# Randomized Complete Block Design (RCBD)

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## Introduction

Blocking is used to handle nuisance factors (effects on responses but not of interest) that are known and controllable. If all treatments are applied within each block and are compared within blocks, the design is (complete) blocking. Randomization is restricted because only occurs within blocks.

Pros: Eliminate between-block variation from experimental error variance.

Cons: Degrees of freedom

## Statistical Model: Two-Way ANOVA

$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$, $i = 1, 2, ..., a$ and $j = 1, 2, ..., b$ where $a$ is the number of treatments and $b$ is the number of experimental units.

$\mu =$ grandmean, $\tau_i = i$th treatment effect, $\beta_j = j$th block effect, $\epsilon_{ij} \sim N(0, \sigma^2)$

$\sum_{i=1}^{a} \tau_i = 0$; $\sum_{j=1}^{b} \beta_j = 0$

### Parameter Estimates

$\hat{\mu} = \bar{y}_{..}$; $\hat{\tau}_i = \bar{y}_{i.} - \bar{y}_{..}$; $\hat{\beta}_j = \bar{y}_{.j} - \bar{y}_{..}$; $\hat{\epsilon}_{ij} = y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..}$

### Sum of Squares

$SS_T = SS_{Treatment} + SS_{Block} + SS_E = \sum\sum(y_{ij} - \bar{y}_{..})^2 = (\sum\sum y_{ij}^2) - y_{..}^2/N$ with df $= ab - 1$

$SS_{Treatment} = b\sum(\bar{y}_{i.} - \bar{y}_{..})^2 = b\sum\hat{\tau}_i^2 = \frac{1}{b}(\sum y_{i.}^2) - y_{..}^2/N$ with df $= a - 1$

$SS_{Block} = a\sum(\bar{y}_{.j} - \bar{y}_{..})^2 = a\sum\hat{\beta}_i^2 = \frac{1}{a}(\sum y_{.j}^2) - y_{..}^2/N$ with df $= b - 1$

$SS_E = \sum\sum(y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 = \sum\sum\hat{\epsilon}_{ij}^2$ with df $(a-1)(b-1)$

### Mean Squared

$MS_{Treatment} = SS_{Treatment}/(a-1)$ and expected value is $\sigma^2 + \sigma_{\tau\beta}^2 + b\sum_{i=1}^{a}\tau_i^2/(a-1)$

$MS_{Block} = SS_{Block}/(b-1)$ and expected value is $\sigma^2 + a\sum_{i=1}^{b}\beta_i^2/(b-1)$

$MS_E = SS_E/((a-1)(b-1)) = \sigma^2 + \sigma_{\tau\beta}^2$ and expected value is $\sigma^2$

$\sigma_{\tau\beta}^2 = 0$ if there is no interaction.

### Hypothesis Testing

Usually tests treatment effect, block effects are not usually of interest.

$H_0 : \tau_1, \tau_2, ..., \tau_a = 0$ vs. $H_1$: at least one is not

$F_0 = \frac{MS_{Treatment}}{MS_E} = \frac{SS_{Treatment}/(a-1)}{SS_E/((a-1)(b-1))} \sim F_{\alpha,a-1,(a-1)(b-1)}$

### ANOVA Table

■ **TABLE 4.2**

**Analysis of Variance for a Randomized Complete Block Design**

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | $F_0$ |
|---|---|---|---|---|
| Treatments | $SS_{\text{Treatments}}$ | $a-1$ | $\frac{SS_{\text{Treatments}}}{a-1}$ | $\frac{MS_{\text{Treatments}}}{MS_E}$ |
| Blocks | $SS_{\text{Blocks}}$ | $b-1$ | $\frac{SS_{\text{Blocks}}}{b-1}$ | |
| Error | $SS_E$ | $(a-1)(b-1)$ | $\frac{SS_E}{(a-1)(b-1)}$ | |
| Total | $SS_T$ | $N-1$ | | |

Figure 1: RCBD Table

## Checking Assumptions

### Checking Normality

Histogram, QQ plot of residuals, Shapiro-Wilk Test

### Checking Constant Variance

Residual plots: residuals vs ($\hat{y}_{ij}$, blocks, treatments)

### Checking Additivity (no interaction between treatment and block effects)

Residuals vs $\hat{y}_{ij}$, Tukey's One-degreee Freedom Test of Non-additivity

#### Tukey's Test for Non-additivity

To check for interaction fully needs $(a-1)(b-1)$ degrees of freedom (needs replicates).

$y_{ij} = \mu + \tau_i + \beta_j + \gamma\tau_i\beta_j + \epsilon_{ij}$

$H_0 : \gamma = 0$ vs. $H_1 : \gamma \neq 0$

$SS_N = \frac{[\sum_i \sum_j y_{ij} y_{i.} y_{.j} - y_{..}(SS_{Trt} + SS_{Blk} + y_{..}^2/ab)]^2}{abSS_{Trt}SS_{Blk}}$, with df $= 1$

$SS'_E = SS_E - SS_N$, with df $= (a-1)(b-1) - 1$

$F_0 = \frac{SS_N}{SS'_E/[(a-1)(b-1)-1]} \sim F_{\alpha,1,(a-1)(b-1)-1}$

Note: Convenient Procedure Fit additive model, acquire $\hat{y}_{ij}$, $q_{ij} = \hat{y}_{ij}^2$, then fit the model with $q_{ij}$, and then check type III SS for $q_{ij}$

# Post-ANOVA Treatments Comparison

Similar to Completely Randomized Design (CRD) procedure. Replace $n$ by $b$ and degrees of freedom for error is $(b-1)(a-1)$