Two-Level Fractional Factorial Design

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Introduction

It may not be possible to perform a full factorial design. Often, only lower order effects are important according to hierarchical ordering principle and effect sparsity principle. Confounding (aliasing) will happen because we are using a subset. 2^{k-1} is the one-half fractional factorial design, 2^{k-2} is the one-quarter fractional factorial, etc.

Process

Select $k-1$ factors to form a 2^{k-1} full factorial. The remaining factor is confounded(aliased) with a high order interaction of the selected $k-1$ factors. The chosen level combinations should form a half of the 2^k design, and the product of the columns of the factors equals 1 or I. This is called the defining relation or the highest order interaction is called a defining word (contrast).

There are other ways to alias that using a lower-order interaction as the defining relation. Not preferred since using the higher order for the defining relation has a higher resolution. Resolution is the number of factors present in the higher order interaction and written in Roman numerals.

Regression Model

Models are just as good as full models.

General 2^{k-p} Design

ith effect estimate: Effect$_i = \frac{Contrast_i}{N/2}$

General 2^{k-1} Design

Has k factors with the kth factor aliazed with ABC. . . J. The defining relation is I = ABC. . . JK with resolution k. The 2^k factorial effects are partitioned into 2^{k-1} groups each with two aliased effects. Only one effect from each group should be included in ANOVA or Regression model.

General 2^{k-2} Design

■ **TABLE 8.8**

Alias Structure for the 2_{IV}^{6-2} Design with $I = ABCE = BCDF = ADEF$

$A = BCE = DEF = ABCDF$	$AB = CE = ACDF = BDEF$
$B = ACE = CDF = ABDEF$	$AC = BE = ABDF = CDEF$
$C = ABE = BDF = ACDEF$	$AD = EF = BCDE = ABCF$
$D = BCF = AEF = ABCDE$	$AE = BC = DF = ABCDEF$
$E = ABC = ADF = BCDEF$	$AF = DE = BCEF = ABCD$
$F = BCD = ADE = ABCEF$	$BD = CF = ACDE = ABEF$
	$BF = CD = ACEF = ABDE$
$ABD = CDE = ACF = BEF$	
$ACD = BDE = ABF = CEF$	

Figure 1: Quarter Table