Generalised Shear with Unilateral Contact in Cellular Solids

Alex Safar and L. Angela Mihai
Work Funded by EPSRC and Cardiff University
E-mail: safarat@cardifff.ac.uk
School of Mathematics, Cardiff University, UK

EPSRC Engineering and Physical Sciences Research Council

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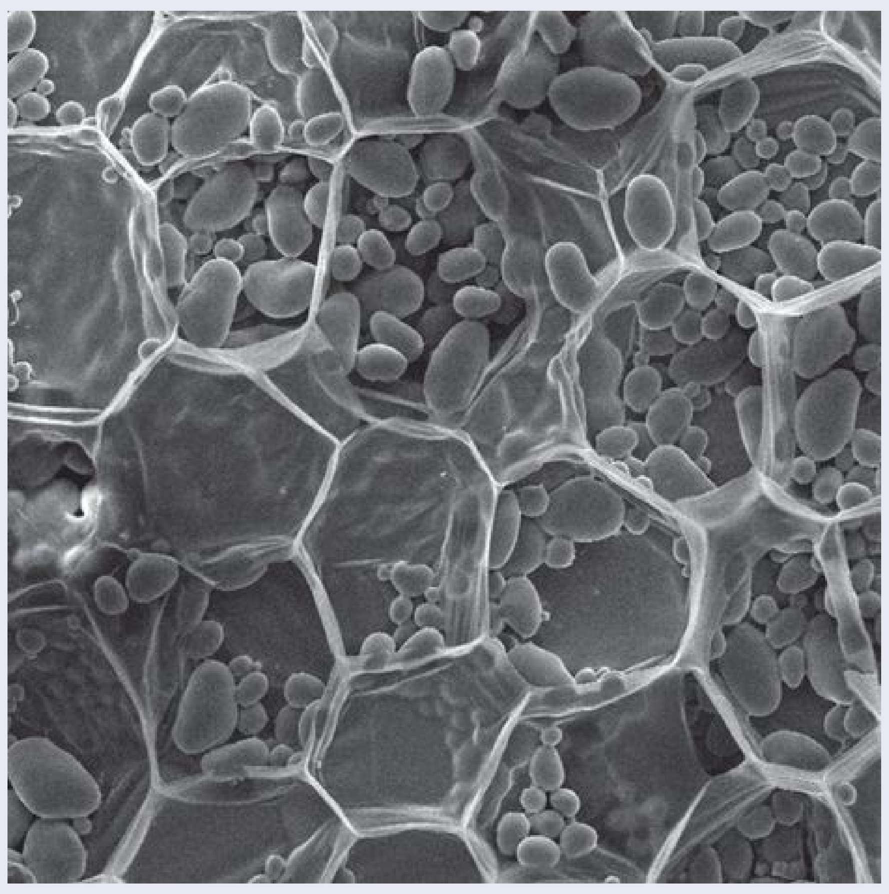
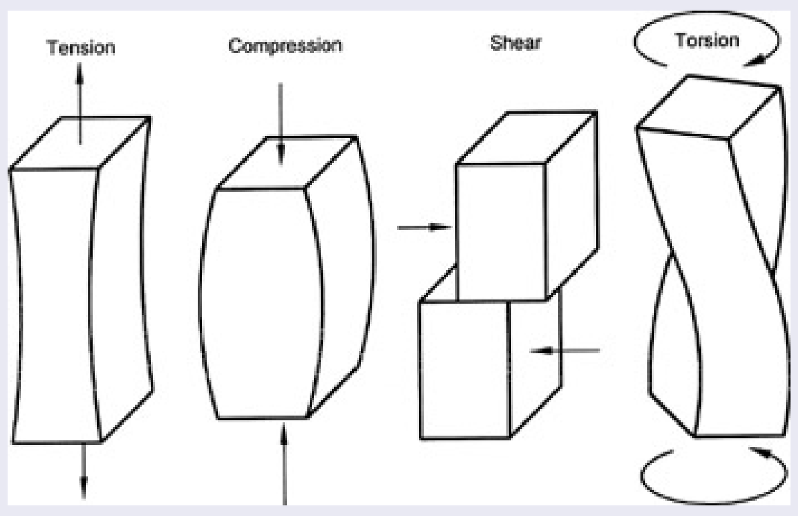
Introduction



Scanning electron micrographs of potato parenchyma

Flesh of the apple and potato consist of a collection of fluid-filled cells bound together by an adhesive substance [1].



