Regressão Linear Múltipla

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4. Considerando dados categóricos
5. Retirando Variáveis
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7. Na prática: Divisão Treino/Teste
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In [1]:
```
#!pip install statsmodels
```

In [2]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.linear_model import LinearRegression
import statsmodels.api as sm

sns.set() # Definir o tema do Matplotlib como sendo o tema padrão do Seaborn
```

In [3]:
```
df = pd.read_csv('fish.csv')
```

In [4]:
```
df.head()
```

Out[4]:

	Species	Weight	Length1	Length2	Length3	Height	Width
0	Bream	242.0	23.2	25.4	30.0	11.5200	4.0200
1	Bream	290.0	24.0	26.3	31.2	12.4800	4.3056
2	Bream	340.0	23.9	26.5	31.1	12.3778	4.6961
3	Bream	363.0	26.3	29.0	33.5	12.7300	4.4555
4	Bream	430.0	26.5	29.0	34.0	12.4440	5.1340

In [5]:
```
df.shape
```

Out[5]: (159, 7)

In [6]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 159 entries, 0 to 158
Data columns (total 7 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   Species   159 non-null    object
 1   Weight    159 non-null    float64
 2   Length1   159 non-null    float64
 3   Length2   159 non-null    float64
 4   Length3   159 non-null    float64
 5   Height    159 non-null    float64
 6   Width     159 non-null    float64
dtypes: float64(6), object(1)
memory usage: 8.8+ KB
```

In [7]:
```
df.describe().T
```

Out[7]:

	count	mean	std	min	25%	50%	75%	max
Weight	159.0	398.326415	357.978317	0.0000	120.00000	273.0000	650.0000	1650.000
Length1	159.0	26.247170	9.996441	7.5000	19.05000	25.2000	32.7000	59.000
Length2	159.0	28.415723	10.716328	8.4000	21.00000	27.3000	35.5000	63.400
Length3	159.0	31.227044	11.610246	8.8000	23.15000	29.4000	39.6500	68.000
Height	159.0	8.970994	4.286208	1.7284	5.94480	7.7860	12.3659	18.957
Width	159.0	4.417486	1.685804	1.0476	3.38565	4.2485	5.5845	8.142

In [8]:
```
df['Species'].unique()
```

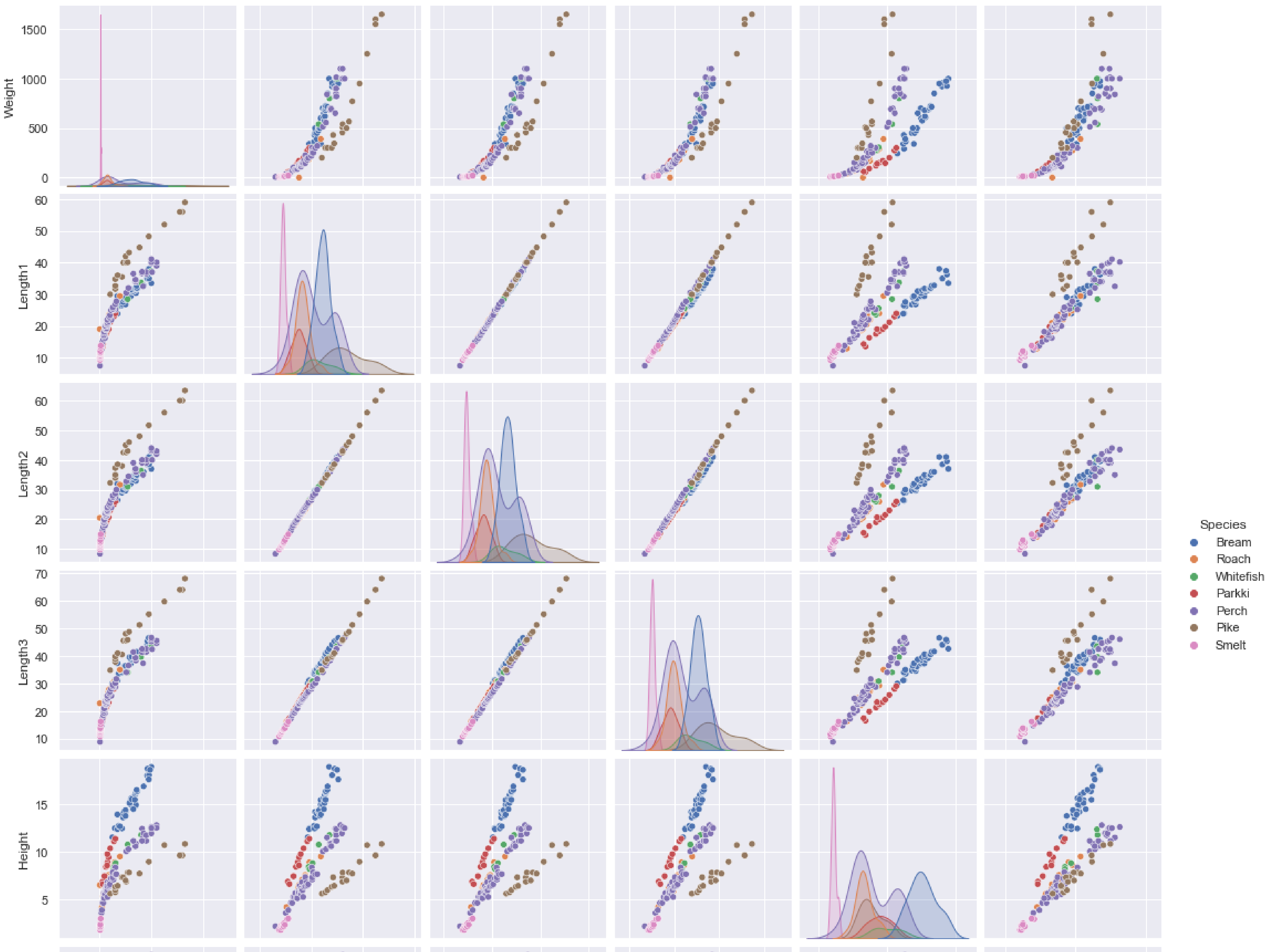
Out[8]:
```
array(['Bream', 'Roach', 'Whitefish', 'Parkki', 'Perch', 'Pike', 'Smelt'],
      dtype=object)
```

Análise Exploratória dos Dados

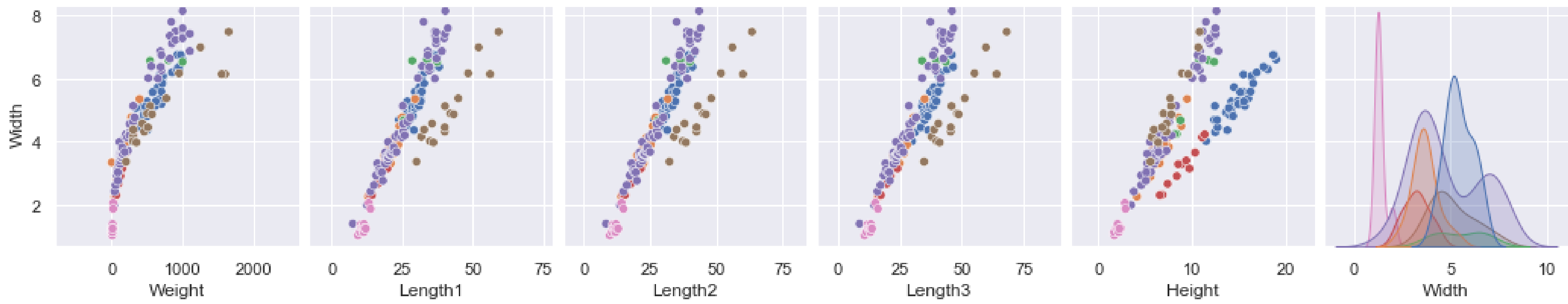
> Como os dados se relacionam?

In [9]:
```
sns.pairplot(df, hue='Species')
```

Out[9]:
```
<seaborn.axisgrid.PairGrid at 0x250fe3d1790>
```

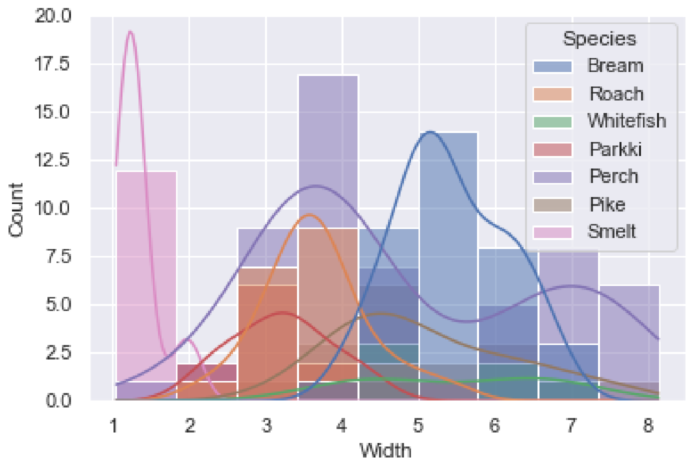


Weight
Length1
Length2
Length3
Height
Species
Bream
Roach
Whitefish
Parkki
Perch
Pike
Smelt



