

# CSI 5325 - Assignment 0

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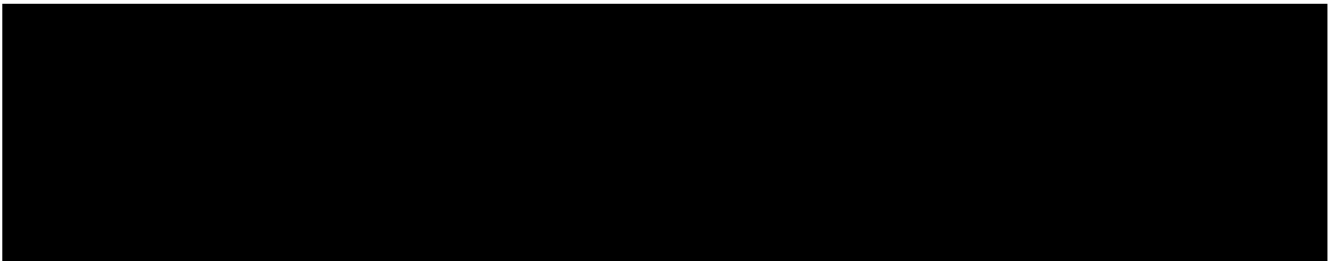
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## 1 Simple exercises for understanding.

1. Find the value  $x$  that maximizes  $f(x) = -3x^2 + 24x - 30$ .



$$x = 4$$



2. Find the partial derivatives of  $g(x)$  with respect to  $x_0$  and  $x_1$ :  $g(x) = 3x_0^3 - 2x_0x_1^2 + 4x_1 - 8$ .

- (a) With respect to  $x_0$ , treat  $x_1$  as a constant:

$$\frac{\partial g}{\partial x_0} = 9x_0^2 - 2x_1^2$$

- (b) With respect to  $x_1$ , treat  $x_0$  as a constant:

$$\frac{\partial g}{\partial x_1} = 4 - 4x_0x_1$$

3. What is the value of  $AB^T + C^{-1}$  when  $A = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ ,  $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ .

$$AB^T + C^{-1} = \begin{bmatrix} 27 & 62 \\ 44 & 107.5 \end{bmatrix} \quad (\text{Matlab.})$$

4. Write down the mathematical definitions of the simple Gaussian, multivariate Gaussian, Bernoulli, binomial and exponential distributions.

- (a) Simple Gaussian,  $\mu$  is the expected value, and  $\sigma^2$  is the variance.

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- (b) Multivariate Gaussian,  $\mathbf{x}$  is a  $k$ -dimensional vector of random variables,  $\boldsymbol{\mu}$  is a vector of their expected values respectively, and  $\boldsymbol{\Sigma}$  is the  $k \times k$  covariance matrix.

$$f_{\mathbf{x}}(x_1, \dots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma}|}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$

- (c) Bernoulli,  $P(X = 1) = 1 - Pr(X = 0) = 1 - q = p$ ,  $k \in \{0, 1\}$ .

$$f(k; p) = p^k (1 - p)^{1-k}$$

- (d) Binomial, the probability of getting exactly  $k$  successes in  $n$  trials with each trial having probability  $p$ .

$$f(k; n, p) = \binom{n}{k} p^k (1 - p)^{n-k}$$

- (e) Exponential,  $\lambda$  is the rate parameter.

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

5. What is the relationship between the Bernoulli and binomial distributions? The Bernoulli distribution is a special case of the binomial distribution with  $n = 1$ .

6. Suppose that random variable  $X \sim N(1, 3)$ . What is its expected value?

$$\mu = 1$$

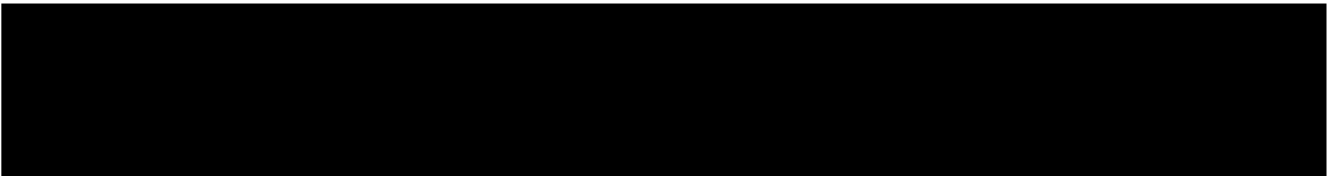
7. Suppose that random variable  $Y$  has distribution

$$p(Y = y) = \begin{cases} e^{-y} & y \geq 0 \\ 0 & y < 0 \end{cases}$$

- (a) Verify that  $\int_{y=-\infty}^{\infty} p(Y = y) = 1$

$$\begin{aligned} \int_{y=-\infty}^{\infty} p(Y = y) &= \int_{-\infty}^0 0 \, dy + \int_0^{\infty} e^{-y} \, dy \\ &= -e^{-y} \Big|_0^{\infty} \\ &= 0 - (-1) \\ &= 1 \end{aligned}$$

- (b) What is  $\mu_Y = E[Y] = \int_{y=-\infty}^{\infty} p(Y = y)y \, dy$ ? (The expected value of  $Y$ .)



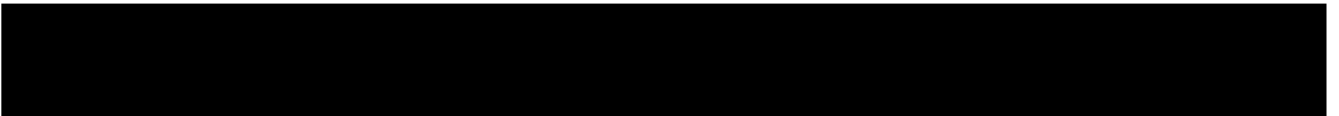
$$\mu_Y = 1$$

- (c) What is  $\sigma^2 = Var[Y] = \int_{y=-\infty}^{\infty} p(Y = y)(y - \mu_Y)^2 \, dy$ ? (The variance of  $Y$ .)



$$\sigma^2 = 0$$

8. What is  $E[Y|Y \geq 10]$ ? (The expected value of  $Y$ , given that  $Y \geq 10$ .)



$$E[Y|Y \geq 10] = 0.0004993992$$