CS5727: Project

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Question 1

We define the first set of variables for the client and the depot locations, we define binary variables for the locations of the depot and the client. We also evaluate the Euclidean distance between each potential set of coordinates. Below is the set of variables:

$$C = \text{number of clients}$$

$$c_i = client :: \sum_{i=1}^{C} c_i = C$$

$$D = \text{maximum number of depots} = \frac{C}{n_{min}}$$

$$x_{cd} = \begin{cases} 1, & \text{if depot } d \text{ services client } c \\ 0, & \text{otherwise} \end{cases}$$

$$y_d = \begin{cases} 1, & \text{if depot } d \text{ exists} \\ 0, & \text{otherwise} \end{cases}$$

$$k_{c_1 c_2} = \text{Euclidean distance from client } c_1 \text{ to client } c_2$$

Objective function:

$$\min \left(\sum_{d=1}^{D} y_d \right)$$

Constraints:

$$\sum_{c=1}^{C} x_{cd} \leq n_{max} * y_d \ \forall d \in 1...D \tag{1a}$$

$$\sum_{c=1}^{C} x_{cd} \geq n_{min} * y_d \ \forall d \in 1...D \tag{1b}$$

$$\sum_{d=1}^{D} x_{cd} = 1 \ \forall c \in 1...C \tag{1c}$$

$$k_{c_1 c_2} \times x_{c_1 d} \times x_{c_2 d} \leq d_{max} \ \forall c_1 \in 1...C, \ \forall c_2 \in 1...C, \ \forall d \in 1...D \tag{1d}$$

In the evaluation (available in the ipynb) we plotted the locations of the various depots and their locations. In these Instance 3 was infeasible as a solution, the solutions for instance 1 and 2 are below. The depot locations, y_i are labelled in the key and the clients they service are labeled with the same color.

