CSE 242: Assignment 3

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Question 1

1.1 A linearly separable training data is what makes the perceptron algorithm converge. So as long as the data can be separated by a hyperplane, then the algorithm converges.

1.2

	X_1	X_2	X_3	y
$\boxed{1}$	1	0	1	+1
$\boxed{2}$	0	-1	1	-1
$\boxed{3}$	1	1	1	+1
$\boxed{4}$	-1	2	0	-1

Initially $w = (0, 0, 0)$.

$w_1+\Delta w_1,\ w_2+\Delta w_2,\ w_3+\Delta w_3$

After the first example. $w = (1, 0, 1)$;

After the second example, $w = (1, 1, 0)$;

After the third example, $w = (1, 1, 0)$;

After the fourth example, $w = (2, -1, 0)$.

Explaination:

1) $a = w_1 x_1 + w_2 x_2 + w_3 x_3$

$= 0 \cdot 1 + 0 \cdot 0 + 0 \cdot 1$

$ya = +1 \cdot 0 = \boxed{0}$ update needed

$\Delta w_1 = y \cdot x_1$

$= +1 \cdot 1$

$\boxed{\Delta w_1 = 1}$

$\Delta w_2 = y \cdot x_2$

$= +1 \cdot 0$

$\boxed{\Delta w_2 = 0}$

$\Delta w_3 = y \cdot x_3$

$= +1 \cdot 1$

$\boxed{\Delta w_3 = 1}$

2) $a = w_1 x_1 + w_2 x_2 + w_3 x_3$

$= 1 \cdot 0 + 0 \cdot -1 + 1 \cdot 1$

$ya =$ [illegible] update needed

$ya = -1 \cdot +1 = \boxed{-1}$ update needed

$\Delta w_1 = y \cdot x_1$

$= -1 \cdot 0$

$\boxed{\Delta w_1 = 0}$

$\Delta w_2 = y \cdot x_2$

$= -1 \cdot -1$

$\boxed{\Delta w_2 = 1}$

$\Delta w_3 = y \cdot x_3$

$= -1 \cdot 1$

$\boxed{\Delta w_3 = -1}$

$\boxed{3}$ $a = w_1 x_1 + w_2 x_2 + w_3 x_3$

$= 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 1$

$ya = 1 \cdot 2 = \boxed{2}$ No update needed

$\boxed{4}$ $a = w_1 x_1 + w_2 x_2 + w_3 x_3$

$= 1 \cdot (-1) + 1 \cdot 2 + 0 \cdot 0$