In [10]:
```
sns.histplot(data=df, x='Width', hue='Species', kde=True)
```

Out[10]:
```
<AxesSubplot:xlabel='Width', ylabel='Count'>
```



In [11]:
```
# Matriz de correlação
df.corr()
```

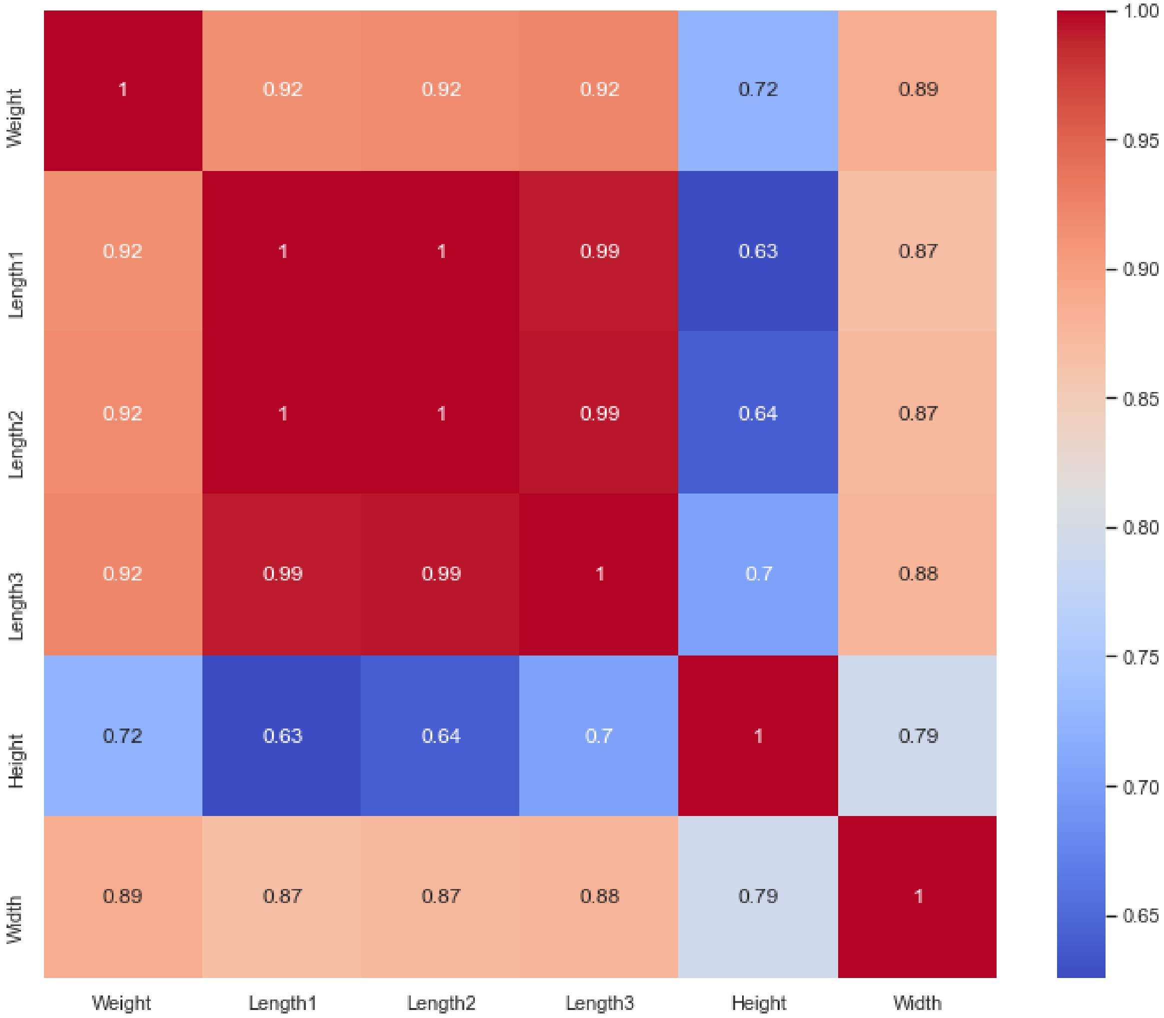
Out[11]:

	Weight	Length1	Length2	Length3	Height	Width
Weight	1.000000	0.915712	0.918618	0.923044	0.724345	0.886507
Length1	0.915712	1.000000	0.999517	0.992031	0.625378	0.867050
Length2	0.918618	0.999517	1.000000	0.994103	0.640441	0.873547
Length3	0.923044	0.992031	0.994103	1.000000	0.703409	0.878520
Height	0.724345	0.625378	0.640441	0.703409	1.000000	0.792881
Width	0.886507	0.867050	0.873547	0.878520	0.792881	1.000000

In [12]:
```
plt.figure(figsize=(12,10))
```

```python
sns.heatmap(df.corr(), cmap='coolwarm', annot=True)
```

Out[12]: <AxesSubplot:>



Definir a Regressão Linear Múltipla

In [13]:
```python
df.head()
```

Out[13]:

	Species	Weight	Length1	Length2	Length3	Height	Width
0	Bream	242.0	23.2	25.4	30.0	11.5200	4.0200
1	Bream	290.0	24.0	26.3	31.2	12.4800	4.3056
2	Bream	340.0	23.9	26.5	31.1	12.3778	4.6961
3	Bream	363.0	26.3	29.0	33.5	12.7300	4.4555
4	Bream	430.0	26.5	29.0	34.0	12.4440	5.1340

In [14]:

```
y = df['Weight'] # Variável resposta
X = df.drop(['Weight', 'Species'], axis=1) # Variáveis explicativas
```

In [15]:

```
y
```

Out[15]:

```
0      242.0
1      290.0
2      340.0
3      363.0
4      430.0
       ...
154     12.2
155     13.4
156     12.2
157     19.7
158     19.9
Name: Weight, Length: 159, dtype: float64
```

In [16]:

```
X.head()
```

Out[16]:

	Length1	Length2	Length3	Height	Width
0	23.2	25.4	30.0	11.5200	4.0200
1	24.0	26.3	31.2	12.4800	4.3056
2	23.9	26.5	31.1	12.3778	4.6961
3	26.3	29.0	33.5	12.7300	4.4555
4	26.5	29.0	34.0	12.4440	5.1340

Scikit Learning

Etapas de utilização de um modelo de ML

1. Preparar o X (variáveis explicativas) e o y (variável resposta)
2. Fitar o modelo (treinar): calcular os coefientes da Regressão Linear.

3. Predizer outros valores de X, utilizando o modelo treinado.

In [17]:
```
lr = LinearRegression()
```

In [18]:
```
lr.fit(X, y)
```

Out[18]:
```
LinearRegression()
```

In [19]:
```
lr.intercept_ # intercepto (Beta 0)
```

Out[19]:
```
-499.58695535694164
```

In [20]:
```
lr.coef_
```

Out[20]:
```
array([ 62.35521443,  -6.52675249, -29.02621861,  28.29735132,
        22.47330665])
```

Regressão Linear Simples

$$ y = \beta_0 + \beta_1 \cdot x $$

Regressão Linear Múltipla

$$ y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 $$

In [21]:
```
y[0]
```

Out[21]:
```
242.0
```

In [22]:
```
X.loc[0] # A primeira linha do DF X
```

Out[22]:
```
Length1    23.20
Length2    25.40
Length3    30.00
Height     11.52
Width       4.02
Name: 0, dtype: float64
```

In [23]:
```
lr.intercept_ + lr.coef_[0] * X.loc[0]['Length1'] + lr.coef_[1] * X.loc[0]['Length2'] + lr.coef_[2] * X.loc[0]['Length3'] + lr.coef_[3] * X.loc[0]['Height'] +
```

Out[23]:
```
326.81612777212615
```

Predição para todos os dados do Dataset

In [24]:
```
X.head()
```

Out[24]:

	Length1	Length2	Length3	Height	Width
0	23.2	25.4	30.0	11.5200	4.0200
1	24.0	26.3	31.2	12.4800	4.3056
2	23.9	26.5	31.1	12.3778	4.6961
3	26.3	29.0	33.5	12.7300	4.4555
4	26.5	29.0	34.0	12.4440	5.1340

In [25]:

```
y
```

Out[25]:

```
0      242.0
1      290.0
2      340.0
3      363.0
4      430.0
       ...
154     12.2
155     13.4
156     12.2
157     19.7
158     19.9
Name: Weight, Length: 159, dtype: float64
```

In [26]:

```
y_pred = lr.predict(X) # Valor predito/estimado do meu Y (Peso dos peixes)
```

In [27]:

```
X.head()
```

Out[27]:

	Length1	Length2	Length3	Height	Width
0	23.2	25.4	30.0	11.5200	4.0200
1	24.0	26.3	31.2	12.4800	4.3056
2	23.9	26.5	31.1	12.3778	4.6961
3	26.3	29.0	33.5	12.7300	4.4555
4	26.5	29.0	34.0	12.4440	5.1340

In [28]:

```
y_pred
```

Out[28]:
```
array([ 326.81612777,  369.57859339,  370.82418025,  439.05613854,
        444.16916821,  466.12531938,  496.15134252,  473.62398623,
        511.95189321,  540.44545602,  536.53040775,  551.80179482,
        540.3150434 ,  556.0586456 ,  584.39982147,  607.28049765,
        608.21493286,  602.63015994,  645.87687859,  629.752138  ,
        636.94759733,  671.89658763,  645.00106119,  651.90331312,
        670.71263903,  666.68027257,  688.73713396,  698.76730011,
        720.43978287,  796.13010587,  803.62924782,  807.54330665,
        867.35964459,  911.56438073,  894.45974647,  -89.13240595,
         34.59573548,   76.61669964,   91.9448575 ,  141.5233585 ,
        148.11845226,  135.97771966,  132.18181968,  160.54902275,
        197.22186967,  182.36104273,  197.43654712,  210.10228652,
        225.46574556,  221.7914616 ,  249.24362069,  286.00318882,
        331.92493631,  344.59908139,  505.82109937,  301.84880663,
        305.71765485,  373.5551134 ,  539.81247674,  695.31306022,
        798.77573778,   13.38283184,   23.99680891,   96.17908694,
        152.41893132,  192.66943342,  192.27759854,  219.17494849,
        241.65405111,  298.44378665,  365.57045736,  402.31855666,
       -250.77187008,  -91.48700757,  -38.54162814,   19.85042062,
         24.77285292,   58.20999915,   80.13844941,   90.61111209,
         92.29357535,  101.29545207,  135.98567529,  136.79043594,
        138.51392229,  163.12202729,  171.28725007,  160.62810848,
        173.9278636 ,  167.27798604,  167.86562382,  182.36631538,
        208.85263622,  190.61210644,  177.77282219,  223.29495289,
        265.45783158,  241.77314035,  272.75253811,  267.22058995,
        295.89108639,  369.56514031,  411.50260444,  366.12664052,
        362.85546671,  374.59364283,  395.07208356,  435.57517102,
        443.59186169,  620.34496346,  644.92999622,  716.12966982,
        712.40160009,  725.52056816,  713.93316545,  717.91289343,
        803.68733248,  770.19311624,  846.27084699,  809.69340158,
        807.1691595 ,  833.8228606 ,  783.19613944,  880.14086852,
        890.72452163,  920.25772656,  927.61035407,  947.51616468,
        383.5635105 ,  412.9323248 ,  451.29129927,  539.96640029,
        599.58148752,  574.21202976,  699.68727978,  689.87959129,
        726.40822496,  739.34856983,  810.28881195,  832.64029335,
        966.66723972, 1103.88858947, 1152.75248089, 1152.75248089,
       1265.84301875, -224.38232605, -199.39843558, -200.96951453,
       -178.04747934, -177.75895011, -180.77278307, -160.73343833,
       -163.38304118, -162.0175998 , -160.60811584, -145.53543881,
       -137.84341252,  -83.27975801,  -82.00569369])
```

