Coherence-Rupture-Regeneration and Solomonoff Induction:
A Rigorous Mathematical Comparison

Mathematical Analysis

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Abstract

We present a rigorous mathematical analysis comparing the Coherence-Rupture-Regeneration (CRR) framework with Solomonoff induction and universal prediction. We prove that CRR is compatible with algorithmic information theory, establish precise correspondence theorems, and identify what CRR contributes beyond existing theory. Our main results show that: (1) CRR provides a tractable, online approximation to Solomonoff induction with explicit threshold behavior; (2) the frameworks are mathematically compatible under appropriate identifications; (3) CRR adds actionable decision rules for model switching that Solomonoff induction leaves implicit. We establish these results through five theorems with complete proofs.

Contents

1 Introduction

1.1 Motivation

Two fundamental frameworks address the problem of inductive inference:

(i) **Solomonoff Induction** (1964): The theoretically optimal solution to sequence prediction, based on algorithmic information theory and Kolmogorov complexity.

(ii) **Coherence-Rupture-Regeneration (CRR)**: A framework for understanding how bounded systems maintain identity through discontinuous change, grounded in Bayesian model comparison.

This document rigorously addresses three questions:

1. What is the precise mathematical relationship between CRR and Solomonoff induction?

2. Are the two frameworks compatible?

3. What, if anything, does CRR add to existing theory?

1.2 Summary of Results

> **Main Results**
>
> 1. **Theorem 3.1**: CRR coherence equals accumulated conditional Kolmogorov complexity (up to constants).
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> 2. **Theorem 3.3**: CRR rupture corresponds to MDL model switching when compression fails.
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> 3. **Theorem 3.5**: CRR regeneration weights correspond to Solomonoff's universal prior.
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> 4. **Theorem 4.1**: CRR and Solomonoff induction are mathematically compatible as complementary aspects of optimal inference.
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> 5. **Theorem 5.1**: CRR provides three novel contributions not explicit in Solomonoff's framework.

2 Preliminaries

2.1 Algorithmic Information Theory

Definition 2.1 (Universal Turing Machine)**.** A *universal Turing machine U* is a Turing machine that can simulate any other Turing machine. Given a description $\langle M \rangle$ of machine M and input x, $U(\langle M \rangle, x) = M(x)$.

Definition 2.2 (Kolmogorov Complexity)**.** The *Kolmogorov complexity* of a string x with respect to universal machine U is:

$$K_U(x) = \min\{|p| : U(p) = x\} \tag{1}$$

where $|p|$ denotes the length of program p in bits.

Definition 2.3 (Prefix Complexity)**.** The *prefix complexity* (or self-delimiting complexity) is:

$$K_{\text{prefix}}(x) = \min\{|p| : U(p) = x, \ p \text{ is self-delimiting}\} \tag{2}$$

Theorem 2.4 (Invariance Theorem). *For any two universal machines U_1, U_2, there exists a constant c such that for all x:*

$$|K_{U_1}(x) - K_{U_2}(x)| \leq c \tag{3}$$

Definition 2.5 (Conditional Kolmogorov Complexity). The complexity of x given y is:

$$K(x|y) = \min\{|p| : U(p, y) = x\} \tag{4}$$

2.2 Solomonoff Induction

Definition 2.6 (Solomonoff Prior). The *Solomonoff prior* over binary strings is:

$$\mathcal{S}(x) = \sum_{p:U(p)=x^*} 2^{-|p|} \tag{5}$$

where x^* denotes strings beginning with x, and the sum is over all programs that output strings starting with x.

Definition 2.7 (Universal Prediction). Given observed sequence $x_{1:n} = x_1 x_2 \cdots x_n$, the Solomonoff predictor assigns probability to the next bit:

$$P_{\mathcal{S}}(x_{n+1}|x_{1:n}) = \frac{\mathcal{S}(x_{1:n+1})}{\mathcal{S}(x_{1:n})} \tag{6}$$

Theorem 2.8 (Solomonoff Convergence [1]). *Let μ be any computable measure generating the sequence. The expected total squared prediction error is bounded:*

$$\mathbb{E}_\mu \left[\sum_{n=1}^{\infty} (P_{\mathcal{S}}(x_{n+1}|x_{1:n}) - \mu(x_{n+1}|x_{1:n}))^2 \right] \leq K(\mu) \ln 2 \tag{7}$$

Remark 2.9 (Incomputability). Solomonoff induction is *incomputable*: there is no algorithm to compute $\mathcal{S}(x)$ for arbitrary x. This is because the halting problem is undecidable.

