Assignment 0

Aleksandr Salo

Due September 2, 2014

Summarization of chapters 0 and 1

Theorem 0.20 $\forall A, B : \overline{(A \cup B)} = \overline{A} \cap \overline{B}$

Theorem 0.21 For every graph G, the sum of the degrees of all the nodes in G is an even number

Theorem 0.22 For each even number n greater than 2, there exists a 3-regular graph with n nodes

Theorem 0.24 $\sqrt{2}$ is irrational

Theorem 0.25 Mortgage monthly payments for each $t \geq 0$,

$$P_t = PM^t - Y(\frac{M^t - 1}{M - 1})$$

Definition 1.5 A finite automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where
1. Q is a finite set called the **states,**
2. Σ is a finite set called the **alphabet,**
3. $\delta : Q \times \Sigma \to Q$ is the **transition function,**
4. $q_0 \in Q$ is the **start state,** and
5. $F \subseteq Q$ is the set of **accept states.**

Definition 1.16 A language is called **regular language** if some finite automaton recognizes it.

Definition 1.23 Let A and B be languages. We define the regular operations **union, concatenation,** and **star** as follows:

- **Union:** $A \cup B = \{x | x \in A \text{ or } x \in B\}$
- **Concatenation:** $A \circ B = \{xy | x \in A \text{ and } x \in B\}$
- **Star:** A*$= \{x_1 x_2 ... x_k | k \geq 0 \text{ and each } x_i \in A\}$

Theorem 1.25 The class of regular languages is closed under the union operation. In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Theorem 1.26 The class of regular languages is closed under the concatenation operation. In other words, if A_1 and A_2 are regular languages, so is $A_1 \circ A_2$.

Definition 1.37 A **nondeterministic finite automaton** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where
1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta : Q \times \Sigma_\epsilon \to \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

Theorem 1.39 Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Corollary 1.40 A language is regular if and only if some nondeterministic finite automaton recognizes it.

Theorem 1.45 The class of regular languages is closed under the union operation.

Theorem 1.47 The class of regular languages is closed under the concatenation operation.

Theorem 1.49 The class of regular languages is closed under the star operation.

Definition 1.52 Say that R is a **regular expression** if R is
1. α for some α in the alphabet Σ,
2. ϵ,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions,
6. $(R_1{*})$, where R_1 is regular expressions.

In items 1 and 2, the regular expressions α and ϵ represent the languages $\{\alpha\}$ and $\{\epsilon\}$, respectively. In item 3, the regular expression $\emptyset$ represents the empty language. In items 4, 5 and 6, the expressions represent the languages obtained by taking the union or concatenation of the languages R_1 and R_2, or the star of the language R_1, respectively.

Theorem 1.54 A language is regular if and only if some regular expression describes it.

Lemma 1.55 If a language is described by a regular expression, the it is regular.

Lemma 1.60 If a language is regular, then it is described by a regular expression.

Definition 1.64 A **generalized nondeterministic finite automaton** is a 5-tuple $(Q, \Sigma, \delta, q_{start}, q_{accept})$, where
1. Q is a finite set of states,
2. Σ is a input alphabet,
3. $\delta : (Q - \{q_{accept}\}) \times (Q - \{q_{start}\}) \to \mathcal{R}$ is the transition function,
4. q_{start} is the start state, and
5. q_{accept} is the accept states.

Claim 1.65 For any GNFA G, $CONVERT(G)$ is equivalent to G.

Theorem 1.70 Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xy^iz$, satisfying the following conditions:
1. for each $i \geq 0, xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Proof of the Theorem 0.22

Let n be an even number greater than 2. Construct graph G = (V, E) with n nodes as follows. The set of nodes of G is V = {0, 1, ..., n - 1}, and the set of edges of G is set

$$E = \{\{i, i+1\} | \text{ for } 0 \leq i \leq n-2\} \cup \{\{n-1, 0\}\}$$
$$\cup \{\{i, i+n/2\} | \text{ for } 0 \leq i \leq n/2-1\}$$

Picture the nodes of this graph written consequently around the circumference of a circle. In that case, the edges described in the top line of E go between adjacent pairs around the circle. The edges described in the bottom line of E go between nodes on opposite sides of the circle. This mental picture clearly shows that every node in G has degree 3.

Exercises

0.1
a. set of odd natural numbers
b. set of even integers
c. set of some even natural numbers
d. set of some natural numbers
e. set of strings with binary palindromes
f. set of integers

0.2
a. {1, 10, 100}
b. $\{n > 5 | n \in \mathbb{Z}\}$
c. {1, 2, 3, 4}
d. {aba}
e. $\{\epsilon\}$
f. $\emptyset$

0.5
Power set has all the combinations of set's elements including empty set and set itself.
Let us experiment:

P({1, 2}) = {{1, 2}, {1}, {2}, {}} - 4 elements
P({1, 2, 3}) = {{1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, {}} - 8 elements
P({1, 2, 3, 4}) = {{1, 2, 3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1}, {2}, {3}, {4}, {}} - 16 elements

Here we can clearly see the tendency that number of elements in power set is equal to the 2^n, n - set's cardinality.

0.6
a. $f(2) = 7$
b. domain of f is: {1, 2, 3, 4, 5}, range is: {6, 7}
c. $g(2, 10) = 6$
d. domain of g is: $\{x, y | x \in \{1, 2, 3, 4, 5\}, y \in \{6, 7, 8, 9, 10\}\}$, range is: {6, 7, 8, 9, 10}
e. $g(4, f(4)) = 8$