In [29]:
```
y
```

Out[29]:
```
0      242.0
1      290.0
2      340.0
3      363.0
4      430.0
       ...
154     12.2
155     13.4
156     12.2
157     19.7
158     19.9
Name: Weight, Length: 159, dtype: float64
```

In [30]:
```python
from sklearn.metrics import r2_score

r2_score(y, y_pred)
```

Out[30]:
```
0.8852867046546207
```

In [31]:
```python
lr.score(X, y)
```

Out[31]:
```
0.8852867046546207
```

Statsmodels

In [32]:
```python
X
```

Out[32]:

	Length1	Length2	Length3	Height	Width
0	23.2	25.4	30.0	11.5200	4.0200
1	24.0	26.3	31.2	12.4800	4.3056
2	23.9	26.5	31.1	12.3778	4.6961
3	26.3	29.0	33.5	12.7300	4.4555
4	26.5	29.0	34.0	12.4440	5.1340
...	...	...	...	...	...
154	11.5	12.2	13.4	2.0904	1.3936
155	11.7	12.4	13.5	2.4300	1.2690
156	12.1	13.0	13.8	2.2770	1.2558
157	13.2	14.3	15.2	2.8728	2.0672
158	13.8	15.0	16.2	2.9322	1.8792

159 rows × 5 columns

In [33]:

```
x = sm.add_constant(X)

model = sm.OLS(y, x).fit()

model.summary()
```

Out[33]:

OLS Regression Results

Dep. Variable:	Weight	**R-squared:**	0.885
Model:	OLS	**Adj. R-squared:**	0.882
Method:	Least Squares	**F-statistic:**	236.2
Date:	Thu, 11 Nov 2021	**Prob (F-statistic):**	4.95e-70
Time:	00:42:54	**Log-Likelihood:**	-987.96
No. Observations:	159	**AIC:**	1988.
Df Residuals:	153	**BIC:**	2006.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-499.5870	29.572	-16.894	0.000	-558.010	-441.164
Length1	62.3552	40.209	1.551	0.123	-17.081	141.791
Length2	-6.5268	41.759	-0.156	0.876	-89.025	75.971
Length3	-29.0262	17.353	-1.673	0.096	-63.309	5.256
Height	28.2974	8.729	3.242	0.001	11.052	45.543
Width	22.4733	20.372	1.103	0.272	-17.773	62.720

Omnibus:	20.989	**Durbin-Watson:**	0.424
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	27.307
Skew:	0.792	**Prob(JB):**	1.18e-06
Kurtosis:	4.269	**Cond. No.**	315.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Análise do Erro da Regressão

In [34]:
```
model.resid
```

Out[34]:
```
0      -84.816128
1      -79.578593
2      -30.824180
3      -76.056139
4      -14.169168
          ...
154    172.808116
155    158.935439
156    150.043413
157    102.979758
158    101.905694
Length: 159, dtype: float64
```

In [35]:
```
y - y_pred # cálculo do resíduo
```

Out[35]:
```
0      -84.816128
1      -79.578593
2      -30.824180
3      -76.056139
4      -14.169168
          ...
154    172.808116
155    158.935439
156    150.043413
157    102.979758
158    101.905694
Name: Weight, Length: 159, dtype: float64
```

1. A média do erro deve ser zero

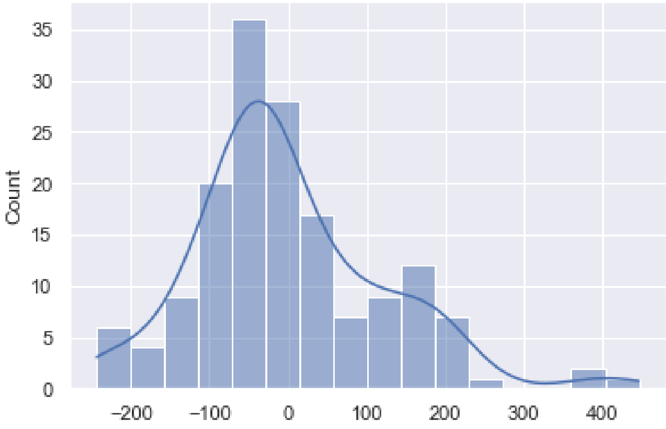
In [36]:
```
model.resid.mean()
```

Out[36]:
```
8.231570561447199e-14
```

2. A distribuição dos erros deve seguir uma distribuição normal

In [37]:
```
sns.histplot(model.resid, kde=True)
```

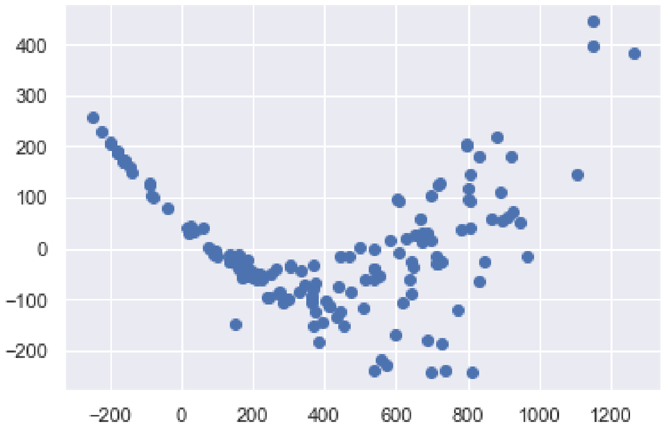
Out[37]:
```
<AxesSubplot:ylabel='Count'>
```



3. Erro deve ser descorrelacionado

In [38]:
```
plt.scatter(y_pred, model.resid)
```

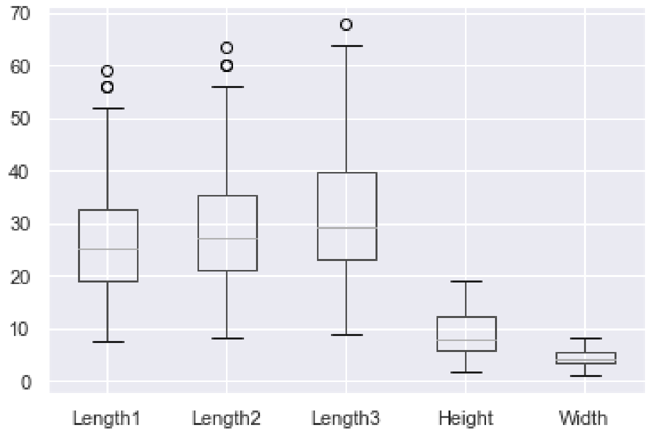
Out[38]: <matplotlib.collections.PathCollection at 0x250819fe580>



Removendo Outliers

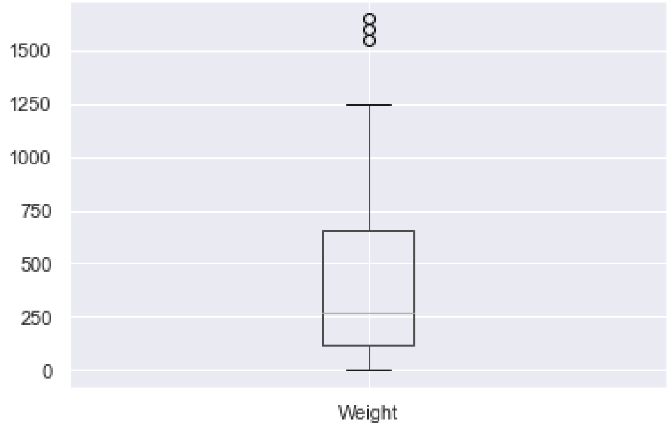
In [39]:
```
df.drop(['Species', 'Weight'], axis=1).boxplot()
```

Out[39]: <AxesSubplot:>



In [40]:
```
df[['Weight']].boxplot()
```

Out[40]:
```
<AxesSubplot:>
```



In [41]:
```
def calc_min_and_max_range(dados_coluna):
    Q1 = dados_coluna.quantile(q=0.25) # Primeiro quartil
    Q3 = dados_coluna.quantile(q=0.75) # Terceiro quartil

    IQR = Q3 - Q1 # Intervalo interquartílico

    inf = Q1 - 1.5 * IQR
    sup = Q3 + 1.5 * IQR

    return inf, sup
```

In [42]:
```
df.columns
```

Out[42]:
```
Index(['Species', 'Weight', 'Length1', 'Length2', 'Length3', 'Height',
       'Width'],
      dtype='object')
```