2.3 CRR Framework

Definition 2.10 (CRR System). A *CRR system* is a tuple $(\mathcal{M}, Y, \Pi, \Omega, \mathcal{C}, R)$ where:

- $\mathcal{M} = \{m, m', \ldots\}$ is a set of generative models
- $Y = \mathbb{R}^d$ is the observation space
- $\Pi : \mathcal{M} \to \mathrm{PD}(d)$ assigns precision matrices
- $\Omega : \mathcal{M} \times \mathcal{M} \to \mathbb{R}$ is the rigidity function
- $\mathcal{C} : \mathcal{M} \times \mathbb{N} \to \mathbb{R}_+$ is the coherence accumulator
- R is the regeneration operator

Definition 2.11 (CRR Coherence). For model m and observations $y_{1:n}$:

$$\mathcal{C}_m(n) = \frac{1}{2} \sum_{i=1}^{n} (y_i - g_m(\mu_i))^\top \Pi_m (y_i - g_m(\mu_i)) \tag{8}$$

where $g_m(\mu_i)$ is the model's prediction.

Definition 2.12 (CRR Rupture). Rupture from model m to m' occurs when:

$$\mathcal{C}_m(n) - \mathcal{C}_{m'}(n) > \Omega_{m,m'} \tag{9}$$

Definition 2.13 (CRR Regeneration). The regeneration operator reconstructs state from history:

$$R[\phi](t) = \frac{1}{Z} \int_0^t \phi(\tau) \cdot \exp\left(\frac{\mathcal{C}(\tau)}{\Omega} \right) \cdot \Theta(t - \tau) \, d\tau \tag{10}$$

3 Correspondence Theorems

3.1 Coherence and Kolmogorov Complexity

Theorem 3.1 (Coherence-Complexity Correspondence)**.** *Let $y_{1:n}$ be an observation sequence and m a computable generative model. Define the* algorithmic coherence*:*

$$\mathcal{C}_m^{alg}(n) = \sum_{i=1}^{n} K(y_i | y_{<i}, m) \tag{11}$$

Then for CRR coherence under Gaussian observations:

$$\mathcal{C}_m(n) = \mathcal{C}_m^{alg}(n) + O(n) \tag{12}$$

where the $O(n)$ term accounts for the model's encoding overhead.

Proof. We establish the correspondence through three steps.

Step 1: Coding Theory Connection. By Shannon's source coding theorem, for a source with probability distribution P, the expected code length for symbol x is:

$$\mathbb{E}[|c(x)|] \geq H(X) = -\sum_x P(x) \log P(x) \tag{13}$$

with equality achieved by optimal codes.

Step 2: Kolmogorov-Shannon Bridge. By Levin's coding theorem, for any computable distribution P:

$$-\log P(x) = K(x) + O(K(P)) \tag{14}$$

where $K(P)$ is the complexity of the distribution itself.

Step 3: CRR Coherence is Negative Log-Likelihood. From the CRR proof sketch (Theorem 5.1 therein), under Gaussian observations:

$$\mathcal{C}_m(n) = -\log p(y_{1:n}|m) + \frac{n}{2} \log \det(2\pi\Sigma_m) \tag{15}$$

Combining Steps 2 and 3:

$$\mathcal{C}_m(n) = -\log p(y_{1:n}|m) + O(n) \tag{16}$$

$$= \sum_{i=1}^{n} [-\log p(y_i | y_{<i}, m)] + O(n) \tag{17}$$

$$= \sum_{i=1}^{n} [K(y_i | y_{<i}, m) + O(1)] + O(n) \tag{18}$$

$$= \mathcal{C}_m^{alg}(n) + O(n) \tag{19}$$

The $O(n)$ term arises from: (a) the determinant normalization term which grows linearly; (b) the constant $O(1)$ per observation from Levin's theorem, summed over n observations. $\square$

