Assignment 1

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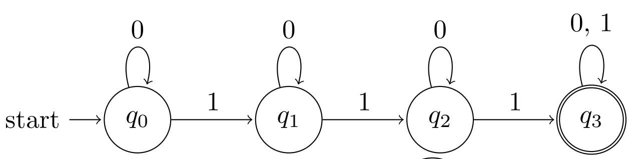
Due September 9, 2014

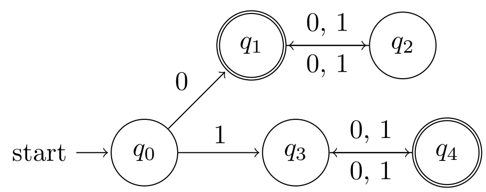
Sipser exercises

1.6

b.



e.



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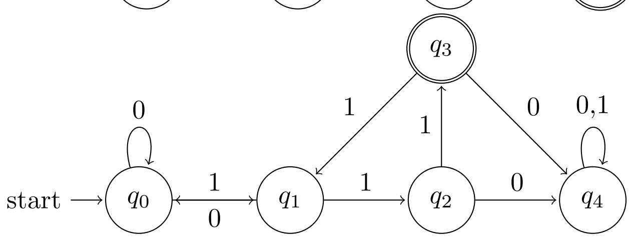
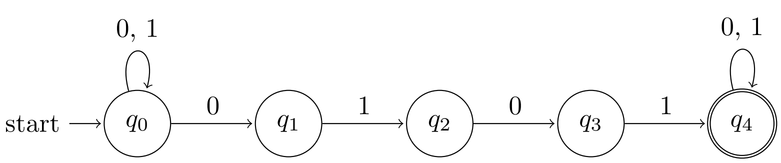


1.7

b.



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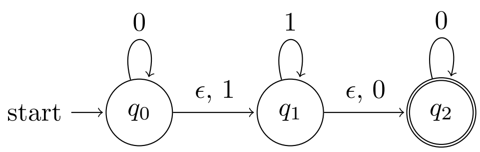
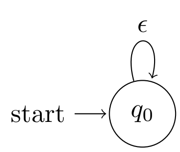
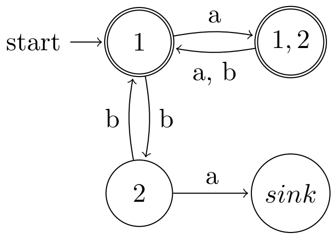
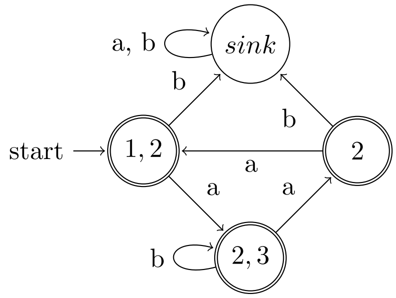


g.



1.16

a. 

b. 

Proofs

Theorem 0.12 Show that every graph with two or more nodes contains two nodes that have equal degrees.

Proof outline
0. Prove Handshaking lemma
1. Show that sum of degrees in the graph is equal to double of number of edges according to the Handshaking lemma
2. Find the minimum sum of degrees that all different
3. Find maximum number of edges possible in simple graph
4. Show contradiction

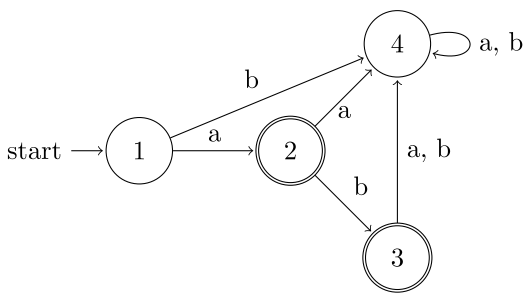
Handshaking Lemma: proof by construction
1. In simple not directed graph G each edge is adjacent to exactly two vertices.
2. The degree of each vertex is the number of edges to which it is incident.
3. Thus when summing up the degrees of all vertices in a G, we are counting all the edges of G twice.

Theorem: proof by contradiction
1. According to the handshaking lemma, the $\Sigma deg(v) = 2|E|$
2. Assume that all the nodes have different degrees, then minimum sum of all degrees is equal to $\frac{n(n-1)}{2}$
3. Thus, $\Sigma deg(v) = 2|E| = \frac{n(n-1)}{2}$ hence $Min|E| = n(n-1)$
4. Any two vertices in a simple graph could be connected with only one edge, hence the maximum number of edges in a complete simple graph is number of all pairs of vertices divided by two, namely: $Max(|E|) = \frac{n(n-1))}{2}$
5. Result in (4) contradicts the formula we got in (3). Thus, if we assume that graph have all verticies with different degrees it should have at least twice as much edges that it possibly could.

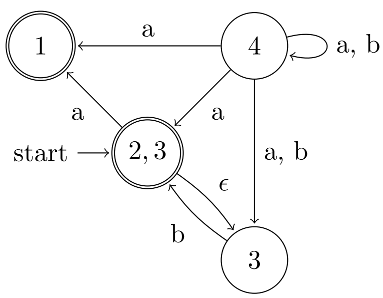
Theorem 1.31 For any string $w = w_1 w_2 ... w_n$, the **reverse** of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_{n..} w_1 w_2$. For any language A, let $A^{\mathcal{R}} = \{w^{\mathcal{R}} | w \in A\}$. Show that if A is regular, so is $A^{\mathcal{R}}$.

Proof outline
1. Show what it means for language to be regular
2. Prove by construction: define NFA with the same set of states, but with the reverse delta function.
3. Show the equality of power NFA and DFA
4. Illustrate the solution.

Proof by construction
1. The language is regular if there exists a finite automaton that recognizes it.
2. Thus for any regular language A we have a 5-tuple automaton $M_A = (Q_A, \Sigma_A, \delta_A, q0_A, F_A)$
3. Let us construct the automaton M_R that will accept the $A^{\mathcal{R}}$ as following:
$Q_R = Q_A$
$\Sigma_R = \Sigma_A$
δ_R : For each $q \in Q$ and $\alpha \in \Sigma$: $\delta_R = \delta_A^{-1}(q_1, \alpha)$
$q0_R = \{f_1 \cup f_1 \cup ... f_n | f_i \in F_A\}$
$F_R = q0_A$.
4. We already know that for any NFA there exists DFA that recognize the same regular language.
5. Illustration



M_A **with** $\Sigma = \{a, ab\}$



M_R **with** $\Sigma = \{a, ba\}$