Character recognition with Hopfield networks

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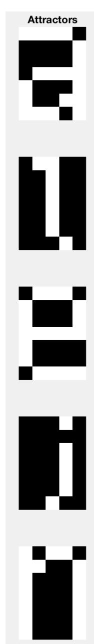
Data preparation

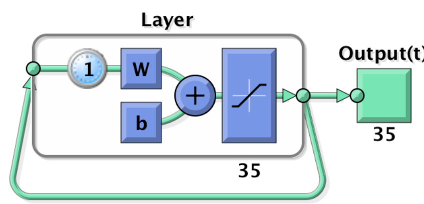
Before starting to solve the tasks, the dataset to use must be constructed. For this exercise, we are asked to pre-pend the lowercase characters of our first and last name to the set of capital characters given in `prprob`. As my name is Alejandro Rodriguez, the characters I have to pre-pend are: `a, l, e, j, n, d, r, o, i, g, u, z`. Once this is done, it's time to solve the three proposed problems.

Task 1: Retrieve the first 5 characters.

The first 5 characters of the alphabet are `a, l, e, j` and `n`, this is, the first five characters that appear in my first name and last name. After loading this characters, the pixels whose value is 0 must be changed to -1. This has to be done because Hopfield networks normally have units that take on values of 1 or -1 as convention.

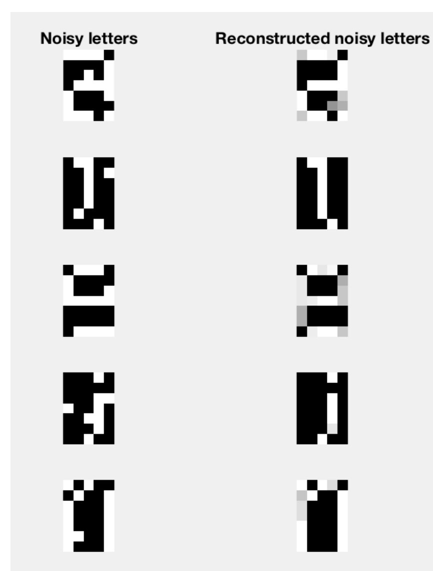


With this five characters as attractor states, a Hopfield neural network is created. Then, three random pixels of each character are inverted (from 1 to -1 and vice versa). The objective of this inversion is to check if the network is able to reconstruct the distorsioned characters.

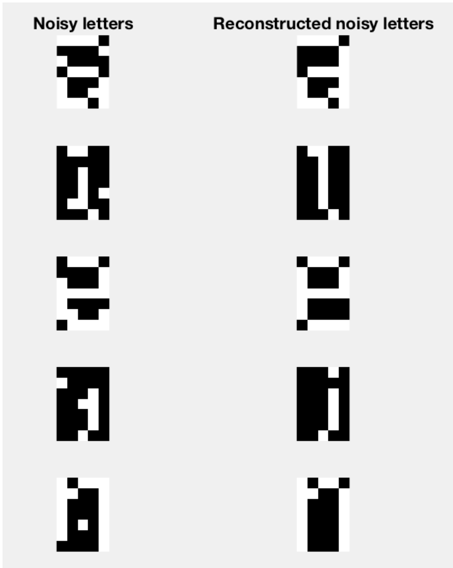


Executing the neural network with the noisy characters as input shows the following results:

- With a small number of steps: it can be seen that the characters are being attracted by the attractor states, but they still contain some noise.



- When the number of steps is increased: with just 5 steps, the characters are perfectly reconstructed.



Sporious patterns are local energy minima that are created during training, in addition to the intended target patterns. They are activity patterns that have not been explicitly embedded in the synaptic matrix, but are nonetheless stable. They are in other words "unwanted" attractor states that, by virtue of a finite overlap with the "wanted" attractor states, come about as a local minimum in the energy function. They can be composed of various combinations of the original patterns or simply the negation of any pattern in the original pattern set.

