

```
/* dataset for montgomery problem 2.8. */
data mont2_8;
INPUT x;
datalines;
9.37
13.04
11.69
8.21
11.18
10.41
13.15
11.51
13.21
7.75
;
run;

/* draw histogram with a normal density curve, and a kernel density curve */
proc sgplot data=mont2_8;
    histogram x;
    density x;
    density x / type=kernel;
run;

/* Within each type group, get basic descriptive info
for the response variable "y" */

proc univariate data=mont2_8;
    var x;
run;

/* one sample t-test (e.g., H0=75), with conf. interval included in the results */
proc ttest data=mont2_8 H0=10 alpha=0.05;
    var x;
run;

/* Within each type, use test statistics to check the normality assumption */
proc univariate data=mont2_8 Mu0=10 NORMALTEST;
    var x;
run;
```

**2.8.** Consider the following sample data: 9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, and 7.75. Is it reasonable to assume that this data is a sample from a normal distribution? Is there evidence to support a claim that the mean of the population is 10?

**2.9.** A computer program has produced the following output for a hypothesis-testing problem:

```
Difference in sample means: 2.35
Degrees of freedom: 18
Standard error of the difference in sample means: ?
Test statistic:  $t_0 = 2.01$ 
P-value: 0.0298
```

- (a) What is the missing value for the standard error?
- (b) Is this a two-sided or a one-sided test?
- (c) If  $\alpha = 0.05$ , what are your conclusions?
- (d) Find a 90% two-sided CI on the difference in means.

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2. Montgomery 2.9

a.) The missing standard error value can be computed by the following,

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\bar{y}_1 - \bar{y}_2}{SE}$$

$$SE = \frac{2.35}{2.01} = 1.1692$$

b.) This is a one-sided test,  $t_0 = +2.01$  (right tail). A P value for a two sided t-test with  $t_0 = +2.01$  is .059662, the reported value is half of that. The associated P value ( $P(T \geq t_0 | H_0)$ ) for 18 DF is 0.0298.

c.) For an  $\alpha = 0.05$  level, we would reject the null hypothesis that the means are the same and conclude that there is a difference in the two samples as the P value of 0.0298 is less than the  $\alpha$  limit. **Remember, the results were for the one-sided test**

d.) A 90% confidence interval ( $\alpha = 0.10$ ) is calculated per the following

$$(\bar{y}_1 - \bar{y}_2) - t_{\alpha/2, n_1+n_2-2} \cdot SE \leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + t_{\alpha/2, n_1+n_2-2} \cdot SE$$

$$2.35 - t_{0.05, 18} \cdot 1.1692 \leq \mu_1 - \mu_2 \leq 2.35 + t_{0.05, 18} \cdot 1.1692$$

(one tailed  $t_{0.1/2, 18}$  is  $t_{0.05, 18} = 1.734$ )

$$2.35 - 1.734 \cdot 1.1692 \leq \mu_1 - \mu_2 \leq 2.35 + 1.734 \cdot 1.1692$$

$$0.3226 \leq \mu_1 - \mu_2 \leq 4.3774$$

```
/* Montgomery 2.26 data */
data mont2_26;
input x type @@;
datalines;
65 1 64 2
81 1 71 2
57 1 83 2
66 1 59 2
82 1 65 2
82 1 56 2
67 1 69 2
59 1 74 2
75 1 82 2
70 1 79 2
;
run;

/* two-sample t-test, confidence interval for the difference of two group means are included in the result */
proc ttest data=mont2_26 alpha=0.05;
class type;
var x;
run;
```

2.26. The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the mean and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

- (a) Test the hypothesis that the two variances are equal. Use  $\alpha = 0.05$ .
- (b) Using the results of (a), test the hypothesis that the mean burning times are equal. Use  $\alpha = 0.05$ . What is the  $P$ -value for this test?
- (c) Discuss the role of the normality assumption in this problem. Check the assumption of normality for both types of flares.

a.) We want to test the following

$$H_0 : \sigma_1 = \sigma_2$$

$$H_1 : \sigma_1 \neq \sigma_2$$

It will be appropriate to conduct an  $F$  test to test equal variance. Reference the output in figure 2 for the output of the two sample t-test of the Montgomery 2.26 data. From “Equality of Variances” output in Figure 2, the value of the test statistic  $F = 1.02$  with an associated  $P$  value of 0.9744, therefore we fail to reject the null and conclude  $H_0$ , the two variances are equal.

b.) With equal variance now assumed, we aim to test the following

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

From output in Figure 2, the value of the test statistic with equal variance assumed is  $t_0 = 0.05$  with an associated  $P$  value of 0.9622, therefore we fail to reject the null and conclude  $H_0 : \mu_1 = \mu_2$ , the means are equal.

c.) The Q-Q normal probability plots in Figure 2 indicate that each data set are approximately normal (do not exhibit any significant deviation from the straight line), and as indicated in the problem description the samples are from two different formulations (random samples drawn from independent populations). Since there are no major deviations from normality in either dataset, the t-test is appropriate; small to moderate deviations from normality will have little impact on its performance.

Please include at what significance level your conclusions are based on

better to also include the test results for normality checking