1. **Montgomery 4.1**
The statistical model is RCBD

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij} \begin{cases} i = 1, 2, ...a \\ j = 1, 2...b \end{cases}$$

Table from Montgomery 4.1

Source	DF	SS	MS	F	P
Treatment	4	1010.56	?	29.84	?
Block	?	?	64.765	?	?
Error	20	169.33	?		
Total	29	1503.71			

a.) Below calculate "?" ANOVA entries.
Calculate Factor DF_{Block}

$$DF_{Block} = DF_{Total} - DF_{Error} - DF_{Treatment} = 29 - 20 - 4 = 5$$

Calculate SS_{Block}

$$SS_{Block} = SS_{Total} - SS_{Treatment} - SS_{Error} = 1503.71 - 1010.56 - 169.33 = 323.82$$

Calculate $MSE_{Treatment}$

$$MSE_{Treatment} = \frac{1}{DF_{Treatment}} \cdot SS_{Treatment} = \frac{1010.56}{4} = 252.64$$

Calculate MSE

$$MSE = \frac{1}{DF_{Error}} \cdot SS_E = \frac{169.33}{20} = 8.467$$

**F and P value apply to all blocks/treatments, so "?" on "Block" line are N/A.
Use online F-ratio calculator for P value for $F(4, 20)$ with F ratio $= 29.84$

$$P < 0.0001$$

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b.) The number of blocks used in the experiment is $DF_{Block} + 1 = 6$. Similarly, there were $DF_{Treatment} + 1 = 5$ treatments within each block, for a total of $6 * 5 = 30 = DF_{Total} + 1$ observations.

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c.) The ANOVA above supports the hypothesis test

$$H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0$$
$$H_a : \text{ at least one is different}$$

The P value $P < 0.0001$ indicates significance, that we reject H_0 and conclude that the means of all $a = 5$ treatments are not equal.

```
/* import data */

PROC IMPORT DATAFILE="/home/u63048916/STAT571B/Homework/Homework 3/Q4-9.csv"
    DBMS=CSV
    OUT=mont4_9
    REPLACE;
    GETNAMES=YES;
RUN;

title 'RCBD ANOVA';
proc glm data=mont4_9;
class Oil Truck;
model Consumption = Truck Oil;
output out=diag r=res p=pred;
Run;

/* check normality */
title 'Check normality of residuals';
proc univariate data=diag normal;
var res;
qqplot res / normal (mu=est sigma=est);
Run;

/* check outliers */
title 'Check outliers';
data outlier;
set diag;
stdres=res/0.022974;
run;

proc print data=outlier;
run;

/* check constant variance using graph*/
title 'residual plot: res vs predicted value ';
proc sgplot data=diag;
scatter x=pred y=res;
refline 0;
run;

title 'residual plot: res vs Oil ';
proc sgplot data=diag;
scatter x=Oil y=res;
refline 0;
run;

title 'residual plot: res vs Truck ';
proc sgplot data=diag;
scatter x=Truck y=res;
refline 0;
run;

/* check additivity for RCBD design (with one replicate)*/
title 'Check Additivity';
data two;
set diag;
q=pred*pred;
run;

title 'Interaction ANOVA';
proc glm data=two;
class Oil Truck;
model Consumption = Truck Oil q/ss3;
run;

/* pairwise compsisons */
title 'Pairwise comparisons';
proc glm data=mont4_9;
class Oil Truck;
model Consumption = Truck Oil;
lsmeans Oil / alpha=0.05 adjust=tukey;
output out=diag r=res p=pred;
Run;
```

4.9. The effect of three different lubricating oils on fuel economy in diesel truck engines is being studied. Fuel economy is measured using brake-specific fuel consumption after the engine has been running for 15 minutes. Five different truck engines are available for the study, and the experimenters conduct the following randomized complete block design.

	Truck				
Oil	**1**	**2**	**3**	**4**	**5**
1	0.500	0.634	0.487	0.329	0.512
2	0.535	0.675	0.520	0.435	0.540
3	0.513	0.595	0.488	0.400	0.510

(a) Analyze the data from this experiment.

(b) Use the Fisher LSD method to make comparisons among the three lubricating oils to determine specifically which oils differ in brake-specific fuel consumption.

(c) Analyze the residuals from this experiment.