$ya = -1 \cdot +1 = \boxed{-1}$ update needed

$\Delta w_1 = y \cdot x_1$

$= -1 \cdot -1$

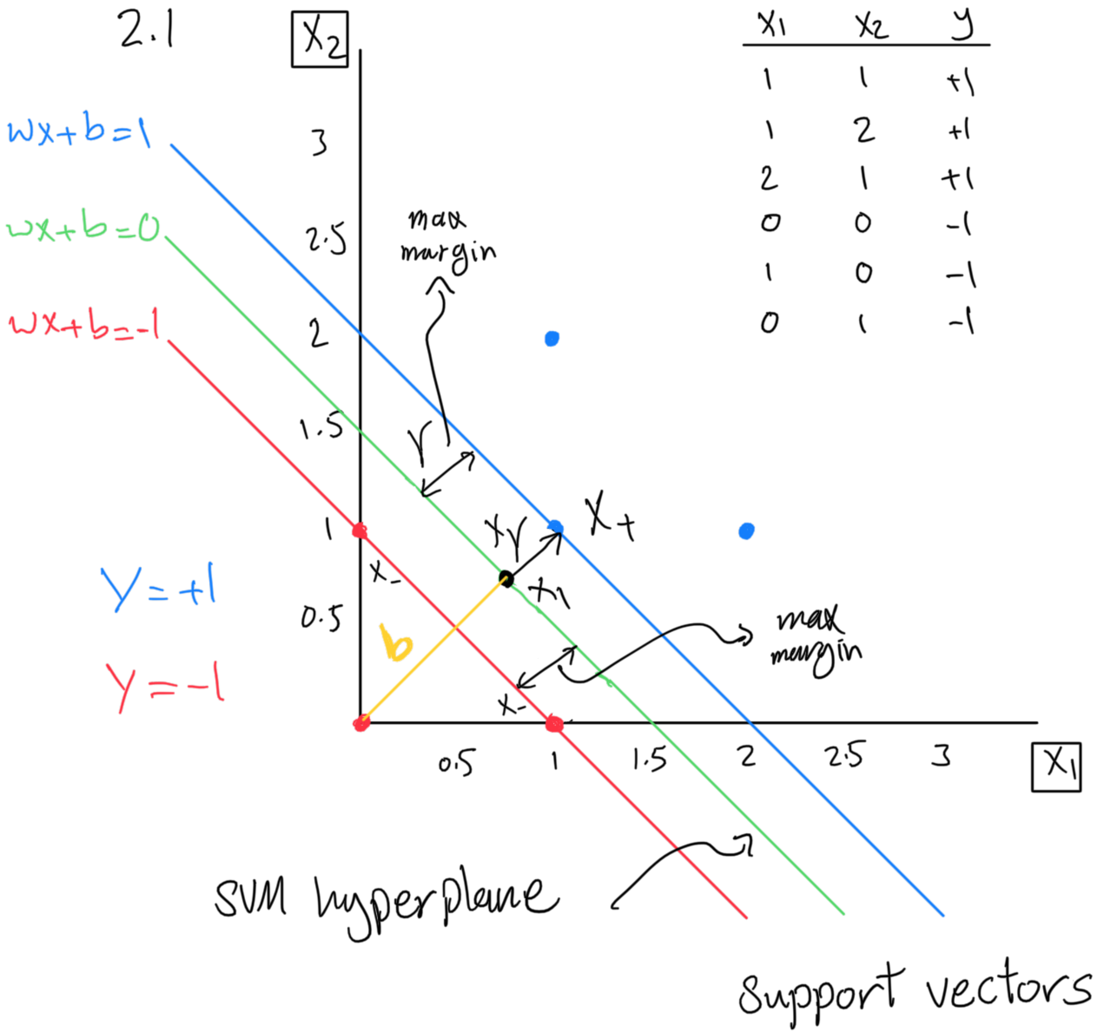
$\boxed{\Delta w_1 = 1}$

$$\Delta w_2 = y \cdot x_2$$
$$= -1 \cdot 2$$
$$\boxed{\Delta w_2 = -2}$$
$$\Delta w_3 = y \cdot x_3$$
$$= -1 \cdot 0$$
$$\boxed{\Delta w_3 = 0}$$

Question 2

2.1

x_1	x_2	y
1	1	+1
1	2	+1
2	1	+1
0	0	-1
1	0	-1
0	1	-1



2.2

The support vectors x_+ and x_- lie on the lines $w \cdot x + b = 1$ and $w \cdot x + b = -1$ respectively

- Using the corresponding values to the equation $w \cdot x + b = 1$ for positive labels and corresponding values to the equation $w \cdot x + b = -1$ for negative labels, I chose the values that lie on the support vectors.

- $w_1 x_1 + w_2 x_2 + b = 1$

  point $(1,1)$ for $w \cdot x + b = 1$

  point $(1,0)$ and $(0,1)$ for $w \cdot x + b = -1$

① $w_1(1) + w_2(1) + b = 1$

② $w_1(1) + w_2(0) + b = -1$

③ $w_1(0) + w_2(1) + b = -1$

① $w_1 + w_2 + b = 1$

② $w_1 + b = -1 \Rightarrow \boxed{w_1 = -b-1}$

③ $w_2 + b = -1 \Rightarrow \boxed{w_2 = -b-1}$

placing w_1 and w_2 values in first equation

$$-b-1-b-1+b=1$$

$$-b-2=1 \Rightarrow \boxed{b=-3}$$

Since values of w_1 and w_2 equal to $-b-1$ then

$$w_1 \text{ and } w_2 = -(-3)-1$$

$$\boxed{w_1=2} \quad \boxed{w_2=2}$$

Finding the norm of w using the norm equation

$$||w|| = \sqrt{w_1^2+w_2^2}$$

$$||w|| = \sqrt{(2)^2+(2)^2} = \sqrt{4+4}$$

$$\boxed{||w|| = \sqrt{8}}$$

And since the geometric margin is

$$\frac{1}{\|w\|}$$

$$\boxed{\text{My geometric margin is } \frac{1}{\sqrt{8}}}$$