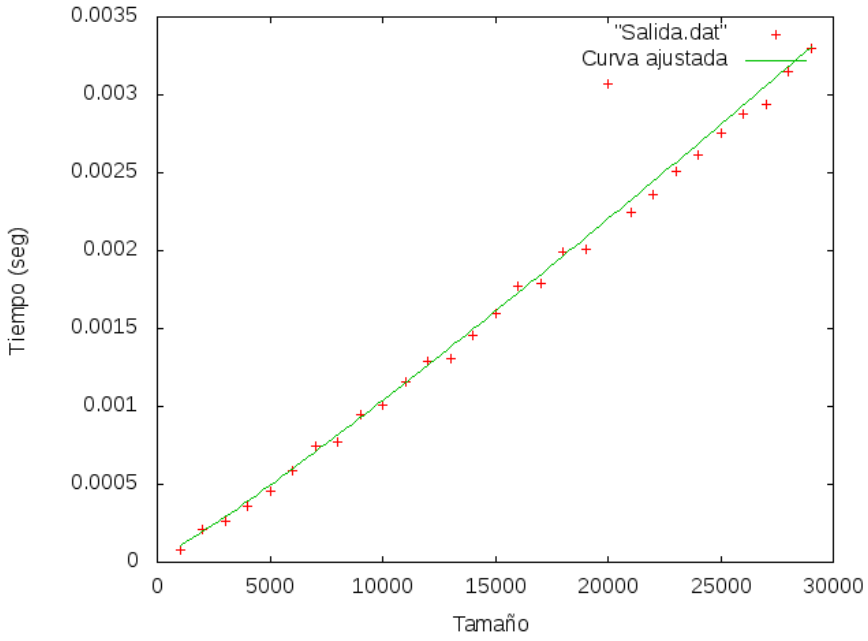
```
degrees of freedom    (FIT_NDF)                        : 27
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.000238253
variance of residuals (reduced chisquare) = WSSR/ndf   : 5.67647e-08

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 1.20462e-08      +/- 3.492e-10    (2.899%)
b               = 5.12778e-05      +/- 8.623e-05    (168.2%)


correlation matrix of the fit parameters:

               a      b
a               1.000
b              -0.858  1.000
```

2.2.2. Quicksort

Eficiencia empírica



Eficiencia híbrida

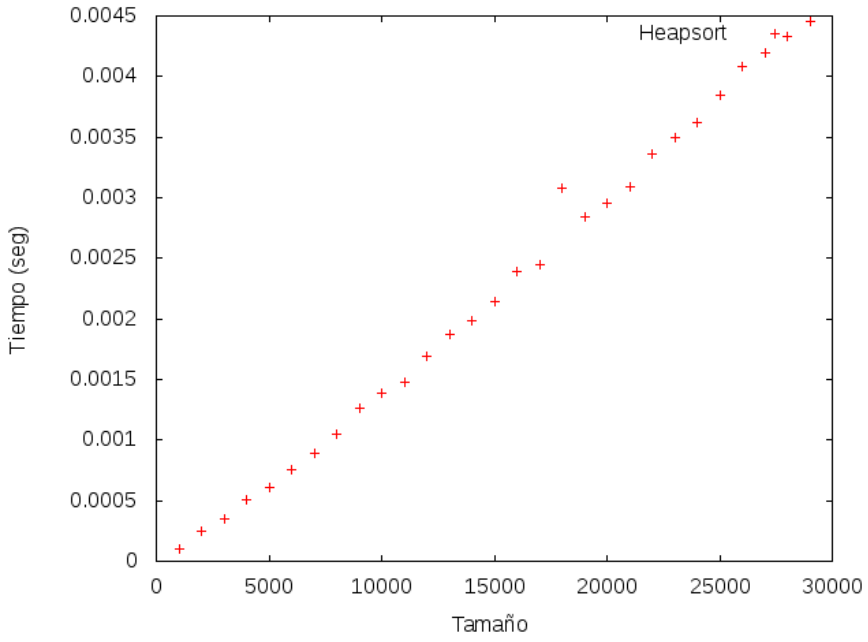



```
*******************************************************************************
Sun Mar 11 18:46:56 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 29
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 1.76788e+12       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 174587

initial set of free parameter values

a               = 1
b               = 1

After 9 iterations the fit converged.
final sum of squares of residuals : 8.20476e-07
rel. change during last iteration : -1.81563e-08
```