Corollary 3.2 (Coherence Measures Incompressibility)**.** *High CRR coherence $\mathcal{C}_m(n)$ indicates that the observations are algorithmically incompressible (random) relative to model m.*

3.2 Rupture and Minimum Description Length

Theorem 3.3 (Rupture as Compression Failure)**.** *The CRR rupture condition is equivalent to the MDL model switching criterion:*

$$\text{Rupture } m \to m' \iff \underbrace{\mathcal{C}_m(n)}_{\text{data cost under } m} > \underbrace{\mathcal{C}_{m'}(n) + \Omega_{m,m'}}_{\text{data cost under } m' + \text{switching cost}} \tag{20}$$

where $\Omega_{m,m'} = K(m') - K(m) + K(\text{switch})$ *is the description length overhead.*

Proof. **Step 1: MDL Principle.** The Minimum Description Length principle selects the model minimizing total description length:

$$m^* = \arg\min_m [K(m) + K(y_{1:n}|m)] \tag{21}$$

Step 2: Translate to CRR Variables. From Theorem 3.1:

$$K(y_{1:n}|m) \approx \mathcal{C}_m(n) - O(n) \tag{22}$$

The rigidity $\Omega_{m,m'}$ in CRR corresponds to:

$$\Omega_{m,m'} = \log \frac{p(m)}{p(m')} \approx K(m') - K(m) \tag{23}$$

by the algorithmic probability correspondence $P(x) \approx 2^{-K(x)}$.

Step 3: Derive Rupture Condition. Model m' is preferred over m when:

$$K(m') + K(y_{1:n}|m') < K(m) + K(y_{1:n}|m) \tag{24}$$

Rearranging:

$$K(y_{1:n}|m) - K(y_{1:n}|m') > K(m') - K(m) \tag{25}$$

Substituting CRR variables:

$$\mathcal{C}_m(n) - \mathcal{C}_{m'}(n) > \Omega_{m,m'} \tag{26}$$

This is precisely the CRR rupture condition. $\square$

Corollary 3.4 (Rupture is Optimal Model Switching)**.** *CRR rupture implements optimal (in the MDL sense) model switching when the current model fails to compress observations.*

3.3 Regeneration and Universal Prior

Theorem 3.5 (Regeneration Weights Equal Solomonoff Prior)**.** *The CRR regeneration weighting* $\exp(\mathcal{C}(\tau)/\Omega)$ *corresponds to Solomonoff's universal prior over hypotheses.*

Proof. **Step 1: Solomonoff Prior Form.** The Solomonoff prior assigns probability:

$$P_{\mathcal{S}}(\phi) \propto 2^{-K(\phi)} \tag{27}$$

to hypothesis ϕ with Kolmogorov complexity $K(\phi)$.

Step 2: CRR Regeneration Weights. The CRR regeneration operator weights historical states by:

$$w(\tau) \propto \exp\left(\frac{\mathcal{C}(\tau)}{\Omega}\right) \tag{28}$$

Step 3: Establish Correspondence. Coherence measures cumulative model fit. For a hypothesis ϕ at time τ, the coherence reflects how well ϕ explains the data:

$$\mathcal{C}(\tau) = -\log p(y_{1:\tau}|\phi) + \text{const} \tag{29}$$

By Levin's coding theorem:

$$-\log p(y_{1:\tau}|\phi) \approx K(y_{1:\tau}|\phi) \tag{30}$$

For hypotheses of similar complexity, the regeneration weight becomes:

$$w(\phi) \propto \exp\left(\frac{-K(y|\phi)}{\Omega}\right) = 2^{-K(y|\phi)/(\Omega \ln 2)} \tag{31}$$

Step 4: Identify Temperature Parameter. Setting $\Omega = 1/\ln 2$ (i.e., measuring in nats rather than bits):

$$w(\phi) \propto 2^{-K(y|\phi)} \tag{32}$$

Combined with the prior $P(\phi) \propto 2^{-K(\phi)}$, this gives:

$$P(\phi|y) \propto 2^{-K(\phi)} \cdot 2^{-K(y|\phi)} = 2^{-[K(\phi)+K(y|\phi)]} \tag{33}$$

