Hyperelastic Multiscale Models for Stretch-Dominated Cellular Materials, Lattices and Foams

Alex Safar and L. Angela Mihai

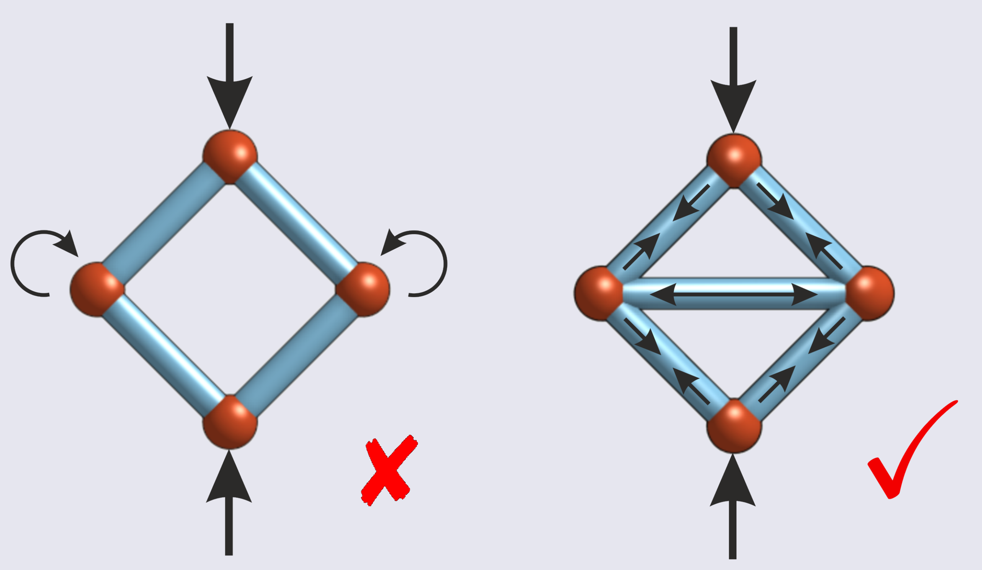
E-mail: safarat@cardifff.ac.uk
School of Mathematics, Cardiff University, UK

Introduction

Cellular materials are computationally expensive to model, so multiscale methods are developed. We can use such models to understand the parameter space and design materials with desirable properties.

Stretch-Dominated Behaviour

The dominant behaviour of a structure can be either through stretching of the cell walls, or bending. If rotation of the joints is restricted by the architecture (as below, right), then the walls will deform by triaxial stretch only.



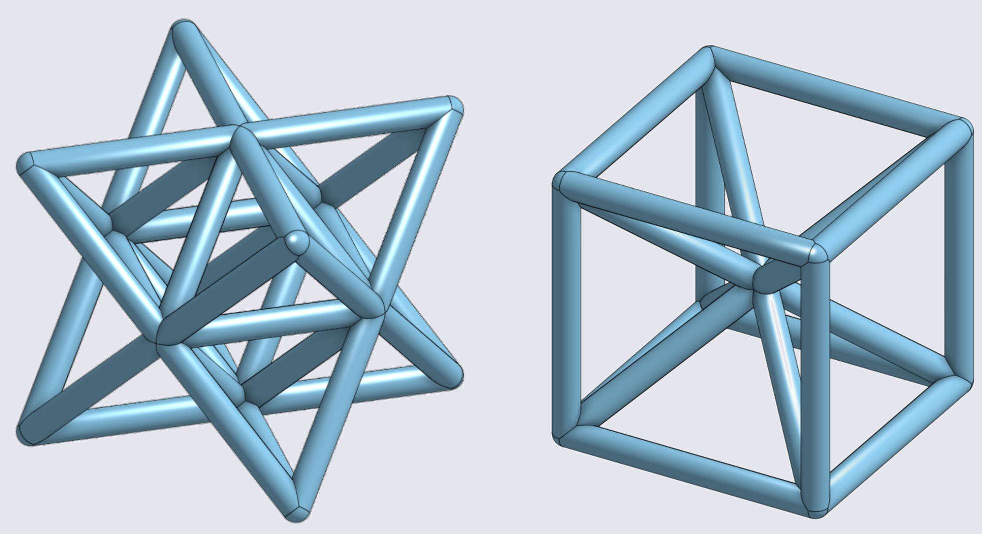
The 3D Maxwell stability criterion states that if

$$b - 3j + 6 \geq 0$$

then the structure will be stretch-dominated, where b and j are the number of beams and joints, respectively [1].

