```
/* Montgomery 13.2 */
proc import datafile="/home/u63048916/STAT571B/Homework/Homework 7/Q13-2.xlsx"
    dbms=xlsx
    out=randr
    replace;
    getnames=yes;
run;

/* 2.1: both factors are random : */

/*  by GLM */
proc glm data=randr;
class Inspector Part;
model Impedance=Inspector|Part;
random Inspector Part Inspector*Part/test;
run;

/*  by proc mixed and method of "type1" */
proc mixed data=randr CL covtest method=type1;
class Inspector Part;
model Impedance=;
random Inspector Part Inspector*Part;
run;

/*  by proc mixed and default method: REML */
proc mixed data=randr CL covtest method=REML;
class Inspector Part;
model Impedance=;
random Inspector Part Inspector*Part;
run;
```

**13.2.** An article by Hoof and Berman ("Statistical Analysis of Power Module Thermal Test Equipment Performance," *IEEE Transactions on Components, Hybrids, and Manufacturing Technology* Vol. 11, pp. 516–520, 1988) describes an experiment conducted to investigate the capability of measurements in thermal impedance (C°/w × 100) on a power module for an induction motor starter. There are 10 parts, three operators, and three replicates. The data are shown in Table P13.2.

**■ TABLE P13.2**
**Power Module Thermal Test Equipment Data for Problem 13.2**

| | Inspector 1 | | | Inspector 2 | | | Inspector 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Test | | | Test | | | Test | | |
| Part No. | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| 1 | 37 | 38 | 37 | 41 | 41 | 40 | 41 | 42 | 41 |
| 2 | 42 | 41 | 43 | 42 | 42 | 42 | 43 | 42 | 43 |
| 3 | 30 | 31 | 31 | 31 | 31 | 31 | 29 | 30 | 28 |
| 4 | 42 | 43 | 42 | 43 | 43 | 43 | 42 | 42 | 42 |
| 5 | 28 | 30 | 29 | 29 | 30 | 29 | 31 | 29 | 29 |
| 6 | 42 | 42 | 43 | 45 | 45 | 45 | 44 | 46 | 45 |
| 7 | 25 | 26 | 27 | 28 | 28 | 30 | 29 | 27 | 27 |
| 8 | 40 | 40 | 40 | 43 | 42 | 42 | 43 | 43 | 41 |
| 9 | 25 | 25 | 25 | 27 | 29 | 28 | 26 | 26 | 26 |
| 10 | 35 | 34 | 34 | 35 | 35 | 34 | 35 | 34 | 35 |

**(a)** Analyze the data from this experiment, assuming that both parts and operators are random effects.

**(b)** Estimate the variance components using the analysis of variance method.

**(c)** Estimate the variance components using the REML method. Use the confidence intervals on the variance components to assist in drawing conclusions.

ANOVA

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 29 | 4023.733333 | 138.749425 | 271.47 | <.0001 |
| Error | 60 | 30.666667 | 0.511111 | | |
| Corrected Total | 89 | 4054.400000 | | | |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Inspector | 2 | 39.266667 | 19.633333 | 38.41 | <.0001 |
| Part | 9 | 3935.955556 | 437.328395 | 855.64 | <.0001 |
| Inspector*Part | 18 | 48.511111 | 2.695062 | 5.27 | <.0001 |

| Source | Type III Expected Mean Square |
|---|---|
| Inspector | Var(Error) + 3 Var(Inspector*Part) + 30 Var(Inspector) |
| Part | Var(Error) + 3 Var(Inspector*Part) + 9 Var(Part) |
| Inspector*Part | Var(Error) + 3 Var(Inspector*Part) |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Error: MS(Inspector*Part) | | | | | |
| Inspector | 2 | 39.266667 | 19.633333 | 7.28 | 0.0048 |
| Part | 9 | 3935.955556 | 437.328395 | 162.27 | <.0001 |
| Error | 18 | 48.511111 | 2.695062 | | |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Inspector*Part | 18 | 48.511111 | 2.695062 | 5.27 | <.0001 |
| Error: MS(Error) | 60 | 30.666667 | 0.511111 | | |

Figure 13.2.1
Analysis of Variance Method

| Cov Parm | Estimate | Standard Error | Z Value | Pr Z | Alpha | Lower | Upper |
|---|---|---|---|---|---|---|---|
| Inspector | 0.5646 | 0.6551 | 0.86 | 0.3888 | 0.05 | -0.7194 | 1.8486 |
| Part | 48.2926 | 22.9067 | 2.11 | 0.0350 | 0.05 | 3.3962 | 93.1890 |
| Inspector*Part | 0.7280 | 0.3011 | 2.42 | 0.0156 | 0.05 | 0.1379 | 1.3181 |
| Residual | 0.5111 | 0.09332 | 5.48 | <.0001 | 0.05 | 0.3682 | 0.7575 |

| Source | DF | Sum of Squares | Mean Square | Expected Mean Square | Error Term | Error DF | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|
| Inspector | 2 | 39.266667 | 19.633333 | Var(Residual) + 3 Var(Inspector*Part) + 30 Var(Inspector) | MS(Inspector*Part) | 18 | 7.28 | 0.0048 |
| Part | 9 | 3935.955556 | 437.328395 | Var(Residual) + 3 Var(Inspector*Part) + 9 Var(Part) | MS(Inspector*Part) | 18 | 162.27 | <.0001 |
| Inspector*Part | 18 | 48.511111 | 2.695062 | Var(Residual) + 3 Var(Inspector*Part) | MS(Residual) | 60 | 5.27 | <.0001 |
| Residual | 60 | 30.666667 | 0.511111 | Var(Residual) | . | . | . | . |

REML Method

-1: Need to check assumptions

| Covariance Parameter Estimates | | | | | | | |
|---|---|---|---|---|---|---|---|
| Cov Parm | Estimate | Standard Error | Z Value | Pr > Z | Alpha | Lower | Upper |
| Inspector | 0.5646 | 0.6551 | 0.86 | 0.1944 | 0.05 | 0.1344 | 67.3537 |
| Part | 48.2926 | 22.9067 | 2.11 | 0.0175 | 0.05 | 22.7624 | 162.52 |
| Inspector*Part | 0.7280 | 0.3011 | 2.42 | 0.0078 | 0.05 | 0.3717 | 2.0164 |
| Residual | 0.5111 | 0.09332 | 5.48 | <.0001 | 0.05 | 0.3682 | 0.7575 |

What does significance mean?

Figure 13.2.3

a.) The ANOVA is run for both factors treated as random effects. Inspector, Part, and interaction Inspector*Part are each individually significant at the $\alpha = 0.05$ level.

b.) See figure 13.2.2. Variance estimates calculated from Analysis of Variance method table.

$$\hat{\sigma}^2 = MS_{Error} = 0.511$$

$$\hat{\sigma}^2_\tau = \frac{MS_{Inspector} - MS_{Inspector*Part}}{j*n} = \frac{19.633 - 2.695}{10*3} = 0.5646$$

$$\hat{\sigma}^2_\beta = \frac{MS_{Part} - MS_{Inspector*Part}}{i*n} = \frac{437.33 - 2.695}{3*3} = 48.293$$

$$\hat{\sigma}^2_{\tau\beta} = \frac{MS_{Inspector*Part} - MS_{Error}}{n} = \frac{2.695 - 0.511}{3} = 0.728$$

These results are confirmed by estimates in table figure 13.2.2.

c,)See figure 13.2.3, REML method produces same variance estimates. However, the Inspector term is not significant at the $\alpha = 0.05$ level.