EC330 Applied Algorithms and Data Structures for Engineers
Fall 2018

Homework 1

Out: September 6, 2018
Due: September 13, 2018

This homework has a written part and a programming part. Both are due at 8:59 am on September 13. You should hand in the written part right before the lecture starts. Make sure you write your name, your BU ID, and your answers clearly on every page. For the programming part, submit your code on Blackboard (detailed instructions in Problem 5).

1. Sums [15 pt]

Provide a closed-form solution to the following problems. Make sure you show the steps.

a) $\sum_{i=1}^{100}(\frac{1}{3})^i$

b) $\sum_{i=0}^{\infty}(\frac{2}{7})^i$

c) $\sum_{i=1}^{N}(i^3 + 5i^2 - 7i + 10)$

2. Exponents and Logs [15 pt]

a) $x^1 \cdot x^2 \cdot x^3 \cdots x^{50}$

b) $log_x x^{2x}$

c) $log_{77}(77^{77} \cdot 77)$

3. Combinatorics [10 pt]

a) How many 10-digit *hexadecimal* numbers do not contain A, B or C?

b) How many integral solutions of $x_1 + x_2 + x_3 = 30$ satisfy $x_1 \geq 2$, $x_2 \geq 0$ and $x_3 \geq -3$?

4. Induction [20 pt]

a) Proposition: All students love homework equally.
Proof:

- Base Case. For $n = 1$, a single student clearly loves homework exactly as much as him- or herself, so the base case holds.
- Inductive Hypothesis. Assume any k students love homework equally.
- Inductive Step. Consider $k + 1$ students.
  i. By the inductive hypothesis, the first k students all love homework equally.
  ii. By the inductive hypothesis, the last k students all love homework equally.
  iii. Therefore, all $k + 1$ students love homework equally, and the proposition holds by the principle of induction.

Clearly, not all students love homework equally. Which of the following is true? Briefly explain your choice. **[5 pt]**

1) The proof of the base case is incorrect.
2) The inductive hypothesis does not hold.
3) The application of the inductive hypothesis in step i is incorrect.
4) The application of the inductive hypothesis in step ii is incorrect.
5) The inductive step logic in step iii is flawed for at least one value of *k*.

b) Proposition: Consider the function f defined as follows.

$$f(x) = x \qquad x = 1, 2, 3$$
$$f(x) = f(x-1) + f(x-2) + f(x-3) \qquad x \in \mathbb{N} \text{ and } x > 3$$

Show that $\forall x \in \mathbb{N}, f(x) < 2^x$. **[15 pt]**

Hint: You can follow the proof structure in part a).

5. Programming [40 pt]

Make sure to write your name and BU ID in a comment at the top of the program, along with your collaborator's name and BU ID, if any.

a) You are given an array of integers. Suppose every integer appears twice in this array except for one. Write a C++ program that finds the integer that occurs only once. **[20 pt]**

You are given a header file and a sample test program. Your job is to implement the function *findSingle*. Your program must compile and run on the lab computers using the command-line interface as follows.

> g++ -std=c++11 (-o myProgram) testProblem5a.cpp Problem5a.cpp

Submit your solution in a single file *Problem5a.cpp.* The most efficient solution(s) will receive an extra credit of 0.1 point for the final class grade (out of 100).

b) Write a C++ program that keeps summing the digits of a positive integer until the sum becomes a single digit. **[20 pt]**

Example:

Input: 12345
Output: 6
Explanation: 1+2+3+4+5 = 15 → 1+5 = 6

Another example:

Input: 982018
Output: 1
Explanation: 9+8+2+0+1+8 = 28 → 2+8 = 10 → 1+0 = 1

Again, you are given a header file and a sample test program. Your job is to implement the function *sumDigit*. Similar to a), your program must compile and run on the lab computers using the command-line interface as follows.

> g++ -std=c++11 (-o myProgram) testProblem5b.cpp Problem5b.cpp

Submit your solution in a single file *Problem5b.cpp.* The most efficient solution(s) will receive an extra credit of 0.1 point for the final class grade (out of 100).