# CSI 5325 - Assignment 0

James Boer

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## 1 Simple exercises for understanding.

1. Find the value $x$ that maximizes $f(x) = -3x^2 + 24x - 30$.

[redacted]

$$x = 4$$

[redacted]

2. Find the partial derivatives of $g(x)$ with respect to $x_0$ and $x_1$: $g(x) = 3x_0^3 - 2x_0x_1^2 + 4x_1 - 8$.

   (a) With respect to $x_0$, treat $x_1$ as a constant:

   $$\frac{\partial g}{\partial x_0} = 9x_0^2 - 2x_1^2$$

   (b) With respect to $x_1$, treat $x_0$ as a constant:

   $$\frac{\partial g}{\partial x_1} = 4 - 4x_0x_1$$

3. What is the value of $AB^T + C^{-1}$ when $A = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

   $$AB^T + C^{-1} = \begin{bmatrix} 27 & 62 \\ 44 & 107.5 \end{bmatrix} \quad \text{(Matlab.)}$$

4. Write down the mathematical definitions of the simple Gaussian, multivariate Gaussian, Bernoulli, binomial and exponential distributions.

   (a) Simple Gaussian, $\mu$ is the expected value, and $\sigma^2$ is the variance.

   $$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(b) Multivariate Gaussian, $\mathbf{x}$ is a k-dimensional vector of random variables, $\boldsymbol{\mu}$ is a vector of their expected values respectfully, and $\boldsymbol{\Sigma}$ is the $k \times k$ covariance matrix.

$$f_{\mathbf{x}}(x_1, \ldots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma}|}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$

(c) Bernoulli, $P(X = 1) = 1 - Pr(X = 0) = 1 - q = p$, $k \in \{0, 1\}$.

$$f(k; p) = p^k (1-p)^{1-k}$$

(d) Binomial, the probability of getting exactly $k$ successes in $n$ trials with each trial having probability $p$.

$$f(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

(e) Exponential, $\lambda$ is the rate parameter.

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

5. What is the relationship between the Bernoulli and binomial distributions? The Bernoulli distribution is a special case of the binomial distribution with $n = 1$.

6. Suppose that random variable $X \sim N(1, 3)$. What is its expected value?

$$\mu = 1$$

7. Suppose that random variable $Y$ has distribution

$$p(Y = y) = \begin{cases} e^{-y} & y \geq 0 \\ 0 & y < 0 \end{cases}$$

(a) Verify that $\int_{y=-\infty}^{\infty} p(Y = y) = 1$

$$\begin{aligned} \int_{y=-\infty}^{\infty} p(Y = y) &= \int_{-\infty}^{0} 0\, dy + \int_{0}^{\infty} e^{-y}\, dy \\ &= -e^{-y}\Big|_0^{\infty} \\ &= 0 - (-1) \\ &= 1 \end{aligned}$$

(b) What is $\mu_Y = E[Y] = \int_{y=-\infty}^{\infty} p(Y = y) y\, dy$? (The expected value of $Y$.)

[redacted]

$$\mu_Y = 1$$

(c) What is $\sigma^2 = Var[Y] = \int_{y=-\infty}^{\infty} p(Y = y)(y - \mu_Y)^2\, dy$? (The variance of $Y$.)

[redacted]

$$\sigma^2 = 0$$

8. What is $E[Y|Y \geq 10]$? (The expected value of $Y$, given that $Y \geq 10$.)

[redacted]

$$E[Y|Y \geq 10] = 0.0004993992$$