Figure 1: Instance 1

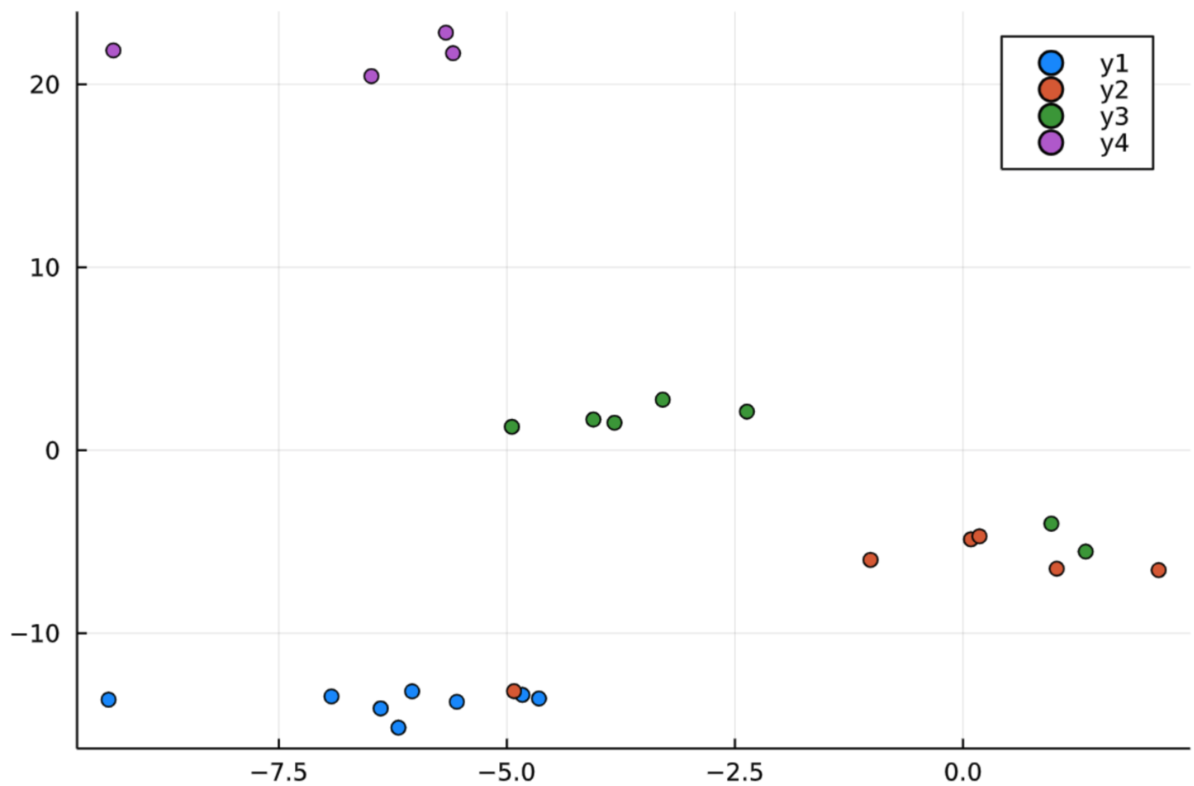
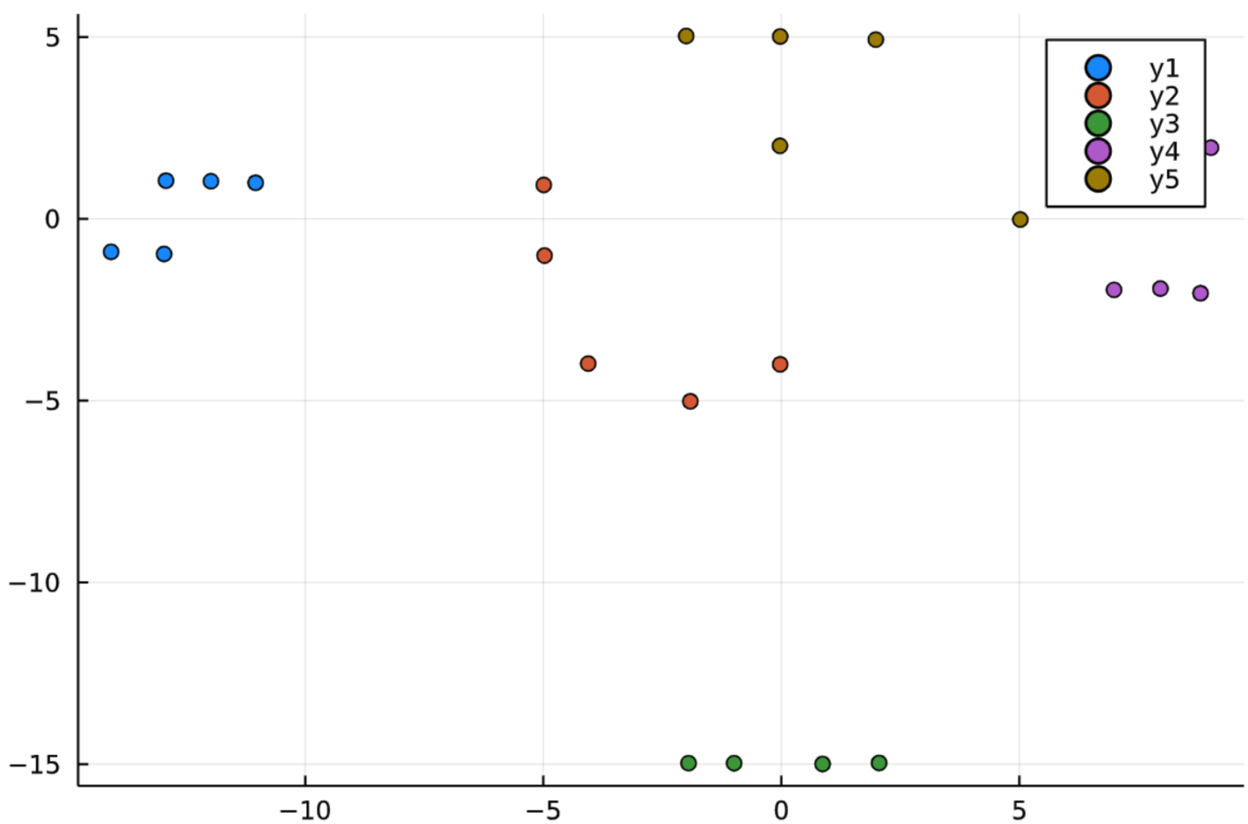


Figure 2: Instance 2



Question 2

For this question we tried different variations for the values of n_{min}, n_{max} & d_{max}. We saw some trends in the data, with the following data. Below are the trends of the various instances with given equations.