In [43]:
```
colunas = ['Weight', 'Length1', 'Length2', 'Length3']
```

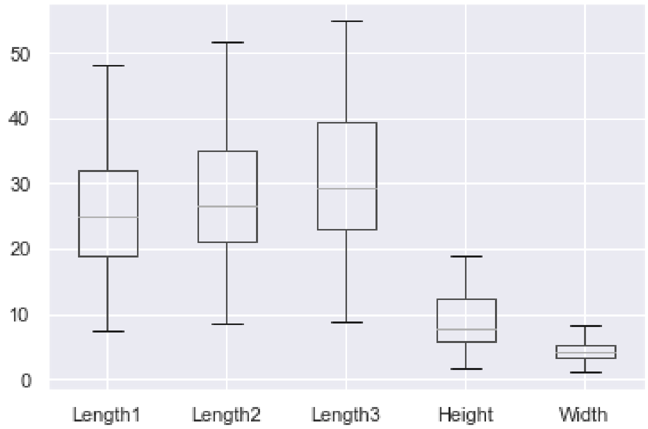
In [44]:
```
for coluna in colunas:
    inf, sup = calc_min_and_max_range(df[coluna])

    df = df[(df[coluna] > inf) & (df[coluna] < sup)]
```

In [45]:
```
df
df.shape
```

Out[45]:
```
(155, 7)
```

In [46]:
```
df.drop(['Species', 'Weight'], axis=1).boxplot()
```

Out[46]:
```
<AxesSubplot:>
```



In [47]:
```
df[['Weight']].boxplot()
```

Out[47]:
```
<AxesSubplot:>
```

Modelo de Regressão sem Outliers

In [48]:
```
X = df.drop(['Species', 'Weight'], axis=1)
y = df['Weight']
```

In [49]:
```
X
```

Out[49]:

	Length1	Length2	Length3	Height	Width
0	23.2	25.4	30.0	11.5200	4.0200
1	24.0	26.3	31.2	12.4800	4.3056
2	23.9	26.5	31.1	12.3778	4.6961
3	26.3	29.0	33.5	12.7300	4.4555
4	26.5	29.0	34.0	12.4440	5.1340
...	...	...	...	...	...
154	11.5	12.2	13.4	2.0904	1.3936
155	11.7	12.4	13.5	2.4300	1.2690
156	12.1	13.0	13.8	2.2770	1.2558
157	13.2	14.3	15.2	2.8728	2.0672
158	13.8	15.0	16.2	2.9322	1.8792

155 rows × 5 columns

In [50]:
```
y
```

Out[50]:
```
0      242.0
1      290.0
2      340.0
3      363.0
4      430.0
       ...
154     12.2
155     13.4
156     12.2
157     19.7
158     19.9
Name: Weight, Length: 155, dtype: float64
```

Sklearn

In [51]:
```python
lr = LinearRegression()
```

In [52]:
```python
lr.fit(X, y)
```

Out[52]:
```
LinearRegression()
```

In [53]:
```python
lr.score(X, y)
```

Out[53]:
```
0.9069290835432218
```

In [54]:
```python
y_pred = lr.predict(X)
```

In [55]:
```python
r2_score(y, y_pred)
```

Out[55]:
```
0.9069290835432218
```

Statsmodels

In [56]:
```python
x = sm.add_constant(X)

model = sm.OLS(y, x).fit()
```

In [57]:
```python
model.summary()
```

Out[57]:

OLS Regression Results

Dep. Variable:	Weight	**R-squared:**	0.907
Model:	OLS	**Adj. R-squared:**	0.904
Method:	Least Squares	**F-statistic:**	290.4
Date:	Thu, 11 Nov 2021	**Prob (F-statistic):**	6.45e-75
Time:	00:43:33	**Log-Likelihood:**	-925.92
No. Observations:	155	**AIC:**	1864.
Df Residuals:	149	**BIC:**	1882.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-407.8029	25.200	-16.183	0.000	-457.598	-358.008
Length1	117.1278	32.241	3.633	0.000	53.419	180.837
Length2	-58.9144	33.359	-1.766	0.079	-124.832	7.003
Length3	-38.8324	13.769	-2.820	0.005	-66.041	-11.624
Height	39.4419	7.022	5.617	0.000	25.566	53.318
Width	56.3974	16.515	3.415	0.001	23.763	89.032

Omnibus:	9.828	**Durbin-Watson:**	0.544
Prob(Omnibus):	0.007	**Jarque-Bera (JB):**	10.392
Skew:	0.609	**Prob(JB):**	0.00554
Kurtosis:	2.646	**Cond. No.**	307.

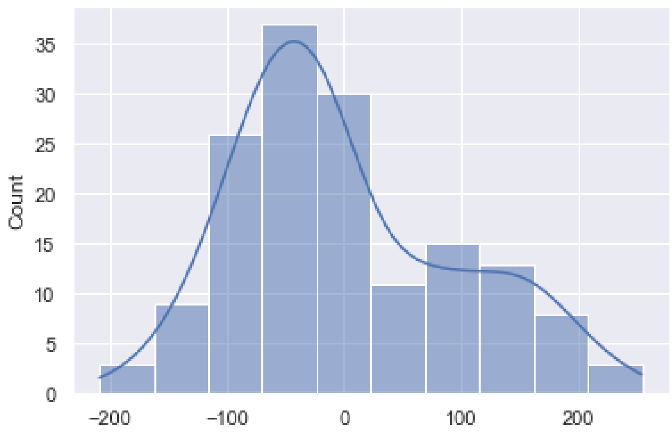
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [58]:

```
sns.histplot(model.resid, kde=True)
```

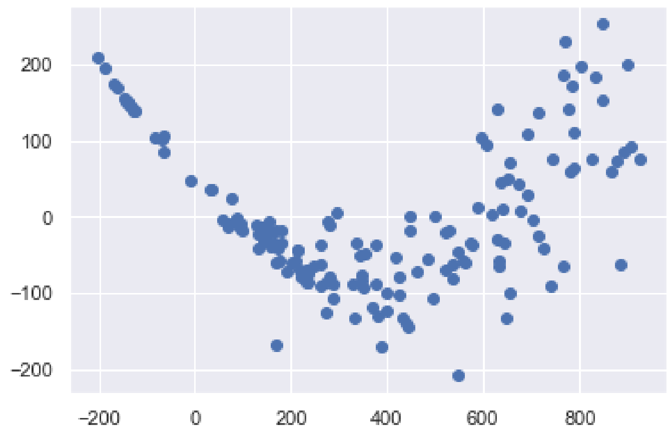
Out[58]:

```
<AxesSubplot:ylabel='Count'>
```



In [59]:
```
plt.scatter(y_pred, model.resid)
```

Out[59]:
```
<matplotlib.collections.PathCollection at 0x25082462520>
```



Considerando os Dados Categóricos

In [60]:
```
df['Species'].unique()
```

Out[60]:
```
array(['Bream', 'Roach', 'Whitefish', 'Parkki', 'Perch', 'Pike', 'Smelt'],
      dtype=object)
```

In [61]:
```
pd.get_dummies(df, drop_first=True)
```

Out[61]:

	Weight	Length1	Length2	Length3	Height	Width	Species_Parkki	Species_Perch	Species_Pike	Species_Roach	Species_Smelt	Species_Whitefish
0	242.0	23.2	25.4	30.0	11.5200	4.0200	0	0	0	0	0	0
1	290.0	24.0	26.3	31.2	12.4800	4.3056	0	0	0	0	0	0
2	340.0	23.9	26.5	31.1	12.3778	4.6961	0	0	0	0	0	0
3	363.0	26.3	29.0	33.5	12.7300	4.4555	0	0	0	0	0	0
4	430.0	26.5	29.0	34.0	12.4440	5.1340	0	0	0	0	0	0
...	...	...	...	...	...	...	...	...	...	...	...	...
154	12.2	11.5	12.2	13.4	2.0904	1.3936	0	0	0	0	1	0
155	13.4	11.7	12.4	13.5	2.4300	1.2690	0	0	0	0	1	0
156	12.2	12.1	13.0	13.8	2.2770	1.2558	0	0	0	0	1	0
157	19.7	13.2	14.3	15.2	2.8728	2.0672	0	0	0	0	1	0
158	19.9	13.8	15.0	16.2	2.9322	1.8792	0	0	0	0	1	0

155 rows × 12 columns

In [62]:
```
df['species'] = df['Species'].map({
    'Bream': 0,
    'Roach': 1,
    'Whitefish': 2,
    'Parkki': 3,
    'Perch': 4,
    'Pike': 5,
    'Smelt': 6
})
```

In [63]:
```
df.head()
```

Out[63]:

	Species	Weight	Length1	Length2	Length3	Height	Width	species
0	Bream	242.0	23.2	25.4	30.0	11.5200	4.0200	0
1	Bream	290.0	24.0	26.3	31.2	12.4800	4.3056	0
2	Bream	340.0	23.9	26.5	31.1	12.3778	4.6961	0
3	Bream	363.0	26.3	29.0	33.5	12.7300	4.4555	0
4	Bream	430.0	26.5	29.0	34.0	12.4440	5.1340	0

In [64]:
```
X = df.drop(['Species', 'Weight'], axis=1)
```

In [65]:
```
X.head()
```

Out[65]:

	Length1	Length2	Length3	Height	Width	species
0	23.2	25.4	30.0	11.5200	4.0200	0
1	24.0	26.3	31.2	12.4800	4.3056	0
2	23.9	26.5	31.1	12.3778	4.6961	0
3	26.3	29.0	33.5	12.7300	4.4555	0
4	26.5	29.0	34.0	12.4440	5.1340	0