Deformation modes of rectangular cell walls

In this study we are interested in the mechanical response of such tissues when subject to shear deformation.

Physical Properties

Cell properties determine tissue behaviour and applied external forces change the cell responses as deformation progresses [2]. These relations lead to nonlinear mechanical behaviour and the requirement for a multiscale approach.

Tissue properties (macro-scale):
Tissue stress-strain relation (stiffness), load at failure (strength).

Cell properties (micro-scale):
Cell size and geometry, wall thickness and elasticity, core pressure, intercellular contact.

Modelling and Analysis

For the analysis of cellular materials we use the framework of finite elasticity which is useful for predicting large strain deformations in soft materials [3]. For numerical solutions, the chosen software is FEBio, a Finite Element (FE) suite for solving nonlinear large deformation problems in biosolid mechanics [4].

An approximate 3D geometry of tetrakaidecahedral cells

Elastostatic Equilibrium

For a deformed body, the equilibrium state in the absence of body load is described in terms of the Cauchy stress $\boldsymbol{\sigma}$ by the Eulerian field equation:

$$\operatorname{div} \boldsymbol{\sigma} = \mathbf{0} \tag{1}$$

The above governing equation is completed by a constitutive law for $\boldsymbol{\sigma}$, depending on material properties, and supplemented by boundary conditions.

Contact Conditions

Unilateral (non-penetrative) contact conditions, in the absence of friction:

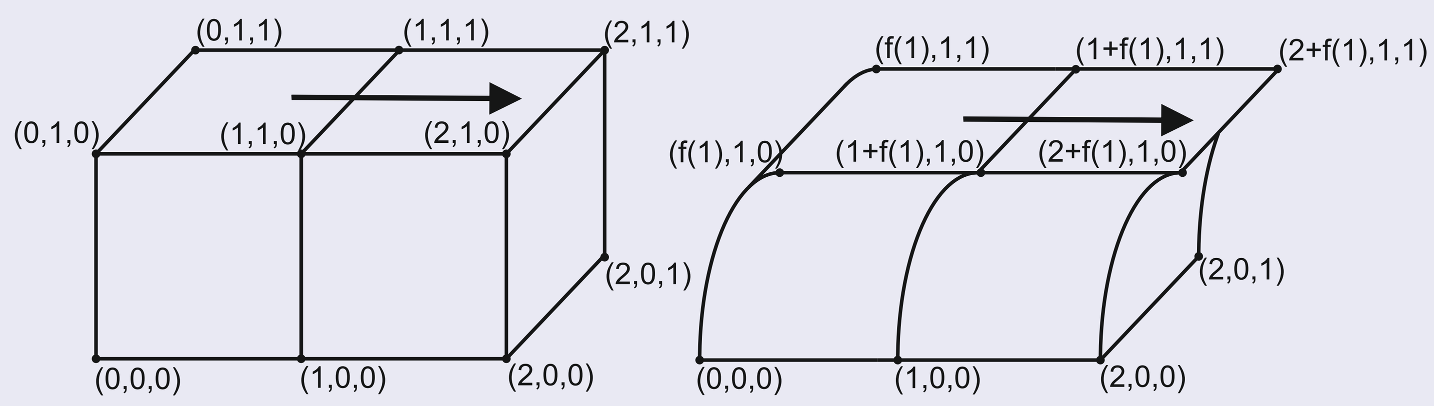
$$\boldsymbol{u}(\boldsymbol{X}) \cdot \boldsymbol{n} \leq d \tag{2}$$

$$\boldsymbol{g}_c(\boldsymbol{X}) \cdot \boldsymbol{n} \leq g \tag{3}$$

$$(\boldsymbol{u}(\boldsymbol{X}) \cdot \boldsymbol{n} - d)(\boldsymbol{g}_c(\boldsymbol{X}) \cdot \boldsymbol{n} - g) = 0 \tag{4}$$

