Домашняя работа.

TVD.

$$f_{i-\frac{1}{2}} = \begin{cases} f_{i-1} + \varphi(r_{i-1})\dfrac{f_i - f_{i-1}}{2}, & \dfrac{\partial\varphi}{\partial f} > 0 \\ f_i - \varphi(r_i)\dfrac{f_i - f_{i-1}}{2}, & \dfrac{\partial\varphi}{\partial f} < 0 \end{cases}$$

$$r_i = \frac{f_i - f_{i-1}}{f_{i+1} - f_i}$$

- $\varphi(r) = \dfrac{r+|r|}{1+|r|} \quad \leftarrow \quad \dfrac{\partial\varphi}{\partial f} < 0$

$$\varphi(r_i) = \frac{\dfrac{f_i - f_{i-1}}{f_{i+1} - f_i} + \left|\dfrac{f_i - f_{i-1}}{f_{i+1} - f_i}\right|}{1 + \left|\dfrac{f_i - f_{i-1}}{f_{i+1} - f_i}\right|}$$

$$r_i > 0: \varphi(r_i) = \frac{f_i - f_{i-1} + f_i - f_{i-1}}{f_{i+1} \cancel{- f_i} \cancel{+ f_i} - f_{i-1}} =$$

$$= \frac{2(f_i - f_{i-1})}{f_{i+1} - f_{i-1}} \Rightarrow \frac{\partial\varphi}{\partial f} < 0: \underbrace{f_{i-1/2} = f_i - \frac{(f_i - f_{i-1})^2}{f_{i+1} - f_{i-1}}}$$

$$r_i < 0: \varphi(r_i) = 0 \Rightarrow \frac{\partial\varphi}{\partial f} < 0: \underbrace{f_{i-1/2} = f_i}$$

$$\frac{\partial\varphi}{\partial f} > 0: \varphi(r_{i-1}) = \frac{\dfrac{f_{i-1} - f_{i-2}}{f_i - f_{i-1}} + \left|\dfrac{f_{i-1} - f_{i-2}}{f_i - f_{i-1}}\right|}{1 + \left|\dfrac{f_{i-1} - f_{i-2}}{f_i - f_{i-1}}\right|}$$

$r_{i-1} > 0$: $\varphi(r_{i-1}) = \frac{2(f_{i-1} - f_{i-2})}{f_i - f_{i-2}}\,r$;

$r \Rightarrow f_{i-1/2} = f_{i-1} + \frac{\cancel{2}(f_{i-1} - f_{i-2})(f_i - f_{i-1})}{(f_i - f_{i-2})\cancel{\cdot 2}} =$

$= \frac{f_i f_{i-1} - \cancel{f_{i-1} f_{i-2}} + f_i f_{i-1} - f_{i-1}^2 - f_i f_{i-2} + \cancel{f_{i-1} f_{i-2}}}{f_i - f_{i-2}} =$

$= \frac{2 f_i f_{i-1} - f_{i-1}^2 - f_i f_{i-2}}{f_i - f_{i-2}}$

$r_{i-1} < 0$: $f_{i-1/2} = f_{i-1}$.

- $\varphi(r) = \max\left(0, \min\left(2r, \frac{1+r}{2}, 2\right)\right)$

$2r = \frac{2(f_i - f_{i-1})}{f_{i+1} - f_i}$; $\frac{1+r}{2} = \frac{1 + \frac{f_i - f_{i-1}}{f_{i+1} - f_i}}{2} = \frac{f_{i+1} - f_{i-1}}{2(f_{i+1} - f_i)}$

$\min(\ldots) = 2r$: $2r < 2 \Rightarrow r < 1$;

$2r < \frac{1+r}{2} \Rightarrow 4r < 1 + r \Rightarrow r < \frac{1}{3}$

$\frac{1}{3} < 1 \Rightarrow r < 1/3$

$\min(\ldots) = \frac{1+r}{2}$: $\frac{1+r}{2} < 2r \Rightarrow \begin{cases} r > \frac{1}{3} \\ r < 3 \end{cases}$

$\frac{1+r}{2} < 2 \Rightarrow$

$$\min(\ldots) = 2 \Rightarrow \begin{cases} r > 1 \\ r > 3 \end{cases} \Rightarrow r > 3$$

$$\varphi(r) = \max\left(0, \min\left(2r, \frac{1+r}{2}, 2\right)\right)$$

$\varphi = 0$	$\varphi(r) = 2r$	$\varphi(r) = \frac{1+r}{2}$	$\varphi = 2$
$f_{i-1/2} = \begin{cases} f_{i-1} \\ f_i \end{cases}$	1	2	$f_{i-1/2} = \begin{cases} f_i \\ f_{i-1} \end{cases}$

r

1: • $\frac{\partial \varphi}{\partial f} > 0$: $f_{i-1/2} = f_{i-1} + \frac{f_{i-1} - f_{i-2}}{\cancel{f_i - f_{i-1}}} \cancel{(f_i - f_{i-1})} =$

$= 2f_{i-1} - f_{i-2}$

• $\frac{\partial \varphi}{\partial f} < 0$: $f_{i-1/2} = f_i - \frac{(f_i - f_{i-1})^2}{f_{i+1} - f_i}$

2 • $\frac{\partial \varphi}{\partial f} > 0$: $f_{i-1/2} = f_{i-1} + \frac{1 + \frac{f_{i-1} - f_{i-2}}{f_i - f_{i-1}}}{4} (f_i - f_{i-1}) =$

$= f_{i-1} + \frac{1}{4} \cancel{(f_i - f_{i-1})} \frac{f_i - f_{i-1} + f_{i-1} - f_{i-2}}{\cancel{f_i - f_{i-1}}} =$

$= f_{i-1} + \frac{1}{4}(f_i - f_{i-2})$

• $\frac{\partial \varphi}{\partial f} < 0$: $f_{i-1/2} = f_i - \frac{1 + \frac{f_i - f_{i-1}}{f_{i+1} - f_i}}{4} (f_i - f_{i-1}) =$

$= f_i - \frac{1}{4} \frac{(f_{i+1} \cancel{- f_i} \cancel{+ f_i} - f_{i-1})(f_i - f_{i-1})}{f_{i+1} - f_i} = f_i - \frac{(f_{i+1} - f_{i-1})(f_i - f_{i-1})}{4(f_{i+1} - f_i)}$

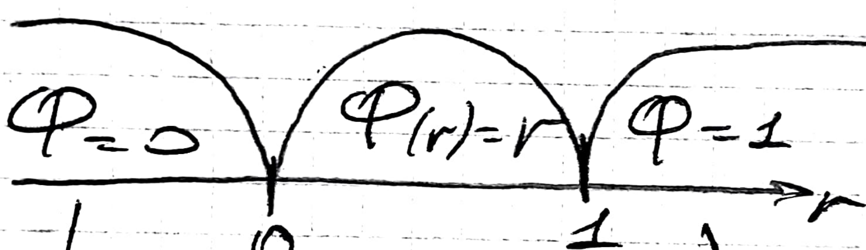
• $\varphi(r) = \max(0, \min(1, r))$

$\min(1,r) = \begin{cases} 1 \Rightarrow r > 1 \\ r \Rightarrow r < 1 \end{cases}$

$\max(0,1) = 1 \ \forall r$

$\max(0,r) = \begin{cases} r, & r > 0 \\ 0, & r < 0 \end{cases}$ $\Rightarrow$

$r \Rightarrow \max(0, \min(1,r)) = \begin{cases} 0, & r < 0 \\ r, & 0 < r < 1 \\ 1, & r > 1 \end{cases}$



$\varphi = 0: f_{i-1/2} = \begin{cases} f_{i-1}, & \frac{\partial\varphi}{\partial f} > 0 \\ f_i, & \frac{\partial\varphi}{\partial f} < 0 \end{cases}$ $\quad f_{i-1/2} = \frac{f_i + f_{i-1}}{2}$

• $\frac{\partial\varphi}{\partial f} > 0 \rightarrow$

$\varphi(r) = r: \quad f_{i-1/2} = f_{i-1} + \frac{f_{i-1} - f_{i-2}}{\cancel{f_i - f_{i-1}}} \frac{\cancel{f_i - f_{i-1}}}{2} =$

$= \frac{3f_i - f_{i-2}}{2}$

• $\frac{\partial\varphi}{\partial f} < 0: \quad f_{i-1/2} = f_i - \frac{(f_i - f_{i-2})^2}{2(f_{i+1} - f_i)}$