There are three different types of spurious states:

1. For each stored pattern, its negation is also an attractor

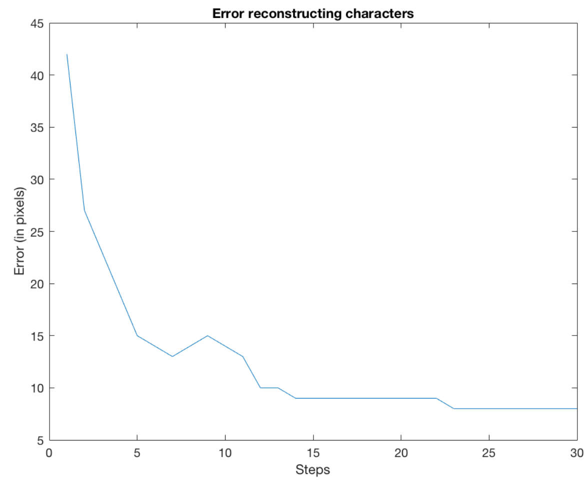
2. Any linear combination of an odd number of stored patterns. give rise to the so-called mixture states.

3. for large p, we get local minima that are not correlated to any linear combination of stored patterns.

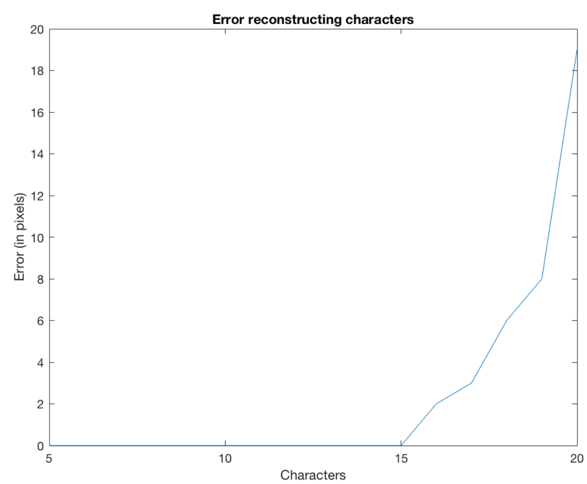
In this case, the existence of spurious patters is not noticeable. According to Hebb rule, a Hopfield network can store up to $0.138N$ uncorrelated patterns, being N the number of neurons. Replacing N by its value in this problem, 35, gives that the uncorrelated patterns that can be stored is almost 5, which is the number of charaters used in this exercise.

Task 2: Critical loading capacity

A Hopfield neural network exceed the loading capacity when the number of learned patterns over the number of units p/N is greater than the critical capacity $\alpha \approx 0.138$. First, a number $p = 20$ characters is selected, and the error in the reconstruction is calculated as a function of the number of steps used to reconstruct the character.



The purpose of this plot is to see how the approximation becomes more accurate when the number of steps is increased. Now, let's see what happens when the number of characters varies while keeping the number of steps fixed at 15.



The critical capacity of a Hopfield network, as stated before, is p/N, where p is the set of patterns to be memorized and N the number of neurons. The number of neurons is determined by the number of pixels in each image. The dimension of each image is 7×5, which means that the number of neurons of the network is 35.

Now, the critical capacity can be expressed as $p/35$.

It can be seen in the previous plot that when the number of patterns p has values below 15, there is no error in the reconstruction, but when it is increased, an error in the reconstruction appears.

The loading capacity can be expressed then as $15/35 = 0.428$, which is much higher than the one obtained using Hebb-rule.

This is an interesting phenomena. Nevertheless, it has an explanation. It could happen that the spurious states and the real attractors overlap, which means that the reconstruction of the character will be correct even if the pixel is reconstructed by an spurious state. Hebb rule states that it can be stored $0.138N$ uncorrelated patterns, which in this case would be $0.138 \times 35 \approx 5$ patterns. It can be then concluded that the charaters are not uncorrelated patterns.

Task 3: Retrieve 25 characters

Appendix