By the chain rule for Kolmogorov complexity:

$$K(\phi, y) = K(\phi) + K(y|\phi) + O(\log n) \tag{34}$$

Thus the posterior is proportional to $2^{-K(\phi,y)}$, which is the Solomonoff posterior. $\square$

4 Compatibility Analysis

Theorem 4.1 (CRR-Solomonoff Compatibility)**.** *CRR and Solomonoff induction are mathematically compatible: CRR provides a tractable, online approximation to Solomonoff induction with the following correspondences:*

Solomonoff Induction	*CRR Framework*
Universal prior $2^{-K(m)}$	*Model prior $p(m)$, rigidity Ω*
Cumulative surprise $\sum K(y_i\|y_{<i}, m)$	*Coherence $\mathcal{C}_m(n)$*
Posterior update	*Rupture condition*
Mixture over all programs	*Regeneration over history*
Incomputable	*Computable (with fixed model class)*

Proof. We verify compatibility along four dimensions.

Dimension 1: Prior Structure. Solomonoff assigns $P(m) \propto 2^{-K(m)}$. CRR uses prior odds in the rigidity:

$$\Omega_{m,m'} = \log \frac{p(m)}{p(m')} \tag{35}$$

These are compatible when $p(m) \propto 2^{-K(m)}$, which is the algorithmic probability distribution.

Dimension 2: Likelihood Accumulation. Solomonoff accumulates $-\log P_{\mathcal{S}}(y_i|y_{<i})$. CRR accumulates:

$$\mathcal{C}_m(n) = \sum_i -\log p(y_i|y_{<i}, m) + \text{const} \tag{36}$$

These match by Levin's theorem (up to constants).

Dimension 3: Model Switching. Solomonoff implicitly switches model weights continuously via the mixture:

$$P_{\mathcal{S}}(y_{n+1}|y_{1:n}) = \sum_m w_m(n) \cdot p(y_{n+1}|m) \tag{37}$$

where $w_m(n) \propto p(m) \cdot p(y_{1:n}|m)$.

CRR makes this explicit via threshold-crossing rupture:

$$\text{Switch when } \mathcal{C}_m(n) - \mathcal{C}_{m'}(n) > \Omega_{m,m'} \tag{38}$$

Both implement Bayesian model comparison; CRR discretizes the continuous reweighting into discrete switches.

Dimension 4: Reconstruction. Solomonoff's posterior is:

$$P(m|y_{1:n}) \propto 2^{-K(m)} \cdot 2^{-K(y_{1:n}|m)} \tag{39}$$

CRR regeneration weights by:

$$w(\tau) \propto \exp(\mathcal{C}(\tau)/\Omega) \tag{40}$$

By Theorem 3.5, these correspond under appropriate parameter identification.

Therefore, CRR and Solomonoff induction are compatible frameworks—CRR is a tractable approximation that discretizes and makes explicit what Solomonoff leaves continuous and implicit. $\qquad\square$

5 What CRR Contributes Beyond Solomonoff

Theorem 5.1 (CRR's Novel Contributions). *CRR provides three contributions not explicit in Solomonoff induction:*

(i) **Actionable Decision Rule**: *An explicit threshold for when to switch models*

(ii) **Online Tractability**: *A computable approximation for finite model classes*

(iii) **Multiscale Structure**: *A framework for understanding nested cycles at different temporal scales*

Proof. We establish each contribution.

Contribution (i): Actionable Decision Rule.

Solomonoff induction maintains a mixture over all programs, with weights evolving continuously. It does not specify when to "commit" to a particular model.

CRR provides an explicit rule: switch from m to m' when

$$\mathcal{C}_m(n) - \mathcal{C}_{m'}(n) > \Omega_{m,m'} \tag{41}$$

This is equivalent to the Sequential Probability Ratio Test (SPRT), which is *optimal* in the Wald sense—minimizing expected sample size for given error probabilities.

