Character recognition with Hopfield networks

Alejandro Rodríguez Salamanca - r0650814 - Erasmus

Data preparation

Before starting to solve the tasks, the dataset to use must be constructed. For this exercise, we are asked to pre-pend the lowercase characters of our first and last name to the set of capital characters given in `prprob`. As my name is Alejandro Rodriguez, the characters I have to pre-pend are: `a, l, e, j, n, d, r, o, i, g, u, z`. Once this is done, it's time to solve the three proposed problems.

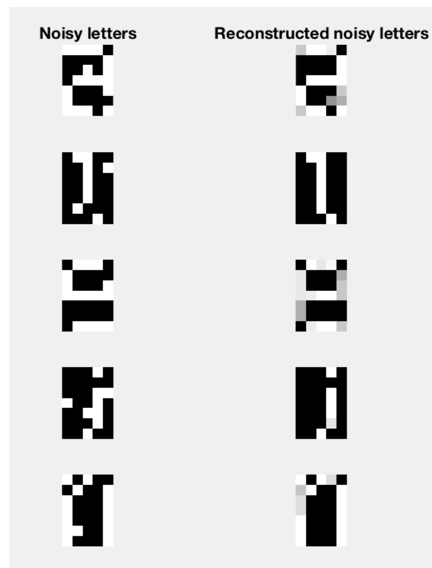
Task 1: Retrieve the first 5 characters.

The first 5 characters of the alphabet are `a, l, e, j` and `n`, this is, the first five characters that appear in my first name and last name. After loading this characters, the pixels whose value is 0 must be changed by -1. This has to be done because Hopfield nets normally have units that take on values of 1 or -1 as convention.

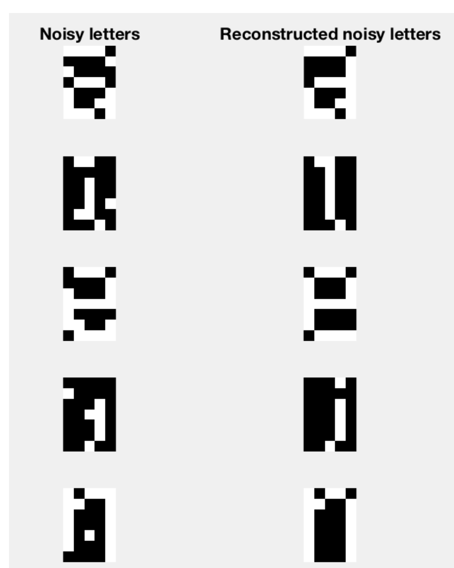
With this five characters as attractor states, a Hopfield neural network is created. Then, three random pixels of each character are inverted (from 1 to -1 and viceversa). The objective of this inversion is to check if the network is able to reconstruct the distorsioned characters.

Executing the neural network with the noisy characters as input shows the following results:

- With a small number of steps: it can be seen that the characters are being attracted by the attractor states, but they still contain some noise.



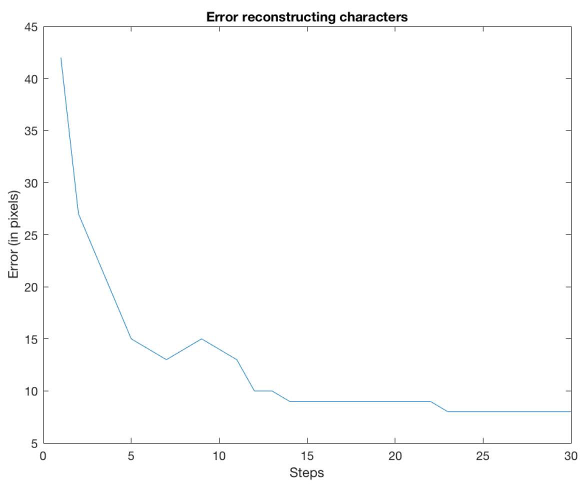
- When the number of steps is increased: with just 5 steps, the characters are perfectly reconstructed.



Sporious patters are local energy minima that are created during training, in addition to the intended target patterns. They are activity patterns that have not been explicitly embedded in the synaptic matrix, but are nonetheless stable. They are in other words "unwanted" attractor states that, by virtue of a finite overlap with the "wanted" attractor states, come about as a local minimum in the energy function. In this case, the existence of sporious patters is not noticeable.

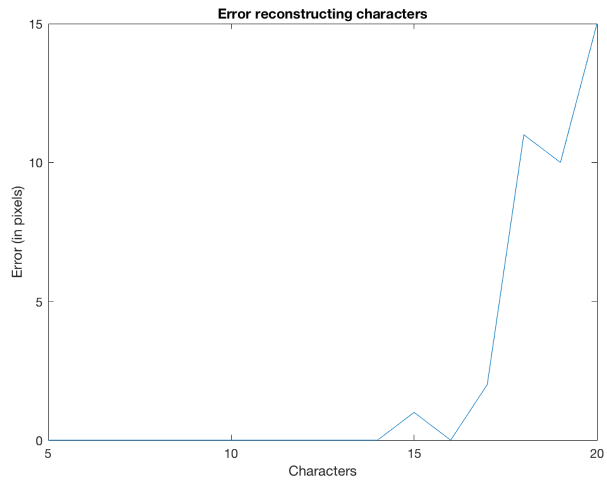
Task 2: Critical loading capacity

A Hopfield neural network exceed the loading capacity when the number of learned patterns over number of units p/N is lower than the critical capacity $\alpha \approx 0.138$. First, a number $P = 20$ characters is selected, and the error in the reconstruction is calculated as a function of the number of steps used to reconstruct the character.



Appendix

The purpose of this plot is to see how the approximation becomes more accurate when the number of steps is increased. Now, let's see what happens when the number of characters varies while keeping the number of steps fixed at 15.



The critical capacity of a Hopfield network, as stated before, is p/N, where p is the set of patterns to be memorized and N the number of neurons. The number of neurons is determined by the number of pixels in each image. The dimension of each image is 7×5, which means that the number of neurons of the network is 35.

Now, the critical capacity can be expressed as $p/35$.

Task 3: Retrieve 25 characters