Open & Empty Cells

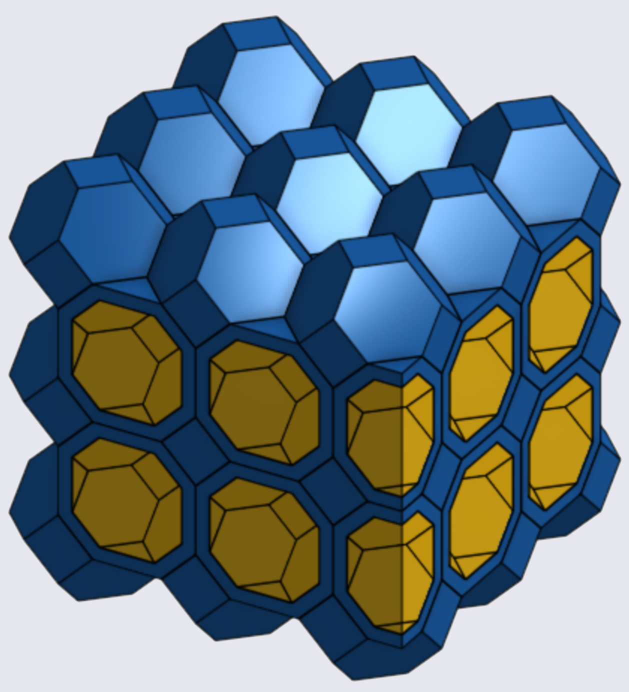
The tetrahedron, octet-truss (left) and body centred cubic (right) satisfy the 3D criterion [2].



Lattices have applications as high stiffness, low density sandwich panels and as scaffolds in tissue engineering [3].

Closed & Fluid-Filled Cells

Many biological tissues have stretch-dominated closed cells, such as skin, brain matter and most fruit/vegetables [4,5].



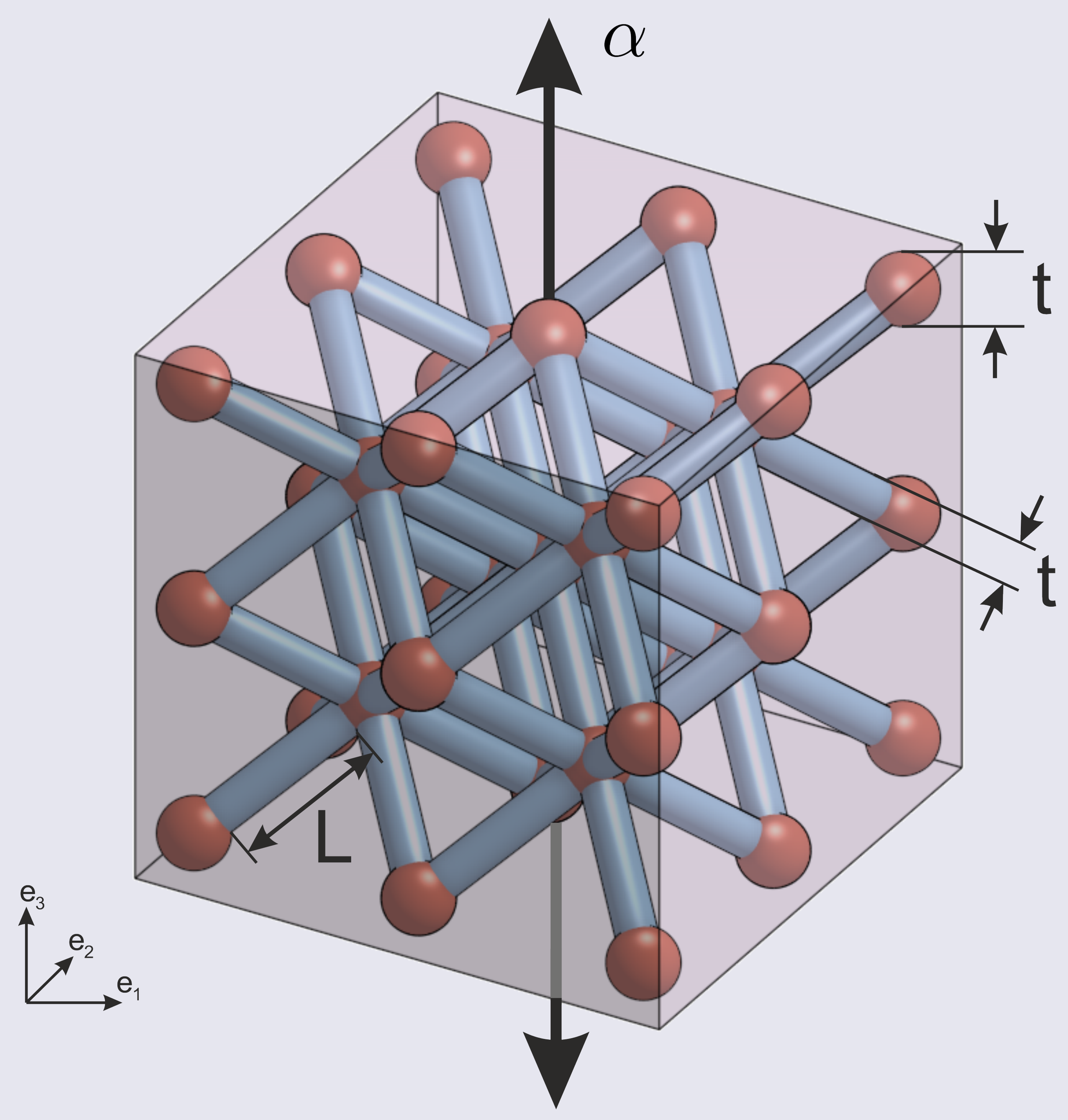
The tetrakaidecahedron is a space filling polyhedra that is stretch-dominated when a cell core is present [2].