Where $\boldsymbol{u}(\boldsymbol{X}) \cdot \boldsymbol{n}$ is the normal distance of a material point from an obstacle, $d \geq 0$ is the relative distance which cannot be exceeded by contact points, $\boldsymbol{g}_c(\boldsymbol{X}) \cdot \boldsymbol{n}$ is the normal contact force at a material point and $g \geq 0$ is the cohesion parameter. The complementarity condition (4) states that, if the objects are in contact, then there may be contact forces, otherwise there are no contact forces.

Generalised Shear



Parallel blocks in contact, subject to shear.
Left: Reference configuration (X,Y,Z) **Right**: Current configuration (x,y,z)

Hyperelastic material: Mooney-Rivlin strain energy density function is used to represent the solid material:

$$W(I_1, I_2) = \frac{\mu_1}{2}(I_1 - 3) + \frac{\mu_2}{2}(I_2 - 3) \tag{5}$$

Where μ_1 and μ_2 are material dependant constants and I_1 and I_2 are principal invariants of the deformation.

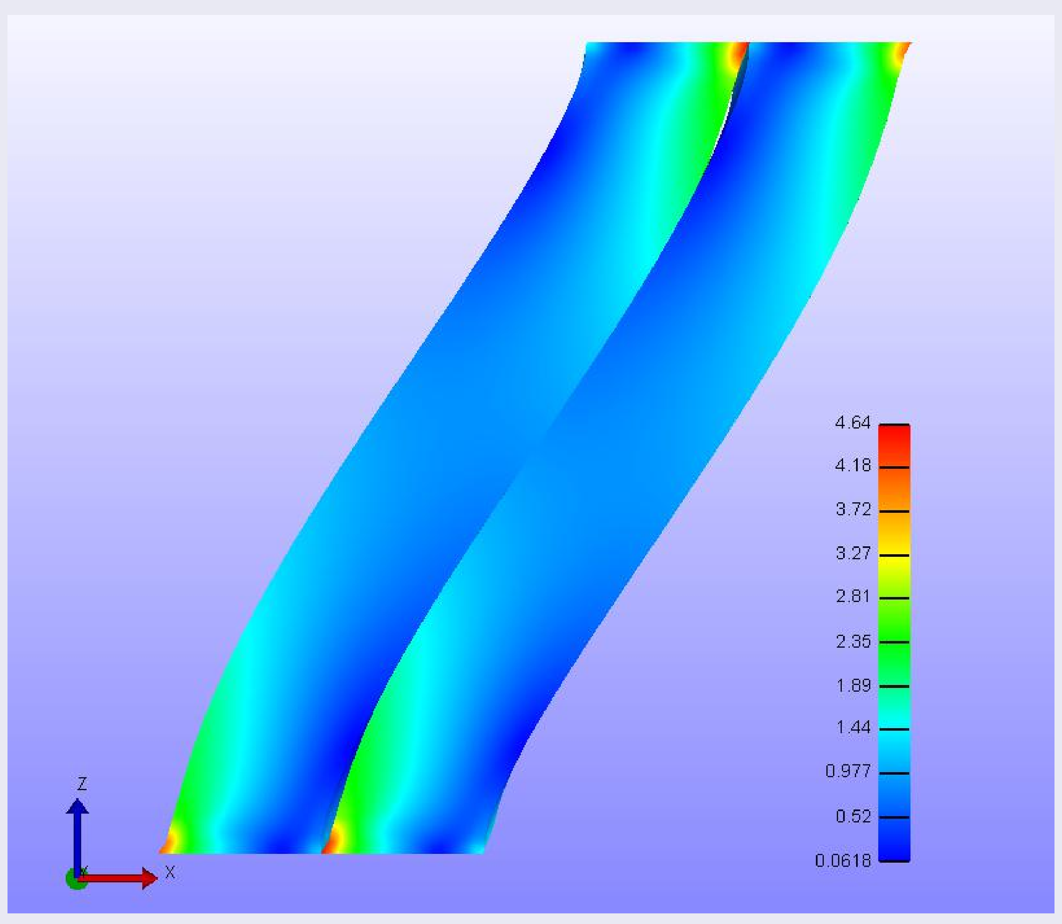
Finite deformation: Gradient tensor for generalised shear is given by [5]:

$$\mathrm{F} = \begin{bmatrix} 1 & f'(y) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

If $f(0) = 0$ and $f(1) = k$, then $f(y) = ay^2 + by$ where $a + b = k$.