Proof of SPRT equivalence: The log-likelihood ratio is:

$$\Lambda_n = \log \frac{p(y_{1:n}|m')}{p(y_{1:n}|m)} = \mathcal{C}_m(n) - \mathcal{C}_{m'}(n) \tag{42}$$

The SPRT stops when $\Lambda_n > A$ (accept m') or $\Lambda_n < -B$ (accept m). Setting $A = \Omega_{m,m'}$ recovers CRR rupture.

Contribution (ii): Online Tractability.

Solomonoff induction is incomputable (requires solving the halting problem). CRR restricts to a finite model class $\mathcal{M}$ and becomes fully computable:

- Coherence: $O(n \cdot |\mathcal{M}|)$ to track all models

- Rupture check: $O(|\mathcal{M}|^2)$ pairwise comparisons

- Regeneration: $O(n)$ weighted integration

This is the standard Bayesian model comparison, but CRR adds:

1. Explicit threshold semantics (not just posterior weights)

2. Connection to stopping time theory (Wald, martingales)

3. MaxEnt derivation of regeneration weights

Contribution (iii): Multiscale Structure.
Solomonoff induction operates at a single scale. CRR introduces multiscale coupling:

$$L^{(n+1)}(t) = \sum_{t_k \in T^{(n)}} \lambda^{(n)} \cdot R^{(n)}(t_k) \cdot \delta(t - t_k) \tag{43}$$

This shows that:

- What appears as smooth accumulation at scale $n + 1$ is counting ruptures at scale n

- The rigidity scales: $\Omega^{(n+1)} \approx \pi \cdot \Omega^{(n)}$ (conjectured)

- Higher scales become more regular: $\mathrm{CV}^{(n+1)} \approx \mathrm{CV}^{(n)} / \sqrt{M^{(n)}}$

This multiscale structure has no analogue in standard Solomonoff induction. $\qquad\square$

6 Rigidity Parameter Analysis

6.1 The Parameter in Both Frameworks

Definition 6.1 (Algorithmic Rigidity)**.** In algorithmic terms, the rigidity between models is:

$$\Omega_{m,m'} = K(m') - K(m) + K(\text{switch}) \tag{44}$$

where $K(\text{switch})$ is the complexity of the model-switching mechanism.

Proposition 6.2 (Rigidity Bounds)**.** *For computable models m, m' in a universal coding scheme:*

$$0 \leq \Omega_{m,m'} \leq K(m') + O(1) \tag{45}$$

Proof. Lower bound: If $K(m') \leq K(m)$, then $\Omega_{m,m'} \geq K(\text{switch}) \geq 0$.

Upper bound: We can always describe the switch as "output m' and ignore m," which costs $K(m') + O(1)$. $\qquad\square$

6.2 The $= 1/$ Conjecture

CRR conjectures a universal rigidity value $\Omega^* = 1/\pi \approx 0.318$. We analyze this from the Solomonoff perspective.

Proposition 6.3 (Information-Theoretic Interpretation of $= 1/$)**.** *If $\Omega = 1/\pi$, then at the rupture threshold:*

$$Bayes\ Factor = e^{\Omega} = e^{1/\pi} \approx 1.375 \tag{46}$$

This is a weak preference (barely above 1), suggesting CRR rupture is "early" relative to strong Bayesian evidence.

Remark 6.4 (Geometric Origin)**.** The information geometry derivation suggests $\Omega = \pi/\sqrt{\kappa}$ where κ is the curvature of the statistical manifold. For $\kappa = \pi^2$, we get $\Omega = 1/\pi$. This connects to the Bonnet-Myers theorem's diameter bound.

Remark 6.5 (Open Problem)**.** Neither framework provides a first-principles derivation of $\Omega = 1/\pi$. This remains the key open problem in CRR theory.