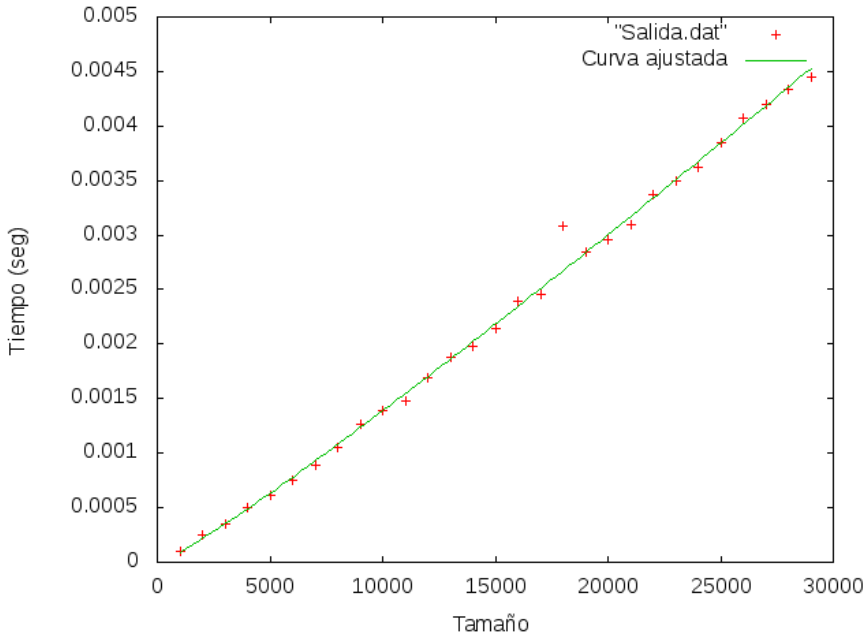
```
degrees of freedom    (FIT_NDF)                        : 27
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.000174322
variance of residuals (reduced chisquare) = WSSR/ndf   : 3.0388e-08

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a                 = 7.62021e-09      +/- 2.555e-10    (3.353%)
b                 = 2.72734e-05      +/- 6.309e-05    (231.3%)


correlation matrix of the fit parameters:

                a      b
a                1.000
b               -0.858  1.000
```