Multiscale Method

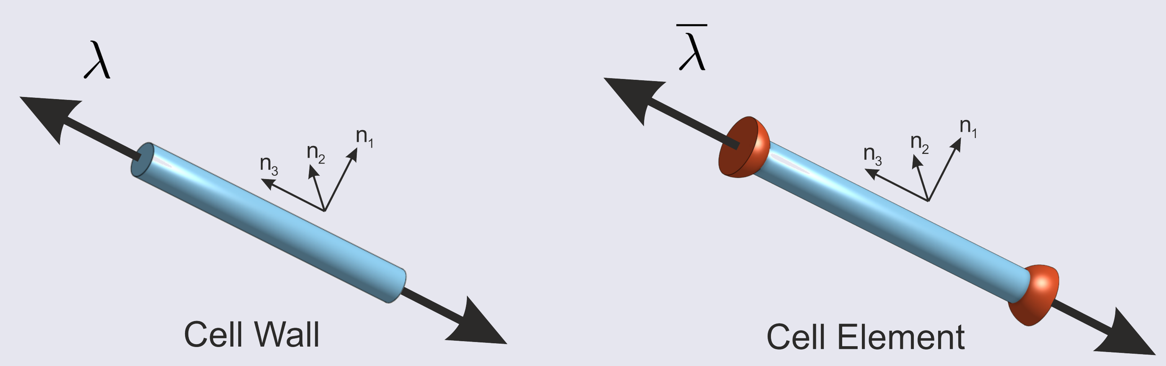
Consider a structure of uniform, arbitrarily orientated cell walls, described by an isotropic nonlinear hyperelastic model [4,5],

$$\mathcal{W}_w(I_1, I_2, I_3)$$

and the structure is subject to a principal stretch α.



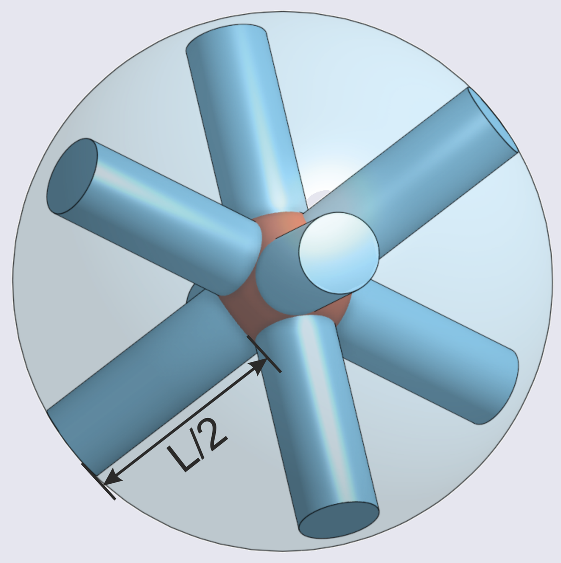
Walls and elements are subject to triaxial stretches λ and $\overline{\lambda}$,



and are related by the assumption cell joints do not deform.

$$\lambda = \overline{\lambda}\left(1 + \frac{t}{L}\right) - \frac{t}{L}$$

Volume ratio ρ, of cell wall (blue) in a representative volume.