7 Comparison Table

Table 1: Detailed Comparison of Solomonoff Induction and CRR

Aspect	Solomonoff Induction	CRR Framework
Foundation	Algorithmic information theory	Bayesian model comparison
Prior	$P(m) = 2^{-K(m)}$	$P(m)$ arbitrary; $\Omega = \log(p(m)/p(m'))$
Update mechanism	Continuous mixture reweighting	Discrete threshold-crossing
Decision rule	None explicit (mixture only)	Rupture when $\mathcal{C}_m - \mathcal{C}_{m'} > \Omega$
Computability	Incomputable	Computable for finite $\mathcal{M}$
Optimality	Optimal prediction (Solomonoff thm)	Optimal switching (SPRT/Wald)
Scale structure	Single scale	Multiscale with coupling
Temporal structure	Continuous	Discontinuous (C-R-R cycles)
Memory	Implicit in weights	Explicit via regeneration operator
Key parameter	Universal machine U	Rigidity Ω

8 Limitations and Open Questions

8.1 Limitations of CRR Relative to Solomonoff

1. **Model Class Restriction**: CRR requires specifying $\mathcal{M}$ in advance; Solomonoff considers all computable hypotheses.

2. **No Universal Convergence Guarantee**: Solomonoff's theorem guarantees convergence to any computable truth. CRR guarantees optimal switching within $\mathcal{M}$, but may fail if the truth is outside $\mathcal{M}$.

3. **Threshold Discretization**: CRR's discrete ruptures may miss the continuous optimality of Solomonoff's mixture.

8.2 Advantages of CRR Over Solomonoff

1. **Computability**: CRR is implementable; Solomonoff is not.

2. **Actionability**: CRR provides explicit decision rules; Solomonoff provides only probabilities.

3. **Multiscale Analysis**: CRR extends to nested temporal scales; Solomonoff does not.

4. **Physical Interpretation**: CRR connects to thermodynamics, ergodic theory, and information geometry.

8.3 Open Questions

1. **Universal** : Can $\Omega = 1/\pi$ be derived from algorithmic information theory?

2. **Convergence Rate**: What is the convergence rate of CRR relative to Solomonoff as $|\mathcal{M}| \to \infty$?

3. **Continuous Limit**: Does CRR with $\Omega \to 0$ (frequent ruptures) converge to Solomonoff's mixture?

4. **Multiscale Kolmogorov**: Is there an algorithmic interpretation of CRR's scale coupling?

9 Conclusion

We have established a rigorous mathematical relationship between CRR and Solomonoff induction:

1. **Compatibility**: The frameworks are mathematically compatible. CRR coherence corresponds to accumulated Kolmogorov complexity, rupture to MDL model switching, and regeneration to the universal prior.

2. **Complementarity**: Solomonoff provides theoretical optimality guarantees; CRR provides tractable, actionable decision rules.

3. **Extensions**: CRR adds three elements not in Solomonoff: explicit thresholds, computable approximations, and multiscale structure.

The key insight is that **CRR is to Solomonoff induction as SPRT is to Bayesian updating**: both pairs share the same theoretical foundation, but the former provides actionable stopping rules while the latter provides continuous probability updates.

For practitioners, CRR offers a computationally tractable framework grounded in optimal inference theory. For theorists, the CRR-Solomonoff correspondence suggests deep connections between discontinuous change, information compression, and bounded rationality.

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A Proof Details

A.1 Levin's Coding Theorem

Theorem A.1 (Levin). *For any computable probability distribution P and string x:*

$$-\log P(x) = K(x) + O(K(P)) \tag{47}$$

Proof Sketch. The upper bound $-\log P(x) \leq K(x) + O(K(P))$ follows from Shannon-Fano coding: we can encode x using $\lceil -\log P(x) \rceil$ bits plus the program for P.

The lower bound $K(x) \leq -\log P(x) + O(K(P))$ follows from the fact that prefix-free codes correspond to probability distributions (Kraft inequality), and the shortest program defines a universal distribution dominating all computable distributions. $\square$

A.2 Chain Rule for Kolmogorov Complexity

Theorem A.2 (Chain Rule). *For strings x, y:*

$$K(x, y) = K(x) + K(y|x^*) + O(\log K(x, y)) \tag{48}$$

where x^* is the shortest program for x.

A.3 SPRT Optimality

Theorem A.3 (Wald-Wolfowitz). *Among all sequential tests with type I error $\leq \alpha$ and type II error $\leq \beta$, the SPRT minimizes the expected sample size under both hypotheses.*

This establishes that CRR rupture, being equivalent to SPRT, implements *optimal* model switching.