

Assignment 7

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1 Textbook exercises (4 points)

def 7.2 $f(n) = O(g(n))$ if positive integers c and n_0 exist such that for every integer $n \geq n_0$: $f(n) \leq cg(n)$.

a. $2n = O(n)$ - TRUE: $c = 3, n_0 = 1$

b. $n^2 = O(n)$ - FALSE: self-evident for any chosen c, n_0

e. $3^n = 2^{O(n)}$ - TRUE: $c = 2, n_0 = 1$

f. $2^{2^n} = O(2^{2^n}) \rightarrow 2^{2^n} = O(2^{2^n})$ - TRUE: $c = 2, n_0 = 1$

2 3SAT is NP (10 points)

$3SAT = \{ \langle \phi \rangle \mid \phi \text{ is a satisfiable 3cnf-formula} \}$. Give two proofs that $3SAT \in NP$. In your first proof, use a poly-time verifier. In your second proof, use a poly-time non-deterministic Turing machine. This is not difficult; but be careful and complete.

Proof 1: poly-time verifier on DTM

1. To prove that 3SAT formula is satisfiable it is sufficient to provide the assignment of values to the variables as the certificate. The quantity of variables can not exceed the quantity of literals in the formula. Thus the assignment length (certificate length) is less than length of the formula itself.
2. To check the certificate we can run a verifier V that runs in poly-time: $V = \text{"on input } \langle w, c \rangle$
 0. Parse the clauses of the formula in w .
 1. Calculate the boolean value of each clause using the variables values from c .
 2. Calculate the resulting boolean value.
 3. If result is TRUE, ACCEPT, otherwise, REJECT."
3. This DTM clearly runs in poly-time. By one scan of the tape we calculate the result of each clause and store it on the auxiliary tape, then calculate formula's result by conjunction the values on the second tape by one more scan.

Proof 2: poly-time decider on NDTM

1. Let us construct a poly-time decider D on NDTM.
2. $D = \text{"on input } \langle w \rangle$, where w is a satisfiable 3cnf-formula,
 1. Non-deterministically select (*all possible*^{1*}) certificate c - combination of assignments of variables.
 2. Run verifier V on input $\langle w, c \rangle$.
 3. If V accepts, ACCEPT
 4. Otherwise, REJECT."

^{1*} Number of combinations of variable assignment is restricted because number of variables in formula is finite, this number less than the number of literals in formula.
3. Clearly this NDTM runs in poly-time. First step is simply a selection. Second runs in poly-time as discussed above.