By frame indifference of rigid body rotation, the principal invariants are the same for both the structure and cell element.

$$\alpha_1 + \alpha_2 + \alpha_3 = i_1 = \overline{i}_1 = \overline{\lambda}_1 + \overline{\lambda}_2 + \overline{\lambda}_3$$
$$\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 = i_2 = \overline{i}_2 = \overline{\lambda}_1\overline{\lambda}_2 + \overline{\lambda}_1\overline{\lambda}_3 + \overline{\lambda}_2\overline{\lambda}_3$$
$$\alpha_1\alpha_2\alpha_3 = i_3 = \overline{i}_3 = \overline{\lambda}_1\overline{\lambda}_2\overline{\lambda}_3$$

Hence,

$$\mathcal{W}_w(I_1, I_2, I_3) = \overline{\mathcal{W}}_w(\overline{i}_1, \overline{i}_2, \overline{i}_3) = \overline{\mathcal{W}}_w(i_1, i_2, i_3)$$

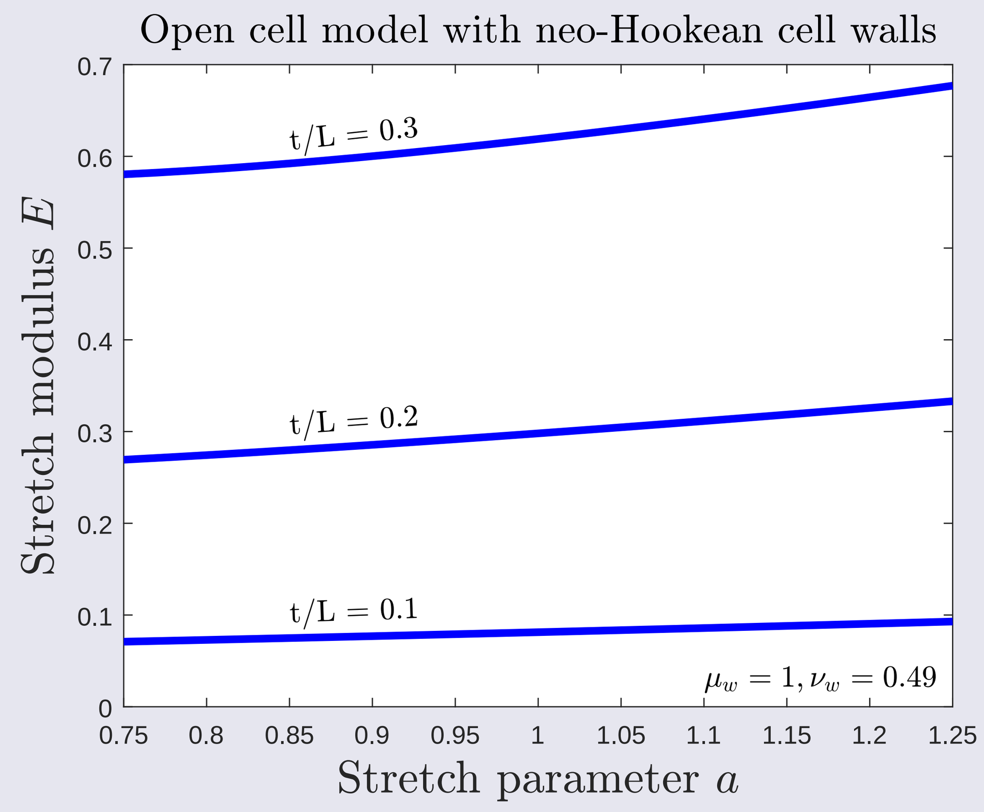
We define the hyperelastic model for an open cell structure.

$$\mathcal{W}^{(o)} = \rho\frac{2}{\pi}\int_0^{\pi/2}\int_0^{\pi/2} \overline{\mathcal{W}}_w \sin\theta \, d\theta d\phi$$

Nonlinear Stretch Modulus

For an isotropic hyperelastic material, the nonlinear stretch modulus is defined in terms of the principal stresses $(\sigma_1, \sigma_2, \sigma_3) = (0, 0, N)$ and the principal stretches $(\alpha_1, \alpha_2, \alpha_3) = (\alpha(a), \alpha(a), a)$, and reflects stiffening or softening under uniaxial loads [4].