FE Simulation of Shearing Walls

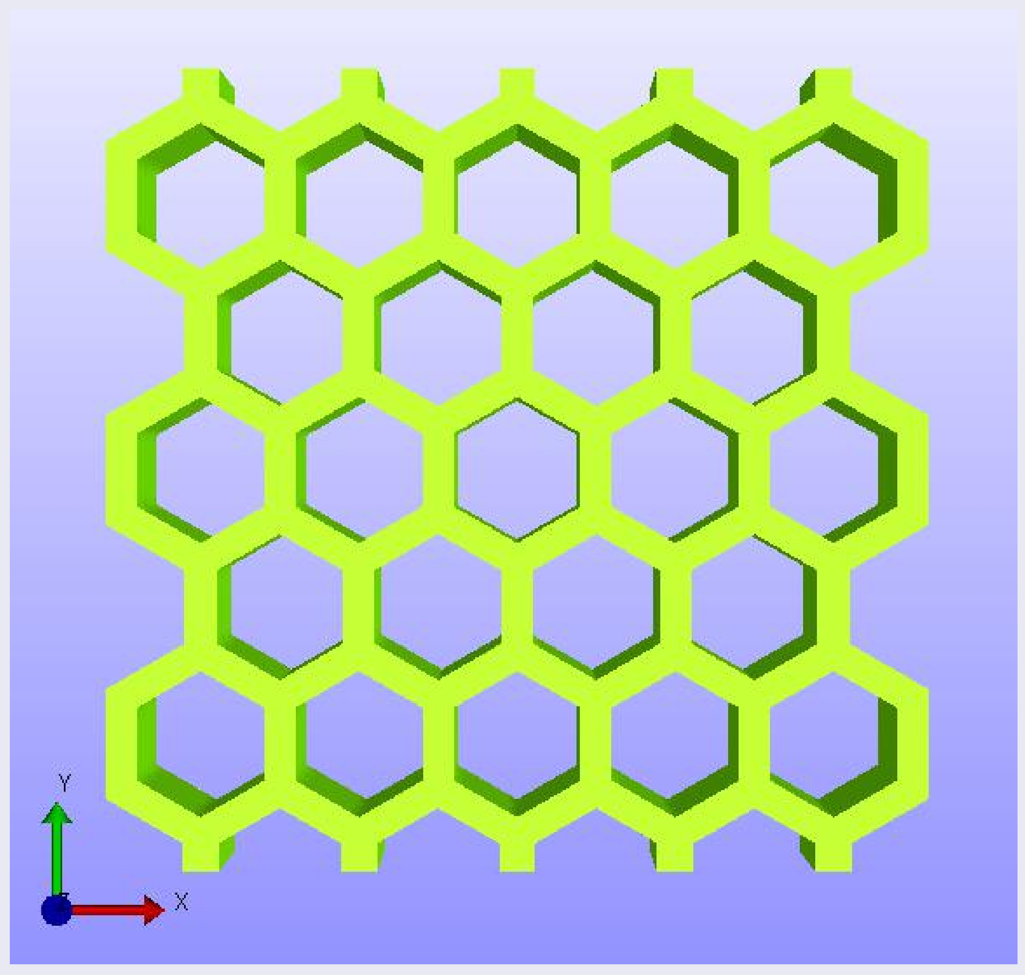
Two columns of Mooney-Rivlin material in unilateral contact subject to generalised shear is a good approximation for the behaviour of cell walls. Analytical solution and numerical simulation show that gaps are created close to the top and bottom corners of the walls, which in a cellular material would mean debonding.