In [66]:
```
y = df['Weight']
```

Avaliando o modelo com a presença dos Dados Categóricos

Sklearn

In [67]:
```
lr = LinearRegression()
```

In [68]:
```
lr.fit(X, y)
```

Out[68]:
```
LinearRegression()
```

In [69]:
```
lr.coef_
```

Out[69]:
```
array([ 82.46690596, -96.25250448,  23.16478763,  36.16223965,
        91.38483721,  46.69845839])
```

In [70]:
```
lr.intercept_
```

Out[70]:
```
-632.5641109755825
```

In [71]:
```
lr.score(X, y)
```

Out[71]:
```
0.9241405510725724
```

In [72]:
```
y_pred = lr.predict(X)
```

In [73]:
```
r2_score(y, y_pred)
```

Out[73]:
```
0.9241405510725724
```

Statsmodels

In [74]:
```
x = sm.add_constant(X)

model = sm.OLS(y, x).fit()

model.summary()
```

Out[74]:

OLS Regression Results

Dep. Variable:	Weight	**R-squared:**	0.924
Model:	OLS	**Adj. R-squared:**	0.921
Method:	Least Squares	**F-statistic:**	300.5
Date:	Thu, 11 Nov 2021	**Prob (F-statistic):**	3.22e-80
Time:	00:43:58	**Log-Likelihood:**	-910.08
No. Observations:	155	**AIC:**	1834.
Df Residuals:	148	**BIC:**	1855.
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-632.5641	45.006	-14.055	0.000	-721.501	-543.627
Length1	82.4669	29.812	2.766	0.006	23.555	141.379
Length2	-96.2525	30.898	-3.115	0.002	-157.310	-35.195
Length3	23.1648	16.433	1.410	0.161	-9.309	55.639
Height	36.1622	6.386	5.662	0.000	23.542	48.782
Width	91.3848	16.133	5.665	0.000	59.504	123.265
species	46.6985	8.059	5.795	0.000	30.773	62.624

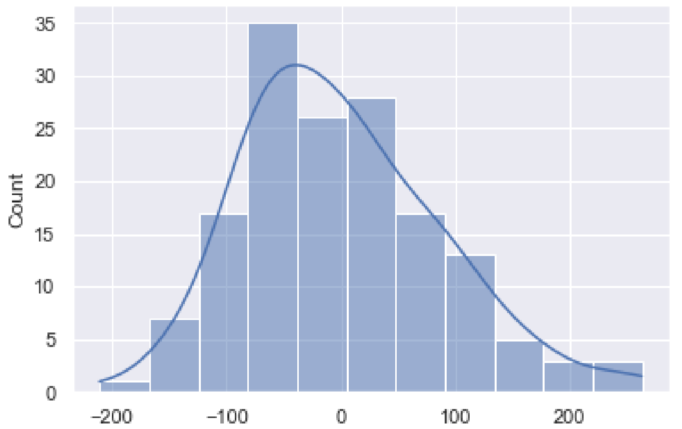
Omnibus:	8.560	**Durbin-Watson:**	0.693
Prob(Omnibus):	0.014	**Jarque-Bera (JB):**	8.485
Skew:	0.561	**Prob(JB):**	0.0144
Kurtosis:	3.236	**Cond. No.**	360.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Análise do Erro

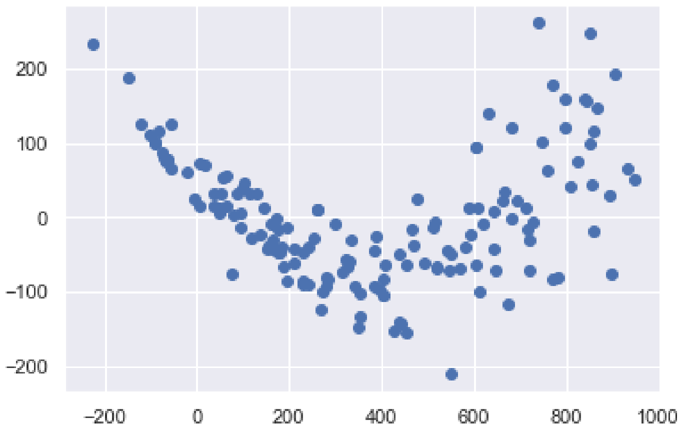
In [75]:
```python
sns.histplot(model.resid, kde=True)
```

Out[75]: <AxesSubplot:ylabel='Count'>



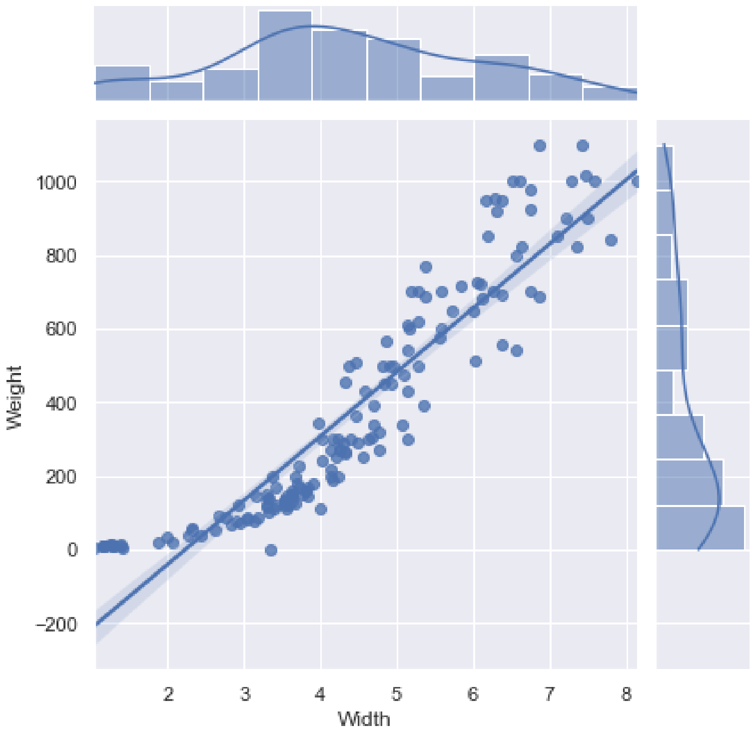
In [76]:
```python
plt.scatter(y_pred, model.resid)
```

Out[76]: <matplotlib.collections.PathCollection at 0x2508254b550>



In [77]:
```python
sns.jointplot(data=df, x='Width', y='Weight', kind='reg')
```

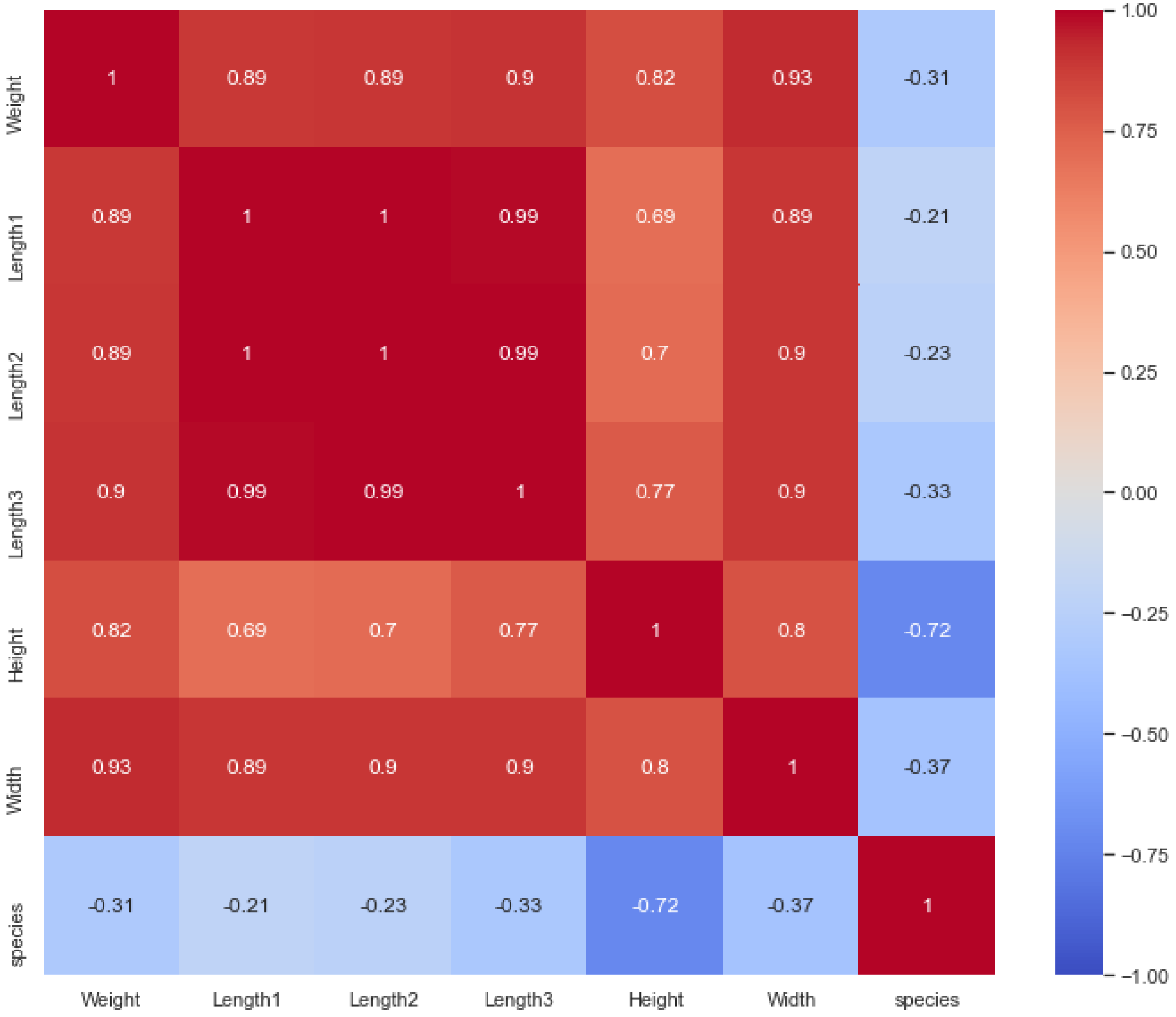
Out[77]: <seaborn.axisgrid.JointGrid at 0x25082559fd0>