(a) Changing n_{min}

For instance 1, while $n_{max} = 8$ & $d_{max} = 10$ we noticed that for instance 1, keeping other values the same, the solution for feasible for n_{min} until 4, post $n_{min} >= 5$ the solution was not feasible

n_{min}	1	2	3	4	≥5
Solutions	4	4	4	4	Infeasible

For instance 2, while $n_{max} = 6$ & $d_{max} = 9$ followed suit similar to instance 1, where after $n_{min} = 4$, the solution is not feasible any more.

n_{min}	1	2	3	4	≥5
Solutions	5	5	5	5	Infeasible

For instance 3, while $n_{max} = 6$ & $d_{max} = 9$ We noticed that as we reduce n_{min}, the solution got feasible, we started with $n_{min} = 0$ the solutions was feasible. We slowly increased the count n_{min} to see when the solution started to get infeasible

n_{min}	1	2	3	≥4
Solutions	6	6	6	Infeasible

(b) Changing n_{max}

For instance 1, while $n_{min} = 2$ & $d_{max} = 10$

n_{max}	2	3	4	5	≥6
Solutions	9	7	6	5	4

For instance 2, while $n_{min} = 4$ & $d_{max} = 9$

n_{max}	4	5	≥6
Solutions	Infeasible	6	5

For instance 3, while $n_{min} = 4$ & $d_{max} = 9$

n_{max}	≥4
Solution	Infeasible

(c) Changing d_{max}

For instance 1, while $n_{min} = 2$ & $n_{max} = 8$

d_{max}	3.2	3.5	4	5	10	15
Solutions	7	6	5	5	4	4

For instance 2, while $n_{min} = 4$ & $n_{max} = 6$

d_{max}	6	7	8	9	10	11
Solutions	Infeasible	6	6	5	5	5

For instance 3, while $n_{min} = 4$ & $n_{max} = 6$ changing d_{max} also helped make the solution feasible. We see that there is no feasible solution until we expand the d_{max} to a value which is greater than 15

d_{max}	9	10	15	20	25
Solutions	Infeasible	5	5	5	5

(d) Additionally for instance 3, as we change increase the values of n_{max} for larger distances the number of optimal clusters continues to reduce. Below is a description where $n_{min} = 2$ and $n_{max} \in \{5, 7, 10, 15, 20\}$ and $d_{max} \in \{9, 15, 20, 25, 45\}$

n_{max}	5	7	10	15	20
d_{max}	9	15	20	25	45
Solutions	5	4	3	2	2

Question 3

If we evaluate the set T to be the set of all the pairs of clients and we can create a set T_1 which is the set of pairs of the clients which are in the same clusters, such that we know $T_1 \subset T$. We have a total of 300 potential pairs when n is 25, given by the formula

$$\frac{n(n-1)}{2}$$

. Variables:

$$C = \text{number of clients}$$
$$D = \text{maximum number of depots} = \frac{C}{n_{min}}$$
$$r = \frac{n(n-1)}{2}$$
$$|T_1| = p \in \{1...r\}$$
$$x^p_{cd} = \begin{cases} 1, & \text{if depot } d \text{ services client c in} |T^p_1| \\ 0, & \text{otherwise} \end{cases}$$
$$y^p_d = \begin{cases} 1, & \text{if depot } d \text{ exists in } |T^p_1| \\ 0, & \text{otherwise} \end{cases}$$
$$k_{c_1 c_2} = \text{Euclidean distance from client } c_1 \text{ to client } c_2$$

We also define an indicator variable, such that we define if the solution has exactly those many pairs i.e.

$$z_p = \begin{cases} 1, & \text{if the optimal solution has p pairs} \\ 0, & \text{otherwise} \end{cases}$$

Objective Function:

$$\min \sum_{p=1}^{r} \frac{z_i}{p} \sum_{c=1}^{n} \sum_{d=1}^{n} \sum c_k{}^n x^p_{cd} x^p_{cc_k}$$

Constraints:

$$\sum_{p=1}^{r} z_p = 1 \tag{2a}$$
$$x^p_{cd} + x^p_{dc} <= 1, \forall c, d \in [1, n], p \in [1...r] \tag{2b}$$
$$\sum_{c=1}^{n} x^p_{cd} >= y^p_d \times n_{min} \forall d \in D, p \in [1, r] \tag{2c}$$
$$\sum_{c=1}^{n} x^p_{cd} <= y^p_d \times n_{max} \forall d \in D, p \in [1, r] \tag{2d}$$
$$\sum_{d=1}^{n} x^p_{cd} = z_i \forall c \in C, p \in [1, r] \tag{2e}$$
$$\sum_{c=1}^{n} \sum_{d=1}^{n} \sum c_k{}^n x^p_{cd} x^p_{cc_k} = p \; times z_p \forall m \in [1...r] \tag{2f}$$
$$\tag{2g}$$

This would be for all the values in:

$$x^p_{cd}, y^p_d, z_p \in 0, 1 \forall c \in C, d \in D, p \in [1..r]$$