2.2.3. Heapsort

Eficiencia empírica



Eficiencia híbrida



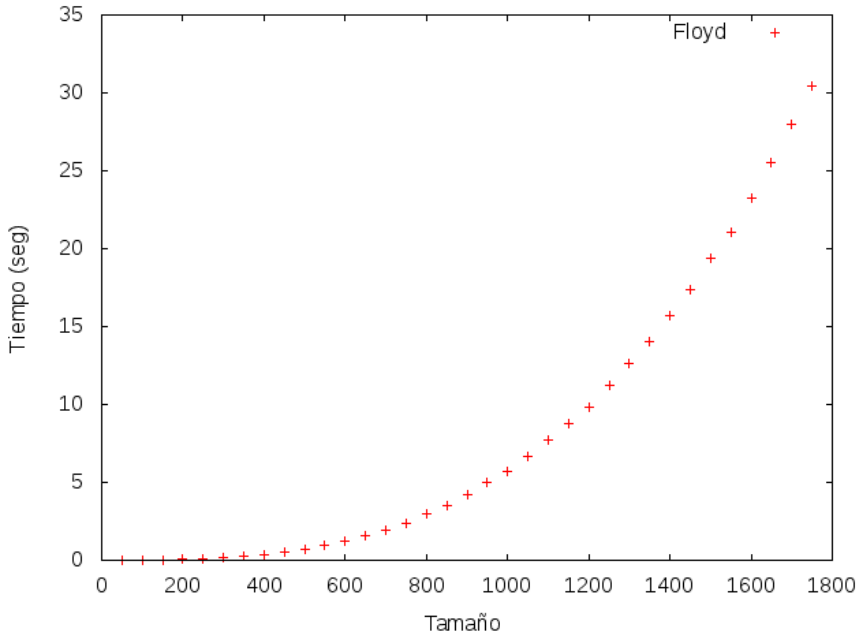

```
*******************************************************************************
Sun Mar 11 18:46:06 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 29
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 1.76788e+12       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda           : 174587

initial set of free parameter values

a               = 1
b               = 1

After 9 iterations the fit converged.
final sum of squares of residuals : 2.06149e-07
rel. change during last iteration : -7.22682e-08
```

```
degrees of freedom    (FIT_NDF)                        : 27
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 8.73794e-05
variance of residuals (reduced chisquare) = WSSR/ndf   : 7.63517e-09

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 1.05692e-08      +/- 1.281e-10    (1.212%)
b               = -1.47074e-05     +/- 3.162e-05    (215%)


correlation matrix of the fit parameters:

               a      b
a               1.000
b              -0.858  1.000
```

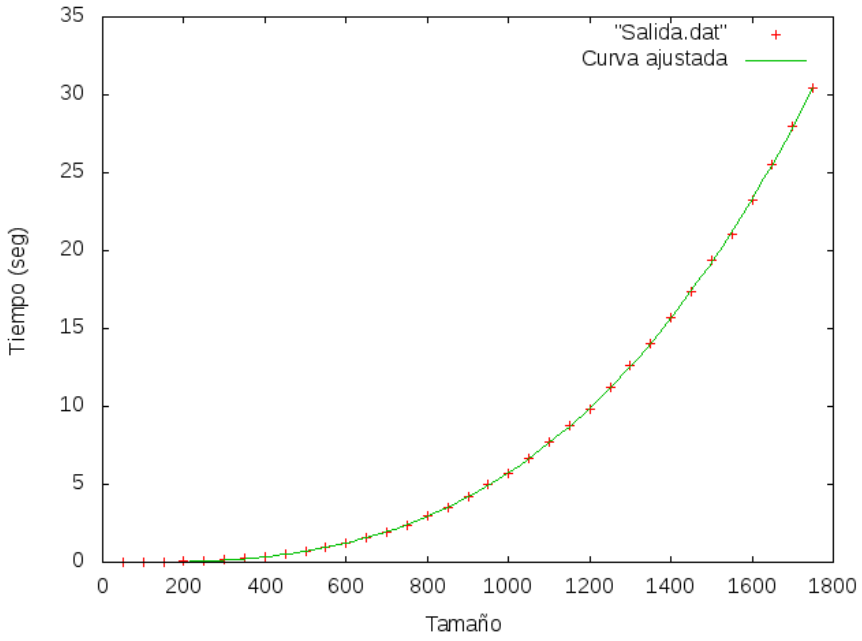
2.3. Algoritmos de orden $O(n^3)$

2.3.1. Floyd

Eficiencia empírica



Eficiencia híbrida



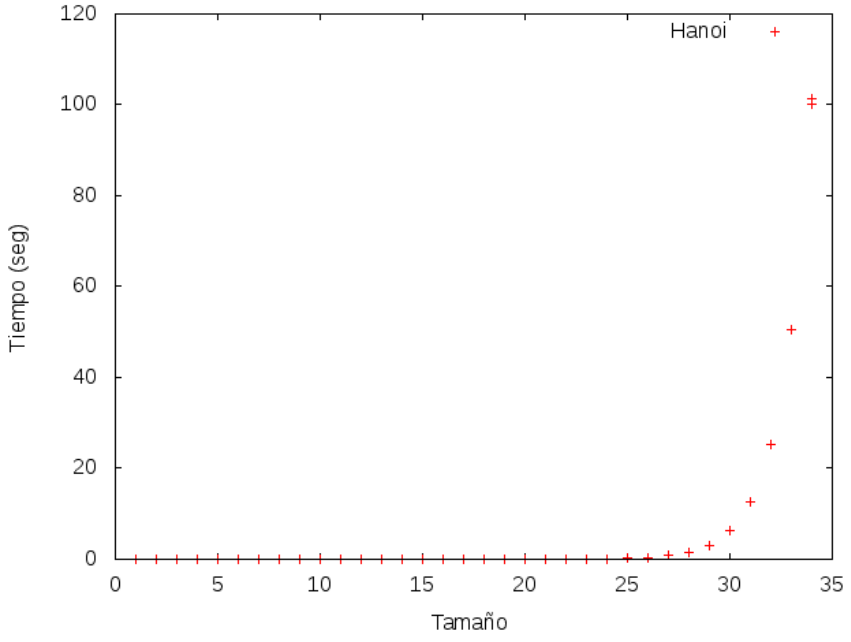

```
*******************************************************************************
Sun Mar 11 17:10:46 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 35
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 1.58595e+20       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 1.06364e+09

initial set of free parameter values

a               = 1
b               = 1
c               = 1
d               = 1

After 13 iterations the fit converged.
```

```
final sum of squares of residuals : 0.0691843
rel. change during last iteration : -6.09799e-13

degrees of freedom    (FIT_NDF)                        : 31
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.0472414
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00223175

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 5.4095e-09       +/- 7.929e-11    (1.466%)
b               = 6.43829e-07      +/- 2.169e-07    (33.7%)
c               = -0.000334536     +/- 0.0001693    (50.61%)
d               = 0.0341415        +/- 0.03569      (104.5%)


correlation matrix of the fit parameters:

               a      b      c      d
a               1.000
b              -0.987  1.000
c               0.923 -0.971  1.000
d              -0.703  0.782 -0.889  1.000
```

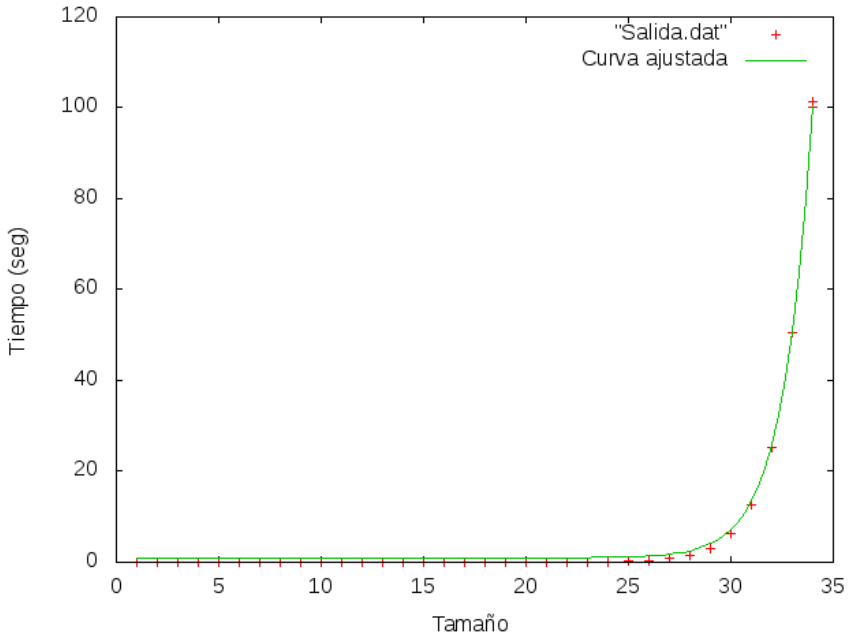
2.4. Algoritmos de orden $O(2^n)$

2.4.1. Hanoi

Eficiencia empírica



Eficiencia híbrida




```
*******************************************************************************
Sun Mar 11 18:20:59 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 34
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 3.93531e+20       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 2.40566e+09

initial set of free parameter values

a               = 1
b               = 1

After 5 iterations the fit converged.
```

```
final sum of squares of residuals : 31.147
rel. change during last iteration : -1.071e-09

degrees of freedom    (FIT_NDF)                        : 32
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.986583
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.973345

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 5.80342e-09      +/- 5.208e-11    (0.8975%)
b               = 1                +/- 0.1772       (17.72%)


correlation matrix of the fit parameters:

               a      b
a               1.000
b              -0.297  1.000


*******************************************************************************
Sun Mar 11 18:22:00 2018


FIT:    data read from 'Salida.dat'
        format = z
        #datapoints = 35
        residuals are weighted equally (unit weight)

function used for fitting: f(x)
fitted parameters initialized with current variable values



 Iteration 0
 WSSR        : 6.88678e+20       delta(WSSR)/WSSR   : 0
 delta(WSSR) : 0                 limit for stopping : 1e-05
 lambda          : 3.1366e+09

initial set of free parameter values

a               = 1
b               = 1

After 5 iterations the fit converged.
final sum of squares of residuals : 31.2957
```

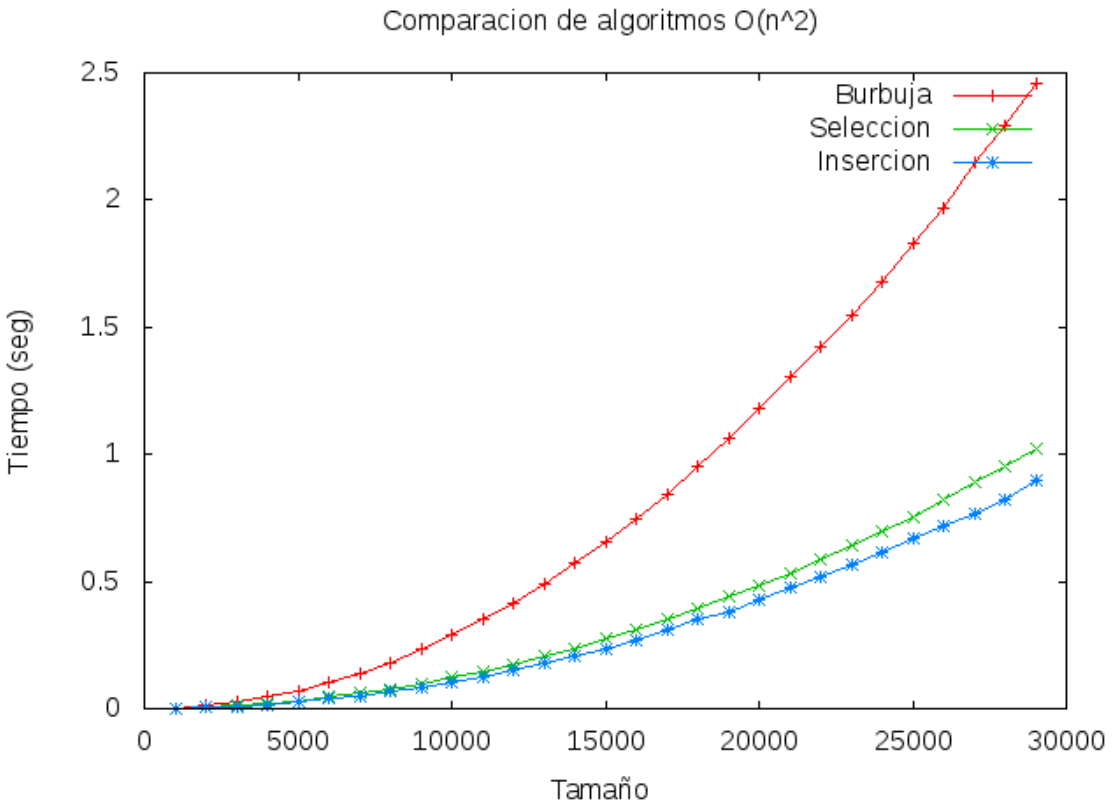
```
rel. change during last iteration : -6.21141e-10

degrees of freedom    (FIT_NDF)                        : 33
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.973835
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.948355

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 5.7907e-09       +/- 3.934e-11    (0.6794%)
b               = 1                +/- 0.1745       (17.45%)


correlation matrix of the fit parameters:

               a      b
a               1.000
b              -0.332  1.000
```

Como hemos podido observar, los resultados de las pruebas de eficiencia híbridas determinan que los resultados empíricos se ajustan correctamente con los teóricos mostrando una total semejanza.

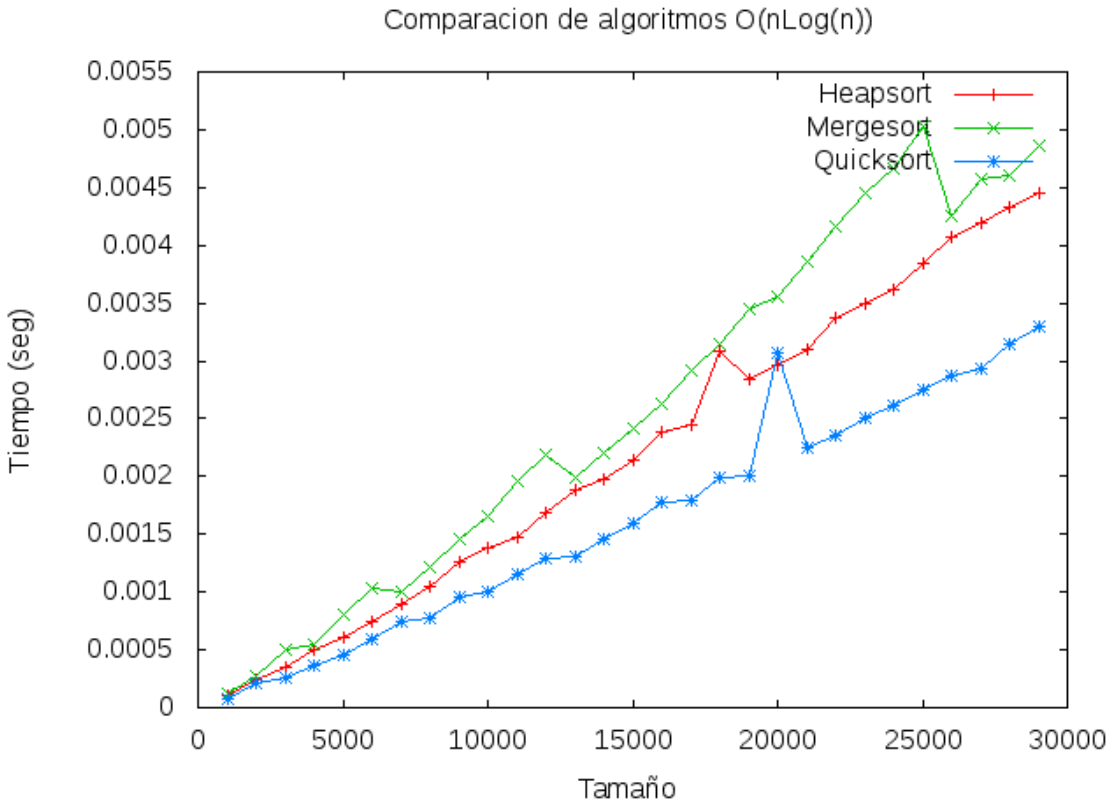
Capítulo 3

Comparación de eficiencias

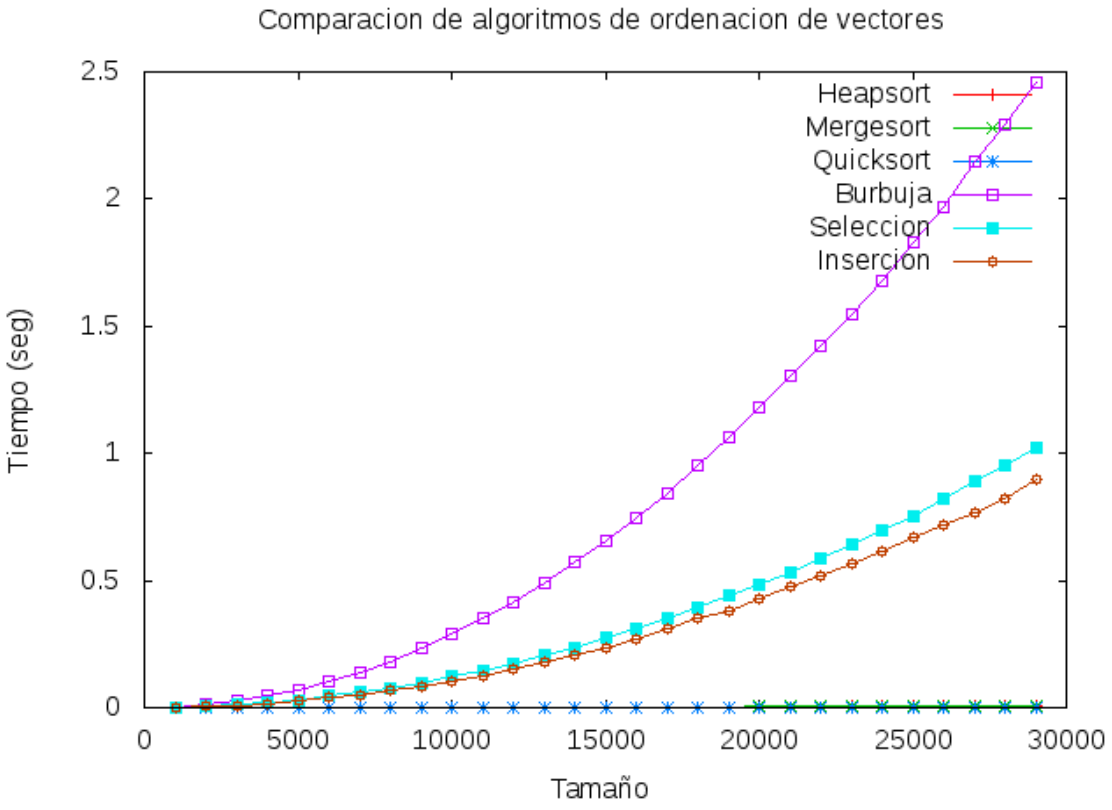
3.0.1. Algoritmos de ordenación



En la imagen 3.0.1 podemos observar la tendencia de los algoritmos O(n^2) en función a los tamaños de los datos de entrada. Podemos apreciar que Burbuja es mas lento que selección e inserción.



En la imagen 3.0.1 podemos observar la tendencia de los algoritmos O(nLOG(n)) en función a los tamaños de los datos de entrada. Podemos apreciar que aunque mantienen una crecida similar Quicksort es el mas eficiente del grupo.



En la imagen 3.0.1 podemos observar la tendencia de los algoritmos en función a los tamaños de los datos de entrada. Podemos apreciar que Quicksort y Mergesort son claramente los mas eficientes y que Burbuja es el menos eficiente. Esta gráfica también deja constancia de las diferencias entre los algoritmos de cada uno de los ordenes de eficiencia.