Remoção de Variáveis Desnecessárias

In [78]:
```
plt.figure(figsize=(12,10))
sns.heatmap(df.corr(), cmap='coolwarm', annot=True, vmin=-1, vmax=1)
```

Out[78]: <AxesSubplot:>



In [79]:
```
df_minimal = df.drop(['Length2', 'Length3'], axis=1)
```

In [80]:
```
df_minimal.head()
```

Out[80]:

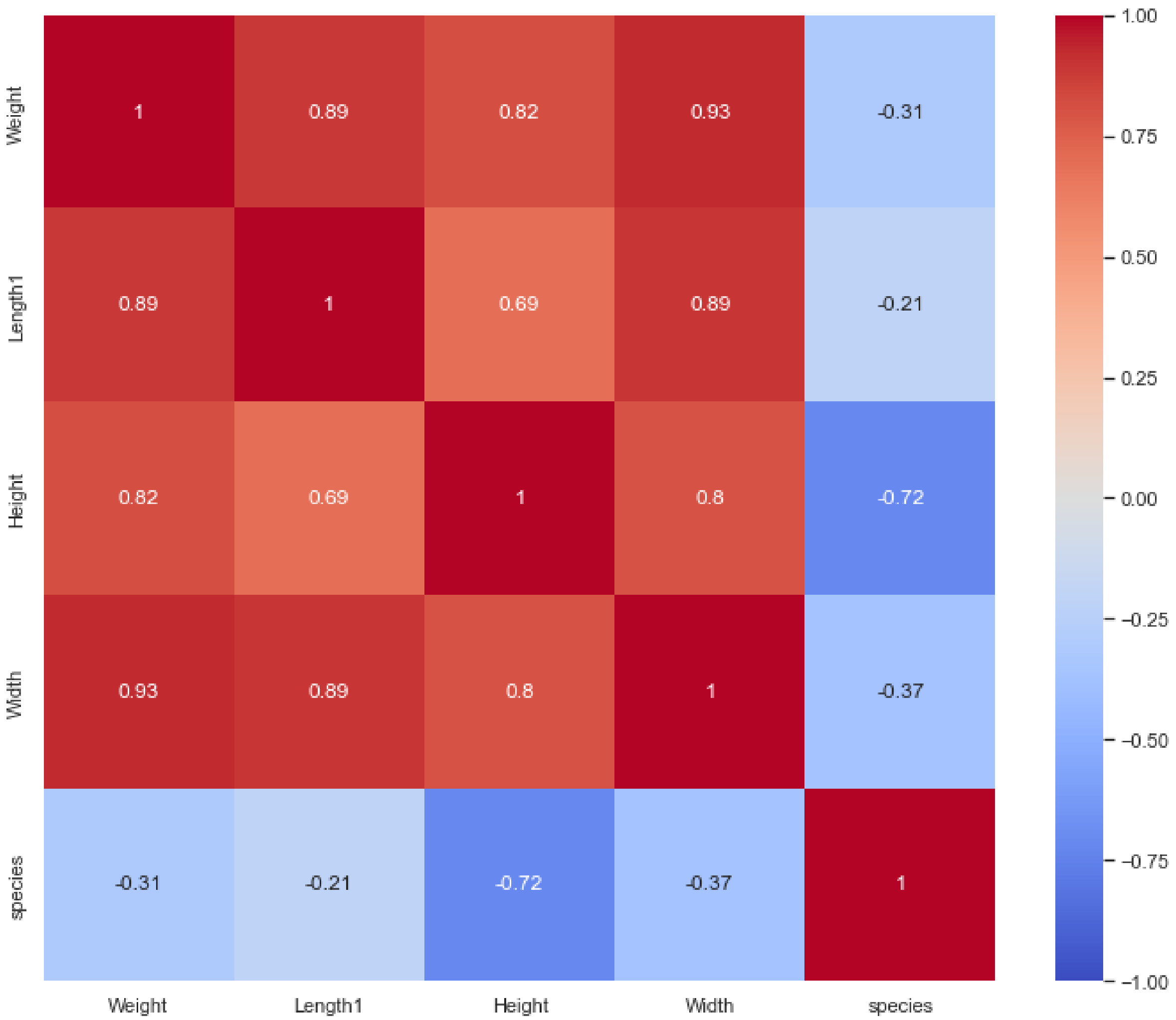
	Species	Weight	Length1	Height	Width	species
0	Bream	242.0	23.2	11.5200	4.0200	0
1	Bream	290.0	24.0	12.4800	4.3056	0
2	Bream	340.0	23.9	12.3778	4.6961	0
3	Bream	363.0	26.3	12.7300	4.4555	0
4	Bream	430.0	26.5	12.4440	5.1340	0

In [107…

```
plt.figure(figsize=(12,10))
sns.heatmap(df_minimal.corr(), cmap='coolwarm', annot=True, vmin=-1, vmax=1)
```

Out[107]:

```
<AxesSubplot:>
```



In [81]:

```
X = df_minimal.drop(['Weight', 'Species'], axis=1)
y = df_minimal['Weight']
```

Sklearn

In [82]:

```
df_minimal.head()
```

Out[82]:

	Species	Weight	Length1	Height	Width	species
0	Bream	242.0	23.2	11.5200	4.0200	0
1	Bream	290.0	24.0	12.4800	4.3056	0
2	Bream	340.0	23.9	12.3778	4.6961	0
3	Bream	363.0	26.3	12.7300	4.4555	0
4	Bream	430.0	26.5	12.4440	5.1340	0

In [83]:
```
lr = LinearRegression()
```

In [84]:
```
lr.fit(X, y)
```

Out[84]:
```
LinearRegression()
```

In [85]:
```
lr.coef_
```

Out[85]:
```
array([ 8.38798067, 38.62601654, 71.33662497, 40.94951776])
```

In [86]:
```
lr.intercept_
```

Out[86]:
```
-616.1785382243611
```

In [87]:
```
lr.score(X, y)
```

Out[87]:
```
0.9191610055608289
```

Statsmodels

In [88]:
```
X.head()
```

Out[88]:

	Length1	Height	Width	species
0	23.2	11.5200	4.0200	0
1	24.0	12.4800	4.3056	0
2	23.9	12.3778	4.6961	0
3	26.3	12.7300	4.4555	0
4	26.5	12.4440	5.1340	0

In [89]:
```
x = sm.add_constant(X)
```

```python
model = sm.OLS(y, x).fit()

model.summary()
```

Out[89]:

OLS Regression Results

Dep. Variable:	Weight	**R-squared:**	0.919
Model:	OLS	**Adj. R-squared:**	0.917
Method:	Least Squares	**F-statistic:**	426.4
Date:	Thu, 11 Nov 2021	**Prob (F-statistic):**	8.25e-81
Time:	00:44:26	**Log-Likelihood:**	-915.00
No. Observations:	155	**AIC:**	1840.
Df Residuals:	150	**BIC:**	1855.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-616.1785	34.377	-17.924	0.000	-684.104	-548.253
Length1	8.3880	1.901	4.413	0.000	4.632	12.144
Height	38.6260	4.395	8.788	0.000	29.941	47.311
Width	71.3366	11.873	6.008	0.000	47.877	94.796
species	40.9495	6.243	6.560	0.000	28.615	53.284

Omnibus:	7.998	**Durbin-Watson:**	0.630
Prob(Omnibus):	0.018	**Jarque-Bera (JB):**	8.391
Skew:	0.566	**Prob(JB):**	0.0151
Kurtosis:	2.875	**Cond. No.**	140.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [90]:
```python
model.resid
```

Out[90]:
```
0      -68.167556
1      -84.332657
2      -57.403232
3      -50.974877
4      -23.007332
          ...
154    106.061108
155    101.354660
156    103.650892
157     21.028195
158     27.312307
Length: 155, dtype: float64
```

