Graeco-Latin Square Design

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Introduction

Graeco-Latin square design is a Latin square design with $p \geq 3$ (except 6) blocking factors and one treatment factor. Appears as a pxp Latin square superposed on a second pxp Latin Square in which the treatments are denoted by Greek/Latin letters.

If two squares when superimposed have the property that each Greek letter appears once and only once with each Latin letter, the two Latin Squares are orthogonal.

Model

$y_{ijkl} = \mu + \alpha_i + \tau_j + \beta_k + \zeta_l + \epsilon_{ijkl}$, $i, j, k, l = 1, 2, ..., p$

Assumptions

$\mu =$ grand mean

$\alpha_i = i$th block 1 effect(row effect), $\sum \alpha_i = 0$

$\tau_j = j$th treatment effect, $\sum \tau_j = 0$

$\beta_k = k$th block 2 effect(column effect), $\sum \beta_k = 0$

$\zeta_l = l$th block 3 effect(Greek letter effect), $\sum \zeta_l = 0$

$\epsilon_{ijkl} \sim N(0, \sigma^2)$(independent)

Completely additive model (no interaction)

Estimates

$\hat{\mu} = \bar{y}_{....}$

$\hat{\alpha}_i = \bar{y}_{i...} - \bar{y}_{....}$

$\hat{\tau}_j = \bar{y}_{.j..} - \bar{y}_{....}$

$\hat{\beta}_i = \bar{y}_{..k.} - \bar{y}_{....}$

$\hat{\zeta}_l = \bar{y}_{...l} - \bar{y}_{....}$

$\hat{\epsilon}_{ijkl} = y_{ijkl} - \bar{y}_{i...} - \bar{y}_{.j..} - \bar{y}_{..k.} - \bar{y}_{...l} + 3\bar{y}_{....}$

Sum of Squares

$SS_{Row} = p\sum(\bar{y}_{i...} - \bar{y}_{....})^2$

$SS_{Treatment} = p\sum(\bar{y}_{.j..} - \bar{y}_{....})^2$

$SS_{Column} = p\sum(\bar{y}_{..k.} - \bar{y}_{....})^2$

$SS_{Greek} = p\sum(\bar{y}_{...l} - \bar{y}_{....})^2$

$SS_E = \sum\sum\hat{\epsilon}_{ijkl}^2$

■ TABLE 4.19

Analysis of Variance for a Graeco-Latin Square Design

Source of Variation	Sum of Squares	Degrees of Freedom
Latin letter treatments	$SS_L = \frac{1}{p}\sum_{j=1}^{p} y_{.j..}^2 - \frac{y_{....}^2}{N}$	$p - 1$
Greek letter treatments	$SS_G = \frac{1}{p}\sum_{k=1}^{p} y_{..k.}^2 - \frac{y_{....}^2}{N}$	$p - 1$
Rows	$SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i...}^2 - \frac{y_{....}^2}{N}$	$p - 1$
Columns	$SS_{\text{Columns}} = \frac{1}{p}\sum_{l=1}^{p} y_{...l}^2 - \frac{y_{....}^2}{N}$	$p - 1$
Error	SS_E (by subtraction)	$(p-3)(p-1)$
Total	$SS_T = \sum_i\sum_j\sum_k\sum_l y_{ijkl}^2 - \frac{y_{....}^2}{N}$	$p^2 - 1$

Note: $N = p^2$

Hypothesis

$H_0 : \tau_1 = \tau_2 = ... = \tau_p = 0$ vs. H_1 : at least one is not

$F_0 \sim F_{\alpha,p-1,(p-1)(p-3)}$