Effective stress of parallel columns in unilateral contact subject to generalised shear

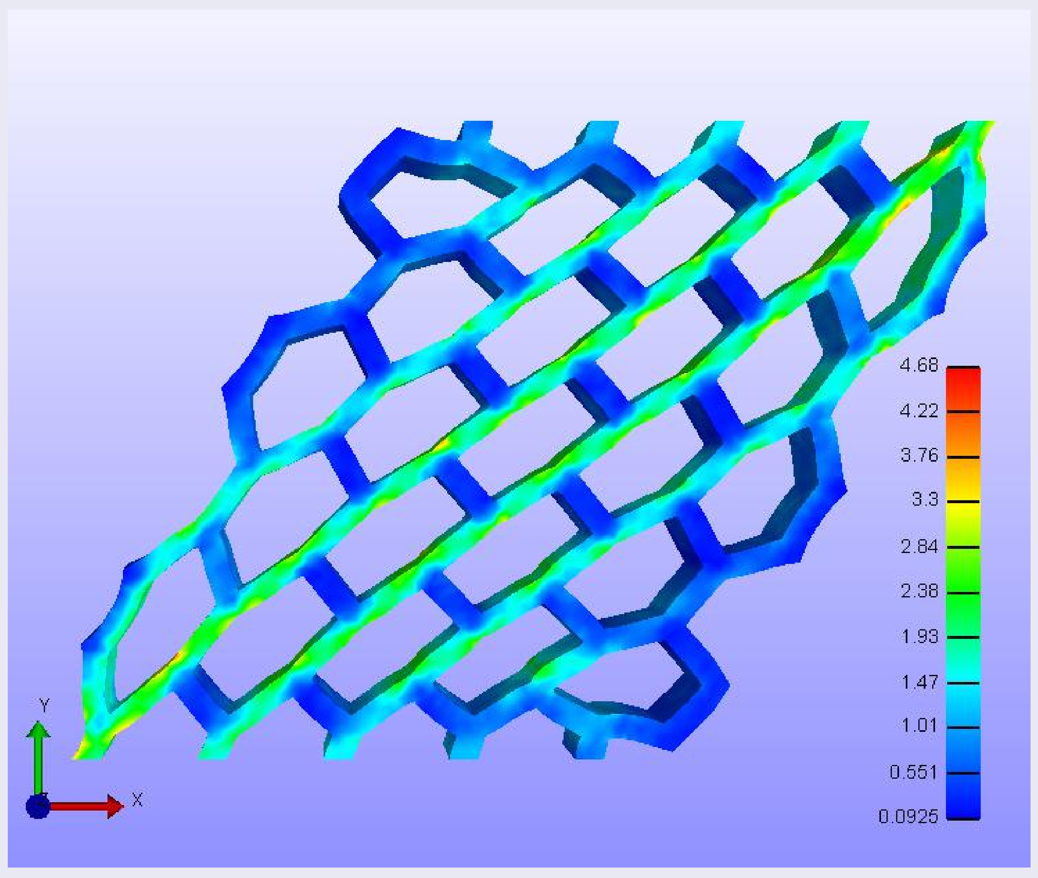
Future Work

Cell debonding is a spontaneous mechanism for crack propagation in many natural structures which has been less investigated to date.



Example FE model of a continuous hexagonal array [A]

Cellular bodies will be modelled as hexagonal (2D) or tetrakaidecahedral (3D) arrays of Neo-Hookean or Mooney-Rivlin material, with or without inclusions. Each cell will have the ability to debond with its neighbours. However, a multiscale solution approach will be developed in order to model large scale structures, with areas where debonding cannot occur described by a continuous hyperelastic material.



Effective stress of [A] under shear deformation

Question: How does debonding propagate under combined loading (e.g. compression and shear), depending on cell size and geometry, cell wall elastic properties, and internal pressure? For example, cell wall thickness and cell pressure change with fruit maturity.

Applications

Agriculture: Damage during handling and transport of fruit and vegetables.
Biomedical: Bio-materials and tissue engineering.

References

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[3] Le Tallec P (1994). Handbook of Numerical Analysis, V3, 465-624.
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