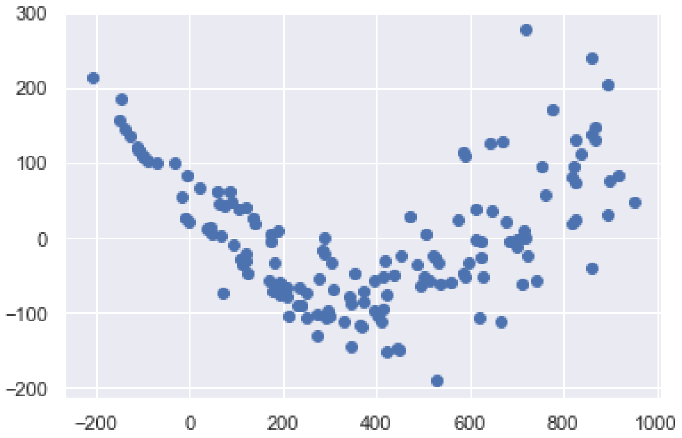
In [91]:
```
y_est = lr.predict(X)
```

In [92]:
```
y_est
```

Out[92]:
```
array([ 310.16755632,  374.33265682,  397.40323192,  413.97487659,
        453.00733204,  485.52995708,  532.86732025,  439.29012903,
        501.82357143,  526.18265575,  537.07166278,  523.12375578,
        470.97316402,  530.54322831,  576.92441571,  624.79490425,
        589.85672805,  586.5836176 ,  613.07854642,  611.50175402,
        628.02828085,  648.07932379,  624.28743343,  685.38595968,
        677.40426383,  714.29573605,  718.66804681,  713.520606  ,
        753.49813303,  868.08794455,  823.96454112,  824.76371276,
        893.3672191 ,  898.67281725,  837.74029923, -145.04278855,
        -30.88456124,   -5.88604729,   20.84436192,   75.85810384,
         73.34562688,   65.45287002,   58.68468898,   87.91672307,
        106.05379254,  140.92411645,   91.11326004,  120.04305249,
        173.92063746,  135.45459912,  190.27479014,  175.09530938,
        289.74447449,  305.95071904,  420.591803  ,  290.38775843,
        285.56883784,  353.48151919,  587.88680125,  671.62218962,
        721.24946268,   50.3640604 ,   45.75639218,  120.11027626,
        185.78433818,  239.39953804,  230.82650397,  272.25812506,
        274.04723852,  345.46110649,  423.49781582,  449.12137737,
       -207.45049724,  -68.64194177,  -15.42977471,   38.55482355,
         66.36367563,  122.17706201,  112.17480776,  127.1722779 ,
        112.00567409,   94.32332155,  180.47146654,  171.50650558,
        188.49683685,  208.19968205,  194.46365694,  184.6914826 ,
        202.84571037,  193.76859648,  213.68113939,  204.29101838,
        240.44496986,  210.15108307,  184.00359642,  236.0682991 ,
        279.44090103,  251.25855359,  294.44642811,  252.0437799 ,
        300.55832641,  329.80931009,  446.95862491,  346.75889692,
        342.29372941,  367.71709212,  366.85257071,  396.91606498,
        415.35552681,  620.17569833,  667.86534501,  820.0989604 ,
        742.75515548,  723.46406939,  702.41317605,  700.35807971,
        818.49718875,  712.1770829 ,  859.21528969,  824.83691226,
        826.46510716,  868.53823349,  761.87613353,  859.13999094,
        861.50858102,  896.35366887,  952.48146887,  916.61686981,
        296.08204213,  371.55530177,  404.92380171,  410.12558549,
        494.39808925,  421.29411413,  513.63823937,  505.80085115,
        591.59759182,  558.96542984,  599.31310016,  644.50408762,
        778.72464597, -150.57804551, -127.68063534, -137.07863975,
        -99.97915787, -109.15318338, -111.79942405,  -98.60115289,
       -107.01174097, -107.788281  ,  -93.86110838,  -87.9546605 ,
        -91.45089221,   -1.32819531,   -7.41230702])
```

In [93]:
```
plt.scatter(y_est, model.resid)
```

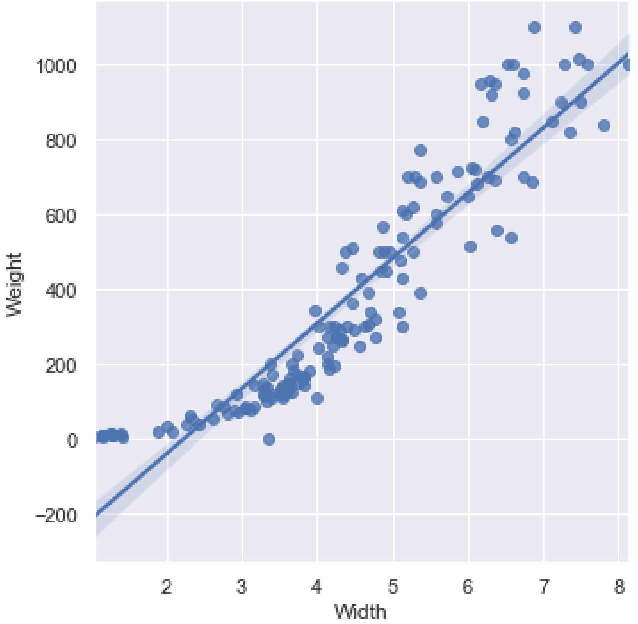
Out[93]:
```
<matplotlib.collections.PathCollection at 0x25083016d00>
```



Transformação de Variáveis

In [94]:
```
sns.lmplot(data=df_minimal, x='Width', y='Weight')
```

Out[94]:
```
<seaborn.axisgrid.FacetGrid at 0x250830573d0>
```



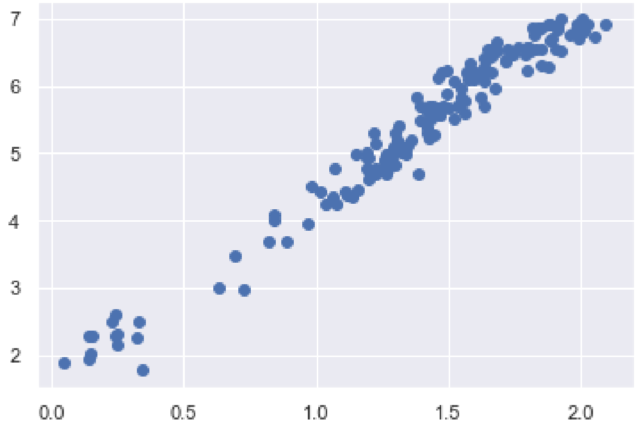
Transformação log & Transformação log-log (duplo)

In [95]:
```
df_minimal.shape
```
Out[95]:
```
(155, 6)
```
In [96]:
```
df_minimal = df_minimal[df_minimal['Weight'] != 0]
```
In [97]:
```
df_minimal.shape
```
Out[97]:
```
(154, 6)
```
In [98]:
```
df_minimal['Width']
```
Out[98]:
```
0      4.0200
1      4.3056
2      4.6961
3      4.4555
4      5.1340
        ...
154    1.3936
155    1.2690
156    1.2558
157    2.0672
158    1.8792
Name: Width, Length: 154, dtype: float64
```
In [99]:
```
np.log(df_minimal['Width'])
```
Out[99]:
```
0      1.391282
1      1.459917
2      1.546732
3      1.494139
4      1.635885
         ...
154    0.331890
155    0.238229
156    0.227773
157    0.726195
158    0.630846
Name: Width, Length: 154, dtype: float64
```
In [100…
```
np.log(df_minimal['Weight'])
```

Out[100]:
```
0      5.488938
1      5.669881
2      5.828946
3      5.894403
4      6.063785
         ...
154    2.501436
155    2.595255
156    2.501436
157    2.980619
158    2.990720
Name: Weight, Length: 154, dtype: float64
```

In [101…
```
plt.scatter(np.log(df_minimal['Width']), np.log(df_minimal['Weight']))
```

Out[101]:
```
<matplotlib.collections.PathCollection at 0x250830ee7f0>
```

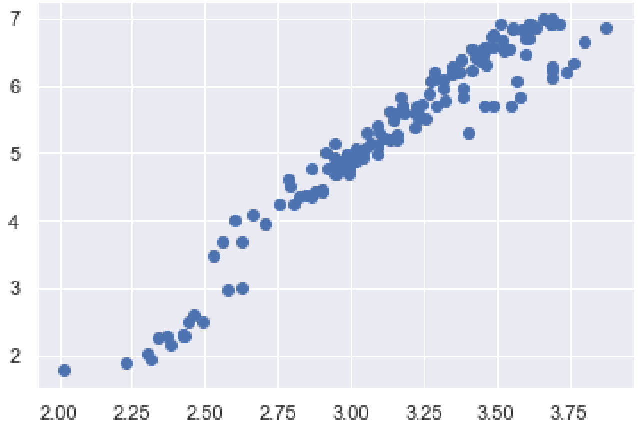


In [102…
```
-np.inf
```

Out[102]:
```
-inf
```

In [103…
```
plt.scatter(np.log(df_minimal['Length1']), np.log(df_minimal['Weight']))
```

Out[103]:
```
<matplotlib.collections.PathCollection at 0x250831480d0>
```

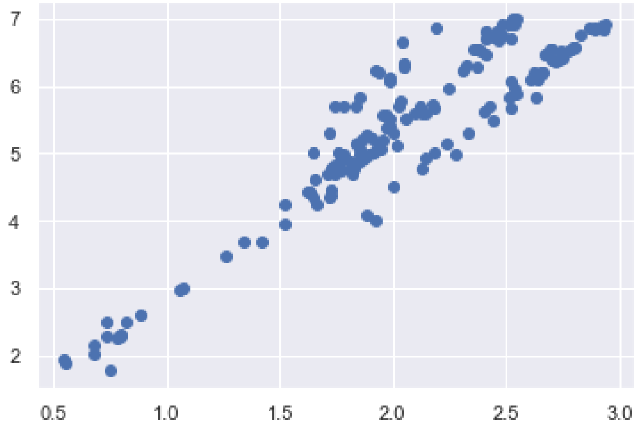


In [104…

```
plt.scatter(np.log(df_minimal['Height']), np.log(df_minimal['Weight']))
```

Out[104]:

```
<matplotlib.collections.PathCollection at 0x2508319b6a0>
```



In [105…

```
df['species']
```

Out[105]:

```
0      0
1      0
2      0
3      0
4      0
      ..
154    6
155    6
156    6
157    6
158    6
Name: species, Length: 155, dtype: int64
```

In [106…
```
df_log = df_minimal.copy()
```

In [107…
```
df_log['Weight'] = np.log(df_log['Weight'])
df_log['Width'] = np.log(df_log['Width'])
df_log['Height'] = np.log(df_log['Height'])
df_log['Length1'] = np.log(df_log['Length1'])
```

