Debonding of Cellular Structures Under Shear Deformation

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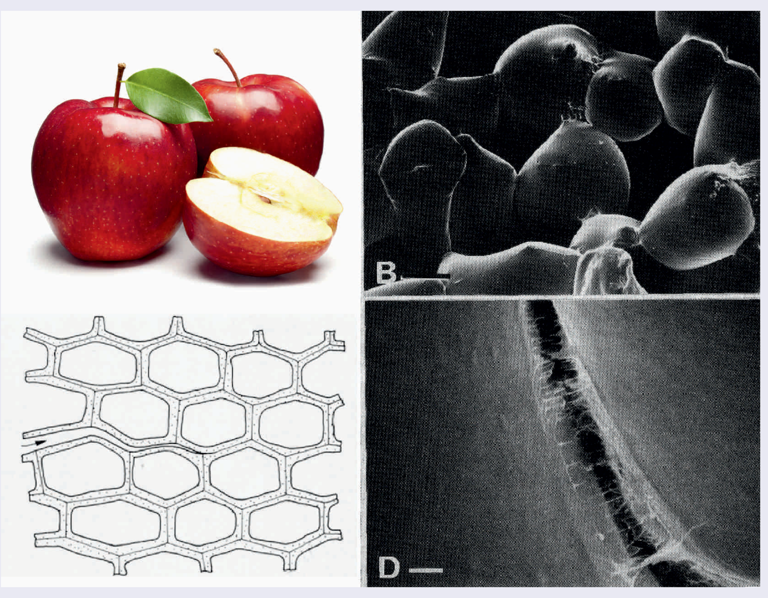
Introduction

Flesh of the apple and potato consist of an assembly of fluid-filled cells with intercellular cohesion.



Fracture surface of apple cells showing cell-cell debonding. Harker & Harlett (1992)

In a fresh, juicy and crispy apple the tissue failure is predominately through cell rupture. Cell-cell debonding becomes the main mode of failure when the cell wall strength increases and intercellular cohesion degrades [1]. This occurs during cold storage and produces a texture known as mealy.

Aim

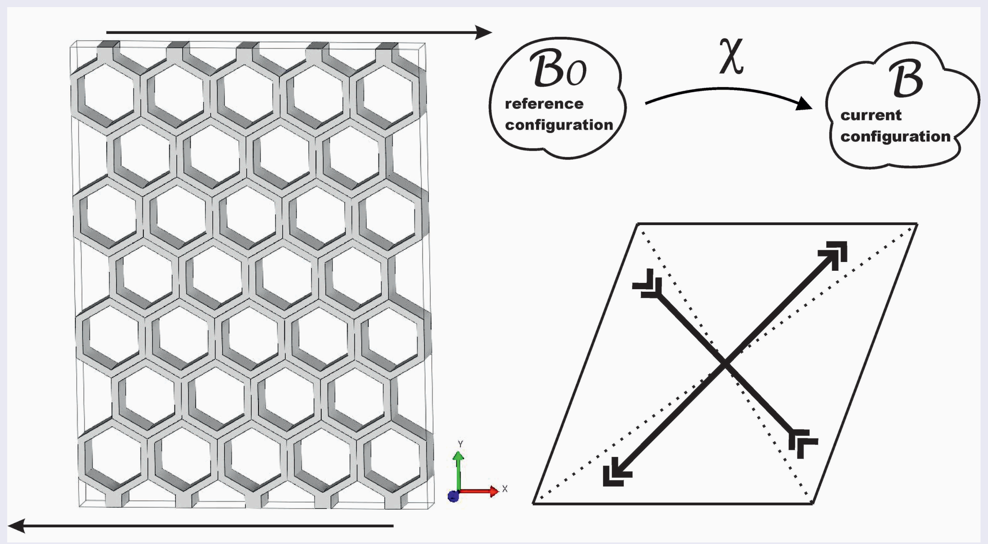
To explore the relationship between microscopic variables and the macroscopic behaviour of debonding when tissue is subject to shear deformation.

Assumptions

Using large strain elasticity theory -

- Elastostatic deformation (no dynamics).
- Cell walls modelled as hyperelastic material.

Uniform cell size, shape and distribution.
Non-penetrative, frictionless and cohesionless contact between cells.



(Right Top) Non-linear elastic deformation. (Left) Geometry and direction of shear. (Right Bottom) Principle stretches.

Elastostatic Equilibrium

Lagrangian equation of non-linear elastostatics, in absence of body forces, where $\mathbf{P}(\mathbf{X})$ is the 1$^{\text{st}}$ Piola-Kirchhoff stress:

$$\text{Div}\,\mathbf{P}(\mathbf{X}) = \mathbf{0} \text{ in } \Omega \qquad (1)$$

Dirichlet boundary condition (fixed displacement):

$$\mathbf{u}(\mathbf{X}) = \mathbf{u}_D \text{ on } \Gamma_D \qquad (2)$$

Neumann boundary condition (fixed surface load):

$$\mathbf{P}(\mathbf{X})\mathbf{N} = \mathbf{g}_N \text{ on } \Gamma_N \qquad (3)$$

Contact Conditions

Non-penetration:

$$\eta(\mathbf{X} + \mathbf{u}(\mathbf{X})) \leq 0 \text{ on } \Gamma_C \qquad (4)$$

η - relative distance to obstacle.
Normal contact pressure:

$$\mathbf{P}(\mathbf{X})\mathbf{N}\cdot\mathbf{N} = \mathbf{g}_C\cdot\mathbf{N} \leq 0 \text{ on } \Gamma_C \qquad (5)$$

Complementarity condition:

$$(\eta(\mathbf{X} + \mathbf{u}(\mathbf{X})))(\mathbf{P}(\mathbf{X})\mathbf{N}\cdot\mathbf{N}) = 0 \qquad (6)$$

Newton's third law (equal and opposite reaction):

$$\mathbf{g}_C(\mathbf{X}) = -\mathbf{g}_C(\mathbf{X}') \text{ if } \chi(\mathbf{X}) = \chi(\mathbf{X}') \text{ on } \Gamma_C \qquad (7)$$

Weak Formulation

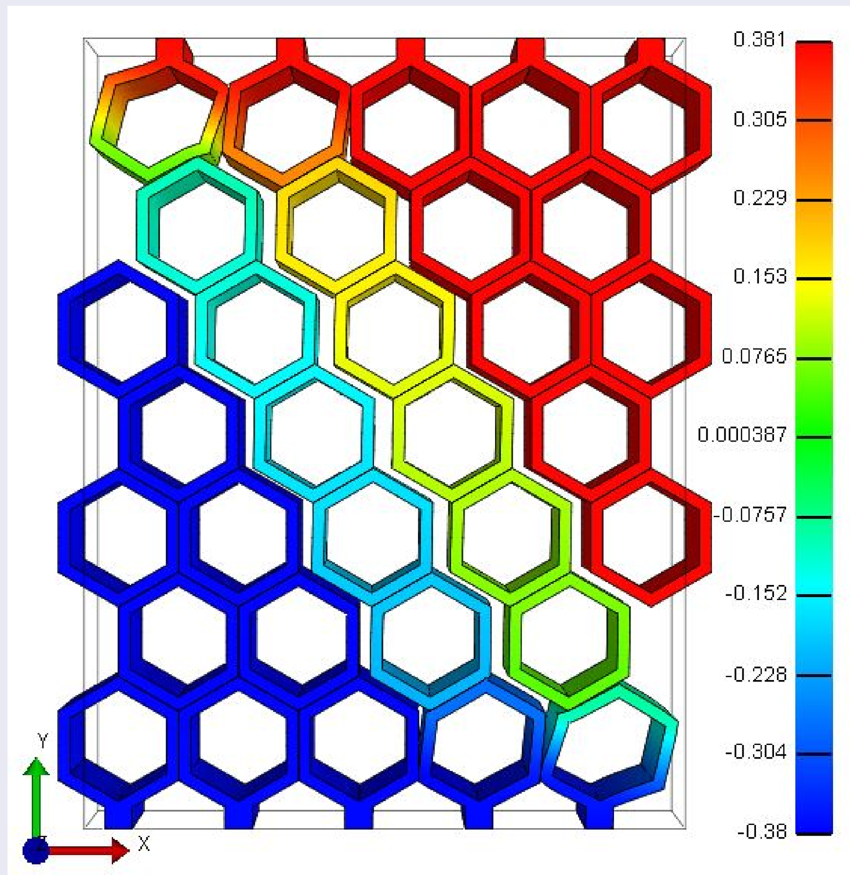
Find the displacement $\mathbf{u} = \mathbf{x} - \mathbf{X}$ such that

$$\int_\Omega \frac{\partial \mathcal{W}}{\partial \mathbf{F}}(\mathbf{X}, \mathbf{F}(\mathbf{u})) : \nabla \mathbf{v} dV = \int_{\Gamma_N} \mathbf{g}_N \cdot \mathbf{v} dA + \int_{\Gamma_C} \frac{\partial \mathcal{W}}{\partial \mathbf{F}} \mathbf{N} \cdot (\mathbf{v} + \mathbf{u}) dA \qquad (8)$$

$\mathcal{W}$ - strain energy density function; $\mathbf{F} = \mathbf{I} + \nabla\mathbf{u}$ - deformation gradient; $\mathbf{v}$ - test function.

Challenge: Detection and resolution of contact and gap openings under contact constraints (4)-(7).

Finite Element Approximation in FEBio

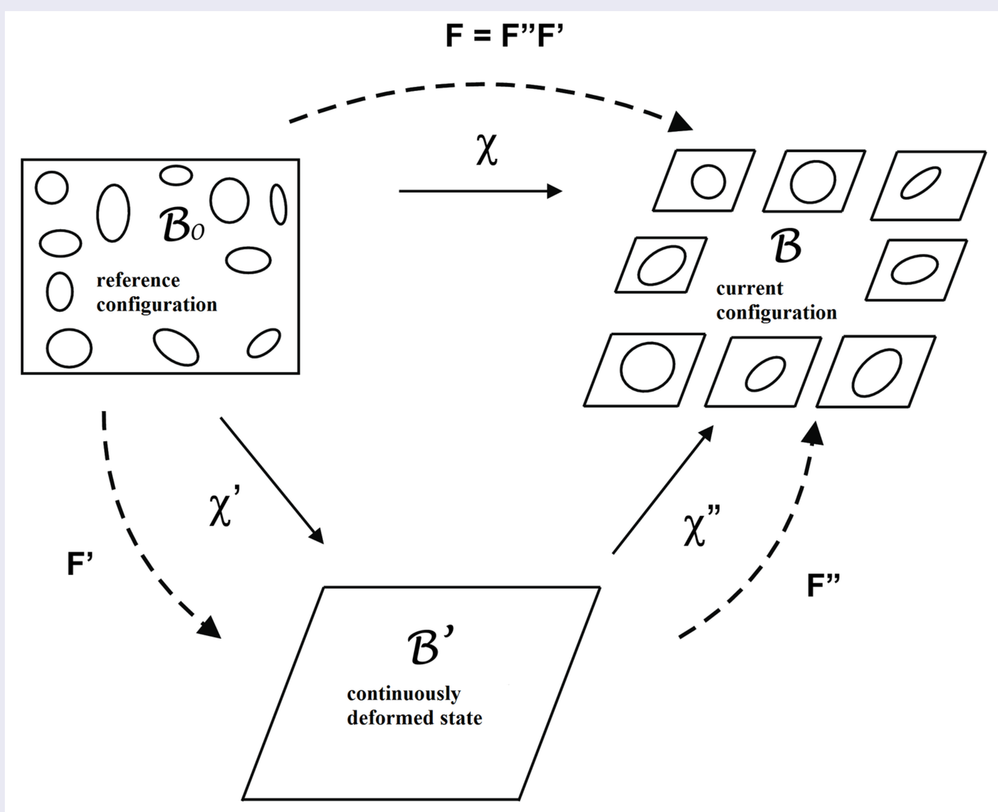


Gap openings during shear deformation (colour chart: X displacement).

For this model, 73 contact interfaces being resolved at every iteration step which leads to slow calculation. A better approach is needed.

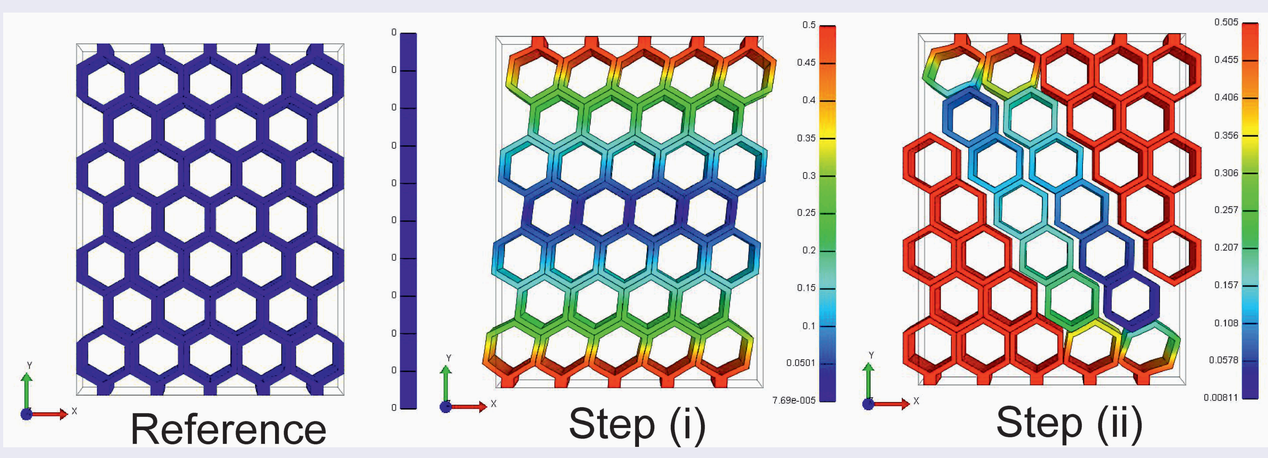
Successive Decomposition Procedure (SDP)

Step (i) - Deform the structure as one continuous, seamless assembly of cells subject to external boundary conditions (1)-(3).
Step (ii) - Take the deform state from (i) and apply contact conditions (4)-(7) at all cell interfaces to allow for possible separation.



Computational efficiency [2]:

- Contact/gap detection is resolved only once in the final iteration step.
- Computing time decreases from 1 hour to 35 sec.



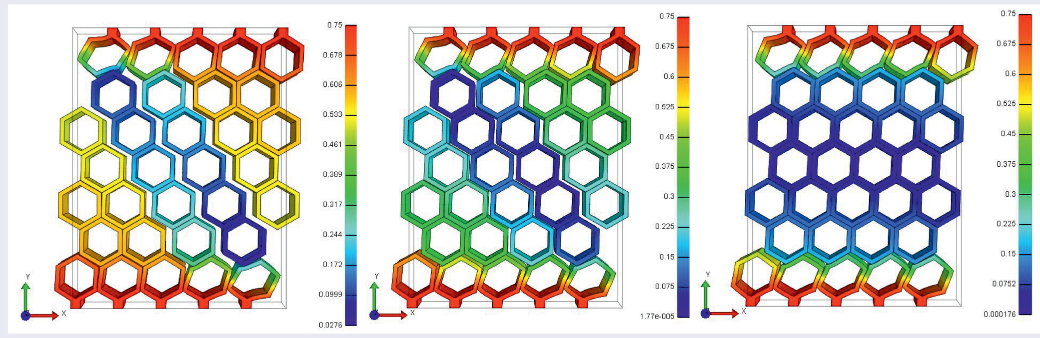
Gap openings during SDP steps (colour chart: X displacement).

Intercellular Cohesion

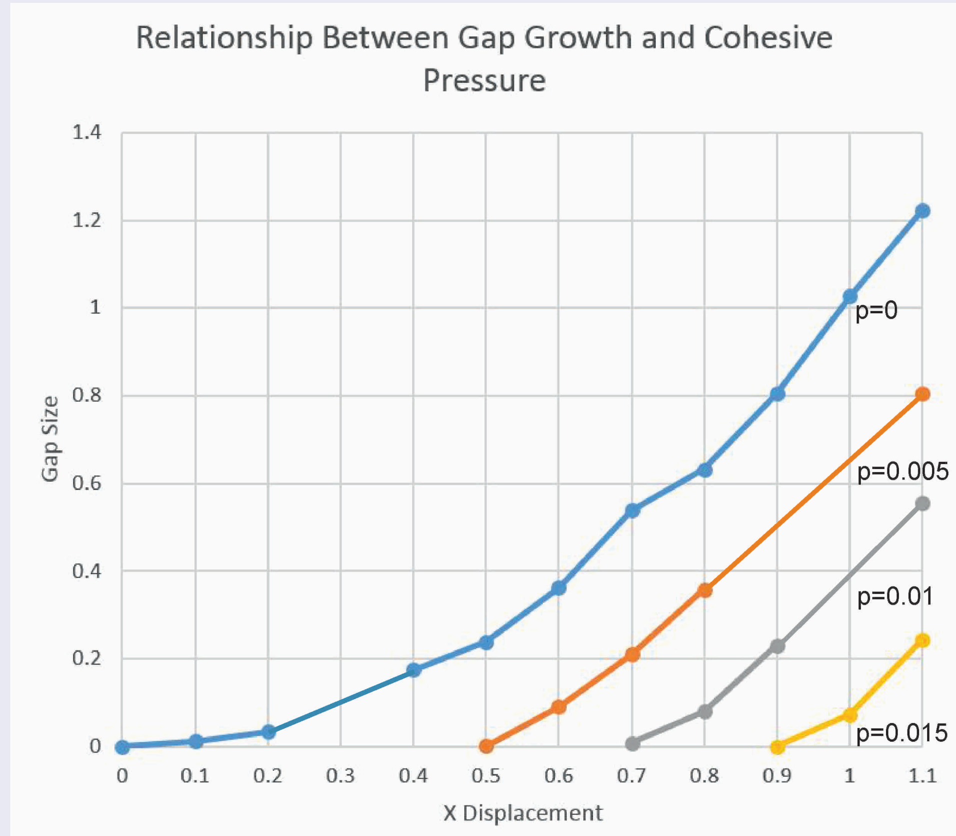
Cohesion on a contact interface requires a tensile force over a critical value to separate faces.

$$\mathbf{P}(\mathbf{X})\mathbf{N}\cdot\mathbf{N} \leq g_{cohesion} \text{ on } \Gamma_C \qquad (9)$$

Amending equations (4)-(6) is computationally expensive. However, normal compressive surface pressure on each side of cell walls has the same cohesive effect and is a standard Neumann boundary condition (3).



Deformation with variable cohesion. (From left to right) Pressure=0.005, 0.01 & 0.015, for fixed X displacement=0.75.

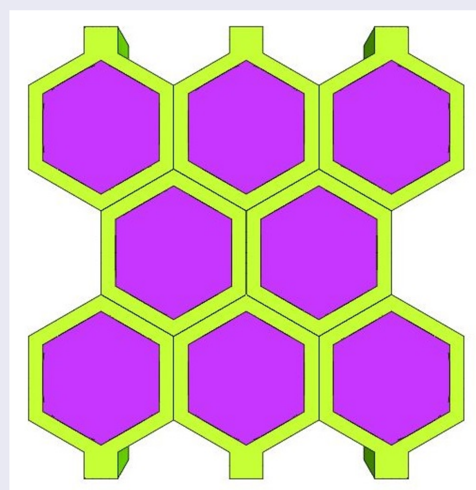


Gap size during deformation where p models cohesion strength.

Gaps initiate at greater deformation, for greater pressure. This is the expected behaviour for cohesion.

Cell Contents

Cell-core effects can be captured in two ways:
1. A weak elastic inclusion provides a volumetric constraint. Here, gaps grow at a slower rate.



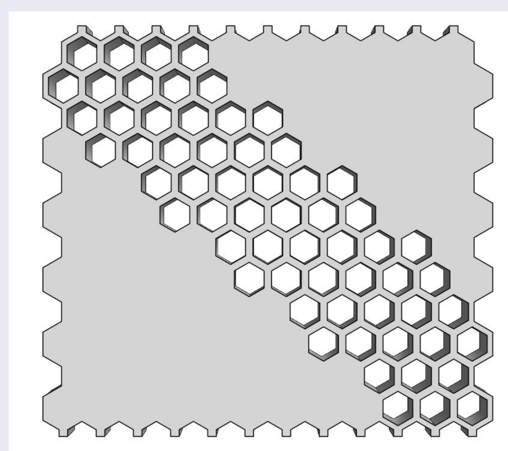
Representative tissue with cell inclusions.

2. Normal pressure on the cell walls represents cell swelling and dehydration. Identical to the above technique for modelling cohesion, cell expansion or retraction will work directly with, or against cohesion.

Multi-Scale Modelling

For structures with a large number of cells, a multi-scale model will be developed whereby the areas where debonding does not occur will be modelled as a continuum [3]. This will improve computational efficiency, and in some cases, will make the difference between finding a solution or not.

Challenge: Find a suitable criterion relating micro- and meso-scale level.



Representative multi-scale geometry.

References

[1] F.R. Harker, I.C. Hallett. 1992. Hort Science 27, 1291-1294.
[2] L.A. Mihai, A. Safar, H.L. Wyatt. 2017. J Eng Math. DOI: 10.1007/s10665-016-9894-2
[3] L.A. Mihai, H.L. Wyatt, A. Goriely. 2017. Proc R Soc A 473, 20170036. DOI: 10.1098/rspa.2017.0036