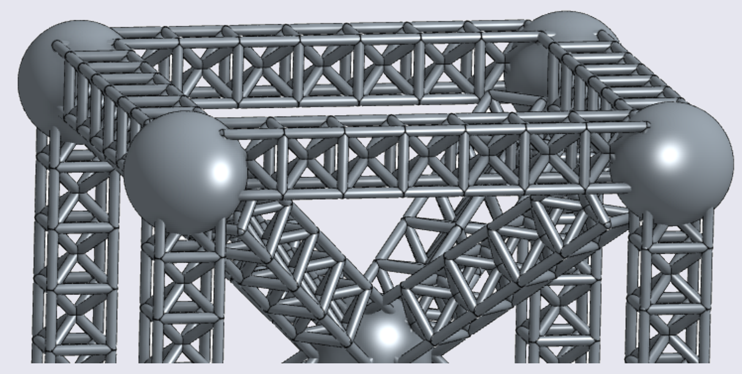
$$E = \frac{N}{\ln a - \ln \alpha(a)}\left(1 - \frac{a\alpha'(a)}{\alpha(a)}\right)$$



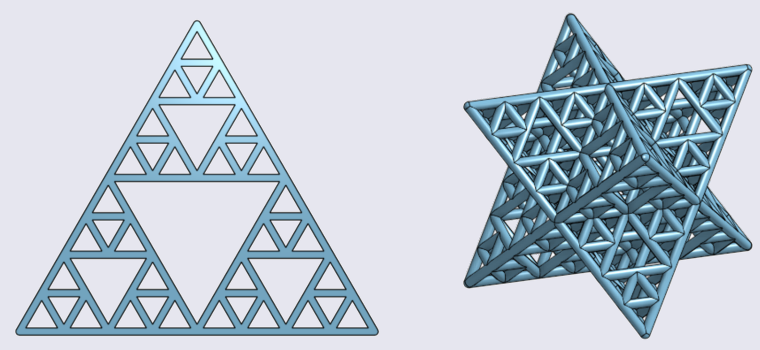
Here, stiffness increases with wall thickness and stretch.

Hierarchical Structures

This cellular material model can be used in structures with a hierarchical architecture [5].



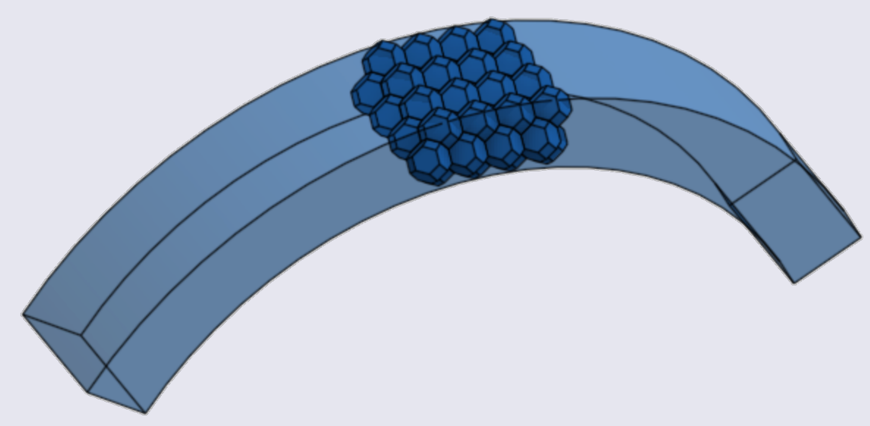
Self-similar or fractal architectures can be captured by iteratively applying the multiscale method, or a cell size factor.



The open and closed cell models can also be combined for a structure analogous to fibre reinforced walls.

Generative Modelling

Micro-scale effects, such as cell-cell debonding, can be studied by modelling explicit microstructures embedded in the bulk continuum material.



This remodelling can be targeted by detecting the areas of greatest stress in the continuum model.

References

[1] Deshpande VS, Ashby MF, Fleck NA. 2001. Foam topology bending versus stretching dominated architectures, Acta Materialia 49, 1035-1040.
[2] Vigliotti A, Pasini D. 2013. Mechanical properties of hierarchical lattices, Mechanics of Materials 62, 32-43.
[3] Gibson LJ, Ashby MF, Harley BA. 2010. Cellular Materials in Nature and Medicine, Cambridge University Press, Cambridge, UK.
[4] Mihai LA, Goriely A. 2017. How to characterize a nonlinear elastic material? A review on nonlinear constitutive parameters in isotropic finite elasticity, Proc. Royal Soc. A 473, 20170607.
[5] Safar A, Mihai LA. 2018. The nonlinear elasticity of hyperelastic models for stretch-dominated architectures, International Journal of Non-Linear Mechanics, in review.