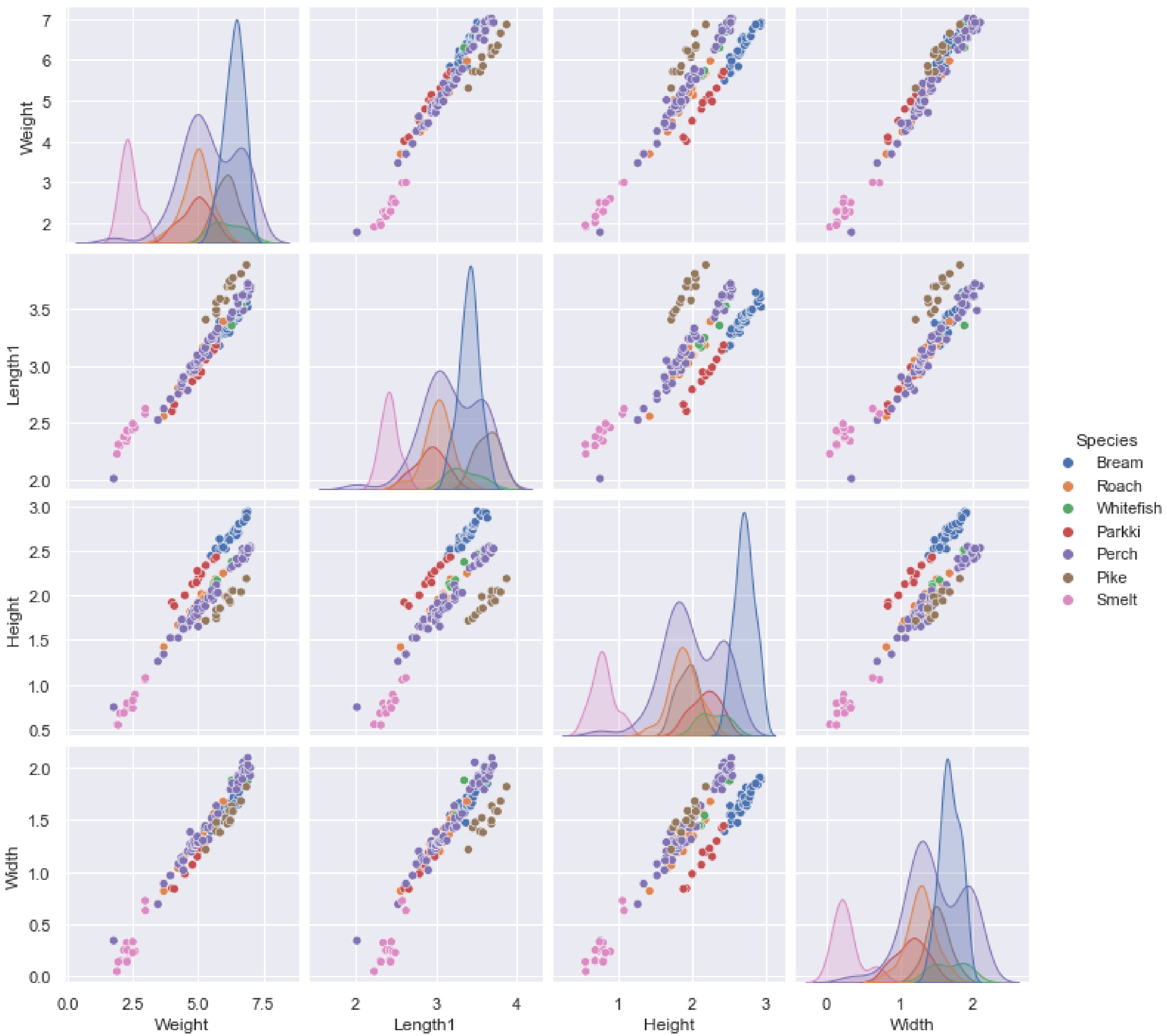
In [108…
```
df_log.head()
```

Out[108]:

	Species	Weight	Length1	Height	Width	species
0	Bream	5.488938	3.144152	2.444085	1.391282	0
1	Bream	5.669881	3.178054	2.524127	1.459917	0
2	Bream	5.828946	3.173878	2.515905	1.546732	0
3	Bream	5.894403	3.269569	2.543961	1.494139	0
4	Bream	6.063785	3.277145	2.521239	1.635885	0

In [109…
```
sns.pairplot(df_log.drop(['species'], axis=1), hue='Species')
```

Out[109]:
```
<seaborn.axisgrid.PairGrid at 0x25082ff27f0>
```



In [110…
```
X = df_log.drop(['Species', 'Weight'], axis=1)
y = df_log['Weight']
```

In [111…
```
X.head()
```

Out[111]:

	Length1	Height	Width	species
0	3.144152	2.444085	1.391282	0
1	3.178054	2.524127	1.459917	0
2	3.173878	2.515905	1.546732	0
3	3.269569	2.543961	1.494139	0
4	3.277145	2.521239	1.635885	0

Sklearn

In [112…
```
lr = LinearRegression().fit(X, y)
```

In [113…
```
lr.coef_
```

Out[113]:
```
array([1.35293214, 0.65645409, 0.98258257, 0.00422266])
```

In [114…
```
lr.score(X, y)
```

Out[114]:
```
0.994751565225584
```

Statsmodels

In [115…
```
x = sm.add_constant(X)
model = sm.OLS(y, x).fit()
```

In [116…
```
model.summary()
```

Out[116]:

OLS Regression Results

Dep. Variable:	Weight	**R-squared:**	0.995
Model:	OLS	**Adj. R-squared:**	0.995
Method:	Least Squares	**F-statistic:**	7060.
Date:	Thu, 11 Nov 2021	**Prob (F-statistic):**	1.04e-168
Time:	00:45:04	**Log-Likelihood:**	145.05
No. Observations:	154	**AIC:**	-280.1
Df Residuals:	149	**BIC:**	-264.9
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-1.6468	0.123	-13.388	0.000	-1.890	-1.404
Length1	1.3529	0.060	22.477	0.000	1.234	1.472
Height	0.6565	0.050	13.150	0.000	0.558	0.755
Width	0.9826	0.062	15.838	0.000	0.860	1.105
species	0.0042	0.007	0.591	0.555	-0.010	0.018

Omnibus:	25.328	**Durbin-Watson:**	1.648
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	57.379
Skew:	-0.688	**Prob(JB):**	3.47e-13
Kurtosis:	5.655	**Cond. No.**	95.1

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [ ]:

```

```

Removendo as espécies

Sklearn

Statsmodels

Na prática

---

1. Processo de treinamento do nosso modelo
2. Processo de teste do nosso modelo (validação)

Divisão treino-teste

- 70% dos dados para treino
- 30% dos dados para teste

In [117…
```
from sklearn.model_selection import train_test_split
```

In [118…
```
X.head()
```

Out[118]:

	Length1	Height	Width	species
0	3.144152	2.444085	1.391282	0
1	3.178054	2.524127	1.459917	0
2	3.173878	2.515905	1.546732	0
3	3.269569	2.543961	1.494139	0
4	3.277145	2.521239	1.635885	0

In [119…
```
y.head()
```

Out[119]:
```
0    5.488938
1    5.669881
2    5.828946
3    5.894403
4    6.063785
Name: Weight, dtype: float64
```

In [120…
```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=101)
```

In [121…
```
X_train
```

Out[121]:

	Length1	Height	Width	species
65	2.912351	2.185242	1.191738	3
99	3.135494	1.862451	1.303863	4
51	3.161247	1.958206	1.362514	1
21	3.446808	2.772189	1.680902	0
22	3.449988	2.742303	1.663945	0
...	...	...	...	...
64	2.862201	2.127303	1.070933	3
71	3.178054	2.430802	1.443147	3
82	2.944439	1.739150	1.268355	4
11	3.356897	2.665240	1.571653	0
96	3.091042	1.986915	1.314530	4

107 rows × 4 columns

In [122…
```
X_test.shape
```
Out[122]:
```
(47, 4)
```

In [123…
```
y_train
```
Out[123]:
```
65    5.010635
99    5.192957
51    5.192957
21    6.529419
22    6.429719
        ...
64    4.787492
71    5.703782
82    4.700480
11    6.214608
96    5.416100
Name: Weight, Length: 107, dtype: float64
```

In [124…
```
y_test.shape
```
Out[124]:
```
(47,)
```

In [125…
```
lr = LinearRegression().fit(X_train, y_train)
```

In [126…
```
lr.score(X_train, y_train)
```

Out[126]:
```
0.9952021736203663
```

In [127…
```
y_pred = lr.predict(X_test)
```

In [128…
```
y_test
```

```
Out[128]: 81     4.442651
107    5.703782
117    6.476972
130    5.703782
32     6.829794
78     4.356709
39     4.787492
92     5.010635
139    6.646391
135    6.234411
42     4.787492
62     4.094345
16     6.551080
97     4.976734
66     4.941642
33     6.882437
4      6.063785
1      5.669881
79     4.382027
85     4.867534
140    6.856462
102    5.703782
109    6.242223
54     5.966147
101    5.384495
31     6.861711
95     5.135798
49     5.081404
119    6.745236
93     4.976734
124    6.907755
43     5.010635
70     5.609472
69     5.298317
56     5.598422
98     5.236442
134    6.122493
25     6.586172
67     5.135798
145    1.902108
75     3.941582
86     4.787492
87     4.787492
113    6.551080
128    5.298317
90     4.700480
44     4.976734
Name: Weight, dtype: float64
```

```
In [129…  np.exp(y_pred)
```

Out[129]:
```
array([  78.1343844 ,  288.14783254,  722.30252206,  297.62647471,
       1112.95920841,   81.1428284 ,  117.01444537,  146.38062859,
        654.67370296,  430.4835345 ,  112.15438134,   56.2929047 ,
        589.69124141,  162.25742895,  139.411906  , 1154.71669054,
        425.71211499,  313.07661826,   86.17408435,  126.60546482,
        921.30580019,  314.34227501,  536.54014054,  427.46001974,
        224.77450991,  966.82953868,  151.65152768,  158.3938401 ,
        914.74903269,  135.71999216, 1032.70877381,  132.22371138,
        269.70437156,  204.01607314,  236.13539951,  190.46667443,
        435.49209381,  767.76995838,  154.2593666 ,    5.91819643,
         54.0103139 ,  123.79930529,  121.44585523,  720.31730078,
        194.49905152,  136.37081301,  137.37494339])
```

In [ ]: