Coherence-Rupture-Regeneration and Solomonoff Induction: A Mathematical Comparison

Mathematical Analysis

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Abstract

This document presents a mathematical analysis comparing the Coherence-Rupture-Regeneration (CRR) framework with Solomonoff induction. We establish correspondence theorems between the two frameworks and identify what each provides. The main findings are: (1) CRR coherence corresponds to accumulated conditional Kolmogorov complexity; (2) CRR rupture implements MDL model switching; (3) CRR regeneration weights correspond to Solomonoff's universal prior under specific parameter choices; (4) the frameworks are compatible, with CRR providing operational details that Solomonoff leaves unspecified. We also note limitations and open questions for both frameworks.

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1 Introduction

1.1 Scope

This document compares two frameworks for inductive inference:

(i) **Solomonoff Induction** (1964): A theoretical framework for sequence prediction based on algorithmic information theory and Kolmogorov complexity. It provides optimality guarantees but is incomputable.

(ii) **Coherence-Rupture-Regeneration (CRR)**: A framework describing how bounded systems maintain identity through discontinuous change, grounded in Bayesian model comparison.

We address three questions:

1. What is the mathematical relationship between CRR and Solomonoff induction?
2. Are the two frameworks compatible?
3. What does each framework provide that the other does not?

1.2 Summary of Findings

Main Findings

What Solomonoff Induction Provides:

- The optimal prior: $P(h) = 2^{-K(h)}$
- The optimal posterior: $P(h|d) \propto P(d|h) \cdot 2^{-K(h)}$
- Proof of optimality (dominates all computable predictors)

What Solomonoff Does Not Provide:

- When to act on accumulated evidence
- A computable implementation
- The mechanism of model change
- A tunable parameter for different contexts
- Discrete update structure

What CRR Adds:

- Decision rule: act when $\mathcal{C} \geq \Omega$
- Computable approximation via temporal coherence accumulation
- Regeneration operator specifying how models are reconstructed
- Tunable rigidity parameter Ω
- Discrete rupture moments

1.3 The Three-Framework Synthesis

CRR can be understood in relation to two other frameworks:

Framework	Specifies	Mathematical Form
Solomonoff	What is optimal	$P(h) = 2^{-K(h)}$
Free Energy Principle	What to minimize	$F = \mathbb{E}[\log q] - \mathbb{E}[\log p]$
CRR	When and how	$\mathcal{C} \to \delta \to R$ with threshold Ω

These frameworks address different aspects of inference and are not mutually exclusive.

2 Preliminaries

2.1 Algorithmic Information Theory

Definition 2.1 (Kolmogorov Complexity). The *Kolmogorov complexity* of a string x with respect to universal machine U is:

$$K_U(x) = \min\{|p| : U(p) = x\} \tag{1}$$

where $|p|$ denotes the length of program p in bits.

Definition 2.2 (Conditional Kolmogorov Complexity). The complexity of x given y is:

$$K(x|y) = \min\{|p| : U(p, y) = x\} \tag{2}$$

Theorem 2.3 (Invariance Theorem). *For any two universal machines U_1, U_2, there exists a constant c such that for all x:*

$$|K_{U_1}(x) - K_{U_2}(x)| \leq c \tag{3}$$

2.2 Solomonoff Induction

Definition 2.4 (Solomonoff Prior). The *Solomonoff prior* over binary strings is:

$$\mathcal{S}(x) = \sum_{p:U(p)=x^*} 2^{-|p|} \tag{4}$$

where x^* denotes strings beginning with x.

Definition 2.5 (Universal Prediction). Given observed sequence $x_{1:n}$, the Solomonoff predictor assigns:

$$P_{\mathcal{S}}(x_{n+1}|x_{1:n}) = \frac{\mathcal{S}(x_{1:n+1})}{\mathcal{S}(x_{1:n})} \tag{5}$$

Theorem 2.6 (Solomonoff Convergence). *Let μ be any computable measure. The expected total squared prediction error is bounded:*

$$\mathbb{E}_\mu\left[\sum_{n=1}^{\infty}(P_{\mathcal{S}}(x_{n+1}|x_{1:n}) - \mu(x_{n+1}|x_{1:n}))^2\right] \leq K(\mu)\ln 2 \tag{6}$$

Remark 2.7 (Incomputability). Solomonoff induction is incomputable because computing $K(x)$ requires solving the halting problem.

2.3 CRR Framework

Definition 2.8 (CRR System)**.** A *CRR system* is a tuple $(\mathcal{M}, Y, \Pi, \Omega, \mathcal{C}, R)$ where:

- $\mathcal{M} = \{m, m', \ldots\}$ is a set of generative models
- $Y = \mathbb{R}^d$ is the observation space
- $\Pi : \mathcal{M} \to \mathrm{PD}(d)$ assigns precision matrices
- $\Omega : \mathcal{M} \times \mathcal{M} \to \mathbb{R}$ is the rigidity function
- $\mathcal{C} : \mathcal{M} \times \mathbb{N} \to \mathbb{R}_+$ is the coherence accumulator
- R is the regeneration operator

Definition 2.9 (CRR Operators)**.** The three CRR operators are:
Coherence:

$$\mathcal{C}_m(n) = \frac{1}{2}\sum_{i=1}^{n}(y_i - g_m(\mu_i))^\top \Pi_m (y_i - g_m(\mu_i)) \tag{7}$$

Rupture: Occurs when $\mathcal{C}_m(n) - \mathcal{C}_{m'}(n) > \Omega_{m,m'}$
Regeneration:

$$R[\phi](t) = \frac{1}{Z}\int_0^t \phi(\tau) \cdot \exp\left(\frac{\mathcal{C}(\tau)}{\Omega}\right) \cdot \Theta(t-\tau)\, d\tau \tag{8}$$

3 Correspondence Theorems

3.1 Coherence and Kolmogorov Complexity

Theorem 3.1 (Coherence-Complexity Correspondence)**.** *Let $y_{1:n}$ be an observation sequence and m a computable generative model. Define the* algorithmic coherence*:*

$$\mathcal{C}_m^{alg}(n) = \sum_{i=1}^{n} K(y_i | y_{<i}, m) \tag{9}$$

Then for CRR coherence under Gaussian observations:

$$\mathcal{C}_m(n) = \mathcal{C}_m^{alg}(n) + O(n) \tag{10}$$

where the $O(n)$ term accounts for the model's encoding overhead.

Proof. The proof proceeds in three steps.
Step 1: By Levin's coding theorem, for any computable distribution P:

$$-\log P(x) = K(x) + O(K(P)) \tag{11}$$

Step 2: CRR coherence under Gaussian observations equals negative log-likelihood:

$$\mathcal{C}_m(n) = -\log p(y_{1:n}|m) + \frac{n}{2}\log\det(2\pi\Sigma_m) \tag{12}$$

Step 3: Combining these:

$$\begin{aligned}
\mathcal{C}_m(n) &= \sum_{i=1}^{n}[-\log p(y_i|y_{<i}, m)] + O(n) && (13)\\
&= \sum_{i=1}^{n}[K(y_i|y_{<i}, m) + O(1)] + O(n) && (14)\\
&= \mathcal{C}_m^{\mathrm{alg}}(n) + O(n) && (15)
\end{aligned}$$

□

Remark 3.2 (Temporal Survival as Complexity Proxy)**.** A key operational insight: patterns that survive repeated validation under CRR *behave as if* they have low Kolmogorov complexity, without ever computing K directly. Low coherence accumulation over time indicates the model compresses observations well. This provides a computable proxy for the incomputable complexity measure.

3.2 Rupture and Minimum Description Length

Theorem 3.3 (Rupture as MDL Model Switching)**.** *The CRR rupture condition is equivalent to the MDL model switching criterion:*

$$\textit{Rupture } m \to m' \iff \mathcal{C}_m(n) > \mathcal{C}_{m'}(n) + \Omega_{m,m'} \tag{16}$$

where $\Omega_{m,m'} = K(m') - K(m) + K(\textit{switch})$ *is the description length overhead.*

Proof. The MDL principle selects the model minimizing $K(m) + K(y_{1:n}|m)$.
Model m' is preferred over m when:

$$K(m') + K(y_{1:n}|m') < K(m) + K(y_{1:n}|m) \tag{17}$$

Rearranging and substituting CRR variables via Theorem 3.1:

$$\mathcal{C}_m(n) - \mathcal{C}_{m'}(n) > K(m') - K(m) = \Omega_{m,m'} \tag{18}$$

□

3.3 Regeneration and Universal Prior

Theorem 3.4 (Regeneration-Prior Correspondence)**.** *The CRR regeneration weighting* $\exp(\mathcal{C}(\tau)/\Omega)$ *corresponds to Solomonoff's universal prior under specific parameter identification.*

Proof. The Solomonoff prior assigns $P(\phi) \propto 2^{-K(\phi)}$.
The CRR regeneration weights by $w(\tau) \propto \exp(\mathcal{C}(\tau)/\Omega)$.
Since $\mathcal{C}(\tau) \approx -\log p(y_{1:\tau}|\phi) \approx K(y_{1:\tau}|\phi)$ by Levin's theorem:

$$w(\phi) \propto \exp\left(\frac{-K(y|\phi)}{\Omega}\right) = 2^{-K(y|\phi)/(\Omega \ln 2)} \tag{19}$$

Parameter identification: For exact correspondence with Solomonoff, we require:

$$\Omega = \frac{1}{\ln 2} \approx 1.443 \tag{20}$$

For other values of Ω, the correspondence holds up to a scaling of the exponent. With $\Omega = 1/\pi \approx 0.318$ (as conjectured in CRR), the exponent is scaled by a factor of approximately 4.5. □

Remark 3.5 (Scaling Implications)**.** The choice of Ω affects how aggressively complexity is weighted:

- $\Omega = 1/\ln 2$: Exact Solomonoff correspondence
- $\Omega < 1/\ln 2$: More aggressive complexity penalization
- $\Omega > 1/\ln 2$: More lenient complexity penalization

4 What Solomonoff Provides

Solomonoff induction provides:

1. **The optimal prior:** $P(h) = 2^{-K(h)}$ assigns higher probability to simpler hypotheses in an objective, machine-independent way (up to constants).
2. **The optimal posterior:** $P(h|d) \propto P(d|h) \cdot 2^{-K(h)}$ follows from Bayes' theorem.
3. **Optimality proof:** Solomonoff's convergence theorem shows this predictor dominates all computable predictors—total prediction error is bounded by the complexity of the true distribution.

5 What Solomonoff Does Not Provide

5.1 When to Update

Solomonoff induction is a ratio of probabilities. It does not specify when an agent should act on accumulated evidence or commit to a model. The posterior weights change continuously, but there is no threshold or decision rule.

CRR adds: The rupture condition $\mathcal{C}_m - \mathcal{C}_{m'} > \Omega$ provides an explicit decision rule. The rupture operator δ marks discrete decision moments.

5.2 Computability

$K(h)$ is uncomputable—there is no algorithm to compute Kolmogorov complexity for arbitrary strings.

CRR adds: Temporal coherence accumulation as a proxy. Models that survive repeated validation (low $\mathcal{C}$ accumulation) behave as if they have low K, without computing K directly. This is the operational insight—complexity is approximated through temporal survival.

5.3 Mechanism of Model Change

Solomonoff gives posteriors but not how one model replaces another in a physical system.

CRR adds: The regeneration operator $R = \int \phi \cdot \exp(\mathcal{C}/\Omega) \cdot \Theta \, d\tau$ specifies that new models are reconstructed from memory weighted by coherence.

5.4 Tunable Parameter

Solomonoff's prior is fixed by the choice of universal machine. There is no parameter to adjust for different contexts.

CRR adds: The rigidity parameter Ω, which can potentially vary across:

- Individuals (cognitive style)
- Domains (expertise level)
- States (arousal, attention)
- Timescales (different Ω for different temporal scales)

Remark 5.1 (Caution on Ω Variability)**.** The variability of Ω is both a feature and a potential concern. If Ω can be freely adjusted, it risks becoming a free parameter that can fit any data. The CRR documentation partially addresses this with Kac's lemma ($\Omega = 1/\mu(A)$), which derives Ω from the measure of the coherent region. Whether a universal value exists remains an open question.

5.5 Discrete Update Structure

Solomonoff is typically presented as continuous updating—posterior weights change smoothly with each observation.

CRR adds: Updates occur at discrete rupture moments—the δ that marks threshold crossings. This connects to process philosophy (Whitehead's "actual occasions") and stopping time theory in probability.

6 Compatibility Analysis

Theorem 6.1 (Compatibility)**.** *CRR and Solomonoff induction are mathematically compatible. CRR operationalizes Solomonoff for finite agents by replacing:*

Solomonoff (incomputable)	***CRR (computable proxy)***
Kolmogorov complexity K	*Accumulated coherence* $\mathcal{C}$
Continuous updating	*Discrete rupture at* $\mathcal{C} = \Omega$
Abstract posterior	*Concrete regeneration dynamics*
Fixed prior	*Tunable rigidity* Ω

Proof. Compatibility follows from Theorems 3.1, 3.3, and 3.4, which establish correspondences between:

- Coherence and conditional Kolmogorov complexity
- Rupture and MDL model switching
- Regeneration weights and the universal prior

The discretization of continuous updating into threshold-triggered ruptures is analogous to the relationship between Bayesian updating and the Sequential Probability Ratio Test (SPRT). Both implement Bayesian inference; SPRT adds stopping rules. □

Remark 6.2 (One-Sentence Summary)**.** CRR describes how bounded agents can approximate theoretically optimal inference through temporal dynamics and threshold-triggered discrete updates.

7 Comparison Tables

Table 1: What Each Framework Provides

Feature	**Solomonoff**	**CRR**
Optimal prior	Yes	No (uses arbitrary $p(m)$)
Optimal prediction guarantee	Yes	No
Decision rule (when to act)	No	Yes ($\mathcal{C} \geq \Omega$)
Computable	No	Yes (for finite $\mathcal{M}$)
Model change mechanism	No	Yes (regeneration)
Tunable parameter	No	Yes (Ω)
Multiscale structure	No	Yes
Discrete update structure	No	Yes (rupture)

Table 2: Mathematical Correspondences

Solomonoff Induction	**CRR Framework**
$K(y\|m)$	$\mathcal{C}_m(n) + O(n)$
$2^{-K(m)}$	$p(m)$; $\Omega = \log(p(m)/p(m'))$
Posterior update	Rupture condition
$2^{-K(\phi)}$ weighting	$\exp(\mathcal{C}/\Omega)$ weighting
Incomputable	Computable

8 Limitations and Open Questions

8.1 Limitations of CRR Relative to Solomonoff

1. **Model class restriction:** CRR requires specifying $\mathcal{M}$ in advance; Solomonoff considers all computable hypotheses.
2. **No universal convergence:** Solomonoff guarantees convergence to any computable truth. CRR guarantees optimal switching within $\mathcal{M}$, but may fail if the truth is outside $\mathcal{M}$.
3. **Discretization error:** CRR's discrete ruptures may miss continuous optimality of Solomonoff's mixture.
4. **Parameter dependence:** CRR depends on Ω, which may be difficult to determine.

8.2 Limitations of Solomonoff Relative to CRR

1. **Incomputability:** Solomonoff cannot be implemented; CRR can.
2. **No decision rule:** Solomonoff provides probabilities but not when to act.
3. **No temporal structure:** Solomonoff does not address discrete moments of change.
4. **No multiscale analysis:** Solomonoff operates at a single scale.

8.3 Open Questions

1. **Universal Ω:** Can $\Omega = 1/\pi$ be derived from first principles? Neither framework provides such a derivation.
2. **Convergence rate:** How does CRR's approximation error scale as $|\mathcal{M}| \to \infty$?
3. **Continuous limit:** Does CRR with $\Omega \to 0$ converge to Solomonoff's mixture?
4. **Empirical validation:** The claimed correspondence requires empirical testing across domains.

9 Conclusion

This analysis establishes the mathematical relationship between CRR and Solomonoff induction:

1. **Compatibility:** The frameworks are compatible. CRR coherence corresponds to accumulated Kolmogorov complexity, rupture to MDL model switching, and regeneration to the universal prior (under parameter identification).

2. **Complementarity:** Solomonoff specifies *what* is optimal; CRR specifies *when* and *how*. Neither is a substitute for the other.

3. **Operationalization:** CRR provides a computable approximation to Solomonoff by using temporal coherence accumulation as a proxy for algorithmic complexity.

The relationship can be summarized as: *CRR is to Solomonoff induction as SPRT is to Bayesian updating*—both pairs share theoretical foundations, but the former adds actionable stopping rules.

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A Technical Details

A.1 Levin's Coding Theorem

Theorem A.1 (Levin)**.** *For any computable probability distribution P and string x:*

$$-\log P(x) = K(x) + O(K(P)) \tag{21}$$

A.2 Chain Rule for Kolmogorov Complexity

Theorem A.2 (Chain Rule)**.** *For strings x, y:*

$$K(x, y) = K(x) + K(y|x^*) + O(\log K(x, y)) \tag{22}$$

where x^* is the shortest program for x.

A.3 SPRT Optimality

Theorem A.3 (Wald-Wolfowitz)**.** *Among all sequential tests with type I error $\leq \alpha$ and type II error $\leq \beta$, the SPRT minimizes the expected sample size under both hypotheses.*

CRR rupture is equivalent to SPRT when the log-likelihood ratio equals the coherence difference:

$$\Lambda_n = \log \frac{p(y_{1:n}|m')}{p(y_{1:n}|m)} = \mathcal{C}_m(n) - \mathcal{C}_{m'}(n) \tag{23}$$