Latin Square Design

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Introduction

Latin square design is a block design with two blocking factors and one treatment factor. Represented by a *pxp* grid with the rows and columns as blocks. The cells have letters which are the treatments. Every row and column contains all the Latin letters. This design blocks two nuisance factors and has two restrictions on randomization.

Model with No Replication

$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \epsilon_{ijk}$, $i, j, k = 1, 2, ..., p$

Assumptions

μ - grand mean

α_i - ith block 1 effect (ith row effect): $\sum_i^p \alpha_i = 0$

τ_j - jth treatment effect: $\sum_j^p \tau_j = 0$

β_k - kth block 2 effect (kth row effect): $\sum_k^p \beta_k = 0$

$\epsilon_{ijk} \sim N(0, \sigma^2)$: (Normality, Independence, Constant Variance)

Completely additive model (no interaction)

Estimates of Parameters

$\hat{\mu} = \bar{y}_{...}$

$\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...}$

$\hat{\tau}_j = \bar{y}_{.j.} - \bar{y}_{...}$

$\hat{\beta}_k = \bar{y}_{..k} - \bar{y}_{...}$

$\hat{\epsilon}_{ijk} = y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}$

Sum of Squares

$SS_{Rows} = p\sum(\bar{y}_{i..} - \bar{y}_{...})^2$

$SS_{Treatment} = p\sum(\bar{y}_{.j.} - \bar{y}_{...})^2$

$SS_{Columns} = p\sum(\bar{y}_{..k} - \bar{y}_{...})^2$

$SS_E = \sum\sum \hat{\epsilon}_{ijk}^2$

$SS_T = \sum\sum(y_{ijk} - \bar{y}_{...})^2$

ANOVA Table

■ **TABLE 4.10**

Analysis of Variance for the Latin Square Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}} = \frac{1}{p}\sum_{j=1}^{p} y_{.j.}^2 - \frac{y_{...}^2}{N}$	$p-1$	$\frac{SS_{\text{Treatments}}}{p-1}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Rows	$SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i..}^2 - \frac{y_{...}^2}{N}$	$p-1$	$\frac{SS_{\text{Rows}}}{p-1}$	
Columns	$SS_{\text{Columns}} = \frac{1}{p}\sum_{k=1}^{p} y_{..k}^2 - \frac{y_{...}^2}{N}$	$p-1$	$\frac{SS_{\text{Columns}}}{p-1}$	
Error	SS_E (by subtraction)	$(p-2)(p-1)$	$\frac{SS_E}{(p-2)(p-1)}$	
Total	$SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$	p^2-1		

Note: $N = p^2$

Hypothesis Testing

$H_0 : \tau_1 = \tau_2 = ... = \tau_p = 0$ vs. H_1 : at least one is not

$F_0 \sim F_{\alpha,p-1,(p-1)(p-2)}$